**N. N. Gor'kavyĭ and A. M. Fridman.** *Resonance nature of the rings of Uranus and prediction of new satellites of Uranus.* The ten narrow, eccentric, and widely separated rings of Uranus are strikingly different from the wide circular ring of Saturn, split up by several gaps. Immediately after the discovery of the rings of Uranus in 1977, an intensive search began for explanations for the unusual properties of the rings.

Dermott and Gold[1] proposed that the rings of Uranus correspond to the position of the three-frequency resonances from Ariel-Titanus and from Ariel-Oberon—the known large satellites of Uranus. This hypothesis, which explained five of the nine rings, was criticized, since there are stronger resonances in the zone of the rings, for example, from Ariel-Miranda. A year later Steigmann[2] modified the Dermott-Gold hypothesis, having studied the corresponding positions of five rings of Uranus with the three-frequency resonances from Ariel-Miranda and Miranda-Uranus VI (hypothetical satellite on an orbit with a radius of 105 221 km). Specialists turned their attention to other hypotheses for three reasons: 1) this hypothetical satellite was not discovered by Voyager 2; 2) there are no three-frequency resonances with the four remaining rings; and 3) there are no answers to the question of why no rings were observed at locations of stronger resonances? Such, for example, were the hypotheses put forth in 1979 regarding the existence in each ring of internal satellites, determining the dynamics of the particles of the rings.[3,4] The most popoular hypothesis, however, was that of Goldreich and Temaine[5] regarding the existence of a pair of satellites around each ring—"shepherds," which do not allow the particles in the rings to spread out because of mutual collisions (a year later, in 1980, the "shepherd" satellites were discovered near the narrow F ring of Saturn).

It was shown by Gor'kavyĭ *et al.*[6] that the rings lie in the zone of intensive collisional destruction of the particles, which collide as a result of the differential rotation of the rings with significant velocities $\sim \Omega a$ ($a$ is the size of the particles and $\Omega$ is the angular velocity of orbital rotation). The formation of satellites in the zone of the rings is forbidden and they must lie outside the boundary of this zone—in the region of lower collisional velocities. Rings and satellites can coexist only in a narrow transitional zone between the regions of the rings and the satellites. For this reason, it is difficult to agree both with the hypothesis of "shepherd" satellites and with the model of internal satellites, predicated on the presence of a large number of satellites in the entire zone of the rings. On the other hand, at the beginning of the 1980s small narrow (often eccentric) rings, whose position correlates well with the Lindblad [of the $n$: $(n+1)$ type] resonances of lowest order ($n = 1,2,3,\ldots$) from external satellites were discovered in Saturn's rings. The authors[7,8] proposed the hypothesis that the rings of Uranus have a resonant character. According to this hypothesis the positions of the rings are determined by the lowest (1:2, 2:3, 3:4) Lindblad resonances from a series of small undiscovered satellites outside the outer boundary of the rings. Indeed, between the rings and the nearest known satellite of Uranus—Miranda—there is a significant (about 80 000 kilometers) space which cannot be empty because of the continuous distribution of matter in the protodisk. Analogous series of small satellites have been discovered in recent years beyond the outer boundary of the rings of Saturn and Jupiter. A compelling reason for suggesting this hypothesis was the surprising regularity, discovered by the authors, in the radial distribution of the rings of Uranus: the rings can be divided into pairs, for each of which there is an outer orbit which is in a resonance ratio of the type 1:2, 2:3, or 2:3, 3:4 with the given pair of rings. This makes it possible to calculate the orbits of the most probable satellites.[2)] Figure 1 illustrates the hypothetical system of satellites, proposed by the authors, and the position of their resonances in the zone of the rings. The satellite V is an exception: it only has one resonance in the zone of the rings, but then it fulfills an additional function: it "herds" the ring $\varepsilon$—the widest and most elliptical of all the rings (in Fig. 1 it is not the rings themselves that are shown, but rather the zones of their eccentric motion; the rings themselves are very narrow—from 600 meters up to 100 km). Figures 2a–d show the sequence of gradual narrowing of the most probable regions of the satellites: a) the zone of the satellites, which have at least one resonance in the region of the rings (we have in mind only the resonances of the type under study); b) is the region of the satellites with two resonances in the zone of the rings; c) contains four regions, which can contain satellites with resonances in two groups of rings; d) contains five narrow zones of the most probable position of the satellites—with two resonances near pairs of rings. The dots indicate the chosen positions of the hypothetical satellites (from the requirement that the resonances be as close as possible to the rings).

