Toward the realization of electromagnetic collapse

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The problems along the road to achieving an extreme compression of plasma in high-current channels are analyzed. The present ideas regarding the physical nature of the equilibrium of a plasma in a pinch system are outlined. The problem of the macroscopic stability of a relativistic pinch is discussed. Solving this problem requires deriving a relativistic two-fluid electromagnetic hydrodynamics. The dynamics of the contraction of pinches, in whose terminal stage radiation plays a particularly important role, is analyzed. The system can radiate off more energy than it acquires by heat evolution, and if it does the result should be a collapse. The practical significance of achieving electromagnetic collapse goes beyond the fusion problem, and involves such fields as the collective acceleration of charges, the production of "cumulative" jets, and the development of intense sources of induced radiation over a broad spectral range, from microwaves to γ rays. For physicists, pinches represent a unique opportunity for producing and studying in the laboratory matter in an extreme state such as nature furnishes only deep in the interiors of stars.

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1. INTRODUCTION

The physics of high energy densities has an interesting history and presents broad opportunities for studying nature and for various practical applications. Progress toward the achievement of extreme energy densities is inseparably linked with the technology of producing strong fields and ultrahigh pressures.

The recent developments in the physics of high energy densities have their origin back in Kapitsa's now classic work[1] on the development of pulsed strong magnetic fields. Progress in the development of the explosive method for concentrating energy ("cumulation") stems from studies by Sakharov.[2] In the 1970s, progress was stimulated in the technology of high pressures, in the megabar range,[3] when Ashcroft[4] pointed out the possibility of achieving a metastable state of metallic hydrogen with a high superconducting transition temperature. The concentration of energy is crucial to directions which are presently being pursued to achieve controlled thermonuclear fusion. Curiously, the application of intense laser beams to plasmas produces ultrastrong magnetic fields.[5]

In nature, high densities of matter occur in the interiors of planets and stars as the result of gravitational compression of massive bodies. The Newtonian gravitational potential satisfies the same equation as is satisfied by the scalar potential of an electric field. Would it be possible to use electromagnetic forces instead of gravitational forces and thereby reduce the scale dimensions required for producing ultradense matter and to achieve extreme compression in the laboratory? The ratio of the gravitational force acting between two protons to the force of their electrostatic interaction is very small, $F_{gr}/F_{el} = -G(m_H/e_H)^2 = -0.8\times 10^{-36}[e_H = 4.8\cdot 10^{-10}\,\text{esu}, m_H = 1.66\cdot 10^{-24}\,\text{g}, G = 6.67\cdot 10^{-8}\ \text{cm}^3/(\text{g}\cdot\text{s})]$. Consequently, if the electromagnetic interaction were used to compress matter, the

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amount of matter which would be required in order to reach the pressures produced by gravitation at, say, the center of the sun would be only something of the order of 10^{-3} g (in comparison with mass of the sun, $m_{\odot} = 2\cdot 10^{33}$ g).

In a state of such extreme compression, the energy of the system per charge would be of the order of the rest mass of the electron. This energy is considerably greater than the binding energy of the outer electrons in atoms. During strong compression, the atomic bonds of electrons with nuclei are broken, and the matter is a fully ionized plasma, generally not neutral. If the plasma is charged, the net charges will clearly repel each other by virtue of their electrostatic forces, and it would not be possible to achieve compression of the plasma as a whole by means of electrostatic forces alone.

We would be dealing with a different situation if the charges of one sign were in motion with respect to the charges of the other sign in the plasma. If the average (drift) velocity of the electrons with respect to the subsystem of ions is nonzero, a current will flow through the plasma, and a magnetic field will arise. The interaction of parallel currents through the magnetic field which they produce is an attraction. The magnetic attraction of moving charges may prove stronger than the electrostatic repulsion even if the plasma contains a net charge. The ultimate result is that the collective interaction of the charges of the plasma will be a mutual attraction. These forces will cause the plasma channel of the current to contract (a pinch effect) to the point that the pressure becomes high enough to establish an equilibrium between forces.

Studies of self-contracting streams of charged particles, begun by Bennett[6] and revived by Budker,[7] have now been taken up extensively in connection with the problem of controlled fusion. The reason is that the problems along the road to achieving pronounced electromagnetic self-contraction are similar in many respects to the problems of controlled fusion. There is accordingly the opportunity to work from a limited amount of material to analyze certain aspects of the problem of developing fusion reactors.

Back in the very infancy of fusion research, Kurchatov gave a now famous lecture[8] reporting significant progress toward the compression and heating of a plasma in a high-current pinch. The earliest experiments revealed an emission of neutrons.[9] It appeared at the time that the problem would be simply one of satisfying the conditions for equilibrium of plasma column in the magnetic field produced by a current flowing through the plasma itself, at values of the linear charge density N and of the current I corresponding to the temperature required for a reaction. The magnetic field would prevent the plasma from expanding in the radial direction and would thereby allow the hot plasma to exist for the time necessary. In actuality, the discharge of a high current is always a rapidly evolving process, having little in common with a state of equilibrium.

The theory of the evolution of a pinch began in a study by Leontovich and Osovets, who studied the dynamics of the contraction of a discharge channel, allowing for the inertia of ions. At a high current, the force of the magnetic compression is initially considerably higher than the plasma pressure. The magnetic force compresses the discharge, and the pressure rises. At a certain time, the plasma pressure becomes equal to the pressure exerted by the magnetic field of the current. By this time, however, the charges have acquired a significant kinetic energy in the radial direction, and the contraction of the current channel continues inertially in the direction opposite the resultant force until the radial velocity vanishes. At this turning point, the plasma pressure exceeds the field pressure, and the system begins to expand. If the current is maintained for a considerable length of time in the circuit, the system will exhibit oscillations, and it is not obvious at the outset that the system will ultimately arrive at a state of equilibrium.

The problem of confining hot plasmas stimulated research on equilibrium plasma configurations[10] and their stability.[11] The transition from Z-pinches to systems with a more complicated geometry[12] forced the development of general methods for studying the stability of a plasma in a magnetic field. An energy principle for studying the magnetohydrodynamic (MHD) stability of plasmas was developed.[13] It was applied successfully to various complicated systems.[14–16]

Analysis of the balance between the radiated energy and the heat evolved in the plasma by the current became a crucial step forward toward an understanding of the physics of pinch systems. This analysis was carried out in 1957 by Pease[17] and Braginskiĭ.[18] It turned out that if the current exceeds a certain critical value (the Pease-Braginskiĭ current) the current channel will contract without bound. The important conclusion was reached that if the current is high enough the plasma will become a collapsing plasma.[19,20]

In the 1960s there was a transition from nonrelativistic to relativistic velocities in the streams of charged particles. There were rapid developments in the physics and technology of intense relativistic electron beams.[21,22] A relativistic electron beam containing positive ions is a self-contracting (pinch) system. At a relativistic drift velocity, however, this system cannot be regarded as a common, electrically neutral, conducting fluid characterized by a hydrodynamic flow velocity which is the same over the entire system. A description of the plasma of relativistic pinch systems thus requires going beyond classical MHD, discarding the assumption of quasineutrality, and carrying out a self-consistent calculation of the charge density and electric field on the basis of the Poisson equation. As a result, a new branch of research was founded and developed: the physics of charged plasmas.[23]

The 1968 discovery[24] in research on pinches of the formation in a low-inductance vacuum spark of so-called plasma points or micropinches, in which the state of matter approaches that in the interior of stars,[25] made a large impression. Micropinches exhibit all the characteristics of a highly compressed plasma: a burst of x-ray emission, high electron and ion temperatures, multiply charged ions, the production of directed streams of accelerated charges, neutron emission, explosive nature of electron emission, and emission of hard x rays at an intensity which falls off slowly with increasing x-ray energy (in a power-law, rather than exponential, fashion).

All these phenomena are observed in a wide variety of high-current devices: direct current discharges,[9,26] Filippov[27] and Mather[28] plasma foci, the electrical explosion of wires,[29,30] and low-inductance vacuum diodes.[31,32] The implication is that the events which occur in a plasma compressed greatly by a current are of a common nature—a nature which, unfortunately, cannot yet be regarded as finally resolved. High-current electric discharges have been under study since before the turn of the century,[33] and over the entire history of this research not one of the phenomena listed above was predicted beforehand. An electric discharge has proved to be "very inconvenient" to describe theoretically, while experimentally it is an extremely interesting phenomenon.

The properties of pinch systems which are of the greatest physical interest are manifested in the stage of maximum compression and heating of the plasma. In the stage of the emission of neutrons and accelerated charges the plasma density is already so high that the scale time for relaxation through Coulomb collisions in each of the subsystems (electrons and ions) is much shorter than the scale time for the existence of the compressed state, as estimated from the duration of the x-ray emission. Under such conditions a system should rapidly reach equilibrium, and its evolution will be a slow change in the equilibrium state.

Today we are seeing a reexamination of the established ideas regarding the nature of the pinch effect and regarding the role which pinches play in the fusion problem.[34] In the stage of the maximum compression and heating, the change in the state of a plasma focus frequently occurs in a completely regular way, and there are no MHD instabilities.[35,36] It appears that the properties of the plasma in the state of maximum compression are only slightly affected by the particular history of the development of the current channel by the complex processes by which the current shell is formed[37,38] and by the particular features of the motion of the shell toward the discharge axis.[39–43] Naturally a concentration of energy has to of course occur.

It is thus time to return to the original approach to the theory of pinches, i.e., to studying the physical nature of the plasma equilibrium in the channel of an intense current, but at a modern level, with allowance for the changes in the understanding of equilibrium configurations.[44] In this direction, for the very first time in research on high-current discharges, the theoreticians have the opportunity not only to construct a systematic and natural explanation of the entire set of events accompanying a pinch effect but also to predict the possibility of reaching as state of a hot, condensed plasma: a plasma compressed by collective-interaction forces to the point of a quantum degeneracy of electrons.[45] For the first time, theory has outstripped experiment: A degeneracy of electrons during the contraction of a current channel has not yet been directly observed. The actual realization of a contraction of a pinch to the state of condensed matter is a most interesting problem in experimental plasma physics.

The electromagnetic self-contraction of charged-particle beams opens up some new opportunities for reaching extreme states of superdense matter.[34,44] A well-conceived program to achieve electromagnetic collapse will make it possible to increase the maximum energy concentration attainable by several orders of magnitude. The practical importance of this field of research goes beyond the problem of controlled fusion reactions. Studies of electromagnetic collapse are pertinent to a broad spectrum of problems: the transport of energy over large distances by intense relativistic electron beams, collective acceleration of charges,[46–51] the development of intense new sources of electromagnetic radiation over a broad spectral range,[22] including free-electron lasers, x-ray lasers,[52] and even γ-ray lasers.[53] Advancing toward progressively higher energy densities opens up some new opportunities for physical research and technical applications which can hardly be completely foreseen today. In addition to the practical applications, the production of extremely high pressures, ultrastrong fields, and huge concentrations of energy in the laboratory is of much general scientific interest.

2. PHYSICAL NATURE OF THE EQUILIBRIUM OF A RELATIVISTIC Z-PINCH

2.1. Subsystems of ions and electrons

A self-compression of a plasma by electromagnetic forces can occur only if a current is flowing. The direction of the current becomes a special direction, and in the simplest geometry a pinch may be thought of as a cylindrically symmetric system which is uniform in the direction along the current and in the azimuthal direction. When a current is flowing, the drift velocity v_0 (the velocity of the relative motion of the electron and ion subsystems) is nonzero. Depending on the magnitude of the drift velocity, either a directed beam of particles (in the case $v_0 \gg \Delta v$, where Δv is the dispersion of the velocities of the charges) or simply a plasma with a current (in the case $v_0 \ll \Delta v$) may be a pinch system.

In order to avoid making any assumptions with respect to a model and thus to study the properties of pinch systems to the fullest extent possible, we will focus on those cases in which the plasma can be regarded as a set of two subsystems (electrons and ions), which are interacting with each other to a slight extent through collisions. In contrast, we do not assume that the interaction of the subsystem through the electromagnetic field produced by the charges themselves (the collective-interaction field) is slight. A situation of this sort occurs when the drift velocity is either very high or very low in comparison with the velocity spread of the charges. In the case $v_0 \gg \Delta v$ the relaxation to equilibrium separately in the electron subsystem and in the ion subsystem occurs much more rapidly than the overall plasma can reach equilibrium through electron-ion collisions. The reason is the rapid decrease in the Coulomb cross section with increasing relative velocity of the colliding charges. In the opposite limit of a low drift velocity, $v_0 \ll \Delta v$, a current flow causes a slight deviation of the plasma as a whole from its equilibrium state. In this case both the electrons and the ions can again be regarded as nearly equilibrium subsystems. Because of the significant difference in the masses, the rate of electron-ion collisions is much smaller than the rate of electron-electron collisions or that of ion-ion collisions.[55]

Under the conditions we are considering each subsystem is at thermal equilibrium in the field of the collective-interaction forces. The electromagnetic field of the collective interaction serves as an external field with respect to both the electrons and the ions during the formation of the equilibrium structure of the plasma. The plasma as a whole, on the other hand, is not in a state of thermal equilibrium, since the subsystems of electrons and ions are moving with respect to each other at a velocity v_0. The particular case of a complete equilibrium $v_0 = 0$, is of no interest in research on electromagnetic collapse.

A description of a pinch system as a set of two thermodynamically equilibrium subsystems is good in the stage of the maximum compression of the plasma, as long as experiments show that the relaxation times are short in comparison with the duration of the x-ray burst,[32] and instabilities do not have time to affect the structure of the current channel at its regions of maximum contraction.[35,36]

The existence of thermal equilibrium in each of the subsystems eliminates arbitrariness in the choice of a distribution function. In the comoving frames of reference K'_{α}, the charge distribution functions f'_{α} depend only on the total energy (Sec. 4 in Ref. 56): $f'_{\alpha} = F(E'_{\alpha}/T_{\alpha})$. The nature of the function F is determined by the statistics of the charges:

$$F(x) = F_{\pm}(x) = [\exp(A_{\alpha} \mp x) \pm 1]^{-1}. \qquad (1)$$

The plus sign is used for fermions, and the minus sign for bosons; E'_{α} and T_A are the total energy and temperature of the charges of species α in frame of reference K'_{α}; and α, here and everywhere below, takes on the values i.e., for ions and electrons, respectively. The scalar A_{α} is related to the chemical potential of the corresponding subsystem: $A_{\alpha} = -\mu_{\alpha}/T_{\alpha}$. It can be expressed in terms of the temperature and the number of particles through a normalization condition (Sec. 55 in Ref. 56). The limiting case $A_{\alpha} \gg 1$ corresponds to classical Boltzmann statistics, and in this case we have $F_{\pm}(x) \rightarrow \widetilde{A}_{\alpha} e^{-x}, \widetilde{A}_{\alpha} = e^{A_{\alpha}}$.

