Superstrings: a new approach to a unified theory of fundamental interactions

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The basic ideas of the superstring approach in the physics of elementary particles are presented at a rather simple theoretical level. The theory of interacting superstrings with dimensions of the order of the Planck dimension is presently regarded as a fundamentally new and extremely promising approach to a unification of all the fundamental interactions (strong, electromagnetic, weak, and gravitational) in a common theoretical framework. Although this field is still in a stage of rapid development, several results of definite interest have been obtained. The classical dynamics and the quantum dynamics of boson and spin strings are described. Such strings arise primarily in hadron physics. How a critical dimensionality of space-time and tachyon states arise upon the quantization of string models is demonstrated. A superstring is a supersymmetric generalization of very simple models of a relativistic string. The total action of a superstring is examined. The field theory of superstrings is discussed, as are various methods for introducing internal symmetries in string theories. The low-energy (field) limit in superstring theory is studied. The cancellation of anomalies and the problem of divergences are discussed. Compactification in superstring theories is discussed, as is a phenomenology which arises here for the modern physics of elementary particles. Some cosmological consequences of the superstring approach are outlined.

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It can be shown convincingly that reality cannot at all be represented by a continuous field.
A. Einstein
(*as translated back from the Russian*)

1. INTRODUCTION

The construction of physical theories always includes two stages: First, new theories are developed to describe a new region of physical phenomena; then, different theories or models are combined into a common theoretical framework. The second stage reflects the physicist's conviction that the physical world is a world of a universal relationship and unity. Success in combining several physical theories on a common basis requires penetrating deeper into the nature of the physical phenomena and identifying the more fundamental behavior. The classic example here is Maxwell's electromagnetic theory, which combined electrical and magnetic phenomena in an orderly framework.

In the first third of this century, physicists made a major effort to combine gravitation and electromagnetism on a geometric basis.[2,3] Einstein's theory of gravitation linked the gravitational field with a geometric characteristic of the space-time continuum: its curvature. It was therefore natu-

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ral to attempt to relate also the electromagnetic field with geometric characteristics of space-time. Efforts in this direction were the conformally invariant theory of gravitation and electromagnetism developed by Weyl[4] and Einstein's unified field theories.[5] A new idea in this area was proposed by Kaluza and Klein.[6] In their approach, space-time was regarded not as four-dimensional but as five-dimensional; the fifth dimension was compactified: $M^5 \sim M^4 \times S^1$. The curvature of the four-dimensional submanifold, M^4, was, as before, identified with the gravitational field, while the components of the metric tensor $g_{\mu 5}$ with $\mu = 0, 1,2,3$ were identified with the electromagnetic potential. The isometry group of the compact manifold S^1 determines a gauge group, in this case the U(1) group of a Maxwell field.

Developments in quantum mechanics and elementary particle physics demonstrated the limitations of this approach in the construction of a unified field theory. It became clear that a unified theory of the fundamental interactions would actually have to incorporate not only gravitation and electromagnetism but all fields whose quanta are elementary particles.

The following four types of fundamental interactions (or four types of forces) are presently known[!!]: the strong interaction (in which hadrons are involved), the electromagnetic interaction, the weak interaction, and the gravitational interaction. Let us briefly characterize each of these interactions.

According to the present understanding, hadrons are made up of quarks, of which there are six in the energy range presently attainable ($\sim 10^2$ GeV). These quarks are grouped in pairs: (u,d), (c,s), (t,b). The u, c, and t quarks have a charge of $+2/3$, while the d, s, and b quarks have a charge of $-1/3$. Baryons consist of three quarks; e.g., the proton is $p = uud$, the neutron is $n = udd$, and the λ particle is $\lambda = uds$. Mesons consist of two quarks: $\pi^+ = u\bar{d}$, $K^+ = u\bar{s}$, $J/\psi = c\bar{c}$. Each quark can be in one of three color states according to the $SU(3)_c$ group (yellow, blue, red). The physically observable hadrons are singlets in terms of the color quantum number. A dynamic theory describing hadron physics is quantum chromodynamics (QCD).[7,8] In this theory, the interaction between quarks is mediated by gluon fields: massless vector mesons with a non-Abelian gauge group SU(3). Within the framework of QCD, it is possible to explain successfully a fundamental feature of the behavior of quarks: the fact that they essentially do not interact with each other at short range (asymptotic freedom). The reason for this situation is a decrease in the effective chromodynamic interaction constant with increasing energy. A question which remains unresolved in QCD, however, is whether quarks exist in a free state.

The hypothesis of quark confinement is used in order to carry out specific calculations in QCD. This hypothesis eliminates the question of why the strong interactions, which are mediated by massless gluons, have a finite radius of action. The use of asymptotic freedom and the hypothesis of quark confinement makes it possible to construct a QCD description of processes with large transverse momenta, the production of lepton pairs, and string processes in e^+e^- annihilation, i.e., reactions in which the details of the formation of the final states from the quarks and gluons are not important. A description of the mass spectrum of hadrons and of exclusive processes turns out to lie beyond the capabilities of QCD as it exists today.

The electromagnetic and weak interactions have been successfully combined into the Glashow-Weinberg-Salam unified theory of the electroweak interaction[137-139] on the basis of the $SU(2) \times U(1)$ gauge group. The observation of intermediate vector bosons W^+, W^- and Z^0 in colliding protons are regarded as the "square roots" of translation transformations.

Supersymmetry combines bosons and fermions into common supermultiplets. At present we have no experimental confirmation of supersymmetry (i.e., it can only be broken), but from the theoretical standpoint the supersymmetric field models have several indisputable advantages. Foremost among them is the mutual cancellation of many ultraviolet divergences in these models.

In theories with a dimensional interaction constant, such as gravitation, the only way to remove divergences is to have them cancel each other out. In supergravity this cancellation has been demonstrated at the single- and two-loop levels. In higher-order perturbation theories, this cancellation of divergences probably does not occur. The supersymmetric grand unified theories also incorporate gravitation. Figure 1 shows the ranges of applicability of the fundamental physical theories and the corresponding symmetry groups.

Recently, Kaluza and Klein's ideas that all the internal symmetries (including gauge symmetries) in four-dimensional Minkowski space are generated by generally covariant space-time symmetries of a D-dimensional world ($D = 4 + k$) have been revived in the framework of supersymmetry.[13] Supersymmetry imposes a natural restriction on the dimensionality D: $D < 11$.

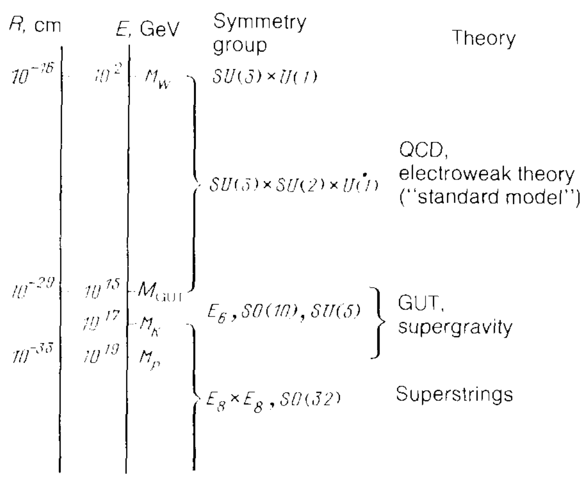

FIG. 1. Basic theoretical models in the physics of elementary particles, their ranges of applicability (in terms of energy E and distance R), and the corresponding fundamental symmetry groups.

The theoreticians are presently being attracted to superstring theories[15,16] as extremely strong candidates for the role of unifying all the fundamental interactions, including gravitation.

Superstrings are one-dimensional relativistic entities with a length on the order of the Planck dimension (10^{-33} cm). In addition to their linear dimensions, superstrings are characterized by spin (fermion) degrees of freedom, which are distributed along the strings. The number of physical fermion degrees of freedom is precisely equal to the number of boson degrees of freedom, so that the overall theory has supersymmetry.

A systematic quantum theory of superstrings is formulated in a ten dimensional Minkowski space-time. For several reasons, for which the detailed mechanisms are not yet completely clear, six of the dimensions undergo a compactification to a Planck level: $M^{10} \to M^4 \times K^6$. An extremely attractive idea here is that the theory of superstrings will make it possible to choose a compact manifold K^6 in an essentially unambiguous way and that the topological properties of this manifold will determine the basic features of the low-energy dynamics of superstrings, which describes the modern physics of elementary particles. In the low-energy limit ($E < 10^{19}$ GeV), the superstring theories become supergravity and the Yang-Mills supersymmetry theory.

Superstring theory thus organically incorporates supersymmetry, Kaluza and Klein's idea regarding the multidimensionality of our space-time world, and also the idea of nonlocality of the entities which are the mediators of the ton-antiproton beams[9] is direct experimental confirmation of this theory. A characteristic feature of this theory is the presence of chiral fermions (fermion fields are eigenvectors of the projection operators $1 \pm \gamma_5$). This circumstance is manifested as a breaking of P-invariance in weak processes.

A unified description of three interactions—strong, electromagnetic, and weak—is the goal of grand unified theories (GUT).[10,11] In these theories, one chooses a sufficiently large gauge group—SU(5), SO(10), or E_6—as a fundamental group, and QCD and the electroweak theory are "installed" in this gauge model. The fundamental fermion fields are quarks and leptons, which are grouped in generations. A pair of leptons is added to each of the pairs of quarks listed above; the u and d quarks with the electron and the electron neutrino form the first generation; the c and s quarks with the muon and with the muon neutrino form the second generation; and, finally, the t and b quarks along with the tau lepton and the corresponding neutrino are grouped in a third generation. The masses of the quarks and leptons increase with increasing index of the generation. An important consequence of a GUT is the prediction that the proton is unstable.[135,136]

With increasing energy, the effective constants of the weak and electromagnetic interactions should increase, while the chromodynamic interaction constant should decrease, so that at an energy of 10^{15} GeV (the energy scale of grand unification) all three constants should become equal. According to the GUT, we cannot expect to find any fundamentally new physics over the vast energy range from 10^2 to 10^{15} GeV (the "grand desert"). Unfortunately, the GUT give us no principle for immediately choosing among the various possible gauge groups. Furthermore, these theories generally contain a large number of numerical parameters which are related to the Higgs sector, to the constants of the Yukawa interaction.

The masses which arise in GUT span a hugh range, from a few electron volts (the possible mass of neutrinos) to 10^{15} GeV. So far, the unified theories give us no satisfactory explanation for this hierarchy of masses. Gravitation does not fit into a GUT in a natural way, since the mediators of the interaction in these theories are gauge vector fields of spin 1, while the quanta of the gravitational field (gravitons) have spin 2.

Incorporating gravitation in a unified theory of the fundamental interactions will first require deriving a quantum theory of gravitation. In this arena, definite hope is pinned on supergravity.

Supergravity is a theory with a localized gauge supersymmetry. The transformations of supersymmetry[12-14] mix boson (B) and fermion (F) fields: $\delta B = \bar{\varepsilon} F$, $\delta F = \partial B \cdot \varepsilon$, where ε is a spinor transformation parameter. Supersymmetry is a space-time symmetry, since the repeated application of a transformation results in a translation: $(\delta_1 \delta_2 + \delta_2 \delta_1) B = a^\mu \partial_\mu B$ where $a^\mu = \bar{\varepsilon}_2 \gamma^\mu \varepsilon_1$, and γ^μ are the Dirac matrices. On this basis, supersymmetry transforma- fundamental interactions. Important properties of superstring theories are the absence of anomalies (i.e., a breaking at the quantum-mechanical level of classical symmetries in the theory (gauge symmetry and Lorentz invariance) and the possibility of eliminating divergences in the theory. Remarkably, superstring theories leave us essentially no latitude in choosing a fundamental gauge group. There are only two possibilities: the SO(32) group and the $E_8 \times E_8$ group.

The list of papers devoted to superstrings is extremely long.[17,18] It is thus impossible to discuss them all completely. The purpose which we do intend to achieve in this review is to present, at a rather simple theoretical level, the basic ideas of the superstring approach in elementary particle physics.