Figure 2e shows ten satellites, discovered in January of 1986 by the American space probe Voyager 2, which passed near Uranus. As expected, all satellites were located outside the outer boundary of the rings (except one, the smallest one, discovered in the "transitional" zone near the outer boundary of the rings).

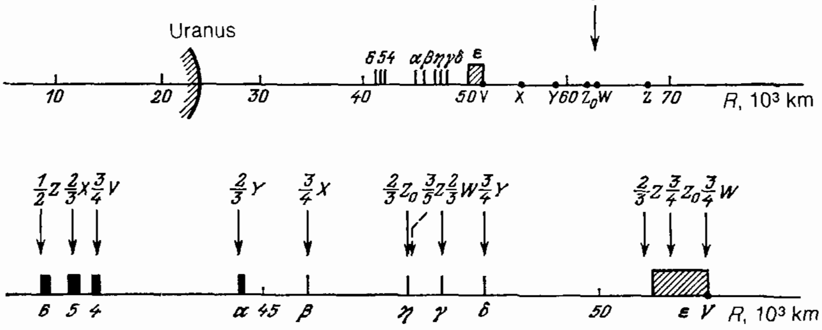

FIG. 1. The general arrangement of the rings of Uranus and of the proposed satellites. This figure is taken from Ref. 7 and contains the additional satellite $z_0$ (with the radius of the orbit equal to 61 860 km) from the first, unpublished variant of the work published in Ref. 7. The rings are denoted by lines (or rectangles), whose width corresponds to the spreading of the position of the rings owing to the eccentricity; the arrows mark the resonance orbits from the outer satellites in the zone of the rings.

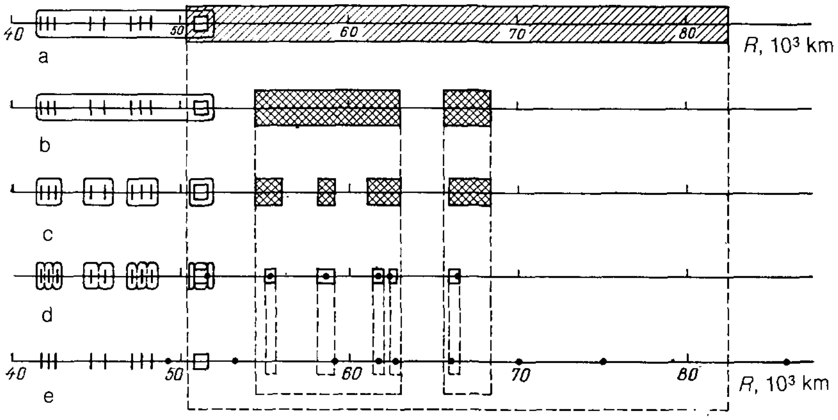

FIG. 2. Algorithm for separating the most probable zones of undiscovered satellites and comparison of their positions with the discovered satellites. The zone of the satellites having the following: one resonance of the type 1:2, 2:3, 3:4 in the zone of the rings (a); two resonances in the zone of the rings (b); resonances in three groups of rings and resonances near two rings (c); the dots indicate the proposed satellites, whose position is chosen from the requirement that the resonance be as close as possible to the rings (d); Fig. 2e indicates the satellites discovered by Voyager 2.

Table I shows the Voyager 2 data on the ten new satellites of Uranus and their comparison with the predicted orbits. We compare below the assumptions of the hypothesis with the Voyager observations obtained 6 months after the publication of these data.

The confirmation of the hypothesis of the resonance nature of the rings of Uranus proves that the resonance effect of the outer satellite gives rise to the appearance of a narrow ring. The formation of narrow rings of Uranus evidently begins with the appearance of a series of spiral density waves and flexural waves at the resonance points of the circumplanetary protodisk.

The spiral density waves, propagating outward from the resonant orbit, serve as barriers in the path of fine dust, moving toward the planet under the action of, for example, the Poynting-Robertson effect or friction against the gas. The dust can stop at the outer edge of the region perturbed by the spiral wave, forming annular condensations. Thus the distance between the resonance and the ring cannot exceed the characteristic scale of damping of the spiral wave (which in the rings of Saturn reaches several hundreds of kilometers).

The flexural waves, propagating toward the planet, can be regions of elevated dust velocity. In this case, the dust can accumulate on the internal edge of the flexural wave—on the boundary of the unperturbed zone.

