Instantons versus supersymmetry

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The present review is a revised version of lectures given at the Bakuriani (Georgia) Workshop on High Energy Physics (January 1985). A discussion is given of the recently discovered phenomenon of instanton-generated dynamical symmetry breaking in supersymmetric gauge theories with matter. For a definite choice of the matter multiplets, the gauge invariance is necessarily spontaneously broken, the gauge bosons acquire mass, the variation of the coupling constant ceases, and a weak-coupling regime sets in. This sometimes also involves spontaneous breakdown of the supersymmetry. A description is given of the fundamental aspects as well as the specific dynamical scenarios realized in the most typical models.

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1. INTRODUCTION

Supersymmetry (SUSY)[1-3] is at present the most actively investigated and rapidly growing trend in theoretical elementary-particle physics. There is a steady and profuse flow of studies devoted to supersymmetric theories and their possible applications (see, for example, the reviews of Refs. 4–11). One of the most important problems whose solution is expected from supersymmetric models is the problem of the hierarchy of masses. The experimentally observed and theoretically existing (hypothetical) scales are the rungs of a gigantic ladder, each step along which takes us up by many orders of magnitude: from several tens of electron volts—the neutrino mass (?)—up to 10^{19} GeV—the Planck mass.

Supersymmetry offers hope[12] for an explanation of the hierarchy of masses, for two interrelated reasons. First, owing to the absence of renormalizations in perturbation theory[13]—the so-called nonrenormalization theorems—the fields are protected from acquiring large masses in the loop diagrams. Therefore, if a mass difference is introduced in the Lagrangian from the outset, it is preserved in all orders in the coupling constant and need not be sustained in an artificial manner. Second, and more importantly, one can expect the symmetry to be broken dynamically by effects which are exponential in the inverse coupling constant, since the above-mentioned theorems hold only in perturbation theory. Thus, the theory contains a natural small parameter, and one can hope for an explanation of the mechanism which gives rise to the hierarchy of masses.

It is the problem of dynamical breakdown of SUSY to which the present review is devoted. Thus, from the flow of contemporary literature on supersymmetry we have chosen only one little stream (the review is based on Refs. 14–20), which, however, may in due course become a deep river if the hopes for an explanation of the hierarchy of masses are justified. At any rate, we are dealing with a very beautiful theoretical phenomenon.

From the technical point of view, the most important step is the use of instantons.[21] Hitherto, instantons, which are very instructive for an understanding of the structure of non-Abelian theories, have not had reliable practical applications (see the review of Ref. 22). Here we deal with the case in which the instanton contribution is well determined and leads to a qualitative effect—spontaneous breakdown of the color symmetry or supersymmetry.

We shall remind the reader of some well-known examples of spontaneous symmetry breaking, noting the analogies with, and differences from, the phenomenon which we intend to discuss in detail.

At first sight, supersymmetric gauge theories are very similar to "ordinary" quantum chromodynamics. In the simplest case, the only difference refers to the representation of the color group with respect to which the fermions transform. In the case of supersymmetric gluodynamics, the fermions, the so-called gluinos λ^a_α, transform according to the same adjoint representation as the gluons. However, if matter fields are added, then in addition to the matter fermions (quarks) their scalar partners (scalar quarks) are also introduced.

In the case of quantum chromodynamics, we know from the phenomenology of the strong interactions that chiral symmetry is spontaneously broken (see, for example, the textbooks of Ref. 23). Namely, whereas it is possible to perform parity-changing chiral rotations in the strong-inter-

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action Lagrangian in the limit of massless u and d quarks, the physical spectrum is noninvariant with respect to these rotations: there is no degeneracy with respect to the parity. The spontaneous symmetry breaking is characterized by the appearance of a quark condensate:

$$\langle \overline{q^\alpha} q^\alpha \rangle \neq 0, \tag{1.1}$$

where q^α are the quark fields, $q^\alpha = u^\alpha, d^\alpha$, and α is the color index.

In supersymmetric theories, we shall also be concerned with the calculation of various condensates in the vacuum and thus learn about spontaneous breakdown of a given symmetry. In particular, we shall discuss the gluino condensate:

$$\langle \lambda^a \lambda^a \rangle \neq 0, \tag{1.2}$$

where a is the color index, $a = 1, 2, 3$ in the case of the group SU(2).

However, in the case of quantum chromodynamics we are dealing with a theory with strong coupling, so that there are no consistent methods of calculating the condensates. For example, the quantity $\langle \bar{q} q \rangle$ is extracted from the phenomenology of pion interactions. On the other hand, in supersymmetric theories (more precisely, in the variants discussed below), all the phenomena make themselves felt when the constant is small. More importantly, in the case of quantum chromodynamics the color symmetry is not broken, whereas supersymmetric theories are characterized above all by dynamical breakdown of the color symmetry.

