S. P. Mikheev and A. Yu. Smirnov. *Neutrino oscillations in a medium with variable density.* Neutrino oscillations[1] are modified in a medium.[2] In the case when two neutrinos are mixed $\mathbf{v}_\alpha = (\nu_e, \nu_\mu)$ the Schroedinger equations have the form $i\dot{\mathbf{v}}_\alpha = \hat{M}\mathbf{v}_\alpha$, and in addition the diagonal elements of the evolution matrix $\hat{M}$ contain terms which are proportional to the density of the matter ρ. The main concepts are the mixing angle θ_m and the characteristic states of the neutrino in the medium ν_{im} $(i = 1,2)$. ν_{im} are defined as the states which diagonalize the evolution matrix: $\mathbf{v}_\alpha = \hat{S}\mathbf{v}_m$, $\hat{S}^{-1}\hat{M}\hat{S} = \hat{M}^{\rm diag}$. θ_m is the angle in the unitary matrix $\hat{S}$, relating ν_{1m}, ν_{2m} and ν_e, ν_μ. The medium alters the mixing of the neutrino. For $\rho = 0$ ν_{im} differ from ν_i—the states with definite masses and $\theta_i = \theta(\rho(t)) \neq \theta_m$.

The effect of the medium has a resonant character.[3] The dependence $\sin^2 2\theta_m$ on the density or neutrino energy is a Breit-Wigner peak, whose maximum $\sin^2 2\theta_m = 1$ is reached at $l_\nu = l_0 \cos 2\theta$ (the condition of resonance), l_0 is the characteristic length for the matter $(l_0 \propto (G_F \rho)^{-1})$, l_ν is the length of the oscillations in the vacuum $(l_\nu = 4\pi E/\Delta m^2)$, and G_F is the Fermi constant. The half-width of the peak is given by $\Delta\rho_R = \rho_R \sin 2\theta$ (or $\Delta E_R = E_R \sin 2\theta$), where ρ_R (E_R) is the resonance density (energy), for which the resonance condition holds. The appearance of the resonance is determined by the fact that the mixed neutrinos are essentially a system of weakly coupled oscillators (the stiffness of the coupling is determined by θ). The medium alters differently the characteristic frequencies of the oscillators, and at resonance these frequencies are equal. The effective density-dependent masses of ν_e and ν_μ coincide at resonance.[4] The manifestation of the resonance depends on the nature of the variation of the density of the medium.

The angle θ_m determines the flavor (i.e., the ν_e and ν_μ composition) of the characteristic states ν_{im}, θ_m and therefore the flavor depends on the density. As ρ decreases from $\rho \gg \rho_R$ to $\rho \ll \rho_R$ the angle θ_m decreases from $\pi/2$ to θ. Correspondingly (for small θ) the flavor ν_{im} changes almost completely. If $\nu_{1m}(\rho \gg \rho_R) = \nu_\mu$, then $\nu_{1m}(\rho \ll \rho_R) = \nu_e$. This property is the basis of the neutrino transformations under discussion.

The dynamics of the oscillations follows from the equations for the characteristic states: $i\dot{\mathbf{v}}_m = (\hat{M}^{\rm diag} + \dot{\theta}_m \sigma_2)\mathbf{v}_m$, σ_2 is a Pauli matrix.[5,6] The off-diagonal terms, describing the transitions $\nu_{1m} \leftrightarrow \nu_{2m} - \dot{\theta}_m$, are proportional to $\dot{\rho}$. The picture of the oscillations is as follows, depending on the rate of change of the density.

In a medium with a constant density $(\dot{\theta}_m = 0)$ ν_{im} evolve independently. The admixtures of ν_{im} in the neutrino state do not change with time, the flavor ν_{im} is conserved. Based on this, the general character of the oscillations turns out to be the same as in a vacuum. In a medium with $\rho = \rho_R$ the depth of the oscillations with arbitrarily small θ is maximum, $A_P = 1$. If neutrinos with a continuous energy spectrum are generated, then in the energy range $E_R - \Delta E_R \div E_R + \Delta E_R$ the oscillations will be resonantly amplified.[3]

In a medium with a slowly varying density, the adiabatic regime is realized.[3,5–10] The condition of adiabaticity $|\dot{\theta}_m|^2 \ll |M_1^{\rm diad} - M_2^{\rm diad}|^2 = 4\pi^2/l_m^2$ (l_m is the length of the oscillations in matter), which in the region of the resonance assumes the form $2\Delta r_R > l_m$ (Δr_R is the spatial width of the resonance layer[3]), means that ν_{im} evolve independently, and the transitions $\nu_{1m} \leftrightarrow \nu_{2m}$ can be neglected. Admixtures of ν_{im} in a given neutrino state are conserved and equal the admixtures at the moment the neutrinos are generated; the flavor ν_{im} changes in accordance with the change in the density. The probability of observing at time t a neutrino of the starting type is a quasiperiodic function, oscillating around the average value $\bar{P}(t)$ with a depth $A_P(t)$. At the same time, $\bar{P}$ and A_P are universal functions of one variable $n = (\rho - \rho_R)/\Delta\rho_R$ and one parameter—the values of n at the initial moment (Fig. 1), the values of $\bar{P}$ and A_P do not