An analogous mechanism of growth resulting from the flux of fine dust, was studied by the authors for the case of spontaneous annular fluctuations of the disk.[10] Taking into account the nondiffusion motion of fine dust leads to the following dispersion relation for annular perturbations of the disk with rare particle collisions[10]:

$$\gamma = -Dk^2 + AK + B,$$

$$A \approx \left(\frac{\partial N^-}{\partial T_0} - \frac{\partial N^+}{\partial T_0}\right) \frac{T_0}{3\Omega^2} \cdot 2\pi G,$$

$$B = -\left(\frac{\partial N^-}{\partial \sigma_0} - \frac{\partial N^+}{\partial \sigma_0}\right), \; T_0 \equiv c^2,$$

where $D = 6\nu > 0$ is the positive diffusion coefficient ($\nu$ is the coefficient of kinematic viscosity), $N^+$ is the rate of increase of the surface density $\sigma_0$ of the disk accompanying the absorption of dust, $c^2$ is the square of the variance of the velocities of chaotic motion of particles, and $k$ is the wave vector. For $A > 0$, $B > 0$ the initial perturbation of the disk grows ($\gamma > 0$), and actively absorbs the settled dust. The large-scale stratification (from 50 to 1000 km) of the rings of Saturn forms according to this scheme; in a sufficiently dense disk it could happen that $D < 0$, and then a short-wave

TABLE I.

| Radii of orbits of predicted satellites $R_{pr}$, km | Radii of orbits of discovered satellites* $R_d$, km | Accuracy of the agreement of the orbits $R_{pr} - R_{d}$, km | Number and type of resonances in the zone of the rings from the satellite | Diameter of the satellite, km |
|---|---|---|---|---|
| | 85 980 | | | |
| | 75 100 | | | |
| | 69 920 | | | |
| 66 450 | 66 090 | +360 | 2 (1 : 2, 2 : 3) | 100 |
| | 64 350 | | 1 (2 : 3) | 80 |
| 62 470 | 62 700 | −230 | 2 (2 : 3, 3 : 4) | 50 |
| 61 860 | 61 750 | +110 | 2 (2 : 3, 3 : 4) | 80 |
| 58 600 | 59 100 | −500 | 2 (2 : 3, 3 : 4) | 50 |
| 55 380 | | | | |
| 51 580 | 53 300 | −1720 | 1 (3 : 4) | 25 |
| | 49 300 | | 0 | 15 |

*The orbits are determined to within about 50 km.

instability with a scale of statification of several hundreds of meters develops.[10]

The evolution of the annular condensation, induced by the resonance perturbation from the satellite, has not yet been studied in adequate detail.

The eccentricity of the rings can counteract the diffusion spreading of the rings. Indeed, particles can escape from a circular ring, and under mutual collisions can cross over to neighboring quasicircular orbits, not intersecting the "mother" ring. For particles escaping from an elliptical ring such close orbits do not exist. All orbits of escaped particles necessarily intersect with the ring, as a result of the differential precession of the orbit in the nonspherical field of Uranus, i.e., the particles will be efficiently returned to the ring. The differential precession of the ring itself is stabilized by self-gravitation for a surface density of the ring of about $25\ \mathrm{g/cm}^3$.[10]

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2) The radii of the orbits of only those satellites which determine the position of two rings at the same time can be determined with the greatest probability.

[1] S. F. Dermott and T. Gold, Nature **267**, 590 (1977).

[2] G. A. Steigmann, Nature **274**, 454 (1978).

[3] T. C. van Flandern, Science **204**, 1076 (1979).

[4] S. F. Dermott, T. Gold, and A. Sinclair, Astron. J. **84**, 1225 (1979).

[5] P. Goldreich and S. Tremaine, Nature **277**, 97 (1979).

[6] N. N. Gor'kavyĭ and A. M. Fridman, Pis'ma Astron. Zh. **11**, 628 (1985) [Sov. Astron. Lett. **11**, 264 (1985)].

[7] N. N. Gor'kavyĭ and A. M. Fridman, *ibid.*, 717 [Sov. Astron. Lett. **11**, 302 (1985)].

[8] N. N. Gor'kavyĭ and A. M. Fridman, Astron. Tsirk., No. 1391, 1 (1985).

[9] V. L. Afanas'ev, N. N. Gor'kavyĭ, M. A. Smirnov, and A. M. Fridman, *ibid.*, 3.

[10] N. N. Gor'kavyĭ and A. M. Fridman, Nonlinear Waves: Structures and Bifurcations (in Russian), ed. by A. V. Gaponov-Grekhov and M. I. Rabinovich, Nauka, M. (1986).