2. RELATIVISTIC STRINGS IN HADRON PHYSICS

2.1. Nambu-Goto action for a boson string

The superstring is a natural generalization of the relativistic string model which originally arose in hadron physics (see, for example, the reviews in Refs. 19–21) as the dynamic basis of dual-resonance models.[22,23] Superstring theory is based on an apparatus which was developed for describing hadron string models. In the dual approach it is assumed that the hadron spectrum in the tree approximation is equidistant and consists of an infinite number of resonances of zero width. This spectrum is generated by a denumerably infinite set of creation and annihilation operators $a_{n\mu}^+$, $a_{n\mu}$, $n = 1,2,3,\ldots$, each of which is a Lorentz vector. A set of operators of this sort can be found by quantizing a relativistic entity of finite size which is extended in one dimension (strings or filaments). A direct generalization of an ordinary linear string to the relativistic case would not be appropriate from the standpoint of the dual models, since in this case the quadratic Lagrangian does not yield limitations on the physical vectors of states which could be identified with the Virasoro conditions in dual models. Accordingly, Nambu[24] and Goto[25] have proposed for a relativistic boson string an action which is proportional to the area of the world surface in space-time which is swept out by the string as it moves:

$$S=-\gamma\int\limits_{\tau_1}^{\tau_2}d\tau\int\limits_0^{\pi}d\sigma\,(\det\|h_{\alpha\beta}\|)^{1/2}, \qquad (1)$$

where $h_{\alpha\beta}=\partial_\alpha x^\mu\partial_\beta x_\mu$ is an induced metric on the world surface of the string; $x^\mu(\tau,\sigma)$ are the coordinates of the string; $\alpha,\beta=0,1$, $\mu=0,1,...,D-1$, D is the dimensionality of the space-time with the metric signature $(+,-,-,...)$; and γ is a constant with a dimensionality $[M]^2$, which is determined in the dual models by the universal slope of Regge paths, α': $\gamma=(2\pi\alpha')^{-1}$. The parameter $u^0=\tau$ is an evolution parameter, while the variable $u^1=\sigma$ numbers the points along the string. By virtue of the reparametrized invariance of action (1), orthonormal gauge conditions can be imposed on the dynamic variables of the string:

$$(\dot{x}\pm x')^2=0,\quad \dot{x}=\partial_\tau x,\quad x'=\partial_\sigma x. \qquad (2)$$

As a result, the equations of motion of the string are linearized:

$$\ddot{x}^\mu-x''^\mu=0. \qquad (3)$$

If the string is open, the boundary conditions

$$x'^\mu(\tau,0)=x'^\mu(\tau,\pi)=0 \qquad (4)$$

should hold at its ends. For a closed string $(0\leqslant\sigma\leqslant2\pi)$, the coordinates of the string obey the periodicity condition

$$x^\mu(\tau,0)=x^\mu(\tau,2\pi). \qquad (5)$$

2.2. Covariant quantization; Virasoro algebra; critical dimensionality of space-time; tachyons

A solution of equations of motion (3) which satisifies boundary conditions (4) can be represented by the Fourier series

$$x^\mu(\tau,\sigma)=Q^\mu+P^\mu\frac{\tau}{\pi\gamma}+\frac{i}{(\pi\gamma)^{1/2}}\sum_{n\neq0}e^{-in\tau}\frac{\alpha_n^\mu}{n}\cos(n\sigma), \qquad (6)$$

where P^μ is the total momentum of the string, and Q^μ are the coordinates of the "center of mass" of the string at $\tau=0$. Since the $x^\mu(\tau,\sigma)$ are real, the amplitudes α_n^μ obey the condition $\alpha_{-n\mu}=\alpha_{n\mu}^+$. In the quantum theory, these amplitudes are interpreted as Fok operators which obey the commutation relations

$$[\alpha_{m\mu},\alpha_{n\nu}]=-mg_{\mu\nu}\delta_{n+m,0},\quad [Q_\mu,P_\nu]=-ig_{\mu\nu}, \qquad (7)$$

where $g_{\mu\nu}=\mathrm{diag}\,(1,-1,-1,...)$.

Orthonormal gauge conditions (2) lead to the Virasoro operators

$$L_n=-\frac{1}{2}\sum_{m=-\infty}^{+\infty}:\alpha_{n-m}\alpha_m:,\qquad n=0,\pm1,\pm2,\ldots$$

$$L_n^+=L_{-n},\quad \alpha_0^\mu=P^\mu(\pi\gamma)^{-1/2}, \qquad (8)$$

which satisfy the algebra

$$[L_n,L_m]=(n-m)L_{n+m}+\frac{D}{12}n(n^2-1)\delta_{n+m,0}. \qquad (9)$$

The operators $L_0,L_{\pm1}$ form a subalgebra in the Virasoro algebra which is isomorphic with the Lie algebra of the SU(1,1) group. An important point here is the appearance of a c-number term (central charge, Schwinger term) in commutation relation (9). In the classical theory, where Poisson brackets play the role of the commutation relations, there is no such anomaly:

$$\{L_n,L_m\}=(-i)(n-m)L_{n+m}. \qquad (9')$$

The algebra (9') is isomorphic with the Lie algebra of a conformal group on a plane. In classical mechanics, the L_n generate holomorphic mappings $z\to f(z)$ and can be represented as differentiation operators $L_n\sim ie^{iz}\partial/\partial z$. If the anomalies in (9) do not cancel out, the conformal symmetry is broken in the quantum theory (Subsection 2.3).

The appearance of the anomalous Schwinger term in (9) is a consequence of the switch to the normal product of the operators α_n^μ in L_n. The simplest way to derive this term is to use the Wick theorem to calculate the expectation value over the vacuum of the commutation relation (9), if we note that the pairing of the operators $\alpha_k^\mu\alpha_j^\nu$ is equal to $-g^{\mu\nu}\theta(k)k\delta_{k+j,0}$.

The temporal components α_{n0}^+, $n>0$, acting on the vacuum, will lead to vectors of states with a negative norm. Only those states $|\phi\rangle$ are physical which satisfy the conditions

$$L_0|\phi\rangle=\alpha(0)|\phi\rangle,\quad L_n|\phi\rangle=0,\quad n=1,2,\ldots. \qquad (10)$$

It has been shown[27,28] that the solution of these equations does not contain states with a negative norm if $1<D<25$, while $\alpha(0)<1$. A physical space of states with a positive norm (i.e., if we eliminate states with a zero norm) can be constructed only in a space-time with dimensionality $D=26$ and if $\alpha(0)=1$ (the Veneziano dual model). This result means that the ground state of a string is a tachyon, since for the square mass of the string we find from (10)

$$M^2=P^2=-\pi\gamma\sum_{m\neq0}:\alpha_{-m}\alpha_m:-2\pi\gamma\alpha(0). \qquad (11)$$

Infinite-dimensional Virasoro algebra (9) belongs to the Kac-Moody affine algebras, the theory of which has recently undergone a development.[29] These algebras may be thought of as a generalization to the infinite-dimensional case of the classical theory of Lie algebras. This expansion preserves to a maximal extent such concepts in the theory of Lie algebras as root diagrams, senior weight, the Cartan matrix, etc. The results derived in the theory of Kac-Moody algebras have made it possible to construct a new proof of the theorem that there are no "ghosts" in a relativistic string model.[30]

The theory of a closed boson string contains two independent sets of oscillator operators α_n and β_n:

$$x^\mu(\tau,\sigma)=Q^\mu+P^\mu\frac{\tau}{2\pi\gamma}+\frac{i}{2(\pi\gamma)^{1/2}}\sum_{n\neq 0}\frac{1}{n}e^{in\tau}(\alpha_n^\mu e^{-in\sigma}+\beta_n^\mu e^{in\sigma}). \quad (12)$$

Accordingly, the set of Virasoro operators is also doubled:

$$L_n=-\frac{1}{2}\sum_{m=-\infty}^{+\infty}:\alpha_{n-m}\alpha_m:,\quad \tilde{L}_n=-\frac{1}{2}\sum_{m=-\infty}^{+\infty}:\beta_{n-m}\beta_m:, \quad (13)$$

where $\alpha_0^\mu=\beta_0^\mu=P^\mu/2(\pi\gamma)^{1/2}$. The physical state vectors obey the gauge conditions

$$L_n|\phi\rangle=\tilde{L}_n|\phi\rangle=0,\quad n\geqslant 1, \quad (14)$$

$$(L_0-\tilde{L}_0)|\phi\rangle=0 \quad (14')$$

and the mass-shell condition

$$[L_0+\tilde{L}_0-\alpha(0)]|\phi\rangle=0. \quad (15)$$

The Virasoro-Shapiro dual model[22,23] corresponds to a closed string. There are no ghosts states in this model if we have $D=26$ and $\alpha(0)=2$. The ground state is thus again a tachyon. The first excited state describes a massless spin-two particle.

From (14') and (15) we find $[L_0-(\alpha(0)/2)]|\phi\rangle=0$ and $[\tilde{L}_0-(\alpha(0)/2)]|\phi\rangle=0$. Condition (14') may also be thought of as a requirement that the theory be invariant under finite shifts of the parameter σ: $\sigma\to\sigma+\sigma_0$. This interpretation is based on the equality

$$\exp[2i\sigma_0(L_0-\tilde{L}_0)]\,x^\mu(\tau,\sigma)\exp[-2i\sigma_0(L_0-\tilde{L}_0)]=x^\mu(\tau,\sigma+\sigma_0). \quad (16)$$

In the theory of closed strings one can introduce the concept of oriented and unoriented strings. If we require that the theory be invariant under reflection of the parameter $\sigma(\sigma\to-\sigma)$, then a closed string of this sort would be called "unoriented." In view of (12), this approach is equivalent to requiring invariance under replacement of the operators α_n^μ by β_n^μ and vice versa. The theory of an unoriented string thus has only states which are symmetric with respect to α and β (a bounded Virasoro-Shapiro dual model). The theory of an oriented string includes a complete set of states (both symmetric and antisymmetric with respect to the operators α and β).

The constant $\alpha'=(2\pi\gamma)^{-1}$ specifies the energy scale in dual strings. Hadron physics requires $\alpha'\sim 1\ (\mathrm{GeV})^{-2}$. The same quantity determines the length scale of a dual string: $L\sim(\alpha')^{1/2}\sim 10^{-13}$ cm. In the tree approximation, string models predict a linear relationship between the spin of a J-state and the square of its mass, M^2 (linear Regge paths). The energy of a relativistic string is proportional to its length, $E\sim L$, so that we have $M^2\sim L^2$. The angular momentum of a rotating string which is a straight line segment is proportional to L^2. We thus have $J\sim M^2$. The linearity of Regge paths forbids (in particular) the existence of light hadron states with large spins (the quantum string theory leads to the limitation $J\leqslant\alpha' M^2$ for an open string or $2J\leqslant\alpha' M^2$ for a closed one). These properties of the hadron spectrum are supported by experiment.

2.3. BRST formalism in a covariant quantum string theory

A covariant quantization of a relativistic string without gauge fixing is set forth in a simple and compact way in Refs. 31–33 by means of the Becchi-Rouet-Stora-Tyutin technique[34,35] (the BRST formalism). This formulation of the quantum mechanics of a string proves useful in deriving a covariant quantum field theory of interacting strings.

This formalism can be described as follows. In gauge and in generally covariant theories, constraints $\phi_a(q,p)$ in which there are "primary" constraints in Dirac's termonology,[36]

$$[\varphi_a,\varphi_b]=f_{ab}^c\varphi_c, \quad (17)$$

it is necessary to introduce dynamic variables (fields) which correspond to Faddeev-Popov ghosts in the derivation of a covariant quantum theory. With each constraint φ_a one associates a canonically-conjugate pair of ghost fields c_a and $\bar{c}_a$:

$$[c_a,\bar{c}_b]=\delta_{ab},\quad [c_a,c_b]=[\bar{c}_a,\bar{c}_b]=0. \quad (18)$$

If some of the constraints are fermion constraints, the algebra (17) is a gauge algebra. The ghost variables are fermion variables if the corresponding constraint is a boson constraint, and vice versa. We then introduce a BRST-charge operator,[37,38]

$$Q=\sum_a\varphi_a c_a-\frac{1}{2}\sum_{a,b,c}f_{ab}^c c_a c_b\bar{c}_c. \quad (19)$$

This is a nilpotent operator; i.e., its anticommutation relation (or its Poisson brackets in classical mechanics) vanishes by virtue of Eqs. (17) and (18) and the Jacobi identity for the structure constants f_{ab}^c (for simplicity, we are assuming that the f_{ab}^c do not depend on the canonical variables q and p). We use Q to construct some new constraints $\tilde{\varphi}_a$, which take into account the presence of ghosts in the theory

$$\tilde{\varphi}_a=[Q,\bar{c}_a]=\varphi_a+\sum f_{ab}^c\bar{c}_c c_b. \quad (20)$$

The constraints $\tilde{\varphi}_a$ again satisfy the algebra (17) and are BRST-invariant:

$$[Q,\tilde{\varphi}_a]=0, \quad (21)$$

since

$$Q^2=0. \quad (22)$$

In going over to a quantum theory, we must order the operators in $\varphi_a(q,p)$ and Q. Equations (17) and (22) must then be rechecked, since anomalies may appear.

At the classical level, the constraints $L_n=0$ in string theory are primary constraints, since they satisfy the closed algebra (9'). We denote c_n and $\bar{c}_n$ the corresponding fermion operators of the ghosts,

$$[c_m, c_n]_+ = \delta_{n+m,0}, \quad [c_m, c_n] = [\bar{c}_m, \bar{c}_n] = 0. \tag{23}$$

The operators c and $\bar{c}$ are individually Hermitian:

$$c_m^+ = c_{-m}, \quad \bar{c}_m^+ = \bar{c}_{-m}. \tag{24}$$

The BRST charge in the theory of a boson string is given by the expression

$$Q = \sum_m L_{-m}c_m - \frac{1}{2}\sum_{m,n} (m-n) : c_{-m}c_{-n}\bar{c}_{m+n} : -\alpha(0)\, c_0. \tag{25}$$

As before, the term width $\alpha(0)$ in (25) has arisen from the normal ordering of the operators α_n in L_0. The square of the Hermitian operator Q must vanish, according to (22). This vanishing is obviously possible only in a space of states with a nonpositive-definite norm.

The quantum Virasoro operators are now defined by

$$\tilde{L}_m = \{Q, \bar{c}_m\} = L_m + \sum (m-n) : \bar{c}_{m+n}c_{-n} : -\alpha(0)\, \delta_{m,0}. \tag{26}$$

The anomalous term in the algebra of the operators $\tilde{L}_m$ is

$$\frac{1}{12} D(m^3 - m) + \frac{1}{6}(m - 13m^3) + 2\alpha(0)\, m. \tag{27}$$

This term vanishes if $D = 26$ and $\alpha(0) = 1$. Consequently, it is only in this case that the quantum theory of a boson string is conformally invariant.

