Emission of acoustic transition waves

V. I. Pavlov and A. I. Sukhorukov

M. V. Lomonosov Moscow State University

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Transition radiation is emitted when physical objects (particles, electromagnetic pulses, etc.) that have no eigenfrequencies move uniformly through inhomogeneous or nonstationary media. The effect is kinematic in character and is observed for waves of any physical nature. This review paper is devoted to the current state of the theory of acoustic transition radiation and transition scattering in media whose motion can be described by the hydrodynamic equations. The most characteristic problems in the theory of acoustic transition radiation are discussed, including the crossing of separation boundaries by a source, emission in a nonstationary medium, and so on. A number of diffraction problems of the theory are examined (e.g., the emission of sound by a source moving near inhomogeneities of different type). The last section is devoted to acoustic transition scattering. Particular attention is devoted to methodological aspects.

CONTENTS

INTRODUCTION

When physical objects that do not have eignefrequencies (we shall refer to them simply as sources) move uniformly through homogeneous elastic media, the only possible form of acoustic radiation is the emission of Mach waves which appear at ultrasonic source velocities. On the other hand, in inhomogeneous media, in nonstationary media, or in the neighborhood of such media, the motion of a source is accompanied by the emission of the so-called transition radiation,[1–3] even when the source moves with subsonic velocity.

The transition radiation effect was discovered in electrodynamics in the middle forties by V. L. Ginzburg and I. M. Frank, who considered the crossing of the separation boundary between two media by a moving charge.[4] A subsequent paper[5] considered x-rays emitted in the direction of motion by a relativistic charge, and showed that the intensity of this radiation increased linearly with increasing energy of the charge. The effect thus became the basis for a possible transition counter, capable of high efficiency at high charged-particle energy. As a result, there was a considerable growth in interest in transition radiation. At present, this problem is attracting an enormous number of papers in electrodynamics (see Refs. 1–10 and the references cited therein).

Transition radiation is produced not only in electrodynamics but also in acoustics, ordinary and magnetic hydrodynamics, superfluid hydrodynamics, and so on. It is clear that transition radiation is a universal phenomenon: it can occur in fields of any type. In general, transition radiation can coexist and interfere with waves produced as a result of the accelerated motion of a source, or motion with velocity exceeding the phase velocity of waves in the medium.

We shall review papers devoted to the theory of acoustic transition radiation in hydrodynamic media. Since, as noted above, we shall be concerned with general wave phenomena,

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it will be clear that acoustic transition radiation will have much in common with electromagnetic transition radiation, for which the transition effect was first discovered. We shall give a brief description of methods of solution and of the results obtained for a number of acoustic problems, many of which have close electrodynamic analogs. The problems presented below are of definite practical significance and, at the same time, relatively simple. Our aim was to avoid the influence of extraneous factors and try to understand the physics of the phenomenon. At the same time, it was important to take into account the obvious connection between the present review and the reviews previously published in this Journal of the analogous problems in electrodynamics.

We must first emphasize the specificity of problems relating to moving sources of acoustic radiation. It is well-known that the motion of a macroscopic body in an elastic medium is accompanied by the appearance of hydrodynamic stresses which are nonlinear when the source displacement velocity is high (they produce vortices, turbulence, and, at ultrasonic velocities, shock waves, etc.). The classical wave equation will therefore provide us with a correct description of either microscopic sources (beams of charged or neutral particles, bodies in tenuous media, and so on) or the so-called "nonmaterial" sources, i.e., regions of hydrodynamic stress transported through the medium and capable of being produced, for example, by electromagnetic fields (through the release of heat or striction), moving vortices, turbulent fluxes, and so on, or, in the case of crystals, by dislocations. Finally, linear theory can be used to examine the motion of slender "wings" through a gas, at a small angle to the direction of flow (small "angle of attack"), and extended bodies of revolution.[11,12] The shape of such bodies must be elongated in the direction of motion, smoothly brought to a point at the rear, and rounded for subsonic and brought to a point (with a small tip angle) for ultrasonic motion. When these conditions are satisfied, it may be considered that the gas flow is potential, which remains the case for low-intensity shock waves produced during the ultrasonic motion of the body.[1)]

It appears that the emission of acoustic transition waves was first discussed by Dokuchaev,[14] who gave the solution of, in a sense, the simplest problem, namely, that of the crossing of the separation boundary between two gaseous media by a body whose linear dimensions were much smaller than the mean free path of the molecules of the medium.

The emission of acoustic transition radiation by dislocations moving in crystals has been examined in a number of papers published in the last few years. When a dislocation crosses a plane of discontinuity in the elastic moduli of a crystal, this should be accompanied by the emission of sound by a mechanism that must be regarded as a transition mechanism.[15] A well-known analog of this effect is the emission of sound by dislocations as they emerge on the surface of a crystal.[2)] This type of emission was given a theoretical base in Ref. 15, and was subsequently discovered and investigated experimentally.[16–19] The authors of Ref. 17 succeeded in detecting, in a pure form, the sound accompanying the emergence of dislocations on the surface, having excluded the nonstationary displacement of dislocations in the interior of the crystal, which produces sound by analogy with the emission of electromagnetic bremsstrahlung by accelerated charges. The spatial distribution of acoustic radiation from dislocation clusters was measured in Ref. 18, while acoustic pulses generated by individual dislocations emerging on the surface were recorded in Ref. 19.

The emission of acoustic transition radiation by a charged particle crossing the separation boundary between two media with different acoustoelectric properties was investigated in Ref. 20. This radiation is due to the motion of the separation boundary under the influence of the electric field of the particle.

There is undoubted interest in transition radiation in the form of the numerous types of surface acoustic waves in solids. Here, we note the emission of Rayleigh waves by dislocations emerging on the surface of the crystal,[21] and the analysis of Gulyaev-Blyusteĭn waves[22] generated when a charged-particle beam crosses the separation boundary between a piezoelectric medium and a vacuum.

Some problems in the theory of transition radiation in hydrodynamics are investigated in Refs. 23–27. The emission of acoustogravitational transition radiation by a mass source crossing the separation boundary between two media, in each of which the pressure and density vary according to the barometric law because of the influence of the gravitational field, was considered in Ref. 28.

One of the factors contributing to the recently growing interest in this range of acoustic problems is that it is now possible to use moving opticoacoustic sources to generate sound.[29,31] Such sources take the form of propagating stresses produced by inhomogeneous heating of the medium due to the absorption of optical radiation, or by electrostriction. They can be created, for example, by scanning the surface of a liquid with a laser beam,[32] or by producing a moving light spot in a nonlinear medium.[33] One of the obvious advantages of optoacoustic sources is that they do not involve flow effects, which facilitates the comparision between theory and experiment. These developments have acted as a stimulus to studies of different aspects of the theory of emission of sound by moving sources[29,34–38] and, in particular, the emission of acoustic transition radiation by optoacoustic sources.[39–44]

It is well-known that the thermal mechanism is the most significant among those responsible for the optoacoustic effect in optically absorbing media and for low (in comparison with the latent heat of vaporization) densities of energy released in the medium. There have been considerable advances in recent years in the theory of optical generation of sound[29–31] by the thermal mechanism. The production of sound in a liquid by laser pulses was one of the questions investigated in detail. The essential point for our purposes is that a laser pulse propagating in a medium is also an optoacoustic source moving with the velocity of light. The crossing of the separation boundary between two media by a pulse of this type is accompanied by the emission of acoustic transition radiation. The generation of sound in a liquid half-space by laser pulses may therefore be looked upon as a form of acoustic transition radiation (see, for example, Refs. 30

and 45–53).

It is thus clear that the optoacoustic source is of particular interest in the acoustics of moving media. If, at the same time, the generation of acoustic disturbances is dominated by the thermal mechanism, the source can also be referred to as thermo-optic or thermal. Without loss of generality, we shall then speak of the *thermal source*, and will include charged-particle beams in this designation. Actually, the thermal mechanism of generation of acoustic disturbances is also found to predominate in the case of heavy charged particles traveling through condensed media. At the cost of considerable simplification, it may be considered that a particle moving through a medium is surrounded by an "overheated region," the linear dimensions of which are largely determined by the process of hydrodynamic relaxation.[54]

In addition to the sources listed above, there is also a source of acoustic disturbances that is characterized by a force that is opposite to the drag resistance. This mechanism is significant in aeroacoustics and is discussed in Ref. 14. When an object with linear dimensions l that do not exceed the mean free path λ travels through a gaseous medium, it is found that free molecular flow takes place. The drag force acting on the body moving with constant velocity $\mathbf{V}$ is then given by

$$\mathbf{F}^* = -\alpha S\rho_0 s\mathbf{V}$$

in the case of subsonic motion ($V<s$), and by

$$\mathbf{F}^* = -\alpha S\rho_0 \mid \mathbf{V} \mid \mathbf{V}$$

in the case of ultrasonic motion ($V>s$). In these expressions, α is a numerical coefficient that depends on the nature of collisions between the atoms (or molecules) of the medium and the surface, and also the shape of the body, S is the effective cross-sectional area, ρ_0 is the equilibrium density of the medium, and s is the velocity of sound. The moving body produces a force $\mathbf{F} = -\mathbf{F}^*$ on particles in the medium that lie in its immediate neighborhood. If we confine our attention to hydrodynamic disturbances with characteristic linear dimensions exceeding the mean free path and therefore much greater than the linear dimensions of the body, the force $\mathbf{f}$ acting on the body per unit volume can be written in the form

$$\mathbf{f} = \mathbf{F}\delta(\mathbf{x} - \mathbf{V}t - \mathbf{x}_0). \qquad \text{(I.1)}$$

A moving distribution of mass sources can often act as a source of hydrodynamic disturbances. The continuity equation then acquires the additional term

$$m = \mu g(\mathbf{x} - \mathbf{V}t - \mathbf{x}_0), \qquad \text{(I.2)}$$

where the function g describes the source distribution and $\mu(t)$ is its time dependence.

Finally, we must mention one further mechanism of the generation of sound which is of importance in aeroacoustics.

Lighthill[55] has proposed an approach to the determination of acoustic radiation generated by a turbulent flow occupying a relatively small region in a homogeneous medium in which the velocity of sound and the equilibrium density are constants. He obtained an equation of the form

$$\Delta p' - \frac{1}{s^2}\frac{\partial^2 p'}{\partial t^2} = -\frac{\partial^2}{\partial x_i \partial x_j} T_{ij}, \qquad \text{(I.3)}$$

where p' represents pressure variations, s is the velocity of sound, $T_{ij} = \rho v_i v_j + \delta_{ij}(p' - s^2\rho') - e_{ij}$ is the Lighthill turbulent stress tensor, and e_{ij} is the viscous stress tensor.

This equation has the same structure as the wave equation for the acoustic field generated by a quadrupole source. It is the starting point of the analysis of the generation of aeroacoustic noise by, for example, a turbulent flow of air (or some other nonstationary flow with high Reynolds numbers) of relatively small spatial extent (such as a jet). It is often possible to reduce the analysis to the approximation[56,57] in which $T_{ij} \sim \rho v_i v_j$ in the region of the jet and is zero elsewhere.

We are thus already in a position to identify in a preliminary manner some of the most essential features of transition radiation in acoustics.

First, in contrast to electrodynamics, there is a great variety of sources of acoustic disturbances (thermal, mass, force, etc. mechanisms).

Second, the crossing of the separation boundary between media by a source is often accompanied by a change in the source itself (for example, in the case of a thermal source, by a change in its power output, characteristic linear dimensions, and thermophysical parameters). It will be shown below that the radiation emitted in this way interferes with the transition radiation. In addition, Mach waves (Cherenkov radiation) are emitted when the source travels with ultrasonic velocity.

It is clear that interference between three types of radiation can then take place.

Finally, a further important aspect of the difference between electrodynamic and acoustic problems is the approach to the problem. In electrodynamics, it is the radiative energy loss by a moving point particle that is of the greatest interest, and the basic characteristics of the transition radiation are its spectral density (or the spectral power), its angular distribution, and so on.

In acoustics, on the other hand, it is often important to have the space-time description of the pressure distribution because acoustic sensors usually measure not the intensity of the radiation but, directly, the pressure in the acoustic wave.

Next, in contrast to electrodynamics, the size of the source often plays an important role in acoustics. The transition radiation is then an acoustic pulse whose shape is determined by the structure of the moving source itself and is the most important feature of the process.

The structure of our paper is as follows.

Section 1 discusses general questions relating to acoustic transition radiation, and provides a qualitative analysis of the radiation process.

Section 2 discusses certain basic problems in the theory of acoustic transition radiation.

In accordance with the adopted definition, transition radiation will also be assumed to encompass the radiation produced by a source moving in a uniform medium near some particular inhomogeneity (screen, surface irregularity, and so on). As in electrodynamics, this type of radiation is often referred to as diffracted radiation. The corresponding aspects of the problem will be touched upon in Section 3.

Section 4 is devoted to transition scattering. By analogy with transition scattering in electrodynamics, acoustic transition scattering will be defined as the transformation of waves and disturbances of any physical nature into acoustic waves as a result of the interaction of the former with moving or stationary sources.

