B. A. Volkov and O. A. Pankratov. *Inverted contact in semiconductors—a new inhomogeneous structure with a two-dimensional gas of zero-mass electrons.* By varying the composition of semiconductor compounds (e.g., $Cd_{1-x}Hg_xTe$ or $Pb_{1-x}Sn_xTe$), it is possible to select pairs of materials in which the energy levels corresponding to band edges are mutually inverted, and the wave function symmetry of the conduction band in one material is the same as the wave function symmetry of the valence band, and vice versa. The contact of such materials is a new type of an inhomogeneous semiconductor structure.[1] The specifics of such inverted contact are determined by the presence in it of electron states that do not depend on the form of the transition region and that are similar to soliton states in one-dimensional systems. In the contact area these states are characterized by a linear zero-mass spectrum nondegenerate with respect to spin.

For the $Pb_{1-x}Sn_xTe$-type semiconductors the energy spectrum of an inverted contact is described in the two-band approximation by the Dirac Hamiltonian:

$$\hat{H} = \begin{pmatrix} \varepsilon_g/2 & \hat{\sigma}\hat{\mathbf{p}} \\ \hat{\sigma}\hat{\mathbf{p}} & -\varepsilon_g/2 \end{pmatrix}, \qquad (1)$$

where the role of the rest mass mc^2 is played by the half-width of the forbidden energy-gap $\varepsilon_g/2$, that depends on the coordinate z, $\hat{\sigma}$ is a vector formed by the Pauli matrices, and the anisotropy of the effective mass is taken into account in the operator $\mathbf{p} = -i\hbar(v_\perp\nabla_x, v_\perp\nabla_y, v_\parallel\nabla_z)$. The two components of the Dirac Ψ-function satisfy the equation

$$\left[\hat{p}_z^2 + \frac{1}{4}\varepsilon_g^2 + \frac{1}{2}\hbar v_\parallel\frac{\partial\varepsilon_g}{\partial z} + \hbar^2 v_\perp^2 k_\perp^2 - \varepsilon^2\right]\Psi_{1,2} = 0. \qquad (2)$$

The equation for other two components $\Psi_{3,4}$ is obtained from (2) by changing the sign of ε_g (the vector $\mathbf{k}_\perp$ lies in the

plane of the contact). If the signs of the ε_g are different on both sides of the contact, then there exists a solution localized at the contact for $\Psi_{1,2}$ (if $\varepsilon_g(-\infty)<0, \varepsilon_g(\infty)>0$) or for $\Psi_{3,4}$ (for the opposite case) that corresponds to the zero mode of the supersymmetric quantum mechanics

$$\Psi_{1,2} \sim \exp\left(-\frac{1}{2\hbar v_\parallel}\int_0^z \varepsilon_g(z)\,dz + i\mathbf{k}_\perp \mathbf{r}\right). \quad (3)$$

The energy of this state

$$\varepsilon_\perp(\mathbf{k}_\perp) = \pm\hbar v_\perp k_\perp \quad (4)$$

depends linearly on $k_\perp$, with the Kramers degeneracy being absent for $k_\perp \neq 0$. The spectrum of an inverted contact thus consists of two branches—a three-dimensional Dirac-type branch $\varepsilon_D(k_\perp)$ and a two-dimensional branch of the Weil-type fermions $\varepsilon_W(k_\perp)$ (Fig. 1).

When the width of the transition region exceeds the critical value $l_0 = 2\hbar v_\parallel \varepsilon_g(\infty) \sim 30$ Å (for $\varepsilon_g \sim 0.1$ eV), in addition to the state (3) there appear other, doubly degenerate states that also lie in the forbidden energy gap and that are shown in Fig. 1 by the dashed line.