To calculate the charge and current densities we need to know the electron and ion distribution functions not in different frames of reference but in the same frame of reference K (the laboratory frame). Making use of the relativistic invariance of the distribution function,[57] we can find f_{α} in frame K by simply expressing the energy E'_{α} in terms of the energy E_{α} and the generalized momentum $\mathbf{P}_{\alpha}$ by means of the equations of a Lorentz transformation: $E'_{\alpha} = \gamma_{\alpha}(E_{\alpha} - \mathbf{P}_{\alpha}\mathbf{v}_{0\alpha}$, where $\gamma_{\alpha} = [1 - (v_{0\alpha}^2/c^2)^2]^{-1/2}$ is the relativistic factor, and $v_{0\alpha}$ is the velocity of frame K'_{α} with respect to K. The total energy of a charge of species α in the electromagnetic field φ. $\mathbf{A}$ is[58] $E_{\alpha} = e_{\alpha}\varphi + \varepsilon_{\alpha}[\mathbf{P} - (e_{\alpha}/c)\mathbf{A}]$, where $\varepsilon_{\alpha}(\mathbf{p}) = (m_{\alpha}^2 c^4 + c^2\mathbf{p}^2)^{1/2}$, and c is the velocity of light.

Since the electrons and ions of a plasma are thermodynamically equilibrium subsystems, their state is determined completely by the numbers of particles N_{α} (per unit length of the pinch); the temperatures T_{α}; and the velocities $v_{0\alpha} = \beta_{\alpha} c$ of the subsystems. The state of a pinch system is described by the six parameters

$$N_{\alpha},\ T_{\alpha},\ \beta_{\alpha}, \quad \alpha = \mathrm{i},\ \mathrm{e}, \qquad (2)$$

where one of the two parameters β_{α} can be made equal to zero through an appropriate choice of frame of reference. To the extent to which the electron and ion subsystems are close to a state of complete thermal equilibrium, the temperatures T_{α} and the velocities $v_{0\alpha}$ do not depend on the spatial coordinates.

One hears the objection[59] that a spatial homogeneity of the temperatures and velocities would amount to, so to say, additional conditions with respect to the complete system of equations describing the pinch system, so that the set of possible equilibrium configurations is restricted by this homogeneity. Actually, the spatial homogeneity of the temperatures and velocities does not represent additional conditions with respect to the equations describing a pinch system. We are dealing here with a system under conditions such that the uniformity of the parameters over the cross section follows from the equations themselves. The homogeneity of the velocities follows from the Navier-Stokes equations, and that of the temperatures follows from the heat transfer equations. Consequently, no restriction of any sort is imposed on the set of possible equilibrium configurations. One could in principle write and solve a complete system of equations including the Navier-Stokes equations and the heat transfer equations for both subsystems. Clearly, however, the solutions of these equations will be close to equilibrium solutions to the extent that the relaxation times within the subsystems are short. The thermodynamic limit has the further advantage that it allows us to obtain the necessary information on the properties of the system essentially without carrying out any calculations.

On the other hand, what one must guard against is an overly extensive use of the conditions of the spatial homogeneity of the parameters T_{α} and $v_{0\alpha}$, if only under the guise of choosing a model.[60] Even if this approach is justified in the stage of maximum compression of a pinch, in the initial stage of the formation and motion of the current shell there is no basis of any sort for such an assumption.

Under conditions such that the plasma can be represented as a set of subsystems which are separately in thermal equilibrium, the problem of the structure of the current channel splits in two. The first problem is that of studying all possible equilibrium distributions of the field and of the charge densities at fixed but initially unknown values of the parameters in (2). In the general case of a charged plasma, this problem reduces to one of studying a system of coupled equations of electrostatics and magnetostatics. In the simplest geometry, the solutions of this system describe all possible configurations of a cylindrically symmetric plasma of a pinch system which are mechanically in equilibrium, for fixed values of the current $I = \Sigma_{\alpha} e_{\alpha} N_{\alpha} v_{0\alpha}$, the charge per unit length of the plasma, $Q = \Sigma_{\alpha} e_{\alpha} N_{\alpha}$, and the temperatures T_{e} and T_{i}.

The second part of the problem is to determine the values of the parameters in (2) for a specific system which includes the plasma itself of the discharge channel, the medium surrounding this channel, and the external circuit, on whose parameters the current depends. Both the instantaneous values of the parameters in (2) and their time evolution are determined in each case by the individual features of the apparatus used to produce and study the discharge. The first part of the problem—that of studying the physical nature of the equilibrium of the plasma in the channel of a high current—is a universal problem. Changes in the state of the current channel which occur slowly in comparison with the relaxation times of the subsystems should be smooth transitions through states of mechanical equilibrium of the plasma of the pinch system. For this reason, a study of the equilibrium structures of self-contracting charged-particle beams is a universal approach to the study of the dynamics of both relativistic beams and simply of plasmas with a high current in the stage of a pronounced contraction of the discharge channel.

2.2. Equilibrium equations; their general properties; confinement conditions

In the steady state, the field potentials φ and $\mathbf{A}$ satisfy the equations of electrostatics and magnetostatics,

$$\frac{\partial^2 \mathbf{A}}{\partial x_k^2} = -\frac{4\pi}{c}\,\mathbf{j}, \qquad \frac{\partial^2 \varphi}{\partial x_k^2} = -4\pi\rho. \tag{3}$$

The current density $\mathbf{J}$ and the charge density ρ are expressed in the standard way by means of the charge distribution functions in the laboratory frame found above. To study equilibrium structures it is convenient to switch from φ and $\mathbf{A}$ to the functions U_α, $U_\alpha = e_\alpha(\varphi - \boldsymbol{\beta}_\alpha \mathbf{A})$, $\boldsymbol{\beta}_\alpha = \mathbf{v}_{0\alpha}/c$. In the case of a cylindrically symmetric plasma, the U_α are the potentials of forces which are acting on the charges of the subsystem α. The equilibrium distributions of the potentials $U_\alpha(r)$ and of the densities $n_\alpha(r)$ are described by the equations

$$\frac{1}{r}\frac{d}{dr}r\frac{dU_\alpha}{dr} = -4\pi e_\alpha \sum_\beta e_\beta(1-\beta_\alpha\beta_\beta)\,n_\beta, \tag{4}$$

$$n_\alpha = \frac{g_\alpha}{(2\pi\hbar)^3}\int F\left(\frac{\gamma_\alpha}{T_\alpha}[\varepsilon_\alpha(p) - pv_{0\alpha} + U_\alpha]\right) d^3p, \tag{5}$$

where g_α is the g-factor; for electrons we have $g_e = 2$. The function F is given by expression (1), while the scalar A_α in (1) is related to the number of particles per unit length of the channel N_α, by the normalization condition

$$N_\alpha = 2\pi\int_0^\infty n_\alpha(r)\,r\,dr. \tag{6}$$

These equations are derived in detail in Refs. 44 and 61. From the condition that the field and the charge densities must be finite on the axis we find $dU_\alpha(0)/dr = 0$.

As a first step in the study of a system, one assumes that the external fields are small in comparison with the fields generated by the charge and current of the pinch itself.

If the drift velocity is nonrelativistic, the equilibrium of the pinch is studied by the equations of magnetostatics alone.[10,62] The fact that the Debye length r_D is small in comparison with the radius of the plasma column, r_0, makes it possible in this case to determine correctly the structure of the plasma and the field in the region with $r_D \ll r_0$, i.e., everywhere except at the periphery of the pinch. Equations (4) and (5), on the other hand, make it possible to calculate correctly the field throughout space, including at the periphery of the pinch. At first glance this might appear to be a rather minor detail in the case $v_0 \ll c$, but actually the field at the periphery of the discharge determines whether charges are drawn into the discharge or repelled from it. By describing correctly the field at the periphery of the current channel, the equations make it possible to find the plasma confinement conditions.

Integrating Eqs. (4), and noting that the total current and the charge of the system are finite, we find $rU'_\alpha = 2E_\alpha$, $r \to \infty$,

$$E_\alpha = -e_\alpha \sum_\beta e_\beta N_\beta(1-\beta_\alpha\beta_\beta), \tag{7}$$

from which we conclude that far from the axis the potentials U_α increase logarithmically: $U_\alpha = 2E_\alpha \ln r$, $r \to \infty$. (For a real current with a channel of length l and radius r_0, $l \gg r_0$, the logarithmic growth of the potentials occurs in the intermediate region $l \gg r \gg r_0$.) By virtue of the meaning of the potentials U_α, the factors E_α in (7) are the energies of the interaction of a charge of species α with the field set up by all the other charges. If $E_\alpha > 0$, the interaction is an attraction, while if $E_\alpha < 0$ it is a repulsion.

The four parameters N_α, β_α can evidently be chosen in such a way that they simultaneously satisfy the two inequalities $E_i > 0$, $E_e > 0$. In this case, the collective interaction is an attraction for both ions and electrons. In particular, if the ions are a whole are at rest ($\beta_i = 0$, $\beta_e = \beta$) in the laboratory frame, these inequalities can be thought of as restrictions on the number of ions:

$$N_e > ZN_i > N_e(1-\beta^2), \tag{8}$$

where Z is the ion charge. We see that in the presence of a current the condition $\beta^2 > 0$ holds, and for N_i there is a range of values for which both ions and electrons at the periphery are attracted toward the central part of the discharge.

As they move in the logarithmically increasing potential, charges having a large total energy penetrate a large distance. However, there are few such charges, and regardless of the degree of compression at the discharge axis the remote charges have a Boltzmann distribution. For the charge densities we find from Eqs. (4) and (5) a power-law decay along the radius: $n_\alpha \sim r^{-2K_\alpha}$, $r \to \infty$, $K_\alpha = E_\alpha/T_{\alpha\perp}$, where $T_{\alpha\perp} = T_\alpha/\gamma_\alpha$ is the effective temperature in the transverse direction in the laboratory frame. (We recall that the temperatures T_α are determined in the comoving frames of reference K'_α.) From the conditions that the total current and the charge must be finite we conclude that the integrals in (6) converge; here the charge densities must fall off more rapily than r^{-2}. We thus have confinement conditions for the electrons and the ions[45]:

$$E_\alpha > T_{\alpha\perp}, \quad \alpha = \mathrm{i},\ \mathrm{e}. \tag{9}$$

With $\beta_i = 0$, $\beta_e = \beta$ we have $E_i = Ze^2(N_e - ZN_i)$ and $E_e = e^2[ZN_i - (1-\beta^2)N_e]$. The energy of the collective attraction must exceed the energy of the thermal dispersal of the charges in the radial direction. These physically transparent conditions for the confinement of charges of both subsystems hold for any value of β, including the limit $\beta \ll 1$, where the equilibrium of the pinch can be studied by the equations of classical MHD.[10,62] Confinement conditions (9) cannot be derived, however, without taking the electrostatic field of the charge separation into account completely.

2.3. The approximation of Boltzmann statistics

The approximation of the classical statistics of ideal gases is used to describe plasmas which have not yet reached the extreme compressed state. In the limit of Boltzmann statistics, Eqs. (4) and (5) become[45]

$$\frac{1}{r}\frac{d}{dr}r\frac{dU_\alpha}{dr}=-2e_\alpha\sum_\beta\frac{e_\beta N_\beta}{\nu_\beta}(1-\beta_\alpha\beta_\beta)e^{-U_\beta/T_{\beta\perp}},$$

$$\nu_\beta=\int_0^\infty e^{-U_\beta/T_{\beta\perp}}r\,dr. \tag{10}$$

Making use of gauge invariance, we can write the boundary conditions as

$$U_\alpha(0)=\frac{dU_\alpha}{d\ln r}\Big|_{r=0}=0.$$

The parameters in (2) appear explicitly in Eqs. (10).

The most important property of Eqs. (10) is gauge invariance. From these equations we can find only the relative distribution of the field (and of the charge densities) along the radius; the value of the discharge radius itself is not determined by Eqs. (10). Solutions of Eqs. (10) are $U_\alpha = U_\alpha(r/r_0)$, where r_0, the discharge radius, is an arbitrary scaling parameter. The physical reason for the gauge invariance is that at fixed values of the parameters in (2) both the gas pressure $p_\alpha = n_\alpha T_{\alpha\perp}$ and the field pressure $p_f = (H_\varphi^2 + E_r^2)/8\pi$ are inversely proportional to the square of the pinch radius.

A detailed study[44] of Eqs. (10) shows that equilibrium can occur only if the parameters in (2) satisfy the relation[44,63,64]

$$\frac{1}{2}\Big(\sum_\alpha e_\alpha N_\alpha\beta_\alpha\Big)^2=\frac{1}{2}\Big(\sum_\alpha e_\alpha N_\alpha\Big)^2+\sum_\alpha N_\alpha T_{\alpha\perp}. \tag{11}$$

The physical meaning of this relation is clear. The energy of the magnetic compression of a unit length of the current channel, $(1/2)(I^2/c^2) = (1/2)(\Sigma_\alpha e_\alpha N_\alpha B_\alpha)^2$, at equilibrium—on the left of this equation—must be exactly balanced by the energy of the space charge,

$$\frac{1}{2}Q^2=\frac{1}{2}\Big(\sum_\alpha e_\alpha N_\alpha\Big)^2,$$

and the total energy of the thermal expansion.

Balance condition (11) generalizes the well-known Bennett equilibrium condition[6] to the case of an arbitrary relativistic drift velocity. Relativistic beams are not electrically neutral, and at $\beta\sim 1$ the energy of the space charge may constitute a significant fraction of the energy of the magnetic compression. From the conditions for confinement of ions, $E_i > T_{i\perp}$, and from the attraction condition for electrons, $E_e > 0$, we find inequalities which place lower and upper bounds on the space charge per unit length of the plasma column, Q:

$$\frac{T_\perp}{Ze}<Q<\frac{\beta I}{c}. \tag{12}$$

Balance condition (11) can be rewritten as

$$\frac{I^2}{2c^2}-\sum_\alpha N_\alpha T_{\alpha\perp}=\frac{Q^2}{2}.$$

Using the inequality on the right in (12), we see that in the limit of a nonrelativistic drift velocity $\beta \ll 1$ the energy of the space charge becomes negligibly small, and Eq. (11) becomes the Bennett relation

$$\frac{I^2}{c^2}=2\sum_\alpha N_\alpha T_\alpha,\qquad \beta\ll 1. \tag{13}$$

We will assume that the ions as a whole are at rest in the laboratory frame K: $\beta_i = 0$, $\beta_e = \beta$. Of the five parameters $N_e, N_i, T_{e\perp}, T_i, \beta$, characterizing the state of the pinch system at equilibrium, only four, say N_α and $T_{\alpha\perp}$, are independent in the approximation of Boltzmann statistics. The fifth parameter, β, and, correspondingly, the current $I = eN_e\beta c$ are determined unambiguously in terms of N_α, $T_{\alpha\perp}$ by balance condition (11):

$$I=I_{cr}(N_\alpha, T_{\alpha\perp})=c\Big(Q^2+2\sum_\alpha N_\alpha T_{\alpha\perp}\Big)^{1/2}$$

$$=c\Big[\Big(\sum_\alpha e_\alpha N_\alpha\Big)^2+\sum_\alpha N_\alpha T_{\alpha\perp}\Big]^{1/2}. \tag{14}$$

In the approximation of Boltzmann statistics, the set of equilibrium configurations which satisfy Eqs. (10) is a four-parameter set. Five parameters must satisfy one exact equation, (11)—the balance condition—and two inequalities, (9)—the confinement conditions. In general, numerical methods would have to be used to integrate Eqs. (10). This question is taken up in Refs. 63 and 65–67.