1. EMISSION OF SOUND BY MOVING SOURCES

A. Basic equations and energy characteristics of the radiation

Let us write down the complete set of linearized hydrodynamic equations, together with the equation of state and the equation of thermal conduction in a liquid:

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p' + \eta \Delta \mathbf{v} + \left(\zeta + \frac{\eta}{3}\right) \nabla \operatorname{div} \mathbf{v} + \mathbf{f}, \quad (1.1)$$

$$\frac{\partial \rho'}{\partial t} = -\rho_0 \operatorname{div} \mathbf{v} + m, \quad (1.2)$$

$$\rho_0 T_0 \frac{\partial \sigma'}{\partial t} = \varkappa \Delta T' + Q, \quad (1.3)$$

$$\rho' = \frac{1}{s^2} p' - \frac{\rho_0 \beta T_0}{c_p} \sigma', \quad (1.4)$$

$$T' = \frac{\beta T_0}{\rho_0 c_p} p' + \frac{T_0}{c_p} \sigma'; \quad (1.5)$$

where $\mathbf{v}$ is the velocity of the liquid, p', ρ', T', and σ' are the pressure, density, temperature, and entropy variations about their equilibrium values p_0, ρ_0, T_0, and σ_0, and η and ξ are the shear and volume viscosities. The quantities Q, $\mathbf{f}$, and m characterize the effect of the corresponding thermal, force, and mass sources in the medium (Q is the power density of thermal sources due to absorption in the liquid, for example, the absorption of optical radiation, etc., $\mathbf{f}$ is the force per unit volume of the medium, and m is the mass released per unit volume per unit time). Finally, s is the velocity of sound, β is the thermal expansion coefficient, and c_p is the specific heat at constant pressure.[3)]

We shall assume that, at each instant of time, the sources are localized in a limited region of space. Let $l_{\min}$ be the smallest characteristic linear dimension of this region (for example, for a thermal source, $l_{\min}$ is the minimum dimension of the region of absorption of the radiation that generates the sound waves). Provided the conditions

$$\omega \gg \left\{\frac{\chi}{l_{\min}^2}, \quad \frac{\nu}{l_{\min}^2}\right\}, \quad (1.6)$$

are satisfied, the basic set of equations becomes much simpler and can be reduced to the following equation for the pressure variations:

$$\Delta p' - \frac{1}{s^2} \frac{\partial^2 p'}{\partial t^2} + 2\delta \Delta \frac{\partial p'}{\partial t} = \operatorname{div} \mathbf{f} - \frac{\beta}{c_p} \frac{\partial Q}{\partial t} - \frac{\partial m}{\partial t}, \quad (1.7)$$

where ω is the frequency of the radiated sound wave, $\nu = [(4\eta/3) + \zeta]/\rho_0$ is the kinematic viscosity, $\chi = \varkappa/\rho_0 c_p$, $\delta = (1/2s^2)[\nu + \varkappa(c_p/c_V - 1)]$, and c_V is the specific heat at constant volume.

Condition (1.6) represents the situation where thermal conduction and viscosity have little effect on the efficiency of the sound-generating mechanism and provides a lower bound for the acoustic frequency ω. The same processes restrict the acoustic frequencies to the region below the upper bound, determined by the condition that the absorption of the generated waves must be small:

$$\omega \ll \left\{\frac{s^2}{\chi}, \quad \frac{s^2}{\nu}\right\}. \quad (1.8)$$

When this condition is met (and it is valid for most liquids up to hypersonic frequencies), dissipation in the wave zone can be neglected and the law of conservation of energy can be written in the following form for the original system[58]

$$\frac{\partial}{\partial t}\left[\int d\mathbf{x} \left(\frac{\rho_0 v^2}{2} + \frac{p'^2}{2\rho_0 s^2}\right)\right] + \oint_{\Sigma} d\Sigma p' \mathbf{v}$$

$$= \int d\mathbf{x} \left[\mathbf{v}\mathbf{f} + \frac{\beta}{\rho_0 c_p} p'Q + \frac{1}{\rho_0} p'm\right]. \quad (1.9)$$

The first and second terms in this expression represent the time dependence of the acoustic energy density and its flux through the bounding surface Σ, while the quantity on the right-hand side is the work done by radiation reaction forces. We emphasize at once that, for the transition effect discussed below, the radiated energy will not, as a rule, be equal to the work done on the source by the field.[4)] This is so because, as the source travels from one medium to the other, its intrinsic (accompanying) field is found to vary, i.e., the energy of this field varies. The analogous problem of macroscopic renormalization of the mass (charge) of a particle crossing the separation boundary between two media is discussed in detail in Ref. 1.

We therefore begin with one of the well-known methods for the evaluation of the spectral energy density of the radiation.

For large distances $|\mathbf{x}|$ from the region in which our process originates (we shall assume that these distances are large in comparison with the characteristic spatial scale of the phenomenon), the field may be regarded as a plane wave in small regions of space. The radiated waves transport a definite energy, the energy flux being given by the vector $\mathbf{S} = p'\mathbf{V}$ which, in a plane wave, is given by $\mathbf{S} = (p')^2\mathbf{n}/\rho_0 s$, where $\mathbf{n}$ is the unit vector drawn from the origin to the point of observation. The intensity radiated into the solid-angle element $d\Omega = \sin\theta\, d\theta\, d\varphi$ is, as usual, defined as the amount of energy flowing per unit time through the area element $d\Sigma = |\mathbf{x}|^2 d\Omega$ on a spherical surface of radius $|\mathbf{x}|$, centered on the origin. The total amount of energy passing through the element $d\Sigma$ is

$$\mathscr{E} = \int_{-\infty}^{\infty} dt \frac{(p')^2}{\rho_0 s} |\mathbf{x}|^2 d\Omega. \quad (1.10)$$

Since

$$\int_{-\infty}^{\infty} dt\, (p')^2 = 2 \int_0^{\infty} \frac{d\omega}{2\pi} |p_\omega|^2, \quad \omega > 0, \quad (1.11)$$

we have

$$\mathscr{E} = \int_0^{\infty} d\omega\, d\Omega \left(\frac{1}{\pi \rho_0 s} |p_\omega|^2 |\mathbf{x}|^2\right), \quad (1.12)$$

i.e., the quantity

$$d\mathscr{E}_{\mathbf{n},\omega} = \frac{1}{\pi\rho_0 s} |\mathbf{x}|^2 |p_\omega|^2 d\omega\, d\Omega = \mathscr{E}_{\mathbf{n},\omega}\, d\omega\, d\Omega \quad (1.13)$$

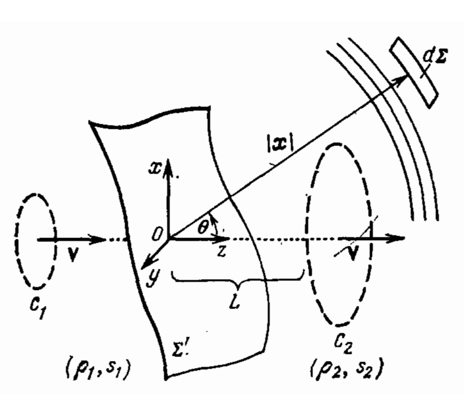

FIG. 1. Crossing by a source of the separation boundary Σ' between media with different acoustic parameters, and the subsequent rearrangement of the source "jacket" with the emission of sound.

can be interpreted as the energy radiated into the solid-angle element $d\Omega$ in the direction $\mathbf{n}$ in the form of waves within the frequency interval $d\omega$. The quantity $\mathscr{E}_{\mathbf{n},\omega}$ then characterizes the angular distribution of the spectral energy density of the radiation.

B. Zone in which radiation originates

If the process in which we are interested takes place in a system consisting of contacting regions in each of which (1.1)–(1.5) with the corresponding values of the equilibrium parameters are valid, we must taken into account the boundary conditions as well. It is precisely the presence of the boundaries that is usually the reason for the emission of transition radiation (Fig. 1).

The point is that a source moving uniformly with subsonic velocity is surrounded by an accompanying region of hydrodynamic perturbations, i.e., a kind of jacket that is never detached from it. The distribution of this "jacket" around the source is determined both by its velocity V and by the equilibrium parameters characterizing the medium, for example, the velocity of sound, the density, and so on. (Lines of equal sfress $C_{1,2}$ can be drawn around the source, as shown in Fig. 1.) As it eneters a region of space in which these parameters have values differing from the original values, the "jacket" begins to change its shape, tending to the distribution corresponding to the new medium parameters. This process cannot occur instantaneously because the acoustic signal has a finite velocity of propagation and takes a certain time τ to develop. During this time, the source moves through a distance $L_f = V\tau$. It is clear that the distance L_f over which the rearrangement of the field accompanying the source is practically completed can be referred to as the formation length.

This length can also be defined as follows. Because of energy conservation (and the fact that the boundary conditions must be satisfied), the rearrangement of the field accompanying the source gives rise to an additional field, namely, the radiation field. The distance over which the radiation field and the field accompanying the source become separated by an amount of the order of one wavelength may be looked upon as the characteristic length for the development of the process.[1]

The formation length is therefore given by

$$L_f = \frac{\lambda/2}{|(s/V) - \cos\theta|}, \tag{1.14}$$

where θ is the angle between the direction of emission of the radiation and the motion of the source. On the other hand, for a source of finite size, the pulse of transition radiation with characteristic duration $\tau_{\max}$ (determined by the parameters of the problem) is formed over the length

$$L_f = \frac{s\tau_{\max}}{|(s/V) - \cos\theta|}. \tag{1.15}$$

We note that (1.14)–(1.15) are valid for both subsonic and ultrasonic motion of the source.

To conclude this section, let us briefly consider the conditions under which the parameters characterizing the source (velocity, linear dimensions, values of Q and m, and so on) remain unaltered as the source moves through uniform media, and change only when the source crosses a separation boundary. This occurs when the characteristic length L, over which a change in the source parameters actually takes place,[5)] is much greater than the formation length L_f:

$$L \gg L_f. \tag{1.16}$$

Since the variation in the source over the length L is accompanied by the emission of waves with the characteristic Doppler frequency[59]

$$\omega_D = \frac{2\pi V/L}{|1 - (V/s)\cos\theta|}, \tag{1.17}$$

it follows that, in accordance with (1.14) and provided $L \gg L_f$, the transition radiation and the radiation due to the time dependence of the source will have different frequencies and can be analyzed independently.[40] We note that the effect of absorption features of a condensed medium on the generation of sound was discussed in Ref. 60.

Finally, we must consider whether the velocity V of the source can be regarded as constant in the presence of radiation (and, consequently, whether there is an associated change in intrinsic source energy). Of course, the situation here is exactly the same as in electrodynamics:[1-3] the source velocity can be regarded as constant (in the absence of external forces) because it is assumed that the intrinsic source energy is infinite. On the other hand, the source velocity can be altered by external forces, in which case this change (which gives rise to bremsstrahlung) must, of course, be taken into account.

2. ACOUSTIC TRANSITION RADIATION

A. Transition radiation from a plane separation boundary

1. We begin with the most characteristic problem, namely, that of the properties of acoustic transition radiation emitted when a thermal source crosses the plane separation boundary between two media. This problem was solved in Ref. 14 for a small body interacting with a medium through a force opposite to the drag force. Detailed analysis of the transition radiation produced when a plane separation boundary is crossed by an extended opticoacoustic source is given in Ref. 40. We shall largely follow the latter treatment.

We shall consider that the source moves in the positive

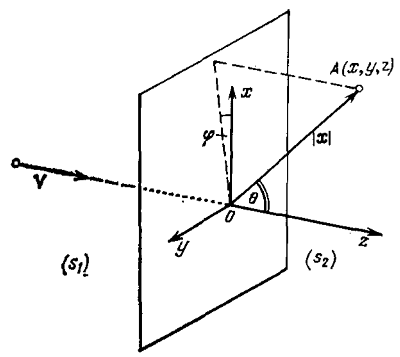

FIG. 2. Crossing of a plane separation boundary by a source.

direction of the z-axis and is symmetric relative to this axis. The separation boundary lies in the x, y plane. The velocity of sound s_n and the equilibrium values of the medium parameters (density ρ_n, volume expansion coefficient β_n, specific heat c_{pn}, $n = 1,2$; Fig. 2) undergo a sudden change across this plane.

The basic equation for the problem is the wave equation

$$\Delta p_n - \frac{1}{s_n^2}\frac{\partial^2 p_n}{\partial t^2} = -\frac{\partial D_n}{\partial t}, \tag{2.1}$$

where $D_n = \beta_n Q_n / c_{pn}$ and $Q_n(\mathbf{x} - \mathbf{V}_n t)$ is the amount of heat liberated per unit volume per unit time.

Equation (2.1) must be augmented with boundary conditions that follow from the requirement that the medium must be continuous and the pressures must be equal on the separation boundary, where the first of these conditions demands that the normal component of velocity must be continuous across the separation boundary. In the linear approximation, the boundary conditions on $z = 0$ have the form

$$\frac{1}{\rho_1}\frac{\partial p_1}{\partial z} = \frac{1}{\rho_2}\frac{\partial p_2}{\partial z}, \quad p_1 = p_2. \tag{2.2}$$

Before we proceed to a presentation of the solution of our problem, we must consider one general point. Strictly speaking, transition radiation is the radiation due to a variation in acoustic parameters along the trajectory of the source. The simultaneous change in the thermophysical parameters β_n, c_{pn} and the quantity Q_n on the right-hand side of (2.1), which characterize *the source*, contributes the component of the resultant radiation that is due to the *rearrangement of the source*. The radiation due to the change in the velocity V_n of the source can be referred to as bremsstrahlung by analogy with electrodynamics. Nevertheless, the emission of sound due to the rearrangement of the source and the bremsstrahlung radiation can be looked upon as effects that are essentially related to transition radiation, and it is interesting to examine their effect on the radiation pattern in this relatively simple formulation of the problem.