In this sense, the situation is more reminiscent of the Glashow-Weinberg-Salam model of the electroweak interactions, in which the "color" scalar field forms a vacuum condensate (see, for example, Ref. 23). In this model, the potential energy of the scalar field has the form

$$V_{\text{pot}}(\varphi) = C(\bar{\varphi}\varphi - v^2)^2, \tag{1.3}$$

where φ is the doublet of scalar fields and v is a constant. Then it is obvious that the minimum of the energy corresponds to a nonzero expectation value of the scalar field:

$$(\bar{\varphi}\varphi)_{\text{vac}} = v^2. \tag{1.4}$$

If v is large, we are dealing with a classical field and can speak of the average value, not only of the square of the field, but of the field itself. The vacuum expectation value of the electrically neutral component of the Higgs field φ is nonzero.

The theories discussed here also exhibit spontaneous breakdown of the color symmetry, and the scalar color field "drops out." In contrast to the theory of the electroweak interactions, however, the existence of a potential energy of the form (1.3) in the Lagrangian is not postulated. The effective potential is generated dynamically by the instantons. It is this meaning of the words "dynamical symmetry breaking" which is frequently encountered in the present paper.

We shall discuss mainly the case in which the vacuum expectation value of the scalar field is large, the gauge fields acquire a large mass, and the effective coupling constant is always small. One may ask how all these small or large quantities appear parametrically if there is no constant of the type v in the Lagrangian. The answer to this question in supersymmetric theories is quite unexpected for anyone whose "intuition" is derived from the experience of working with quantum chromodynamics.

Namely, the set of Lagrangian dimensional parameters in supersymmetric theories is the same as in QCD. These are the parameter Λ, which determines the value of the effective coupling constant, and the masses of the particles of matter (quarks):

$$\Lambda, \quad m_1, \quad m_2, \quad \ldots .$$

In the simplest case of quarks with a single flavor, the vacuum expectation value of the scalar field φ_{vac} can be estimated as

$$\varphi_{\text{vac}} \sim \Lambda \left(\frac{\Lambda}{m_1}\right)^{1/4}, \tag{1.5}$$

and φ_{vac} is large, $\varphi_{\text{vac}} \gg \Lambda$, if $m_1 \ll \Lambda$! There is nothing like this in QCD: Here the masses of the u and d quarks can be assumed to be zero with good accuracy, and no quantities become infinite.

The mechanism which gives rise to the condensate (1.5) can be explained as follows. In the massless limit, the lowest state of the theory—the vacuum—is not fixed at all in the framework of perturbation theory. There are directions in the color space in which the scalar field can take an arbitrary value without increasing the energy of the state. This property of the potential energy is built into the supersymmetry of the theory and is not fortuitous, and it is maintained in all orders of perturbation theory (see the nonrenormalization theorems mentioned above).

For this reason, the value of the vacuum field φ_{vac} is determined by small perturbations. These perturbations include the mass term in the Lagrangian (if $m \ll \Lambda$, the mass term is small) and instanton effects. Instantons are distinguished by the fact that they represent the principal nonperturbative contribution and break the degeneracy of the vacuum with respect to the quantity φ. It is in this way that the unexpected results of the type (1.5) arise. We stress again that all the calculations are in essence simple and can be performed in full.

Dynamical breakdown of color symmetry occurs in a rather large class of models, the simplest of which was described briefly above—supersymmetric QCD with a single quark flavor. The breakdown of supersymmetry itself takes place in theories with so-called chiral matter, i.e., in models in which the numbers of left- and right-handed fermions in the Lagrangian are different. The fact that the chirality of matter is a necessary condition for the breakdown of supersymmetry has long been known—from consideration of the so-called Witten index.[24] However, it has only now been demonstrated explicitly that this same condition is also sufficient, at least in certain cases.

The plan of the review is as follows. In Sec. 2 we describe supersymmetric quantum chromodynamics—a theory which is discussed in some detail in the present review. Section 3 is devoted to instantons in supersymmetric gauge theories. This section lays the groundwork for the discussion in Sec. 4 of the physical effects mentioned above—spontaneous breakdown of color symmetry and supersymmetry.

2. SUPERSYMMETRIC QUANTUM CHROMODYNAMICS

a) Description of the model

To begin with, we give general information about supersymmetric quantum chromodynamics—the composition of the particles and the structure of the interaction vertices. We consider here the simplest variant: the gauge group SU(2), with one flavor of matter.

The gauge sector of the model includes three gluons and three spinor superpartners—gluinos—for whose description it is possible to use either a four-component Majorana (real) field or a two-component Weyl (complex) field.

With regard to matter, we recall that the Dirac quark field in ordinary QCD is equivalent to two chiral fields—one left-handed and one right-handed field, both of which transform according to the fundamental representation of the group $SU(2)_c$, i.e., are color doublets. If we go over from particles to antiparticles, the right-handed doublet can be rewritten as a left-handed antidoublet. Further, since all the representations of the group SU(2) are (pseudo)real, the antidoublet is in essence identical to the doublet and, thus, the Dirac quark reduces to two left-handed fields—color doublets.

