Chromodynamics as a theory of the strong interactions[1)]

I. V. Andreev

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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A popular exposition of the present status of quantum chromodynamics is given. The main features of the theory are described, including the regime of asymptotic freedom at small distances and the character of the symmetry breaking at large distances. The properties of the vacuum and the theoretical methods which are used are discussed briefly.

CONTENTS

1. INTRODUCTION

The past 15 years have completely changed the appearance of strong-interaction theory, as well as elementary-particle physics in general. By the beginning of the seventies, the SLAC accelerator was used to accumulate convincing evidence that pointlike constituents—quark—partons—exist inside nucleons. After this, events began to move rapidly. In 1972 quantum chromodynamics—a non-Abelian gauge theory of quarks and gluons—was proposed. In 1973 asymptotic freedom of the non-Abelian theory—its decisive property—was discovered. The charmed c quark was discovered in 1974, and the b quark in 1977–1978. At about this same time, quarks began to be "seen" in the form of jets of particles in e^+e^- annihilation, and in 1979 the gluon was "seen" (three-jet events). The same stormy development took place in parallel in weak-interaction physics. In 1973 the so-called neutral currents were detected experimentally, and the discovery of the c quark was followed here by the establishment of a new unified gauge theory of the electroweak interactions. Striking confirmation of this theory was provided by the discovery in 1983 of the carriers of the weak interactions—the intermediate vector bosons $W^\pm$ and Z^0 with the masses predicted previously.

What is the picture at the present time? We can say that the world around us is constructed from quarks and leptons interacting with gauge fields (see, for example, Ref. 1). The strong interactions are inherent in the quarks. The existence of five species (flavors) of quarks u, d, s, c, b (in the order of increasing mass) is now firmly established, and there is evidence for the existence of at least one more heavy t quark. Each of these quarks experiences the same strong interactions.

Each quark with a given flavor can occur in three physically equivalent color states, or, as we say, has three colors, $q = (q_1, q_2, q_3)$. The antiquarks $(\bar q_1, \bar q_2, \bar q_3)$ possess the three complementary colors. In the free state, one observes only colorless hadrons, in which the colors of their constituent quarks and antiquarks are compensated.

The quarks interact through eight massless vector fields A_μ^a $(a = 1,...,8)$, which are usually written in the form of four matrices A_μ. Weak excitations of the field A_μ^a (the individual field quanta) are called gluons, which emphasizes that their role is to "glue together" the quarks in mesons and baryons. The color structure of the gluon can be represented roughly as a combination $q_i \bar q_j$ (more precisely, eight combinations formed from the nine quantities $q_i \bar q_j$ correspond to gluons, and one corresponds to a colorless state). When an interaction occurs, a quark q_1 can emit a gluon $(q_1\bar q_2)$ and be converted into a quark q_2; at the same time, its partner in the interaction, a quark q_2, is converted into a quark q_1.

The theory of the interaction of quarks and gluons in known as quantum chromodynamics (QCD). Chromodynamics is based on the principle of local color symmetry. This means that the color states of the individual quarks can be changed independently without any physical consequences, in particular without destroying the colorless character of the hadrons. Clearly, this is possible only if there is a gluon field which is capable of assuming the extra color. The equivalence of different color states can be formulated mathematically as invariance of the theory with respect to transformations from the group SU(3), and the parameters of the group transformations can depend on the point of space-time. Such theories are known as gauge theories. In our case, the transformations of the group do not commute with each other, and we are dealing with a non-Abelian gauge theory. Local gauge invariance is also a property (an Abelian gauge theory). It is this property of electrodynamics, whose existence was practically ignored for a century, that has now become the guiding principle for the construction of new theories.

The principle of local gauge invariance makes it possible to reconstruct uniquely the chromodynamics Lagrangian $\mathcal{L}_S$, which is similar to the electrodynamics Lagrangian but takes into account the color degrees of freedom:

$$\mathcal{L}_S = i\bar q \gamma^\mu \left(\partial_\mu - ig\,\frac{\lambda^a}{2} A_\mu^a\right) q - m\bar q q - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a ,$$

where g is a coupling constant, λ^a are color Hermitian 3×3 matrices analogous to the 2×2 Pauli spin matrices, and the intensity of the gluon field

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

differs from that of the electric and magnetic fields of elec-

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trodynamics by additional terms nonlinear in the gauge field and containing the structure constants f^{abc} of the group SU(3). The presence of these nonlinear terms, which are required for gauge invariance of a non-Abelian theory, implies a self-interaction of gluons, which in turn leads to the most important property of chromodynamics—the effect of antiscreening of the charge.

2. PHYSICS AND SMALL DISTANCES

Antiscreening of the charge means that the effective charge of the quarks and gluons is high at large distances and becomes small as the distances decrease. This leads to completely different physics at small and large scales.