In a magnetic field H directed perpendicular to the plane of the contact, the spectrum (4) is split into the Landau levels

$$\varepsilon(n) = \pm(2n\varepsilon_g(\infty)\hbar\omega_c)^{1/2}, \quad n = 0, 1, 2, \ldots, \quad (5)$$

where ω_c is the cyclotron frequency for the conduction electrons. For weak fields $\varepsilon_g(\infty)\hbar\omega_c \ll T^2$ the two-dimensional states (3) give a diamagnetic contribution to the magnetic susceptibility

$$\chi = -\frac{e^2 v_\perp}{48\pi\hbar v_\parallel c^2}\frac{\varepsilon_g(\infty)}{T\operatorname{ch}^2(\mu/2T)}. \quad (6)$$

The quantity χ here is normalized to the contact unit volume $(sl_0)^{-1}$, μ is the chemical potential. It can be seen from (6) that $|\chi|$ is a nonmonotonic function of temperature (Fig. 2), reaching the maximum

$$|\chi_{max}| = \frac{e^2 v_\perp^2}{96\pi\hbar v_\parallel c^2}\frac{\varepsilon_g(\infty)}{\mu}. \quad (7)$$

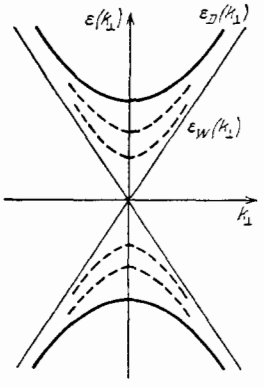

FIG. 1

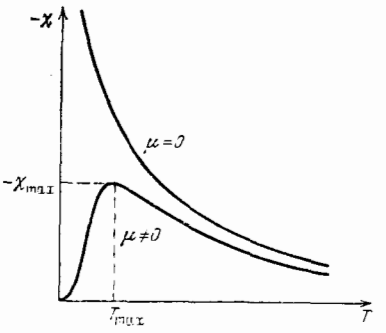

FIG. 2

at the temperature $T \approx |\mu|/2\sqrt{3/5}$. χ_{max} is of the same order of magnitude as Landau diamagnetism increased by the factor $\varepsilon_g(\infty)/\mu$.

In the area of strong fields the dependence of a magnetic moment on the field has a saw-tooth shape, in accordance with the results of Peierls.[2] The oscillation period in $1/H$ is equal to $e/2\pi\hbar c n_s$ (n_s is the concentration of two-dimensional zero-mass electrons). The oscillations of these electrons must start before the oscillations of the de Haas–van Alfen band electrons and can end beyond the ultraquantum limit for Dirac electrons.

In conclusion we must note that structures with a Weil-type branch of elementary excitations have already been produced. First of all, these are heterocontacts and CdTe–HgTe superlattices.[3] In these semiconductors the energy levels Γ_6 and Γ_8 are mutually inverted. Because of the asymmetry of such an inverted contact the Weil spectrum has termination points merging with the energy bands of light electrons in HgTe. Moreover, for $k_\perp \neq 0$ a hybridization of the Weil branch with the energy band of heavy holes takes place.

Theoretical studies of similar structures carried out abroad recently by the authors of Ref. 3 together with Bastard (see Ref. 4), and later by Lin-Lin and Sham[5] are based on a Luttinger Hamiltonian which takes into account only the band multiplet Γ_8. For the inverted CdTe–HgTe contact, however, where the terms Γ_6 and Γ_8 cross over, one should use not the Luttinger Hamiltonian but the Cane Hamiltonian. For this reason the approach of Bastard and Sham in principle cannot lead to the correct conical form of the two dimensional spectrum in a single contact.

In a recent work by Cade[6] a sharp CdTe–HgTe contact and a rectangular CdTe–HgTe–CdTe quantum well were studied numerically using a Cane model. Calculations[6] have confirmed the presence of a conical point of the Weil branch of the spectrum for $k_\perp = 0$ and its strong hybridization with the band of heavy holes for $k_\perp \neq 0$.

At the same time, numerical calculations cannot demonstrate the main property of a Weil branch, i.e., its universality caused by supersymmetry. As can be seen from formulas (2)–(4), the presence of the Weil branch of the spectrum is dictated only by the supersymmetry of the Hamiltonian (2), i.e., is determined only by the global characteristics of the function $\varepsilon_g(z)$, that plays the role of a superpotential (and namely, only by the signs of its asymptotes for

$z \to \pm \infty$).[7] For this reason the Weil states (at least for sufficiently small $k_\perp$) are stable with respect to random fluctuations of $\varepsilon_g(z)$ caused by technological inhomogeneities.

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