In order to describe a single flavor in the supersymmetric variant, it is necessary to introduce two chiral (left-handed) superfields S_1 and S_2 which transform according to the fundamental representation of the group $SU(2)_c$. In what follows, we shall use the notation S_f^α, where $\alpha = 1, 2$ is the color index ($f = 1, 2$). In terms of components, the chiral field has the usual form

$$S = \varphi(x_L) + \sqrt{2}\theta^\gamma\psi_\gamma(x_L) + \theta^2 F(x_L), \qquad (2.1)$$

where the indices α and f are omitted, θ^γ is a two-component Grassmann spinor, and F is an auxiliary field, introduced in the Lagrangian without a kinetic term. The fields of the quark and scalar quark are denoted by ψ and φ, respectively. The coordinates θ and x_L are the standard coordinates on which the chiral fields depend. We give their transformation law under superdisplacements with parameters ε and $\bar{\varepsilon}$:

$$(x_L)_{\alpha\dot{\alpha}} = x_{\alpha\dot{\alpha}} - 2i\theta_\alpha\bar{\theta}_{\dot{\alpha}}, \qquad \delta(x_L)_{\alpha\dot{\alpha}} = -4i\theta_\alpha\bar{\varepsilon}_{\dot{\alpha}},$$
$$\delta\theta_\alpha = \varepsilon_\alpha, \qquad (2.2)$$

where $\alpha, \dot{\alpha} = 1, 2$ are Lorentz indices.

If we employ the superfield language, the Lagrangian of the model can be represented in the very compact form

$$\mathcal{L} = \frac{1}{2g^2}\,\mathrm{Tr}\int d^2\theta W^2 + \frac{1}{4}\int d^2\theta d^2\bar{\theta}\bar{S}^f l^V S^f + \left(\frac{m}{4}\int d^2\theta S^{\alpha f} S_{\alpha f} + \text{H.c.}\right), \qquad (2.3)$$

where V is a superfield of general form containing a gluon four-potential, and W_α is the chiral superfield of the intensities,

$$W_\alpha(x_L, \theta) = \frac{1}{8}\bar{D}^2(e^{-V}D_\alpha e^V)$$
$$= [i\lambda_\alpha(x_L) - \theta_\alpha D(x_L) - i\theta^\beta G_{\alpha\beta}(x_L) + \theta^2\mathscr{D}_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}(x_L)],$$
$$G_{\alpha\beta} = -\frac{1}{2}(\partial_{\alpha\dot{\alpha}}A^{\dot{\alpha}}_\beta + \partial_{\beta\dot{\alpha}}A^{\dot{\alpha}}_\alpha + [A_{\alpha\dot{\alpha}}, A^{\dot{\alpha}}_\beta]),$$
$$A_{\alpha\dot{\alpha}} = (\sigma_\mu)_{\alpha\dot{\alpha}}A_\mu, \quad \sigma_\mu = (1, \boldsymbol{\sigma}). \qquad (2.4)$$

Further, the fields V and W_α are matrices in the color space; for example, $V \equiv V^a\tau^a/2$, where τ^a are the Pauli matrices. The coupling constant g is included in the normalization of the field V. The symbol D_α denotes the spinor derivative, and $\mathscr{D}_{\alpha\dot{\alpha}}$ denotes the covariant derivative.

We note that the studied model *with one flavor* possesses global SU(2) invariance associated with the field transformations $S_1 \leftrightarrow S_2$. The symmetry holds even in the presence of a mass term [see (2.3)]. This SU(2) group, which we shall call the flavor group, owes its existence to the fact that the representations of the color group $SU(2)_c$ are (pseudo)real. All the indices corresponding to the SU(2) groups (color, Lorentz, and flavor) can be lowered and raised by means of the ε symbol according to the general rules.

If we go over to the components, the Lagrangian (2.3) includes: a) the kinetic terms of the gluon, gluino, quark, and scalar quark; b) the matter mass terms; c) the normal gauge vertices; d) a coupling of the quark, gluino, and scalar quark of the type $\bar{\varphi}T^a\psi\lambda^a$, where $T^a = \tau^a/2$ are generators of $SU(2)_c$ in the fundamental representation; e) the square of the D terms,

$$\Delta\mathcal{L}_D^{(W)} = \frac{1}{2g^2}D^a D^a.$$

We call attention to the unusual—positive—sign of this term in the Lagrangian. It originates from $W^2|_F$; the kinetic term of the field D^a is absent, and it can be eliminated by making use of the equation of motion. In pure gluodynamics $D^a = 0$, but the introduction of matter in the form $\bar{S}e^V S$ adds to the Lagrangian the term

$$\Delta\mathcal{L}^{(S)} = D^a\bar{\varphi}^f T^a\varphi^f. \qquad (2.4')$$

Finally, after eliminating the auxiliary field D^a, we arrive at the following expression for the self-action of the scalar fields:

$$V_{pot} = \frac{1}{2g^2}D^a D^a, \quad D^a = -\frac{g^2}{2}(\bar{\varphi}_1\tau^a\varphi_1 + \bar{\varphi}_2\tau^a\varphi_2), \qquad (2.5)$$

where we have taken into account the fact that the generators of the group SU(2) reduce here to $\tau^a/2$.