At small distances r or at large momentum transfers Q, the effective charge tends to zero:

$$\left(\frac{g^2}{4\pi}\right)_{\rm eff} \approx \frac{1}{(11/4\pi)\ln(Q^2/\Lambda^2)} .$$

This property has become known as asymptotic freedom. (The expression given here does not take into account the quark contribution. The quantity Λ characterizes the scale of the strong interactions; its value depends on the method of regularization of the theory. In the regime of asymptotic freedom, frequent use is made of the value $\Lambda = \Lambda_{\rm MOM} \approx 200\text{–}400$ MeV.) At small distances, the quarks and gluons look like practically free particles, and all the processes in which they participate can be calculated by means of perturbation theory, using the initial Lagrangian $\mathscr{L}_S$. The masses of the quarks u, d, and s are small, so that they can be neglected (so-called current quarks).

This approach encompasses a large part of high-energy physics—hard processes, for which large momentum transfers are characteristic. The classical example of hard processes is deep inelastic scattering of leptons (electrons, muons, and neutrinos) on nucleons, the study of which led to the idea of partons (i.e., practically free quarks and gluons inside the nucleon) and stimulated the creation of chromodynamics.

The measurement of the momentum of the scattered leptons in deep inelastic processes makes it possible to determine experimentally the distribution of the current quarks and the gluons with respect to the fraction x of the momentum carried in a rapidly moving nucleon (the so-called nucleon structure functions). An example of this distribution for the valence quarks $u_v(x) + d_v(x)$, antiquarks $\bar{q}(x)$, and gluons $G(x)$ at scales $r \approx 0.11$ F (which corresponds to a squared momentum transfer $Q^2 = 5$ GeV^2) is given in Fig. 1.[2] The distribution of the quarks and antiquarks has now been determined quite reliably, whereas for the gluon distribution there remains some uncertainty. This is due to the fact that gluons do not interact directly with leptons and must be distinguished by making use of additional arguments based on QCD and invoking a more complete set of experimental data, some of which have low accuracy. The structure functions of mesons have now also been measured.

Inclusion of chromodynamical corrections calculated by means of perturbation theory leads to a change in the parton distributions as the test momentum Q varies (violation of scaling). As Q increases, we penetrate deeper inside a

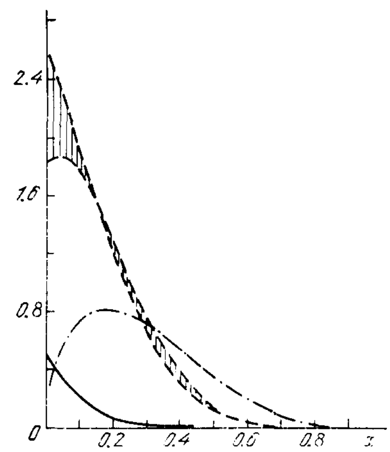

FIG. 1. Distribution of partons for $Q^2 = 5$ GeV^2. The chain curve is the valence-quark distribution $x[u_V(x) + d_V(x)]$, the continuous curve is the antiquark distribution $x\bar{q}(x)$, the broken curve is the gluon distribution $xG(x)$, and the crosshatched region shows the uncertainty in the gluon distribution.

quark and should observe an increase in the number of quark-antiquark pairs and gluons forming its polarization cloud, with a simultaneous decrease in the momentum fraction x carried by each parton. The experimental data on the violation of scaling in deep inelastic processes are, on the whole, in good agreement with the predictions of the calculations.

It is worth mentioning several unexpected results obtained in experiments on deep inelastic scattering on nuclei. It has been found that the distribution functions of the quarks in medium and heavy nuclei (Al, Fe, Cu) differ appreciably from the nucleon structure functions (the EMC effect[3,4]). This probably means that heavy nuclei cannot be regarded as aggregates of immutable nucleons. Many mechanisms responsible for this effect have been proposed: six-quark bags, diquarks, pions in nuclei, and percolation of quarks from nucleon to nucleon in large nuclei, but as yet the situation is still not entirely clear (see Ref. 5).

Another important trend in the physics of hard processes is the study of hadronic jets produced in $\bar{p}p$ and e^+e^- collisions with the highest energies. Jets are now becoming an important source of our knowledge of hadronic structure and are widely used to search for new particles.

According to chromodynamics, jets are produced by quarks and gluons. In $\bar{p}p$ collisions, two partons are scattered with large momentum transfer and then give rise to two jets of hadrons emitted at large angles to the direction of motion of the colliding particles (Fig. 2a). By a "jet" we mean here a group of several particles emitted into a narrow cone of angles. A striking example of such an event obtained by the UA2 group at the SPS collider is given in Fig. 2b, which shows the angular distribution of the transverse flux of energy of the produced particles (borrowed from Ref. 6). The measured spectra of the particles (the number of particles as a function of their transverse momentum) are in good agreement with the predictions of chromodynamics.