Equations (10) are invariant under the interchange of electrons and ions. As a consequence of this symmetry there exist distributions of the charges in the field such that the radial profiles of the properties are the same for the two species of particles. If the parameters in (2) satisfy the equalities $E_i/T_i = 2, E_e/T_{e\perp} = 2$. [in this case, balance condition (11) is satisfied identically], system (10) degenerates into a single equation and can be integrated analytically. The degerate solution was found by Bennett[6]:

$$U_\alpha^B=E_\alpha\ln\Big[1+\Big(\frac{r}{r_0}\Big)^2\Big],$$

where r_0 is an arbitrary scaling parameter with the meaning of the radius of the plasma column. The Bennett solution is a three-parameter solution since the five original parameters must satisfy two equalities.

A "model" assumption which has been adopted in certain studies in order to solve the equations analytically is the assumption that the radial profiles of the electron and ion densities differ by only a constant factor[68]: $n_i = fn_e$, = const. The role of the ions is thereby reduced to simply one of causing a partial charge neutralization. This assumption will clearly narrow the region of parameters and will make it possible to find only degenerate solutions.

Let us reinterpret the properties of equilibrium configurations in the approximation of Boltzmann statistics for the particular example of the dependence of the pinch radius on the current. It is found that at fixed values of the numbers of particles and of the temperatures equilibrium is possible only at the unique value of the current in (14), while the pinch radius can be arbitrary. Actually, of course, the radius of the discharge channel cannot be completely arbitrary. In order to reach an understanding of how the discharge radius nevertheless depends on the current, we consider which assumptions are violated at very large and very small values of the radius r_0.

A real discharge channel has a finite length l. Clearly, the condition $r_0 \ll l$ holds, for otherwise the approximation of an infinite plasma column, uniform along the current, would be incorrect. As the pinch radius increases, an increasingly important role is played by the structural features of the particular apparatus, the image fields near metal walls, and the external fields used to confine the plasma. At large radii r_0 the external fields are no longer negligibly weak in comparison with the pinch field. Consequently, Eqs. (10), which incorporate the field of only the collective interaction, are valid only to the extent that the external factors are unimportant for plasma confinement. We denote by d a dimension of the plasma channel which is characteristic of the given apparatus at a low current, when confinement is achieved without the help of the magnetic field of the current. The condition for the applicability of Eqs. (10) at large radii is then $r_0 \ll d$. If r_0 is not small in comparison with d, external factors may also contribute substantially to the plasma confinement, so that equilibrium will be possible at values of the current below I_{cr} in (14). At $I \langle I_{cr}$, however, the equilibrium configurations are no longer described by Eqs. (10), which take into account only the field of the collective interaction and which ignore other factors which are capable of confining the plasma. Consequently, under the condition $I < I_{cr}$ we enter a region of parameter values, $r_0 \sim d$, in which not only the field of the collective interaction but also external factors play a role in shaping the equilibrium structure.

We now assume that, at fixed values of N_α and T_α, the current exceeds I_{cr} in (14). In this case the magnetic energy of the compression is so high that the energy of the space charge and that of the thermal motion of the particles are insufficient to counter the magnetic energy. Eqs. (10) have no solutions in the case $I > I_{cr}$, but this circumstance is a consequence of the use of Boltzmann statistics to describe the electron subsystem. If we replace (10) by the more general equations (4) and (5), we find that equilibrium is possible even in the case $I > I_{cr}$, but the equilibrium configurations in this case are of such a nature that the plasma turns out to be very highly compressed: to the point of electron degeneracy. When an electron gas is compressed to degeneracy, an additional exchange repulsion of electrons arises by virtue of the Pauli principle and increases the pressure of the electrons. It is this pressure of the Fermi gas of electrons which balances the excess energy of the magnetic compression at $I > I_{cr}$. The plasma is compressed to the point of electron degeneracy; the radius of the plasma column is of the order of or even less than the atomic radius a. Equilibrium configurations with $I > I_{cr}$ form a complete five-parameter family in the space of the parameters N_α, T_α, β. The properties of equilibrium configurations in the case of compression

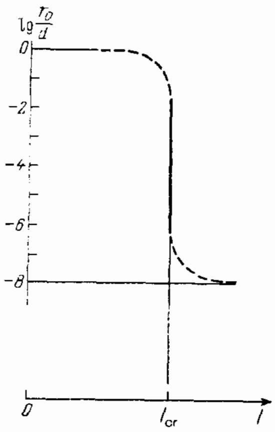

FIG. 1. Equilibrium pinch radius as a function of the current at fixed values of the parameters N_α and $T_{\alpha\perp}$. Here I_{cr} is that value of the current, given in (14), at which the balance described by (11) is reached among the energy of the magnetic compression, the electrostatic repulsion and the thermal dispersal of the particles in the approximation of Boltzmann statistics. The solid vertical line corresponds to solutions of Eqs. (10) in their range of applicability. The dashed lines show the function $r_0(I)$ schematically at $I < I_{cr}$ and $I > I_{cr}$ [see (15) and (16); $d/a = 10^8$].

to the point of electron degeneracy are described in Refs. 44, 45, 63, 67, and 69.

The conditions for the applicability of Eqs. (10) for describing the equilibrium of a self-contracting beam of charged particles are therefore

$$a \ll r_0 \ll d.$$

Here r_0 is the radius of the plasma column, a is the radius at which the electron degeneracy is important, and d is the radius which the plasma column would have in the given apparatus in the absence of a compression of the current by the magnetic field. The pinch radius is shown schematically as a function of the current in Fig. 1. The solid vertical line corresponds to solutions of Eqs. (10) in their region of applicability, $a \ll r_0 \ll d$, while the dashed lines show $r_0(I)$ at $r \sim d$ and at $r \sim a$ (for fixed values of N_α, $T_{\alpha\perp}$). Under these conditions we have $I < I_{cr}$ and $I > I_{cr}$, respectively.

At the current $I = I_{cr}$ there is a transition from confinement of the plasma by the external factors to a confinement due to a Fermi exchange interaction of electrons and a compression to the point of degeneracy. The value of the current in (14), $I = I_{cr}$, is the boundary for electromagnetic collapse from the standpoint of the force equilibrium of the pinch system.

2.4. Equilibrium near the collapse boundary; electrically neutralized discharge

The behavior of the pinch radius r_0 as a function of the parameters of the pinch system near the collapse boundary, with $|I - I_{cr}| \ll I_{cr}$, is amenable to analytic study.[44] In the case $I < I_{cr}$ this behavior is conveniently studied on the example of an electron beam whose charge is completely neutralized by ions: $ZN_i = N_e \equiv N$. This case is also of independent interest. Charge neutralization usually results from the buildup of ions in the potential well of the negative charge of an electron beam.[70]

If the ions as a whole are at rest, the ion subsystem in an electrically neutral current channel can never be in a state of thermal equilibrium when subject to solely the forces of a collective interaction. A thermal equilibrium, the ion temperature is uniform over the beam cross section. If $T_i \neq 0$, and if $ZN_e = N_i$, condition (9), for ion confinement, does not hold. This conclusion means that the collective interaction forces by themselves are insufficient for a steady-state current flow in the case $ZN_i = N_e$. There is a further need for a contribution from the interaction of charges with the surrounding medium.

In general, thermal equilibrium within the subsystems for specific devices is achieved only to some degree of accuracy. Accordingly, it is worthwhile to analyze pinch systems in the case in which the state of the electrons or ions deviates from thermodynamic equilibrium. Kovalenko *et al.*[72] have studied the quasiequilibrium states of a self-contracting electron-ion beam under conditions such that a few charges of one species are not confined and escape from the system. The region of parameter values consistent with such quasiequilibrium states is naturally wider than that determined by the balance condition (11). Here we will consider a slightly different situation: that in which the number of particles is conserved, while the energy of the ion subsystem is lost in collisions with atoms of the surrounding gas.

As a strong current flows through a plasma, the discharge channel is strongly compressed by magnetic forces which are acting on electrons. Despite the neutrality of the channel as a whole, a pronounced electric polarization arises in the plasma. The magnetic compression force compresses the electrons to a greater extent than the ions—to the extent that the electric field which results from the charge separation offsets the compressional force. If the ions as a whole are immobile, the potential U_i is equal to the scalar potential of the field. In the case of a strong current the polarization is so strong that only a small fraction of the total number of ions has a total energy above the limiting value of the potential, $U_\infty = U_i(\infty)$ [we are assuming $U_i(0) = 0$]. We therefore have $U_\infty \gg T_i$, and even a slight radial decrease in the ion temperature due to a transfer of energy from ions to atoms of the surrounding medium in collisions will be sufficient to cause a rapid decrease in the ion density along the radius.[71] The function $T_i(r)$ can be expanded in a power series in r:

$$T_i = T_{i0} - \frac{1}{2}\left|T_i''(0)\right| r^2.$$

We then find equations of the type in (10), with the one difference that U_i/T_i is replaced by $(U_i/T_{i0}) + (r^2/d^2)$ in the exponential functions. Here $d = (2T_{i0}^2/U_\infty |T_i''(0)|)^{1/2}$ is a scale dimension of the region in which the ion density falls off because of cooling in collisions with surrounding atoms. By virtue of the inequality $U_\infty \gg T_{i0}$, the temperature itself falls off only negligibly over distances of the order of d—justifying the expansion of the temperature in powers of the radius.

In the case of a strong current, with $ZN_i = N_e$, the ion subsystem is in a state of local thermal equilibrium (a nearly complete equilibrium), so that the interaction of ions with the surrounding medium can be taken into account in a study of the structure by means of simply a single phenomenological parameter d. The interaction of the ions with the surrounding medium leads to a decrease in the ion density in proportion to $\exp(-r^2/d^2)$ and thus a confinement of the ions. The energy balance under these conditions is achieved at currents $I < I_{cr}$, and it becomes possible to study the properties of the pinch as the collapse boundary is approached from below, with $(I_{cr}/I) - 1 \ll 1$.

It turns out[44,71] that in the limit $I \to I_{cr} - 0$ the electrons and some of the ions, $\widetilde{N}_i < N_i = N/Z$, are localized near the axis in a region $r \sim r_0$, considerably smaller than d: $r_0 \ll d$. This central region turns out to be negatively charged, and its charge, $Q = e_e N_e + e_i \widetilde{N}_i$, is precisely that which confines ions compressed to a radius r_0: $|e_i Q| = T_i$. The rest of the ions, whose number is $\frac{N}{Z} - \widetilde{N}_i$ are at distances $r \sim d$ from the axis and produce a positively charged shell. The deviation of the current from I_{cr} can be characterized by the dimensionless parameter $\Delta = e^2(I_{cr} - I^2)/T_i^2 c^2$, $e_i = e$. In the limit $I \to I_{cr} - 0$, the radius r_0 of the compressed core decreases sharply[44]: $r_0 \sim d \exp(-\pi\Delta^{-1/2})$ or

$$r_0 \sim d \exp\left[-\frac{\pi T_i c}{e}(I_{cr}^2 - I^2)^{-1/2}\right], \qquad I \to I_{cr} - 0. \qquad (15)$$

This expression describes the dependence of the discharge radius on the current, $r_0(I)$, as the boundary of electromagnetic collapse is approached. The radius of the region through which the electrons move decreases very sharply (Fig. 1). Expression (15) continues to apply until degeneracy must be taken into account.

The dependence of the discharge radius on the current beyond the collapse boundary was found in Ref. 44. We denote by a the radius of the condensed plasma. The typical value of a depends on the parameters of the pinch system in (2). This dependence is given in Ref. 44. As moderately relativistic energies, $\beta \sim 1$, $I \sim I_A = 17$ kA, we have the estimates $n \sim (mc/\hbar)^3$, $N \sim mc^2/e^2$, and from the relation $N \sim na^2$ we find a in order of magnitude: $a \sim (e^2/\hbar c)^{1/2}\hbar^2/me \sim 10^{-9}$ cm, $\beta \sim 1$, $I \sim I_A$. As we approach the collapse boundary from the high-current side, $I \to I_{cr} + 0$, the discharge radius r_0 increases rapidly[44]:

$$r_0 \sim a\left(1 - \frac{I_{cr}^2}{I^2}\right)^{-1/2}, \qquad \frac{I}{I_{cr}} - 1 \ll 1. \qquad (16)$$

Expressions (15) and (16) give an idea of the current dependence of the equilibrium pinch radius near the boundary for electromagnetic collapse. This behavior is shown in Fig. 1. The solid vertical line $I = I_{cr}$, $a \ll r_0 \ll d$, corresponds to an intermediate region of a transition from a diffuse plasma, which cannot be confined without the help of external factors, to a plasma which is extremely compressed by the dominant magnetic forces. This intermediate region is described by classical equations (10), which give gauge-invariant solutions at a fixed value of the current in (14).

This has been a summary of our present understanding of the dynamic nature of the plasma equilibrium in a straight pinch system. The equilibrium is analyzed with the help of the entire apparatus of electrostatics and magnetostatics, as is required for relativistic fluxes of charges. For pinch systems with a nonrelativistic drift velocity (which are usually studied by MHD in the approximation of quasineutrality), the general analysis which has been carried out has also yielded some new results. In particular, the confinement conditions in (19) cannot be found in ordinary MHD. We see why Bennett equilibrium condition (13) and the corresponding gauge-invariant equilibrium configurations correspond to the boundary for electromagnetic collapse, i.e., to the transition from a diffuse plasma (in the formation of whose structure instrumental factors play a role) to an extremely compressed plasma, for which the excess magnetic pressure is balanced by the pressure of the Fermi gas of electrons.

This analysis of the equilibrium is a solid basis for studying the stability and dynamics of highly compressed pinch systems.