This information is sufficient to understand the aspect of the models which is the basis of the entire range of phenomena under consideration.

We shall assume that the scalar fields are independent of x, and we shall suppose at first that $m = 0$ (effects due to the introductiom of a small matter mass will be taken into account later). Then it is easy to see that the minimum of the potential ($V_{min} = 0$) is attained not only for zero values of $\varphi_{1,2}$ but also along entire directions, which we shall call troughs (the term "flat directions" is used in the literature). In fact, let

$$\varphi_1 = v\begin{pmatrix}1\\0\end{pmatrix}, \quad \varphi_2 = v\begin{pmatrix}0\\1\end{pmatrix}. \tag{2.6}$$

It is obvious that for any v the values of D^a are equal to zero. Moreover, if $m = 0$, then the values of the F terms are also equal to zero.

In addition, the infinite degeneracy of the vacuum—the existence of the trough (2.6)—holds not only at the classical level but also with allowance for all orders of perturbation theory. This fact follows from the renormalizability of the theory and from the nonrenormalization theorems,[13] which say that the entire effect of perturbation theory reduces to a renormalization of the wave functions of the fields.

In this respect, supersymmetric theories are fundamentally different from nonsupersymmetric ones. In the ordinary theory, say, with

$$\mathscr{L} = |\partial_\mu \varphi|^2 + \bar{\psi} i \hat{\partial} \psi + (g\varphi\bar{\psi}\psi + \text{H.c.}),$$

we could also assume that at the classical level the mass of the field φ and its self-action are absent. In other words, $V_{\text{pot}} = 0$ at the classical level. However, both a mass and a self-action would necessarily occur even at the single-loop level, so that it is necessary to write the corresponding counterterms in the Lagrangian from the outset, thus automatically destroying the indifferent equilibrium in the potential energy.

Thus, in supersymmetric theories with matter it is possible to carry out quantization and to develop perturbation theory near any point of the trough as soon as there is an indifferent equilibrium in the perturbation theory. The central question is whether the degeneracy is removed when nonperturbative effects are included.

Before answering this question, we shall discuss the structure of the model with $v \neq 0$ in somewhat greater detail. It is obvious that the regime with $v \neq 0$ corresponds to spontaneous breakdown of the color symmetry, when, as usual, there is a rearrangement of the spectrum: the gauge bosons become massive, "eating up" certain scalar fields, which change into their longitudinal components. Owing to the supersymmetry, the growth of the mass of the vector fields is accompanied by a growth of the mass of their spinor partners as well.

We recall that in the ordinary (nonsupersymmetric) SU(2) model with one complex Higgs field (a color doublet), which is equivalent to four real fields, three of them can be made to *vanish identically* by using the gauge invariance of the model, while the remaining, fourth field, developing a vacuum expectation value v, gives a mass $m_V = gv$ to the three vector bosons. The deviation of this field from v represents a physical Higgs particle.

In the supersymmetric variant, instead of gauge freedom we have freedom with respect to supergauge transformations

$$S \to e^{i\Lambda} S,$$

where Λ is the chiral superfield, $\Lambda = \Lambda^a T^a$, and the lower component Λ^a is an arbitrary *complex* function $\omega^a(x_L)$. Using this freedom, in the topologically trivial sector it is always possible to assume that three of the four chiral superfields $S^{\alpha f}$ are identically equal to zero,

$$S_f^\alpha(x_L) = \delta_f^\alpha \phi(x_L), \tag{2.7}$$

where $\phi(x_L)$ is a singlet chiral superfield without color or flavor indices. Then the three chiral superfields go over into the "longitudinal" components of the vector superfield V^a, which becomes massive if the vacuum value of ϕ is nonzero. Thus, the relation (2.7) is an analog of the unitary gauge in the ordinary Higgs model. Substituting (2.7) into the Lagrangian (2.3), we see that all the particles in the vector supermultiplet become massive,

$$m_V = g\langle\phi\rangle \equiv gv, \tag{2.8}$$

while the singlet superfield ϕ remains massless.

The possibility of using the supergauge condition to dispose of the fields "eaten" by the Higgs mechanism is a general one. In fact, we see that the coordinate corresponding to motion along the bottom of the trough is the color and flavor invariant $S_f^\alpha S_\alpha^f = 2\phi^2$.