It should be said that with increase of the energy the fraction of these processes rises sharply and that in $\bar{p}p$ colli-

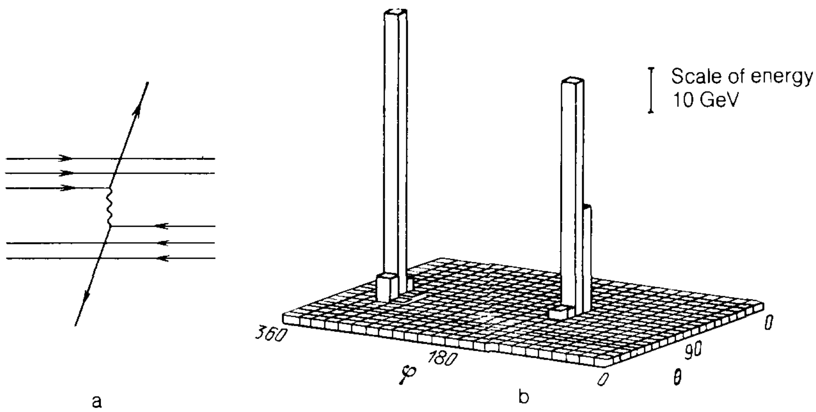

FIG. 2. a) Example of scattering of partons in a $\bar{p}p$ collision, leading to the production of jets; b) a two-jet event in a $\bar{p}p$ collision at total energy of 540 GeV. The polar angle θ, the azimuthal angle φ, and the transverse energy are plotted along the axes.

sions they now comprise about 20% of all the events, determining to a great extent the growth of the total interaction cross sections.

In e^+e^- annihilation, jets arise from fragmentation of a quark and an antiquark, as well as from bremsstrahlung of hard gluons (Fig. 3a). An advantage of this process is that there are no hadrons in the initial state and that the jets must emerge in the purest form. Experiment actually shows that jet configurations are present. An example of a three-jet event corresponding to the diagram of Fig. 3a is given in Fig. 3b (borrowed from Ref. 7).

Jets observed experimentally under different conditions are found to be surprisingly similar in their main characteristics. This raises the question of criteria by which one might be able to distinguish quark and gluon jets. The point is that chromodynamics predicts perfectly definite differences between these two kinds of jets. For example, gluon jets should be considerably softer (in energy) and broader (in angle) than quark jets, and the multiplicity of particles in them should be greater (asymptotically, by a factor 9/4). The solution of this problem depends above all on the difficulties associated with measuring the momenta of the jets and separating them from the background particles with small transverse momenta. Efforts are now being made to resolve this difficulty by separating the individual modes of fragmentation and studying the correlations of particles with definite quantum numbers.

On the other hand, there is the problem of giving a sufficiently complete theoretical description of the process of fragmentation, including the evolution of jets at the parton level and the transition from quarks and gluons to colorless hadrons. Here there exist at present two main approaches. One of them, the more phenomenological one, is based on the idea of strings joining the produced quarks and antiquarks (the so-called Lund model[8]). The energy stored in a string is transferred to hadrons as a result of fluctuations and rupture of strings. A gluon is regarded here as a sharp bend in a string and practically as an equivalent $q\bar{q}$ pair (Fig. 4a). In another approach,[9] the early stages of the process of fragmentation are described in the language of perturbative chromodynamics, and it is assumed that the conversion of partons into hadrons takes placed at a comparatively late stage and has little influence on the angular and energy characteristics of the process (Fig. 4b).

Although they are apparently very different, the two approaches give similar results. For example, allowance for the interference of the soft gluons in the quark-gluon picture of the evolution of jets can reproduce the effects of the string model (see Ref. 10). It is possible that what is operating here is duality—a principle which has demonstrated its power many times and which makes it possible to describe the same quantities in two different languages. In any case, many more studies are required in order to exclude by trail and error numerous models which claim to describe the process of fragmentation. A special role will necessarily be played here by the search for delicate effects which are specific for particular models. From a more general standpoint, we encounter here questions of the physics of large distances,

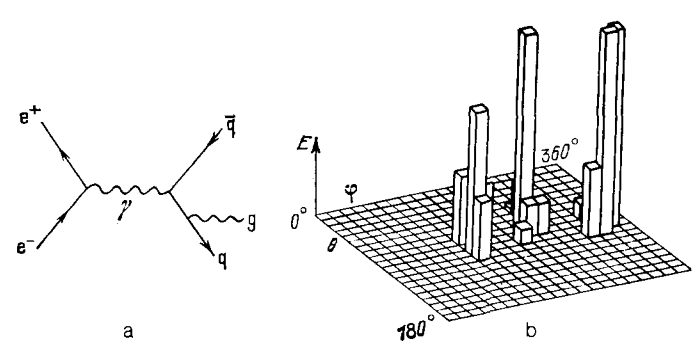

FIG. 3. a) Emission of a hard gluon accompanying the production of a quark-antiquark pair in e^+e^- annihilation; b) a three-jet event in e^+e^- annihilation, detected at the PETRA accelerator at total energy of 38 GeV.

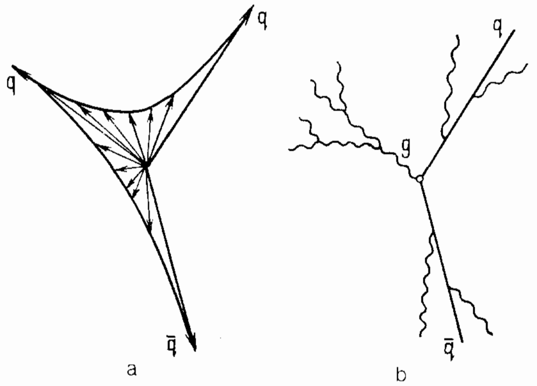

FIG. 4. Hadronization in a three-jet $q\bar{q}g$ process. a) The Lund model (the arrows show the distribution of momenta, to which it is necessary to add a constant component in the transverse direction); b) the model of perturbative evolution with soft decoloration of the partons.

where the main problems of chromodynamics are concentrated.

