# Cooling atoms by means of laser radiation pressure

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The present state of research on the cooling of atomic ensembles by means of laser radiation pressure is discussed. First the basic theory of resonance radiation pressure is reviewed. The effect of recoil on the drift and fluctuations of the momentum of an atom interacting with resonance radiation is examined. There is an analysis of the forces acting on an atom in some simple light fields: a traveling plane wave, a Gaussian beam, and counterpropagating Gaussian beams. The review then focuses on the monochromatization of atoms by resonance radiation pressure and related methods for laser cooling of atoms. The roles played by the radiation pressure force and by momentum diffusion in shaping the narrow velocity distributions of the ensemble of atoms are analyzed. The lowest temperature obtainable in the radiation cooling of atoms is estimated. The basic experimental methods for laser cooling of atomic beams are described. Problems of the three-dimensional cooling of atomic ensembles and problems of localizing cold atoms in light fields, steady-state magnetic fields, and electrostatic fields are discussed. The review concludes with a discussion of some applications of cold atoms in high-precision spectroscopy, frequency standards, and atomic physics.

This review covers the basic theory of resonant radiation pressure, longitudinal cooling of atomic beams, radiation collimation and transverse cooling of atomic beams, three-dimensional radiation cooling of atoms, the localization of cold atoms, and some applications of cold atoms.

## 1. INTRODUCTION

Laser radiation has a high effective temperature $T_{\text{rad}}$, which is determined by the average number $(\bar{n})$ of photons in one degree of freedom of the field (in one laser oscillation mode). From the Bose-Einstein relation we have

$$\bar{n} = \left[\exp\left(\frac{\hbar\omega}{k_B T_{\text{rad}}}\right) - 1\right]^{-1}, \tag{1.1}$$

and thus

$$k_B T_{\text{rad}} \approx \bar{n}\hbar\omega, \text{ where } \bar{n} \gg 1.$$

This is the basis for the many applications of laser light for heating matter, over a range stretching from the reasonance excitation of atoms and molecules to laser thermonuclear fusion.

Is the inverse process possible? Can laser radiation instead be used to cool matter? In general, thermodynamics does not rule out this possibility if energy is conserved and if the entropy of the closed system consisting of the matter and the radiation increases. As matter is cooled, its energy and entropy decrease, so that a cooling is possible if the energy and the entropy of the radiation increase after the interaction with the matter. We recall that the energy and entropy of radiation are given by[1]

$$E_{\text{rad}} = N\hbar\omega, \tag{1.2}$$

$$S_{\text{rad}} = G\,[(\bar{n}+1)\ln(\bar{n}+1) - \bar{n}\ln\bar{n}]; \tag{1.3}$$

Here $N = G\bar{n}$ is the number of photons of the radiation, and $G$ is the number of degrees of freedom of the radiation, which is given by the Rayleigh-Jeans law

$$G = \frac{\omega^2\,\Delta\omega}{(2\pi c)^3}\,\Omega V, \tag{1.4}$$

where $V$ and $\Omega$ are the volume and solid angle filled by the radiation, and $\Delta\omega$ is the spectral interval of the radiation. Qualitative discussions based on relations (1.2)–(1.4) recall the discussion in the pages of this Journal of the question of increasing the brightness and temperature of a laser beam in an interaction with matter.[2]

We will assume here that there is no real absorption of photons in the matter and that the matter does not amplify the radiation; i.e., the matter simply scatters the incident radiation. Under these assumptions we have $N = \text{const}$, and from (1.2) we find that the frequency of the radiation must increase after the scattering $(\omega \to \omega + \delta\omega)$ if the matter is to be cooled. In other words, the laser beam must *be scattered* with an anti-Stokes frequency shift by the matter. Only under this condition is a cooling of matter by electromagnetic radiation possible. This circumstance was pointed out many years ago by Kastler[3] and Zel'dovich.[4] The necessary increase in the energy of the radiation does not lead to any new condition, because the entropy of laser radiation is always small (it is strictly zero for monochromatic radiation), while the entropy of scattered radiation is quite high because of the finite spectral interval of the scattered radiation and the filling of the entire solid angle $\Omega = 4\pi$ by the scattered radiation.

In 1975 Wineland and Dehmelt[5,6] and Hänsch and Schawlow[7] pointed out two interaction mechanisms in which these general thermodynamic requirements would be met and which would result in a cooling by laser radiation of

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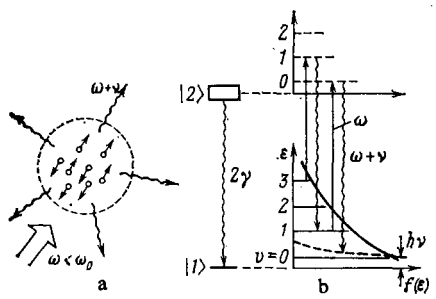


FIG. 1. a—Scattering of resonant radiation by ions localized in an electromagnetic confinement system; b—transitions responsible for the cooling of ions.

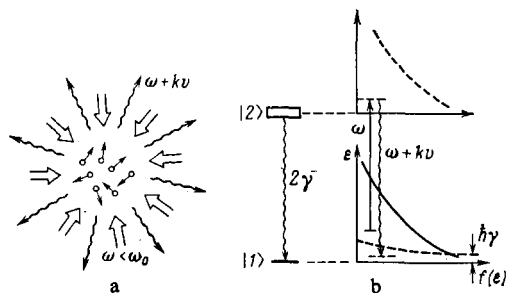


FIG. 2. a—Scattering of resonance laser radiation by freely moving atoms; b—transitions responsible for the cooling of atoms. The dashed curve is the distribution of cold atoms in the kinetic energy $\varepsilon$.

atomic ions[5,6] localized in electromagnetic confinement systems and a cooling of freely moving atoms.[7] The suggestions of Refs. 5–7, which were developments of the earlier ideas of Kastler and Zel'dovich regarding the possible cooling of matter by means of electromagnetic radiation, have led in recent years to the development of laser methods for producing cold atoms and atomic ions.

Let us take a qualitative look at the elementary processes which underlie the cooling of localized atomic ions and free atoms by laser radiation. Figure 1 explains the laser cooling of two-level atomic ions which are executing an oscillatory motion in an electromagnetic confinement system. From the quantum-mechanical standpoint, the oscillatory motion of ions at a frequency $\nu$ corresponds to a distribution of ions in vibrational energy levels $\varepsilon_v = \hbar\nu(v + 1/2)$ (Fig. 1b). In the scattering of low-frequency laser radiation, of frequency $\omega = \omega_0 - \nu$, for example, by the resonance electronic transition $|1\rangle - |2\rangle$, with frequency $\omega_0$ and natural linewidth $\gamma \ll \nu$, the ion initially undergoes a transition $|1,v\rangle \rightarrow |2,v-1\rangle$ and then spontaneously reverts to the lower state by virtue of the transition $|2,v-1\rangle \rightarrow |1,v-1\rangle$. As a result, some of the kinetic energy of the ion ($\hbar\nu$) is transferred to the resonantly scattered laser radiation. The reradiation of photons from the directed laser beam into the solid angle of $4\pi$ sr and the increase in the spectral width of the scattered radiation, to the natural linewidth $\gamma$ of the resonance transition, cause an increase in the entropy of the scattered radiation. This elementary process makes it possible to cool localized atomic ions to extremely low energies. Under the condition $\nu \gg \gamma$, under which the arguments above are valid, the limiting energy of the cold ions is close to the energy of zero-point vibrations, $\langle\varepsilon\rangle \approx (1/2)\hbar\nu$, as follows from the transition scheme in Fig. 1b. The corresponding temperature of the cold ions for typical vibration frequencies, $\nu \approx 10$ MHz, is $T \approx (1/2)\hbar\nu/k_B \sim 10^{-3}$ K.

Laser cooling of ions can also be explained in a simple way in classical terms. The oscillatory motion of an ion at a frequency $\nu$ causes side components to arise near the frequency $\nu$ of any optical spectral line of the ion. These side components are of the same nature as in the spectrum of a frequency-modulated oscillation at a high modulation index (i.e., when the frequency deviation $\delta\omega$ is less than or comparable to the modulation frequency $\nu$). Laser radiation excites an electronic state at frequencies of the low-frequency components of the spectral line, and the photons spontaneously reradiated into $4\pi$ sr have a spectrum which is symmetric with respect to the frequency $\omega_0$. Correspondingly, an increase in the frequency of the scattered radiation means that the laser radiation is cooling the ions.

The idea of laser cooling of atomic beams has been implemented experimentally[8,9] in the cooling of Mg II and Ba II ions to temperatures $\sim 10^{-2}$ K. The success of these experiments was attributable in large measure to the prolonged interaction of the ion in the confinement system with the laser radiation; this prolonged confinement was required for the repeated reradiation of a large number of photons by a single ion, so that a significant loss of kinetic energy could be achieved.

In the case of freely moving atoms the absorption spectrum is broadened inhomogeneously by the Doppler effect. This circumstance can also be exploited for laser cooling of atoms. Figure 2 shows the atomic cooling scheme corresponding to the idea of Hänsch and Schawlow.[7] Here the laser radiation isotropically illuminates part of the low-frequency half of the Doppler absorption line at some frequency $\omega$. Radiation with a wave vector $\mathbf{k} = \mathbf{n}\,\omega/c$ and a frequency $\omega < \omega_0$ can be absorbed only by atoms which are moving opposite to the photon which is absorbed:

$$\omega - \omega_0 = \mathbf{kv}, \tag{1.5}$$

in which case the Doppler effect compensates for the difference between the atomic transition frequency $\omega_0$ and the photon frequency $\omega$. The photons which are reradiated into the solid angle of $4\pi$ sr have, on the average, a frequency $\omega_0 > \omega$. As a result, part ($kv$) of the kinetic energy of the atom is transferred to the scattered radiation. In other words, in each event in which a directed photon is absorbed and reradiated isotropically on the average, the kinetic energy of the atom decreases, on the average, by an amount

$$\Delta\varepsilon = kv. \tag{1.6}$$

These arguments in terms of energies correspond to the following microscopic picture of the cooling of atoms. During the absorption of an oppositely propagating photon, the velocity of the atom decreases by an amount equal to the recoil velocity:

$$\Delta v = v_{\text{rec}} = \frac{\hbar k}{M}, \tag{1.7}$$

where $M$ is the mass of the atom. When the absorption and spontaneous emission of photons are repeated, the velocities of all the atoms decrease continuously, so that the radiation cools a gas of atoms. From this microscopic standpoint, the slowing of an individual atom in a resonance radiation field, under certain conditions (which always are satisfied quite well in situations of practical interest), may be thought of as the result of the effect on the atom of a resonance radiation pressure force. For this reason, the cooling of free atoms in a resonance radiation field is essentially always a cooling by *resonance radiation pressure*.

The cooling of atoms by resonance radiation pressure continues until fluctuations of the momentum of the atom come into play; these fluctuations are unavoidably present in the stochastic process of the reradiation of a large number of photons. The fluctuational heating of the cold atoms leads to the attainment of a steady-state temperature, whose minimum value is[10]

$$T_{\min}=\frac{\hbar\gamma}{k_B}\sim 10^{-4}-10^{-3}\text{K}. \tag{1.8}$$

Three-dimensional laser cooling of an atomic gas in the geometry shown in Fig. 2 has not yet been implemented experimentally. The problem is one of technical difficulties in irradiating a gas in the large volume which is required for arranging the interaction of freely moving atoms with laser beam over a lengthy cooling time $\tau_{\text{cool}}\gtrsim(v_0/v_{\text{rec}})\gamma^{-1}\approx 10^{-3}$ s, where $v_0$ is the average thermal velocity. Laser radiation pressure has been used successfully in several experiments to cool sodium atom beams by a counterpropagating resonant laser wave tuned to an absorption frequency of the fast atoms.[11-17] In the first experiment,[11-13] a pulsed scanning of the laser frequency was arranged in order to maintain continuously a resonance with the absorption frequency of the atoms in the beam as they were cooled. It then became possible to lower the temperature of the relative motion of the atoms to 1.5 K at a strong saturation of the resonance transition, with most of the atoms in the Doppler line entrained in the interaction.[14,15] When a spatial scanning of the frequency of the atomic transition along the path of the atoms to be slowed was used, it was possible to achieve a beam temperature of [16,17] 0.07 K. Finally, transverse irradiation has recently resulted in a cooling (radiation collimation) of a beam of Na atoms to a temperature of[18] $3.5\cdot10^{-3}$ K.

The cooling of free atoms by laser radiation pressure has now become the subject of an active research program in many laboratories concerned with the production of ultranarrow atomic resonances without Doppler broadening, atomic frequency standards, etc. (see the proceedings of a special symposium on this problem[19]). The reason is that the cooling of free atoms to very low temperatures dramatically weakens the fundamental restrictions on the width and shift of spectral lines: The linear Doppler effect and even the quadratic Doppler effect cancel out, as does transit-time broadening. Consequently, this extremely interesting problem, at the meeting point of atomic physics and laser physics, deserves a detailed discussion. We would like to call attention to a review of the problem of the laser cooling of ions[20] and some reviews of the more general problem of the effect of resonance radiation pressure on the motion of atoms.[21,27]

In the present review we discuss the present state of research on the cooling of neutral atoms by the pressure exerted by resonance laser radiation. Section 2 begins with a discussion of elements of the theory of resonance radiation pressure. Expressions for the radiation pressure force in some simple laser field configurations are given. These expressions are used in Sections 3 and 4 to analyze methods for longitudinal and transverse cooling of atomic beams. The experiments which have been carried out are also discussed. Section 5 deals with an important direction for future research: the three-dimensional cooling of an atomic gas. Section 6 discusses the present state of the problem of the localization of cold atoms. The solution of this problem will make it possible to develop a unique research method: spectroscopy of a single atom which remains for an unlimited time in the observation region. The final section of this review discusses the potential applications of cold atoms, which are a large part of the motivation for progress in this field of physics.

## 2. BASIC THEORY OF RESONANCE RADIATION PRESSURE

The elementary events which give rise to resonance radiation pressure are the absorption and emission of photons by an atom. As a result of each elementary interaction event, the atom acquires a recoil momentum. When an average is taken over a large number of these elementary events in the classical limit ($\hbar\to 0$), we find the concepts of a radiation pressure force and a momentum diffusion. Let us examine some simple results on resonance radiation pressure which can be extracted from an analysis of the influence of the recoil effect on the motion of an atom.

### a) Recoil effect in the absorption (or emission) of a photon

For a nonrelativistic atomic velocity $\mathbf{v}_0$, the momentum and energy conservation laws in the absorption ( + ) and emission ( − ) of a photon by an atom are

$$M\mathbf{v}_0\pm\hbar\mathbf{k}=M\mathbf{v},\quad \frac{1}{2}M\mathbf{v}_0^2\pm\hbar\omega=\frac{1}{2}M\mathbf{v}^2\pm\hbar\omega_0, \tag{2.1}$$

where $\mathbf{v}$ is the velocity of the atom after the absorption (emission) of a photon with a wave vector $\mathbf{k}$ and a frequency $\omega=kc$, and $\omega_0=2\pi\nu_0$. The change in the kinetic energy of the atom is given by an expression which follows from (2.1):

$$\Delta E_{\text{kin}}=\frac{1}{2}M(\mathbf{v}^2-\mathbf{v}_0^2)=\pm\hbar\mathbf{k}\mathbf{v}_0+R. \tag{2.2}$$

The energy of the absorbed (emitted) photon is

$$\hbar\omega=\hbar\omega_0+\hbar\mathbf{k}\mathbf{v}_0\pm R, \tag{2.3}$$

where $\mathbf{R}$ is the recoil energy, given by

$$R=\frac{\hbar^2k^2}{2M}. \tag{2.4}$$

According to (2.2), the change in the kinetic energy of the atom is the sum of the Doppler energy shift of the photon and the recoil energy.