It is instructive to examine how the degrees of freedom are redistributed when the color symmetry is spontaneously broken. Before the symmetry breaking, we had three massless gluons (six degrees of freedom) and three massless gluinos (six degrees of freedom), four complex scalar fields (eight degrees of freedom), and four Weyl matter fermions (eight degrees of freedom). After the spontaneous symmetry breaking, there are three gluons, three real scalar fields, and three Dirac spinors, all with mass (12 boson and 12 fermion degrees of freedom), one complex scalar field, and one Weyl spinor with zero mass (two boson and two fermion degrees of freedom).

Thus, if $v \neq 0$, the theory splits into two sectors—the sector of massive particles, which form SU(2) triplets, and the sector of massless particles. In this last sector, both the color and the flavor SU(2) groups are realized trivially, since the massless particles are singlets with respect to both groups.

At low energies ($\ll m_V$), the superfield ϕ is sterile, and, in particular, its vacuum value is not fixed—for any constant ϕ the energy is equal to zero, which reflects the presence of troughs. In this language, the question of whether of not the degeneracy is removed by nonperturbative effects can be formulated as follows: does there exist in the low-energy Lagrangian for ϕ, which is obtained after an integration over all the heavy degrees of freedom, a nonzero superpotential?

b) Instantons and the dynamics of flat directions

It remains for us to demonstrate the emergence in SQCD of an effective potential which removes the continuum degeneracy and fixes the value of ϕ, $\phi_{\text{vac}} = v \neq 0$. It turns out that the problem of the effective potential can be investigated almost completely without performing any calculations. By analyzing only the general properties of the model, it is possible to fix the functional form of the superpotential, apart from a numerical constant, whose specific value is in general not very important (though the fact that it is nonzero is important—a fact established only by direct calculation; see Sec. 3).

First of all, on what variables can the effective potential depend? Since the vector superfields are massive, in the low-energy region there remains a dependence only on ϕ. As in any low-energy expansion, in the leading approximation the dependence on the derivatives of the superfield ϕ can be neglected.

The ensuing analysis will be clearer if we temporarily dispense with the unitary gauge and return to a gauge of general form. In this gauge, the required variable must be constructed from the superfields S_1 and S_2, and it must obviously be invariant with respect to the symmetries present in the model—the color and flavor SU(2) groups.

The only invariant which can be constructed from the matter superfields in this case is

$$I = S^{\alpha f} S_{\alpha f} \qquad \alpha, f = 1, 2, \tag{2.9}$$

which is equal to $2\phi^2$.

The effective superpotential, if it exists, must have the form

$$\mathcal{L}_{\text{eff}'} = \int d^2\theta f\,(I\,(x_L,\,\theta)) + \text{H.c.},$$

where f is some function.

The form of the function f is readily established if we recall that the required SQCD Lagrangian possesses an additional invariance associated with axial rotations of the gluino and matter fields. At the classical level, there exist two conserved axial currents. One of them, the so-called R current J^R_μ, is the superpartner of the energy-momentum tensor and the supercurrent and corresponds to the following rotations:

$$\lambda_\alpha \to e^{i\vartheta}\lambda_\alpha, \quad \psi^f_\alpha \to e^{-(i/3)\vartheta}\psi^f_\alpha, \quad \varphi^f \to e^{(2i/3)\vartheta}\varphi^f.$$

In the superfield language, the last two transformations are equivalent to a simultaneous phase transformation of the superfields S and the parameter θ:.

$$S^f \to \exp\left(\frac{2i}{3}\vartheta\right) S^f, \quad \theta_\alpha \to \exp\,(i\vartheta)\,\theta_\alpha.$$

The other current J^M_μ affects only the matter fields; the transformations corresponding to it are

$$\psi^f_\alpha \to e^{i\gamma}\psi^f_\alpha, \quad \varphi^f \to e^{i\gamma}\varphi^f,$$

or, in the superfield notation, $S^f(x_L,\theta) \to \exp(i\gamma) S^f(x_L,\theta)$. The quantum effects destroy the conservation of both currents because of the well-known triangle anomalies. However, one linear combination of the two currents remains anomaly-free. Without plunging into a discussion of the anomalies—the interested reader should turn to the special literature—we point out that the strictly conserved (with allowance for the anomalies) current for the gauge group SU(2) has the form

$$J^R_\mu - \frac{5}{3} J^M_\mu. \tag{2.10}$$

Thus, the effective SQCD Lagrangian must be invariant with respect to the transformations

$$S^f \to e^{-i\vartheta} S^f, \quad \theta_\alpha \to e^{i\vartheta}\theta_\alpha. \tag{2.11}$$

At the same time, the variable I goes over into $\exp(-2i\vartheta)I$, and the only possible invariant expression for the superpotential obviously reduces to

$$\mathcal{L}_{\text{eff}} = \int d^2\theta \cdot \text{const} \cdot [I\,(x_L,\,\theta)]^{-1} \equiv \int d^2\theta\, \frac{C\Lambda^5}{S^{\alpha f}S_{\alpha f}}; \tag{2.12}$$

here the factor Λ^5 has been written on the basis of dimensional arguments, and C is a numerical constant.