3. PHYSICS OF LARGE DISTANCES AND THE PHASES OF QCD

At large distances, the interaction of quarks and gluons becomes stronger, nonperturbative effects become important, and problems characteristic of strongly interacting systems arise. In fact, we are dealing with a problem of statistical physics in a four-dimensional space and for fields with a very complex internal structure. Just one gluon field is characterized by more than 30 invariants ($G^a_{\mu\nu}G^a_{\mu\nu}$, $e^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$, $f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$, $G^a_{\mu\nu}G^a_{\rho\sigma}G^b_{\mu\nu}G^b_{\rho\sigma}$, etc.). According to the general approach, it is necessary to find the ground state (the vacuum), the excitations over it (mesons, baryons), and the interaction of the excitations.

The first and basic problem is to understand the structure of the vacuum state. It is by no means trivial: it turns out that in the vacuum a number of symmetries of the initial Lagrangian are broken and that there exist condensates of numerous invariants. More specifically, two fundamental phenomena occur—violation of scale invariance and violation of chiral symmetry.

If the initial masses m_q of the light quarks are neglected (which in a first approximation is permissible), the chromodynamics Lagrangian $\mathscr{L}_S$ possesses scale symmetry, i.e., invariance with respect to a scale transformation of the coordinates. This symmetry is broken at large distances—there is a characteristic scale $l \sim 1\ F$, whose presence is signaled by the appearance of a nonzero vacuum expectation value of the trace of the energy—momentum tensor of the gluon field:

$$\langle T^\mu_\mu\rangle \approx -\frac{9}{32\pi^2}\langle (gG^a_{\mu\nu})^2\rangle_0.$$

(In the case of scale invariance, which occurs at the classical level, $T^\mu_\mu = 0$. This invariance is broken by the quantum corrections, allowance for which gives a nonaveraged form of the equality given here.[11] The nonzero vacuum value $\langle T^\mu_\mu\rangle_0$ was established by means of sum rules (see below).) In other words, the vacuum is populated by gluon fields and has a nonzero (negative) energy density ε and an additional pressure p not present in the classical peturbative vacuum. By virtue of Lorentz invariance, the energy-momentum tensor of the vacuum state takes the form

$$T^{\mu\nu} = g^{\mu\nu}\varepsilon, \quad p = -\varepsilon \approx 0.5\ \text{GeV/F}^3.$$

In addition, massless quarks have an additional symmetry which does not exist for massive quarks, since it is possible to make independent transformations of right- and left-handed quarks, i.e., states with spin directed parallel and antiparallel to the momentum (chiral symmetry). This symmetry is also broken at large scales, as is signaled by a nonzero vacuum expectation value $\langle\bar{q}q\rangle_0 \approx -(0.25\ \text{GeV})^3$ (see Ref. 12). In other words, the vacuum is populated by $q\bar{q}$ pairs, which give an additional negative contribution to the energy density. The quarks, as quasiparticles belonging to the hadrons, then acquire a rather large effective mass $m^*_q \approx 300$–350 MeV, as is indicated to us by the success of nonrelativistic composite models of the hadrons, i.e., for the light quarks the mass becomes ten times as large. The Goldstones bosons which appear when chiral symmetry is broken are pseudoscalar mesons (pions). Their actual mass is due solely to the small nonzero masses of the initial quarks:

$$m^2_\pi \approx -2(m_u + m_d)\langle\bar{q}q\rangle_0(133\ \text{MeV})^{-2}.$$

In order to clarify the general picture, we can cite a somewhat exaggerated realization of chromodynamics—the so-called bag model.[13,14] According to this model, a hadron is a bubble of definite radius R in the QCD vacuum, in which practically free quarks ($q\bar{q}$ or qqq; see Fig. 5) are confined. The quarks obey the Dirac equation with the condition that their flux through the boundary S is equal to zero: $n^\mu\bar{q}\gamma_\mu q|_S = 0$. The interaction of the quarks is regarded as a perturbation, which gives a contribution ΔE_g to the total energy depending on their spins σ^i_j and colors λ^a_j:

$$(\Delta E_g)_{jk} \sim \frac{1}{R}(\sigma^i\lambda^a)_j(\sigma^i\lambda^a)_k.$$

An energy density B inside the bubble, in addition to that of the vacuum, is also introduced. All this is equivalent to the problem of current quarks in a scalar potential well with infinitely high walls but with an additional boundary condition requiring equality of the pressures of the Dirac quark field from inside the hadron and of the QCD vacuum from outside the hadron (this condition restores the conservation law for the energy-momentum tensor in the model: $\partial_\mu T^{\mu\nu} = 0$).