The infinite number of conditions on the physical state vectors of the string, (10), are replaced by a single condition in the BRST formalism:

$$Q\,|\,\phi\rangle = 0. \tag{28}$$

All solutions (28) which do not contain excitations of ghosts satisfy conditions (10). We can thus use the "no-ghosts" theorem from covariant string theory.

2.4. Lightlike gauge

If we do not require explicit relativistic invariance, we can derive a quantum string theory by eliminating dependent dynamic variables from the theory.[30] For this purpose, orthonormal gauge conditions (2) should be supplemented with lightlike gauge conditions,

$$nx = nP\frac{\tau}{\gamma\pi} + nQ, \tag{29}$$

where n is a constant isotropic vector with $n^2 = 0$. Orthonormal gauge conditions (2) do not yet completely fix the set of parameters τ, σ on the world surface of the string. Equations (2) and (3) allow a transformation to the new variables, $\tilde{\tau}, \tilde{\sigma}$ in accordance with the formulas $\tilde{\tau} \pm \tilde{\sigma} = f_\pm(\tau \pm \sigma)$, with arbitrary functions $f_\pm$. Using these transformations, one can always satisfy gauge conditions (29). Using (2) and (29), we can express two components of the vector $x^\mu(\tau,\sigma)$ in terms of the remaining transverse components, which are quantized as independent dynamic variables. We assume that the vector n^μ has the components $n^\mu = 1, 0, ..., 0, -1)$; in terms of the light-cone variables $x^\pm = (x^0 \pm x^{D-1})/\sqrt{2}$, $x^\mu = (x^\pm, \mathbf{x}_\perp)$ we then find

$$\dot{x}^- = \frac{\pi\gamma(\dot{\mathbf{x}}_\perp^2 + \mathbf{x}_\perp'^2)}{2P^+}, \quad x'^- = \frac{\pi\gamma\dot{\mathbf{x}}_\perp\mathbf{x}'_\perp}{P^+},$$
$$\dot{x}^+ = \frac{P^+}{\pi\gamma}, \qquad x'^+ = 0. \tag{30}$$

The dynamics of the independent components of $\mathbf{x}_\perp(\tau,\sigma)$ is determined by the quadratic action of the string in the lightlike gauge:

$$S = \frac{\gamma}{2}\int_{\tau_1}^{\tau_2} d\tau \int_0^\pi d\sigma\,(\dot{\mathbf{x}}_\perp^2 - \mathbf{x}_\perp'^2). \tag{31}$$

The transverse components in expansion (6), $\mu = 1, 2, ..., D-2$, constitute a solution of equations of motion (3) for the independent variables $\mathbf{x}_\perp(\tau,\sigma)$. In terms of Fourier amplitudes, Eqs. (30) can be rewritten as

$$\alpha_k^{(+)} = 0, \quad k \neq 0, \quad \alpha_k^{(-)} = \frac{\pi\gamma}{P^+} L_{n\perp} - \delta_{n,0}\alpha(0),$$
$$n = 0, \pm 1, \pm 2, \ldots, \tag{32}$$

where the $L_{n\perp}$ are Virasoro operators constructed from transverse Fourier amplitudes:

$$L_{n\perp} = \frac{1}{2}\sum_{m=-\infty}^{+\infty} : \alpha_{n-m\perp}\alpha_{m\perp} :, \quad \alpha_{0\perp} = \frac{\mathbf{P}_\perp}{(\pi\gamma)^{1/2}},$$
$$\alpha_0^{(\pm)} = \frac{P^\pm}{(\pi\gamma)^{1/2}}. \tag{33}$$

In particular, for the mass of the string as a whole we find the following expression from (32) for $n = 0$:

$$M^2 = P^2 = -\mathbf{P}_\perp^2 + 2P^+P^- = \pi\gamma\sum_{m\neq 0} : \alpha_{-m\perp}\alpha_{m\perp} : -2\pi\gamma\alpha(0). \tag{34}$$

In the case in which we are interested, the norm of the state vectors is evidently positive, since these vectors are constructed by the action on the vacuum exclusively of the operators $\alpha_{n\perp}^+, n \geqslant 1$.

The basic problem in the noncovariant approach is in proving the relativistic invariance of the theory in the quantum case. To construct this proof it is necessary to show that the generators of the Poincaré group, P_μ and $J_{\mu\nu}$ constructed by means of the dynamic string variables satisfy the well-known commutation relations. The generator of translations is the total momentum of the string, P_μ, while the angular-momentum tensor of the string,

$$J_{\mu\nu} = \gamma\int_0^\pi (x_\mu\dot{x}_\nu - x_\nu\dot{x}_\mu)\, d\sigma = Q_\mu P_\nu - Q_\nu P_\mu$$
$$-\frac{i}{2}\sum_{n\neq 0}\frac{1}{n}(\alpha_{-n\mu}\alpha_{n\nu} - \alpha_{-n\nu}\alpha_{n\mu}), \tag{35}$$

is the generator of Lorentz rotations. It turns out that all the commutation relations have the correct values except for[39]

$$[J^{-i}, J^{-j}]$$
$$= \frac{2}{(P^+)^2}\sum_{m=1}^\infty \left\{m\left[1 - \frac{1}{24}(D-2)\right] + \frac{1}{m}\left[\frac{1}{24}(D-2) - \alpha(0)\right]\right\}$$
$$\times(\alpha_{-m}^i\alpha_m^j - \alpha_{-m}^j\alpha_m^i), \tag{36}$$

where $i,j = 2,3, ..., D-1$, and D is the dimensionality of space-time. An algebra of the Poincaré group requires $[J^{-i}, J^{-j}] = 0$. Consequently, the only possibility for reconciling a given theory with relativistic invariance is to require $\alpha(0) = 1$ and $D = 26$. It follows in particular that the ground state of the string has an imaginary mass (is a tachyon) according to (34). The first excited state, $\alpha^i_{-1}|0\rangle$, describes a massless vector particle with spin 1. In the real hadron spectrum there is no such state. This fact, along with the nonphysical dimensionality of space-time and the tachyon states, is a basic disadvantage of the dual string approach to hadron physics.

Methods have been proposed for quantizing a relativistic string without restricting the dimensionality of space-time and without tachyon states.[40,41] In that approach, however, one loses the connection of the string models with the dual-resonance approach.

2.5. String model of hadrons and quantum chromodynamics

The concept of a relativistic string arises in hadron physics, in addition to dual models, and also within the framework of quantum chromodynamics. It is extremely likely that when the distance between quarks approaches the size of a hadron ($\sim 10^{-13}$ cm) the configurations of the gluon fields which are favored from the energy standpoint are those in which these fields do not fill the entire space (as in electrodynamics) and are instead concentrated along lines connecting quarks.[42–44] The energy of two quarks coupled by a gluon-field tube of this sort is proportional to the distance between the quarks. The attractive forces between quarks thus do not fall off with distance; they instead remain constant. Accordingly, no external agent of any sort can break this bond and produce a free quark. The reason for the appearance of gluon-field configurations which are localized along a line is the existence of vacuum fields in QCD,[45] which create an external pressure on the gluon tube. Such localized configurations of the gluon field are modeled by a relativistic string (the transverse dimensions of the gluon tube are assumed to be infinitely small). A relativistic string is considerably simpler than such a complex quantum-field model as chromodynamics. Furthermore, the string model reproduces the basic predictions found in the field approach. In particular, a relativistic string coupling massive particles gives rise to a potential between the particles which increases linearly with the distance[46] (the quark confinement potential).

2.6. Spinning string; dynamic variables and covariant quantization

The dual-resonance models incorporating fermion operators (the Neveu-Schwarz model[47] and the Ramond model[48]) were historically the first to be constructed. Later on, a spinning string was proposed as a dynamic basis for these models.

In a spinning string, the boson coordinates $x^\mu(\tau,\sigma)$ are supplemented with spin variables $S^\mu_\alpha(\tau,\sigma)$, which are Grassmann (anticommuting) quantities already at the classical level. With respect to the index μ, they transform as a Lorentz D-dimensional vector while with respect to the index α they transform as a two-dimensional spinor in (τ,σ) space. Physically, one can interpret the variables $S^\mu_\alpha(\tau,\sigma)$ as variables describing the spin distribution along the string.

Equations of motion (3) and orthonormal gauge conditions (2) are generalized in the theory of a spinning string in the following way:

$$\ddot{x}^\mu - x''^\mu = 0,\quad \dot{S}^\mu_1 = S'^\mu_1,\quad \dot{S}^\mu_2 = -S'^\mu_2, \tag{37}$$

$$\begin{gathered}\dot{x}^2 + x'^2 + 2i\,(S^\mu_1 S'_{1\mu} + S^\mu_2 S'_{2\mu}) = 0,\\ \dot{x}_\mu x'^\mu + i\,(S^\mu_1 S'_{1\mu} - S^\mu_2 S'_{2\mu}) = 0,\end{gathered} \tag{38}$$

$$(\dot{x}_\mu + x'_\mu)\,S^\mu_1 = 0,\quad (\dot{x}_\mu - x'_\mu)\,S^\mu_2 = 0. \tag{39}$$

Consequently, x^μ and S^μ obey the free d'Alembert and Dirac equations, respectively; they interact only by virtue of auxiliary conditions (38) and (39). The boundary conditions in the theory of a spinning string are written as

$$\begin{gathered}x'^\mu(\tau,0) = x'^\mu(\tau,\pi) = 0,\\ S^\mu_1(\tau,0) = S^\mu_2(\tau,0),\quad S^\mu_1(\tau,\pi) = \varepsilon S^\mu_2(\tau,\pi),\end{gathered} \tag{40}$$

where $\varepsilon = -1$ for the Neveu-Schwarz dual string and $\varepsilon = +1$ for the Ramond model.

Equations of motion (37) are found from the following action in orthonormal gauge (38), (39):

$$S = -\frac{\gamma}{2}\int_{\tau_1}^{\tau_2} d\tau \int_0^\pi d\sigma\,(\partial_\alpha x^\mu \partial_\alpha x_\mu + i\bar{S}^\mu \rho^\alpha \partial_\alpha S_\mu), \tag{41}$$

where ρ^α, $\alpha = 0,1$ are two-dimensional Dirac γ matrices for the space τ,σ, given by

$$\begin{gathered}\rho^0 = -i\sigma_2,\quad \rho^1 = \sigma_1,\quad \rho^5 = \sigma_3,\quad \rho^\alpha\rho^\beta = \eta_{\alpha\beta} + \rho^5\varepsilon_{\alpha\beta},\\ \eta_{\alpha\beta} = \mathrm{diag}\,(1,\,-1),\quad \varepsilon_{01} = 1.\end{gathered} \tag{42}$$

The spin variables S^μ_1, S^μ_2 can be represented as a two-component column S^μ, where $\overline{S}_\mu = S^T_\mu \rho^0$. Auxiliary conditions (38) and (39), which do not follow from the action (41), mean that the symmetric energy-momentum tensor $T_{\alpha\beta}$ vanishes in the τ,σ parameter space,

$$T_{\alpha\beta} = \partial_\alpha x^\mu \partial_\beta x_\mu - \frac{1}{2}\eta_{\alpha\beta}\,(\partial_\rho x)^2 + \frac{i}{4}\,\overline{S}\,(\rho_\alpha\partial_\beta + \rho_\beta\partial_\alpha)\,S = 0, \tag{43}$$

as does the supercurrent density,

$$J_\alpha = (\partial_\beta x^\mu)\,\rho^\beta\rho_\alpha S_\mu = 0. \tag{44}$$

The solution of the equations of motion for x^μ is given as before by expansion (6); for the spin variables we have

$$\begin{gathered}S^\mu_1(\tau,\sigma) = \frac{1}{2\sqrt{\pi}}\sum_k b^\mu_k \exp[-ik(\tau+b)],\\ S^\mu_2(\tau,\sigma) = \frac{1}{2\sqrt{\pi}}\sum_k b^\mu_k \exp[-ik(\tau-\sigma)],\quad b^\mu_k = \overset{*}{b}{}^\mu_{-k}.\end{gathered} \tag{45}$$

If $\varepsilon = -1$ in boundary conditions (40), then the summation in (45) is over half-integers, while if $\varepsilon = +1$ it is over integers.

Constraints (38) and (39) lead to the gauge operators

$$\begin{gathered}G_n = -\frac{1}{2}\sum_{k=-\infty}^{+\infty} :\alpha_{n-k}\alpha_k: - \frac{1}{2}\sum_{k=\infty}^{+\infty} k : b_{n-k} b_k :,\\ H_n = -\sum_{k=-\infty}^{+\infty} b_{n-k}\alpha_k,\quad [b^\mu_n, b^\nu_m] = -g^{\mu\nu}\delta_{n+m,\,0}.\end{gathered} \tag{46}$$

As in the boson case, the algebra of these operators is unclosed at the quantum level because of the appearance of anomalous terms which depend on the dimensionality of space-time. In the case of Ramond's dual model ($\varepsilon = +1$), for example, we have

$$
\begin{aligned}
&[G_n, G_m]_- = (n-m)\,G_{n+m} + \frac{D}{8}\,n^3\delta_{n+m,\,0},\\
&[H_n, H_m]_+ = 2G_{n+m} + \frac{D}{2}\,n^2\delta_{n+m,\,0},\\
&[G_n, H_m]_- = \left(\frac{n}{2}-m\right)H_{n+m}.
\end{aligned}
\tag{47}
$$

Those state vectors which satisfy the conditions

$$G_n\,|\,\phi\rangle = 0,\quad H_n\,|\,\phi\rangle = 0,\quad n = 0,\,1,\,\ldots, \tag{48}$$

are regarded as physical. A Hilbert space of state vectors with a positive definite norm can be constructed only for $D = 10$. In the theory of a spinning string, the operator representing the square of the mass is given by

$$\alpha' M^2 = m,\quad \begin{array}{ll} m = 0,\,1,\,\ldots, & \text{if}\ \ \varepsilon = 1,\\ m = -\frac{1}{2},\,0,\,\frac{1}{2},\,1,\,\ldots, & \text{if}\ \ \varepsilon = -1. \end{array} \tag{49}$$

Consequently, there are no tachyon states in a spinning string corresponding to the Ramond dual model.