It is this factor Λ^b (b is the first coefficient of the Gell-Mann–Low function, equal to 5 in the SU(2) model with one flavor) which enters into the expression for the single-instanton measure. Thus, there is every reason to expect that the superpotential (2.12) obtained from the analysis of the symmetries oi the model is indeed generated in the single-instanton approximation. In Sec. 3 we describe the instanton calculus developed by us, by means of which it is easy to examine explicitly the appearance of the superpotential (2.12) and to find in principle the value of the numerical constant C. Since it is important fcr us only that $C \neq 0$, in the ensuing equations we shall assume that $C = 1$.

Returning to the unitary gauge, we see that the superpotential (2.12)

$$\mathcal{L}_{\text{eff}} \sim \int d^2\theta\, \frac{\Lambda^5}{\phi^2\,(x_L,\,\theta)} + \text{H.c.} \tag{2.13}$$

leads to the appearance of a nonzero F term of the superfield ϕ, namely,

$$\bar{F} = \frac{\Lambda^5}{\varphi^3}, \quad F = \frac{\Lambda^5}{\bar{\varphi}^3}, \tag{2.14}$$

which corresponds to the potential energy

$$V_{\text{pot}} = |F|^2 = \frac{\Lambda^{10}}{|\varphi|^6}. \tag{B}$$

The instanton contribution to the energy has slightly bent the flat bottom of the trough, raising slightly the coordinate origin (Fig. 1). In other words, the solution with a zero value of the scalar field in the vacuum is unstable, and the theory pushes itself away from the origin.

In the limit of massless matter, the vacuum state does not exist at all. Indeed, the larger the vacuum field $|\varphi|$, the lower the potential energy, and the true supersymmetric minimum $V_{\text{pot}} = 0$ is attained for infinite values of the condensates.

If we wish to have a normal theory with a vacuum state, we must "block the exits from the troughs" by somehow raising slightly the bottom of the trough for large values of the scalar field. In the model under consideration, stabilization is easily achieved by introducing a small mass term $mS^2|_F$ [see (2.3)]. Then instead of (2.14) we have (Fig. 2)

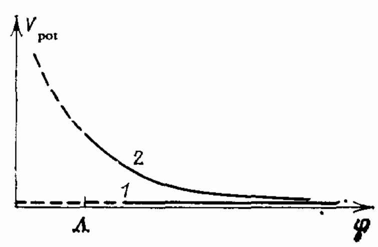

FIG. 1. SQCD with one massless flavor: the potential energy corresponding to the self-action of the scalar field along the bottom of the trough. Curve 1 is for perturbation theory, and curve 2 is with allowance for the effective potential induced by the instanton.

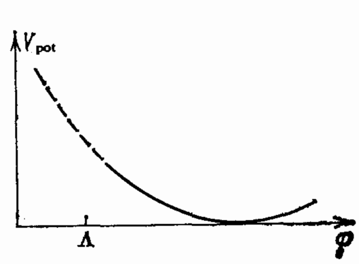

FIG. 2. Potential energy (see Fig. 1) after the introduction of the matter mass term ($m \ll \Lambda$).

$$\bar{F} = -m\varphi + \frac{\Lambda^5}{\varphi^3}, \qquad V_{\text{pot}} = \left| m\varphi - \frac{\Lambda^5}{\varphi^3} \right|^2. \tag{2.15}$$

If $m \ll \Lambda$, the vacuum value of the field φ, which minimizes the potential (2.15) ($V_{\text{pot}} = 0$, and the SUSY is not broken!), is large,

$$\varphi^2_{\text{vac}} = \pm \Lambda^{5/2} m^{-1/2} \gg \Lambda^2, \tag{2.16}$$

and this justifies the assertions formulated above—the spontaneous breakdown of the color symmetry and the validity of the method of effective potentials and of the entire calculation as a whole.

We note that the solution for the gauge-invariant quantity $\langle S^{\alpha f} S_{\alpha f} \rangle$ is doubly degenerate. The twofold degeneracy is in complete agreement with the fact that the Witten index in the SU(2) model is equal to two. This degeneracy reflects the spontaneous breakdown of the discrete symmetry Z_2 inherent in the model.