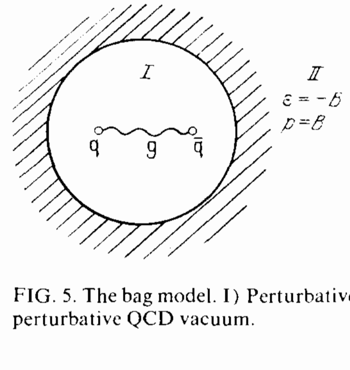

FIG. 5. The bag model. I) Perturbative vacuum inside a hadron; II) nonperturbative QCD vacuum.

It is possible to calculate in the model the levels E_i of the quarks in the potential well. This gives the hadron masses

$$M = \sum_i E_i + \frac{4}{3}\pi R^3 B + \Delta E_g - \frac{Z}{R},$$

where the last term parametrizes the contribution of Casimir effects (i.e., the change in the energy density of the perturbative vaccum due to presence of the walls) and the elimination of the center-of-mass motion. With four adjustable parameters, the bag model gives the spectrum of mesons and baryons, and reproduces well the magnetic moments of the hadrons. To be sure, it should be noted that the volume energy density B in the model is practically an order of magnitude smaller than the chromodynamical vacuum energy density ε, which calls into question the use of the idea of current quarks inside a hadron.

In the course of time, the bag model has been modified in order to ensure conservation of the axial quark currents. In other words, it was necessary to ensure that the quarks could flip their spin on being reflected from the wall of the potential well (massless fermions must conserve helicity, i.e., the projection of the spin onto the direction of motion). This required the introduction of a phenomenological pion field. As a result, the situation became much more complicated, and the conclusions became more ambiguous. In any case, the simple bag model illustrates well the new picture which chromodynamics offers for the vacuum and the hadrons.

The asymptotically free and the confining phases of chromodynamics which occur in the bag model can manifest themselves not only when we investigate the response of the system at small and large scales, but also as possible macroscopic states (see Refs. 15–18). There is general agreement that with increasing density or temperature there occurs an "ionization"—the formation of a quark-gluon plasma, in which the quarks and gluons interact perturbatively. Intuitively, this is quite clear: with increasing density the hadrons must fuse into a single common "bag" containing the quarks and gluons, and with increasing temperature the confining configuration of the gluon field is no longer so preferable statistically.

The striking general agreement extends also to the values of the parameters of the transition and to the form of the phase diagram (see Fig. 6). The critical baryon density in Fig. 6 is approximately $n_c \approx 0.5$ baryon/F^3, and the temperature of the transition to the state of the quark-gluon plasma is $T_c \approx 150$–200 MeV. The energy density at the points of the phase transition is only 3–4 times as large as the nuclear density,

$$\varepsilon_c \approx (3\text{–}4)\cdot 0.15\ \mathrm{GeV/F^3},$$

both at $n = n_c$, $T = 0$, when $\varepsilon_c \approx m_N n_c$, and at $n = 0$, $T = T_c$, when in the presence of a sharp jump in the energy density in the region $T \approx T_c$ the quantity ε_c takes a value close to half of the energy of an ideal quark-gluon gas, $\varepsilon_c \approx 0.5\sigma_{qg} T_c^4$. This corresponds approximately to the energy density inside a typical hadron. Such values are obtained from the bag model, from matching of the state of hadronic matter to the state of the quark-gluon plasma, and from calculations on a lattice (see Refs. 16 and 17).

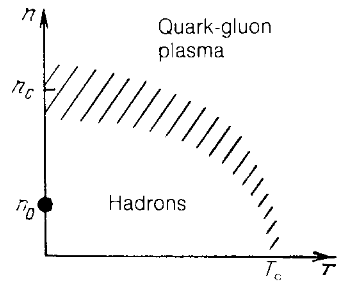

FIG. 6. Phase diagram of chromodynamics. The temperature T and the baryon density n are plotted along the axes. The region in which it is assumed that chiral symmetry is broken in the quark-gluon phase is cross-hatched; n_0 is the state of nuclear matter.

Besides the early Universe in the first 10^{-5}–10^{-4} sec of its existence and perhaps neutron stars, the new state of matter might be formed in a collision of ultrarelativistic heavy ions (see Ref. 18). It is assumed that when nuclei pass through one another the degrees of freedom with small longitudinal momenta interact strongly, leading to formation of a plasma. According to estimates, for nuclei with initial energy $E \gtrsim 30$ GeV/nucleon there appears a state with energy density $\varepsilon \gtrsim (1$–$10)$ GeV/F^3. This should be sufficient for a phase transition.

Consideration has been given to possible signals indicating production of a quark-gluon plasma, such as the production of photons and e^+e^- pairs or enormous multiplicities of produced particles. Studies have also been made of more exotic possibilities, for example, the production of metastable clusters of quark-gluon matter and new states containing a greater number of quarks than the ordinary hadrons. All this is very promising, and this is the case for the theory as well. It remains only to obtain irrefutable experimental evidence for production of the plasma. It is possible that decisive results will be obtained in forthcoming experiments using the SPS accelerator at CERN, where a beam of oxygen ions with energy 200 GeV/nucleon is supposed to be generated at the end of 1986.