Since the problem is homogeneous in time and in x, y, it is convenient to use the Fourier transformation with respect to time and the Hankel transformation with respect to the transverse coordinate r:

$$\begin{aligned} p(x, t) &= \int_0^\infty d\varkappa \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \varkappa J_0(\varkappa r)\, e^{-i\omega t}\bar{p}_{\omega,\varkappa}(z), \\ \bar{p}_{\omega,\varkappa}(z) &= \int_0^\infty dr \int_{-\infty}^{\infty} dt\, r J_0(\varkappa r)\, e^{i\omega t} p(x, t). \end{aligned} \tag{2.3}$$

Substituting (2.3) in (2.1) and (2.2), we obtain the following set of equations for the Fourier components $\bar{p}(z)$ (the subscripts ω and $\varkappa$ are omitted for brevity):

$$\frac{d^2\bar{p}_n}{dz^2} + \left(\frac{\omega^2}{s^2} - \varkappa^2\right)\bar{p}_n = \frac{i\omega\bar{D}_n(\omega/V_n, \varkappa)}{V_n}\, e^{i\omega z/V_n}, \tag{2.4}$$

$$\frac{1}{\rho_1}\frac{\partial\bar{p}_1}{\partial z}\bigg|_{z=0} = \frac{1}{\rho_2}\frac{\partial\bar{p}_2}{\partial z}\bigg|_{z=0}, \quad \bar{p}_1|_{z=0} = \bar{p}_2|_{z=0}, \tag{2.5}$$

where

$$\bar{D}_n\left(\frac{\omega}{V_n}, \varkappa\right) = \frac{1}{2\pi}\int d\mathbf{x}\, D_n(\mathbf{x}, t=0)\, e^{-i\mathbf{k}\cdot\mathbf{x}}, \quad \mathbf{k} = \left\{\varkappa, \frac{\omega}{V_n}\right\}.$$

The solution of (2.4) is the sum of the particular solution defining the intrinsic field of the source in the medium:

$$\bar{p}_n^{\text{intr}} = \frac{i\omega}{V_n}\frac{\bar{D}_n(\omega/V_n, \varkappa)}{\mu_n^2 - (\omega^2/V_n^2)}\, e^{i\omega z/V_n}, \tag{2.6}$$

and the general solution for the radiation field:

$$\bar{p}_n^{\text{tr}} = a_n e^{\pm i\mu_n z}, \text{where } \mu_n = \sqrt{(\omega^2/s_n^2) - \varkappa^2}. \tag{2.7}$$

The amplitudes a_n are found from the boundary conditions (2.5), and are given by

$$\begin{aligned} a_1 &= \frac{i\omega\rho_2}{\rho_1\mu_2 + \rho_2\mu_1}\left[\frac{\rho_1/\rho_2}{\omega + \mu_2 V_2}\bar{D}_2\left(\frac{\omega}{V_2}, \varkappa\right)\right. \\ &\qquad \left. - \frac{\omega - \mu_2 V_1\rho_1/\rho_2}{\omega^2 - \mu_1^2 V_1^2}\bar{D}_1\left(\frac{\omega}{V_1}, \varkappa\right)\right], \\ a_2 &= \frac{i\omega\rho_1}{\rho_1\mu_2 + \rho_2\mu_1}\left[\frac{\omega + \mu_1 V_2\rho_2/\rho_1}{\omega^2 - \mu_2^2 V_2^2}\bar{D}_2\left(\frac{\omega}{V_2}, \varkappa\right)\right. \\ &\qquad \left. - \frac{\rho_2\rho_1}{\omega - \mu_1 V_1}\bar{D}_1\left(\frac{\omega}{V_1}, \varkappa\right)\right]. \end{aligned} \tag{2.8}$$

The field $\bar{p}_n^{tr}$ is the transition radiation field but only when (2.7) describes a propagating wave, i.e., when $\omega^2/s_n^2 > \varkappa^2$. When $\omega^2/s_n^2 < \varkappa^2$, the field $\bar{p}_n^{tr}$ decays exponentially with distance from the boundary and, in this case, we must put $\mu_n = i\sqrt{\varkappa^2 - (\omega^2/s_n^2)}$.

2. To obtain the space-time description of the radiated field, we must invert the Fourier-Hankel transformation. The main contribution of the integral with respect to $\varkappa$ is found by the method of steepest descents. To be specific, let us consider the Gaussian distribution D_m:

$$D_m(\mathbf{x} - \mathbf{V}t) = (\gamma_m\rho_m / l_m b_m^2\pi^{3/2})\exp\left[-\frac{(z - V_m t)^2}{l_m^2} - \frac{r^2}{b_m^2}\right],$$

$$\gamma_m = \frac{\beta_m q_m}{\rho_m c_{pm}},$$

Without reproducing in detail the complete calculation, we give the final expression for the pressure in the wave zone relative to the "track" of the source on the boundary:[40 6)]

$$p_n^{\text{tr}}(|\mathbf{x}|, \theta_n, t)$$

$$= \frac{\rho_n W_n(\theta_n)}{4\pi^{3/2}|\mathbf{x}|}\left[\gamma_2 f_2^{(n)}(\theta_n)\frac{e^{-[t - (|\mathbf{x}|/s_n)]^2/(\tau_{l_2}^2 + \tau_{b_{2n}}^2)}}{\sqrt{\tau_{l_2}^2 + \tau_{b_{2n}}^2}}\right.$$

$$-\gamma_1 f_1^{(n)}(\theta_n)\;\frac{e^{-[t-(|\mathbf{x}|/s_n)]^2/(\tau_{l_1}^2+\tau_{b_{1n}}^2)}}{\sqrt{\tau_{l_1}^2+\tau_{b_{1n}}^2}}\Bigg]; \tag{2.9}$$

where

$$W_n(\theta_n)=\frac{2\cos\theta_n}{\cos\theta_n+Z_{n\alpha}\sqrt{1-(s_\alpha/s_n)^2\sin^2\theta_n}}$$

is the wave transmission coefficient of the boundary $(\alpha = 3-n)$, $\tau_{lm} = l_m/V_m$ is the time taken by the source to cross the boundary, $T_{bmn} = (b_m/s_n)\sin\theta_n$ is the travel time of sound over the projection of the lateral dimension of the source b_m onto the direction of observation, and $f_m^{(n)}$ are functions defining the direction of emission. For example, for the second medium $(n = 2)$,

$$f_1^{(2)}(\theta_2)=\frac{1}{1-(V_1/s_1)\sqrt{1-(s_1/s_2)^2\sin^2\theta_2}},$$

$$f_2^{(2)}(\theta_2)=\frac{1+Z_{21}(V_2/V_1)\sqrt{1-(s_1/s_2)^2\sin^2\theta_2}}{1-(V_2/s_2)^2\cos^2\theta_2}, \tag{2.10}$$

where θ_1, θ_2 are the angles between the direction of observation and the vector $-V_1$ (for the first medium) and V_2 for the second, respectively, and $Z_{n\alpha} = \rho_n s_n/\rho_\alpha s_\alpha$.

Analysis of (2.9) shows that the transition-radiation pulses are also Gaussian and their duration is determined by the longest of the times $\tau_{l_1}, \tau_{b_{1n}}$ or $\tau_{l_2}, \tau_{b_{2n}}$, and the polarity of the pulses may be different.

We note that, when $\tau_{lm} \gg \tau_{bmn}$, the source can be described by a δ-function in the lateral direction. For point sources, the pulses of sound take the form of δ-functions at the point of observation.

3. We have described the radiation pattern when the motion of the source is subsonic in both media. We must now consider the opposite case, namely, that of a source crossing the separation boundary with velocities s_1 and s_2 exceeding the velocity of sound. To be specific, we shall suppose that $V_1 = V_2 = V > s_1 > s_2$. The source then radiates the Mach wave p_1^{Ma} in the first medium (see Ref. 40), which propagates at an angle $\theta_1^{\mathrm{Ma}} = \arccos(s_1/V)$. This wave is reflected from the boundary and provides an additonal contribution to the transition radiation at angles $\theta_1^{\mathrm{Ma}} < \theta_1 < \pi/2$. It is given by

$$p_{(1\to1)}^{\mathrm{Ma}}(|\mathbf{x}|,\ \theta_1,\ t)=V(\theta_1^{\mathrm{Ma}})p_1^{\mathrm{Ma}}(|\mathbf{x}|,\ \theta_1,\ t,\ \theta_1^{\mathrm{Ma}}), \tag{2.11}$$

where $V(\theta_1^{\mathrm{Ma}})$ is the reflection coefficient for a plane wave incident on the boundary at the angle θ_1:

$$V(\theta_1)=\frac{Z_{21}\cos\theta_1'-\sqrt{1-(s_2/s_1)^2\sin^2\theta_1}}{Z_{21}\cos\theta_1+\sqrt{1-(s_2/s_1)^2\sin^2\theta_1}}. \tag{2.12}$$

In the second medium, we have not only the transition radiation but also the Mach wave p_2^{Ma} propagating at the angle $\theta_2^{\mathrm{Ma}} = \arccos(s_2/V)$, as well as the Mach wave $p_{(1\to2)}^{\mathrm{Ma}}$, arriving from the first medium:

$$p_{(1\to2)}^{\mathrm{Ma}}(|\mathbf{x}|,\ \theta_2,\ t)=W_1(\theta_1^{\mathrm{Ma}})\,p_1^{\mathrm{Ma}}(|\mathbf{x}|,\ \theta_2,\ t,\ \theta_{(1\to2)}^{\mathrm{Ma}}); \tag{2.13}$$

where $W_1(\theta)$ is the transmission coefficient of the boundary [see (2.12)]. The Mach wave arriving from the first medium provides a contribution at angles $\pi/2 > \theta_2 > \theta_{(1\to2)}^{\mathrm{Ma}} = \arcsin[(s_2/s_1)\sin\theta_1^{\mathrm{Ma}}]$ but, when $(s_2/s_1)\sin\theta_1^{\mathrm{Ma}} > 1$, the Mach wave in the first medium experiences total internal reflection at the boundary.

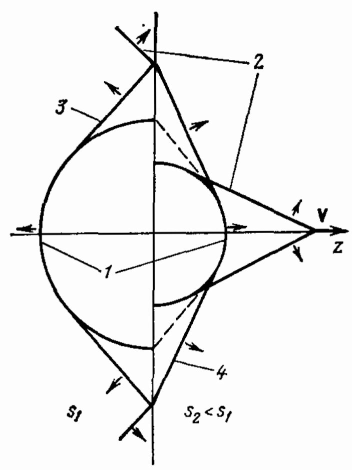

FIG. 3. Disposition of the wave fronts of radiated acoustic waves for $V_1 = V_2 = V > s_1 > s_2$.[40] 1—$p_1^{tr}, p_2^{tr}$; 2—$p_1^{\mathrm{Ma}}, p_2^{\mathrm{Ma}}$; 3—$p_{(1\to1)}^{\mathrm{Ma}}$; 4—$p_{(1\to2)}^{\mathrm{Ma}}$.

Figure 3 shows the disposition of the wave fronts for this case after source crosses the separation boundary. Arrows show the directions of propagation of the waves.

4. We shall now obtain the expression for the angular distribution of the spectral density. This is of interest in the present context because it will enable us to demonstrate clear analogies with the electrodynamic case. It is readily found once we have expression (2.9) for the pressure, which is determined by the method of steepest descents. It is, however, possible to start directly with the expressions (2.8) for the amplitude $a_n\left(\frac{\omega}{V},\varkappa\right)$, following the method described in Refs. 1–3. The principle of this method is that, to find the radiated energy, it is sufficient to evaluate its asymptotic value for $t\to\infty$. Substituting the expansion (2.3) into the expression for the wave energy $\mathscr{E}_n = \int d\mathbf{x}(p_n^{tr})^2/\rho_n s_n^2$ and integrating with respect to r, and then once with respect to $\varkappa$, we obtain

$$\mathscr{E}_n=\frac{1}{2\pi\rho_n s_n^2}\int\limits_{-\infty}^{\infty}dz\int\limits_{\varkappa^2<\omega^2/s_n^2}d\varkappa\iint\limits_{-\infty}^{\infty}d\omega'\,d\omega''\varkappa\,(\bar{p}_n\bar{p}_n^*)\,e^{-i(\omega'-\omega'')t}$$

$$=\frac{1}{\rho_n s_n^2}\int\limits_{\varkappa^2<\omega^2/s_n^2}d\varkappa\varkappa\iint\limits_{-\infty}^{\infty}d\omega'\,d\omega''\,(a_n'a_n''^*)\,\delta\,[\mu_n(\omega')-\mu_n(\omega'')]. \tag{2.14}$$

Since

$$\delta\,[\mu_n(\omega')-\mu_n(\omega'')]=\delta\,(\omega'-\omega'')\left(\frac{d\mu_n}{d\omega}\right)^{-1},$$

the radiation energy is given by

$$\mathscr{E}_n=\frac{1}{\rho_n}\int\limits_{\varkappa^2<\omega^2/s_n^2}d\varkappa\int\limits_{-\infty}^{\infty}d\omega\frac{\varkappa}{\omega}\sqrt{\frac{\omega^2}{s_n^2}-\varkappa^2}\,|a_n|^2. \tag{2.15}$$

Finally, let us introduce the angle θ_2 between the vectors $\mathbf{n}$ and $\mathbf{V}$ (forward emission) and the angle θ_1 between $\mathbf{n}$ and $-\mathbf{V}$ in accordance with the expression $\sin^2\theta_n = \varkappa^2 s_n^2/\omega^2$.[7]

The final result is

$$\mathscr{E}_n = \int_0^\infty d\omega \int_0^{\pi/2} d\theta_n \cdot 2\pi \sin\theta_n \cdot \mathscr{E}_{n,\,\omega}, \qquad (2.16)$$

where

$$\mathscr{E}_{n,\,\omega} = \frac{1}{\pi\rho_n s_n^3}\,\omega^2 \cos^2\theta_n\,|a_n|^2, \qquad (2.17)$$

and the amplitudes a_n are given by (2.8), in which $\varkappa^2 = (\omega^2/s_n^2)\sin^2\theta_n$, where the angles θ_n for $n = 1$ and $n = 2$ lie in the range between 0 and $\pi/2$.[8)]

It follows from (2.8) and (2.17) that, when the source crosses the separation boundary between media with different acoustic resistance $\rho_n s_n$, the resulting transition radiation will interfere with the radiation due to the rearrangement of the source itself on the boundary (change in γ_n and the characteristic dimensions l_n, b_n of the source). When the crossing of the boundary by the source is accompanied by a change in its velocity V_n, bremsstrahlung radiation is also produced, and interference between all these types of radiation takes place.

5. Let us now use the above relationships to consider the case where the free surface of a liquid is crossed by a source with a Gaussian distribution and subsonic velocity. From (2.8) and (2.17), we then have

$$\mathscr{E}_{n,\,\omega} = \frac{\rho_0\gamma^2}{4\pi^3 s}\,\frac{(V/s)^2\cos^2\theta}{[1-(V/s)^2\cos^2\theta]^2}\,\exp\left(-\frac{\omega^2 b^2}{2s^2}\sin^2\theta - \frac{\omega^2 l^2}{2V^2}\right), \qquad (2.18)$$

where $0 \leqslant \theta \leqslant \pi/2$. Analysis of this expression for low velocities $V \ll s$ shows that this is dipole radiation for $l \gg b$, i.e., $\mathscr{E}_{n,\omega} \sim \cos^2\theta$. The characteristic length of the acoustic wave is then much greater than the characteristic linear dimensions of the region in which the sound wave is generated: $\lambda \sim sl/V \gg l \gg b$.