A quantitative idea of the exchange of momentum and energy during the absorption (emission) of a photon can be

found in the example of a Na atom which resonantly absorbs radiation at a wavelength $\lambda = 5890$ Å in a transition from the 3S ground state to the 3P excited state. The change in the velocity of the Na atom during the absorption (emission) of a photon at this wavelength is $v_{res} = \hbar k/M = h/M\lambda = 3$ cm/s. In frequency units, the recoil energy is $R/h = h/2M\lambda^2 = 25$ kHz. By way of comparison, the average thermal velocity of a Na atom at 300 K is $\bar{v} = 5 \cdot 10^4$ cm/s. At this velocity, the Doppler frequency shift of the photon is $\Delta\nu = kv/2\pi = 850$ MHz. The natural linewidth of the 3S–2P transition is 10 MHz.

This example clearly illustrates that the recoil effect due to the absorption or emission of an optical photon is very small. For an atom with an average thermal velocity the typical relative change in velocity (or momentum) during the single absorption (or emission) of an optical phonon is $v_{rec}/\bar{v} \approx 10^{-4}$.

How does the recoil effect influence the motion of an atom under conditions such that the atom absorbs and emits a large number of photons?

#### b) Fluctuations and drift of the atomic momentum during repeated reradiation of photons

We consider the absorption and reemission of photons in an electric dipole transition of an atom between the (lower) $|1\rangle$ ground level and the $|2\rangle$ excited level. The excited level decays only to the ground level, in a process accompanied by the emission of a spontaneous photon (Fig. 3a). This two-level atom decays radiatively to its original state $|1\rangle$ after each excitation to the upper state $|2\rangle$, so that the atom undergoes a continuous resonance interaction with the laser radiation.

If conditions allow a prolonged interaction of the atom with the radiation, the change in the momentum of the atom is caused by the joint effects of the recoil of the stimulated and spontaneous transitions. In each stimulated absorption (or emission) of a photon, the atom acquires a recoil momentum $\hbar k = \hbar\omega_0/c$ along the wave vector of the radiation ($\hbar\mathbf{k}$ during absorption and $-\hbar\mathbf{k}$ during emission of a photon). The sequence of stimulated transitions is a random sequence by virtue of the statistical nature of the spontaneous relaxation of the atom to its ground state. During spontaneous emission, whose direction fluctuates, an atom acquires a recoil momentum which has a fixed magnitude $\hbar\omega_0/c$ but a random direction. For these reasons, the combined effects of the recoil of the stimulated and spontaneous transitions always cause the change in the momentum of the atom to be a complicated *stochastic* process (Fig. 3b). In turn, the stochastic nature of the change in the atom's momentum causes the change in the atom's coordinate to be a stochastic process.

What is the average change in the momentum of the atom, i.e., the change over a time interval in which the atom experiences the recoil effect many times? In this analysis, the role of the elementary processes (stimulated absorption, stimulated emission, and spontaneous emission) which change the momentum of the atom can be summarized as follows: The stimulated absorption on the average increases, and the stimulated emission on the average reduces, the momentum of the atom along the direction of the radiation vector $\mathbf{k}$. Since the direction of spontaneous emission is stochastic, this emission does not change the momentum of the atom on the average, but it does allow a relaxation of the atom to its ground state. By virtue of the spontaneous relaxation, some of the momentum acquired by the atom during the stimulated absorption of photons turns out to be a net momentum acquired during the stimulated emission of photons. As a result, the momentum of the atom on the average undergoes a *systematic drift* in the direction of the vector $\mathbf{k}$.

These qualitative arguments show that the motion of an atom in a resonance light field is the sum of a drift along the direction of the radiation wave vector and a fluctuational motion. In cases of practical importance, the motion of an atom in the light field can be treated at the classical level. In such cases the drift of the atomic momentum is a result of the application of the radiation pressure force, and the fluctuations of the atomic momentum reduced to a diffusion.

Under what conditions can the motion of an atom in a light field be treated classically?

#### c) Conditions under which the motion of an atom can be treated classically

The change in the momentum of an atom in a resonance light field is the sum of discrete steps whose magnitude is the photon momentum $\hbar k$. Accordingly, a measure of the quantum fluctuations of the atomic momentum is the momentum of the photon.

The change in the average momentum of an atom under the influence of the force due to radiation pressure should be regarded as substantial if this change causes the atom to deviate from resonance with the radiation. A disruption of the resonance nature of the interaction of the atom with the radi-

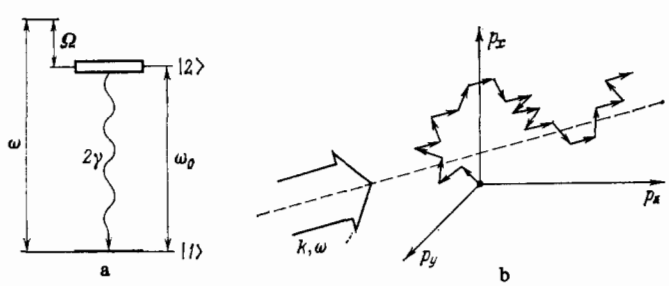


FIG. 3. a—Two-level scheme for the resonance interaction of an atom with monochromatic radiation; b—illustration of the stochastic change in the momentum of an atom in a resonance radiation field with a wave vector $\mathbf{k}$. The initial momentum of the atom is chosen to be zero.

ation occurs when the momentum changes by an amount

$$\Delta p_{\text{res}} \approx \frac{M\gamma}{k}, \tag{2.5}$$

where $\gamma$ is the half-width of the line of the atomic transition. Using (2.5), we can find a first condition under which the motion of the atom can be treated classically from the requirement that the momentum of the photon be small in comparison with $\Delta p_{\text{res}}$ (Ref. 28):

$$\hbar k \ll \frac{M\gamma}{k}. \tag{2.6a}$$

This condition can be rewritten as

$$R = \frac{\hbar^2 k^2}{2M} \ll \hbar\gamma. \tag{2.6b}$$

This condition always holds for allowed dipole transitions of atoms, since we would typically have $\gamma \approx 2\pi\cdot(10^7$–$10^8)$ Hz, while the recoil energy would be of the order of $R/\hbar \approx 2\pi\cdot(10^4$–$10^5)$ Hz.

A second condition which must be satisfied if the motion of the atom is to be treated classically stems from the obvious requirement that the small-scale fluctuational changes in the momentum by an amount $\hbar k$ must be smoothed out over a time interval $\Delta t$, over which the classical motion of the atom is examined. Since the scale time of the momentum fluctuations is $\tau_{\text{sp}} = -(2\gamma)^{-1}$, a classical description of the motion is legitimate only over time intervals[29]

$$\Delta t \gg \gamma^{-1}. \tag{2.7}$$

#### d) Radiation pressure force and gradient force

##### *1) Plane light wave*

Let us examine the expression for the radiation pressure force exerted on a two-level atom as it interacts with a plane light wave. For this purpose we determine the change $(\Delta \langle \mathbf{p} \rangle)$ in the average momentum of the atom due to stimulated and spontaneous transitions over a classical time interval $\Delta t$. Denoting by $\mathbf{p}_0$ the initial momentum of the atom, we can write its momentum at the end of the time interval $\Delta t$ as[30]

$$\mathbf{p} = \mathbf{p}_0 + \hbar\mathbf{k}(N_+ - N_-) + \sum_s \hbar\mathbf{k}_s. \tag{2.8}$$

The second term here determines the change in the momentum due to stimulated transitions involving the absorption and emission of photons with a wave vector $\mathbf{k}$. The quantities $N_+$ and $N_-$ are the numbers of photons which are involved in stimulated absorption and emission over the interval $\Delta t$. The third term reflects the change in the momentum during spontaneous decays accompanied by the emission of photons with wave vectors $\mathbf{k}_s$ ($|\mathbf{k}| = |\mathbf{k}_s| = \omega_0/c$).

From (2.8) we find the average momentum to be

$$\langle \mathbf{p} \rangle = \langle \mathbf{p}_0 \rangle + \hbar\mathbf{k}(\langle N_+ \rangle - \langle N_- \rangle), \tag{2.9}$$

where we have made use of the fact that the spontaneous photons do not contribute to the average momentum over the interval $\Delta t$, when the number of spontaneous photons is quite large. From the definition of the force we then find

$$\mathbf{F} = \frac{\Delta\langle \mathbf{p} \rangle}{\Delta t} = \frac{\langle \mathbf{p} \rangle - \langle \mathbf{p}_0 \rangle}{\Delta t} = \hbar\mathbf{k}\frac{\langle N_i \rangle}{\Delta t}, \tag{2.10}$$

where $\langle N_i \rangle = \langle N_+ \rangle - \langle N_- \rangle$ represents the average number of photons which are scattered by the atom in stimulated transitions during the time interval $\Delta t$.

We now introduce the probabilities for stimulated absorption, $W_{\text{abs}}$, and stimulated emission, $W_{\text{em}}$, of a photon by a two-level atom, along with the probability for spontaneous emission, $W_{\text{sp}} = 2\gamma$. We denote by $n_1$ and $n_2$, respectively, $(n_1 + n_2 = 1)$, the relative probabilities for finding the atom in levels $|1\rangle$ and $|2\rangle$. The average numbers of photons absorbed and emitted in stimulated transitions are then

$$\langle N_+ \rangle = n_1 W_{\text{abs}} \Delta t, \quad \langle N_- \rangle = n_2 W_{\text{em}} \Delta t. \tag{2.11}$$

The steady-state populations satisfy the balance condition

$$n_2 = n_1 \frac{W_{\text{abs}}}{W_{\text{em}} + W_{\text{abs}}}. \tag{2.12}$$

Using these relations, we can express the radiation pressure force in terms of the relative population of, say, the upper level, $n_2$:

$$\mathbf{F} = \hbar\mathbf{k}(n_1 W_{\text{abs}} - n_2 W_{\text{em}}) = 2\hbar\mathbf{k}\gamma n_2. \tag{2.13}$$

The steady-state population $n_2$ is (see, e.g., Ref. 31)

$$n_2 = \frac{1}{2}\frac{G}{1 + G + [(\Omega - \mathbf{kv})^2/\gamma^2]}, \tag{2.14}$$

where $G$ is the saturation parameter of the atomic transition, and $\Omega = \omega - \omega_0$. Finally, we find the expression originally derived by Ashkin[32] for the radiation pressure force:

$$\mathbf{F} = \hbar\mathbf{k}\gamma\frac{G}{1 + G + [(\Omega - \mathbf{kv})^2/\gamma^2]}. \tag{2.15}$$

Although this derivation of the expression for $\mathbf{F}$ has not been rigorous, the result is the same as that of the rigorous derivation and is exact.

The force in (2.15) has a Lorentzian dependence on the projection of the velocity $\mathbf{v}$ onto the wave vector $\mathbf{k}$ which is typical of a resonance interaction of a two-level atom with a monochromatic field (Fig. 4a). The force reaches a maximum exactly at resonance, with $\mathbf{kv} = \Omega$. If there is a strong saturation ($G \gg 1$), the force tends toward a maximum value $F_{\max} = \hbar k\gamma$.

##### *2) Gaussian light beam*

Most of the suggestions and experiments on laser cooling of atoms have been based on the properties of the radiation force which is exerted on an atom in a laser beam. Let us examine the features of the radiation force for the two simplest types of laser fields: the field of a Gaussian light beam and the field of two counterpropagating Gaussian beams. Everywhere below we assume that the atom is a two-level atom (Fig. 3a).

We assume that the field of the Gaussian light beam is given by

$$\mathbf{E} = \frac{1}{2}\mathbf{e}E_0 \exp[i(kz - \omega t)] + \text{c.c.} = \mathbf{e}E_0 \cos(kz - \omega t); \tag{2.16}$$

here $\mathbf{e}$ is the unit polarization vector,

$$E_0 = \mathscr{E}_0 e^{-\rho^2/2q^2}$$

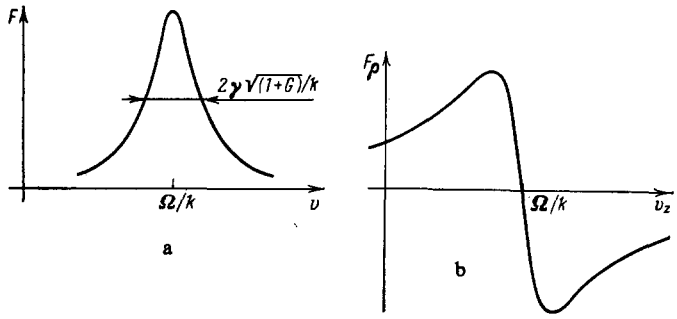


FIG. 4. a—Radiation pressure force for the case of a traveling plane wave as a function of the projection of the velocity of the atom onto the wave vector $\mathbf{k} = k\mathbf{e}_z$; b—the gradient force as a function of the projection of the velocity of the atom onto the $z$ axis. The light beam is propagating in the positive $z$ direction.

is the field amplitude of the beam, which depends on the transverse cylindrical coordinate $\rho$, and $q$ is the beam radius. We assume that the Gaussian beam is slightly divergent. For this reason, we have omitted from (2.16) a dependence of the phase on the transverse cylindrical coordinate $\rho$.

Calculations show that the radiation force in field (2.16) has components along and across the beam axis:

$$\mathbf{F} = \mathbf{e}_z F_z + \mathbf{e}_\rho F_\rho; \tag{2.17}$$

here $F_z$, the longitudinal component of the radiation force, is the radiation-pressure force and is given by (2.15),

$$F_z = \hbar k\gamma \frac{G(\rho)}{1+G(\rho)+[(\Omega-kv_z)^2/\gamma^2]}. \tag{2.18}$$

The transverse component is[33,34]

$$F_\rho = (\hbar\rho/q^2)\frac{\Omega-kv_z}{1+G(\rho)+[(\Omega-kv_z)^2/\gamma^2]}. \tag{2.19}$$

In (2.18) and (2.19), $\Omega = \omega - \omega_0$ is the difference between the field frequency and the field of the atomic transition, and $\mathbf{e}_\rho$ is a unit vector along the radial coordinate $\boldsymbol{\rho} = x\mathbf{e}_x + y\mathbf{e}_y$. The saturation parameter is

$$G(\rho) = \frac{1}{2}\left(\frac{d\mathscr{E}_0}{\hbar\gamma}\right)^2 e^{-\rho^2/q^2}, \tag{2.20}$$

where $d$ is a maxtrix element of the atomic dipole moment.

In contrast with the longitudinal component of the radiation force in (2.18), the $v_z$ dependence of the transverse component of the force is described by a dispersion curve (Fig. 4b). The transverse component of the radiation force is frequently called the "gradient force." The physical reason for the existence of a gradient force is the effect of a spatially nonuniform optical field on the atomic dipole moment which is induced by high-frequency oscillations of the field.[35] The modulus of the gradient force reaches a maximum at a frequency deviation $\Omega = kv_z \pm \gamma\sqrt{1+G}$:

$$|F_\rho|_{\max} = \frac{1}{2}\frac{\hbar\gamma\rho}{q^2}\frac{G(\rho)}{\sqrt{1+G(\rho)}}. \tag{2.21}$$

As the saturation parameter $G(\rho)$ is increased, the maximum value of $|F_\rho|$ increases in proportion to $G^{1/2}$.

### 3) Counterpropagating Gaussian beams

The field of two counterpropagating, slightly divergent Gaussian beams can be written

$$\mathbf{E} = \frac{1}{2}\mathbf{e}E_0[\exp(ikz - i\omega t) + \exp(ikz + i\omega t) + \text{c.c.}]$$
$$= 2\mathbf{e}E_0 \cos kz \cos\omega t, \tag{2.22}$$

where the notation is the same as in (2.16). The radiation force for field (2.22) can also be written in the form in (2.17), but now the longitudinal component $F_z$ (the radiation pressure force) and the transverse component $F_\rho$ (the gradient force) are determined by infinite converging fractions.[36] The expression for the radiation force takes this mathematical form because of multiresonance processes in which the atom interacts with the counterpropagating waves. These processes are responsible for a multiresonance structure of the radiation force (Fig. 5). The appearance of multiresonance structures can be explained qualitatively using the example of the radiation pressure force as follows.