4. THE QCD VACUUM

A good model of the vacuum is required for a sound approach to chromodynamics. Contemporary methods make it possible to perform calculations if the vacuum is dominated by some classical field configuration and if the quantum corrections to it are comparatively small. In practice, we are content with approximate test states in which the classical configurations of the gluon field dominate in individual regions of space-time. The calculations are carried out in four-dimensional Euclidean space.

The simplest assumption is the spontaneous generation in the vacuum of slowly fluctuating quasi-Abelian magnetic fields B (chromoelectric fields should produce $\bar{q}q$ pairs). Such a variant aroused considerable interest at one time, since a minimum was discovered in the vacuum energy for a nonzero constant field B,[19]

$$\varepsilon(B) = \frac{1}{2}B^2 + \frac{11g^2}{32\pi^2}B^2 \ln\frac{gB}{\mu^2}, \qquad \varepsilon_{\min} < 0,$$

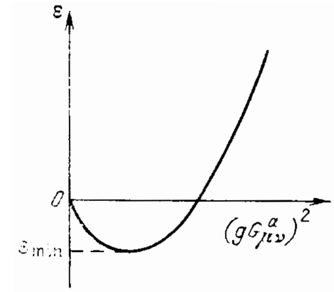

FIG. 7. Vacuum energy density as a function of a slowly varying field.

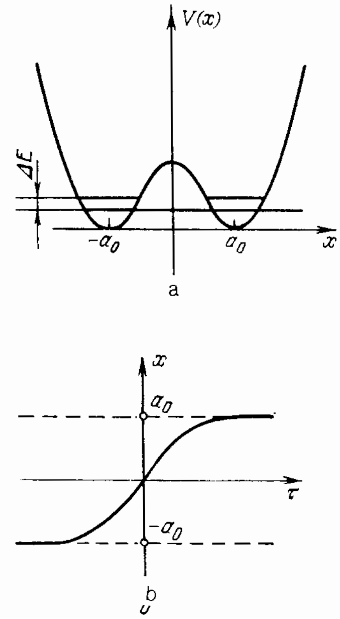

FIG. 8. a) Energy reduction ΔE due to the effect of tunneling in a potential with equivalent minima; b) classical solution in imaginary time, analogous to a QCD instanton.

(Fig. 7), although this minimum is not guaranteed theoretically (the correction terms are large at the minimum). Of course, it was assumed that in sufficiently large regions of space the competition between the energy and the statistical weight leads to a variable field and ensures the invariance of the vacuum.

Subsequently, it was found that a configuration with a constant magnetic field is unstable in one of the modes (there is so far no complete agreement on this point) and that more intricate fluxes of the magnetic field are energetically more favorable. This approach leads to a stochastic configuration with zero mean value of the individual field components. This is in fact necessary for the invariance of the vacuum; however, such states are difficult to describe.

On the whole, the situation here is not completely clear. There must be some truth in the expression for the vacuum energy given above, since, first, it is possible to avoid the instability by introducing self-dual fields instead of the magnetic fields and, second, the result agrees with general arguments concerning the form of the trace of the energy-momentum tensor of the gluon field. In any case, this model assumes a sufficiently large scale of fluctuations to be able to regard the important vacuum fields as constant.

The most popular model is the instanton vacuum (see, for example, Ref. 20). This model exploits the topological properties of a gauge field, which have acquired enormous popularity among physicists. When at the spatial infinity a Yang-Mills field reduces to a pure gauge, $A_\mu = i\Omega\partial_\mu\Omega^+(x)$, the gauge matrix $\Omega(x)$ can bring about a topologically nontrivial mapping of a remote sphere of the coordinate space onto the space of the parameters of the gauge group (more precisely, its subgroup SU(2)). In this case, we say that the field possesses a topological charge ν corresponding to the multiplicity of the mapping.

Topologically nontrivial classical solutions in Euclidean space possessing a finite action are known as instantons (see the review of Ref. 22). The simplest solution

$$A^a_\mu = 2\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}$$

gives an instanton with dimension ρ and center at the point x_0 and with topological charge $\nu = 1$. The role of the coefficients $\eta_{a\mu\nu}$ is that they ensure a correlation of the space-time and gauge components of the field A^a_μ. This solution describes tunneling of the field in Euclidean space (i.e., for imaginary time) between topologically distinct states of the classical vacuum (states with $G_{\mu\nu} = 0$ for $x \to \infty$).

It is most important that excitations such as instantons reduce the energy of the vacuum, just as sub-barrier motion between two potential wells reduces the energy of the ground state in quantum mechanics. This is illustrated in Fig. 8. Here the analog of two different zero fields at infinity is the pair of coordinates $-a_0$ and a_0 of the potential-energy minima, and the analog of the QCD instanton is the solution in the imaginary time τ which takes the particle from one extremum to the other (see Fig. 8b). The reduction of the energy due to tunneling gives rise to the idea of the instanton vacuum.