For velocities approaching the velocity of sound, $(1 - V^2/s^2 \ll 1)$, the radiation is concentrated in the narrow angular range $0 < \theta^2 \lesssim 1 - V^2/s^2 \ll 1$ containing the direction of motion of the source and, for $l \gg b$, Eq. (2.18) can be written in the form

$$\mathscr{E}_{n,\,\omega} \approx \frac{\rho_0\gamma^2}{4\pi^3 s}\,\frac{1}{[1-(V^2/s^2)+\theta^2]^2}\,e^{-\omega^2 l^2/2V^2}, \qquad (2.19)$$

The total radiated energy turns out to be

$$\mathscr{E} = \pi \int_0^\infty d\theta^2 \int_0^\infty d\omega\,\mathscr{E}_{n,\,\omega} = \frac{\rho_0\gamma}{2(2\pi)^{3/2} l}\,\frac{1}{1-(V^2/s^2)}. \qquad (2.20)$$

It is interesting to compare the effects of the transition radiation when the source crosses the free surfce of a liquid and when the source is suddenly brought to rest in a uniform medium. In the latter case, we find form (2.8) and (2.17) that

$$\mathscr{E}_{n,\,\omega} = \frac{\rho_0\gamma^2}{16\pi^3 s}\,\frac{(V/s)^2\cos^2\theta}{[1-(V/s)\cos\theta]^2}\,\exp\left(-\frac{\omega^2 b^2}{2s^2}\sin^2\theta - \frac{\omega^2 l^2}{2V^2}\right), \qquad (2.21)$$

where, in contrast to (2.18), the angle θ lies in the range $0 \leqslant \theta \leqslant \pi$. This explains the fact that, as follows from a comparison of (2.18) and (2.21), for transition-radiation intensity within the solid angle $d\Omega$ for low source velocity ($V \ll s$) is greater by a factor of four than the intensity radiated when the source is suddenly stopped. On the other hand, for velocities $V \to s$, the radiation emitted in both of these cases is concentrated in the small angular range $\theta^2 \ll 1$ containing the direction of motion of the source, and the energy parameters turn out to be the same.

Comparison of (2.17)–(2.21) with the analogous electrodynamic problem[1–3] demonstrates both the similarity due to the common wave character of the phenomenon and the differences due to the specific wave polarization and dispersion properties. In particular, for an ultrarelativistic charge, the angular distribution of the radiation is strongly peaked in the direction of motion. This can also occur in the acoustic case (depending, however, on the degree of rearrangement of the source) for velocites $V \to s_2$ ($V < s_2 < s_1$). On the other hand, for low velocities ($V \ll s_n$ in acoustics and $V \ll c$ in electrodynamics), there is an essential difference between the angular distributions. For example, in the electrodynamic case, there is no radiation in the direction of motion of the charge. Moreover, in electrodynamics, the relationship between the transition radiation emitted by a charge entering a perfect conductor from vacuum and the radiation when the charge is suddenly stopped in a vacuum is analogous to that examined above (in the case of acoustic transition radiation emitted when the free surface of a liquid is crossed by a moving source and the radiation emitted when the source is suddenly stopped in a uniform medium).

Finally, there is an essential difference due to the specific nature of acoustics, namely, the absence of significant dispersion in most liquids. This is why the integral

$$\mathscr{E} = \int_0^\infty d\omega\,\mathscr{E}_\omega,$$

evaluated for a point source, will diverge, and the total energy of transition radiation must be estimated by taking the upper limit to be $\omega_{max} \simeq s/a$, where a is the characteristic linear dimension of the source. On the other hand, in electrodynamics, the corresponding integrals can be evaluated, even for a point source, when the permittivity is given by the plasma formula.[9)]

B. Generation of sound in a liquid by laser pulses. Interpretation of the effect in terms of the theory of transition radiation

The generation of sound by laser pulses in liquids has been investigated in terms of the thermal mechanism in a large number of theoretical and experimental papers (see, for example, Refs. 46–53). The most important results were recently reviewed in Refs. 29 and 30.

A laser pulse propagating through a weakly absorbing liquid is a thermo-optic thermal source moving with the velocity of light. Transition radiation is produced when the source crosses the surface of a liquid.[39] However, the phrase, "transition radiation," has not been used in the theory of sound generation because the velocity of light was assumed to be infinite in comparison with the velocity of sound. Sound generation was looked upon as the consequence of the time dependence of thermal sources acting in the liquid, whose density was taken to be

$$Q(\mathbf{x}, t) = A\mu I(x, y)e^{-\mu z}f(t), \tag{2.22}$$

where A is the transmission coefficient for optical radiation of the surface of the liquid, μ is the light absorption coefficient of the liquid, $I(x,y)$ is a function describing the lateral intensity distribution in the laser beam, and $f(t)$ describes the time dependence of the intensity $[\max f(t) = 1]$.

To illustrate the connection between sound generation in a liquid by laser pulses in the case of the thermal mechanism and the theory of transition radiation, we must modify the above approach to some extent. We shall use (2.22) with $f(t)$ replaced with $f(t-z/c)$ as the thermal-source power density, where c is the velocity of light. We have already shown that the transition radiation and the radiation due to the time dependence of the moving source (in the present case, due to absorption) may be considered independently for $L \gg L_f$, where L is the length over which a significant change occurs in the source (for a laser pulse, $L \sim \mu^{-1}$ and $L_t = c\tau_{max}/|1-(c/s)\cos\theta|$ is the formation length of transition radiation, i.e., an acoustic pulse of characteristic length τ_{max}). In the case of a laser pulse ($c \gg s$), the above condition is equivalent to $\tau_{max} \ll \tau_\mu$, where $\tau_{max} = \max\{\tau_p \tau_b\}$ (see subsection A), τ_p is the laser pulse length, and $\tau_b = (b/s)\sin\theta$ and $\tau_\mu = (1/\mu s)\cos\theta$ are, respectively, the characteristic times of delay of sound waves from elementary thermal sources lying in the horizontal and vertical cross sections of the region of effective heat release (Fig. 4). Consequently, under the above conditions, the contribution of transition radiation can be examined separately on the assumption of an unaltered laser pulse. The corresponding expression for the pressure in the case of the Gaussian intensity distribution $I(x,y) = I_0\exp(-r^2/b^2$ is then found from (2.9), where we must substitute $\gamma_1 = 0$, $\gamma_2 = (\pi^{3/2}c\tau_p b^2)\times(\beta/\rho_0 c_p)(\mu A I_0)$, $\rho_1 s_1 \ll \rho_2 s_2 = \rho_0 s$, $V_1 = V_2 \gg s$).[10] In the case of a laser pulse of arbitrary shape, described by the function $f(t-|\mathbf{x}|/c)$, we have for $\tau_p \gg \tau_b$, $t_\mu \gg \tau_p$

$$p(\mathbf{x}, t) = -\frac{A\beta I_0 b^2}{2|\mathbf{x}|c_p\tau_\mu} f\left(t - \frac{|\mathbf{x}|}{s}\right). \tag{2.23}$$

The exponential decay of the laser pulse in the liquid can also be taken into account by modifying somewhat the calculation given in Subsection A. (The source function D is then no longer a function of only the difference $x - Vt$.) Under the same conditions, we find that

$$p(\mathbf{x}, t) = -\frac{A\beta I_0 b^2}{2|\mathbf{x}|c_p\tau_\mu}\left[f\left(t - \frac{|\mathbf{x}|}{s}\right) - \pi\frac{\sigma}{\tau_\mu}e^{-|t-|\mathbf{x}|/s|\tau_\mu}\right], \tag{2.24}$$

where

$$\sigma = \int_{-\infty}^{\infty} dt f(t) \ll \tau_\mu.$$

Equation (2.24) is identical with that given in Ref. 47 except for the fact that, here, we have omitted terms involving higher powers of the parameter $(\mu c\tau_p)^{-1}$.

Thus, in this case, the acoustic signal is the sum of two pulses, the first of which has negative polarity and a shape reproducing the envelope of the laser pulse. Comparison of (2.23) and (2.24) shows that this pulse is, in fact, the pulse of transition radiation.[39,40] The additional positive term in (2.24) that is proportional to τ_p/τ_μ, is due to the attenuation of the laser pulse as it propagates through the liquid (radiation due to the time dependence of the source).

A similar analysis can be performed for other ratios of the characteristic times of the problem. In particular, when $\tau_\mu \gg \tau_b \gg \tau_p$, the contribution of the transition radiation can again be treated separately. In terms of the notation introduced above, we find from (2.9) that

$$p(\mathbf{x}, t) = -\frac{A\beta I_0 b^2\tau_p}{2|\mathbf{x}|c_p\tau_\mu\tau_b}e^{-(t-|\mathbf{x}|/s)^2/\tau_b^2}. \tag{2.25}$$

The shape of the transition-radiation pulse is now determined by the lateral intensity distribution in the laser pulse, and is independent of the shape of this pulse. When the attenuation of the laser pulse during its propagation in the liquid is taken into account, the expression for the pressure again acquires an additional positive term proportional to the small parameters τ_b/τ_μ, and is significant for $|t-(|\mathbf{x}|/s)| > \tau_b$ (see Fig. 5). The final result is, of course, identical with that obtained by a different method in Ref. 47 (see also the reviews given in Refs. 29 and 30).[11]

We note in conclusion that, when the times τ_p and τ_b are comparable with τ_μ, the transition radiation interferes with the radiation due to the attenuation of the laser pulse by absorption.

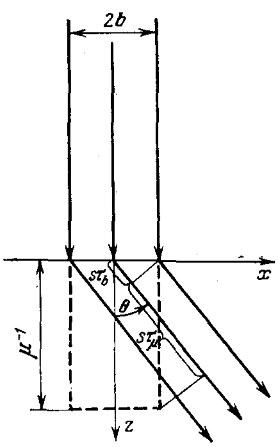

FIG. 4. Region of heat release for sound generation by laser pulses in a liquid in the case of the thermal mechanism.

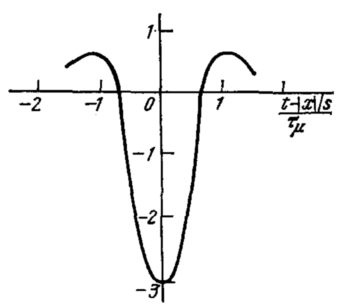

FIG. 5. Shape of the acoustic signal generated by a short laser pulse ($\tau_\mu \gg \tau_b \gg \tau_p$).[29]

C. Acoustic transition radiation generated by a thermo-optic source produced by laser-beam scanning of the separation boundary between two media

This type of transition radiation is of considerable interest because it may be possible to produce practical sources moving with arbitrary (including near-sonic) velocities. Theoretical calculations of the transition radiation emitted in this case are performed in Refs. 40 and 44. These calculations differ from the analysis given in Subsection A by the fact that the source treated in Refs. 40 and 44 is asymmetric relative to the direction of motion, and the presence of the free (or fixed) surface of the liquid must be taken into account.

Sound generation produced when a laser beam crosses the sharp boundary of an opaque screen covering part of the surface of a uniform liquid[39–43] is a well-known analog of this effect. The transition radiation then takes the form of a single acoustic pulse whose shape depends significantly on the ratio of the characteristic times of the problem, and the amplitude is proportional to the Doppler factor

$$J=\frac{1}{1-(V/s)\cos\theta}. \qquad (2.26)$$

Thus, for ultrasonic source of velocities, the pulse changes its polarity as the direction of observation passes through the direction of propagation of the Mach wave. This is explained by the fact that the source overtakes the sound waves produced by it in directions of observation for which $\cos\theta > s/V$. This property of the transition radiation was predicted in Refs. 39 and 40 and confirmed experimentally in Ref. 41.

The authors of Refs. 41–44 have carried out a number of detailed experiments with acoustic transition radiation produced by thermo-optic sources. The properties of the transition radiation produced by a laser beam crossing the sharp boundary of an opaque screen covering part of the free surface of a liquid were investigated in Refs. 41–43 while, in Ref. 44, the source was produced by scanning a laser beam over the free surface of two liquids (ethanol and water) separated by an acoustically thin transparent film. In the latter case, the radius of the light spot on the surface of the liquid was 0.5 cm and the lateral beam distribution was nearly Gaussian. The laser pulses ($\lambda = 1.06\ \mu$m) had the dumbbell shape of width $\tau \approx 150\ \mu$s at half height and total energy $\mathscr{E} \approx 0.6$ J. The scan velocity was varied between 1.05×10^5 and 1.7×10^5 cm/s, and the laser absorption coefficients due to the absorbing impurity were the same ($\mu = 0.6$ cm^{-1}). At the distance $R = 0.6$ m from the point of crossing of the boundary by the source, the transition radiation recorded at $\theta = 21°$ had the shape of an N-pulse which began with the rarefaction phase with amplitudes $p \sim 11 \pm 3$ Pa (Fig. 6). The results of these experimental studies are in good agreement with the theoretical model of the effect.[40,44]

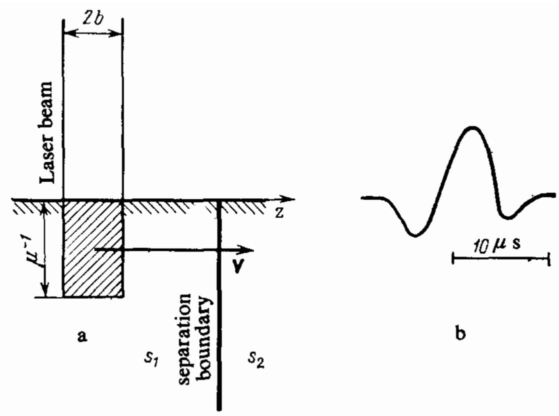

FIG. 6. Acoustic transition radiation from a thermo-optic source produced by scanning a laser beam over the separation boundary between two media[44]: a—geometry of the problem; b—shape of observed acoustic pulse.