The spectrum of a spinning string can be pruned so that the numbers of bosons and fermions are equal at each mass level, i.e., the spectrum becomes supersymmetric.[49]

2.7. Noncovariant quantization

In the theory of a spinning string, as in the boson case, one can separate the dynamic variables into groups of dependent and independent variables, and one can construct a quantum theory in terms of exclusively the independent (transverse) degrees of freedom.[15] For this purpose, lightlike gauge (29) is also extended to the spin variables:

$$n_\mu S_1^\mu = n_\mu S_2^\mu = 0. \tag{50}$$

It follows that

$$S_1^+(\tau,\,\sigma) = S_2^+(\tau,\,\sigma) = 0. \tag{51}$$

It turns out to be possible to introduce this gauge because Eqs. (37), (38), and (39) are invariant under (pseudo-) conformal transformations of the parameters τ and σ:

$$
\begin{aligned}
&\tilde\tau \pm \tilde\sigma = f_\pm(\tau \pm \sigma),\quad \tilde x^\mu(\tilde\tau,\,\tilde\sigma) = x^\mu(\tau,\,\sigma),\\
&\tilde S_1^\mu(\tilde\tau,\,\tilde\sigma) = S_1^\mu(\tau,\,\sigma)\,(\dot f_+)^{-1/2},\quad \tilde S_2^\mu(\tilde\tau,\,\tilde\sigma) = S_2^\mu(\tau,\,\sigma)\,(\dot f_-)^{-1/2},
\end{aligned}
\tag{52}
$$

and under supersymmetric transformations of a special type which mix boson and fermion variables,

$$
\begin{aligned}
&\tilde S_1^\mu = S_1^\mu + \varepsilon f(\tau,\,\sigma)\,(\dot x^\mu + x'^\mu),\\
&\tilde S_2^\mu = S_2^\mu + \varepsilon g(\tau,\,\sigma)\,(\dot x^\mu - x'^\mu),\\
&\tilde x^\mu = x^\mu - 2i\varepsilon\,(f S_1^\mu + g S_2^\mu),
\end{aligned}
\tag{53}
$$

where $f(\tau,\sigma)$ and $g(\tau,\sigma)$ are functions which anticommute with S^μ and obey the equations

$$\dot f - f' = 0,\quad \dot g + g' = 0. \tag{54}$$

Equations (38), (39), and (51) can be used to express the variables $x^-(\tau,\sigma)$ and $S_1^-(\tau,\sigma)$, $S_2^-(\tau,\sigma)$ in terms of $\mathbf{x}_\perp(\tau,\sigma)$ and $\mathbf{S}_{\perp 1}(\tau,\sigma)$, $\mathbf{S}_{\perp 2}(\tau,\sigma)$, in complete analogy with (30). The dynamics of the independent variables is specified by the action of a spinning string in the lightlike gauge, as in (41):

$$S = -\frac{\gamma}{2}\int_{\tau_1}^{\tau_2} d\tau \int_0^\pi d\sigma\,[(\partial_\alpha \mathbf{x}_\perp)^2 + i\bar{\mathbf{S}}_\perp \rho^\alpha \partial_\alpha \mathbf{S}_\perp]. \tag{55}$$

The solution of equations of motion (37) for the independent variables consist of the transverse components $\mu = 1,2, \ldots, D-2$ in (6) and (45).

The Lorentz generators J^{-i} are given by

$$
\begin{aligned}
J^{-i} = Q^- P^i - Q^i P^- - \frac{i}{P^+}\sum_{n=1}^{\infty}\frac{1}{n}\,(\alpha_n^i G_{-n\perp} - \alpha_{-n}^i G_{n\perp})\\
+ \frac{1}{P^+}\sum_{n>0}(b_n^i H_{-n\perp} - b_{-n}^i H_{n\perp}),
\end{aligned}
\tag{56}
$$

where $G_{n\perp}$ and $H_{n\perp}$ are gauge operators (46) for the transverse variables. The condition that the theory be Lorentz-invariant requires

$$[J^{-i},\,J^{-j}] = 0. \tag{57}$$

This condition holds only if $D = 10$ and $\alpha(0) = 0(\varepsilon = 1)$ and $\alpha(0) = -1/2(\varepsilon = -1)$.

The mass of a spinning string is expressed in terms of the independent variables as follows:

$$
\begin{aligned}
\alpha' M^2 = \sum_{n>0}(:\boldsymbol{\alpha}_{-n\perp}\boldsymbol{\alpha}_{n\perp}: + n:\mathbf{b}_{-n\perp}\mathbf{b}_{n\perp}:)\\
+\left\{\begin{array}{ll} 0, & \varepsilon = +1,\\ -\frac{1}{2}, & \varepsilon = -1. \end{array}\right.
\end{aligned}
\tag{58}
$$

Tachyons appear in the spectrum of dual strings because of the incorporation of the energy of the zero-point vibrations of the harmonic oscillators which describe the dynamics of the string.[50] When these vibrations are taken into account, the classical expression in the formula for the square of the mass of a boson string, (21), i.e.,

$$\sum_{n=1}^{\infty}\sum_{i=1}^{D-2}\alpha_{-n}^i\alpha_n^i = \sum_{n=1}^{\infty}\sum_{i=1}^{\infty} n a_n^{+i} a_n^i,\quad (n)^{1/2} a_n^i = \alpha_n^i,\quad n \geqslant 1, \tag{59}$$

must be replaced in the quantum theory by the operator

$$\sum_{n=1}^{\infty} n\sum_{i=1}^{D-2}\left(a_n^{+i}a_n^i + \frac{1}{2}\right) = \frac{D-2}{2}\sum_{n=1}^{\infty} n + \sum_{n=1}^{\infty}\sum_{i=1}^{D-2} n a_n^{+i} a_n^i. \tag{60}$$

We thus have

$$2\alpha(0) = -(D-2)\sum_{n=1}^{\infty} n.$$

This divergent series must be regularized. Comparing it with the Riemann ζ function[51]

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}, \tag{61}$$

which can be analytically continued to the point $s = -1$, and which has the value $\zeta(-1) = -1/12$, we find

$$\alpha(0) = \frac{D-2}{24}. \tag{62}$$

A renormalization procedure of this sort is consistent with the relativistic invariance of the quantum theory of a boson string, which requires $D = 26$ and $\alpha(0) = 1$, as was shown earlier.

We now see why the theory of a spinning string yields a quantum solution without tachyons. The energy of the zero-point vibrations of the fermion operators $\mathbf{b}_{in}^{+}, \mathbf{b}_{in}$ is negative and equal in magnitude to the energy of the zero-point vibrations of the boson operators $\mathbf{a}_{in}^{+}, \mathbf{a}_{in}$ (Ref. 52 and 53). As a result, they cancel out, since the number of independent boson and fermion degrees of freedom in a spinning string is identical and equal to 8.

2.8. Total action for a spinning string

A shortcoming of the spinning-string theory described above is that only equations of motion (37) are derived by a variational approach from action (41); gauge conditions (38) and (39) are postulated additionally. We are interested in the total action for a spinning string, from which we would be able to obtain the entire dynamics by means of a variational principle, i.e., equations of motion (37) and gauge conditions (38), (39) or (43), (44). Such an action has been constructed[54–59] by expanding the symmetry of the theory under transformations (52), (53) to generally covariant transformations in the two-dimensional space τ,σ and under local supersymmetric transformations. Nambu-Goto action (1) proved inconvenient for this generalization, since the coordinates of the string appear in it in a nonpolynomial form. For a boson string, a new action has been proposed which is quadratic in the coordinates of the string[54]:

$$\begin{gathered} S' = -\frac{\gamma}{2} \iint\limits_{\Omega} d^2u \mid g \mid^{1/2} g^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x_\mu, \\ u = (u^0, u^1), \quad u^0 = \tau, \quad u^1 = \sigma. \end{gathered} \tag{63}$$

Here $x^\mu(u)$ are as before the coordinates of the string, $g_{\alpha\beta}(u)$ is an auxiliary field of a second-rank symmetric tensor, specified in Ω, and the constant γ has the dimensionality of a mass square. Below we will set it equal to 1. Action (63) is invariant under global Poincaré transformations

$$\delta x^\mu = \omega^\mu{}_\nu x^\nu + a^\mu, \quad \delta g_{\alpha\beta} = 0, \tag{64}$$

under generally covariant transformations of the parameters u^0, u^1,

$$\delta x^\mu = \xi^\alpha \partial_\alpha x^\mu, \quad \delta g_{\alpha\beta} = \xi^\gamma \partial_\gamma g_{\alpha\beta} + \partial_\alpha \xi^\gamma \cdot g_{\gamma\beta} + \partial_\beta \xi^\gamma \cdot g_{\alpha\gamma}. \tag{65}$$

and under Weyl transformations

$$\delta g_{\alpha\beta} = \lambda(u)\, g_{\alpha\beta}, \quad \delta x^\mu = 0. \tag{66}$$

At the classical level, S' is completely equivalent to Nambu-Goto action (1). A variation of S' with respect to x^μ and $g_{\alpha\beta}$ yields the equations of motion

$$\frac{1}{\mid g \mid^{1/2}} \frac{\partial}{\partial u^\alpha} (g^{\alpha\beta} \mid g \mid^{1/2} \partial_\beta x^\mu) = 0, \tag{67}$$

$$\frac{\delta S'}{\partial g^{\alpha\beta}} = \frac{1}{2} \mid g \mid^{1/2} T_{\alpha\beta}$$

$$= -\frac{1}{2} \mid g \mid^{1/2} \left(\partial_\alpha x^\mu \partial_\beta x_\mu - \frac{1}{2} g_{\alpha\beta} \partial_\gamma x^\mu \partial_\rho x_\mu g^{\gamma\rho} \right) = 0; \tag{68}$$

here $T_{\alpha\beta}$ is a symmetric "metric" energy-momentum tensor of the fields $x^\mu(u)$. From (68) we find that the auxiliary field $g_{\alpha\beta}(u)$ is equal (within an arbitrary factor) to the metric on the world surface of the string:

$$g_{\alpha\beta} = \partial_\alpha x^\mu \partial_\beta x_\mu. \tag{69}$$

In this case, (67) reduces to the equations of motion found from Nambu-Goto action (1), and (63) becomes the same as (1).

It is of interest to see how orthonormal gauge conditions (2) arise when we work with action (63). Making use of the invariance of S' under transformations (65) we can always put the second-rank tensor field $g_{\alpha\beta}$ in diagonal form: $g_{\alpha\beta}(u) = \exp[\varphi(u)]\eta_{\alpha\beta}, \eta_{\alpha\beta} = \mathrm{diag}(1, -1)$. The equations of motion for $g_{\alpha\beta}(u)$, (68), then reduce to (2). The tensor field $g_{\alpha\beta}(u)$ in a classical string theory based on action (63) thus plays the role of Lagrange multipliers. At the classical level, actions (1) and (63) are therefore completely equivalent. Upon quantization of action (63), however, as Polyakov[40] has shown, it becomes possible in principle to construct a noncontradictory quantum theory of a relativistic string which differs from the standard approach at $D \neq 26$. At the quantum level, we can give up the invariance of the string theory under Weyl transformations (60) [Nambu-Goto action (1) does not have this invariance] and take into account the conformal anomaly. As a result, the conformally planar part of the metric $\varphi(u)$ becomes a dynamic variable which obeys a nonlinear two-dimensional Liouville equation, $\partial^2\varphi + \mu_0^2 e^\varphi = 0$.

Volkov and Zheltukhin[60] have proposed yet another action for a closed Nambu-Goto string, one which is quadratic in the coordinates of the string and which allows a supersymmetric generalization.