In the case of a single traveling wave, the radiation pressure force contains a single resonance (Fig. 4a), which corresponds to an exact resonance of the atom with the field: $kv_z = \Omega$. In counterpropagating waves, with a slight saturation of the atomic transition, two resonances arise: one for each traveling wave, $\pm kv_z = \Omega$. These may be called "first-order resonances."

As the saturation parameter in the velocity dependence of the force increases, higher-order resonances appear because of the nonlinear interaction of the atom with the two counterpropagating waves. These resonances can be seen quite well on the dotted and dashed lines in Fig. 5. The simplest of these resonances are second-order resonances (Fig. 6a). These resonances arises when the absorption of photons by one traveling wave is accompanied by a simultaneous emission of photons into the other traveling wave. Since the frequency of one wave in the rest frame of the atom is $\omega \pm kv_z$, while the frequency of the other wave is $\omega \mp kv_z$, energy conservation,

$$(\omega \pm kv_z) - (\omega \mp kv_z) = 0, \tag{2.23}$$

tells us that the second-order resonances arise at a zero velocity ($kv_z = 0$),

Next come the third-order resonances (Fig. 6b). These resonances are caused by the nonlinear interaction in which the absorption of photons from the two propagating waves occurs at the same time as the emission of photons into one of the waves. According to the energy conservation law written in the rest frame of the atom,

$$(\omega \pm kv_z) - (\omega \mp kv_z) + (\omega \pm kv_z) = \omega_0, \tag{2.24a}$$

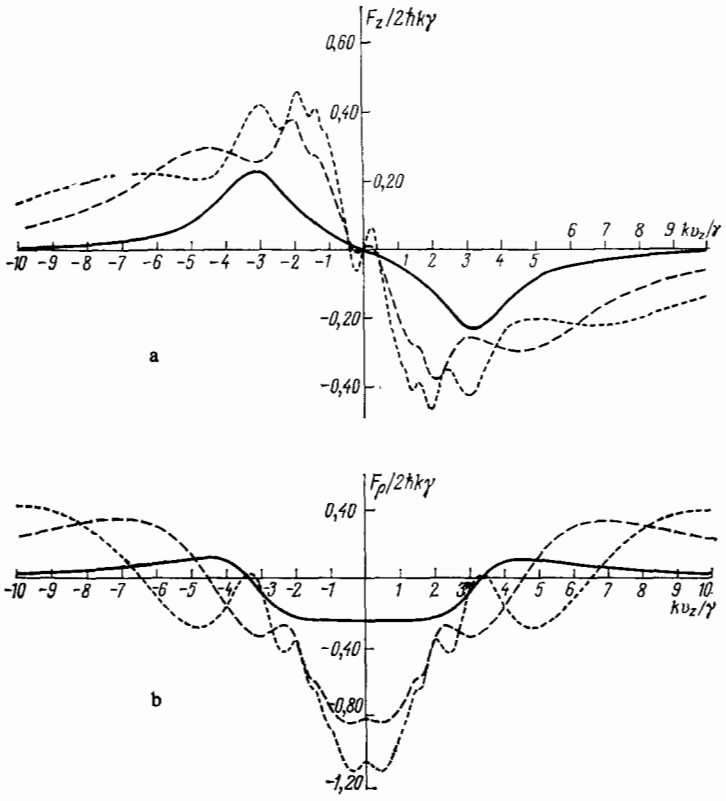


FIG. 5. a—Radiation pressure force; b—gradient force for the field in (2.22) as functions of the velocity projection $v_z$ for a frequency deviation $\Omega = 3\gamma$. The saturation parameter is $G = 1$ (solid line), $G = 9$ (dashed line), or $G = 25$ (dotted line). The transverse coordinate is $\rho = kq^2$.

the third-order resonances are centered at velocities

$$\pm kv_z = \frac{\omega - \omega_0}{3}. \tag{2.24b}$$

Higher-order resonances arise in an analogous way.

In general, the reasons for the appearance of resonances of odd and even orders can be seen from these examples to be nonlinear processes which either change the internal state of the atom or leave it unchanged. Those processes in which the absorption (or emission) of $n + 1$ photons from one wave is accompanied by the emission (or absorption) of $n$ photons into the other wave alter the internal state of the atom and give rise to resonances of order $2n + 1$, which are localized at velocities

$$kv_z = \pm \Omega/(2n + 1). \tag{2.25}$$

Those processes in which $n$ photons are absorbed from one wave and $n$ photons are emitted into the other wave do not alter the internal state of the atom and give rise to resonances of order 2n, which are localized near zero velocity.

An important consequence of the multiresonance processes is that the sign of the radiation pressure force changes near the velocity $v_z = 0$ in the case of pronounced saturation, as can be seen from the dashed line in Fig. 5a. For example, if the frequency deviation is chosen to be negative, the sign of the force near $v_z = 0$ at small values of $G$ will be opposite to the sign of the velocity $v_z$, while at high values of $G$, on the contrary, the sign of the force near $v_z = 0$ will be the same as the sign of $v_z$. A change in the sign of the radiation pressure force occurs only at a sufficiently large value of the saturation parameter $G$, such that multiresonance processes become so significant that the atom is effectively involved in a stimulated scattering of both of the counterpropagating waves.

Simple analytic expressions for the radiation force in the field of two counterpropagating waves can be derived in the approximation of slight saturation, in which saturation parameter (2.20) satisfies the condition

$$G \ll \sqrt{1 + \frac{\Omega^2}{\gamma^2}}. \tag{2.26}$$

In this approximations, the radiation pressure force is given by[10,37]

$$F_z = \hbar k \gamma G \frac{L_- - L_+}{1 + G(L_- + L_+)}, \tag{2.27}$$

where

$$L_\pm = \frac{\gamma^2}{\gamma^2 + (kv_z \pm \Omega)^2}. \tag{2.28}$$

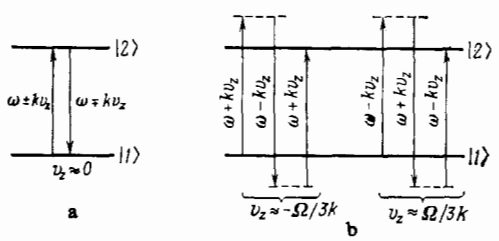


FIG. 6. Schemes of resonance processes of second (a) and third (b) orders.

The gradient force in approximation (2.26) is[33,34]

$$F_p = \frac{\hbar p}{q^2} G \frac{(\Omega - kv_z) L_- + (\Omega + kv_z) L_+}{1 + G(L_- + L_+)}. \quad (2.29)$$

Approximate expressions (2.27) and (2.29) are acceptable approximations of the exact value of the radiation force even if the saturation of the atomic transition is pronounced, and condition (2.26) does not hold. The primary distinction between the approximate and exact expressions is that the approximate expressions incorporate only the first-order resonances.

#### e) Momentum diffusion

We turn now to the fluctuation processes which cause a diffusion of the atomic momentum against the background of its drift. There are two such processes.

One of these fluctuation processes consists of fluctuations in the direction of the spontaneous emission of the photons. Since the change in the momentum of an atom is related unambiguously to the momentum of the spontaneously emitted photon,

$$\Delta \mathbf{p} = \mathbf{p}' - \mathbf{p} = -\hbar \mathbf{k}_s, \quad (2.30)$$

the fluctuations in the direction of the spontaneous emission always lead to *fluctuations in the direction of the recoil momentum* (Fig. 7).

The second process is caused by fluctuations in the number of photons which are scattered by the atom.[30,38] Since a recoil momentum $\hbar\mathbf{k}$ ($-\hbar\mathbf{k}$) is associated with each stimulated absorption (or emission) of a photon, the momentum of the atom fluctuates by the amount $\pm\hbar\mathbf{k}$ upon each unit fluctuational change in the number of scattered photons. This fluctuation process causes the momentum of the atom to vary only along the wave vector $\mathbf{k}$, and always by a discrete quanity $\hbar\omega/c$.

The existence of these two fluctuation processes follows directly from relations (2.8) and (2.9). The difference between these two expressions determines the fluctuational deviation of the momentum of the atom from its expectation value:

$$\Delta \mathbf{p} = \mathbf{p} - \langle \mathbf{p} \rangle = (\mathbf{p}_0 - \langle \mathbf{p}_0 \rangle) + \hbar \mathbf{k} \Delta N_i + \sum_s \hbar \mathbf{k}_s. \quad (2.31)$$

Here $\Delta N_i = N_i - \langle N_i \rangle$ is the deviation of the number of photons ($N_i = N_+ - N_-$) scattered by the atom from the expectation value $\langle N_i \rangle = \langle N_+ \rangle - \langle N_- \rangle$. The second term in (2.31) incorporates fluctuations in the number of scattered photons, while the third term incorporates fluctuations in the direction of the spontaneous emission of photons.

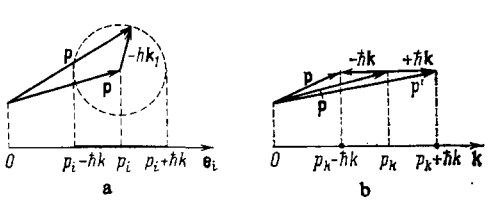


FIG. 7. Fluctuations caused in an atom's momentum by (a) fluctuations in the direction of the spontaneous emission photons and (b) fluctuations in the number of photons scattered during stimulated transitions.

For classical motion of an atom over an interval $\Delta t$ satisfying condition (2.7), these fluctuation processes cause a diffusive broadening of the momentum distribution. The broadening is diffusive because of the large number of photons which are scattered by an atom during a classical time interval (2.7). Each of the fluctuation processes causes a corresponding type of diffusive broadening of the momentum distribution. The fluctuations in the number of scattered photons are the reason for *directed diffusion*, while fluctuations in the direction of the spontaneous photons are the reason for *anisotropic diffusion*. These diffusion processes are determined quantitatively by corresponding diffusion tensors.

For the simplest case of the interaction of a two-level atom with resonance radiation with the wave vector $\mathbf{k}$ and a frequency $\omega = kc$, we can evaluate the diagonal elements of the momentum-diffusion tensor in a simple way. We define the diagonal elements of the diffusion tensor by ($i = x, y, z$)

$$D_{ii} = \frac{1}{2} \frac{\langle (\Delta p_i)^2 \rangle}{\Delta t}. \quad (2.32)$$

If, for definiteness, we assume that the radiation is propagating along the $z$ axis ($\mathbf{k} = k\mathbf{e}_z$), we can write the following expression on the basis of (2.31):

$$D_{ii} = D_{ii}^{a} + D_{ii}^{d} = \frac{1}{2} \hbar^2 k^2 \left( \alpha_{ii} \frac{\langle N_s \rangle}{\Delta t} + \frac{\langle (\Delta N)^2 \rangle}{\Delta t} \delta_{zi} \right); \quad (2.33)$$

here $a_{ii} = \langle \cos^2 \vartheta_i \rangle$ is the expectation value of the square of the cosine of the angle $\vartheta_i$, which determines the projection of the momentum $\hbar\mathbf{k}_s$ onto the axis $i = x, y, z$. The indices "a" and "d" specify anisotropic and directed diffusion.

To find the elements of the momentum diffusion tensor we need to determine the mean square value of the fluctuations in the number of photons which are involved in the stimulated scattering, $\langle (\Delta N_i)^2 \rangle$; this value appears in (2.33). It depends on the statistics of the number of scattered photons. It can be estimated by assuming that the scattering of the photons is a completely random process. In this case the probability $P(N_i)$, for the stimulated scattering of $N_i$ photons over the interval $\Delta t$, must obey a Poisson distribution:

$$P(N_i) = \frac{\langle N_i \rangle^{N_i}}{N_i!} e^{-\langle N_i \rangle}. \quad (2.34)$$

For Poisson statistics the relation $\langle (\Delta N_i)^2 \rangle = \langle N_i \rangle$ holds. Furthermore, for our two-level atom we have $\langle N_i \rangle = \langle N_s \rangle = 2\gamma n_2 \Delta t$; i.e., the expectation values of the numbers of photons involved in stimulated scattering and spontaneous emission are identical. Using the latter relations, we finally find[28,38]

$$D_{ii} = \frac{1}{2} \hbar^2 k^2 \gamma (\alpha_{ii} + \delta_{zi}) \frac{G}{1 + G + [(\Omega - \mathbf{kv})^2/\gamma^2]}. \quad (2.35)$$

Approximate expression (2.35) is quite close to the exact value, derived in Refs. 29 and 39–41. A deviation of (2.35) from the exact value is caused by the deviation of the statistics of the photons involved in the stimulated scattering from Poisson statistics.

Near a resonance ($\Omega = \mathbf{kv}$) and with a pronounced saturation of the atomic transition ($G \gg 1$) we have

$$D_{ii} \approx \hbar^2 k^2 \gamma. \tag{2.36}$$

For optical transitions of atoms, the elements of the velocity diffusion tensor, found by replacing the recoil momentum by the recoil velocity $v_{rec} = \hbar k/M$,

$$C_{ii} = \frac{D_{ii}}{M^2} \approx \gamma v_{rec}^2, \tag{2.37}$$

are of the order of $10^9$ cm$^2$/s$^3$.

### 3. LONGITUDINAL COOLING OF ATOMIC BEAMS

We begin with a consideration of the use of radiation pressure to slow the motion of atoms along the axis of an atomic beam. We first offer a qualitative analysis of the deformation of the velocity distribution of an atomic beam on the basis of the simple expressions derived for the force in Section 2. In those cases in which the diffusion is important, we will use a stochastic Langevin equation to derive the motion of the atoms.

#### a) Monochromatization of atomic velocities

We begin with some basic estimates dealing with the change in the velocities of an ensemble of two-level atoms in the field of a plane, monochromatic, traveling light wave. In this case an atom experiences a radiation pressure force (2.15), which has a significant effect on an atom if its velocity lies in the interval

$$| kv_z - \Omega | \leqslant \gamma_S = \gamma (1 + G)^{1/2}. \tag{3.1}$$

The change in the velocity of an atom in interval (3.1) occurs over the scale time ($\tau_f$) for the resonance interaction of the atom with the wave. This time can be found from the equation of motion of the atom under the influence of the force in (2.15):

$$\tau_f \approx \frac{2(1+G)^{3/2}}{G} \frac{\hbar}{R}. \tag{3.2}$$

At a moderate saturation of the atomic transition, with $G \approx 1$ (here the wave intensity required for an allowed transition of the atom would be $I \approx 0.1$ W/cm$^2$), the time $\tau_f$ is of the order of $10^{-5}$–$10^{-6}$ s. The change in the velocity of the atom over the time $\tau_f$ is $\gamma_S/k \approx 10^2$–$10^3$ cm/s. Over this time, an atom with a velocity of the order of the average thermal velocity, $\bar{v} \approx 10^5$ cm/s, travels a distance $l = \bar{v}\tau_f \approx 0.1$–$1$ cm.