D. Transition radiation produced when the source crosses inhomogeneities of finite size

When the source crosses inhomogeneities of finite size, the ratio of the characteristic dimension d of the inhomogeneity to the formation length L_f becomes important. After passing through the inhomogeneity, the source returns to the original medium and, if $d \ll L_f$, the change in its intrinsic field at the boundaries will be unimportant, i.e., there will be little transition radiation. Of course, in general, the radiation pattern will also depend on the shape of the inhomogeneity. Some of the properties of the transition radiation produced when the source crosses a spherical separation boundary between two media were discussed in Ref. 62. It was shown that, when $d \gg L_f$ (d is the diameter of the sphere), and the velocity of the source approaches the velocity of sound s_n ($\rho_1 s_1 \sim \rho_2 s_2$, so that there is little reflection from the boundary), the radiation emitted is the same as when an infinite plane layer of thickness d is crossed.

E. Transition radiation from a viscous layer. Effect of dissipation on the formation of the transition radiation

Dissipative processes have so far been ignored in our discussion of the different manifestations of the transition effect. Actually, the absorption of sound due to viscosity and, even more so, the absorption due to thermal conduction are small in ordinary liquids. However, the effect of dissipation on the formation of transition radiation may turn out to be significant in some cases. The point is that the process responsible for the generation of transition radiation depends on the ratio of three characteristic scales, namely, the formation length L_f, the characteristic linear dimensions d of inhomogeneities in the medium in which the process develops, and the dissipation length L_d which is determined by the viscosity and thermal conductivity of the medium and represents the length over which there is a significant dissipative reduction in the amplitude.

In a perfect medium, $L_d \to \infty$ and the formation of the transition radiation is determined by the ratio of L_f and d: when $L_f \gg d$, there is little transition radiation and the field accompanying the source does not change; if, on the other hand, $L_f \lesssim d$), the transition radiation effect is well defined.

Consider a model in which a point thermal source crosses a thin infinite layer of a viscous liquid in which transverse waves may be present. Even when the source is potential (for example, a thermal or a mass source), the intrinsic longitudinal field will transform into a vortical (rotational) component on the boundaries of the viscous layer.[12)] Although the equation for the vortical component, which has the form of the equation of thermal conduction, does not then contain external sources, the generation of transverse waves occurs as a result of the appearance of shear stresses on the boundaries (in the region where they are crossed by the source). This fact is reflected mathematically in the boundary conditions relating the potential and the vortical components of the fields.

Thus, the emission of transverse transition waves should take place in this system when viscosity is significant in at least one of the two adjacent media. It is well-known that transverse waves are rapdily attenuated in a liquid but, nevertheless, their energy must be taken into account in the overall balance. We shall not examine this question here, and will confine ourselves to an estimate of the effect of dissipative processes on the emission of acoustic transition radiation.

Calculations performed at near-sonic source velocity V $(1 - V^2/s_1^2 \ll 1)$, where $s_2 = s_1 + \Delta s$, $|\Delta s| \ll s_2$ (little reflection from the boundaries) for negligible source changes $(\gamma_1 \simeq \gamma_2)$ show that the leading contribution to the transition-radiation energy in the case of a thin layer with $d \gg L_d$ is

$$\mathscr{E}_{n,\omega} \approx \frac{\rho_0\gamma^2\omega^2}{\pi^5 s_1^3}\left|\frac{1}{L_{f2}^{-1}+L_d^{-1}} - \frac{1}{L_{f1}^{-1}}\right|^2 \sin^2\frac{\pi d}{2L_f}, \qquad (2.27)$$

where d is the layer thickness,

$$L_{fn} = \frac{\pi V/\omega}{1-(V^2/s_n^2)+\theta^2}$$

is the formation length in media 1 and 2,

$$L_d = \frac{2s_2^2/\omega^2}{\nu+\chi\,[(c_p/c_V)-1]}$$

is the dissipation length for a wave mode of frequency ω, ν is the kinematic viscosity, $\chi = \varkappa/\rho_0 c_p$, and $\varkappa$ is the thermal conductivity.

It turns out that if, in addition to the condition $d \ll d_d$, we have $L_{f2} \ll L_d$, the transition radiation is concentrated within the small angular angle $\theta^2 \ll 1$ containing the direction of motion of the source. It also follows from (2.27) that, when the formation length L_{f2} is much greater than the layer thickness d, then $\mathscr{E}_{n,\omega} \sim (d/L_{f2})^2$, i.e., as expected, the energy yield is small. When $L_f \ll d$, the factor $\sin^2\left(\frac{\pi d}{2L_{f_2}}\right)$ oscillates rapidly with the angle θ, and its average value is 1/2. It is then readily seen by comparing (2.27) and (2.17) that the transition-radiaton energy from the layer is approximately equal to the sum of the energies radiated from the two boundaries (for $L_{f_2} \ll L_d$). The maximum amount of radiation is produced when the interference condition $d = (2n+1)L_{f_2}$[13] is satisfied. When the layer viscosity is low and $L_{f_2} \ll L_d$, the energy of the transition radiation is reduced to some extent with practically no effect on the directivity of the emission. In the opposite case, $L_{f_2} \gg L_d$, the radiation is no longer concentrated in the small angular range $\theta^2 \ll 1$.

Let us now consider the case $L_d \ll d$ for which viscosity has a considerable effect. If the formation length L_{f_2} of radiation from the first boundary is small in comparison with the dissipation length L_d, the influence of the first boundary will be negligible because phonons can be formed and then absorbed in the layer. If, on the other hand, the formation length L_{f_2} is comparable with the phonon absorption length L_d, "emitted phonons" cannot be distinguished from "virtual phonons." One can then only speak of the *total* energy lost by the source because radiative losses cannot be uniquely defined.

F. Transition radiation in a nonstationary medium

As noted in the Introduction, a moving source can also generate acoustic transition radiation in a spatially inhomogeneous medium whose properties are functions of time. This effect will be illustrated below for a thermal source and a sharp change in the medium parameters at a particular time. Suppose that, at $t = 0$, thre is a sudden change in the velocity of sound, the medium density, the thermal expansion coefficient, the thermal capacity of the medium, and the power and characteristic linear dimensions of the source from $s_1, \rho_1, \beta_1, c_{p1}, q_1, l_1, b_1$ to $s_2, \rho_2, \beta_2, c_{p2}, q_2, l_2, b_2$, respectively. To determine the characteristics of the resulting radiation, we must solve the wave equation (2.1) for $t < 0$ and $t > 0$, and then match the two solutions at $t = 0$. The matching conditions which follows directly from (1.1) and (1.2), when f, m, η, ζ are all zero, consist of the continuity of displacement and velocity at $t = 0$. The Fourier components of the velocity potential ($\mathbf{v} = \nabla\varphi$) are then given by

$$\frac{\partial^2\varphi_{\mathbf{k}n}}{\partial t^2} + k^2 s_n^2 \varphi_{\mathbf{k}n} = -\frac{\overline{D}_n(\mathbf{k})\, s_n^2}{\rho_n} e^{-i(\mathbf{kV})t}, \qquad (2.28)$$

$$\rho_1 \left.\frac{\partial\varphi_{\mathbf{k}1}}{\partial t}\right|_{t=0} = \rho_2 \left.\frac{\partial\varphi_{\mathbf{k}2}}{\partial t}\right|_{t=0}, \quad \varphi_{\mathbf{k}1}|_{t=0} = \varphi_{\mathbf{k}2}|_{t=0}, \qquad (2.29)$$

where

$$\begin{aligned}
\varphi_{\mathbf{k}}(\mathbf{k},\ t) &= \int d\mathbf{x}\, e^{-i\mathbf{kx}}\varphi(\mathbf{x},\ t),\\
\varphi(\mathbf{x},\ t) &= \int \frac{d\mathbf{k}}{(2\pi)^3} e^{i\mathbf{kx}}\varphi_{\mathbf{k}}(\mathbf{k},\ t),\\
\overline{D}_n(\mathbf{k}) &= \int d\mathbf{x}\, e^{-i\mathbf{kx}} D_n(\mathbf{x},\ t=0).
\end{aligned} \qquad (2.30)$$

The solutions of (2.28) are

$$\begin{aligned}
\varphi_{\mathbf{k}1} &= -\frac{\overline{D}_1(\mathbf{k})/\rho_1}{k^2-[(\mathbf{kV})^2/s_1^2]} e^{-i(\mathbf{kV})t}, && t<0,\\
\varphi_{\mathbf{k}2} &= -\frac{\overline{D}_2(\mathbf{k})/\rho_2}{k^2-[(\mathbf{kV})^2/s_2^2]} e^{-i(\mathbf{kV})t} + a_+ e^{-iks_2t} + a_- e^{iks_2t}, && t>0.
\end{aligned} \qquad (2.31)$$

For $t < 0$ only the particular solution of (2.28) is the solution and there is no radiation. When $t > 0$, two waves are produced with amplitudes a_+ and a_- and propagate along the vectors $\mathbf{k}$ and $-\mathbf{k}$, respectively.

By satisfying conditions (2.29) we obtain the following expressions for the amplitudes a_+ and a_-:

$$a_\pm = \frac{1}{2k^2}\left\{\frac{[1 \pm (\mathbf{n}V/s_2)]\,\overline{D}_2/\rho_2}{1-[(\mathbf{n}V)^2/s_2^2]} - \frac{[1 \pm (\rho_1/\rho_2)\,\mathbf{n}V/s_2]\,\overline{D}_1/\rho_1}{1-[(\mathbf{n}V)^2/s_1^2]}\right\}, \qquad (2.32)$$

where $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$ is a unit vector in the direction of $\mathbf{k}$. Hence it follows that $a_-(-\mathbf{k}) = a_+^*(\mathbf{k})$ so that, for given $\mathbf{k}$, the amplitude a_+ determines the forward emission, i.e., emission into the cone $0 \leqslant \psi \leqslant \pi/2$, where ψ is the angle between the vectors $\mathbf{k}$ and $\mathbf{V}$, and a is the backward-emission amplitude $\pi/2 \leqslant \psi \leqslant \pi$).

To obtain the space-time description of the radiated field, we use the inverse Fourier transformation. Assuming a Gaussian distribution for the source, we obtain

$$p(|\mathbf{x}|, \ \theta, \ t) = \frac{\rho_2}{4\pi^{3/2}|\mathbf{x}|}\left[\gamma_2 f_2(\theta)\frac{e^{-[t-(|\mathbf{x}|/s_2)]^2/(\tau_{l_2}^2+\tau_{b_2}^2)}}{\sqrt{\tau_{l_2}^2+\tau_{b_2}^2}} - \gamma_1 f_1(\theta)\frac{e^{-[t-(|\mathbf{x}|/s_1)]^2/(\tau_{l_1}^2+\tau_{b_1}^2)}}{\sqrt{\tau_{l_1}^2+\tau_{b_1}^2}}\right], \tag{2.33}$$

where

$$f_1(\theta) = \frac{1+Z_{12}(V/s_1)\cos\theta}{1-(V/s_1)^2\cos^2\theta}, \quad f_2(\theta) = \frac{1}{1-(V/s_2)\cos\theta}, \tag{2.34}$$

$\tau_{lm} = (l_m/s_2)\cos\theta$, $\tau_{b_m} = (b_m/s_2)\sin\theta$ are the characteristic times for the acoustic waves to traverse a distance equal to the projections of the longitudinal and transverse linear dimensions of the source along the direction of observation, and $Z_{12} = \rho_1 s_1/\rho_2 s_2$, $m = 1.2$.

In accordance with (2.33), the momentum of the transition radiation in a nonstationary medium can be represented by the sum of two momenta of Gaussian form whose length is determined by the longer of the times τ_{l_1}, τ_{b_1} and τ_{l_2}, τ_{b_2}.

The angular distribution of the radiation spectal energy density can readily be found directly from (2.32). Substituting the expansion given by (2.30) into the expression for the wave energy $\mathscr{E} = \int d\mathbf{x}\, \frac{\rho_2}{s_2}\left(\frac{\partial\varphi}{\partial t}\right)^2$, we obtain

$$\mathscr{E} = \frac{\rho_2}{(2\pi)^6 s_2^2}\int_{t\to\infty} d\mathbf{x} \int\int d\mathbf{k}'\, d\mathbf{k}'' e^{i(\mathbf{k}'+\mathbf{k}'')\mathbf{x}}\frac{\partial\varphi_{\mathbf{k}'}}{\partial t}\frac{\partial\varphi_{\mathbf{k}''}}{\partial t}$$

$$= \frac{\rho_2}{4\pi^3}\int dk \int_0^\pi d\theta \sin\theta\, 2\pi k^4 |a_\pm|^2 = \int_0^\infty d\omega \int_0^\pi d\theta \sin\theta \cdot 2\pi \mathscr{E}_{\mathrm{n},\,\omega}. \tag{2.35}$$

Hence, since $k = \omega/s_2$, we find that

$$\mathscr{E}_{\mathrm{n},\,\omega} = \frac{\rho_2}{4\pi^3 s_2^5}\omega^4 |a_\pm|^2. \tag{2.36}$$

For angles $0 \leqslant \theta \leqslant \pi/2$, expressions (2.33) and (2.36) describe the emission of radiation in forward directions, whereas for $\pi/2 \leqslant \theta \leqslant \pi$ they describe emission in backward directions.

It is well-known[1] that, in electrodynamics, the transition radiation emitted by a relativistic particle at the separation boundary between two media is practically the same as in the case of a sharp change in the permittivity with time.[14] In acoustics, this coincidence also occurs for $V \to s_2 (V < s_2 < s_1)$, but only when the changes involve the velocity of sound alone. Equations (2.17) and (2.36) then reduce to the following form for $l \gg b$:

$$\mathscr{E}_{\mathrm{n},\,\omega} = \frac{\rho_0\gamma^2}{4\pi^3 s_2}\left|\frac{1}{1-(V^2/s_2^2)+\theta^2} - \frac{1}{1-(V^2/s_1^2)+\theta^2}\right|^2 e^{-\omega^2 l^2/2s_2^2}. \tag{2.37}$$

The emission is concentrated in the narrow angular range $0 < \theta^2 < 1 - V^2/s_2^2 \ll 1$ in the direction of motion of the source.