The total action for a spinning string[54,57] is found by introducing fermion fields in a supersymmetric fashion in (63). The metric tensor $g_{\alpha\beta}(u)$ is "split up" in the standard way with the help of moveable-reference vectors $V_a^\alpha(u)$: $g^{\alpha\beta} = V_a^\alpha V_b^\beta \eta^{ab}, \eta^{ab} = \mathrm{diag}(1, -1)$. The action is written

$$\begin{aligned} S_{sp} = -\frac{\gamma}{2} \int d^2u V \Big[& g^{\alpha\beta} \partial_\alpha x^\mu \partial_\beta x_\mu + i V_a^\alpha \bar{S}^\mu \rho^a \partial_\alpha S_\mu \\ & + 2 V_a^\alpha V_b^\beta \bar{\psi}_\alpha \rho^b \rho^a S^\mu \left(\partial_\beta x_\mu + \frac{1}{2} \bar{S}_\mu \psi_\beta \right) \Big], \end{aligned} \tag{70}$$

where $V = \det \| V_a^\alpha \|$, and $\psi_\alpha(u)$ is a spin 3/2 field. This action is invariant under local two-dimensional Lorentz transformations in the τ,σ space; under Weyl transformations,

$$x \to x, \quad S \to \lambda^{-1/2} S, \quad V_a^\alpha \to \lambda V_a^\alpha, \quad \psi_\alpha \to \lambda^{1/2} \psi_\alpha; \tag{71}$$

and under local supersymmetric transformations,

$$\begin{gathered} \delta x^\mu = i\bar{\varepsilon}(u) S^\mu, \quad \delta S = (\partial_\alpha x + i\bar{S}\psi_\alpha)\, \rho^a V_a^\alpha \varepsilon(u), \\ \partial V_a^\alpha = -2i\bar{\varepsilon}(u)\, V_b^\alpha V_a^\beta \rho^b \psi_\beta, \quad \delta\psi_\alpha = -D_\alpha \varepsilon(u), \end{gathered} \tag{72}$$

where D_α is a covariant derivative of the spinor in two dimensions. The Rarita-Schwinger field $\psi_\alpha(u)$ and the moveable reference $V^\alpha_a(u)$ serve as auxiliary variables. By virtue of the invariance of the theory under supersymmetric transformations (72), we can always choose a gauge in which the relations

$$V^\alpha_a(u) = h(u)\delta^\alpha_a, \quad \psi_\alpha(u) = 0 \tag{73}$$

hold (a superconformal gauge). As a result, the equations of motion for x^μ and S^μ which follow from (70) reduce to (37); the equations of motion for the moveable reference V^α_a give us gauge conditions (38) in the form in (43); and the Euler equations $\delta S_{\rm sp}/\delta\psi^\alpha = 0$ reduce to conditions (39) in the form in (44).

3. DYNAMICS OF SUPERSTRINGS AND THEIR RELATIONSHIP WITH ELEMENTARY PARTICLE PHYSICS

3.1. The superstring; action functional and dynamic variables in the lightlike gauge

A superstring[15,16,61–63] is a modification of a spinning relativistic string. In this model, a different description is used for the spin variables which are distributed along the string.[62] In a spinning string, this description is made by means of the Grassmann variables $S^\mu_A(\tau,\sigma)$, $\mu = 0, 1, \ldots, D-1$, $A = 1,2$ which constitute a Lorentz vector in a D-dimensional Minkowski space and a Majorana spinor in a two-dimensional space which is tangential to the world surface of the string. The supersymmetry in the spinning string actually refers to the two-dimensional space τ,σ, rather than to the enveloping ten-dimensional Minkowski space. We can attempt to expand this supersymmetry. A direct construction of an $(N=2)$-supersymmetric string theory through a doubling of the number of fields[58] leads to a model with a critical dimensionality $D = 2$. This case is obviously of no physical interest.

Another approach is to seek new representations of a fermion field specified along a string. For supersymmetry in the theory, the number of physical degrees of freedom of the fermion field must be equal to the number of boson degrees of freedom, i.e., to the number of transverse components of the vector $x^\mu(\tau,\sigma)$ (this number is eight in the case of a ten-dimensional space-time). The simplest way to construct a superstring theory in the lightlike gauge is to work exclusively with the independent dynamic variables. The Lie algebra of the SO(8) group has three real, nonequivalent, eight-dimensional representations, one vector representation, and two spinor representations. Consequently, a switch from a spinning string to a superstring is made by means of the substitution

$$S^i_A(\pi,\sigma) \to Q^a_A(\tau,\sigma), \tag{74}$$

where $i = 1,2,\ldots,D-2$; $A = 1,2$ is the spin index in the two-dimensional space τ,σ; and a is an eight-digit spin index with respect to the SO(8) group. Substituting (74) into (55), we find[61]

$$S = \frac{T}{2}\int\int d\tau\, d\sigma\, [\eta^{\alpha\beta}\partial_\alpha x^i\partial_\beta x^i + i\bar{Q}^a\rho^\alpha\partial_\alpha Q^a]. \tag{75}$$

All the equations from the noncovariant quantum theory of a spinning string are brought over to superstring theory by substitution (74). Again, there is relativistic invariance only in a ten-dimensional space-time, and the ground state of the theory is massless (there are no tachyons). We use boundary conditions as in the case of Ramond's spinning string.[40] Consequently, the expansion of the Grassmann variables is carried out in terms of integer modes:

$$\begin{aligned} Q^a_1(\tau,\sigma) &= \frac{1}{2\sqrt{\pi}}\sum_n Q^a_n \exp[-ik(\tau+\sigma)],\\ Q^a_2(\tau,\sigma) &= \frac{1}{2\sqrt{\pi}}\sum_n Q^a_n \exp[-ik(\tau-\sigma)]. \end{aligned} \tag{76}$$

In the quantum theory, the operators Q^a_n obey the anticommutation relations

$$[Q^a_n, Q^b_m]_+ = \delta_{n+m,0}\delta_{ab}. \tag{77}$$

The operators $Q^a{}_{-n}$, $n>0$ are creation operators. When they act on boson states, they convert the latter into fermion states; the numbers of boson states and fermion states are the same in each supermultiplet with a fixed mass.

To test the relativistic invariance of the theory of Ref. 15, we need to construct the dependent operators x^-, Q^-, and J^{i-} and to make sure that the Poincaré algebra is satisfied with $D = 10$.

3.2. Total action for a superstring

A total action[64] from which both dynamic equations and gauge conditions follow has been constructed for a superstring, by analogy with a spinning string [see (70)]. The complete set of dynamic variables for a superstring consists of the space-time coordinates $x^\mu(\tau,\sigma)$ and the anticommuting Grassmann variables $\theta^a_A(\tau,\sigma)$. $A = 1,2$. The action is constructed in a ten-dimensional Minkowski space. The spinor index a takes on $2^5 = 32$ values, since the dimensionality of the Dirac γ matrices in a D-dimensional space-time is 2^k, where k is the greatest integer in $D/2$.

It is required that the variables θ^a_A be a Majorana-Weyl spinor in the index a:

$$h^{ab}\theta^b_A = 0, \quad \bar\theta_A = -\theta_A\gamma^0, \quad A = 1, 2, \tag{78}$$

where h represents the Weyl projection operator $h = (1\pm\gamma_{11})/2$. A condition of this sort can be satisfied only in a space-time with $D = 2 \pmod 8$, i.e., $D = 2, 10, \ldots$. Under conditions (78), we have two possibilities: θ^a_1 and θ^a_2 may have the same chirality or opposite chiralities. The complex variables $\theta^a_A(\tau,\sigma)$, $A = 1,2$, contain 2^7 independent real functions. Conditions (78) and also the boundary conditions and the Dirac equation for θ^a_A reduce the number of independent fermion degrees of freedom to $2^3 = 8$.

The dynamics of a free superstring is determined by the action

$$S = \frac{T}{2}\int\int d\tau\, d\sigma\, (\mathcal{L}_1 + \mathcal{L}_2), \tag{79}$$

where

$$\mathcal{L}_1 = -\frac{1}{2\pi}(-g)^{1/2} g^{\alpha\beta}\pi^\mu_\alpha\pi_{\mu\beta}, \quad \pi^\mu_\alpha = \partial_\alpha x^\mu - i\bar\theta_A\gamma^\mu\partial_\alpha\theta_A, \tag{80}$$

$$
\mathscr{L}_2 = -i\varepsilon^{\alpha\beta}\partial_\alpha x^\mu (\bar{\theta}_1\gamma_\mu\partial_\beta\theta_1 - \bar{\theta}_2\gamma_\mu\partial_\beta\theta_2) + \varepsilon^{\alpha\beta}\bar{\theta}_1\gamma_\mu\partial_\alpha\theta_1\bar{\theta}_2\gamma_\mu\partial_\beta\theta_2,
$$
$$
\alpha, \beta = 0, 1, \quad \mu = 0, 1, \ldots, 9. \tag{81}
$$

In addition to local reparametrized invariance, this action has global ($N = 2$) supersymmetry under the transformations

$$
\begin{gathered}
\delta\theta_A = \frac{1}{4}\omega_{\mu\nu}\gamma^{\mu\nu}\theta_A + \varepsilon_A, \\
\delta x^\mu = \omega^\mu{}_\nu x^\nu + a^\mu + i\bar{\varepsilon}_A\gamma^\mu\theta_A, \quad \delta g^{\alpha\beta} = 0.
\end{gathered} \tag{82}
$$

The boundary conditions on θ_A lower this symmetry to $N = 1$. Action (79) is also invariant under local supersymmetric transformations.

The total action of the superstring, (79), leads to primary and secondary constraints. An important point is that it is not possible to identify explicitly all the primary constraints by a Lorentz-invariant method.[64–68] An analogous situation prevails for a supersymmetric massless point particle.[66]

Local supersymmetry and a reparametrized invariance nevertheless make it possible to go over to the lightlike gauge, $x^+ \sim \tau, \gamma^+\theta_A = 0$. In this gauge, the remaining independent variables satisfy free equations of motion generated by action (75).

The second term in the total action of the superstring, (79), which is generated by L_2, is analogous to the auxiliary Wess-Zumino term in the action for nonlinear sigma models.[69] This increment leads to an additional local fermion symmetry in superstring action (79); this local symmetry is characteristic exclusively of two dimensions.

To unify all the fundamental interactions on the basis of a superstring theory we need to consider the fact that the dimensional constant T in the superstring action (75) or (79) satisfies the order-of-magnitude relation $T^{-1/2} \sim 10^{-33}$ cm, i.e., the superstrings must have dimensions in the Planck range.

3.3. Field theory of superstrings

Up to this point we have been talking exclusively about a first quantization of string models, i.e., about the quantum mechanics of strings. A complete quantum theory of strings requires a description of the processes by which strings are created and annihilated and of their mutual conversions; in other words, we need a second-quantized field theory of relativistic strings.

Attempts to reproduce dual loop diagrams in string models have shown that the interaction of strings may be exceedingly specific: The strings must interact in a strictly local way, at one point. For example, the ends of an open string can be joined together, with the result that an open string becomes a closed string, etc.

The second-quantized string functional $\Psi[x(\sigma)]$ is conveniently expanded in the eigenstates of the operator representing the squared mass of the string, M^2. For an open boson string we have[79]

$$
\Psi[x(\sigma)] = \{\varphi(x) + A^{(1)}_\mu(x)\alpha^\mu_{-1} + h_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1} + A^{(2)}_\mu(x)\alpha^\mu_{-2} + \ldots\}\,|\,0\rangle.
$$

The coefficients in this expansion are ordinary local fields. Conditions (10), brought over to the field theory of strings,

$$
L_n\Psi[x(\sigma)] = \delta_{n,0}\Psi[x(\sigma)], \quad n = 0, 1, 2, \ldots,
$$

give us equations of motions for these fields:

$$
\begin{gathered}
L_0\varphi(x) = -\alpha' P^2\varphi(x) = \alpha'\partial^2\varphi(x) = \varphi(x), \\
(\alpha'\partial^2 - 1 + n)A^{(n)}_\mu(x) = 0, \quad \partial^\mu A^{(1)}_\mu(x) = 0, \\
(\alpha'\partial^2 + 1)h_{\mu\nu}(x) = 0,
\end{gathered}
$$

etc. Accordingly, scalar field $\varphi(x)$ is the field of a tachyon with a squared mass $(\alpha')^{-1}$; $A_\mu(x)$ is the electromagnetic field, etc. In a field theory of open strings there is no symmetric, massless second-rank tensor field which can be identified with the gravitational field. Gravitation is described by closed strings.

A complete quantum field theory of interacting strings has yet to be derived. The first efforts in this direction were taken[70–72] back in the early 1970s, at which time relativistic strings were regarded as the dynamic basis of dual-resonance models in hadron physics. The analogs of Feynman diagrams in a field theory of strings are two-dimensional surfaces (the world surfaces of the strings), which may in general have a complicated topological structure. In terms of Feynman path integration it is necessary to carry out a summation over all these surfaces for given initial and final configurations of the strings (Fig. 2). This step requires determining the measure in the space of two-dimensional surfaces.[40,130–132] This problem has been solved[133] in general form for surfaces of type p in the theory of closed boson strings. A Riemann surface of type p is topologically equivalent to a sphere with p handles. It describes a p-loop string diagram. For $p = 2,3$, explicit expressions have been derived.[130–133] In this approach we must deal with the fundamental question of which weights we should assign to the contributions from the surfaces of various types to an amplitude or to a partition function. A guiding principle here should be the requirement that the string amplitude be unitary.

A simpler situation is that in which the functional integration is carried out only over physical transverse variables of the string. It was in the lightlike gauge that the theory of

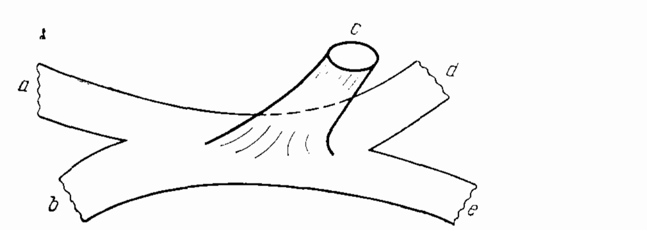

FIG. 2. Interaction of open and closed relativistic strings, describing the process $a + b \to c + d + e$. The two open strings a and b, whose ends on becoming connected, form a single open string, which generates a closed string c and two open strings d and e.