We now consider the case of an atomic ensemble, in which all the atoms have positive initial velocities $v_z$. The ensemble is interacting with a light wave which is propagating along the $z$ axis (Fig. 8a). In this case the radiation pressure force increases the velocities of the resonant atoms with time, leading to a dip in the velocity distribution of the atoms, centered at $v_{res} = \Omega/k$. The formation of this dip is in turn accompanied by the formation of a peak in the accelerated atoms, centered at a velocity above $v_{res}$. If the interaction lasts a sufficiently long time, essentially all the atoms increase in velocity, so that a narrow, monovelocity distribution of atoms always forms from an initial broad distribution. Analogously, in the propagation of a light wave opposite to an atomic beam, a dip forms at the resonance velocity $v_{res} = -\Omega/k$ in the velocity distribution of the atoms (Fig. 8b). In this case a narrow velocity distribution of atoms forms at a velocity below $v_{res}$. The effect of the radiation pressure force is thus to form from any broad initial distribution a narrower velocity distribution, i.e., to *monochromatize the atoms* in velocity space.[29,42–49]

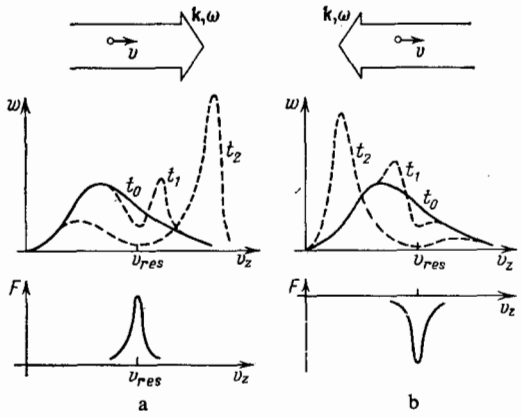


FIG. 8. Sketch of the deformation of the velocity distribution $w(v_z)$ of an ensemble of atoms in the field of a traveling light wave; plot of the radiation pressure force $F$ as a function of the velocity projection $v_z$ ($t < t_1 < t_2$). a—The atoms are moving in the direction of the wave vector of the light wave; b—the atoms are moving in the opposite direction.

A quantitative description of the velocity monochromatization of atoms by the radiation pressure force $F = MA_z$ ($M$ and $A_z$ are the mass and acceleration of the atom) can be found from the Liouville equation describing the evolution of an atomic velocity distribution $w = w(v_z, t)$:

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial v_z}(A_z w) = 0. \tag{3.3}$$

We assume that the initial velocity distribution of the atoms corresponds to a relaxation of the atoms in the beam from a source with a temperature $T$:

$$w(v_z, 0) = 4\pi^{-1/2}\bar{v}^{-3} v_z^2 \exp(-v_z^2/\bar{v}^2), \quad v_z \geqslant 0, \tag{3.4}$$

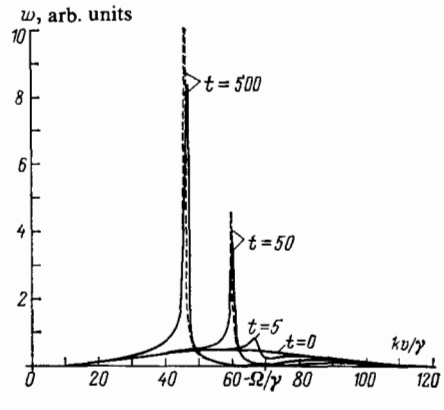


FIG. 9. Evolution of the velocity distribution of a spatially uniform atomic beam in a counterpropagating light wave. The frequency deviation is $\Omega = -70\gamma$, and the saturation parameter is $G = 10$. The time and velocity units are $(kv_{rec})^{-1}$ and $\gamma/k$. Dashed lines—Solutions of the Liouville equation; solid lines—solutions of the Fokker-Planck equation.

where $\bar{v} = (2k_B T/M)^{1/2}$ is the mean velocity of an atom.

We consider the case of the slowing down of an atomic beam in a light wave propagating in the direction opposite to that of the beam. In this case the frequency deviation $\Omega$ is negative, and the acceleration of an atom, $A_z$, is

$$A_z = -\gamma v_{\text{rec}} \frac{G}{1+G+[(\Omega+kv_z)^2/\gamma^2]}. \tag{3.5}$$

Figure 9 shows the results of a numerical solution of Liouville equation (3.3) for this case. The mean velocity has been chosen to be $\bar{v} = 48\gamma/k$. We see from Fig. 9 that the deformation of the velocity distribution actually occurs over a time of the order of $\tau_f$. The numerical values of the quantities indicated in Fig. 9 depend on the parameters of the atomic transition. For example, let us assume that the counterpropagating light wave is interacting with a beam of Ca atoms at the 4S-4P resonance transition. In this case the dimensional values are

$$\gamma/k = 740 \text{ cm/s}, \ (kv_{\text{rec}})^{-1} = 2{,}9\cdot 10^{-6} \text{ s},$$
$$\bar{v} = 3{,}5\cdot 10^4 \text{ cm/s at } T = 300 \text{ K}.$$

### b) Role of momentum diffusion

In all cases, the monochromatization of the atomic velocities proceeds only up to a certain limit, set by velocity diffusion. In order to incorporate the effect of velocity diffusion on the evolution of the atomic velocity distribution we need to solve the corresponding Fokker-Planck equation.[45,29] For a spatially uniform, one-dimensional ensemble of atoms in the field of a traveling plane wave, the Fokker-Planck equation is

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial v_z}(A_z w) = \frac{\partial^2}{\partial v_z^2}(C_{zz} w), \tag{3.6}$$

where $C_{zz}$ is the velocity diffusion coefficient. An approximate value of the latter is given by (2.35).

Figure 9 shows numerical solutions of Eq. (3.6) found for the same parameter values as in the solution of Liouville equation (3.3). Comparison of these solutions with the solutions of Eq. (3.3) shows that the velocity monochromatization of the atoms is disrupted as the diffusion time is increased. The relative contribution of diffusion depends on the time over which the atoms interact with a field and on the width of the initial atomic velocity distribution. In particular, for the rather broad initial distribution chosen in Fig. 9 the contribution of diffusion becomes important at times $t \gg (kv_{\text{rec}})^{-1} = \hbar/2R$.

The minimum width of a velocity distribution in the monochromatization of atoms by a light field has been calculated in many studies.[39,45,46] A simple estimate of the minimum width can be found from the stochastic Langevin equation. Let us consider atoms which at $t = 0$ are at resonance with a wave which is directed opposite to the motion of the atoms (Fig. 10). At $t = 0$ the velocity distribution (for convenience, normalized to a single atom) is a $\delta$-function for the resonant atoms:

$$w(v_z, 0) = \delta(v_z - v_{\text{res}}), \tag{3.7}$$

where $v_{\text{res}} = -\Omega/k$. At $t > 0$, the motion of the resonant

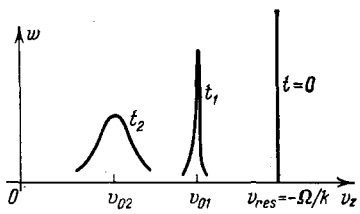


FIG. 10. Shift of the mean velocity $v_0$ and broadening of the velocity distribution of atoms which at $t = 0$ are at resonance with the radiation. The atoms are moving in the direction opposite to the light wave. The mean velocities $v_{01}$ and $v_{02}$ correspond to the times $t_1 < t_2$.

atoms is described by the Langevin equation

$$\frac{dv_z}{dt} = A_z + \xi(t), \tag{3.8}$$

where the acceleration $A_z$ is determined by expression (3.5), and $\xi(t)$ is the stochastic force responsible for the diffusion of atomic velocities, for which the relation $\langle \xi(t) \rangle = 0$ holds.

Equation (3.8) describes the change in the mean velocity $v_0$ and the broadening of the velocity distribution of the resonant atoms. To find the broadening of the velocity distribution, we need to consider these two processes separately. For this purpose it is convenient to introduce the relative velocity $u = v_z - v_0$. The width of the velocity distribution of the monochromatized atoms is small in comparison with the velocity interval $\gamma\sqrt{1+G/k}$, as is clear from the solution of Eq. (3.8). The force $MA_z(v_0 + u)$ can thus be expanded in a series in the velocity $u$. Retaining the first two terms of this series, we find that Eq. (3.8) reduces to an equation for the change in the mean velocity $v_0$.

$$\frac{dv_0}{dt} = -\gamma v_{\text{rec}} \frac{G}{1+G+[(|\Omega|-kv_0)^2/\gamma^2]}, \tag{3.9}$$

and a Langevin equation for the relative velocity $u$,

$$\frac{du}{dt} = -\beta u + \xi(t), \tag{3.10}$$

where the dynamic friction coefficient is

$$\beta = \frac{4R}{\hbar} \frac{G(|\Omega|-kv_0)/\gamma}{1+G+[(|\Omega|-kv_0)^2/\gamma^2]}. \tag{3.11}$$

Equation (3.10) describes the relaxation of the velocity distribution of resonant atoms to a steady-state distribution with a width which depends on the mean velocity $v_0$. The scale time for the relaxation of the velocity distribution is determined by the reciprocal of the friction coefficient, $\tau_v = \beta^{-1}$, according to (3.10). According to (3.11), the time $\tau_v$ is a nonlinear function of the mean velocity. At the mean velocity

$$v_0 = \frac{|\Omega|-\gamma_S}{k} - v_{\text{res}} = \frac{\gamma_S}{k} \tag{3.12}$$

the relaxation time reaches its minimum value

$$\tau_v^m = \frac{\hbar(1+G)^{3/2}}{GR}. \tag{3.13}$$

This time is equal in order of magnitude to the time $\tau_f$ given by (3.2).

Over a time $t \approx \tau_v^m \approx \tau_f$, a steady-state velocity distribu-

tion of the atoms is thus established. The steady-state solution of Eq. (3.10) is the Maxwellian distribution

$$w(u)=\sqrt{\frac{M}{2\pi k_B T_z}}\exp\left(-\frac{Mu^2}{2k_B T}\right) \tag{3.14}$$

with a temperature determined by the velocity diffusion coefficient and the dynamic friction coefficient,

$$T_z=\frac{MC_{zz}}{k_B\beta}. \tag{3.15}$$

At the time at which the narrow distribution of monochromatized atoms is formed, with $t\approx\tau_f\approx\tau_v^m$ and $v_0\approx(|\Omega|-\gamma s)/k$, the temperature in (3.15) is given by

$$T_z\approx\frac{\hbar\gamma_S}{2k_B}. \tag{3.16}$$

The width of the velocity distribution, $\delta v_z$, is related to the temperature $T_z$ by

$$\delta v_z=\sqrt{\frac{2k_B T_z}{M}}.$$

We thus see that the quantity $\delta v_z$ is smaller than the velocity interval $\gamma_S/k$ by a factor $\sqrt{\hbar\gamma_S/R}$, which would ordinarily be of the order of 10–100.

At $v_0=0$, with the retarding force of the radiation pressure stopping atoms, the minimum possible temperature is estimated to be (for $|\Omega|\gg\gamma_S$)

$$T_z=\frac{\hbar|\Omega|}{4k_B}. \tag{3.17}$$

In the case discussed above, of Ca atoms with the parameter values $G=10$ and $|\Omega|=48\gamma$, we would have $T_z\approx 8\cdot10^{-4}$ K at the time at which the narrow velocity distribution is formed, and with $v_0=0$ we would have $T_z\approx10^{-2}$ K.

### c) Difficulties and necessary conditions in experiments

#### 1) Two stages in the evolution of the velocity distribution

The time evolution of the velocity distribution of an atomic beam which is being slowed down by a laser beam can be thought of as consisting of two stages.[29,48,49]

In the first stage, a narrow velocity distribution is established (this is the kinetic stage of the evolution). The scale time for the attainment of this velocity distribution is[29]

$$\tau_1\approx\tau_v^m\approx\frac{(1+G)^{3/2}}{Gkv_{rec}}. \tag{3.18}$$

Over this time, the radiation pressure force changes the velocities of the resonant atoms by an amount of order $\Delta v_z\approx\gamma_S/k$, while the width of the narrow peak in the velocity distribution reaches the value $\delta v_z\approx(2\hbar\gamma_S/M)^{1/2}$.

The second stage in the evolution of the velocity distribution is the slowing down of the atomic ensemble to vanishingly low velocities (the "gas-dynamic" stage of the evolution).[29,28,49] The relative lengths of the first and second stages are determined by the ratio of the characteristic velocity interval of the effect of the radiation pressure force (which is of the order of $\gamma\sqrt{1+G/k}$) and the initial velocity of the atomic ensemble ($v_0\approx|\Omega|/k$) which is at resonance with the laser beam. In the case $|\Omega|\gg\gamma\sqrt{1+G}$, i.e., a case in which the change in the velocities of the atoms in the course of the slowing down is greater than the scale velocity interval for the effect of the radiation pressure force, the time of the second stage is[49]

$$\tau_2\approx\frac{(|\Omega|/\gamma)^3}{3Gkv_{rec}}\gg\tau_1. \tag{3.19}$$

The typical duration of the second stage in the case in which resonance transitions in the atom are used is $\tau_2=10^{-3}$–$10^{-2}$ s, and the corresponding distance over which the light interacts with the beam is $l\approx1$–10 m. Clearly, such large interaction lengths hold little promise for producing slow atoms. In other words, it is necessary to shift the velocity peak effectively from the center of the distribution to vanishingly low velocities.

Let us examine the basic problems which arise in experiments on the production of beams of cooled atoms.

#### 2) Cyclic nature of the interaction

The change in the velocity of an atom during the reemission of $N$ photons in the case in which an atomic beam is subjected to a resonant laser beam propagating in the opposite direction is $\Delta v=Nv_{rec}$. In the case in which the change in the velocity of the atom is comparable to its mean velocity in the beam, $\bar{v}$, the number of re-emitted photons must be of the order of $N\simeq\bar{v}/v_{rec}\approx10^4$–$10^5$. The reason why such a large number of photons must be re-emitted is the small value of the photon's momentum in comparison with the mean thermal momentum of an atom. The large value of the number of re-emitted photons shows that a substantial change in the velocity of an atom can be achieved only if there is a prolonged cyclic interaction of the atom with the resonant radiation. Experimentally the basic factor which leads to a loss of the cyclic nature of the interaction of an atom with radiation is either optical pumping into one of the hyperfine levels of an atom or an instability of the laser frequency.

#### 3) Doppler shift

As atoms are slowed down by a laser beam, there is a change in the Doppler shift of the absorption frequency of an individual atom with respect to the laser frequency. The efficiency of the slowing down of the atoms drops sharply. The scale value of the change in the velocity of an atom in the course of monochromatization is $\Delta v\approx\gamma_S/2\pi\approx10^2$–$10^3$ cm/s, or an insignificant fraction of the width of the original velocity distribution. For a subsequent efficient slowing of the atoms, it is necessary to eliminate the Doppler shift which has occurred.

Three methods have been used to eliminate the Doppler shift during the slowing down of atoms: 1) changing the laser frequency to match the changes in the absorption frequency of the atom; 2) changing the frequency of the atomic transition by means of a magnetic field; and 3) introducing a field-induced increase in the width of the absorption line of the atoms by increasing the laser intensity.

#### 4) Diffusion and geometric dispersal of atoms

As we have already mentioned, a velocity monochromatization is unavoidably accompanied by a diffusion of

atoms in both velocity space and coordinate space. As a result of these effects, the atoms are heated, and the transverse dimension of the atomic beam increases. If the velocities of the atoms are reduced substantially ($\Delta v \approx \bar{v}$), the increase in the transverse dimension of the atomic beam may be significant, as we will show below. Another factor tending to increase the transverse dimension of the beam is the finite divergence of the beam, which determines the transverse component of the velocity of the atoms. A decrease in the longitudinal velocity component increases the time spent in the interaction zone, and, since the transverse velocity of the atoms varies only slightly in the process, an increase in the transit time leads to an increase in the transverse dimension of the atomic beam at the end of the interaction zone.

Both the diffusion and the geometric dispersal of the atoms will cause a transverse displacement of the atoms out of the laser beam and terminate the interaction of these atoms with the laser field. The increase in the transverse dimension of the atomic beam can be estimated in the following simple way. After the scattering $N$ photons by an atom, the transverse velocity of the atom becomes

$$v_\perp \approx \frac{1}{2} v_0 \Delta\varphi + v_{\text{rec}} \sqrt{\frac{N}{3}}, \tag{3.20}$$

where $\Delta\varphi$ is the angular divergence of the atomic beam, and $v_{\text{rec}}$ is the recoil velocity. Correspondingly, the transverse dimension of the atomic beam is given by

$$d = \int_0^{t_{\text{int}}} v_\perp \, dt. \tag{3.21}$$

The number of photons re-emitted by an atom during uniform-deceleration motion is

$$N = \frac{a t_{\text{int}}}{v_{\text{res}}}, \tag{3.22}$$

where $a$ is the acceleration of an atom, and $t_{\text{int}}$ is the time over which the atom interacts with the radiation. According to (3.20)–(3.22), the increase in the transverse dimension of the atomic beam due to diffusion and geometric dispersal is

$$d \approx \frac{1}{2} v_0 \Delta\varphi t_{\text{int}} + \frac{2}{3\sqrt{3}} \sqrt{a v_{\text{rec}} t_{\text{int}}^3}. \tag{3.23}$$

With the typical parameter values $a = 10^8$ cm/s$^2$, $t_{\text{int}} = 10^{-3}$ s, and $\Delta\varphi = 10^{-2}$ rad, we find $d \approx 15$ mm.