The tunneling events can be regarded in the four-dimensional Euclidean space as a gas of excitations. When vacuum expectation values are calculated, it is necessary to take an average over all configurations of instantons and anti-instantons and over their dimensions. This leads to problems associated with instantons with large dimensions, which have an increased statistical weight and can influence each other. As a result, we are concerned more with a liquid than with a gas. Nevertheless, it is possible to develop and refine models of such a vacuum and to estimate observable quantities by means of them (see Ref. 20).

The instanton vacuum also accommodates quarks. At a qualitative level, it is then possible to understand the mechanism of chiral symmetry breaking with the appearance of a condensate $\langle\bar{q}q\rangle_0$, as well as the more specific so-called U(1) problem, i.e., the absence in nature of a flavor-singlet pseudoscalar Goldstone boson.

On the whole, of course, we are at present still far from a quantitative model of the vacuum. Even the characteristic scale of the vacuum fluctuations is not clear. If the characteristic correlation length l is sufficiently large, for example, of the order of the nucleon dimensions $l \sim 1$ F, then the constant-field model may be appropriate. On the other hand, if the characteristic length is much smaller, for example, $l \sim 0.2$ F, then it is natural to begin with the model of the instanton vacuum, for which sharp fluctuations of the field are characteristic.

In view of the complexity and the preliminary character of the explicit models of the QCD vacuum, it becomes desirable to dispense with a detailed description of the configurations of the vacuum fields and to attempt to give a more averaged description by introducing scalar order parameters such as $\chi^4 \sim (T^{\mu}_{\mu})_G$ and $\varphi^3 \sim \bar{q}q$ and regarding them as classical fields corresponding to collective degrees of freedom. This approach is possible because for one or two condensates the Lagrangian of the scalar fields corresponding to them can be reconstructed almost uniquely by requiring that the trace of the energy-momentum tensor be reproduced correctly. For example, for the gluon condensate

$$\mathcal{L}_{\text{eff}} = \frac{1}{2\eta^2}(\partial_\mu \chi)^2 - \frac{1}{4}\cdot\frac{9}{32\pi^2}\chi^4\left(\ln\frac{\chi^4}{\chi_V^4} - 1\right),$$

which corresponds closely to the expression for the effective Lagrangian in the case of generation of gluon fields with a constant intensity (see Fig. 7). In the vacuum the scalar field has a nonzero value χ_V, and the second parameter η is related to the mass of the gluon excitations (glueballs). It is possible to add gluon fields and quarks to this Lagrangian and to attempt to construct hadrons. On the whole, however, the problem of order parameters is quite complex. It is not completely clear how many fields must be introduced for a good description and which of these additional fields are most important.

5. THEORETICAL METHODS OF QCD

Under conditions in which the properties of the vacuum are not known sufficiently well, special importance attaches to indirect methods whose claims are more limited but which have a more solid foundation. These approaches include the method of sum rules, which has been used very widely in recent years (see Refs. 12 and 23–25). The idea of this method is to approach the region of bound states from the direction of asymptotic freedom. Consideration is given to the perturbation of the vacuum by some or other current with definite quantum numbers for large virtualities Q^2 or masses M^2 (the perturbation of the vacuum at small scales). The violation of asymptotic freedom is taken into account by adding nonperturbative power corrections containing the vacuum expectation values of the quark and gluon fields. On the other hand, the excitations induced by the currents are approximated by resonances. Here we encounter one more example of the use of quark-hadron duality.

More specifically, one considers the Fourier transform of a product of currents,

$$\Pi(Q^2) \sim \langle TJ(x)\,J(0)\rangle_F.$$

The current $J(x)$ can be, for example, $\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d$, which distinguishes the ρ-meson channel with the quantum numbers $I = 1$, $J^{PC} = 1$. For sufficiently large Q^2 we can write the operator expansion

$$\Pi(Q^2) = C_{\text{pert}}(Q^2) + C_m\frac{\langle\bar{q}q\rangle_0}{Q^3} + C_G\frac{\langle(G^a_{\mu\nu})^2\rangle_0}{Q^4} + \ldots,$$

in which the correction terms $\sim 1/Q^3$, $1/Q^4$ should be relatively small but nevertheless important. Finally, this expansion is compared with the resonance contribution

$$\sum_i \frac{g_i^2}{M_i^2 - q^2} + \text{background}, \quad q_2 = -Q^2.$$

In actual calculations, use is made of a dispersion representation of the polarization operator $\Pi(Q^2)$ in order to be able to compare the spectral densities of the intermediate states of the two expressions, as well as a Borel transformation for $\operatorname{Im}\Pi(m^2)$ in order to emphasize the contribution of small masses in the intermediate states.

The method of sum rules gives the spectrum of masses M_i of the lowest hadronic states (both mesons and baryons) and their effective coupling constants g_i. It has made it possible to calculate, for example, the pion form factor and the magnetic moments of baryons. It is by the method of sum rules that numerical values have been obtained for the gluon condensates $\langle(G^a_{\mu\nu})^2\rangle$ and $\langle f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}\rangle$. Of course, the possibilities of the method are not unlimited. It can reproduce only the main low-lying resonances, provided that their structure is sufficiently simple, and cannot, for example, reproduce the set of resonances forming a Regge sequence. Nevertheless, the achievements of this limited approach are impressive. They are greater than what could have been expected in advance.