3. DIFFRACTIVE EMISSION OF SOUND

A source moving uniformly in a homogeneous medium is accompanied by a field which is distributed in accordance with some law in the region surrounding the source. It is precisely this field and not the source itself (point or extended) that governs the emission of radiation. Hence, even when there is no change in the properties of the medium along the trajectory of the source, but there are some inhomogeneities in the neighborhood of the trajectory (for example, screens, irregularities, inclusions, and so on), the fields accompanying the source may become modified. If the acoustic parameters of the inhomogeneities are such that total internal reflection takes place at their boundaries, the resulting radiation is usually referred to as diffractive.[15] It is thus clear that diffractive emission can be treated as a variety of transition radiation in its widest sense. Moreover, there is a considerable range of methods available for the solution of diffraction problems. We now proceed to a presentation of the results obtained for some basic problems in acoustic diffractive emission, and to a brief discussion of the methods available for their solution.

A. Emisison of sound by a source traveling near an uneven surface

Suppose that a point mass source moves with constant subsonic velocity $V = \mathbf{Ma}s$ above an uneven but perfectly rigid surface. The deviations of this surface from the xy plane are characterized by the shape function η (Fig. 7). The basic equation for the problem is the Helmholtz equation[63] for the Fourier components φ_ω of the potential:

$$\Delta\varphi_\omega + \frac{\omega^2}{s^2}\varphi_\omega = \frac{\gamma}{\mathrm{Ma}\, s}\delta(y)\,\delta(z-h)\, e^{i(\omega/\mathrm{Ma}\, s)x} \tag{3.1}$$

subject to the boundary condition

$$\left.\frac{\partial\varphi_\omega}{\partial n}\right|_\eta = 0; \tag{3.2}$$

where $\gamma = \mu/\rho_0$, μ is the mass output per unit time, ρ_0 is the equilibrium density of the medium, and, in the case of thermal sources, $\gamma = \beta q/\rho_0 c_p$.

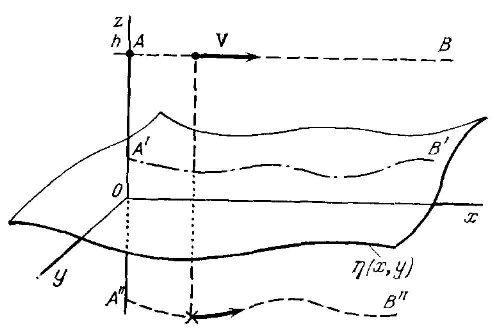

FIG. 7. Source moving above an uneven acoustically rigid surface.

We shall confine our analysis to the case of small surface irregularities η, i.e., $|\nabla_\perp \eta| \ll 1$, where $\nabla_\perp = \mathbf{i}\partial/\partial x + \mathbf{j}\partial/\partial y$ is the two-dimensional differentiation operator. This problem can be solved by the method of small perturbations, the validity of the results being restricted by the condition that the irregularities are small, i.e., $\lambda \gg |\eta|$. Expanding the boundary condition (3.2) into a series around $z = 0$, we can reduce it to the form

$$\left.\frac{\partial \varphi_\omega}{\partial z}\right|_{z=0} = -\left.\frac{\partial^2 \varphi_\omega}{\partial z^2}\right|_{z=0} \eta + \nabla_\perp \eta \cdot \nabla_\perp (\varphi_\omega|_{z=0}) + \ldots \qquad (3.3)$$

The solution of (3.1) with the boundary condition (3.3) will now be written in the form of an expansion in terms of the small parameters $|\eta|/\lambda$:

$$\varphi_\omega = \varphi_\omega^{(1)} + \varphi_\omega^{(2)} + \ldots, \text{where } \varphi_\omega^{(n)} \sim (|\eta|/\lambda)^{n-1}. \qquad (3.4)$$

In the zero-order approximation corresponding to the level surface ($\eta = 0$), we have $\left.\frac{\partial \varphi_\omega^{(1)}}{\partial z}\right|_{z=0} = 0$ and the potential can be expressed in terms of the Macdonald functions:

$$\varphi_\omega^{(1)} = \frac{\gamma}{2\pi \mathbf{Ma}\, s} e^{i(\omega/\mathbf{Ma}\, s)x} [K_0(\alpha\sqrt{y^2 + (z+h)^2}) + K_0(\alpha\sqrt{y^2+(z-h)^2})], \qquad (3.5)$$

where $\alpha = \frac{\omega}{s}\sqrt{\frac{1}{\mathbf{Ma}^2} - 1}$, $\mathbf{Ma} = V/s < 1$.

The expression given by (3.5) determines the perturbation of the medium by the motion of the source. For large values of the argument, the Macdonald function can be replaced with the asymptotic form: $K_0(\zeta) \simeq (\pi/2\zeta)^{1/2} \exp(-\zeta)$. The perturbations of the medium decay exponentially in directions perpendicular to the direction of motion, so that a source moving with constant subsonic velocity in a uniform medium above a level surface will not radiate.

The emitted radiation is due to surface irregularities, which are taken into account in the first approximation in $|\eta|/\lambda$:

$$\Delta\varphi_\omega^{(2)} + \frac{\omega^2}{s^2}\varphi_\omega^{(2)} = 0,$$
$$\left.\frac{\partial \varphi_\omega^{(2)}}{\partial z}\right|_{z=0} = -\left.\frac{\partial^2 \varphi_\omega^{(1)}}{\partial z^2}\right|_{z=0} \eta + \nabla_\perp \eta \cdot \nabla_\perp (\varphi_\omega^{(1)}|_{z=0}). \qquad (3.6)$$

The solution of (3.6) can readily be written down in terms of the Green's function of the second boundary-value problem.

1. Consider the case where a source travels above the corrugated surface $\eta = a \sin \varkappa x$. Analysis shows that radiation is emitted in a given direction at frequencies

$$\omega_m = \frac{m\varkappa \mathbf{Ma}\, s}{1 - \mathbf{Ma} \sin\theta \cos\varphi}, \qquad (3.7)$$

where $m = 1, 2, \ldots$ is the order of the spectrum. The fact that the spectrum is determined by (3.7) and is independent of the structure of the surface, but depends only on its period can readily be understood in terms of the following simple considerations. Let us suppose that the wave $\sim \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t$ is emitted from some point on the surface sinusoid, but only at the time when the source passes above it. The wave $\sim \exp\{i\mathbf{k}\cdot(\mathbf{r} + \mathbf{d}) - i\omega(t + d/V)\}$ is then emitted from the point $\mathbf{r} + \mathbf{d}$ at the time $t' = t + d/V$, where $d = 2\pi/\varkappa$ is the period of the sinusoid. These waves add coherently if the phase difference is a multiple of 2π, i.e., $\mathbf{k}\cdot\mathbf{d} - \omega d/V = 2\pi m$ and, since

$$\mathbf{k}\cdot\mathbf{d} = \frac{\omega}{s}\frac{2\pi}{\varkappa}\sin\theta\cos\varphi,$$

Eq. (3.7) follows immediately. There is another graphic interpretation: the quantity $\varkappa \mathbf{Ma} s$ is the frequency with which the source traverses one period of the surface sinusoid, and the Doppler-shifted radiation frequency is exactly as given by (3.7). It follows from (3.7) that the frequency ω_m can be present only in m directions, where

$$\frac{\omega_m}{\varkappa s}\left(\frac{1}{\mathbf{Ma}} - 1\right) \leqslant m \leqslant \frac{\omega_m}{\varkappa s}\left(\frac{1}{\mathbf{Ma}} + 1\right). \qquad (3.8)$$

Effective generation occurs for $m = 1$, and radiation with $m > 1$ is suppressed exponentially. The linear approximation in $|\eta|/\lambda$ can be used to calculate the emission spectrum, but only in the first order. Since η is periodic, it is interesting to consider the radiated power $\mathscr{E}_n$. Calculations show that this is given by

$$\dot{\mathscr{E}}_n = \frac{\rho_0\gamma^2}{8\pi}\frac{a^2\varkappa^4 s}{\mathbf{Ma}}\frac{(\beta_1^2 - \beta_2/\mathbf{Ma})^2}{\beta_1^2\beta_2^5} e^{-2\varkappa h\beta_1/\beta_2}, \qquad (3.9)$$

where $\beta_1 = \mathbf{Ma}^{-2} - 1 + \sin^2\theta\sin^2\varphi)$, $\beta_2 = \mathbf{Ma}^{-1} - \sin\theta\cos\varphi$.

2. Let us now suppose that the surface is random in x, but its vertical displacement is normally distributed with the joint probability density

$$W(\eta'(x'), \eta''(x'')) = \frac{1}{2\pi\sigma^2(1-C^2)^{1/2}}\exp\left[-\frac{\eta'^2 - 2C\eta'\eta'' + \eta''^2}{2\sigma^2(1-C^2)}\right], \qquad (3.10)$$

where σ^2 is the variance and $C(\rho) = \exp\{-(x' - x'')^2/l^2\}$ is the height correlation coefficient at the distance $\rho = |x' - x''|$. We then have

$$\dot{\mathscr{E}}_{n,\omega} = \frac{\rho_0\gamma^2}{\pi^{3/2}}\frac{\sigma^2 l\omega^4}{\mathbf{Ma}\, s^4}\frac{(\beta_1^2 - \beta_2/\mathbf{Ma})^2}{\beta_1^2} \times \exp\left(-\frac{\omega^2 l^2}{4s^2}\beta_2^2 - \frac{2h\omega}{s}\beta_1\right). \qquad (3.11)$$

The basic properties of the radiation pattern can be seen in Figs. 8–9, which show sections of this pattern by the planes $\varphi = 0, \pi$; $\varphi = \pi/4, 5\pi/4$; $\varphi = \pi/2, 3\pi/2$ at frequency $\omega = 50\ \mathrm{c}^{-1}$ and $\mathbf{Ma} = 0.5$, and also the radiation integrated with respect to the frequency for $\mathbf{Ma} = 0.9$.

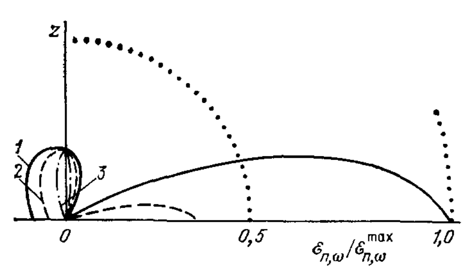

FIG. 8. Section through the radiation pattern at frequency $\omega = 50\ \mathrm{s}^{-1}$ and $\mathbf{Ma} = 0.5$ by the planes $\varphi = 0$, $\pi(1)$, $\varphi = \pi/4$, $5\pi/4$ (2), and $\varphi = \pi/2$, $3\pi/2$ (3).

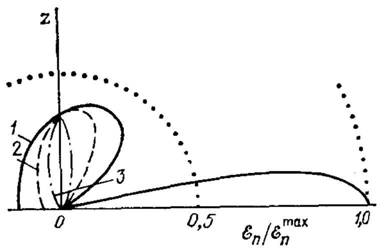

FIG. 9. Section through the frequency-integrated radiation pattern for **Ma** $= 0.9$ by the planes $\varphi = 0, \pi$ (1), $\varphi = \pi/4, 5\pi/4$ (2), and $\varphi = \pi/2, 3\pi/2$ (3).

We note that, in the case of perfectly soft surfaces, the factor $\beta_1^2 - \beta_2/\mathrm{Ma})^2/\beta_1^2$ in (3.9) and (3.11) is replaced with $\cos^2\theta$. This means that the effective radiator is a dipole with a variable dipole moment, moving in the direction perpendicular to its axis. Finally, all types of surface produce an exponential reduction in the intensity of the emitted radiation with increasing height of the source above the surface.

B. Generation of sound by a source moving past a half-plane

Problems involving diffracted radiation can be successfully solved by the Wiener-Hopf method. For example, this method can be used to calculate the emission of sound by a source moving past a half-plane.[24]

Consider an extended streamlined body of length l moving in a uniform medium with constant subsonic velocity $V = \mathbf{Ma}s\,(\mathbf{Ma} < 1)$ past the half-plane $z = 0, y > 0$ in the form of an acoustically rigid screen. The motion of the body is confined to the yz plane and is at an angle ψ to the y axis. The minimum distance between the edge of the screen and the source trajectory will be denoted by a (Fig. 10).

Assuming that the motion of the medium is potential, the basic set of linearized hydrodynamic equations can be reduced to the wave equation for the velocity potential φ.[16] The solution of this equation will be sought in the form of the sum $\varphi = \varphi^{(1)} + \varphi^{(2)}$, where the potential $\varphi^{(1)}$ describes the perturbation field in the medium due to a body moving in infinite space (intrinsic field of the source) and $\varphi^{(2)}$ represents the effect of the half-plane.

The term $\varphi^{(1)}$ is conveniently determined in the cylindrical frame $\{r, \chi, \xi\}$ attached to the moving body, where the ξ axis lies along the axis of the body, the beginning of the body corresponds to $\xi = 0$, and the end to $\xi = l$. The Fourier component of this potential is

$$\varphi_\omega^{(1)} = \frac{\mathrm{Ma}\, s}{2\pi} f(\omega) K_0\left(\frac{\omega r}{s}\sqrt{\frac{1}{\mathrm{Ma}^2} - 1}\right), \tag{3.12}$$

where $f(\omega)$ is determined from the boundary condition on the surface of the body (the flow velocity must be tangential to the surface of the body) and is given by[17]

$$f(\omega) = \frac{1}{2\pi\, \mathrm{Ma}\, s}\int_0^l d\xi\, e^{-i(\omega/\mathrm{Ma}\, s)\xi}\frac{dS(\xi)}{d\xi}; \tag{3.13}$$

Here $S(\xi) = \pi r^2(\xi)$ is the cross section of the body.

The resulting solution can now be transformed back to the original fame $\{x, y, z\}$ and we can use the potential in the form of a Fourier integral with respect to t and x. The radiation field is then described by the equation

$$(\Delta_{y,z} + p^2)\,\varphi_{\omega,q}^{(2)} = 0, \tag{3.14}$$

where $p^2 = \omega^2/s^2 - q^2$ (to be specific, we take $\mathrm{Im}\, p > 0$), together with the boundary condition on the surface of the screen

$$\frac{\partial\varphi_{\omega,q}^{(2)}}{\partial z} = -\frac{\partial\varphi_{\omega,q}^{(1)}}{\partial z} \quad (\text{for } z = 0,\ y > 0) \tag{3.15}$$

and the continuity condition for $\varphi_{\omega,q}^{(2)}$ on its continuation (for $z = 0, y < 0$).