To conclude this subsection, we outline briefly an alternative method for the determination of the vacuum condensates (see Refs. 15, 18, 20, 29, and 30). If we assume that $\varphi_{\text{vac}} = v \neq 0$, the instanton calculus described in Sec. 3 makes it possible to fix the gluino condensate:

$$\langle \lambda^\alpha \lambda_\alpha \rangle = \frac{\Lambda^6}{v^2}. \tag{2.17}$$

The last relation is valid for any $v \gg \Lambda$. In order to find the values of $\langle \lambda\lambda \rangle$ and v^2 individually, we can make use of the supersymmetric Ward identity, the so-called anomalous Konishi relation[25]:

$$\bar{D}^2 \overline{S^{\alpha f}} e^V S^{\alpha f} = 4m S_{\alpha f} S^{\alpha f} + \frac{1}{2\pi^2} \operatorname{Tr} W^2. \tag{2.18}$$

It is obvious from Eq. (2.18) that in the supersymmetric vacuum

$$\langle \lambda^{a\alpha} \lambda^a_\alpha \rangle = \text{const} \cdot m v^2. \tag{2.19}$$

Combining (2.17) and (2.19), we come back to our previously established result for the vacuum scalar field, given in (2.16).

3. INSTANTONS

As we have already mentioned, we intend to discuss instanton effects. In this preparatory section, we shall give a brief description of instantons in supersymmetric theories. Of course, there is no possibility of giving a new account of the whole "instanton alphabet"—this is the subject of a separate review,[22] and we shall concentrate on the aspects which are important for what follows. Somewhat paradoxically, the instanton calculus in supersymmetric theories is simpler than, say, in quantum chromodynamics. In fact, all the nonzero modes, whose analysis requires the greatest expenditure of labor and time, cancel in the supersymmetric calculation, and the problem becomes a purely classical problem—the description of the family of solutions of classical equations of motion in Euclidean space-time. Therefore we hope that the material will be comprehensible even to readers who have not previously been specially concerned with instantons.

The original instanton of Belavin, Polyakov, Schwartz, and Tyupkin[21] is the solution of the duality equations for the gluon field, which we shall give here in a somewhat nonstandard notation:

$$A^{\gamma\delta}_{\alpha\dot{\alpha}} = -i\left[\delta^\gamma_\alpha (x-x_0)^\delta_{\dot{\alpha}} + \delta^\delta_\alpha (x-x_0)^\gamma_{\dot{\alpha}}\right] \frac{1}{(x-x_0)^2+\rho^2}. \tag{3.1}$$

We use a spinor notation for the vector indices; in particular, instead of the four-potential A_μ we have introduced the quantity $A_{\alpha\dot{\alpha}}$ with one dotted and one undotted SU(2) index, $A_{\alpha\dot{\alpha}} \equiv A_\mu (\sigma_\mu)_{\alpha\dot{\alpha}}$, $\sigma_\mu = (1, \boldsymbol{\sigma})$. What is more unusual, a spinor notation is used also for the color index a [$a = 1, 2, 3$ for $SU(2)_c$]. Instead of A^a we have introduced $A^{\gamma\delta}$ according to the rule

$$A^{\gamma\delta} = A^a \left(\frac{\tau^a}{2}\right)^\gamma_\rho \varepsilon^{\delta\rho}. \tag{3.2}$$

The intensity of the gluon field corresponding to the instanton is

$$G^{\gamma\delta}_{\alpha\beta} = -i\left(\delta^\gamma_\alpha \delta^\delta_\beta + \delta^\delta_\alpha \delta^\gamma_\beta\right) \frac{\rho^2}{[(x-x_0)^2+\rho^2]^2}, \qquad G^{\gamma\delta}_{\dot{\alpha}\dot{\beta}} = 0.$$

Further, x_0 and ρ are parameters, which are called the center of the instanton and its dimension (radius), respectively.

The action for the instanton is

$$S_{\text{inst}} = \frac{8\pi^2}{g^2}, \tag{3.3}$$

where g is a coupling constant, so that the instanton contribution to the physical amplitudes is proportional to $\exp(-8\pi^2/g^2)$ and we are dealing with effects which are exponentially small in the inverse coupling constant.

The parameters x_0 and ρ can be introduced directly by solving the classical equations. It is more important here that their existence follows from the symmetry of the classical Lagrangian. The Lagrangian of the gluon field is invariant with respect to the group of conformal transformations, whose generators we denote by $P_{\alpha\dot{\alpha}}$ (displacements in x space) $K_{\alpha\dot{\alpha}}$ (special conformal transformations), $M_{\alpha\beta}$ and $M_{\dot{\alpha}\dot{\beta}}$ (Lorentz rotations), and D (dilatations). We recall again that α and $\dot{\alpha}$ are spinor indices, and the relation with the possibly more customary tensor notation is given, for example, by $P_{\alpha\dot{\alpha}} = P_\mu (\sigma_\mu)_{\alpha\dot{\alpha}}$, where $\sigma_\mu = (1, \boldsymbol{\sigma})$.

The commutation relations between the generators of the group of conformal transformations can be found, for example, in the review of Ref. 5, and we shall not write them here.

In the most general form, the relation between the symmetry of the classical Lagrangian and the parameters of the type x_0 and ρ is as follows. We begin with some solution of the classical equations. Applying to it the symmetry transformations, we obtain a whole family of solutions, corresponding to the introduction of parameters of the type x_0

and ρ or collective coordinates. In our case, x_0 is obviously associated with displacements, while ρ is associated with dilatations or with a change of scale.