3. CORRELATION PROPERTIES OF CHARGES IN PLASMA FLOWS

3.1. Field of a charged current-carrying filament

The electrostatic field which is produced in an equilibrium plasma by test objects is known to fall off rapidly with distance, becoming very weak at distances large in comparison with the Debye-Hückel radius.[73] On the other hand, the magnetic field penetrates freely into a plasma in a state of thermal equilibrium. This situation changes if the plasma is moved away from the state of thermal equilibrium, e.g., if a current flows through it. When a current flows, the electrons as a whole are in motion with respect to the ions, and even if there is equilibrium in each of the subsystems the plasma as a whole will not be an equilibrium system.

The introduction of a test charge or current in the plasma disrupts the balance of forces. In the absence of thermal equilibrium, the electrostatic and magnetostatic perturbations are interrelated and are described by a common system of equations. Because of the relatively free penetration of the magnetic field into the plasma, the perturbations introduced by the test objects acquire a long-range component in addition to the short-range part, which described Debye screening.

Let us assume that there is a thin filament at the axis of a plasma stream. If the charge and current of the filament are small, the field in the plasma can be found by means of linearized equations (4) and (5). This perturbation does not disrupt the symmetry of the initial equilibrium state. This problem was solved analytically in Ref. 61 for a Bennett density profile $n_\alpha^B(r) = (N_\alpha/\pi r_0^2)\times[1 + (r_2/r_0^2]^{-2}$, and the structure of the field of the filament was analyzed in various cases of physical interest. In general, the field is a linear combination of four linearly independent functions, with coefficients which are chosen to suit the boundary conditions. There are four boundary conditions: The electric and magnetic fields near the filament $(r\to 0)$ correspond to the charge and current of the filament, while outside the plasma $(r\to\infty)$ they correspond to the total charge and current of the overall system.

Bennett equilibrium configurations constitute a three-parameter family. The initial parameters adopted in Ref. 61 were $\beta, G_e = e_e^2 N_e \gamma/T_e$, $G_i = e_i^2 N_i/T_i$. An interesting case is that of a high plasma density, $G_e, G_i \gg 1$, in which the Debye length r_D is small in comparison with the pinch radius r_0: $r_0/r_D \sim (G_e + G_i)^{1/2} \gg 1$. The quantity β (the ratio of the drift velocity to the velocity of light is a convenient measure of the deviation of the state of the plasma from a thermodynamically equilibrium state because of the current. Under the conditions G_e, $G_i \gg 1$, the field near the filament is the sum of two parts. One part—a component which falls off exponentially—stems from Debye screening, while the other is an oscillatory function which falls off slowly over distance. With $\beta\sim 1$, both the screening length and the oscillation period are of the order of the Debye length near the filament. The amplitudes for the decreasing and oscillating functions are comparable to magnitude at $\beta\sim 1$. With decreasing β, the index of the decreasing component remains of the order of the Debye length, while the oscillation period of the long-range component increases in proportion to r_D/β. The oscillation amplitude falls off. In the limit $\beta\to 0$, the decreasing component remains, and it describes a Debye screening of electrostatic perturbations, while the oscillatory component becomes a logarithm corresponding to free penetration of the magnetic field of the current of the filament into the equilibrium plasma.

By analyzing the perturbations introduced in a plasma stream by a test filament, we can determine the nature of the stratification of high-current discharges into a set of narrow current channels. This phenomenon is observed both in intense beams of relativistic electrons[74] and in plasma-focus devices.[52,75–78] Various aspects of the stratification of a discharge into channels have been studied theoretically.[79–81]

Let us imagine that the excess magnetic compressional force and the rapid radiative cooling have caused a small part of plasma stream to contract into a thin filament at the axis. The field produced by this filament will determine the correlation properties of the charges of the plasma with respect to the given filament. The field of the filament is an oscillatory function of the radius. As they move in this field and give off energy by radiation, charges may be captured by the filament onto trajectories which are finite along the radius. This process can lead to a stratification of the plasma into concentric tubes whose spatial positions are the minima of the potential set up by the filament. These tubular structures may in turn decay into distinct filaments, in such a way that the mutual energy of their correlation interaction is minimized. We will return to this question of the stratification of a current channel into distinct filaments when we take up the problem of the stability of the pinch.

3.2. Interaction of charges in plasma flows

A current in the plasma is not the only factor which would cause deviations from Debye-Hückel screening. Even if the test charge is moving, the field which it produces falls off in a power-law rather than exponential fashion in the plasma.[82–87] There is no Debye screening in a turbulent plasma[88] or even in the equilibrium plasma in a quantizing magnetic field.[89]

The field of test objects of finite size and of individual charges in a plasma with a current was found by Ignatov and

Rukhadze[89] in the limit in which the drift velocity v_0 is low in comparison with the thermal velocities $v_{T\alpha}$: $v_0 \ll v_{T\alpha}$. The field in the opposite limit, $v_0 \gg v_{T\alpha}$ (the beam situation), was found in Ref. 90.

The field of objects of finite length in a plasma flow depends not only on r but also on z, and it does not correspond to the symmetry of the original state of the system. Here it becomes necessary to solve the complete problem of the flow of streams of charged particles around the test object. For nonrelativistic velocities, $v_0 \propto c$, there is time for the electromagnetic field to adjust to slow variations of the charge and current. The displacement currents can thus be ignored in Maxwell's equations, and we can solve the equations of electrostatics and magnetostatics, (3), supplemented with the equations of two-fluid MHD for ideal electron and ion fluids.

A test object with the charge distribution $\rho(r)$, moving at a velocity $\mathbf{v}_1$ with respect to the electron subsystem, distorts the initial distribution of the charge density. In a frame of reference moving with the test object there is a steady-state flow of two streams of charged particles around this object: an electron stream with a velocity $\mathbf{v}_{e0} = -\mathbf{v}_1$ and an ion stream with $\mathbf{v}_{i0} = -\mathbf{v}_1 - \mathbf{v}_0$. If the size of the object, a_0, is small in comparison with r_0, the radius of the plasma flow $a_0 \ll r_0$, the perturbations will be local: $a_0 \ll k^{-1} \ll r_0$, where $\mathbf{k}$ is the wave vector of the field generated in the plasma by the test object. The locality condition $kr_0 \gg 1$ can be used to represent the Fourier component of the perturbation of the scalar field potential in the form $\varphi(\mathbf{k}) = 4\pi\rho(\mathbf{k})/k^2\varepsilon(\mathbf{k})$, where $\varepsilon(\mathbf{k})$ is the static dielectric permittivity. An expression for this component is given in Ref. 90. The field away from the test object is determined by the zeros of $\varepsilon(\mathbf{k})$. At a low drift velocity, $v_0 \ll s_\alpha$ (s_α is the sound velocity), the corresponding dispersion relation is[90] $k^4 + (k^2/r_D^2) - (\beta^2/r_i^2 r_e^2) = 0$, where r_α is the Debye length of the particles of species α and is defined as the ratio of the sound velocity to the frequency of the plasma waves, $r_D^{-2} = \sum_\alpha r_\alpha^{-2}$. In the case $\beta = v_0/c \ll 1$, this equation has two purely imaginary roots, $k_{1,2}^2 = -r_D^2$, and two real roots, $k_{3,4}^2 = \beta^2/(r_i^2 + r_e^2)$. The imaginary roots provide exponential solutions; the one which decays describes Debye screening. The real roots provide perturbations which fall off slowly and which oscillate in space with a period $(r_i^2 + r_e^2)^{1/2} c/v_0$. The situation here is precisely the same as in the case of a test filament. The oscillating perturbations stem from the relatively free penetration of the magnetic field into the plasma in the case $\beta \ll 1$.

If the drift velocity is large in comparison with the sound velocity, $v_0 \gg s_\alpha$, the following expression is found for the Fourier component of the scalar potential of the field set up by an individual electron[90]:

$$\varphi(\mathbf{k}) = 4\pi e_e \left\{ k^2 + \left[r_e^2 \left(1 - \frac{(\mathbf{k}\mathbf{v}_1)^2}{k^2 s_e^2} \right) \right]^{-1} \right\}^{-1}, \qquad v_1 \sim s_e \ll v_0. \tag{17}$$

This expression differs from that corresponding to ordinary Debye screening, $\varphi(\mathbf{k}) = 4\pi e_e (k^2 + r_e^{-2})^{-1}$.

3.3. Possibility of superconductivity

This analysis of the physical nature of the equilibrium of streams of charged particles shows that if the current in the plasma exceeds the value in (14) the excess force of magnetic compression can be balanced only by the pressure of the Fermi gas of electrons upon compression to a state of condensed matter. If the system radiates off more energy than it acquires, the temperature rise will be limited, and a collapse will be promoted. And in the compressed state, the radiative cooling causes the electron temperature to fall off.

For a degenerate electron gas the sound velocity in (17) can be expressed in terms of the Fermi velocity: $s_e^2 = v_F^2/3$. The energy of the binary interaction of electrons at the Fermi surface ($v_1 = v_F$) is described at large distances by $U = e_e\varphi(0) = 4\pi e^2 r_e^2 (1 - 3\cos^2\chi)$, where χ is the angle between the vector $\mathbf{k}$ and the direction of the velocity $\mathbf{v}_1$. This interaction is anisotropic: a repulsion along the direction in which the charges are moving and an attraction in the transverse direction. Any arbitrarily weak attraction of fermions at large distances will involve a lowering of the temperature at which the spectrum of elementary excitations changes,[91] and the system goes into a superconducting state. The interaction of electrons under consideration here is of the nature of an exchange of plasmons. A plasmon superconductivity mechanism has been studied by Pashitskiĭ with reference to degenerate semiconductors.

The analysis of the electron vertex function in Ref. 90 (by a method which has been used to study pairing with nonvanishing angular momenta[93]) showed that as the electron temperature decreases a plasma compressed to the point of degeneracy by magnetic forces can indeed become unstable with respect to Cooper pairing. At the most favorable densities, $n_e \sim 10^{24} - 10^{25}\ \mathrm{cm}^{-3}$ (where the value of the Coulomb parameter is $e^2/\hbar v_F \sim 1$), the transition temperature is expected to be of the order of 10^2 K. At low densities $T_c \sim 10^{-3} E_F$, where $E_F = m_e v_F^2/2$ is the Fermi energy, while at high densities T_c falls off exponentially with increasing density: $T_c \sim E_F \exp(-9\hbar v_F/e^2)$, $e^2/\hbar v_F \ll 1$, in accordance with the early result found by Abrikosov.[94] Figure 2 is a schematic plot of T_c as a function of n_e ($E_F = (3\pi^2)^{2/3}(\hbar^2/2m_e) n_e^{2/3}$).

Today, energy densities of the order of $10^{23}\ \mathrm{cm}^{-3}$ have been reached in pinch systems.[30,95–97] There have been re-

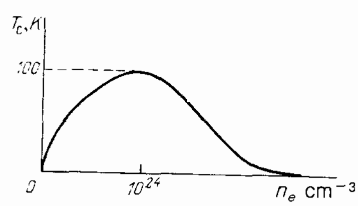

FIG. 2. Schematic plot of the transition temperature T_c as a function of the electron density (or of the Fermi energy). If the excess magnetic force due to the radiative collapse has the effect that the plasma in the pinch is compressed to the point of electron degeneracy, then during the subsequent cooling of the electrons in the course of the emission of radiation there may be a transition to a superconducting state. The maximum value, $T_c \sim 10^2$ K, corresponds to a density of 10^{24}–$10^{25}\ \mathrm{cm}^{-3}$.

ports of densities of[98] 10^{24} cm^{-3} and even[102] 10^{25} cm^{-3}. These figures are well above those in a solid, but it is not yet clear whether radiative cooling of electrons is occurring, and if so, to what temperature. On the other hand, the possibility is not ruled out that before the superconducting transition occurs as a system cools down it may go into some other phase which prevents superconductivity, e.g., a phase with a charge density wave. The estimates of Ref. 90 actually provide a lower boundary on the region of stability of the normal phase, and not the temperature of an actual superconducting transition. The approach of Ref. 90 should be thought of as a formulation of this complex procedure, not its solution. However, the question of possible superconductivity in a pinch system is exceedingly interesting, and further research is necessary, both experimental and theoretical.

4. PROBLEM OF THE STABILITY OF PINCH SYSTEMS

4.1. Two-fluid hydrodynamics of a charged plasma

The pinch effect is a rapidly evolving process for a laboratory observer. The duration of the radiation burst from high-current discharges such as the vacuum spark,[24,31,99,100] the plasma focus,[101–104] and exploding wires[29,30] ranges from microseconds (for microwave emission) to a fraction of a nanosecond (for hard x rays). This fast evolution of a pinch system from the standpoint of our everyday world is nevertheless a slow process on the scale of the time over which the system itself forms: the relaxation times in the subsystems and the scale times for traversal of distances of the order of the radius of the system by the charges. This circumstance has made it possible to carry out a detailed analysis of the equilibrium of a plasma with a strong current in the field of collective-interaction forces. The actual realization of an equilibrium configuration depends strongly on its stability. During rapid Coulomb relaxation within the electron and ion subsystems, it is natural to use a hydrodynamic description of the change in the state of the pinch system. Since electron–ion collisions are of minor importance, we arrive at the approximation of two-fluid hydrodynamics.

Two-fluid hydrodynamics allows for the difference between the velocities of the electron and ion subsystems due to the current. If, on the other hand, the drift velocity is comparable to the velocity of light, the approximation of quasineutrality must be completely abandoned, and the apparatus of the two-fluid hydrodynamics of a charged plasma operates in its full glory.

At a low drift velocity, it is customary to use the simpler equations of classical MHD, where the plasma is treated as a continuous, neutral conducting fluid, and the relative motion of the electron and ion subsystems is ignored. The range of applicability of this approximation has been discussed by Kadomtsev.[105] If we are to be able to ignore the relative motion of the subsystems, the drift velocity must be small in comparison with the hydrodynamic flow velocity of the plasma. If we can ignore electron–ion collisions and also the inertia of electrons, we conclude from the Euler equation that the magnetic field is frozen in the electron fluid. Since we are ignoring drift, we find that the magnetic field is frozen in the plasma.[105]

In those cases in which it is not possible to ignore the relative motion of the subsystems, it becomes necessary to use the two-fluid hydrodynamics of a charged plasma, occasionally even if the drift velocity is low. In a study of equilibrium configurations, for example, the plasma flow velocity is zero in general, while the drift velocity is fundamentally different from zero if a current is flowing. Consequently, ordinary MHD does not give a complete description of the equilibrium of a plasma with a current. In the problem of the dynamics of a pinch system with a macroscopic motion of the plasma, on the other hand, the MHD description may prove to be sufficient.

4.2. Hydromagnetic stability of pinches

An approach to the problem of the stability of pinch systems was formulated by Buneman.[106] That approach is not helpful, however, because of both some serious mathematical difficulties and the impossibility of simultaneously incorporating the entire set of events characteristic of the pinch effect. The kinetic approach to the problem of stability is usually taken in the approximation of a collisionless plasma.[23,80,107,108] This approximation works better in the early stage of the discharge, while the mean free path is greater than the thickness of the current shell. An important disadvantage of the collisionless approach is the model nature of the calculations, because of the arbitrariness in the choice of the distribution function. In the absence of collisions, the charge distribution is an arbitrary function of the integrals of motion.