#### 5) Saturation of the excited transition

Saturation of the transition works in two ways. First, it is only in the saturation regime that the rate of re-emission of photons by an atom reaches a maximum, so that the length of the zone of the interaction with the laser beam reaches a minimum for the given decrease in the velocity of the atoms. Second, a pronounced saturation of the atomic transition will lead to a resonance of the radiation with atoms in a velocity interval comparable to the width of the original velocity distribution.

### d) Experiments on the laser cooling of atomic beams

Most of the experiments which have been carried out on velocity monochromatization and radiative slowing down have used a collinear interaction of the atomic beam with the laser beam, since it is only in this geometry that beams of slow atoms can be formed. Three resonance methods have been used for radiative slowing down; the three differ in the way in which the Doppler shift between the frequency of the atomic transition and the laser frequency is eliminated.

The first report of a slowing down of atoms was in Refs. 11 and 12, where it was demonstrated that it is possible to achieve a pulsed slowing down of an atomic beam by *scanning the frequency of the laser beam* along the Doppler absorption line. In this case the light acts efficiently on the atom by virtue of the continuous adjustment of the laser frequency to match the resonance frequency of the atom as it is being slowed down. This method was developed further in the experiments of Refs. 16, 17, and 50, where the frequency of the atomic transition was continuously adjusted by means of a nonuniform magnetic field, while the laser frequency was held fixed (*scanning the frequency of the atomic transition*). The third method[13–15,51,52] for the radiative slowing down of an atomic beam makes use of an intense laser beam with fixed frequencies in the Doppler absorption line of the atoms (*steady-state monochromatization*).

We will discuss each of these approaches separately.

#### 1) Scanning the laser frequency

When there is a significant deformation of the velocity distribution by the pressure exerted by a laser beam with a scanned frequency $\omega_l$, it is necessary to satisfy the following condition at each instant:

$$\omega_l(t) = \omega_0 - kv. \tag{3.24}$$

This condition ensures that the radiation pressure force in (2.15) will be at its maximum. Differentiating (3.24) with respect to the time, and using expression (2.15) for the radiation pressure force, we find the optimum rate of the frequency scanning to be

$$\frac{d\omega_l}{dt} = \frac{2R\gamma}{\hbar} \frac{G}{1+G}. \tag{3.25}$$

All the research which has been carried out has used beams of sodium atoms interacting with laser radiation at the $3S_{1/2} - 3P_{3/2}$ transition (Fig. 11). A prolonged cyclic interaction of an atom with radiation was achieved in Refs. 11 and 12 through a preliminary optical orientation of the atom by circularly polarized ($\sigma^+$) laser radiation (Fig. 11a). For this purpose the laser beam was adjusted to resonance with the $3S_{1/2}(F=2) - 3P_{3/2}(F'=3)$ transition. The radiation excited transitions involving a change $\Delta m_F = 1$ in magnetic quantum number. As a result of the stimulated and spontaneous transitions, the atom quickly reached the $F=2$, $m_F=2$ sublevel. The scale time for the orientation at a laser beam intensity $I_l \approx 0.1$ W/cm$^2$ was about $100\tau_{\text{sp}} = 1.6\,\mu$s, much shorter than the transit time of the atoms through the interaction zone ($\approx 1$ ms).

The transitions

$$F=2(m_F=2) - F'=3(m'_F = -2, -1, 0, 1, 2)$$

are forbidden by the selection rules in this case. Consequently, an atom which has reached the $F=2$, $m_F=2$ sublevel

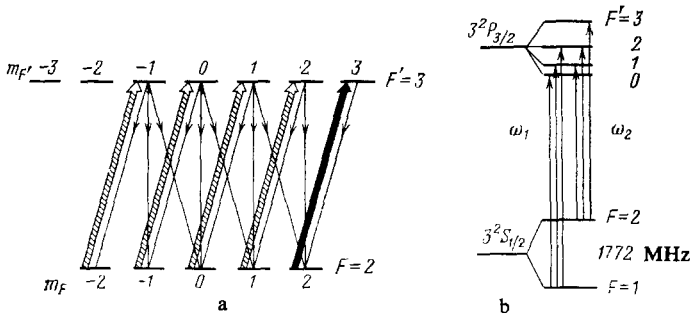


FIG. 11. Ground and first excited levels of the sodium atom and schemes for a cyclic interaction with a laser beam. a—Interaction with a single-frequency beam of polarization $\sigma^+$; b—interaction with a two-frequency beam.

interacts cyclically with the radiation, undergoing transitions between two quantum states.

An interaction of a cyclic nature was arranged in Ref. 53 through a two-frequency excitation of sodium atoms from two hyperfine sublevels of the ground state (Fig. 11b). The two-frequency laser beam was produced as a single-mode laser beam passed through an electro-optic modulator. At the exit from the modulator the fundamental frequency was accompanied by side components differing from the fundamental frequency by the modulation frequency of the crystal.

The experimental arrangement used in Refs. 11, 12, and 53 to monochromatize the velocity distribution of a beam of sodium atoms slowed down by a counterpropagating laser beam is shown schematically in Fig. 12. In this arrangement, the collimated beam of atoms (2) emitted from source 1 is bombarded by a counterpropagating light beam 3, whose frequency is adjusted to resonance with the $3S_{1/2} - 3P_{3/2}$ transition. The velocity distribution of the atoms along the axis of the atomic beam is determined from a fluorescence signal excited by a probe laser beam 4. This probe beam is a single-frequency beam, with a scannable frequency; it propagates at a small angle with respect to the laser beam. To keep the intense laser beam from distorting the fluorescence signal excited by the probe beam, the intense laser beam was periodically interrupted by a mechanical chopper for the time required for measurements. The fluorescence signal was measured only in those time intervals in which the intense beam was cut off by the chopper.

Figure 13 shows an experimental curve of the fluorescence intensity as a function of the frequency of the probe laser beam.[53] Curve 1 is the original velocity distribution of the atomic beam, measured in the $F = 1$ and $F = 2$ sublevels of the ground state. Curve 2 directly reflects the longitudinal velocity distribution of the atoms in the beam which results from the slowing down of the atoms by the counterpropagating laser beam. The arrows show the directions and the range of the scanning of the laser frequency. The mean velocity of the atoms in the narrow velocity distribution is $\bar{v} \approx 2 \cdot 10^4$ cm/s, and the width of the velocity distribution is $\Delta v \approx 2 \cdot 10^3$ cm/s.

### *2) Scanning the frequency of the atomic transition*

Another possibility for prolonging the resonance interaction of the atoms with the laser beam in the slowing down of an atomic beam is to fix the laser frequency and scan the frequency of the atomic transition. The frequency of an atomic transition can be changed either by applying an electric field (by virtue of the Stark effect) or by applying a magnetic field (by virtue of the Zeeman effect). The field must be varied in such a way that the frequency shift of the atomic transition due to the interaction with the electric or magnetic field is equal to the Doppler shift.

This resonance-interaction regime was achieved in

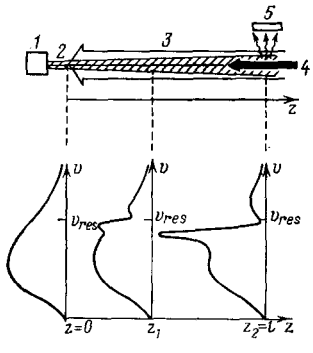


FIG. 12. The experimental arrangement of Refs. 11 and 12 for observing velocity monochromatization of an atomic beam. 1—Beam source; 2—atomic beam; 3, 4—laser beams; 5—fluorescence detector. Shown at the bottom is the deformation of the velocity distribution for two different lengths of the zone in which the atoms interact with the radiation.

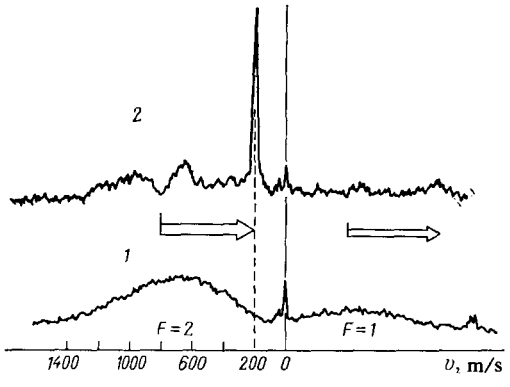


FIG. 13. Radiation slowing-down of sodium atoms during frequency scanning of the laser beam.[53] 1—Original velocity distribution; 2—velocity distribution after the interaction of the atoms with the laser beams; arrows—directions and ranges of the scanning of the laser frequencies.

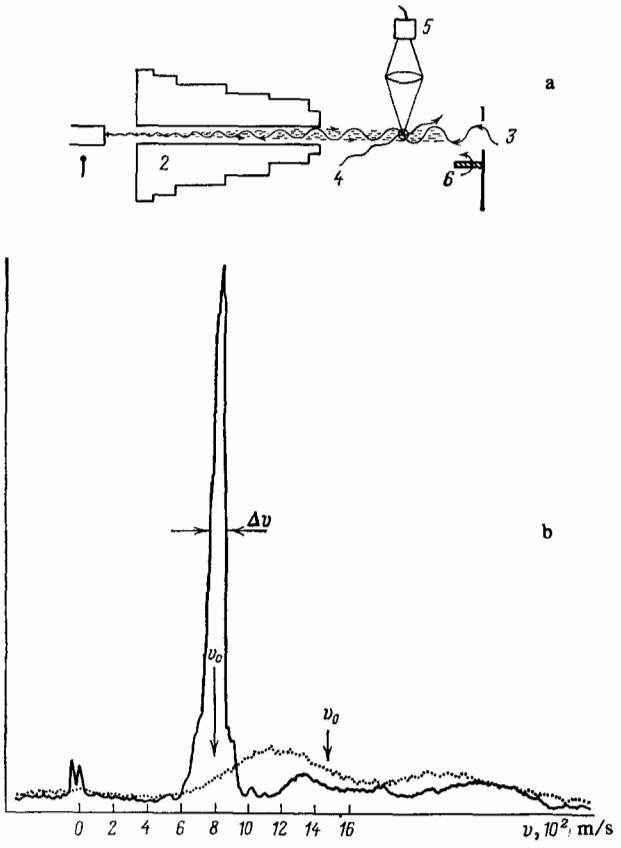


FIG. 14. a: Experimental arrangement for experiments on the slowing-down of sodium atoms during scanning of the frequency of an atomic transition. 1—Source of atoms; 2—solenoid; 3—intense laser field; 4—probe laser beam; 5—atomic detector; 6—mechanical laser-beam chopper. b: Longitudinal cooling of an atomic beam during scanning of the frequency of an atomic transition.[16] Points—Original velocity distribution (the two peaks at the left correspond to zero velocities); $v_0$—velocity of the atoms at resonance with the radiation at the beginning of the interaction; $\Delta v$—width of the peak of the monovelocity atoms; $v_c$—central velocity in the peak.

Refs. 16 and 17 in the following way. A beam of sodium atoms is sent through a nonuniform magnetic field, which varies linearly along the beam axis. The magnetic sublevels are split and shifted in the magnetic field, so that there is a change in the frequency of transitions between sublevels of the ground and excited states.

For the $3S_{1/2} - 3P_{3/2}$ resonance transition of the sodium atom, the constant of the Zeeman frequency shift of the transition is $\gamma_z = 14$ GHz/T in the case of circularly polarized radiation. The change in the absorption frequency of an atom due to the change in the Doppler shift is offset by the magnetic field, which is varied in accordance with $B = B_b + B_0\sqrt{1 - (2az/v^2)}$, where $B_b$ is the static magnetic field, the second term is the spatially varying magnetic field, and $a$ is the acceleration of an atom. For $B_0 = 0.16$ T and $B_b = 0.05$ T, a resonance interaction of the radiation is achieved with atoms with an initial velocity $v_0 = 900$ m/s, over a distance $l = 90$ cm.

In the experiments, the fundamental laser beam (circularly polarized) excites the transition $3S_{1/2}(F = 2, m_F = 2) - 3P_{3/2}(F' = 3, m'_F = 3)$ of the sodium atom. The static magnetic field $B_0$ makes it possible to avoid a parasitic optical pumping to the $F = 1$ sublevel of the ground state (Fig. 11), since in a magnetic field of this magnitude the splitting of the magnetic sublevels is substantially greater than the radiation widths of the sublevels. In this case the probability for the transition $(F = 2, m_F = 2) - (F' = 3, m'_F = 3)$ is many orders of magnitude higher than the probabilities for other possible transitions.[16]

Figure 14a shows the layout of an experimental apparatus used to slow down sodium atoms during a scanning of the frequency of an atomic transition. The retarding laser beam is directed opposite to the atomic beam. The intensity of the laser beam is 30 mW, and the diameter of the laser beam is 5 mm. The atomic beam moves inside the laser beam over the entire interaction distance—from the atomic gun to the detection zone. The velocity distribution of the atoms is measured at the end of the interaction zone with the help of a second laser beam (a single-mode beam whose frequency can be tuned along the Doppler absorption line of the atomic beam). The intense field is cut off by a mechanical chopper during the measurements.

Figure 14b shows the deformation of the velocity distribution. The points show the original velocity distribution. The two peaks at the left in this figure are the fluorescence signals from the atomic beam as it is excited by the perpendicular laser beam. Their positions on the frequency scale indicate a zero longitudinal velocity for the atoms; here $v_0$ is the velocity of the atoms which are at resonance with the radiation at the beginning of the interaction, while $v_c$ is the mean velocity of the narrow velocity distribution formed as a result of the interaction of the atoms with the laser beam. The width ($\Delta v$) of the peak in the velocity distribution is $v_c/10$.

The minimum velocity reached in those experiments was $v_{min} = 4 \cdot 10^3$ cm/s. The minimum width of the velocity distribution was $\Delta v_{min} \approx 10^3$ cm/s, corresponding to an effective temperature of 0.07 K. The authors attribute the limiting value of the velocity, $v_{min}$, to two factors: a) the scattering of the atomic beam by sodium vapor in the chamber and b) velocity diffusion of the atoms. The atomic density at the minimum temperature was $n = 10^5$ atoms/cm$^3$.

One of the basic parameters characterizing the laser slowing down of atoms is the atomic density in a unit velocity interval at low velocities, $dn/dv$. In these experiments, this parameter was $dn/dv \approx 10^2$ (atoms/cm$^3$)/(cm/s).

#### 3) Steady-state monochromatization

During steady-state monochromatization, both the frequency of the laser beam and the frequency of the atomic transition remain fixed in the interaction of the beam atoms with the radiation. Steady-state monochromatization has been implemented with sodium atoms.[14,15] A cyclic interaction of atoms with the radiation has been achieved by using two-mode laser radiation (Fig. 11b). The distance between the axial modes was chosen equal to the distance between the hyperfine components of the ground state. One mode excited

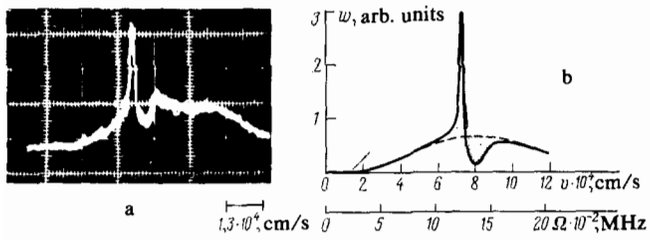


FIG. 15. a—Experimental velocity distribution of an atomic beam after irradiation of sodium atoms by a resonant laser beam (the interaction length is $l = 20$ cm; the intensity of the laser beam corresponds to a saturation parameter $G = 40$); b—theoretical deformation of the velocity distribution found from a solution of the Liouville equation (a horizontal division on the oscilloscope trace corresponds to $1.3 \cdot 10^4$ cm/s, while that on the frequency scale corresponds to 220 MHz).