The solution of (3.14)–(3.15) is found by the Wiener-Hopf method. Let

$$\varphi_{\omega,q}^{(2)} = -\frac{\operatorname{sgn} z}{2\pi i}\int_{-\infty}^{\infty} d\eta\, \frac{F(\eta)}{\sqrt{p^2 - \eta^2}}\, e^{i\eta y + i\sqrt{p^2-\eta^2}|z|}, \tag{3.16}$$

where $F(\eta)$ is a function of the complex variable η, to be determined, and $\mathrm{Im}\,(\sqrt{p^2 - \eta^2}) > 0$ by virtue of the radiation principle. The boundary conditions (3.15) lead us to the following two integral equations for $F(\eta)$:

$$\begin{aligned} &\int_{-\infty}^{\infty} d\eta\, e^{i\eta y} F(\eta) = -\frac{\nu\, \mathrm{Ma}\, s}{2\lambda} f(\omega)\, e^{-\mu y - \lambda a} \quad \text{for} \quad y > 0, \\ &\int_{-\infty}^{\infty} d\eta\, \frac{e^{i\eta y}}{\sqrt{p^2 - \eta^2}} F(\eta) = 0 \quad \text{for} \quad y < 0, \end{aligned} \tag{3.17}$$

where $\mu = \lambda\sin\psi - \dfrac{i\omega}{\mathbf{Ma}s}\cos\psi$, $\nu = \lambda\cos\psi + \dfrac{i\omega}{\mathbf{Ma}s}\sin\psi$, $\lambda = \sqrt{\dfrac{\omega^2}{\mathbf{Ma}^2 s^2} - p^2}$. The final expression for the angular distribution of the spectral density of the emitted radiation is[64]

$$\mathcal{E}_{n,\omega} \approx \frac{\rho_0\, \mathrm{Ma}^2 s^3}{2}\, |f\omega|^2\, \frac{\sin\theta\cdot\exp\left[-2a(\omega/s)\sqrt{\mathrm{Ma}^{-2} - \sin^2\theta}\right]}{(\mathrm{Ma}^{-2} - \sin^2\theta)^{3/2}}. \tag{3.18}$$

The intensity maximum occurs at $\theta = \pi/2$, and there is no radiation at $\theta = 0$ and π. the spectral density decays exponentially with increasing "impact parameter" a.

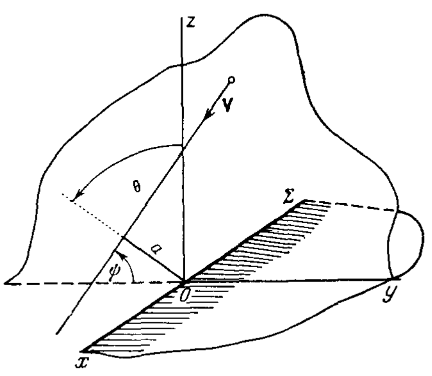

FIG. 10. Motion of a streamlined body past a half-plane along the trajectory $x_0(t) = \{0, -Vt\cos\psi, -Vt\sin\psi + (a/\cos\psi)\}$.

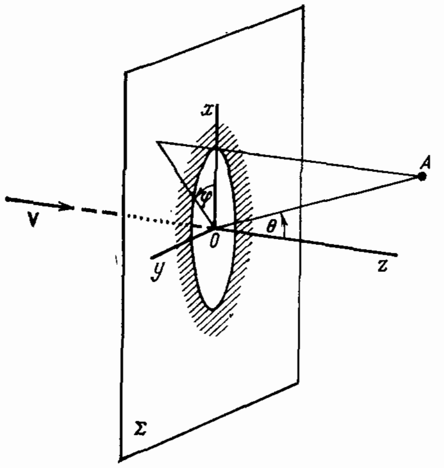

FIG. 11. Passage of a source through a circular aperture of radius R in an acoustically rigid screen Σ. When $V \sim s$, the radiation is largely confined to the forward cone containing the direction of motion.

C. Generation of sound by a source passing through an aperture in a rigid screen

We now consider a problem of the diffractive emission of sound, which admits a relatively simple solution.[66] Suppose that a point source traveling with velocity V approaching the velocity of sound $1 - \mathbf{Ma}^2 \ll 1$ travels along the z axis of a circular aperture of radius R in an infinite acoustically rigid screen Σ (Fig. 11). The perturbation of the medium produced by the source is described by an inhomogeneous wave equation, the solution of which can be written in the form of the sum of the Fourier components $\varphi_\omega = \varphi_\omega^{(1)} + \varphi_\omega^{(2)}$, where $\varphi_\omega^{(1)}$ is the intrinsic field of the source [cf. (3.5) and (3.12)]

$$\varphi_\omega^{(1)} = \frac{\gamma}{2\pi \,\mathrm{Ma}\, s} K_0\left(\frac{\omega r}{s}\sqrt{\frac{1}{\mathrm{Ma}^2} - 1}\right) e^{i(\omega/\mathrm{Ma}\, s)z}, \qquad (3.19)$$

and $\varphi_\omega^{(2)}$ satisfies the homogeneous Helmholtz equation and is determined by its derivative in the plane of the screen Σ. In the wave zone, i.e., for $|\mathbf{x}| \gg R_1$ and $|\mathbf{x}| \gg (\omega/s) R_1^2$ (R_1 is the characteristic dimension of the region in which $\left.\frac{\partial \varphi_\omega^{(2)}}{\partial n}\right|_{z=0}$ is nonzero), we may write

$$\varphi_\omega^{(2)} \approx -\frac{1}{2\pi} \int_\Sigma \mathrm{d}\mathbf{r}' \left.\frac{d\varphi_\omega^{(2)}}{\partial z'}\right|_{z'=0} e^{-i(\omega/s)\mathbf{n}\cdot\mathbf{r}'} \frac{e^{-i(\omega/s)|\mathbf{x}|}}{|\mathbf{x}|}, \qquad (3.20)$$

where $\mathbf{n}$ is the unit vector in the direction of observation. The derivative $\left.\frac{\partial \varphi_\omega^{(2)}}{\partial z}\right|_{z=0}$ in (3.20) is determined from the boundary conditions for $z = 0$. We note that, since $1 - \mathbf{Ma}^2 \ll 1$, the intrinsic field of the source is highly compressed in the direction of its motion, and is not very different in its properties from a set of plane-waves incident on the aperture. The problem can therefore be solved by a method similar to the Kirchhoff method used in the problem of diffraction of high-frequency plane waves by apertures in a screen. It is well-known that, in this case, the resultant field $\varphi_\omega = \varphi_\omega^{(1)} + \varphi_\omega^{(2)}$ in the plane of the aperture ($r < R$) is assumed to be the same as in the absence of the screen:

$$\left.\frac{\partial \varphi_\omega}{\partial z}\right|_{z=0} = \left.\frac{\partial \varphi_\omega^{(1)}}{\partial z}\right|_{z=0},$$

whereas in the shadow ($r > R$)

$$\left.\frac{\partial \varphi_\omega}{\partial z}\right|_{z=0} = 0.$$

This assumption is valid for waves of frequency $\omega \gg s/R$. Evaluation of the integral in (3.20) leads to the following expression for the angular distribution of the spectral energy density:

$$\mathscr{E}_{\mathbf{n},\omega} = \frac{\rho_0 \gamma^2 \mathrm{Ma}}{2\pi^2 s} \frac{D[\zeta, \eta]}{(1 - \mathrm{Ma}^2)^2}, \qquad (3.21)$$

where

$$D[\zeta, \eta] = \frac{\zeta^2}{(1+\eta^2/2)} [J_0(\zeta\eta) K_1(\zeta) - \eta J_1(\zeta\eta) K_0(\zeta)],$$

and $\zeta = (\omega R/s)(\mathbf{Ma}^{-2} - 1)^{1/2}$ and $\eta = \theta(\mathbf{Ma}^{-2} - 1)^{-1/2}$ are dimensionless variables. The total radiated energy is given by the following order-of-magnitude expression:

$$\mathscr{E} \sim \frac{\rho_0 \gamma^2}{R(1 - \mathrm{Ma}^2)^{3/2}}, \qquad (3.22)$$

i.e., it increases with decreasing R as $\mathbf{Ma} \to 1$ until factors such as the finite size of the source, absorption in the medium, and so on, become significant. As R decreases, the fraction of the field accompanying the source that is "cut off" is found to increase, and more radiation is emitted. However, it must be remembered that (3.21) is meaningful for $\omega \gg s/R$, i.e., $\zeta \gg (1 - \mathbf{Ma}^2)^{1/2}$. The validity of the above approximation breaks down for frequencies $\omega \lesssim s/R$.

Figure 12 shows a graph of the spectral density as a function of the variables ζ, η. It follows from this graph that the radiation maximum corresponds to $\zeta_m \simeq 1$, i.e., to the frequency $\omega_m \approx (s/R)(\mathbf{Ma}^{-2} - 1)^{-1/2}$.

The condition $|\mathbf{x}| \gg (\omega/s) R_1^2$ has a simple interpretation in this problem. When a plane wave is diffracted by an aperture, the characteristic scale of the region on which

$$\left.\frac{\partial \varphi_\omega}{\partial z}\right|_{z=0}$$

is significantly different from zero is taken to be the size of the aperture. In the present case, on the other hand, (3.19) shows that this scale is the quantity $R_1 \sim (s/\omega) \times (\mathbf{Ma}^{-2} - 1)^{-1/2}$, so that

$$\frac{\omega}{s} R_1^2 \sim \frac{\mathrm{Ma}^2 s}{\omega(1 - \mathrm{Ma}^2)} \sim \frac{\lambda}{1 - \mathrm{Ma}} \sim L_f, \qquad (3.23)$$

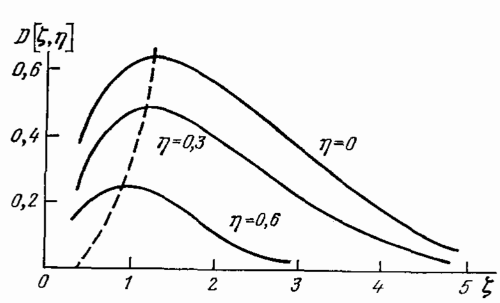

FIG. 12. Angular distribution of radiated spectral power for different values of $\zeta = (\sqrt{1 - \mathbf{Ma}^2}/\mathbf{Ma})\omega R/s$ and $\eta = (\mathbf{Ma}/\sqrt{1 - \mathbf{Ma}^2})\theta$.

where L_f is the formation length. Our condition is then written in the form $|\mathbf{x}| \gg L_f$ and signifies that the point of observation must lie at a distance that is much greater than the formation length.

In the foregoing discussion, we have used solutions of certain characteristic problems to exhibit the general properties of diffracted radiation. The most important of these are as follows. The spectral density (or power) of the radiation is always found to decrease exponentially with increasing $\omega R/s$, where R is the characteristic spatial scale of the problem. For a source traveling above an uneven surface, this parameter is the height of the trajectory above the surface while, in the case of a source passing through a circular aperture in the screen, it is the radius of the screen, and so on. The total radiated energy is inversely proportional to R and depends significantly on the extent to which the velocity of the source approaches the velocity of sound.

Diffracted radiation in systems consisting of periodic structures (periodically uneven underlying surface, periodic lattice of acoustically opaque screens, and so on) exhibits a number of features.[18] Radiation is emitted in a given direction only at a finite number of discrete frequencies, and the spectrum is determined by the period of the structure. Since the emission process is continuous, the energy characteristic is not the radiated energy, but the radiated power. Strictly speaking, we are dealing here not with diffracted radiation, but diffraction scattering, which is a variety of transition scattering.

4. TRANSITION SCATTERING IN ACOUSTICS

The phrase "transition scattering" was first introduced in electrodynamics[67] in connection with the scattering of a permittivity wave by a fixed charge. The use of this terminology is discussed in Refs. 1–3.

We shall now briefly describe the distinctive features of transition scattering.[19]

One of the principal differences between transition scattering and transition radiation is that the former can occur even in the case of a stationary source. An example is the emission of sound when an unmodulated laser beam is incident on the wavy surface of a liquid. The instant at which the source (absorption region) is turned on is accompanied by the appearance of a pulse of transition radiation, and the subsequent continuous emission of sound is due to transition scattering of surface waves into acoustic waves by perturbations produced by the source. If, then, in the general case, transition scattering by the moving source is described by

$$\omega - kV = \omega_0 - k_0 V, \tag{4.1}$$

where ω_0, ω and k_0, k are the frequencies and wave vectors of the incident and scattered waves, respectively, and V is the velocity of the source, then, in the latter case, $V = 0$ and the radiation is emitted without change of frequency.

The continuity of the radiation process gives rise to the second difference between transition scattering and transition radiation, which is pulsed in character. Transition scattering is therefore characterized not by the energy, but by the power of the radiation.

Transition scattering with the emission of acoustic waves can occur for any waves that can exist in the system. Transition scattering of surface waves into acoustic waves, which we have just mentioned, is an example. In principle, transition scattering encompasses a great variety of phenomena involving the transformation of acoustic waves of one type into acoustic waves of another by different inhomogeneities and perturbations. In this sense, transition scattering in solids is a special case. It involves the transformation of volume and surface acoustic waves by a periodically disturbed portion of the surface of a solid,[68] the transformation of Rayleigh waves by surface defects (cracks or irregularities) into longitudinal and transverse volume waves,[69–71] and so on.

Emission by a source moving uniformly in a randomly inhomogeneous medium or in a spatially periodic medium must be looked upon as a variety of transition scattering.[20] These processes are described by (4.1) with $\omega_0 = 0$ and, in electrodynamics, are often referred to as resonance transition radiation, or, simply, resonance radiation.