In the stage of pronounced contraction of the pinch, the relaxation times are significantly shorter than the duration of the emission of the system, and it becomes possible to switch from a kinetic description of the plasma to a hydrodynamic description. The study of the stability of a plasma in a magnetic field in general and of nonrelativistic pinch systems in particular on the basis of the MHD equations has been pursued to the greatest extent.[11] An MHD analysis of the stability of a cylindrical column of a fully ionized plasma with a longitudinal current, distributed uniformly over the cross section, was first carried out by Trubnikov.[109] It was found that an instability with respect to axisymmetric perturbations which are periodic along the current direction occurs in this system, with the consequence that constrictions can form. In a series of studies, Leontovich and Shafranov showed[110–112] that instabilities in which the plasma column coils up or acquires constrictions can be countered by stabilizing facilities such as conducting walls and longitudinal magnetic fields.

The transition from cylindrical discharges to toroidal systems and systems with a more complex magnetic field configuration[12] forced the development of general methods for determining global stability criteria. An energy principle for studying the stability of a plasma in MHD was developed in 1955 and finally formulated in a paper by Bernstein *et al.*[13] If we apply this energy principle to a cylindrical pinch, we find the following stability condition[11] for stability with respect to azimuthally symmetric perturbations ($m = 0$):

$$-\frac{d \ln p}{d \ln r} < 4\gamma(2+\gamma\beta)^{-1}. \qquad (18)$$

Here γ is the adiabatic index, and $\beta = 8\pi p/H^2$ is the ratio of the plasma pressure to the pressure of the magnetic field of the current. For electrons, the index is $\gamma = 5/3$.

Outside the plasma the magnetic field of a straight current falls off with radius in proportion to r^{-1}, while the plasma density and the pressure fall off more rapidly than r^{-2}. These conclusions follow from confinement conditions (9). Consequently, β will vanish outside the plasma, and we find from the stability criterion that the pressure and the density should fall off no more rapidly than $r^{-2\gamma} = r^{-10/3}$ with distance from the axis. One example of an equilibrium configuration is the Bennett distribution, according to which the pressure and the density of the plasma fall off in proportion to r^{-4}. Since we have $4 > 2\gamma = 10/3$, a Bennett plasma is hydrodynamically unstable according to criterion (18). Azimuthally symmetric perturbations are excited spontaneously in such a plasma; these are radial contraction waves, whose growth can give rise to necks, constrictions, and striations. At one point, the effort to avoid the formation of constrictions in the plasma column served as one of the reasons for giving preference to the more complex toroidal geometry of a plasma. A study of the phenomena in the Z-pinch showed, however, that a plasma column which is severely distorted by constrictions may provide a neutron yield greater than that of a system constructed especially to avoid constrictions.[34] There have been reports that in experiments with dense pinches there are no MHD instabilities. The plasma turns out to be stable with respect to azimuthally symmetric ($m = 0$) perturbations[35] throughout the duration of the discharge.[36] The general reexamination of the approach to the fusion problem as a whole which has recently been undertaken cannot pass over the question of the stability of pinches.

We first note that the rapid decay of the density (e.g., the Bennett decay $n \sim r^{-4}$) with distance from the discharge axis quickly goes beyond the accuracy of the quasineutral theory. The theory becomes inapplicable with increasing distance from the discharge axis since the decrease in the density is accompanied by an increase in the Debye length of the plasma. An analysis based on two-fluid hydrodynamics without the quasineutrality condition shows that there are equilibrium configurations for which the charge density of one sign can fall off over radius in proportion to r^{-K}, where K is any number greater than 2. Equilibrium structures, however, are designed in such a way that if the exponent for the ions satisfies $K_i < 4$ the exponent for the electrons must satisfy $K_e > 4$, and vice versa. Is it sufficient that the density of the ions alone or that of the electrons alone fall off more slowly than $r^{-2\gamma}$ with radius? If the answer is yes, then there would be a region of hydromagnetic stability of a plasma column with respect to constrictions: $2 < K < 2\gamma = 10/3$.

Again there is the question of the range of applicability of the MHD approximation, in this case in the problem of the stability of pinch systems. What effects do we lose sight of when we ignore the relative motion of the electrons and ions associated with the current flow? In the linearized equations describing small oscillations, the plasma flow velocity is treated as a small quantity, and it is not clear in comparison with just what are we ignoring the drift velocity.

It is clear from these comments that there is a need for a complete analysis of the stability of pinch systems on the basis of the equations of the two-fluid MHD of a charged plasma.

4.3. Analysis of the stability of a pinch in two-fluid electromagnetic hydrodynamics

In the general case (with $\beta = v_0/c \sim 1$, and with a high current $I \sim I_A = m_e c^3/e = 17$ kA), the electrons appear as a gas of relativistic particles. There is accordingly, much practical interest in studying the stability on the basis of the equations of relativistic electromagnetic hydrodynamics. Such a study has not yet been carried out for pinch systems, and it will have to cope with some serious difficulties. An attempt to derive equations of relativistic electromagnetic hydrodynamics by the apparatus of the general theory of relativity was undertaken by Solov'ev.[114] So far, however, the research on the stability of pinches has been based on a nonrelativistic theory. The two-fluid hydrodynamics of a charged plasma was used to describe nonrelativistic pinch systems in Refs. 61, 64, 90, and 115–117.

Under the condition $v_0 \ll c$ we can ignore retardation effects, and we can assume that the field changes instantaneously to correspond to slow variations in the charge and current distributions. In this approximation the field potentials again satisfy Eqs. (3). To ignore the derivatives of the field potentials with respect to time implies ignoring radiation. The ineffectiveness of electron–ion collisions allows us to eliminate from consideration the basic cause of dissipation in a nonrelativistic plasma: its electrical resistance. If the mean free paths of the charges, l_α (with respect to collisions of electrons with electrons and of ions with ions), are short in comparison with the radius of the plasma column, r_0 $l_\alpha \ll r_0$, the viscosity and the thermal conductivity can also be ignored. These effects arise in a higher-order approximation in the parameters $l_\alpha/r_0 \ll 1$. Slow variations in the state of the current channel can thus be dealt with in the approximation of the hydrodynamics of two ideal charged fluids: electrons and ions.

If we are to be able to treat the electrons and ions as isolated subsystems which interact with each other only through the field of a collective interaction, we must ensure that the length of the discharge channel l and the characteristic frequency ω satisfy the conditions $v_0/\nu_\alpha \ll l \ll v_0/\nu_{ei}$, $\nu_\alpha \gg \omega \gg \nu_{ei}$, where ν_α is the rate of collisions of the charges of species α with each other, and ν_{ei} is the rate at which electrons collide with ions. In the case of a long current channel, $l \gg v_0/\nu_{ei}$, electron–ion collisions would lead to a stopping of the electrons over a distance of the order of v_0/ν_{ei}. In the absence of an electric field to sustain the current, the drift velocity would vanish over this distance, and the energy would be converted into heat. If, instead, an external electric field is applied, the drift velocity and the field distribution over distance will change in such a way that the accelerating force compensates for the friction force between electrons and ions. If, on the other hand, the length of the current channel is so large that the condition $l \gg v_0/\nu_{ei}\delta$ holds (where δ is the fraction of the energy which is transferred to an electron–ion collision), then the electron and ion temperatures will become equal over a distance $v_0/\nu_{ei}\delta$ after the establishment of a steady drift velocity. In this case, a Druyvesteyn-Davydov distribution[118–122] will be established over a distance $l \gg v_0/\nu_{ei}\delta$. The plasma becomes heated to the extent that the electron velocity dispersion becomes comparable to the drift velocity. If the mean free path of the charges remains short, the flow of such a plasma can be described by the equations of ordinary hydrodynamics, and the electrical conductivity plays an important role.

General methods for studying stability have not been developed in two-fluid hydrodynamics, and most of the work has focused on small oscillations. For a system which is uniform along the current direction and also in the azimuthal direction, perturbations of the type $f_\alpha \sim \exp \times [i(\omega t - kz - m\varphi)]$ are considered. We will not write out the equations here or reproduce the calculations, but we do wish to examine the physical side of the matter in more detail and to discuss the results. The detailed calculations can be found in Refs. 116 and 117.

An important aspect of a pinch is the strong magnetic field of the current itself. In cylindrical geometry, the magnetic lines of force run in the azimuthal direction. The asymmetry of the motion along and across the lines of force of the magnetic field of the current must be taken into account at the very outset. As long as the system can be described by classical mechanics, the asymmetry of the motion in the magnetic field will not affect the properties of the unperturbed state, since each of the subsystems is in thermodynamic equilibrium (see the footnote on p. 175 in the Russian original of Ref. 56). Small oscillations, however, move a subsystem away from its equilibrium state, and the effect of the magnetic field on the motion of the elements of the fluid becomes important in a study of stability.

In the case of a low current, the magnetic field is also weak, and the asymmetry disappears. As the current is increased, however, the motion in the r, z plane (perpendicular to the magnetic field) becomes "frozen." The charges are magnetically restrained in the case of a strong current, and their motion in the r, z plane can be ignored. This comment applies primarily to the electrons. For the field, it is a matter of being frozen into the plasma, while for the electrons it is a matter of the magnetic restraint by the strong magnetic field of the current.

In general, the oscillations of a pinch are interrelated oscillations of the densities, pressures, and velocities of the charged fluids and of the electromagnetic field. The oscillations are studied in various limiting cases which are amenable to analytic study.

4.3.1. Oscillations of the azimuthal current

For perturbations which are uniform in the azimuthal direction ($m = 0$), oscillations of the azimuthal components of the velocity (v_φ) and of the field (A_φ) are not coupled to oscillations in other properties. These are oscillations of the azimuthal current of the pinch, similar to the torsional oscillations of a string. In the ordinary MHD of an ideal plasma, there is no such motion at all ($v_\varphi = 0$). It arises in the case at hand as a result of the relative motion of the subsystems.

The spectrum of these oscillations can be calculated analytically for short waves ($kr_0 \gg 1$) and long waves ($kr_0 \ll 1$). Since the short waves are localized, we can seek the r dependence in the form[123] $\exp\{-i\int^r q(r)dr\}$. If the ions as a whole are at rest in the laboratory frame, the spectrum of oscillations in the azimuthal current is found to be

$$\omega = kv_0 \left(1 + \frac{\omega_e^2(r)}{c^2(q^2+k^2)}\right], \quad kr_0 \gg 1;$$

where ω_e is the electron plasma frequency. The phase velocity of these waves is always higher than the drift velocity, while the group velocity can be either positive or negative.

In the limit of long waves, $kr_0 \ll 1$, we are dealing with oscillations of the current channel as a whole. A particular feature of these oscillations is that the oscillations of the charges and the field are spatially separated. The charges are localized in the pinch, $r \lesssim r_0$, while the field makes its greatest contribution from the region $r_0 \ll r \ll 1/k$. The field gives the system a sort of elasticity. The electron component undergoes torsional oscillations as a system which is elastic as a whole. The oscillation spectrum in the case $kr_0 \ll 1$ depends only on the total number (N_α) of particles per unit length: $\omega = kv_0(1 + 2\Lambda e_e^2 N_e/m_e c^2)$, $kr_0 \ll 1$. Here $\Lambda = \ln(1/kr_0) \gg 1$ is a logarithmic factor. The group velocity and phase velocity of the long-wave oscillations of the azimuthal current agree within logarithmic accuracy.

On the whole, the oscillations of the azimuthal current which are not found in ordinary MHD do not lead to an instability of the system for either short or long waves. Long-wave oscillations in two-fluid hydrodynamics are generally stable.[116,117] This is true of all types of oscillations: of both the azimuthal current and other properties.

4.3.2. Stratification of a current channel into tubes and filaments

Let us consider oscillations in properties which are not related to v_φ or A_φ in the case $m = 0$. We assume that the oscillations are uniform along the current ($k = 0$), so that all quantities depend on r alone. Such perturbations do not disrupt the symmetry of the original equilibrium state, and we are dealing with the same situation as in a study of the field of a test filament placed on the current axis. The field of the filament and the oscillations which we are now considering satisfy the same equations.

In the short-wave limit ($qr_0 \gg 1$) the spectrum is[115]

$$\omega^2 = \omega_i^2 \left(q^2 r_D^2 + \frac{\Omega_e^2}{\omega_e^2} - \beta^2\right), \quad qr_0 \gg 1; \tag{19}$$

where r_D is the Debye length, $\beta = v_0/c$, ω_i and ω_e are the ion and electron plasma frequencies, and Ω is the electron Larmor frequency. Here we have the ratio $(\Omega_e/\omega_e)^2 \sim I\beta/I_A$, so that at a low current, $I \lesssim \beta I_A$ where $I_A = m_e c^3/e = 17$ kA is the Alfvén current, in the region $(\Omega_e/$

$\omega_e)^2 \lesssim \beta^2 \ll (r_D/r_0)^2$ we find from (19) a spectrum of sound waves $\omega = qs$, where $\omega = qs$, $s^2 = \omega_i^2 r_D^2$. At $T_i \gg T_e$ the sound velocity is of the order of the ion thermal velocity, while for $T_e \gg T_i$ it increases by a factor of $(T_e/T_i)^{1/2}$. At a high current, $I > \beta I_A$, where β^2 is lower than Ω_e^2/ω_e^2, a gap appears to the wave spectrum: $\omega = (\omega_0^2 + q^2 s^2)^{1/2}$, where $\omega_0^2 = \omega_i^2[(\Omega_e^2/\omega_e^2) - \beta^2]$, $\Omega_e/\omega_e > \beta$. This situation is radically different from ordinary MHD, where the radial oscillations under these conditions reduce to ion acoustic waves, $\omega^2 = q^2[s^2 + (H^2/4\pi\rho_0)]$.

The spectrum (19) indicates that the system is unstable with respect to oscillations with a wave number q satisfying the condition $r_0^{-1} \ll q < q_{cr}$, where $q_{cr} = (\beta^2 - \Omega_e^2/\omega_e^2)^{1/2} r_D^{-1}$, if $\beta > \Omega_e/\omega_e$, i.e., at currents $I \lesssim \beta I_A$. At $\beta > \Omega_e/\omega_e$, at wavelengths $q < q_{cr}$, the oscillation frequency becomes imaginary: $\omega = \pm i\omega_i r_D (q_{cr}^2 - q^2)^{1/2}$. Since we are assuming that the perturbations are localized, the conclusion regarding an instability holds at $q_{cr} r_0 \gg 1$. For a Boltzmann plasma this condition would give us $e^2 N_e \beta^2 \gg T$, where $T = \max(T_e, T_i)$. The quantity $e^2 N_e \beta^2$ is the energy of the magnetic compression per particle. Consequently, if this energy of magnetic compression is considerably greater than the thermal energy, the pinch system will clearly be unstable.