In order to describe the strong interactions, wide use is now also made of more phenomenological approaches which to varying degrees are suggested by chromodynamics. Among them, potential quark models and string models of hadrons are most popular. The method of effective chiral Lagrangians, which is capable of describing the low-energy physics of pions, is to some extent in a class of its own. Here striking results have recently been obtained, making it possible to establish contact between this method and chromodynamics and to describe baryons as soliton solutions.[26] We shall not consider these approaches or the nontraditional formulation of QCD in loop space (see Ref. 27).

However, any picture of chromodynamics would now be incomplete without mention of a new approach which is gradually gaining more and more ground. This is something intermediate between theoretical and experimental physics and deserves to be called computational physics. We are speaking about calculations on a lattice by means of a computer.

This approach was formulated in 1974 (Ref. 28) and actually came to life in 1980,[29] when the first calculations were performed. The four-dimensional continuum of points of the Euclidean space is replaced here by the set of points of a discrete lattice. The step length a of the lattice serves as a cutoff. The connection between the quantity a and the physical scales is made by fixing the coupling constant $g = g(a)$ (after any dimensional observable has been fixed by means of a comparison with experiment). The value $g = 1$ corresponds to a lattice step length $a \approx 0.1$–0.2 F. This is evidently the maximally allowed value of a (and, accordingly, the maximum value of g) which one can take so as to obtain a satisfactory lattice description of continuous QCD. (In order to avoid misunderstandings, we note that the scale factor Λ_L on the lattice is very different from the values which are used in calculations according to perturbation theory: $\Lambda_L \approx 10^{-2}\Lambda_{MOM}$.)

The role of the dynamical variables is now played by the elements of the group SU(3) associated with the links of the lattice,

$$U_{x,\mu} = \exp\left(ig\frac{\lambda^b}{2}A^b_\mu(x)\,a\right),$$

where a link is taken along a direction μ. In order to introduce intensities, it is necessary to traverse a path around a cell along the links, multiplying together the corresponding $U_{x,\mu}$. As a result, we obtain (for small a)

$$U_{x,\mu\nu} \approx \exp\left(ig\frac{\lambda^b}{2}G^b_{\mu\nu}a^2\right),$$

and the action on the lattice can be written in the form

$$S = \frac{6}{g^2}\sum_{x,\,\mu>\nu}\left(1-\frac{1}{3}\mathrm{Re}\,\mathrm{tr}\,U_{x,\mu\nu}\right)\xrightarrow[a\to 0]{}\frac{1}{4}\sum_x a^4(G^a_{\mu\nu}(x))^2.$$

In the limit $a\to 0$, this expression reduces to the standard action of chromodynamics.

Quantum expectation values of observable quantities

$$\langle O\rangle = S\prod_{x,\,\mu} dU_{x,\mu}e^{-S(U)}O(U)\left[\int\prod_{x,\,\mu} dU_{x,\mu}e^{-S(U)}\right]^{-1}$$

now become finite-dimensional integrals, which are usually calculated by the Monte Carlo method, i.e., sets of random values of the variables of integration are formed, the integrands are calculated for each set, and their sum is taken. The number of variables in this problem is, of course, very large. At each point it is necessary to specify four complex 3×3 matrices, and the number of points in the four-dimensional space must be at least 10^4, i.e., we have about 10^6 dynamical variables. For an acceptable machine time, we require here a computer with a speed of the order of 100 million operations per second.

A large number of lattice calculations have now been carried out (see Refs. 30–32). By means of the method described above, calculations have been made of the strength of the interaction between two heavy quarks (the Wilson loop), the masses of low-lying gluonic excitations (glueballs), and the temperature of the transition to the state of the gluon plasma.

If quarks are included in the treatment, the calculations become much more laborious. Therefore it is now customary to make a restriction to a treatment of quarks propagating in the gluon fields without taking into account the polarization of the vacuum due to the production of virtual $q\bar{q}$ pairs. In other words, the action S which determines the weight of a configuration is taken in a purely gluon form. In this way, calculations have been made, for example, of the masses of low-lying mesons and baryons.

Finally, there have also been attempts to make a complete calculation of the dynamics of QCD with allowance for virtual quarks. This is particularly important when considering the effects of chiral symmetry breaking, when a vacuum condensate of $q\bar{q}$ pairs is formed. Because of the nonlocal character of the fermion determinant, it is necessary here to reduce the dimensions of the lattice and lower the quality of the Monte Carlo statistics, so that the results are for the most part preliminary. However, with the present-day rapid development of computational techniques, it will soon be possible to calculate numerically any sufficiently simple quantity. Of course, this will be very helpful in understanding chromodynamics.

On the whole, there is now every reason to suppose that the key physical elements of the theory of strong interactions have already been established. We are now faced with great labor in the development of mathematical methods of calculating a complex dynamical system such as the set of gluon and quark fields.

[1)] An expanded text of a lecture given at a seminar at the Department of Theoretical Physics, P. N. Lebedev Physics Institute, USSR Academy of Sciences, devoted to the memory of I. E. Tamm (1895–1971), in April 1986.

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