A. Transition scattering in a randomly inhomogeneous medium

As already noted, transition scattering is characterized by continuous emission along the entire trajectory of the source. It is clear that, in contrast to transition radiation, the intrinsic energy of the field of the source in the medium is now constant on average. Consequently, the radiated power is wholly determined by the work done by the given source. It can therefore be calculated in terms of the mean-field radiation reaction.

This method was used in Ref. 72, in which an analysis was given of acoustic transition radiation (scattering) in an inhomogeneous gaseous medium, for a point thermal or force source. Both small-and large-scale inhomogeneities were considered. By analogy with the electromagnetic case, the effective permittivity tensor $\varepsilon_{ij}^{\text{eff}}$ was introduced. It characterizes the relationship between the Fourier transform of the average momentum per unit volume of the medium and the velocity $\mathbf{v}$ of the perturbations:

$$\langle(\rho_0 + \delta\rho(\mathbf{r}))\, v_i(\omega, \mathbf{r})\rangle_{\mathbf{k}} = \rho_0 \varepsilon_{ij}^{\text{eff}}(\omega, \mathbf{k})\,\langle v_j(\omega, \mathbf{k})\rangle, \tag{4.2}$$

where $\varepsilon(\mathbf{r}) = \delta\rho(\mathbf{r})/\rho_0$, $|\varepsilon(\mathbf{r})| \ll 1$ and $\delta\rho(\mathbf{r})$ are fluctuations in the gas density around the mean value ρ_0.

We shall now reproduce some of the results reported in Ref. 72 for a thermal source and the exponential correlation function $\varepsilon(\mathbf{r})$.

In the case of small-scale fluctuations ($k_0 l \ll 1$, $k_0 = \omega/s_0$, s_0 is the average velocity of sound and l is the correlation length) and low source velocity ($\mathbf{Ma}^2 \ll 1$), the energy output is very low: for a slow ($\mathbf{Ma}^2 \ll (k_0 l)^2 \ll 1$) replacement of the realizations of $\delta\rho$, the spectal power of the radiation is $\mathscr{E}_\omega \sim \mathbf{Ma}^3$, whereas for a relatively rapid replacement $(k_0 l)^2 \ll (\mathbf{Ma}^2 \ll 1)$, the contributions due to a large number of inhomogeneities tend to cancel out and $\mathscr{E}_\omega \sim (k_0 l)^2/\mathbf{Ma}$. When $(k_0 l)^2 \simeq \mathbf{Ma}^2 \ll 1$, the radiated power has a local maximum due to the synchronization between the longitudinal scale of variation of the intrinsic field of the moving source $\mathbf{Ma}/k_0$ and the correlation length ρ. As the Cherenkov threshold is approached, $\mathscr{E}_\omega$ is found to rise and, at the threshold shifted by fluctuations relative to **Ma** $=1$ toward greater **Ma** by the amount $(2/3)\langle\varepsilon^2\rangle$, the radiated power reaches the maximum value

$$\dot{\mathscr{E}}_\omega \approx \frac{\rho_0\gamma^2\omega}{8\pi s_0}, \tag{4.3}$$

where $\gamma=\beta\, q/\rho_0 c_p$. For a hypersonic source ($\mathbf{Ma}^2 \gg 1$),

$$\dot{\mathscr{E}}_\omega \approx \frac{\rho_0\gamma^2\omega}{4\pi s_0 \mathbf{Ma}}. \tag{4.4}$$

It follows from (4.3) that, in contrast to electrodynamics, Cherenkov radiation appears early in the present situation: it is the dominant effect already at near-threshold velocity.

In the case of large-scale fluctuations ($k_0 l \gg 1$) and $\mathbf{Ma} \ll 1$, we have $\dot{\mathscr{E}} \sim \mathbf{Ma}^3$, and thereafter the radiated power increases with increasing source velocity. At the Cherenkov threshold, it is described by (4.3) and, when $\mathbf{Ma}^2 \gg 1$, by expression (4.4). The synchronization condition $\mathbf{Ma} \approx k_0 l$ can now no longer be satisfied for $\mathbf{Ma}<1$, so that the resonance mechanism is no longer observed for the transition radiation.

B. Transition scattering in a turbulent medium

Transition scattering in a turbulent medium was considered in Ref. 73 for a force source of the form given by (I.1). It was assumed that the frequencies of the turbulent velocity pulsations were small in comparison with the characteristic acoustic frequencies. Dissipative effects and temperature fluctuations were assumed to be negligible. The angular distribution of the spectral power $\dot{\mathscr{E}}_{\mathbf{n},\omega}$ was investigated in the single-scattering approximation. It was shown, in particular, that, for low mass numbers $\mathbf{Ma} \ll 1$, there was a very strong powertype dependence of $\dot{\mathscr{E}}_{\mathbf{n},\omega}$ on **Ma** and the radiation intensity was low. The power emitted in the direction of motion of the source ($\theta=0$) was of the same order as the backward-emitted radiation ($\theta=\pi$); there was no radiation at $\theta=\pi/2$. The transition radiation was emitted with maximum efficiency as the Cherenkov threshold was approached. The radiation was then mostly confined to the forward cone drawn around the direction of motion of the source.

C. Transition scattering of surface waves into acoustic waves

In the practical situations encountered in both electrodynamics and acoustics, transition scattering can occur under a great variety of different conditions. The general principles of transition scattering can be successfully applied both to the interpretation of known physical phenomena and to the analysis of new effects. As an example, we shall consider a possible mechanism proposed in Ref. 74 for the generation of infrasound in the atmosphere/ocean system, the essence of which can be explained in terms of transition scattering.[21)]

Finite local inhomogeneities and considerable local velocities can form in the inhomogeneous air stream above the surface of the ocean. Examples include filamentary and ring vortices, toroidal Hill vortices,[76] and solitary toroidal vortices[77] characterized by an exponential reduction of local velocity at infinity. Since they are located near the surface of the ocean, these vortices can be the source of quite powerful infrasound. The point is that a local vortex has associated with it considerable excess hydrodynamic pressure which acts as the "intrinsic field of the source." Its interaction with surface waves gives rise to the scattering of the latter and to a partial transformation into volume waves: acoustic waves are radiated into the atmosphere and the ocean. This interpretation is in accord with the theory of transition scattering. In the case of a fixed source (vortex), radiation is produced without change of frequency: infrasonic waves are emitted with maximum efficiency.

In the model discussed in Ref. 74, the emitted radiation turns out to be practically isotropic. For realistic values of the vortex parameters, the radiated power may be of the order of a few kilowatts, and this confirms the importance of this mechanism for the generation of infrasound in the atmosphere/ocean system.

CONCLUSIONS

Our review of the theory of acoustic transition radiation is, of course, incomplete. Many aspects of this theory were not touched upon, or were treated inadequately. On the other hand, the subject covers a great variety of different topics and research has only just begun in many of its branches. Some of these new lines of research were enumerated above and were also mentioned in Ref. 1, where attention was drawn to a number of interesting acoustic problems (in particular, acoustic transition scattering in superfluid helium II, acoustic transition radiation, acoustic scattering of different types of wave in solids, and so on).

Of course, in both acoustics and electrodynamics, there is a further transition effect, namely, the analog of transition bremsstrahlung.[1] Transition bremsstrahlung appears in electrodyamics when two charges separated by a short distance travel through a medium. It is due to the scattering of polarization waves produced by one of the charges by the field of the other. This radiation interferes with ordinary bremsstrahlung due to the acceleration of charges but, in contrast to the latter, it does not vanish in the limit of infinite masses of the "colliding" particles. A similar effect is observed in acoustics, for example, when the surface of a liquid is scanned by two laser beams, provided the light spots are near enough to one another.

Obviously, all these transition effects play a definite role in atmospheric acoustics. It is well-known that the emission of electromagnetic waves by charges moving in nonuniform electric and magnetic fields constitutes the electrodynamic analog of the aerodynamic generation of sound during the interaction of entropy perturbations and vortical motions with a nonuniform flow field (see, for example, Refs. 78–80). However, by analogy with the way transition radiation in electrodynamics can coexist with synchrotron radiation (for example, in a strong magnetic field in vacuum[1]), so also can the mechanism of acoustic transition radiation (and acoustic transition bremsstrahlung) be significant in different processes of aerodynamic generation of sound.

We hope that the material presented above will act as a stimulus to the formulation of new problems that will be of interest not only in acoustics but also in other branches of physics.

The authors are indebted to V. L. Ginzburg, at whose suggestion this review was undertaken.

[1)]It is, however, important to remember that this statement applies only to "moderate" ultrasonic velocities of the body for which the associated perturbations of the medium are small. On the other hand, in the case of hypersonic velocities (**Ma**$\gg$1), particle velocity perturbations that are small in comparison with the velocity V of the incident flow may not be small in comparison with the velocity of sound s. The predictions of the linear theory are then invalid even in the case of slender pointed bodies (see, for example, Refs. 11 and 13). Finally, linear theory is also invalid at near-sonic (**Ma**$\to$1) velocities of the body.

[2)]The phrase "transition radiation" is sometimes interpreted in a broader sense: it is extended to the radiation due to the disappearance of the source as it emerges on the surface.

[3)]When striction is taken into account, the right-hand side of (1.1) must be augmented with the term $(\rho_0/8\pi)(\partial\varepsilon/\partial\rho)_T\nabla\langle E^2\rangle$, where ε is the permittivity of the liquid and E is the electic field of the optical radiation (see Ref. 45).

[4)]These two quantities will, of course, be equal in the case of diffracted radiation and transition scattering (Secs. 3–4) when the energy of the intrinsic field of the source in the medium is constant on average.

[5)]In real situations, the moving source always varies with time. For example, when a laser pulse propagates through a medium, it becomes attenuated by absorption. The length L is then $L=\mu^{-1}$, where μ is the absorption coefficient for the laser radiation. In the case of a particle beam, the variation of the source with time may be due to the variation in the energy lost by the particles, i.e., in their velocity (due to deceleration).

[6)]The wave zone is defined by the condition for the validity of the asymptotic form of Hankel functions for large values of their argument. Moreover, the functions $D_m(\varkappa)$ must be smooth. Finally, the condition for the absence of interference between the transition radiation and the intrinsic field of the source is written as $|x|\gg L_{fn}$, where the formation length L_{fn} is given by (1.15). These conditions define the range of validity of (2.9):

$$|\mathbf{x}|\gg\max\left\{\frac{s_n(\tau_{\max})_n}{\sin^2\theta_n},\ \frac{s_n(\tau_{\max})_n}{1-(V_m/s_n)\cdot\cos\theta_n}\right\},$$

where $(\tau_{\max})_n=\max\{\tau_{l_m},\tau_{b_{mn}}\}$; $m,n=1,2$ (see below).

[7)]As before, we include only propagating waves, so that the integral over $\varkappa^2$ in (2.15) is confined to the range $\varkappa^2<\omega^2/s_n^2$.

[8)]It is important to recall that (2.17) will also describe the excitation of lateral waves which decay more rapidly than $|\mathbf{x}|^{-1}$ (see Refs. 61 and 40). A lateral wave is excited in directions $\theta_n>\theta_n^*$ (θ_n^* is the angle of total internal reflection) in the medium in which the velocity of sound is lower. Thus, when $s_1>s_2$, the lateral wave is excited in the second medium in directions $\theta_2>\theta_2^*=\arcsin(s_2/s_1)$. For such angles, $(s_2/s_1)^2-\sin^2\theta_2<0$, so that the sign of the modulus in (2.17) is important. The contribution of the lateral wave is shown by the broken line in Fig. 3.

[9)]We note that, for ultrarelativistic particles, the maximum radiated frequency lies in the region of transparency of any material ($\omega\gg\omega_{pe}$), and the plasma formula $\varepsilon(\omega)=1-(\omega_{pe}^2/\omega^2)$ is valid for all media.[1]

[10)]We recall that the analysis of the emission of transition radiation from the plane separation boundary, given in Subsection A, was based on the assumption that all the source parameters in each of the media remained constant and could change only at the instant of boundary crossing.

[11)]We note that the corresponding expressions in Refs. 29 and 30 contain the quantity

$$\sigma=\int_{-\infty}^{\infty}\mathrm{d}tf(t).$$

Since (2.9) has been obtained for the Gaussian source, it follows that, in our case, $\sigma=\sqrt{\pi}\tau_p$.

[12)]We note that, for example, the force source of the form (I.1) is not potential and the transition radiation is produced both as a result of the change in the longitudinal component of the accompanying field on the separation boundary and as a result of the vortical component.

[13)]The conclusions presented above, are, of course, valid even when there is no viscosity ($L_d\to\infty$). The structure of (2.27) is then, in many respects, similar to that of the corresponding expression for the angular distribution of spectral density in the case of a charge crossing a layer of foreign material.[1] In particular, the radiation pattern again depends significantly on the factor $\sin^2(\pi d/2L_{f2})$. At the same time, and in contrast to the acoustic case, the radiation is not concentrated in the angular range $\theta^2\ll1$ for $d\ll L_{f2}$.

[14)]For ultrarelativistic particles, the main contribution to forward emission is due to high frequencies $\omega\gg\omega_{pe}$, in which case $\mu_1=\mu_2=1$.[1]

[15)]The basic properties of diffracted radiation in electrodynamics are examined in the reviews in Refs. 7 and 8.

[16)]In reality, the motion of the liquid in the case of a streamlined body can be regarded as potential in all space with the exception of a thin layer near the surface of the body and the relatively narrow region of the "wake" behind the body.[11] We note that the Lighthill representation has been used[65] to analyze the emission of sound by a turbulent wake produced at the rear of a streamlined body in a uniform flow.

[17)]In the case of a point thermal or mass source, the function $f(\omega)$ is the solution of the inhomogeneous Helmholtz equation, such as (3.1).

[18)]A recent paper[81] examines the emission of acoustic waves by a body moving uniformly in a tenuous medium above a periodic structure in the form of a set of periodically distributed acoustically opaque bands. The calculation is based on the use of the Huygens-Fresnel principle.

[19)]They are, of course, the same as in electrodynamics.[1–3]

[20)]As already noted in Section 3, the emission of sound by a source moving near an uneven surface or a periodic structure can be referred to as diffractive or transition scattering.

[21)]Known mechanisms for the generation of low-frequency noise in the atmosphere/ocean system are discussed in Ref. 75.

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