In the case $e^2 N_e \beta^2 \sim T$ the boundary wavelength q_{cr}^{-1} is of the order of the pinch radius, $q_{cr} r_0 \sim 1$. In this region the approximation of locality does not apply. Long-wave oscillations ($qr_0 \ll 1$), on the other hand, which represent a motion of the elements of the pinch as a whole, do not drive instabilities.[116] For this instability there is accordingly a threshold in the region $qr_0 \sim 1$. This instability induces cylindrically symmetric excitations with a wavelength $\lambda \sim r_D/\beta$, i.e., with the same scale dimension as that of the oscillations of the field from a test filament discussed in Subsection 3.1. The beam should decay into concentric tubes nested within each other and separated from each other by distances of the order of r_D/β.

These perturbations are associated with oscillations of the plasma in the radial direction: $v_r \neq 0$. Since this is a motion across the magnetic field of the current, however, these oscillations will become frozen with increasing current because of the increasing magnetic restraint of the electrons by the magnetic field of the current. With increasing current, the term $(\Omega_e/\omega_e)^2 \sim \beta I/I_A$ increases, and the instability disappears at $\Omega_e/\omega_e > \beta$. The suppression of an instability by the magnetic field of the current in the plasma is characteristic only of completely symmetric oscillations ($k = 0$, $m = 0$). For $m \neq 0$, we do not have this symmetry. Instabilities with $m > 0$, $k = 0$ would lead to a coiling of the column ($m = 1$) and a decay into filaments ($m \geqslant 2$).

We turn now to two-dimensional perturbations. We assume $k = 0$ but $m \neq 0$. There is an important difference between the cases $m = 0$ and $m \neq 0$ because of the possibility of magnetic restraint of the electrons. With increasing current, the magnetic field H_φ increases, with the result that perturbations of the velocities v_r and v_z decreases in proportion to ω/Ω_e, while v_φ does not change. This asymmetry is seen in its full glory as we go from $m = 0$ to $m > 0$.

For $m = 0$ a rotation through an arbitrary angle φ brings the system back into itself. Consequently, motion in the azimuthal direction in the case $m = 0$ does not change the state of the system, and oscillations which are azimuthally symmetric and uniform along the current are associated with motion only in the r, z plane, which is affected by the magnetic restraint on the electrons. With increasing current, the amplitudes of the oscillations in the r, z plane decrease, the degrees of freedom transverse with respect to the magnetic field "become frozen," and the result is a suppression of the instability. The oscillations with $m \neq 0$, on the other hand, are associated both with motion in the r, z plane and with azimuthal motion. The magnetic field of the current does not affect the azimuthal motion, so that with $m \neq 0$ one degree of freedom—the motion in the azimuthal direction—does not become frozen as the current is increased, and the instability is no longer suppressed as the current is raised.

At $k = 0$, $m \neq 0$, the wave spectrum is described by[116,117]

$$\frac{\omega^2}{\omega_i^2} = Q^2 r_D^2 - \beta^2 + \frac{\Omega_e^2}{\omega_e^2}\left(1 - \frac{m^2\Omega_e^2}{Q^2 r^2 \omega^2}\right)^{-1}, \tag{20}$$

where $Q^2 = q^2 + (m^2/r^2)$. The derivation of (20) made use of the approximation of locality, $Qr_0 \gg 1$, and also the inequalities $I \ll I_A/\beta$ and $\beta \ll 1$. With $m = 0$ and also at a very low current ($\Omega_e/\omega \ll 1$ expression (20) becomes (19). The magnetization of the electrons is determined by the factor Ω_e/ω. At $\Omega_e/\omega \gg 1$, the last term in (20) is negligibly small, and we find

$$\omega^2 = \omega_i^2 (Q^2 r_D^2 - \beta^2), \quad \Omega_e \gg \omega, \quad m \neq 0, \ Qr_0 \gg 1. \tag{21}$$

We see that with $m \neq 0$ the instability is not suppressed as the current is raised. At a high current the plasma should immediately decay into jets, skipping the stage of stratification into concentric tubes. At a low current $I < \beta I_A$, stratification of the pinch into tubes is possible if the threshold for the $m = 0$ instability is lower than that for $m \geqslant 1$ instabilities.

To study the behavior of the system near the instability threshold and thereby to determine the stability criterion, we need to abandon the approximation of locality. For $m \sim 1$ the threshold is in the region $qr_0 \sim 1$. A pinch system at $I \gg \beta I_A$ was studied near the stability threshold on the basis of an analogy with the Schrödinger equation. The stability of the system with respect to the types of excitation considered is determined by the value of the parameter $g = e^2 N_e \beta^2/T = (I\beta/I_A) \times (m_e c^2/T)$—the ratio of the magnetic-compression energy to the thermal energy of the system. For each given plasma density profile, calculations are carried out to find the corresponding monotonic sequence of numbers $g_1, g_2, g_3, \ldots$, which increase with increasing index m in proportion to m^2: $g_m \sim m^2$. If the parameter g turns out to be larger than any of the numbers in this sequence, say $g > gf_{m^*}$, then oscillations proportional to $e^{-im^*\varphi}$ are excited spontaneously in the system. If, on the other hand, we have $g < g_{m^*}$, oscillations with $m \gg m^*$ cannot be excited in the system. Consequently, at $g < g_1$ and at currents $I \gg \beta I_A$ the pinch system is stable.[116,117] This is the criterion for the stability of the pinch system with respect to perturbations with $k=0, m\neq 0$. Under the condition $(m_e c^2/T)\beta I/I_A > 1$, oscillations with wave numbers $m < m^*$ will arise spontaneously in the system:

$$m^* = \frac{m_e c^2}{T}\frac{\beta I}{I_A}. \tag{22}$$

The current channel can decay into m^* distinct filaments, each carrying a current $I_m \sim I_A T/\beta m_e c^2$. The dependence of the number of these filaments on the discharge parameters in (22) has been confirmed experimentally.[78] Segalov *et al.*[78] studied the break up of the current into distinct channels through a densitometer analysis of x-ray photographs. They points out that the number of channels is established in accordance with a balance between the magnetic and gas pressures.

A complex internal structure of the channel of a pinch can also be seen in the damage inflicted on electrodes and special screen-targets[74–77] placed in the path of the charged-particle streams. Nardi *et al.*[77] achieved a high spatial resolution. The nature of the damage to the targets is evidence that the distinct current channels condense into very thin filaments, in which the current is 10^3–10^4 times higher than in the diffuse background.[77] The great variety of tracks left on the targets—ranging from extremely small point defects $\sim 0.5\ \mu$m in size to craters $\sim 300\ \mu$m in size—is evidence that the interaction of the current filaments is complicated and that these filaments decay and form clusters. On the photographs of Ref. 77 one can see both isolated spots and craters in the form of concentric rings.

The maximum instability growth rate is $(\mathrm{Im}\,\omega)_{\max} = \omega_i\beta$, where ω_i is the ion plasma frequency, and $\beta = v_0/c$ is the ratio of the drift velocity to the velocity of light. If β is small, and the state of the pinch is changing relatively rapidly, an instability need not necessarily occur.

In ordinary MHD, in the approximation of an ideal plasma, there is no breakup of the current channel into filaments. In order to explain[79–81] the filaments observed experimentally[74–78] on the basis of MHD it is necessary to take into account dissipative processes which drive thermal instabilities. In two-fluid hydrodynamics, on the other hand, this effect arises even in the absence of dissipation, i.e., even if the higher-order terms in the expansions in the parameters $l_\alpha/r_0 \ll 1$ are ignored.

For a Bennett profile the parameter g is equal to 2, and the first two terms in the g_m series are 1.48 and 4.62. Bennett configurations are accordingly unstable with respect to a displacement of the column as a whole ($k=0$, $m=1$), but they are stable with respect to decay into filaments. With $k\neq 0$, the symmetry of the system is disrupted, but the oscillation spectrum should contain an acoustic branch, which converts into a static displacement in the limit $k\to 0$. This Goldstone mode corresponds to a smooth coiling of the pinch in a manner reminiscent of the oscillations of a stretched string.[172]

A study of perturbations which are not uniform along the current ($k\neq 0$) in two-fluid electromagnetic hydrodynamics is of considerable interest. In ordinary MHD, such perturbations lead to sausage instabilities. Perturbations with $k\neq 0$, $m=0$ were studied in the approximation of two-fluid electromagnetic gas dynamics by Solov'ev.[113,124] It was found that in the two-fluid theory the stability conditions differ from the hydromagnetic conditions (18). For plasma streams with a parabolic current distribution over the cross section, an instability capable of giving rise to constrictions arises after a threshold is reached, when there is a sufficiently large number (N) of charges per unit length of the cylinder. For a hydrogen plasma in the model of incompressible fluids, the stability condition reduces to the inequality[113] $N < 10^{14}\ \mathrm{cm}^{-1}$.

4.4. Macroscopic description of the stability of pinches

The situation regarding the stability of pinches in two-fluid electromagnetic hydrodynamics differs from that in ordinary MHD. Just which of these two types of hydrodynamics should we apply to a pinch system? To answer this question we recall how one makes the transition from the kinetics to macroscopic equations. A detailed derivation of macroscopic transport equations from the kinetic equations is given in the review by Braginskiĭ.[125]

If the system as a whole is in a state of local thermal equilibrium, one finds the equations of ordinary MHD from the kinetic equations. In a state of local thermal equilibrium, the electron subsystem is in equilibrium with the ion subsystem, so this case is the limit of a drift velocity which is small in comparison with the velocity spread of the charges. $v_0 \ll \Delta v$.

The transition to two-fluid hydrodynamics occurs in the opposite limit: $v_0 \gg \Delta v$. A high drift velocity has the consequence that the Coulomb cross sections for the electron–ion collisions become negligibly small in comparison with the electron–electron and ion-ion cross sections. Even before we make the transition to the macroscopic equations, we make use of the inequality $v_0 \gg \Delta v$, which allows us to treat the electron and ion subsystems as if they did not interact with each other by collisions. In the next step, if each subsystem can be assumed to be locally in equilibrium, we make the transition to the macroscopic equations and obtain two-fluid hydrodynamics.

Consequently, if the drift velocity in a given system is small in comparison with the velocity dispersion of the charges, $v_0 \ll \Delta v$, the macroscopic description of the pinch reduces to ordinary MHD. In the opposite limit, $v_0 \gg \Delta v$, we are dealing with two-fluid electromagnetic hydrodynamics.

It is not a simple matter to measure the parameter v_0, which is the drift velocity. It is a simpler matter to measure the total current $I = eN_e v_0$. On the other hand, the temperatures of the charges and thus the thermal velocities Δv are measured experimentally. Since the nature of the instabilities is different in ordinary hydrodynamics and two-fluid hydrodynamics, we can conclude that v_0 and Δv have mutual effects on each other. If constrictions are observed, we are dealing with the case $v_0 < \Delta v$, while if the current channel breaks up into filaments we are dealing with the case $v_0 > \Delta v$.

A question which remains open is how we are to describe a pinch system with $v_0 \sim \Delta v$ at the macroscopic level in the case in which the mean free path of the charges is small in comparison with the scale dimensions.

5. DYNAMICS OF THE CONTRACTION OF A PINCH SYSTEM

5.1. Adiabatic approximation

The list of questions pertinent to the dynamics of pinch systems is quite lengthy. Breakdown of the gas,[38] the formation of[37] and motion of a current shell[39–42] toward the axis, the entrainment of neutral gas[126] the decay of the current shell into distinct glowing fibers,[74–78,127–133] and the explosive nature of the electron emission[134–135] fall far short of exhausting the complete list of phenomena which determine the evolution of pinch systems. From the standpoint of electromagnetic collapse we are interested in a pinch in the stage of pronounced compression. Since the relaxation times are considerably shorter than the x-ray burst, it is natural to use the approximation of hydrodynamics. For systems with a high drift velocity, a two-fluid approach becomes necessary. For relativistic streams it is necessary to use the equations of relativistic electromagnetic two-fluid hydrodynamics. The work in this direction[136,137] is still in a formative stage. Quasisteady compressional plasma flows are presently being studied[138–145] in which the motion of the plasma is driven by the pressure exerted by the magnetic field. As a rule, the calculations use the simpler equations of ordinary hydrodynamics. The difference in the descriptions of steady-state compressional flows by the equations of ordinary and two-fluid MHD was analyzed by Morozov.[144] The difference is that Ohm's law in the ordinary hydrodynamics of an ideally conducting plasma ($\sigma = \infty$) is written $\mathbf{E} + (1/c)[\mathbf{vH}] = 0$, while in two-fluid hydrodynamics it is written $\mathbf{E} + (1,c)\ [\mathbf{v}_e \mathbf{H}] = 0$. Ordinary MHD ignores the Hall effect,[144] because of the relaxation $\mathbf{v}_e = \mathbf{v} - (\mathbf{j}/en)$. Nevertheless, the approximation of ordinary hydrodynamics is valid for studying the dynamics of a pinch system if the drift velocity is low in comparison with the hydrodynamic flow velocity of the plasma.

If there is no dissipation of energy, the compression of the current channel by magnetic forces is an adiabatic process. If we ignore Joule heating, the radiative energy loss, the energy released in fusion reactions, the thermal conductivity, and other energy-exchange processes, the heat transfer equation reduces to the conservation of entropy. In ordinary hydrodynamics, the entropy of an element of the fluid as a whole is conserved. In two-fluid hydrodynamics, with a slight friction between the electron and ion subsystems, the entropy of each subsystem is conserved.

Conservation of entropy has the consequence that the temperature increases during the compression. In the approximation of Boltzmann statistics, the part of the entropy per unit length of the pinch which depends on r_0 and T is $S = N \ln r_0^2 T^{3/2}$, where N is the number of particles per unit length. If N remains constant in the course of the compression, we have $r^2 T^{3/2} = \text{const}$ ($\gamma = 5/3$). Along with the increase in the temperature, there are increases in the pressure in the plasma and in that current I_{cr} in (14) at which the magnetic pressure becomes balanced by the pressure of the plasma and by the electrostatic energy. If we assume that the discharge current remains constant in the course of the compression, we find that the ratio I/I_{cr} decreases. In the plane of r_0/d and I/I_{cr} (Fig. 3), the state of the system is imaged by a dashed line which starts at point A and moves toward decreasing values of both coordinates. The solid line in Fig. 3 is a plot of the equilibrium radius r_0 as a function of the current I (the same as in Fig. 1, but in the plane of r_0/d and I/I_{cr}). The dashed line intersects the equilibrium curve at point B. Point B selects from all configurations in mechanical equilibrium the one for which entropy is conserved, i.e., for which the internal energy does not change.