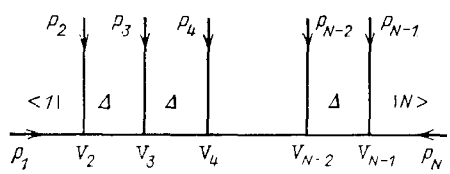

FIG. 3. Tree dual diagram.

interacting strings was constructed in the earliest papers,[70,71,74] which used a functional integration for this purpose. A shortcoming of that approach is that the explicit Lorentz invariance is lost.

Practical calculations are carried out in the field theory of strings by making use of rules for calculating dual tree diagrams and loop diagrams with corresponding corrections for the supersymmetry of the theory. These rules can be outlined schematically in the following way: The N-particle tree amplitude (Fig. 3) is of the form[22,23,75]

$$T_N = \langle 1 | V_2 \Delta \cdot V_3 \cdot \Delta \ \ldots \ \Delta \cdot V_{N-1} | N \rangle, \tag{83}$$

where the propagator

$$\Delta = \frac{\alpha'}{L_0}, \tag{84}$$

describes the propagation of the string, and V is a vertex operator. Loop diagrams are constructed from the tree diagrams by closing the external lines. We should point out that this method can be used to construct string amplitudes only on the mass shell.

Attempts have been made[76,77] to derive a covariant field theory of strings by the BRST formalism, i.e., to carry out a second quantization of a string quantum mechanics constructed in the BRST formalism (Subsection 2.3). The complete string field incorporates dynamic and ghost fields and also some auxiliary fields which are required to close the BRST algebra on the mass shell. Functionals of Grassmann variables arise in this approach. A noncommutative geometry may prove useful here.[78]

Some other approaches to a field string theory are described in Refs. 80 and 81.

3.4. Internal degrees of freedom in string models

Superstring theories, like the simpler string models, do not allow a deviation from the properties of uniformity along the string, since such deviations would immediately lead to a loss of parametrization invariance of the theory and to a violation of the gauge algebra (in the boson case, this is the Virasoro algebra). As was mentioned earlier, in hadron physics the strings are associated with tubes or braids of a gluon field which connect quarks. It would thus appear to be completely natural to attempt to introduce some auxiliary terms in the action of the string to describe point masses, charges, and spins at the ends of a string. However, even such a minimal violation of the uniformity of relativistic strings leads to a fundamental change in their dynamics; e.g., the relationship with dual models is immediately lost.

Internal degrees of freedom are introduced in string models *in a purely kinematic, rather than dynamic, way*. For this purpose one uses the Chan-Paton mechanism, which was developed in dual-resonance models. This mechanism can be summarized by the following recipe.[82]

With each vertex in the string diagram we associate a generator λ_{a_i} of group G. The factors λ_{a_i} are multiplied in the order in which the corresponding vertices appear in the diagram, and a trace is taken over the entire product of matrices:

$$\mathrm{tr}\,(\lambda_{a_1}\lambda_{a_2} \ldots \lambda_{a_N}). \tag{85}$$

For dual amplitudes, the factorization

$$\mathrm{tr}\,(\lambda_{a_1}\lambda_{a_2} \ldots \lambda_{a_N}) = \sum_a \mathrm{tr}\,(\lambda_{a_1}\lambda_{a_2} \ldots \lambda_{a_M}\lambda_a)\,\mathrm{tr}\,(\lambda_a\lambda_{a_{M+1}} \ldots \lambda_{a_N}) \tag{86}$$

of product (85) was an important property; it guaranteed conservation of the duality properties of the amplitude upon the introduction of an internal symmetry. Property (86) is not satisfied by just any arbitrary group G. Chan and Paton[82] proposed this mechanism for the group $G = \mathrm{SU}(N)$. In a quantum field theory of strings, however, this method for introducing internal quantum numbers turns out to be consistent only for the groups $G = \mathrm{SO}(N)$ and $G = \mathrm{USp}(N)$.

Bardakci and Halpern[83] examined a method for introducing internal quantum numbers in dual models which differs from the Chan-Paton technique, but that method has not proved popular. Higher-order representations of the internal symmetry group appear in the amplitudes.

A fundamentally new mechanism for introducing an internal symmetry in a string theory was proposed in Ref. 84 in the construction of a heterotic string (Subsection 3.8). That mechanism is based on the theory of representations of infinite-dimensional Kac-Moody algebras for which the theory has recently been developed.[29]

3.5. Low-energy (local field) limit in the theory of interacting strings

A fundamentally new way of looking at string theories was suggested in 1974 by Scherk and Schwarz.[85] Analyzing the low-energy limit, $\alpha' \to 0$, in the theory of closed strings, which model the path of a pomeron, they showed that a massless spin 2 state behaves in this limit in exactly the same way as a graviton would, i.e., it obeys the same dynamic equations which follow for the quanta of the gravitational field in Einstein's theory. String models are thus candidates for the role of unified theories incorporating gravitation.

In contrast with a local field theory in which each field describes quanta (particles) of only one type, a free superstring carries an infinite number of supermultiplets corresponding to normal vibration modes of the string. The supermultiplet of the ground state is massless; the excited states have masses and angular momenta (spins), which increase without bound. The mass scale here is the tension T of a superstring with the dimensionality $[M]^2$; the value of $T^{1/2}$ is of the same order of magnitude as the Planck mass, $T^{1/2} \sim 10^{19}$ GeV. This situation is quite natural for a model which is to incorporate a quantum theory of gravitation. *In the limit in which the energy is considerably smaller than*

$T^{1/2}$, the states of a superstring with a nonzero mass drop out of the picture, and the effective low-energy field theory incorporates only the massless ground supermultiplet. It is specifically these multiplets that should be filled by the elementary particles which are presently observed experimentally, since they should all be regarded as massless in comparison with the Planck mass ($\sim 10^{-5}$ g).

The low-energy limit in a string theory is constructed in the following way. One first uses the tree approximation to calculate an N-point string amplitude with massless external states in the limit $\alpha' \to 0$ (Subsection 3.3). One then chooses an action for the local field theory which reproduces the string amplitude at the level of the tree Feynman diagrams. The loop diagrams in the field theory constructed in this fashion should reproduce the loop corrections in the string theory. It is not obvious at the outset that a local quantum field model of this sort exists. The existence requires special proof.[86,87] In practice, one usually calculates the vertex for a gauge interaction in the lowest nontrivial order, and a higher-order vertex is reconstructed from gauge invariance.

This procedure is somewhat ambiguous, since the dual diagram technique (there simply are no other developed methods) can be used to calculate string amplitudes only on the mass shell. Consequently, the corresponding low-energy field theory can be derived up to terms which do not contribute to string amplitudes on the mass shell.

Yet another way to find a low-energy field limit in a string theory is based on the method of a background field.[88] One examines the interaction of a string with massless background fields (a graviton, a gravitino, a dilaton, etc.). One then calculates an effective Lagrangian for these fields under certain consistency conditions.

3.6. Classification of superstring theories

The superstring theories[94,95] of type I describe open strings and closed unoriented strings (see Subsection 2.2. regarding the concept of orientation in a theory of closed strings). The ends of open strings carry the quantum numbers of the gauge groups $G = \mathrm{SO}(N)$ or $G = \mathrm{USp}(N)$ in accordance with the Chan-Paton formalism. The unitary groups $\mathrm{SU}(N)$ do not allow a quantum analysis. The theory is locally invariant under two supersymmetries, but the boundary conditions allow only one. The massless states of these strings are states of a Yang-Mills supersymmetric theory in a ten-dimensional space-time with a gauge group G.

Superstring theories of type II describe only closed oriented strings. In the theories of type IIa, the supercharges have opposite chiralities. The low-energy limit of such theories is a nonchiral ($N = 2$) supergravity in a ten-dimensional space-time. In superstring theories of type IIb the supercharges have the same chirality. In the low-energy limit these superstring theories reduce to a chiral ($N = 2$) supergravity,[89–92] which has recently been shown[93] to be free of gravitational anomalies. A shortcoming of superstring theories of type II is the absence of any non-Abelian gauge symmetry. It is thus unlikely that these theories would be able to give us a chiral compactified field theory of physical interest.

A heterotic string (Subsection 3.8) furnishes two more examples of noncontradictory superstring theories with the gauge groups SO(32) and $E_8 \times E_8$.

The fact that string models incorporate gauge theories and a generally covariant theory of gravity is rather unexpected, since neither gauge invariance nor general covariance is built into these models. The basic properties of invariance in a string theory is a reparametrization invariance of the action of the string (a two-dimensional general covariance in τ,σ space). This question would probably have been resolved if it had been found possible to formulate a quantum field theory of strings in a general covariant fashion in the space of string configurations, e.g., in the space of loops in the case of closed strings.

There can be two types of local interactions of superstrings.

Two free ends of two open strings or of the same string may join together. The result is a single open or closed string (Fig. 4). The inverse process is also possible, i.e., the breaking of one open string in two or the breaking of a closed string. Such an interaction of superstrings is a "Yang-Mills" interaction.

A second type of interaction of superstrings is of a "gravitational" nature, when two interior points of two strings or of the same string come in contact (Fig. 5).

Significantly, superstring theory has no contact interactions of a higher order—involving the interaction of three or more points. Vertices containing a large number of gravitons are reproduced in the string approach as low-energy effective vertices which arise in the exchange of massive string modes.

In the low-energy field theory which follows from superstrings with the gauge group SO(32), the Yang-Mills interaction constant g and the Newtonian gravitational constant $\varkappa$ are related by

$$\varkappa = \mathrm{const} \cdot g^2 T, \tag{87}$$

where T is the tension of the superstring.

3.7. Cancellation of anomalies and the problem of divergences

A remarkable property of superstring theories is the absence (i.e., the cancellation) of gauge and gravitational anomalies. This fact, which was established[96] in 1984, attracted even more interest to these theories.

Anomalies in quantum field models stem from the breaking of classical symmetries[97–101] (gauge invariance, Lorentz invariance, etc.) at the quantum level. Anomalies of this sort were first discovered as an inconsistency in the de-

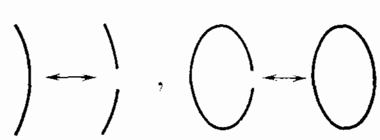

FIG. 4. "Yang-Mills" interaction of superstrings, characterized by the constant g.

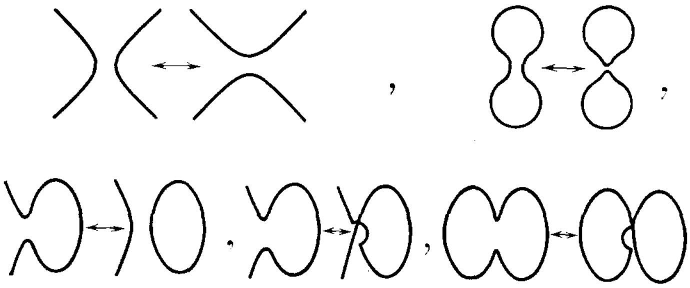

FIG. 5. "Gravitational" interaction of superstrings, characterized by the constant $\varkappa$.

scription of the decay of the π^0 meson into two γ rays in field theories with a pseudoscalar and pseudovector interaction in the calculation of a single-loop diagram with three virtual fermion lines. It was later established that in gauge theories with chiral fermions anomalies are generally manifested as a noninvariance of an effective fermion action $\Gamma_f[A]$ under gauge transformations of the vector potential A.

Gravitational anomalies are evidence of a disruption at the quantum level of the general covariance of the theory or of a local Lorentz invariance. Gauge and gravitational anomalies may be thought of as the violation upon quantization of corresponding conservation laws: the law expressing the conservation of a gauge current or a covariant conservation law for the energy-momentum tensor. Anomalies make a theory inconsistent, since they lead to a violation of unitarity because of the interaction of physical transverse modes of the gauge or gravitational field with nonphysical longitudinal modes. The requirement that there be no anomalies identifies those field theories which will be viable at the quantum level. For example, when quarks and leptons are combined in a generation, anomalies cancel out in the Weinberg-Salam model.

In a Minkowski space of even dimensionality $D = 2n$, gauge anomalies arise in single-loop Feynman diagrams with $n + 1$ and with a larger number of external lines of gauge fields (Fig. 6). Chiral fermions circulate around the loops. If $D = 4k + 2$, the theory may also have gravitational anomalies. A significant point is that the anomalies corresponding to diagrams of higher order are unambiguously determined by the contribution of the lowest anomalous diagrams.

In superstring theories formulated in a ten-dimensional Minkowski space, gravitational, gauge, and mixed anomalies arise in six-vertex loop diagrams. The cancellation of anomalies in superstring models with the SO(32) internal symmetry group has been verified in two ways: at the string level, by a direct calculation of the six-vertex string diagrams and in the low-energy limit at the local quantum field level.

Figure 7 shows some single-loop string diagrams of sixth order. The external wavy lines correspond to Yang-Mills states. These diagrams can have topologies of three types: 1) planar ring diagrams with the topology of a cylinder, in which the external lines are linked with only one boundary (Fig. 7a); 2) nonplanar diagrams with the topology of a Möbius sheet (Fig. 7b); 3) planar diagrams whose external lines approach both boundaries (Fig. 7c). Diagrams of the third type have no anomalies at all.