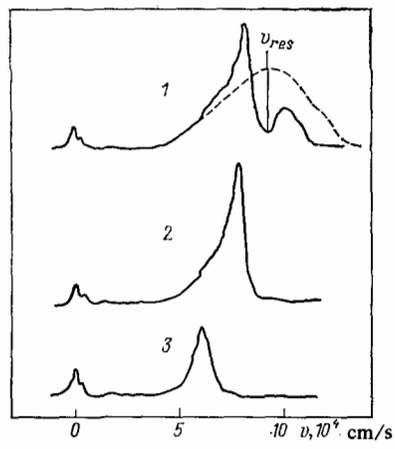


FIG. 16. Steady-state monochromatization of an atomic beam for various durations of the interaction with the laser beam.

sodium atoms on the transition $3S_{1/2}(F = 1) - 3P_{3/2}$, and the other on the transition $3S_{1/2}(F = 2) - 3P_{3/2}$. Clearly, for this type of excitation of the atom, for an arbitrary polarization of the laser beam, there is a reliable excitation of the atom from any sublevel $F = 1, 2$ of the hyperfine structure of the ground state. Both of the modes are tuned within the Doppler absorption lines of these transitions. At the temperatures of the atomic sources which have been used, the overlap of the absorption lines of the transitions $3S_{1/2}(F = 1) - 3P_{3/2}$, $3S_{1/2}(F = 2) - 3P_{3/2}$ could be ignored. The two modes acted on the same velocity group of atoms and deformed the velocity distributions in the $F = 1, 2$ levels identically.

Figure 15a shows an experimental profile of the deformed velocity distribution found during steady-state monochromatization of an atomic beam. This curve directly reflects the longitudinal velocity distribution of the atoms in the beam which results from the nonlinear slowing down of the atoms by the counterpropagating laser beam. To find the dependence $w(v)$, the laser beam was tuned to resonance with the atoms at the maximum of the initial thermal velocity distribution. The deformation of the velocity distribution observed experimentally was caused primarily by the radiation pressure force. The velocity diffusion was of minor importance because of the small transit time of the atoms through the zone of the interaction with the intense laser beam. At an average atomic thermal velcoity $\bar{v} = 8 \cdot 10^4$ cm/s the interaction time is $\tau = 2.5 \cdot 10^{-4}$ s. Over this time, the velocity distribution can broaden the narrow velocity peak by no more than

$$\Delta v_{\text{diff}} \approx \sqrt{C\tau} \approx 300 \text{ cm/s},$$

where we have taken the diffusion coefficient in the form $C \approx \gamma v_{\text{rec}}^2$ for an estimate. On the other hand, the typical velocity interval of the change in the force under the experimental conditions was $5.6 \cdot 10^3$ cm/s. Since the velocity diffusion in the longitudinal direction did not have any significant effect on the deformation of the velocity distribution, the experimental results turned out to agree well with results calculated on the deformation of the distribution from Liouville equation (3.3). In particular, we show for comparison in Fig. 15b the calculated dependence for the same parameter values as for the experimental curve in Fig. 15a. In the experiments of Refs. 14 and 15 the ratio of the width of the initial velocity distribution to the width of the narrow peak of monochromatized atoms was $\mu = \Delta v_{\text{in}} / \Delta v_{\text{fin}} = 19$. This degree of monochromatization corresponds to a lowering of the temperature of the relative motion of the atoms from an initial $T_{\text{in}} = 573$ K to a final $T_{\text{fin}} = T_{\text{in}} / \mu^2 = 1.5$ K. Figure 16 shows examples of the velocity monochromatization of a beam of sodium atoms for various durations of the interaction of the atoms with the radiation. We see that as the duration of the interaction is increased the velocity peak shifts toward zero and simultaneously shrinks.

A detailed experimental study of the velocity monoch-

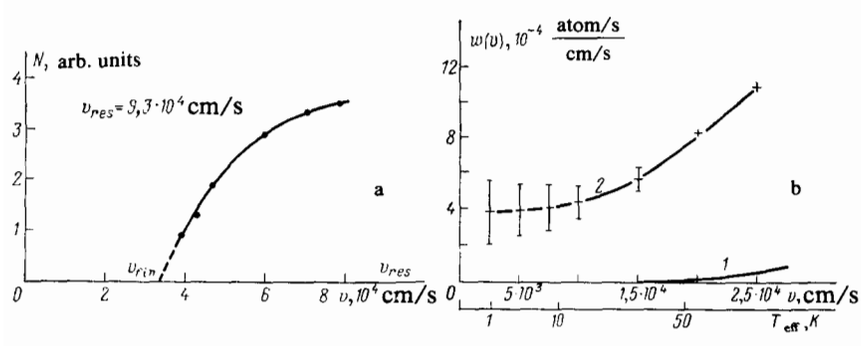


FIG. 17. a—Density of atoms in the peak of the monochromatized velocity distribution of the atomic beam as a function of the mean velocity of the peak during slowing-down; b—low-velocity part of the velocity distribution (2) in a cooled atomic beam. Shown for comparison is the velocity distribution (1) for the original uncooled beam.

romatization in the slowing down of a beam of Na atoms by a resonant laser beam was made in Ref. 52. The primary result of that study was the determination that a significant slowing down leads to a sharp decrease in the intensity of the atomic beam (Fig. 17a). The reason for this radical decrease in the intensity of the beam of slowed-down atoms turned out to be a transverse spreading of the atomic beam due to a velocity diffusion in the transverse direction, which removed atoms from the region of the interaction with the laser beam (which caused the slowing down of the atoms). The optimum conditions on the configuration of the laser beam, the interaction time, and the interaction length—the optimum conditions for achieving low effective temperatures of the atomic beam while keeping the beam intensity high—were found. As a result, an intense, steady-state beam of Na atoms with an effective temperature of 1 K was found. The intensity of this slowed-down atomic beam with $T = 1$ K exceeded the intensity of the original atomic beam by a factor of $3 \cdot 10^3$ (Fig. 17b). At this temperature, the density of atoms in a unit velocity interval is $dn/dv = 2 \cdot 10^3$ (atoms/cm$^3$)/(cm/s).

## 4. RADIATION COLLIMATION AND TRANSVERSE COOLING OF ATOMIC BEAMS

### a) The idea behind the method

In the preceding section we examined the effect of a laser beam on the longitudinal velocity component of the atoms in a beam and the longitudinal cooling of the atomic beam which results. The question of the transverse velocities of the beam atoms, on the other hand, was discussed only in connection with the slight increase in these velocities due to diffusion. It is legitimate to ignore the transverse velocity of the beam atoms in this way only in a certain stage of the longitudinal cooling, in which the longitudinal velocity is substantially higher than the transverse velocity. The divergence of an atomic beam is typically $10^{-2}$–$10^{-3}$ rad, so that when atoms are slowed down by a factor of $10^2$–$10^3$ the longitudinal velocity becomes comparable to the transverse velocity, and the divergence of the atomic beam approaches 1 rad. This circumstance means that in a certain stage of the slowing down of the atomic beam there must be a transverse cooling of the beam.

Despite the complete understanding of the physical ideas underlying methods for radiation cooling of atoms, it was only recently that specific schemes were found for irradiating atoms to achieve transverse cooling of atomic beams. These schemes are based on the use of the forces of resonance radiation pressure which are exerted on atoms in axisymmetric light fields to reduce the velocities of the atoms in the direction transverse with respect to the axis of the atomic beam. Curiously, the possibility of reducing the transverse velocities of an atomic beam through two-dimensional radiation cooling was first mentioned by Hänsch and Schawlow.[7]

Figure 18 shows one of the schemes[54] for transverse radiation cooling of an atomic beam. In this arrangement, the beam of atoms 2 emerging from source 1 is irradiated by an axisymmetric light field 3, whose frequency $\omega$ is red-shifted with respect to the frequency of the atomic transition, $\omega_0$. The axisymmetric field is formed by the reflection of a plane light wave 5 from a conical mirror surface 4 (a reflection axicon). In the axisymmetric field produced by the reflection axicon, an atom whose velocity is directed away from the axis of the cone experiences a radiation pressure force, which is directed toward the axis of the cone if $\omega < \omega_0$. This force causes a rapid contraction of the velocity distribution of the atoms transverse with respect to the axis of the cone in the region of the axisymmetric field. This contraction in turn causes a sharp decrease in the angular divergence of the atomic beam, i.e., a collimation of the beam.

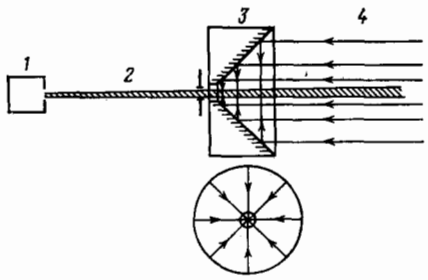


FIG. 18. Radiation atomic-beam collimator. 1—Source of atomic beam; 2—atomic beam; 3—conical axicon; 4—light beam incident on the axicon. The propagation directions of the light field in the axicon are shown at the bottom.

### b) Collimation of a thermal atomic beam

#### *1) Basic estimates*

We provide some simple estimates of the degree of collimation of a beam which is moving at a thermal velocity at room temperature.[54]

The evolution of the transverse atomic velocities in the field of an axicon is determined both by the radiation pressure, which contracts the transverse velocity distribution $w(v_\rho)$, and the diffusion of atomic momenta, which broadens the distribution $w(v_\rho)$. For transverse velocities $v_\rho$ satisfying the condition $|v_\rho| \ll |\Omega|/k$, the radiation pressure force reduces to a friction force:

$$F_\rho = -M\beta v_\rho, \tag{4.1}$$

where the dynamic friction coefficient $\beta$ is given by (3.11). The velocity diffusion coefficient $v_\rho$ agrees with (2.35), (2.37), within a numerical factor determined by the field polarization. Correspondingly, the steady-state distribution of transverse velocities in the axicon is determined by the temperature[10]

$$T = \frac{\hbar\gamma}{2k_B}\left(\frac{\gamma}{|\Omega|} + \frac{|\Omega|}{\gamma}\right). \tag{4.2}$$

Using this temperature, we can find the collimation angle of an atomic beam which determines the free dispersal of the atoms in the direction transverse with respect to the $z$ axis at the exit from the axicon:

$$\theta = \frac{\sqrt{2k_B T/M}}{\bar{v}_z}; \tag{4.3}$$

here $\bar{v}_z$ is the average velocity of the atoms along the axis of the atomic beam. The limiting collimation angle is (for $\Omega = -\gamma$)

$$\theta_{\min} = \frac{\sqrt{2\hbar\gamma/M}}{\bar{v}_z}. \tag{4.4}$$

For a beam of Na atoms irradiated on the 3S–3P transition by a laser beam with a wavelength $\lambda = 5890$ Å, and with $\bar{v}_z = 5 \cdot 10^4$ cm/s, the minimum collimation angle is $\theta_{\min} = 10^{-3}$. These numerical estimates show that the geometric divergence of an atomic beam can be reduced effectively by means of an axisymmetric light field.

The collimation angle $\theta$ characterizes the divergence of an atomic beam only at the exit from the axicon. In the interior of the axicon, the beam divergence is weaker, since it is determined not by the free dispersal of the atoms but by the slow diffusive broadening of the beam.

Let us estimate the divergence of an atomic beam due to the spatial diffusion of atoms. For this purpose we use the Einstein relation to estimate the spatial diffusion coefficient. Assuming the intensity of the collimating radiation to be low ($G \ll 1$), and assuming a frequency deviation $\Omega = -\gamma$, we find

$$D \approx k_B T/\beta \approx \gamma\lambda^2. \tag{4.5}$$

We can use (4.5) to estimate the effective beam divergence which is caused by the diffusion of the atoms in the direction transverse with respect to the $z$ axis. Determining the effective divergence angle $\sigma$ as the ratio of the diffusion width of the beam to the transit distance of the atoms, $z = \bar{v}_z t_{\text{int}}$, we find

$$\sigma = \frac{\sqrt{Dt_{\text{int}}}}{z} \approx \lambda \sqrt{\frac{\gamma}{z\bar{v}_z}}. \tag{4.6}$$

For a beam of Na atoms with $z = 10$ cm the divergence angle is $\sigma \approx 7 \cdot 10^{-5}$, much smaller than $\theta_{\min}$. Consequently, this numerical example shows that the irradiation of a thermal atomic beam by an axisymmetric field makes it possible to transport the atoms through an axicon with a small beam divergence and without loss of atoms.

### 2) Experimental studies

The first experiment on the radiation collimation of a beam of sodium atoms was carried out in Ref. 18 in the arrangement shown in Fig. 18. A two-frequency dye laser tuned to the $D_2$ component of the line of the sodium atom was used for a cyclic interaction of the atoms with radiation. The difference between the two laser frequencies was chosen to be 1772 MHz, so that one frequency excited atoms from the $F = 1$ $(3S_{1/2})$ level to the $F' = 2(3P_{3/2})$ level, while the other excited atoms from the $F = 2(3S_{1/2})$ level to the $F' = 3(3P_{3/2})$ level (Fig. 11). The intensity distribution of the atoms along the transverse coordinate was measured. Figure 19 shows profiles of the atomic beam before and after the interaction with the laser beam. These profiles demonstrate an increase in the intensity of the atoms in the beam and a contraction (collimation) of the atomic beam. Measurements of the diameter of the atomic beam before and

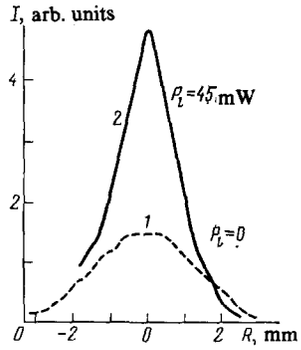


FIG. 19. Intensity distribution of a beam of sodium atoms in the transverse cross section before (1) and after (2) interaction with the field of an axicon.

after interaction with the laser field can be used to calculate the change in the transverse velocity of the atoms in the course of their collimation. For the case in Fig. 19, these calculations show that the transverse velocity of the atoms is reduced from $5.5 \cdot 10^2$ cm/s to 1.6 cm/s. This decrease corresponds to a decrease in the temperature of the transverse motion from 42 mK to 3.5 mK. Estimates from (4.2) yield a fairly close temperature (1.8 mK).

The collimation of an atomic beam is very sensitive to the position of the frequency of the laser field with respect to that of the atomic transition. The degree of collimation was measured as a function of the radiation frequency in Ref. 18. Figure 20 shows how the intensity of atoms at the center of the beam depends on the frequency of the collimating field. We see from this figure that for a negative frequency deviation beam collimation is observed, while for a positive deviation decollimation is observed. This behavior of the curves confirms that the observed effect is in fact due to the radi-

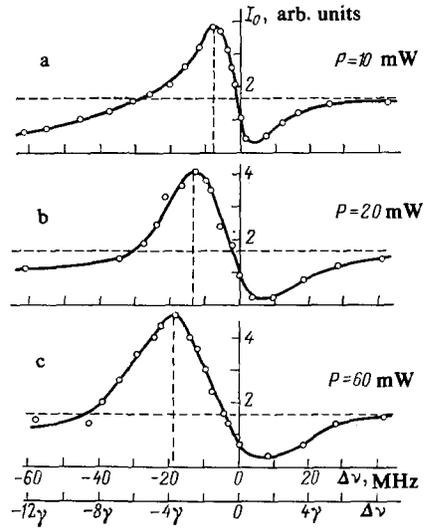


FIG. 20. Radiation collimation of an atomic beam. Intensity of the atoms at the center of a beam bombarded by a laser beam inside a conical axicon as a function of the field frequency.

ation pressure force, which has a dispersive dependence on the frequency of the laser field in the field of the standing light wave.[10] In addition to the radiation pressure force, the gradient force (Sec. 2) may influence the transverse motion of the atoms. This force has been used at large power levels to focus atomic beams.[55-57] Estimates show, however, that the effect of this force under these experimental conditions was negligible. We might also note that, as can be seen from Fig. 20, for each power of the laser beam there exists an optimum value of the frequency of the collimating field. The optimum frequency deviation of the collimating field has a $\sqrt{1+G}$ dependence on the laser intensity, in agreement with the calculations in Ref. 10.