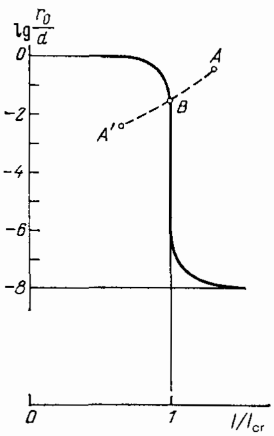

FIG. 3. Dashed lines AA' schematically shows the change in the state of a pinch system which is undergoing adiabatic oscillations near its equilibrium position. The solid line shows the equilibrium radius of the discharge as a function of the current (the same as in Fig. 1).

In the course of an adiabatic compression, the system acquires a kinetic energy of radial motion along region AB, and it moves past point B by inertia. In region BA' the system continues to contract opposing the direction of the resultant force, since now the gas-kinetics pressure outweighs the force of the magnetic compression. After its kinetic energy has been exhausted, the system stops at the turning point A' and then starts to expand. Under adiabatic conditions, the system would continue to execute radial oscillations between points A and A' around the equilibrium position B (Fig. 3).

In 1982, Felber[146] reproduced Kulikovskiĭ's 1957 study[147] which yielded an analytic self-similar solution of the MHD equations of an ideally conducting plasma describing the adiabatic oscillations of a current channel. In the solutions of this type, the contraction velocity is proportional to the radius: $v = \rho\, dr_0(t)/dt$, where $\rho = r/r_0(t)$ is the self-similar variable. The assumption of a uniform deformation makes it possible to satisfy the continuity equations, the conservation of entropy, and the freezing-in of the magnetic fluid for the density, the temperature, and the magnetic field, treated as arbitrary functions of the self-similar variable ρ. Of these three functions, two are determined after substitution into the equation of motion. One remains arbitrary. It is thus possible to see the evolution of the pinch under adiabatic conditions without any assumptions regarding its spatial structure. Because of the separation of variables, the self-similar dynamics of the pinch does not depend on its spatial structure, which is determined by the choice of an arbitrary function. In Felber's study this arbitrary function was cho-

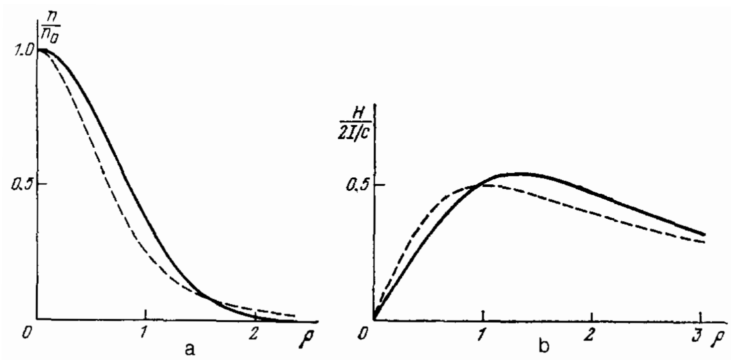

FIG. 4. Felber's self-similar solution[146] describing the oscillations of a pinch about its equilibrium state. The solid lines show the instantaneous distributions of the density (a) and of the magnetic field (b) along the radius. They differ from the Bennett equilibrium distributions, shown by the dashed lines.

sen in such a way that the temperature was uniform over the cross section. In that case the plasma density n^{Flbr} and the magnetic field in the pinch, H^{Flbr},

$$n^{\text{Flbr}} \sim \frac{N}{\pi r_0^2(t)} e^{-\rho^2}, \quad H^{\text{Flbr}} = \frac{2I}{cr_0(t)} \frac{1}{\rho} [1-(1+\rho^2)e^{-\rho^2}]^{1/2},$$

differ from the corresponding Bennett distributions

$$n^B \sim \frac{N}{\pi r_0^2} \left(1+\frac{r^2}{r_0^2}\right)^{-2}, \quad H^B = \frac{2I}{c} \frac{r}{r_0^2+r^2}$$

(Fig. 4). Although the temperature is uniform over the cross section, this self-similar solution is not a thermodynamic-equilibrium solution: Its parameters vary in time. A measure of the deviation from thermal equilibrium is the deviation of the drift velocity from a constant value over the radius (Fig. 5).

The possibility of constructing a self-similar solution[146,147] will facilitate a future numerical analysis of the dynamics of pinch systems in more complicated cases. In particular, the presence of an arbitrary function[59,147] makes it possible to incorporate bremsstrahlung, which leads to radiative collapse in the case of a strong current[173] (more on this below). Velikovich and Liberman are presently studying self-similar solutions of the equations of ideal MHD for the cylindrically symmetric problem in the classes of both subsonic and supersonic flows.[174]

Under adiabatic conditions, the oscillations of a pinch are undamped. The actual evolution of a system depends on dissipative factors: processes which supply energy to the system and by which it is removed from the system, by radiation, heat conduction, and other mechanisms. The heat transfer equations generally do not reduce to conservation of entropy. One should solve the more general equation

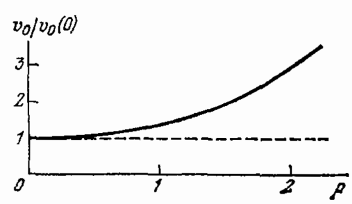

FIG. 5. Plasma executing adiabatic oscillations[146] and which is not in a slowly evolving equilibrium state. A measure of the deviation from equilibrium is the variation of the drift velocity along the radius (the solid line). For a Bennett distribution, this velocity would be constant: $v_0 = \text{const}$.

$$T\frac{dS}{dt} = W - \mathscr{Y}, \tag{23}$$

where W is the power of the heat evolution per unit length of the pinch, and $\mathscr{Y}$ is the rate at which energy is removed from the system, again per unit length. If the incoming energy is balanced by the outgoing energy, it follows that entropy is conserved. In two-fluid hydrodynamics it is necessary to solve equations of this sort for both subsystems, with allowance for the heat exchange between them.

5.2. Radiation from pinch systems

Back in 1956 Budker[7] pointed out the need to take into account the radiation caused by the exertion of forces on electrons by the field of the collective interaction. At high currents these forces increase rapidly in the course of the compression of the plasma.

Electromagnetic emission is usually the primary source of information about the processes which occur in the plasma of an electric discharge. The emission process itself has little effect on the motion of the radiating charge along its path. For the same reason, electromagnetic radiation is ignored in first approximation in a study of the plasma structure. In a pinch system, in contrast, the emission intensity increases rapidly in the course of the compression, and it begins to have a strong effect on the energy balance. The emission becomes not simply a passive consequence of processes which are occurring in the current channel but instead itself plays a governing role in the evolution of the pinch system.

It has usually been assumed that the primary emission mechanism is the bremsstrahlung in collisions of electrons with ions. Kogan and Migdal[148] calculated the bremsstrahlung intensity. While the number of particles per unit length of the pinch does not change in the course of the compression, the intensity of the collisional radiation is proportional to[148] $T^{1/2}/r_0^2$, $v_0 \ll \Delta v$. During adiabatic compression we have $r_0^2 T^{3/2} = \text{const}$, and with decreasing radius the intensity of the bremsstrahlung increases in proportion to $r_0^{-8/3}$. If the drift velocity is high, $v_0 \gg \Delta v$, the relative velocities of the electrons and ions in collisions (of the order of v_0) are considerably higher than the thermal velocities. The bremsstrahlung intensity turns out to be significantly lower than in the case $v_0 \ll \Delta v$. If the discharge radius r_0 or the electron Larmor radius r_H in the magnetic field of the current is shorter than the mean free path of the electrons with respect to collisions with ions, l_{ei}, the collisional radiation becomes less effective than the collisionless radiation.[149] If $r_0 \ll r_H$ ($I/I_A \ll v_{Te}/c$, where v_{Te} is the electron thermal velocity), the emission by electrons which is caused by acceleration in the field of the collective interaction which prevents a radial dispersal of the charges is predominant. In the opposite limit, $r_0 \gg r_H$, the synchrotron radiation of electrons in the magnetic field of the current is predominant. The scale energy of the emitted photons in the case of the synchrotron radiation is

$$\hbar\omega \sim \hbar\Omega\gamma^2 \sim \frac{c\hbar}{r_0} \frac{I}{I_A} \gamma^2,$$

where $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic factor, and $\beta = v_0/c$. In the case of compression to the point of degeneracy (the maximum possible degree of compression at the given current), we have the estimate[117]

$$r_0 \sim \frac{\hbar}{m_e c} \frac{1}{\gamma^2\beta^2\alpha^{1/2}} \frac{I_A}{I}, \qquad I \gg \frac{I_A}{\beta\gamma},$$

where $\alpha = e^2/\hbar c = 1/137$ is the fine-structure constant. In the state of maximum compression, with $\hbar\omega_{max} \sim \alpha^{1/2} m_e c^2 \beta^2 \gamma^4 (I/I_A)^2$, the photon energy increases in proportion to the square of the current. With $\beta \sim 1, \gamma \sim 1, I \sim I_A$ the energy $\hbar\omega_{max}$ is 50 keV: a tenth of the rest energy of an electron. As we go to currents $I \gg I_A$ or values $\gamma \gg 1$, the photon energy increases very rapidly and goes into the γ ray region. If it were possible to show by independent experiments that upon the appearance of hard x radiation a pinch was compressed to the point of electron degeneracy, one could confidently assert that the hard x radiation appears in quantum transitions of electrons between Landau levels in the magnetic field of the current.[69] The energy levels are separated by great distances at the maximum compression of the pinch.

The intensity of the emission from a pinch can be calculated most simply in the case in which the emitted photons escape freely from the plasma without any significant absorption by other charges. How does the transparency of a pinch plasma depend on the current? At very strong currents the plasma is transparent to the radiation, so that the frequency $\omega \sim \Omega$ increases more rapidly than the electron plasma frequency with increasing current. At $(\omega/\omega_e)^2 \sim (\Omega/\omega_e)^2 \sim \beta I/I_A \gg 1$ we have a region of transparency.

At a very low current, the plasma is again transparent. At $I \ll \beta I_A$, the skin depth for the field penetration into the plasma, $\delta \sim c/(\omega\sigma)^{1/2}$ ($\sigma \sim e^2 n/m_e\omega$ is the electrical conductivity at the frequency ω), is considerably larger than the pinch radius r_0: $(\delta/r_0)^2 \sim I_A\beta/I \gg 1$. In the limiting cases $I \ll I_A\beta$ and $I \gg I_A/\beta$ the intensity of the emission from a pinch can thus be calculated as simply the sum of the events of emission by individual electrons. In the intermediate region, $I_A\beta \lesssim I \lesssim I_A/\beta$, the plasma is not transparent to its own emission, and in this region it emits as a blackbody.

5.3. Radiative contraction

The balance between the energy evolved in the plasma by the strong current and that radiated off was studied in 1957 by Pease[17] and Braginskiĭ.[18] This question was later taken up by several other investigators.[19,32,98,117,150–152]

The evolution of a pinch system is governed by energy exchange as well as the equalization of mechanical forces. If the heat evolution exceeds the outward removal of energy, the current channel will expand; in the opposite case, the channel will contract. If Joule heating is the primary mechanism for heat evolution, and if bremsstrahlung in electron–ion collisions may be regarded as the predominant mechanism for the removal of energy, we can equate the two and find the temperature at which there is an energy balance: $T_{cr} \sim 2\beta L^{1/2}$ MeV, where L, the Coulomb logarithm, is a numerical factor of the order of ten. If we substitute this value of the temperature into Bennett condition (13) for mechanical equilibrium, we can find the current at which there is a balance of both forces and energies in the pinch system. This value of the current is independent of the number of particles (N_e), the drift velocity, and the radius r_0; for hydrogen, its numerical value is $I_{PB} \approx 1.7$ MA. The current at which the radiation removes precisely the amount of energy as is dissipated in the form of heat in the system is called the "Pease-Braginskiĭ current." If the current in the pinch is higher than the Pease-Braginskiĭ current, $I > I_{PB}$, the system will continuously lose energy and will contract without bound. The contraction of a system due to radiative energy loss is called "radiative collapse."

If the system is initially not in a state of mechanical equilibrium, its evolution will be determined by both factors: the compression by the excess magnetic forces and energy exchange. When the heat transfer is unimportant, we find adiabatic oscillations of the pinch around its equilibrium position (Fig. 3); an example is the self-similar solution of Refs. 146 and 147. In general, on the other hand, one should solve the heat transfer equation (23) instead of assuming entropy conservation. From the standpoint of collapse, we are interested in the case $\mathscr{Y} > W$, in which the radiative loss is dominant. In this case the system loses energy; the oscillations around the equilibrium position become damped; and the equilibrium radius of the pinch decreases.[173] If on the otherhand condition $\mathscr{Y} < W$ holds, the system will instead expand.

We thus find the following picture of the dynamic compression of the plasma system. At some time which we arbitrarily take as the initial time, the state of the system is imaged by point A in Fig. 3 (or Fig. 6), which does not lie on the force-equilibrium line. If conditions (8) hold, the system is self-contracting. If the system is to be able to contract to a great extent, inequalities (9) must hold with some margin. During compression, the temperatures rise, and as soon as T_e or T_i exceeds the value corresponding to the confinement energy E_α the compression process is disrupted. When the system loses more energy by radiation than is dissipated in it as heat, the temperature increase is bounded; the effect is to promote a collapse.

Let us assume that the conditions at the initial time are such that dissipative processes can be ignored altogether, and the initial stage is an adiabatic process. If, already in the adiabatic stage, one of the temperatures exceeds the corresponding confinement energy, the compression will cease even before it reaches the equilibrium line, and then the plasma will flow out in the radial direction. The situation is different if inequalities (9) hold with an ample margin. In this case the magnetic forces compress the system so strongly that mechanisms for a dissipation of the current into heat and radiative losses come into play, and the confinement conditions are not yet disrupted. Depending on the current, there are two possibilities here.

If the current is below the Pease-Braginskiĭ current, $I < I_{\text{PB}}$, the deviation from adiabatic behavior occurs in the direction of an even greater heating of the plasma. The temperature ultimately exceeds the confinement energy, and the compression gives way to an expansion. This situation corresponds to the line AC in Fig. 6.

When the current exceeds the Pease-Braginskiĭ current, $I > I_{\text{PB}}$, the dominant removal of energy by radiation causes cooling of the plasma. The radiative cooling initially slows the temperature rise during the compression (in comparison with an adiabatic process), and then the heating comes to a complete halt. If the conditions for confinement are satisfied in the stage of maximum heating, they will not be violated during a subsequent compression. The radiation will reduce the energy of the radial oscillations of the system. The compression process, combined with the radiative cooling proceeds until the quantum properties of electrons come into play. In Fig. 6, this process corresponds to the line AD (with the downward arrow).