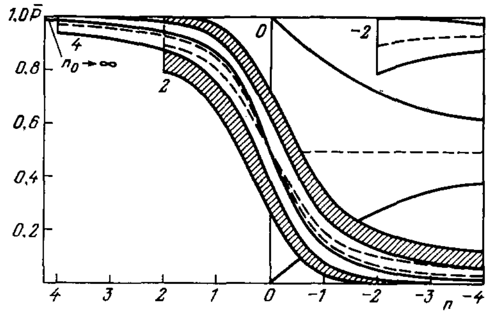

FIG. 1. Dependence of the average probability $\bar{P}$ and depth of oscillations (A_P) on $n = (\rho - \rho_R)/\Delta\rho_R$ for different initial conditions n_0 (numbers on the curves) in the adiabatic regime.

depend on the distribution $\rho(r)$.[5,6,8] The larger the difference between the initial and final densities and the smaller the vacuum mixing, the stronger is the change in the flavors ν_{im} and the more complete is the transition of one type of neutrino into another.[3,7,8] As n_0 increases the depth of the oscillations decreases:

$$A_P \backsim (n_0^2 + 1)^{-1/2},$$

and in the limit $n_0 \to \infty$ the propagation of the neutrino acquires the character of a nonoscillating transformation of one type of neutrino into another.[7,8] In this case, $\nu(t)$ virtually coincides with one of the ν_{im}.

If the density varies rapidly, so that the adiabatic condition is not satisfied, the transitions $\nu_{im} \leftrightarrow \nu_{2m}$ become significant. In the process of propagation both the admixtures of ν_{im} in the neutrino state and the flavor of ν_{im} themselves change. A_P and $\bar{P}$ depend on the density distribution and the phase of the oscillations.

On scales over which the resonance effects of matter are manifested $l_\rho \gtrsim l_0 \rho \approx m_N/G_F \approx 3.5 \cdot 10^9$ g/cm^2, it turns out that the spreading of the wave packets of ν_{im} is significant. The difference in the group velocities of ν_{im} depends on the density and, vanishing, it changes sign for $\rho = \rho_R/\cos^2 2\theta$. In the adiabatic regime the effect (P), summed over the packets, is identical to the average effect neglecting spreading.[5,6]

The effects discussed above are applicable in the following region: the sun,[3,4,5,6,9–12] the earth,[1,3,8] the envelopes and nuclei of collapsing stars,[7,8] and the early universe.[8]

1. The sun. There exists a region of $\sin^2 2\theta$ and Δm^2 (Fig. 2) in which for the standard solar model 2- to 4-fold suppression of the rate of ν capture in the Cl–Ar experiment is achieved. In this case, there arises a definite distortion in the form of the spectrum of boron neutrinos. This can explain Davis' results. For the Ga–Ge experiment the predictions fall in a wide range extending from the standard values up to values which are suppressed by a factor of 10 compared with the standard values. Comparison of the data from the Cl–Ar and Ga–Ge experiments, as well as the measurement of the form of the boron neutrino spectrum by direct electronic methods will make it possible to establish whether or not resonance transformation of neutrinos occurs in the sun and to determine virtually uniquely Δm^2 and $\sin^2 2\theta$.

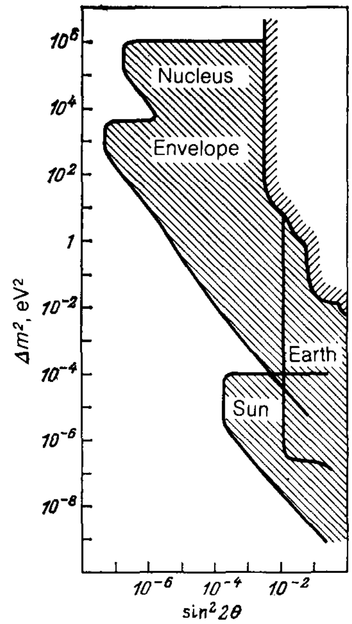

FIG. 2. Region of values of Δm^2 and $\sin^2 2\theta$ in which appreciable transformation of neutrinos in the matter of the sun, the nuclei and envelopes of collapsing stars, and the earth occurs. For the earth the energy range 1 meV–10^5 GeV was used. The shaded region is the region of existing experimental limits.

2. The nuclei and envelopes of collapsing stars (see Fig. 2). The effects depend on the mixing channel, in which the resonance condition holds. For the mixing $\nu_e \leftrightarrow \nu_x$ it is predicted that the ν_e peak from neutronization vanishes; for $\nu_e \leftrightarrow \nu_\mu$ (ν_τ) the spectra ν_e and ν_μ (ν_τ) are "exchanged"; for $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ ($\bar{\nu}_\tau$) an analogous exchange occurs for the spectra $\bar{\nu}_e - \bar{\nu}_\mu$ ($\bar{\nu}_\tau$). If the resonance occurs in the channel $\bar{\nu}_e \leftrightarrow \bar{\nu}_s$ or $\nu_e \leftrightarrow \nu_s$, where ν_s is the sterile state, then virtually complete vanishing of the signal in $\bar{\nu}_e$ or ν_e detectors is possible. In this connection it is important to search for ν bursts, directed toward recording ν_e and $\bar{\nu}_e$ simultaneously.

3. The earth. The region of the strong effect is of the order of $(E/\Delta m^2)$. About one-half of the resonant length of the oscillations fits into the thickness of the earth. The dependence of the suppression factor on $E/\Delta m^2$ and the zenith angle reflects the distribution of matter. In this connection the following are discussed: transillumination of the earth by a neutrino beam from an accelerator,[13] distortions of the spectrum of atmospheric neutrinos,[6] modulation (day-night, etc.) of the neutrino flux from the sun, and the difference in the ν fluxes from gravitational collapses under different conditions.

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