#### c) Transverse cooling of a slowed-down atomic beam

An atomic beam which is slowed down by the pressure of a counterpropagating laser beam always undergoes a transverse spreading because of the diffusion of the atomic velocities in the light field. For this reason, the slowing down of the atomic beam is unavoidably accompanied by a loss of slow atoms. If we use, in addition to the laser beam causing the slowing-down, an axisymmetric light field with wave vectors directed transverse with respect to the beam axis (Fig. 18), we can achieve a significant suppression of the divergence of the atomic beam. Estimates[54] show that when an atomic beam is slowed down in the arrangement of Fig. 18 the minimum width of an atomic beam slowed down to zero mean velocity can be

$$q_{\min} \approx \frac{\hbar |\Omega|}{\gamma} \sqrt{\frac{\hbar |\Omega|}{6RG}}; \qquad (4.7)$$

here $\Omega$ is the frequency deviation, and $G$ is the saturation parameter for the laser beam causing the slowing-down.

For the slowing-down of a beam of sodium atoms to zero mean velocity, the width of the atomic beam reaches $q_{\min} \approx 0.15$ cm for the parameters values $|\Omega|/\gamma = 50$ and $G = 10$, according to (4.7). In the absence of a collimating field, the beam width should be three orders of magnitude greater; i.e., slow atoms are lost if there is no collimating field.

The combination of laser slowing-down with collimation represents an important step toward the production of cold atomic beams.

## 5. THREE-DIMENSIONAL RADIATION COOLING OF ATOMS

#### a) A qualitative look

The one-dimensional and two-dimensional cooling of atoms discussed above can be extended to three dimensions. There are a variety of configurations of optical fields which would cause a three-dimensional cooling of atoms. For example, we could place four light beams with frequencies $\omega_i < \omega_0$ at the vertices of a regular tetrahedron and direct them toward the center. As the number of beams is increased, the optical fields can be made more complex. In particular, six beams could be directed from the centers of the faces of a cube toward the center of the cube.

To obtain a qualitative understanding of the action of such optical fields in cooling an ensemble of atoms we consider the simplest case, of two counterpropagating waves of identical intensities and frequencies; the waves are propagating along one of the coordinate axes, e.g., along $\pm z$ [a field of the type in (2.22)]. Figure 21 shows the radiation pressure force as a function of the velocity projection $v_z$ in the case of two counterpropagating waves, along with the profile of a rather arbitrary initial velocity distribution of the atoms. It can be seen from Fig. 21 that in the case of a negative frequency deviation the radiation pressure force is directed opposite to the velocity of the atoms, while if the frequency deviation is positive the force will be in the same direction as the velocity $v_z$. In the former case, a narrow velocity distribution centered at $v_z = 0$ will therefore eventually be formed from a broad initial velocity distribution. In the latter case, two narrow velocity distributions, which move along the $\pm z$ directions as time elapses, will form.

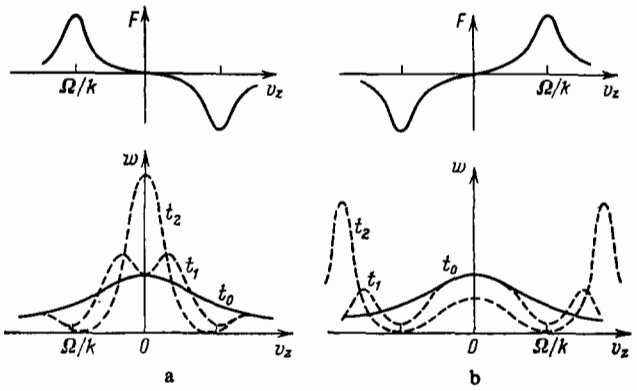


FIG. 21. Radiation pressure force as a function of the velocity projection $v_z$ for the case of two counterpropagating light waves; deformation of the atomic velocity distribution $w(v_z)$ for (a) $\Omega < 0$ and (b) $\Omega > 0$ for durations $t_0 < t_1 < t_2$ of the interaction of the atoms with the radiation.

The case of a negative frequency deviation is of particular interest since in this case there is a decrease in the modulus of the velocity of each atom; in other words, radiation cooling of the atoms along the $z$ axis occurs.[7,10] For the three-dimensional optical fields mentioned above, an analysis of this sort of evolution of the atomic velocities along the coordinate axes leads to analogous conclusions, i.e., to the possibility that the moduli of all three projections of the atomic velocity will be reduced, so that a cooling of an atomic ensemble will indeed occur.

#### b) Steady-state velocity distribution

To derive the temperature of the steady-state atomic ensemble we describe the motion of the cold atoms by means of a stochastic Langevin equation. Following Ref. 10, we consider the case of the cooling of atoms by the field formed by six beams, directed from the centers of the faces of a cube toward the center of the cube (Fig. 22), i.e., a three-dimensional standing light wave:

$$\mathbf{E} = \frac{1}{2} \sum_{\alpha=1-6} \mathbf{e}_\alpha E_0 \exp\left[i(\mathbf{k}_\alpha \mathbf{r} - \omega t)\right] + \text{c.c.}, \qquad (5.1)$$

where $\mathbf{k}_\alpha$ and $\mathbf{e}_\alpha$ are the wave vectors and polarization vectors of each of the six waves.

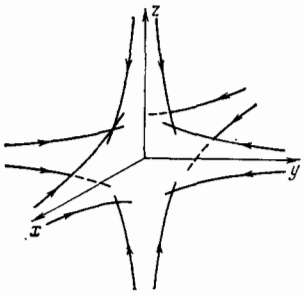


FIG. 22. Centrally symmetric field formed by six laser beams.

We restrict the discussion to the case of a slight saturation, in which the partial saturation parameters $G_\alpha$ satisfy the condition

$$G_\alpha = \frac{1}{2}\left(\frac{\mathrm{d}e_\alpha E_0}{\hbar\gamma}\right)^2 \ll 1. \tag{5.2}$$

In this case we may assume that the radiation pressure force is the sum of the partial forces set up by the six independent waves. Each of the partial forces is determined in the case of a slight saturation by expression (2.18) in the limit $G \ll 1$:

$$\mathbf{F}_\alpha = \hbar\mathbf{k}_\alpha\gamma G_\alpha\,[1 + (\Omega - \mathbf{k}_\alpha\mathbf{v})^2\gamma^{-2}]^{-1}, \tag{5.3}$$

where the parameters $G_\alpha$ may be different because of the different orientations of the polarization vectors of the different waves.

Analogously, the velocity diffusion tensor can be written as six tensors corresponding to the individual waves. In accordance with (2.35) and (2.37), the components of the partial velocity diffusion tensor can then be written

$$C_{ii}^\alpha = \frac{1}{2}\gamma v_{\mathrm{rec}}^2\,(1 + \eta_{ii}^\alpha)\,G_\alpha\,[1 + (\Omega - \mathbf{k}_\alpha\mathbf{v})^2\gamma^{-2}]^{-1}, \tag{5.4}$$

where the quantities $\eta_{ii}^\alpha$ have the same meaning as $\alpha_{ii}$ in (2.35).

For simplicity we consider the case of a symmetric orientation of the polarization vectors $\mathbf{e}_\alpha$, in which case we have $\eta_{ii}^\alpha = 1/3$. We also assume that the atomic velocities are quite small:

$$|\mathbf{v}| \ll \frac{|\Omega|}{k}. \tag{5.5}$$

The forces $\mathbf{F}_\alpha$ and the tensor components $C_{ii}^\alpha$ can then be expanded in power series in $\mathbf{v}$ around $\mathbf{v} = 0$. As a result, the total radiation pressure force, consisting of the sum of the partial forces, is, in first order in $\mathbf{v}$,

$$\mathbf{F} = -M\beta\mathbf{v}, \tag{5.6}$$

where the dynamic friction coefficient $\beta$ is defined by

$$\beta = \frac{8R}{\hbar}\frac{|\Omega|}{\gamma}\,G\left(1 + \frac{\Omega^2}{\gamma^2}\right)^{-1}. \tag{5.7}$$

For the velocity diffusion tensor we find the following expression in the zeroth order in $\mathbf{v}$:

$$C_{ii} = C = \frac{4R\gamma G}{M}\left(1 + \frac{\Omega^2}{\gamma^2}\right)^{-1}. \tag{5.8}$$

The Langevin equation describing the motion of the cold atoms under the influence of friction force (5.6) and the stochastic force responsible for the diffusion of the atomic velocity is

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\beta\mathbf{v} + \xi(t). \tag{5.9}$$

A steady-state solution of Eq. (5.9) is reached in a time $t \gg \beta^{-1}$ and is Maxwellian,

$$w(\mathbf{v}) = \left(\frac{M}{2\pi k_{\mathrm{B}}T}\right)^{3/2}\exp\left(-\frac{M\mathbf{v}^2}{2k_{\mathrm{B}}T}\right) \tag{5.10}$$

with a temperature[10]

$$T = \frac{MC}{\beta k_{\mathrm{B}}} = \frac{\hbar\gamma}{2k_{\mathrm{B}}}\left(\frac{\gamma}{|\Omega|} + \frac{|\Omega|}{\gamma}\right). \tag{5.11}$$

The minimum temperature of the atomic ensemble is reached at $\Omega = -\gamma$ and is[10]

$$T_{\min} = \frac{\hbar\gamma}{k_{\mathrm{B}}}. \tag{5.12}$$

Condition (5.5) holds automatically, as we can now verify.

The radiative cooling of atoms in a resonant field thus makes it possible to lower the temperature of an atomic ensemble to a level determined by the natural width of the atomic transition. At a low saturation level ($G \ll 1$) this minimum temperature does not depend on the field intensity. For a typical value of the natural line width of an optical transition, $\gamma/2\pi \approx 10^7$ Hz, the minimum attainable temperature is of the order of $T_{\min} \approx 10^{-3}$ K. According to (5.9), the scale time required for the attainment of the steady-state temperature is determined by the reciprocal of the dynamic friction coefficient: $\tau_{\mathrm{cool}} \simeq \beta^{-1}$. In particular, the scale time for the cooling of an atomic ensemble with $G \ll 1$ is

$$\tau_{\mathrm{cool}} \approx \frac{2}{G}\frac{\hbar}{R}. \tag{5.13}$$

For Na atoms (the 3S – 3P resonant transition), for example, with $G = 0.1$ this time is $\tau_{\mathrm{cool}} = 3\cdot10^{-5}$ s.

A more detailed discussion of various schemes for the radiation cooling of atomic gases can be found in Refs. 10, 42–45, and 58–65.

## 6. LOCALIZATION OF COLD ATOMS

The successful development of laser methods for cooling atoms has recently attracted research interest to the problem of stably localizing cold atoms in a bounded spatial volume. The successful resolution of this problem might become the next logical step toward the ability to manipulate the velocities and coordinates of individual atoms. The storage of cold atoms is particularly attractive for reaching an ultimate goal of laser spectroscopy: ultrahigh-resolution measurements of the absorption spectra of individual atoms.

The problem of localizing neutral atoms has not yet been finally solved. While in the case of charged atomic particles we have well-developed methods for localizing particles in a nonuniform static electric field in the presence of a static magnetic field (Penning confinement systems) or in a nonuniform rf electric field (rf confinement system),[66] in the case of neutral atoms it has yet to be proved that stable localization is possible at all. On the other hand, the fact that the potential well required for localizing cold atoms would

be quite shallow raises the hope that this problem will also be solved successfully. While at room temperature the depth of the potential well required for spatial localization of atoms would have to be of the order of 0.1–1 eV, a well depth of only $10^{-7}$–$10^{-6}$ eV would be sufficient for localizing atoms with a temperature $\sim 10^{-3}$ K.

The three methods which have now been proposed for localizing neutral atoms are based on the confinement of the atoms in optical, magnetic, and electric fields, respectively. We will discuss each approach separately.

#### a) Problem of a radiation atomic confinement system

The problem of the localization of a neutral atom in an optical field was first formulated by Letokhov.[67] He suggested that atoms might be localized at the nodes or antinodes of a standing light wave whose frequency was far from frequencies of atomic transitions. The idea of the localization of atoms in a region determined by the length of a light wave was subsequently discussed in Refs. 68 and 69.

This idea was subsequently extended to a resonant light wave.[70,71,37] A special study of the case of a resonant standing wave showed, however, that the idea of the trapping of an atom at a node or antinode of a light wave was incompatible with the spectral structure of the energy bands of the translational motion of an atom in the field of a standing wave.[72] Since the widths of the allowed and forbidden energy bands turned out in all cases to be equal in order to magnitude, calculations[72] showed that stable localization of an atom at the node or antinode of a wave was not possible.

Since it is not possible to localize an atom in a region with dimensions of the order of the length of a light wave, attention has turned to the possible localization of atoms in a large region formed by the intersection of light beams.[33,34,39,73–77] All the ideas for developing a radiation confinement system for atoms have been based on the use of radiation forces to cool and localize the atoms in an optical field simultaneously. It has been assumed that the field frequency should be chosen below the frequency of the resonant atomic transition so that the optical field can cool an ensemble of atoms.

Three methods have been proposed for forming a potential well for cold atoms. In the first approach, a gradient force forms a potential well. This approach will use the optical field formed by the intersection of several beams. In particular, it has been suggested that six laser beams be used; they would propagate along the $\pm x$, $\pm y$, and $\pm z$ directions of a Cartesian coordinate system.[33,34] The second suggestion is based on combining the use of the gradient force with the use of the radiation pressure force.[73] In particular, it has been suggested that the optical field set up by two counterpropagating divergent light beams might be used. It has been suggested that the gradient force would confine the atoms in the direction transverse with respect to the common axis of the beams, while the radiation pressure force would confine the atoms in the direction along the axis.[73,74,77] Finally, the third approach[75,76] has a potential well being set up by a radiation pressure force in a centrally symmetric optical field produced by divergent Gaussian beams.

Subsequent study of all these suggestions has shown that none of them solves the problem of stably storing cold atoms. Minogin[78] showed that the potential barrier set up by the gradient force does not exceed the average kinetic energy of cold atoms. Ashkin and Gordon[77] found that the potential of the radiation pressure force in the centrally symmetric fields considered in Refs. 75 and 76 does not have an absolute minimum.

#### b) Localization of atoms in magnetic confinement systems

A second possibility for prolonged storage of cold atoms is to localize them in a nonuniform magnetic field by making use of the effect of the magnetic dipole force on an atom.[79] The fields required for localizing atoms could be produced in toroidal or spherical magnetic confinement systems of the type used to store ultracold neutrons (see the review by Golub and Pendlebury,[80] for example). As an example here we will discuss the conditions for the accumulation and storage of cold $Na^{23}$ atoms in a toroidal magnetic field.[79]

The toroidal system for trapping the atoms is shown schematically in Fig. 23. Six direct currents create a nonuniform field whose modulus increases from the center to the periphery of the cross section of the torus in proportion to the square of the distance from the axial line:

$$H(r) = H_{\text{T}} \frac{r^2}{a^2}, \tag{6.1}$$

where $a$ is the minor radius of the torus, and $H_{\text{T}}$ is the field at the surface of the torus.