The contribution of ring diagrams (Fig. 7a) is written (Subsection 3.4)

$$N \operatorname{tr}(\lambda_1\lambda_2 \ldots \lambda_6) \int d^{10}p \operatorname{Tr}[\Delta\cdot V(1)\, \Delta\cdot V(2) \ldots \Delta\cdot V(6)], \tag{88}$$

where tr means the trace of the matrices in the fundamental representation of the group G of the internal symmetry of the string theory, and Tr means the trace of the matrices in the associated representation of group G. The group factor in front of the integral in (88) is the Chan-Paton factor, and the λ_i are the generators of group G in the fundamental representation (they are anti-Hermitian $N \times N$ matrices). The internal boundary provides a factor $N = \operatorname{tr}(1)$, i.e., the dimensionality of the fundamental representation of group G. A correct calculation requires regularization of the integrand in (88). Two regularizations were used in Ref. 96: a Pauli-Villars regularization and a Gaussian regularization.

The nonplanar diagrams in Fig. 7b contain, instead of the factor N, a factor l, which takes on different values for different groups G:

$$l = \begin{cases} +1, & \mathrm{USp}(N), \\ 0, & \mathrm{U}(N), \\ -1, & \mathrm{SO}(N). \end{cases} \tag{89}$$

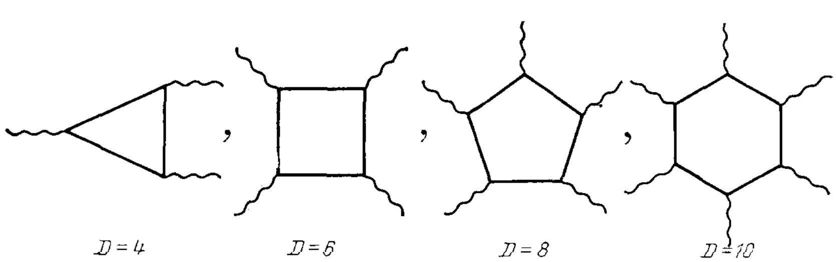

FIG. 6. Single-loop Feynman diagrams which give rise to gauge anomalies for various dimensionalities of space-time, D.

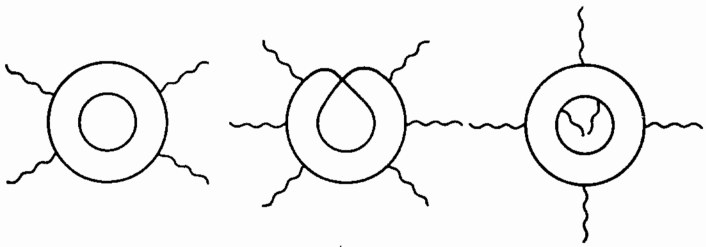

FIG. 7. Single-loop string diagrams with six external lines of massless gauge bosons. a—Planar ring diagram; b—diagram with the topology of a Möbius sheet; c—nonplanar oriented diagram with external lines approaching both boundaries. Gauge anomalies give rise to diagrams of the types in parts a and b.

A mutual cancellation of the anomalous contributions from the diagrams of the first two types is thus possible only for $G = SO(N)$. The contribution of single-loop string diagrams of sixth order turns out to be proportional to $N + 32l$ so that only superstring theories with the SO(32) group are free of anomalies.

In the low-energy field limit, a superstring theory of type I reduces to a $(D = 10, N = 1)$ supergravity and a $(D = 10, N = 1)$ supersymmetric Yang-Mills theory. A supergravitational multiplet contains the following fields. A moving n-hedral e_μ^m, an antisymmetric second-rank tensor $B_{\mu\nu}$, a scalar field φ (a dilaton), a spin-3/2 gravitino ψ_μ^+, a spin-1/2 field λ^- (the $\pm$ represent the chirality of the corresponding field), the vector potential A^a, and its spin-1/2 fermion partner χ^+ (A^a and χ^+ belong to an associated representation of the gauge group). The boson sector of the low-energy superstring theory is described by the action

$$S_0 = \int d^{10}x \cdot e \left(-\frac{1}{2\varkappa^2} R - \frac{1}{\varkappa^2} \varphi^{-2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{4g^2} \varphi^{-1}F^a_{\mu\nu}F^{\mu\nu a} - \frac{3\varkappa^2}{2g^4} \varphi^{-2}H_{\mu\nu\rho}H^{\mu\nu\rho} \right), \tag{90}$$

where

$$e = \det \| e_\mu^m \|, \quad F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu = dA + A \wedge A,$$

and R is the curvature. The 1-form A_μ (a potential) is a matrix representation of the gauge algebra $A = A_\mu^a \lambda^a dx^\mu$ where λ^a are anti-Hermitian matrices in the fundamental representation of the internal symmetry group of the string theory. As Green and Schwarz[96] have shown a cancellation of anomalies at the field level occurs for two gauge groups: SO(32) and $E_8 \times E_8$ which have the same rank, 16.

An important point in the anomaly cancellation mechanism is the need to modify the ordinary $(N = 1)$ supergravity which is interacting with an $(N = 2)$ supersymmetric Yang-Mills theory. In a determination of the intensities H of gauge field B, some additional topological terms must be introduced:

$$H = dB - \omega_{3Y} + \omega_{3L}, \tag{91}$$

where ω_{3Y} is the Yang-Mills Chern-Simons 3-form, and ω_{3L} is a Lorentz Chern-Simons 3-form. For the SO(32) group we have

$$\omega_{3Y} = \mathrm{tr}\left(A \wedge F - \frac{1}{3} A \wedge A \wedge A\right), \tag{92}$$

where tr is the trace of the matrices in the SO(32) vector represenation, and $\wedge$ means the outer product. For ω_{3Y} there is an alternative way to write this:

$$\omega_{3Y} = \frac{1}{30}\mathrm{Tr}\left(A \wedge F - \frac{1}{3} A \wedge A \wedge A\right), \quad d\omega_{3Y} = \frac{1}{30}\mathrm{Tr}\,(F \wedge F). \tag{93}$$

This version is valid for the two groups SO(32) and $E_8 \times E_8$. In (93), Tr means the trace in the associated representation of these groups.

The Lorentz Chern-Simons 3-form ω_{3L} is given by

$$\omega_{3L} = \mathrm{tr}\left(\omega \wedge R - \frac{1}{3}\omega \wedge \omega \wedge \omega\right), \quad d\omega_{3L} = \mathrm{tr}\,(R \wedge R), \tag{94}$$

where $\omega = \omega_\mu dx^\mu$ is a local Lorentz connection, which takes on values in the Lie algebra of the SO(1,9) group and $R = d\omega + \omega \wedge \omega$ is the curvature 2-form. In (94), tr means the trace in the SO(1,9) vector representation.

The anomalous contributions of the single-loop six-vertex diagrams (Fig. 6c) turn out to be proportional to the difference $(n - 496)$, where n is the number of chiral fields with spin 1/2. We thus need 496 spin-1/2 fields to cancel the anomalies. However, the dimensionality of the associated representation of the SO(32) and $E_8 \times E_8$ groups is specifically 496; for the $E_8 \times E_8$ group, the fundamental representation has the same dimensionality. Consequently, for any of these groups there is a cancellation of anomalies at the quantum field level.

An increase in the dimensionality of space-time makes the situation worse regarding ultraviolet divergences in a local quantum field theory, since it increases the degree of the momenta in the numerator of the Feynman integrals. Single-loop calculations[102] furnish some basis for suggesting that superstring theories with the SO(32) and $E_8 \times E_8$ gauge groups are finite in a ten-dimensional Minkowski space; i.e., the divergences in the individual diagrams cancel each other out when the contributions of all diagrams are summed. An important point is that this cancellation occurs exclusively because of the properties of the SO(32) and $E_8 \times E_8$ gauge groups; i.e., these groups are again special cases.

The only type of divergence which might in principle not cancel out upon the summation of individual diagrams is the string analog of ordinary "tadpoles" (lines which emerge from and terminate at the same vertex) in a local field theory.[103–105] In this case, a closed string is emitted into vacuum with a zero momentum and a zero mass (Fig. 8).

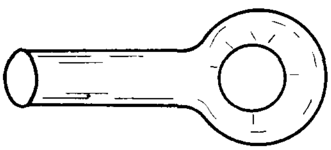

FIG. 8. Single-loop string diagram: an analog of Feynman diagrams with closed lines which begin and end at the same vertex.

We know quite well that the divergences which are generated by "tadpoles" in a local field theory are evidence that the perturbation theory is constructed with respect to a nonphysical vacuum. The vacuum must be redefined by eliminating the contributions of such diagrams.[26] In supersymmetric local field theories, the contribution of such diagrams is automatically zero. If a superstring theory has no anomalies its supersymmetry remains unbroken at the quantum level. We can therefore expect a corresponding situation in superstring theories with the gauge groups SO(32) and $E_8\times E_8$. In superstring theory we thus see a close relationship between the absence of anomalies and the finiteness of the theory. In an ordinary local field theory this relationship is not unambiguous: The absence of anomalies definitely does not mean that the field theory is finite, although there will clearly be no anomalies in a renormalizable theory.

Interestingly, a local field theory with an $N=4$ supersymmetry which is ultraviolet-finite in all orders of perturbation theory was first found as the low-energy limit of a fermion string model.[49,106–109]

3.8. Heterotic string

The anomaly cancellation mechanism which was established by Green and Schwarz in a superstring model leans heavily on the properties of the SO(32) group. These properties are shared by another semisimple Lie group, $E_8\times E_8$. This gauge group, however, cannot be introduced in a string model in the standard way, by means of Chan-Paton matrix multipliers. A new theory of closed strings was derived in Ref. 84. In the low-energy limit, that theory reduces to a ten-dimensional ($N=1$) supergravity which is interacting with a supersymmetric Yang-Mills field with the gauge group $\mathrm{Spin}(32)/Z_2$ or $E_8\times E_8$ [the $\mathrm{Spin}(32)Z_2$ group has the same Lie algebra as the orthogonal group SO(32)]. This theory, which has come to be called the "heterotic string," is a chiral hybrid[2] of a relativistic boson string in a twenty six-dimensional space-time and a superstring model in a ten-dimensional space-time.

This hybridization is based on the following observation. States of closed oriented (type-II) strings (boson or fermion strings) are the direct product of modes which are moving to the left (left-side) or to the right (right-side). The physical variables in a closed boson string are the twenty four transverse components of the position vector of the string, which describe right-side, $x^i(\tau,\sigma)$ and left-side, $\tilde{x}^i(\tau+\sigma)$, modes. The functions $x^i(\sigma)$ and $\tilde{x}^i(\sigma)$ satisfy periodic boundary conditions on the interval $0\leqslant\sigma\leqslant\pi$. A closed superstring contains eight right-side and eight left-side independent boson variables and also eight each of right-side and left-side real fermion variables in the two-dimensional τ,σ space: $S^a(\tau-\sigma)$ and $\tilde{S}^a(\tau+\sigma)$, respectively. The boson variables are determined by the following Fourier expansions:

$$x^i(\tau-\sigma)=\frac{Q^i}{2}+\frac{P^i}{2}(\tau-\sigma)+\frac{i}{2}\sum_{n\neq0}\frac{\alpha_n^i}{n}\exp[-2in(\tau-\sigma)],\quad(95)$$

$$\tilde{x}^i(\tau+\sigma)=\frac{Q^i}{2}+\frac{P^i}{2}(\tau+\sigma)+\frac{i}{2}\sum_{n\neq0}\frac{\beta_n^i}{n}\exp[-2in(\tau+\sigma)],$$

where

$$[\alpha_n^i,\ \alpha_m^j]=[\beta_n^i,\ \beta_m^j]=n\delta^{ij}\delta_{n+m,\,0},\quad [\alpha_n^i,\ \beta_m^j]=0,\quad [Q^i,\ P^j]=i\delta^{ij}.\quad(96)$$

For the fermion variables we have

$$S^a(\tau-\sigma)=\sum_{n=-\infty}^{+\infty}S_n^a\exp[-2in(\tau-\sigma)],\quad(97)$$

where

$$\gamma^+S_n=hS_n=0,\quad [S_m^a,\ S_n^b]=(\gamma^+h)^{ab}\delta_{m+n,\,0},\quad 2h=1\pm\gamma_{11},\quad \gamma^\pm=\gamma^0\pm\gamma^9,\quad(98)$$

and there is an analogous expression for the left-side variables $\tilde{S}^a(\tau+\sigma)$.

Independent dynamic variables of a heterotic string are combined from the variables of the boson string and of the superstring in the following way. From a boson string we take only the left-side variables: eight transverse coordinates $\tilde{x}^i$ and sixteen internal coordinates $\tilde{x}^I$, $I=1,\ldots,16$. From the superstring we take only the right-side variables: eight transverse boson coordinates x^i, $i=1,\ldots,8$, and eight Majorana-Weyl fermion variables S^a. It is then assumed that the internal boson coordinates $\tilde{x}^I$, $I=1,\ldots,16$, are compactified on the special torus T^{16}, whose basis vectors e_i^I, $i=1,\ldots,16$, generate an even integer self-dual lattice.[110–112] By this we mean that the metric

$$g_{ij}=\sum_{I=1}^{16}e_i^Ie_j^I$$

is an integer metric, that it has even diagonal elements, and that its determinant is $\det g=1$. Only two lattices of this type exist: a lattice constructed on the root vectors of the $E_8\times E_8$ group and the lattice of weights of the $\mathrm{Spin}(32)/Z_2$ group.

The components of the total momentum of the string, P^I, $I=1,\ldots,16$, which correspond to internal compactified boson variables, $\tilde{x}^I$, can take on strictly defined values:

$$P^I=\sum_{i=1}^{16}n_ie_i^I,\quad(99)$$

where the n_i are integers.