Two successive stages in the contraction of a micropinch are observed experimentally.[97,98,152–155] In the first stage, constrictions form with the parameter values $r_0 \sim 10^{-2}$ cm, $n_e \sim 10^{20}$–10^{21} cm^{-3}, and $T_e \sim 50$ eV. After 20–30 ns, an intense pulse of x-ray emission occurs, at a time corresponding, within experimental error, to the time of the second contraction, characterized by the parameter values $r_0 \lesssim 10^{-4}$ cm, $n_e \sim 10^{24}$ cm^{-3} and $T_e \sim 1$ keV. If the radiative collapse continues after the second contraction, at a radial velocity $\sim 10^6$ cm/s the collapse from $r_0 = 10^{-4}$ cm to the

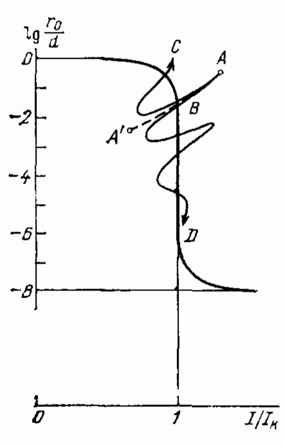

FIG. 6. The deviations from an adiabatic situation increase as the contraction proceeds. When dissipative processes are taken into account, it is concluded that the system not only executes oscillations but also either expands or contracts, depending on the current. Specifically, for $I < I_{\text{PB}}$ the pinch expands, while for $I > I_{\text{PB}}$ radiation causes the system to lose energy, resulting in a contraction. The oscillations around the equilibrium line decay. The behavior of the system in the course of the radiative collapse is shown schematically by line AD.

point of electron degeneracy will occur in 10^{-10} s—beyond the resolving time of the apparatus. For this reason, it would still be premature to assert flatly that the x-ray burst, the emission spectrum of multiply charged ions, the collectively accelerated charges, etc., appear when the plasma is compressed to a condensed state.

The very fact that in the experiments the second contraction is stronger than the first lends credence to the picture of radiative contraction drawn above. These results constitute experimental proof that the conditions for radiative collapse are realized in a micropinch. If, after the second contraction, the confinement conditions are not disrupted, the collapse should continue to the point of electron degeneracy. After this point is reached, the system will cool off by radiation, and the radiation burst will decay. This decay of the burst of x-ray emission of a micropinch corresponds to the process of radiative cooling.[32]

The Pease-Braginskiĭ current is affected by a variety of factors. In particular, the radiative loss turns out to be considerably greater if the system contains heavy ions which emit well.[152] Numerical calculations[151,156] show that a xenon admixture of only a few percent will greatly shift the energy balance and reduce the Pease-Braginskiĭ current. The streams of accelerated electrons and ions which form in the pinch systems also carry off a significant fraction of the energy of this system[31] and thereby promote a collapse. It is an exceedingly complicated problem to calculate exactly the Pease-Braginskiĭ current for a specific system. The difficulties are aggravated by the circumstance that the plasma is optically dense at precisely those currents in which we are primarily interested: $I_A\beta \lesssim I \lesssim I_A/\beta$. Estimates[32,117] based on trapping of the radiation and a possibly induced nature of the emission of photons from a thin surface layer of the plasma show that the Pease-Braginskiĭ current is in the range of several tens of kiloamperes, i.e., lower than in experiments with micropinches in vacuum diodes, not to mention the plasma focus.

One difficulty on the way to realization of a profound electromagnetic collapse is that the ion subsystem gives up energy more slowly than does the electron subsystem. Because of the large difference between the masses, the ions also radiate more weakly, and the exchange of energy with electrons is suppressed. Consequently, if the ions are protons or deuterons, there will be no other energy-removal mechanisms, and the adiabatic nature of the contraction of the ion subsystem will prevent a collapse. If the ions are instead ions of heavy elements, with a complex internal electron structure, there will be yet another mechanism for the removal of energy from ions: line emission.[152] During the contraction and heating, electron shells of heavy ions will be excited. Transitions of electrons will then occur to lower-lying energy states, accompanied by emission of line radiation. This radiation carries off a significant fraction of the energy of the ion subsystem and promotes a collapse. Here we are seeing a difference between plasma-focus systems and low-inductance vacuum diodes. In a focus system, one uses deuterium, while in a vacuum spark one uses the ions of the anode material. The line emission in the latter case may be more than one hundred times greater than the bremsstrahlung. The Pease-Braginskiĭ current is lowered, and a collapse becomes possible at a current of the order of 100 kA.[152]

In 1979, Winterberg[53] suggested producing an ultradense electron-positron plasma by placing electron and positron storage rings in a common magnetic field. The electrons and positrons which are moving opposite to each other form a common ring, which is compressed by radiative collapse to a very small radius. Winterberg showed[53] that if we go to an ultrarelativistic drift velocity $\gamma = [1 - (v_0^2/c^2)]^{-1/2} \gg 1$ the radiative loss will exceed the increase in the energy of thermal motion in collisions if the inequality $I > 9I_A L/4\gamma^2$ holds, where L is the Coulomb logarithm, and $I_A = 17$ kA is the Alfvén current. At ultrarelativistic drift velocities, the radiative collapse may therefore occur at a low current. Assuming $9L/4 \sim 10^2$, i.e., $r_0 \sim 1$ with $\gamma = 10^2$, we find that the minimum current for collapse is 170 A, and at $\gamma = 3\cdot 10^3$ (a typical value for storage rings) we would have $I_{\min} = 0.17$ A.

5.4. Acceleration of charges by pinches

In 1960 Plyutto[157] observed that a high-current spark is a source of fast charged particles. Results obtained on the energy and angular distributions of neutrons provided evidence that directed streams of deuterons form in pinch systems.[9,158] Observations revealed electrons accelerated to energies exceeding the source voltage by one or two orders of magnitude.[158] How do the strong electric fields capable of accelerating charges to this extent arise?

The answer to this question was found a long time ago. The magnetic field which compresses the plasma in the pinch effect acts as a cylindrical piston converging on the axis. For the charges in the plasma this piston is a moving magnetic wall, in collisions with which the charges acquire energy. Until the charge acquires a high energy, a collision with the magnetic wall terminates with a reflection of the charge back toward the axis. After a large number of reflections, the energy of the charge can ultimately exceed the energy height of the magnetic wall, and the latter becomes transparent to the given charge. The accelerated particle will escape from the compression region. The distributions of the accelerated charges in emission angle and energy depend on the geometry of the magnetic field which is compressing the plasma. In the direction along the axis of the discharge, the charges can escape freely, while in the radial direction the energy height of the magnetic wall is of the order of $E_w \sim e^2 N_e \beta^2 \ln(l/r_0) = m_e c^2 (I\beta/I_A) \ln(l/r_0)$, where l/r_0 is the ratio of the length of the discharge to the radius in the zone of maximum compression. If the logarithm is not very large, $\ln(l/r_0) \sim 3-4$, we have $E_w \sim 100 I\beta$ eV, where I is the current in amperes, and $\beta = v_0/c$ is the ratio of the drift velocity to the velocity of light. The energy E_w is the maximum energy to which the charges can be accelerated by the cylindrical magnetic pistons. The charge acceleration process is shown schematically in Fig. 7.

If the mean free path of a charge is large in comparison with the size of the compression region, the acceleration process will play out in its pure form. The charges will be reflect-

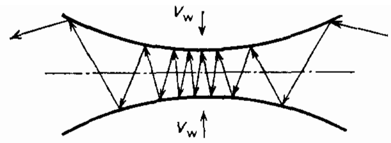

FIG. 7. Magnetic field of the current which is compressing the discharge. It is a cylindrical piston which is convering on the axis. This magnetic field is a moving magnetic wall in collisions with which the charges acquire energy and are accelerated. The motion of the charges in the course of the acceleration is shown schematically here.

ed from the converging magnetic wall until their motion becomes parallel to the current or their energy exceeds a limiting value for the given direction. If the mean free path of the charges is instead small in comparison with the compression region, the energy acquired in collisions with the magnetic wall is expended on heating the plasma. Only the charges of high energy (runaway charges) will be capable of escaping from the compression region.

Although this description of the acceleration of charges in terms of collisions with a magnetic wall or piston is graphic, it is not rigorously correct. Everything said above could be reformulated in terms of an induction electric field, whose component E_z arises as a result of motion of the magnetic field H_φ in the radial direction. It can be shown rigorously[158] that the energy acquired by a charge in collisions with a magnetic wall is precisely equal to the energy acquired in motion in an electric field which arises as a result of motion of a magnetic field. In adiabatic self-similar oscillations of a pinch, the induction electric field is[146] $E_z(r,t) = -(2I/c^2 r_0)\dot{r}_0 R(r/r_0)$, where $R(\rho) = [1 - (1+\rho^2)e^{-\rho^2}]^{1/2}$ is a structure factor, and $r_0(t)$ is the plasma radius, which oscillates with time. The energy acquired by a charge as it traverses the length of the pinch, l, can be estimated to be $E \sim m_e c^2 (I/I_A)(\dot{r}_0/c) l/r_0$. At the ratio $r_0/c \sim 10^{-4}$ and at a current of 150 kA, the ratio of the length of the plasma to the radius would have to be $l/r_0 \sim 10^3$ for acceleration to an energy of 500 keV.

In the early research on pinches[8,9,128,158] it seemed unlikely that the discharge channel could contract to a fraction of a millimeter and still be contracting at a high velocity. In 1925 Kapitsa found that overvoltages arise during a sudden discharge: electric fields exceeding the potential difference of the source.[160] In 1958 Trubnikov[161] explained the acceleration of ions in terms of fields which result from the rupture of a current channel as a result of the onset of sausage instabilities. Fukai and Clothiaux[162] studied the acceleration of charges as a result of the cutoff of a current with reference to the micropinch in a vacuum diode.

In 1975, Cilliers, Datla and Griem[99] pointed out that the current is actually not cut off. There is a slight decrease in the current (by up to 30%) at the time at which the discharge is a source of accelerated charges.

The discovery of the micropinch[24] and subsequent studies of it[31,32,95–100,153,154] showed that the radius of the current channel does not exceed a few microns at the stage of maximum contraction. The time resolution and spatial resolution of even the latest diagnostic equipment cannot keep up with the evolution of the pinch at radii in the submicron region. In a plasma focus one also observes a complex internal structure: discrete, highly contracted current channels.[75–77,130] The formation of a micropinch region has recently been observed in a Z-pinch also.[155] The fact that the current is not cut off completely and also the fact that the contraction occurs at least down to micron radii (if not for the entire channel, at least for some of its filaments) force a reexamination of the mechanism of the acceleration of charges by an induction electric field which arises as the magnetic field of the current contracts to the axis.

The overall picture of emission phenomena in a pinch can be explained in a natural and elegant manner in terms of electromagnetic collapse. As a plasma is compressed to a very small radius in the course of electromagnetic collapse, charges are accelerated by the magnetic field which is converging on the axis. At the same time, there is a heating of the charges, which leads to the appearance of fusion neutrons. The fusion origin of the neutrons is almost entirely beyond doubt. Their source is the plasma channel itself, not exclusively (or even predominantly) collisions of accelerated deuterons with the cathode. For a quarter of a century there has been a dispute over whether the deuterons participating in the reaction $^2D + {}^2D \rightarrow {}^3He + n$ have commensurable energies (the case of a reactor) or whether the energy of the nuclei of one group is significantly lower than that of another (the beam-target case).[163] The acceleration of deuterons along the axis of the filament means that we are actually dealing with a "moving reactor." In other words, the deuterons participating in the nuclear reactions are heated to a great extent and at the same time in motion as a whole. The mechanisms which lead to the special conditions for nuclear fusion reactions in a dense plasma focus have been analyzed by Witalis.[164,165]

A significant fraction of the x-ray emission which arises in the course of the collapse comes from electrons subject to the electromagnetic field of a collective interaction. At a high current, this interaction reduces to synchrotron radiation from electrons in the magnetic field of the pinch itself. During the contraction of the filament, the distances between Landau levels increase, and as the channel becomes more highly compressed the photons emitted from it become harder. Upon compression to the point of electron degeneracy, the energy of the emitted photons reaches the region of hard x rays. Experimentally, the harder photons are emitted from a spatial region which is smaller than that from which low-energy photons are emitted.[52,166] The source of the harder radiation is in a sense submerged in a region which is emitting softer photons. This picture agrees with the mechanism which has been discussed for the x-ray emission, according to which the energy of the x rays increases in the course of the compression.

6. CONCLUSION

At the present diagnostic level we cannot tell whether the radiative collapse comes to a halt after the second contraction of the pinch, at micron or submicron dimensions, or the process continues to the point of electron degeneracy. There is the point of view that the pinch plasma may go into a turbulent state as instabilities develop. Questions of turbulence and anomalous dissipative phenomena are being studied intensely.[167,168] Several pinch experiments exhibit stability throughout the evolution of the highly compressed state.[35,36] So far, we see no fundamental reason why a pinch cannot be compressed to the state of condensed matter. However, as we move to progressively smaller radii we may run into some obstacles of which we are presently unaware. Our task is to identify and overcome the obstacles to compression at each step. We will then ultimately be able to achieve electromagnetic collapse to the point of electron degeneracy in a pinch system. What we need most for success in this effort are a refinement of the conventional methods of plasma diagnostics and the development of fundamentally new methods[169] which are capable of measuring small radii of discharge channels in subnanosecond times. Ya. B. Zel'dovich has offered the interesting suggestion of determining the radius of a collapsing pinch from the scattering of a laser beam.

The achievement of electromagnetic collapse will bring us substantially closer to a solution of the fusion problem. However, the fusion problem does to exhaust the practical significance of the achievement of collapse. There are a wide variety of applications: to the collective acceleration of heavy ions, the production of intense coherent jets of charged and neutral matter, and the development of intense sources of induced radiation over a broad spectrum, from microwaves and visible light to hard x rays and γ rays. An understanding of the processes which occur in directed beams of charged particles will make it easier to solve the problem of transmitting energy over large distances by relativistic electron beams, will make it possible to construct better sources of heavy ions,[170] and will make it possible to develop x-ray sources which operate on the principle of free-electron lasers.[171]

Beyond its practical applications, the achievement of collapse is also general scientific importance. This is a unique possibility for producing and studying matter in a state which occurs in nature if at all, then only in the deep interiors of stars. The problem of electromagnetic collapse combines various fields of modern physics: from plasma physics to solid state physics and from emission phenomena to quantum electrodynamics. We have already traveled a long road, but we still have a long way to go. Our purpose in this paper has been to call the attention of physicists in various fields to the problem of electromagnetic collapse.

I wish to thank Ya. B. Zel'dovich for interest in this problem and for suggesting this review. I also thank L. P. Pitaevskiĭ, A. F. Andreev, and E. D. Korop for discussions of these questions.

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