The motion of a cold atom injected into the magnetic field of a toroidal confinement system depends on the orientation of the dipole moment $\mu$ of the atom with respect to the vector **H** (Fig. 23b). If $\mu$ and **H** are antiparallel, the atom is

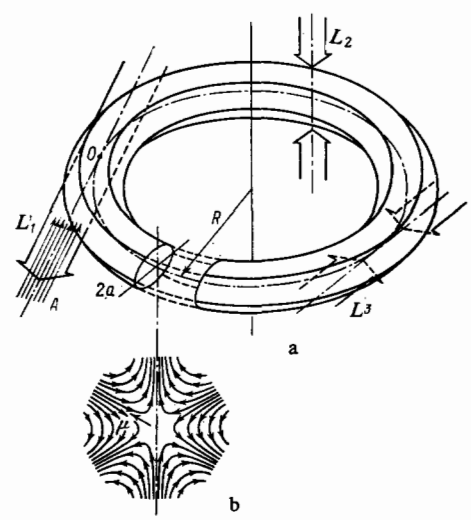


FIG. 23. a: Toroidal magnet ring for storing atoms ($L_1$, $L_2$, $L_3$—laser beams; A—beam of injected atoms). b: Magnetic lines of force in a cross section of the storage ring.

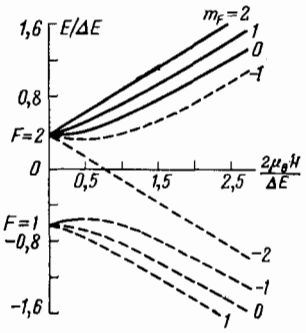


FIG. 24. Energy of the interaction of a $Na^{23}$ atom with the magnetic field, normalized to the hyperfine-structure interval $\Delta E = 1772$ MHz.

attracted toward the center of the torus, while if $\boldsymbol{\mu}$ and $\mathbf{H}$ are parallel the atom is repelled from the magnetic field region.

In the case of $Na^{23}$ atoms, only atoms in the $3S_{1/2}$ ground state in the hyperfine sublevels $F = 1$, $m_F = -1$ and $F = 2$, $m_F = 2$ or 1 can be confined in a toroidal magnetic confinement system (Fig. 24). The confining force acting on the atoms is weakest for atoms in the $F = 1$, $m_F = -1$ state.

Two basic conditions must be met for stable localization of atoms near the axial line of a torus. First, the depth of the potential well must be significantly greater than the average kinetic energy of the cold atoms moving transverse with respect to the axial line of the torus. Second, the centrifugal displacement of the atoms must be small in comparison with the minor radius of the torus. Simple estimates for the case in which the temperature of the transverse motion of the atoms is $T_{tr} = 10^{-3}$ K (the limiting value in laser cooling) and in which the temperature of the longitudinal motion of the atoms is $T_l = 10^{-2}$ K (increased by an order of magnitude because of the need to inject the atoms into the confinement system) show that it is completely feasible to satisfy these conditions. For example, with $H = 600$ G we would use $R = 30$ cm and $\alpha = 3$ cm.

The storage time of cold atoms in a magnetic confinement system is determined by the following factors: the finite height of the potential barrier, collisions with particles of the residual gas, and the formation of molecules in binary collisions. Estimates show that in a comparatively weak magnetic field (hundreds of gauss) and at a low residual gas pressure ($\lesssim 10^{-10}$ torr) the time over which cold atoms can be stored in a magnetic confinement system can be as long as several hours.

For the injection of atoms into magnetic confinement systems, they must first be slowed down by a counterpropagating laser beam (Fig. 23a). The injection of atoms can be time-varying or steady-state. In the time-varying case, the beam of slow atoms is directed into the confinement system with the magnetic field turned off. After the field is turned on to its maximum value $H_m$, the confinement system traps the atoms which were in the axial region of the torus at the time at which the field was turned on. In this approach, only a small number of atoms can be trapped in the system. In the case of steady-state injection, the atoms can be injected by using a counterpropagating laser beam to slow down the injected atomic beam in the region of the injection. Since the levels of an atom are split into several components in the magnetic field of the confinement system, we could evidently use a multifrequency beam to slow down the atoms, in a process in which the atoms undergo a cyclic interaction with the laser beam. In the steady-state regime, the density of cold atoms in the confinement system can be quite high, e.g., of the order of $10^{10}$ atoms/cm$^3$.

### c) Localization of atoms in electrostatic confinement systems

The idea of using an electric field to localize atoms can be outlined as follows[81]: An atom in an electric field experiences a shift $U$ of its energy levels by virtue of the Stark effect. The magnitude of this Stark shift is $U = -(1/2)\alpha E^2$ for a nondegenerate level, where $\alpha$ is the polarizability of the atom. The direction of the force acting on an atom in an electric field, $F = -\nabla U$, depends on the sign of the polarizability of the atom, $\alpha$. If this polarizability is positive, the atom will be attracted into the region of the maximum field. If the polarizability is negative, the atom finds a stable position near a minimum of the field. For the ground state of an atom the polarizability is always positive (the Stark shift is negative), while for the excited states of an atom the polarizability is, on the contrary, negative. Atoms in an excited state should thus tend toward a region with a minimum electric field. A minimum electric field exists, for example, in the field configuration produced by the configuration of electrodes used in rf quadrupole ion confinement systems (Fig. 25). The square of the electric field inside the quadrupole system is given by

$$E^2 = \Phi_0^2 (x^2 + y^2 + 4z^2) R_0^{-4}, \tag{6.2}$$

and the energy of the interaction of an atom with the field is

$$U = -\frac{1}{2}\alpha E^2 = \frac{1}{2} M(\Omega_x^2 x^2 + \Omega_y^2 y^2 + \Omega_z^2 z^2), \tag{6.3}$$

where $\Omega_x$, $\Omega_y$, and $\Omega_z$ are the frequencies of oscillation of the atom along the $x$, $y$, and $z$ axes; $M$ is the mass of the atom;

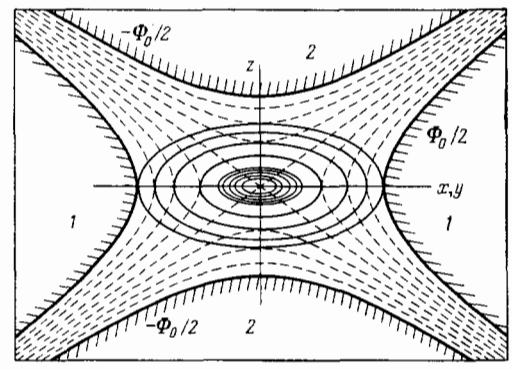


FIG. 25. Diagram of an electrostatic quadrupole confinement system[81,82] for localizing neutral atoms. Dashed lines—Surfaces of hyperbolic electrodes 1, 2; solid lines—cross sections of surfaces $E^2 = \text{const}$ (Refs. 84 and 85).

and $R_0$ is the radius of the annular electrode. For the confinement of atoms in an electrostatic confinement system we would be interested primarily in highly excited Rydberg atoms, since the polarizability of an atom is proportional to the square of the principal quantum number. It was found in Ref. 82 that the maximum depth which can be achieved for the potential well for a sodium atom in the 16P state is $U = 17\ \text{cm}^{-1} = 2.1 \cdot 10^{-3}$ eV. This value is determined by a crossing of levels which differ in the sign of the Stark slope. The field required for producing such a potential well is rather low: $E = 2.5$ kV/cm. A confinement system with these characteristics can capture 0.25% of the atoms of a vapor at a temperature of 500 K. The lifetime of excited Rydberg atoms is in the range $10^{-5}$–$10^{-3}$ s, and this time determines the time over which the atom will be confined in a confinement system of this type. Wing[82] has suggested that the confinement time might be increased by exciting atoms by means of a cw laser beam, and a steady-state population of the excited state might be sustained in this manner.

## 7. CONCLUSION: SOME APPLICATIONS OF COLD ATOMS

A long list of potentially useful applications of the pressure of a resonant laser beam in controlling the motion of atoms has recently been discussed in the literature. Generally speaking, the basic fields of application can be seen clearly from the fact that the diffusion of the atomic velocity which is associated with the radiation pressure opposes a directed change in the atomic velocity under the influence of the radiation pressure force. For this reason it is difficult to expect that the resonance radiation pressure could be used for any significant acceleration of atoms. At the same time, all the examples discussed above show that the resonance radiation pressure can be used exceedingly effectively in cases in which it is necessary to change the velocity of an atom by an amount of the order of the mean thermal velocity at room temperature. In this connection we could expect that the most important applications of resonance radiation pressure will involve slowing-down of the thermal motion of atoms.

In turn, cold atoms are of considerable interest in such research fields as atomic and molecular physics, ultrahigh-resolution spectroscopy, and quantum metrology. The use of cold atoms is extremely interesting for studying collision processes, phenomena associated with the formation of chemical bonds, and condensation. In this field of applications, laser methods for cooling atoms may become an important complement to the widely used methods involving supersonic nozzles for cooling gases. The use of cold atoms in spectroscopy and quantum metrology will make it possible to eliminate some fundamental causes of the broadening and shift of narrow spectral resonances due to finite velocity of motion: the Doppler effects of first and second orders and transit-time broadening. For these reasons, the use of cold atomic particles can sharply improve both the resolution of spectroscopic studies and the accuracy of quantum frequency standards. [83,84]

Other applications follow from the circumstance that cold atoms can be stored and confined for a long time in magnetic confinement systems.[79] The accumulation and storage of localized atoms make it possible to solve the problem of improving the sensitivity of several research methods which are used in atomic physics, spectroscopy, and nuclear physics. In particular, the confinement of atoms in magnetic fields can make it possible to carry out spectroscopic studies of radioactive atoms produced in countable numbers through nuclear reactions.

Let us conclude by taking a more detailed look at some of the ideas which have been advanced in these fields in recent years.

The cooling of atoms opens up several interesting possibilities for producing narrow and, especially, ultranarrow resonances free of Doppler broadening.

First, during the cooling of atoms the Doppler broadening is eliminated for *all* the quantum transitions, not only from the excited transition or associated transitions. This circumstance is of major interest for producing ultranarrow resonances on forbidden quantum transitions of cooled particles. This possibility was discussed for both ions[85] and atoms[10] in the earliest suggestions regarding laser cooling of atomic particles.

Second, in the case of cold atomic particles there is a substantial decrease in the contribution of the quadratic Doppler effect, which is the primary obstacle to the production of ultranarrow and superstable resonances with quality factors $Q > 10^{11}$. For the He–Ne/$CH_4$ laser, for example, the shift and inhomogeneous broadening of the resonance due to the quadratic Doppler effect amount to about 150 Hz at $T = 300$ K. In order to reduce the shift and broadening of the narrow resonance in $CH_4$ at $\lambda = 3.39$ $\mu$m which are caused by the quadratic Doppler effect to a level of $\delta\nu/\nu \approx 10^{-15}$, for example, it will thus be necessary to lower the temperature of the $CH_4$ molecules to

$$T \approx \frac{Mc^2}{k_B}\frac{\delta\nu}{\nu} \approx 0.17\ \text{K}. \qquad (7.1)$$

Such a low translational temperature of free atoms and molecules would probably be unattainable by any methods other than radiation cooling.

Third, the low velocity of the atoms in a cooled beam substantially increases the time over which the atoms interact with a probe light field. There is accordingly a decrease in the transit-time broadening of narrow resonances; this broadening is the basic obstacle to the production of ultranarrow resonances of absorption saturation and two-photon absorption. For example, the transit-time broadening of a narrow resonance in $CH_4$ could be reduced to $\Delta\nu/\nu \approx 10^{-13}$ at a beam diameter $a = 10$ cm by lowering the temperature to

$$T \approx 2\pi^2 \frac{Mv^2a^2}{k_B}\left(\frac{\Delta\nu}{\nu}\right)^2 \approx 0.01\ \text{K}. \qquad (7.2)$$

The increase in the duration of the interaction with a light beam for slow particles would be particularly convenient for use in combination with the method of spatially separated optical fields.[86]

Fourth, ultracold atoms can be accumulated and stored in a magnetic confinement system for neutral atoms. In this case the duration of the interaction of the atomic particles

TABLE I. Characteristics of the cooling and probing quantum transitions of $Mg^{24}$, $Ca^{40}$, and $Sr^{88}$ atoms which would be suitable for achieving ultranarrow resonances by the laser-cooling method.

| Atomic isotope | $Mg^{24}$ | $Ca^{40}$ | $Sr^{88}$ |
|---|---|---|---|
| I. Strong optical transition for cooling | $3^1S_0 - 3^1P_1$ | $4^1S_0 - 4^1P_1$ | $5^1S_0 - 5^1P_1$ |
| 1. Wavelength, Å | 2852 | 4226 | 4607 |
| 2. Natural line half-width, MHz | 39.4 | 17.4 | 12.8 |
| 3. Saturation intensity, $W/cm^2$ | 0.44 | 0.06 | 0.03 |
| 4. Temperature of cooled atoms, K | $2\cdot10^{-3}$ | $8\cdot10^{-4}$ | $6\cdot10^{-4}$ |
| 5. Velocity of cooled atoms, cm/s | 117 | 57 | 34 |
| II. Weak optical transition for detecting the narrow resonance | $3^1S_0 - 3^3P_1$ | $4^1S_0 - 4^3P_1$ | $5^1S_0 - 5^3P_1$ |
| 1. Wavelength, Å | 4571 | 6573 | 6893 |
| 2. Radiation width, Hz | 68 | 420 | 6400 |
| 3. Doppler half-width after cooling, MHz | 2.6 | 8.7 | 4.9 |
| 4. Shift due to the quadratic Doppler effect, Hz | $5\cdot10^{-3}$ | $8\cdot10^{-4}$ | $3\cdot10^{-4}$ |

with the field is essentially unlimited, and the transit-time broadening vanishes completely. By taking this path we could expect to produce ultranarrow spectral resonances in experiments with a minimal number of atomic particles.

For producing very narrow resonances of radiation-cooled atoms, the intercombinational transitions of even isotopes of alkaline earth atoms ($Mg^{24}$, $Ca^{40}$, and $Sr^{88}$), for example, would be completely suitable. These atomic isotopes also have strong allowed transitions suitable for laser cooling. The basic characteristics of the allowed cooling transition and the intercombinational probe transition of these atoms are listed in Table I (Ref. 84).

From this table we see just what laser cooling will achieve immediately in terms of the production of narrow resonances and just what is potentially possible with the help of cooled atoms. We see that a pronounced cooling of atoms reduces the Doppler width of any quantum transition by a factor of about $10^3$, but the residual Doppler broadening is $10^3$–$10^5$ times the limiting width set on the narrow resonances at the probe transition by radiation broadening. This potential possibility should be realized in combination with other methods for eliminating Doppler broadening. For example, one could use the method of saturation of absorption in a beam of cooled atoms. In this case, the width of the resonance will be caused primarily by the finite time over which the atom interacts with the probe field. However, because of the low velocity of the cooled atoms, the diameter ($\alpha$) of the probe beam which would be required to prevent this broadening is comparatively small ($\alpha \gtrsim 2$ cm for $Mg^{24}$, $\alpha \gtrsim 0.14$ cm for $Ca^{40}$, and $\alpha \gtrsim 5\cdot10^{-2}$ cm for $Sr^{88}$). Consequently, the combination of the method of deep laser cooling of atoms with the method of absorption saturation would be quite capable of producing ultranarrow resonances with widths $10^2$–$10^3$ Hz. In cases where it is not possible to achieve the maximum possible cooling of atoms the presence of a nonzero velocity can be offset by increasing the diameter of the probe light beam or even by resorting to the method of spatially separate optical fields.

Another possibility of significant interest is laser cooling in systems for detecting individual atoms. Here we have two methods for increasing the selectivity of the detection.[87] First, the laser slowing-down of the beam of atoms to be detected makes it possible to increase the transit time of the atoms through the detection region. For example, in an arrangement for fluorescent detection an increase in the transit time of the atoms through the laser beam exciting the fluorescence would make it possible to increase the number of scattered photons, with the result that the selectivity of the detection would be sharply increased. Second, a slow beam of atoms to be detected could be directed into a magnetic confinement system. Prolonged confinement of the atoms in the confinement system would also allow a sharp increase in the selectivity of the detection of individual atoms.


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