A hybrid string model of this sort has no tachyon states, it is relativistically invariant in a ten-dimensional Minkowski space, and has supersymmetry. The massless states in this theory form an irreducible multiplet for ($N = 1$, $D = 10$) supergravity and an irreducible multiplet for ($N = 1$, $D = 10$) supersymmetric Yang-Mills theory with the gauge group $\mathrm{Spin}(32)/Z_2$ or $E_8 \times E_8$. It is because of these strictly definite group properties of states that the corresponding gauge symmetry arises in a heterotic string.

This mechanism for generating an internal non-Abelian gauge symmetry, as a result of compactification, is unique to string theories. In the compatification of a local field model on the torus T^{16}, only the trivial Abelian symmetry $[U(1)]^{16}$ could arise.

In the theory of interacting heterotic strings, the gauge constant g and the Newtonian constant $\varkappa$ satisfy a relation different from (87):

$$\varkappa = \mathrm{const} \cdot g\,(T)^{-1/2}. \tag{100}$$

3.9. Compactification in superstring theory; Calabi-Yao manifolds; low-energy phenomenology

Superstring theory, originally formulated in a ten-dimensional Minkowski space, can be a realistic theory only if a dynamic compactification of six dimensions occurs in it. In other words, the vacuum space-time manifold must be of the form $M^4 \times K^6$, where M^4 is a four-dimensional Minkowski space, and K^6 is some compact six-dimensional manifold. At the present state of superstring theory, it is not possible to derive this assertion as a consequence of a solution of dynamic equations. Furthermore, we do not even have proof that a ten-dimensional Minkowski space is the solution of the complete quantum field theory of superstrings. Consequently, the compactification concept must be regarded for the time being as a hypothesis which will have to be justified in the future.

Some simpler questions are presently being studied[113–119]: What might the vacuum state be in principle in superstring theories? How can some grand unified theory or other be inserted into a superstring model? The groups which are being examined as gauge groups in GUT are SU(5), SO(10), and E_6, with respective dimensionalities 24, 45, and 78. None of these groups could occur as the isometry group of a compact manifold K^6, whose dimensionality[120] cannot exceed 21. Accordingly, the gauge groups of a GUT must be built into the string groups of symmetry SO(32) or $E_8 \times E_8$.

We first consider superstrings with the SO(32) group. The group E_6 is not a subgroup of SO(32). Various ways of fitting SU(5) and SO(10) into SO(32) were studied in Refs. 115–117. Some difficulties are unavoidable here. One question, for example, is how to arrange a nontrivial unification of generations.

More attractive from the standpoint of applications is a superstring theory with the gauge group $E_8 \times E_8$ (the heterotic string). The group E_8 contains $SU(3) \times E_6$ as a subgroup.

It turns out that the latitude in the choice of the compact manifold K^6 can be cut down significantly by requiring that the following conditions be satisfied by the compactified superstring theory[113]:

1) The geometry must be of the form $M^4 \times K^6$, where M^4 is a maximally symmetric space-time.

2) In four dimensions, an unbroken ($N = 1$) supersymmetry must exist.

3) The gauge group and the fermion spectrum must be realistic.

The second requirement is dictated by the important role played by supersymmetry in solving the problem of hierarchies and the problem of Dirac's large numbers.[121,122]

As was shown in Ref. 113, the satisfaction of conditions (1)–(3) requires that K^6 be a six-dimensional Calabi-Yao manifold, i.e., a complex three-dimensional Kähler Ricci-planar manifold with the holonomy group SU(3). The existence of such manifolds was proposed by Calabi[123,124] and proved by Yao.[125]

Not just any real manifold of even dimensionality $2n$ can be thought of as a global complex manifold of dimensionality n. For example, the two-dimensional sphere S^2 is a complex manifold. Complex coordinates are introduced on S^2 by means of a stereographic projection. The four-dimensional sphere S^4, however, is not a complex manifold. Furthermore, a complex manifold is called a "Kähler" manifold if all the components of the metric tensor $g_{\alpha\bar\beta}(z)$ are determined by a single function (the Kähler potential) in accordance with

$$g_{\alpha\bar\beta}(z) = \partial_\alpha \partial_{\bar\beta} K(z, \bar z). \tag{101}$$

The Ricci tensor $R_{\alpha\bar\beta}$ for a Kähler manifold is

$$R_{\alpha\bar\beta} = \partial_\alpha \partial_{\bar\beta} \ln \det (g_{\gamma\bar\delta}). \tag{102}$$

If the space is Ricci-planar, then we have

$$R_{\alpha\bar\beta} = 0. \tag{103}$$

The holonomy group[126] is generated by a connection which acts on the manifolds[31]; for a six-dimensional Riemann manifold this group is a subgroup of O(6). For a Calabi-Yao space, the curvature 2-form $R^{mn}_{\alpha\beta}$ takes on values in SU(3).

Proving the existence of a Calabi-Yao space required proving the existence of solutions of some nonlinear partial differential equations which follow from (101), (102), and (103). There is no complete classification of Calabi-Yao spaces.

Structural methods for constructing Calabi-Yao spaces consist of examining submanifolds of a projective complex space CP^n or a factorization of tori. This approach usually leads to Calabi-Yao manifolds with a large (in modulus) Euler characteristic χ. This result is physically unacceptable, since the number of generations in this compactification scheme would be $|\chi|/2$. One possible way to lower $|\chi|$ is to switch from K^6 to K^6/G, where G is a discrete symmetry group of K^6.

Since Calabi-Yao manifolds are Ricci-planar, they do not have continuous symmetries; i.e., for them there are no nonzero Killing vector fields.[40] Consequently, the mechanism of generation as a result of the compactification of the gauge symmetry, which holds in Kaluza-Klein field models,[6,13] will not work here.

The need for a modification by means of Chern-Simons terms of the local supersymmetric field theory which follows from string dynamics in the low-energy limit leads to an interesting relationship between the gravitational field and the gauge field. Specifically, for an unambiguous determination of the intensity in (91)–(94) a necessary condition[113] is $dH = 0$, i.e.,

$$\frac{1}{30}\,\mathrm{Tr}\,(F \wedge F) = \mathrm{tr}\,(R \wedge R). \tag{104}$$

Consequently, the curvature of space-time in string theories requires the existence of a nontrivial gauge field, and vice versa. As was shown in Ref. 113, condition (104) can be satisfied by requiring that the gauge field be equal to the spin connection which is embedded in a corresponding way in SO(32) or $E_8 \times E_8$.

In a string theory with the gauge group $E_8 \times E_8$, after compactification,[113] one of the groups E_8 will be broken to E_6. Quarks and leptons are massless excitations of a superstring. They are described by the zero modes of corresponding wave equations on a Calabi-Yao manifold, and they transform in accordance with representation *27* of the E_6 group. With respect to the unbroken group E_8 they are singlets. The quantum numbers of fermions are determined by the topological invariants of K^6. The number of generations turns out to be $|\chi|/2$, where χ is the Euler characteristic of K^6. In this compactification scenario, the cosmological constant in M^4 is zero. Interestingly, the Yukawa interaction constants, which are adjustable parameters in GUT, are also related to topological characteristics of a Calabi-Yao manifold.

Then comes a breaking of E_6 to $SU(3)_c \times SU(2)_L \times U(1) \times G$, where $G = U(1)$, $U(1) \times U(1)$, $SU(2) \times U(1)$, etc.[114] In addition to the standard model $SU(3)_c \times SU(2)_L \times U(1)$, the group G implies the existence of additional Z^0 bosons in the low-energy superstring theory. There is no basis of any sort for assuming that their mass should be greatly different from the mass of the Z^0 boson in the Weinberg-Salam theory ($\sim 10^2$ GeV).

The prediction of additional Z^0 bosons may run into conflict with the phenomenology of neutral currents. In other words, the E^6 groups of GUT which follow from superstrings are probably too large. These problems could apparently be avoided by using the SO(10) group.

The SO(10) group can be reached in a $E_8 \times E_8$ superstring theory in the following way.[114] As K^6 we take not a Calabi-Yao space but a Ricci-planar compact six-dimensional manifold with holonomy group SO(6). In principle, such manifolds may exist, although specific examples have yet to be constructed. In this case the low-energy phenomenology is determined by the subgroup in E_8 which commutes with SO(6), i.e., the SO(10) group. The supersymmetry is broken even at the tree level; the number of generations remains $|\chi|/2$; and the cosmological term in M^4 is zero.

Other compactification mechanisms,[118,119] in which K^6 is not a Ricci-planar space and instead has a nonzero torsion, have also been discussed. In this case the number of generations is not determined by the Euler characteristic χ.

3.10. Cosmological consequences

In a superstring theory with the gauge group $E_8 \times E_8$ (a heterotic string), the real world is described by the group E_8, while the second group, $E_{8'}$, describes a "shadow" world.[5] This shadow world interacts with ordinary matter only through gravitational forces.[129] As a result, the two worlds, existing in parallel, essentially do not perceive each other after the Planck epoch (the age of the universe is $< 10^{-43}$ s, and its temperature $> 10^{19}$ GeV). In each world, a separate thermodynamic equilibrium is established by virtue of the internal nongravitational interactions of each world. If the ordinary matter and the shadow matter are initially well-mixed, this state of affairs will persist until the nongravitational forces become important on a macroscopic scale. In standard cosmology this time corresponds to a late stage in the formation of galaxies.

Later on, the equilibrium in the spatial distribution of ordinary matter and shadow matter may be disrupted because of the random nature of nongravitational perturbations which are acting independently in each type of matter. In other words, there may be a spatial separation of ordinary matter and shadow matter. In principle, therefore, there may exist galaxies in which either ordinary matter or shadow matter is predominant. A further possibility is the existence of binary stars made up of an ordinary star and a shadow star. Such an entity would be perceived as a single star with a periodic motion. In fact, there are some nearby stars (within < 5 pc) which are thought to have invisible partners. There are of course some less exotic explanations here; e.g., the invisible partner might be a neutron star, a black hole, or even an object of planetary size.

The gauge group of the shadow world, $E_{8'}$, may be broken in precisely the same way as the group of the ordinary world, E_8, is; i.e., the shadow world may be an identical twin of the observable world. However, incorporating shadow matter in an analysis of the primordial nucleosynthesis forbids a complete symmetry between our world and the twin world.[129]

4. CONCLUSION

Superstrings are an organic part of the modern theory of elementary particles. From the standpoint of predictions, they transform into supersymmetric grand unified theories in the low-energy field limit. The primary advantages of the superstring approach are as follows:

1. Superstrings make it possible to unify all the fundamental interactions, including gravitation.

2. They essentially unambiguously fix the fundamental gauge group in a grand unified theory.

3. The four-dimensional nature of our world is treated in the superstring approach as a consequence of the dynamic equations of this theory.

4. In the ideal case, the theory would include only two fundamental parameters: the string tension T and one of the

constants of the Yang-Mills or gravitational interaction of the superstrings.

We should also list the fundamental questions in superstring theory which will have to be solved if that theory is to have a solid basis:

1) Finiteness or renormalizability will have to be proved rigorously in all orders of string perturbation theory.

2) The compactification process requires a dynamic basis.

3) The mechanism for the breaking of supersymmetry at energies $E < 10^2$ GeV must be determined.

4) More-fundamental reasons for the vanishing of the cosmological constant of our four-dimensional world must be found.

Resolving these questions will first require the derivation of a second-quantized field theory of superstrings to make specific calculations possible. A shortcoming of the existing formulations of field theories of strings is that a definite background metric (usually the metric of a ten-dimensional planar Minkowski space) is built into the theory at the outset. However, excitations of superstrings also contain, in particular, the graviton; i.e., the observable physical metric of space-time must be determined dynamically through a solution of the equations of a field theory of superstrings.

In the formulation of a field string theory it would probably be useful to call upon a principle analogous to the equivalence principle in the general theory of relativity, but formulated in the space of string configurations. It has been established quite well that the dynamics of "old" hadron strings was determined to a large extent by the requirement of the duality of the string amplitudes. No analog of the duality principle has been found for superstrings.

1)See, for example, the paper by Fischback *et al.*[140] for a discussion of experimental indications of the possible existence in nature of new long-range forces (a fifth fundamental interaction).

2)The biological term "heterosis" (hybrid strength) refers to the intensification of beneficial properties of a hybrid in comparison with specimens of the original plants or animals.

3)This group is defined in the following way.[134] We take some point of an n-dimensional manifold on which the connection is defined, and we carry out a parallel translation of a vector along a closed contour which begins and ends at this point. As a result of this translation, we find a new vector. If the connection is a Riemann connection, the transformations of the original vector into the final vector form a subgroup in $O(n)$, which is called the "holonomy group" for the given manifold.

4)The equations for the Killing vectors ξ^i are written with the aid of R_{ij} as follows[127]:

$$g^{jk}\xi^i_{;j;k} + R^i_j\xi^j = 0, \quad \xi^i_{;i} = 0,$$

where the semicolon means covariant differentiation with respect to the metric g_{ij}. Consequently, if $R_{ij} = 0$, then each component of a Killing vector must be a harmonic function. If the manifold is compact, and if the metric is positive definite on it, then all harmonic functions on this manifold reduce to constants. In this case the condition $\xi^i_{;i} = 0$ means that ξ^i is a null vector.

5)A shadow universe has also been discussed in the physics of elementary particles and earlier; see Ref. 128, for example.

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