It is easy to see, however, that the number of parameters is smaller than the number of generators of the symmetry group. The reason for this is as follows. In general, there can exist a stationary subgroup of transformations whose action on some classical solution reduces to unity. For the standard instanton which we are discussing at the moment, the stationary subgroup contains the generators $M_{\dot{\alpha}\dot{\beta}}$ of Lorentz rotations—the instanton field has a definite chirality. The stationary subgroup also contains a linear combination of Lorentz rotations $M_{\alpha\beta}$ and global rotations $T_{\alpha\beta}$ in the color space, namely,

$$M_{\alpha\beta} + T_{\alpha\beta}, \tag{3.4}$$

where $T^a = \tau^a/2$.

Finally, the stationary subgroup contains $K_{\alpha\dot{\alpha}} + 2\rho^2 P_{\alpha\dot{\alpha}}$.

In general, we would have to introduce collective coordinates corresponding to global color rotations. However, in the final analysis we shall always consider quantities which are scalars in the ordinary and color spaces, for example, G^2, the square of the gluon field intensity. The generators $T_{\alpha\beta}$ do not act on such objects, and there is no need to include in the analysis the corresponding collective coordinates (the orientation of the instanton in the color space).

We turn now to *supersymmetric gluodynamics*. The symmetry group of the Lagrangian is then extended to the *superconformal group*, the generators enumerated above being supplemented with Q_α and $\bar{Q}_{\dot{\alpha}}$ (the generators of supertransformations) and S_α and $\bar{S}_{\dot{\alpha}}$ (the generators of superconformal transformations). For a more detailed discussion of the group, we refer again to the review of Ref. 5.

The generators $\bar{Q}_{\dot{\alpha}}$ and S_α complement the stationary subgroup—the instanton field is chiral, as we have already mentioned. (The action of S_α is associated with multiplication by $x_{\alpha\dot{\alpha}}$, and $S^\alpha x_{\alpha\dot{\alpha}}$ effectively has the same chirality as $\bar{Q}_{\dot{\alpha}}$.) As to the generators Q_α and $\bar{S}_{\dot{\alpha}}$, their action on the initial boson solution (3.1) is nontrivial and must generate new solutions of the classical equations, which this time are obviously fermion solutions. The classical Dirac equation for the gluino field in an external instanton field has the form

$$\hat{\mathscr{D}}\lambda = 0, \tag{3.5}$$

and its solutions are also called zero fermion modes because of the procedure for calculating the fermion determinant in an external boson field.

It is also easy to obtain the explicit form of these zero modes on the basis of symmetry arguments if we take into account the fact that for supertransformations

$$\delta\lambda_\alpha \sim G_{\alpha\beta}\varepsilon^\beta, \tag{3.6}$$

where ε^β is a parameter of the transformations and $G_{\alpha\beta}$ is the gluon field intensity (in the spinor notation). For superconformal transformations

$$\delta\lambda_\alpha \sim G_{\alpha\beta}x^{\beta\dot{\beta}}\bar{\varepsilon}_{\dot{\beta}}, \tag{3.7}$$

where $\bar{\varepsilon}_{\dot{\beta}}$ is a parameter. It is easy to see directly that after the substitution $G_{\alpha\beta} = (G_{\alpha\beta})_{\rm inst}$ the right-hand sides of (3.6) and (3.7) actually satisfy the Dirac equation (3.5); however, this is also obvious beforehand from the supersymmetry.

We mention that in the case of ordinary chromodynamics the zero modes are sought directly[26] as solutions of the Dirac equation. Here they have a simple geometrical meaning and are related to the symmetry of the classical Lagrangian.

It is clear that the supersymmetry will be realized completely only if we introduce, in addition to x_0 and ρ, fermion collective coordinates of the instanton. In order not to frighten the reader, we note at once that the fermion coordinates have the meaning of the coefficients of the zero modes in the general expansion of the fermion field with respect to the eigenfunctions of the Dirac operator. The "classical" spinor field is then obviously linear in this coordinate, and the calculation is simple. However, we cannot exclude the possibility that there are boson fields, say, quadratic in the fermion coordinates. Such fields correspond to repeated application of the supertransformations, and in what follows we will have explicit examples of this kind. Maintaining the supersymmetry explicitly at each step, we will be able to find at once the entire family of solutions which must be taken into account in the functional integral together with the original instanton. In themselves, the collective coordinates can be introduced by means of the following simple formal procedure. We introduce the generalized displacement operator

$$\mathscr{V}(x_0, \rho, \theta_0, \bar{\beta}) = e^{iPx_0}e^{-i\theta_0 Q}e^{-i\bar{S}\bar{\beta}}e^{iD\ln\rho}, \tag{3.8}$$

where θ_0 and $\bar{\beta}$ are Grassmann collective coordinates. Further, in order to determine the law of transformation of the collective coordinates under superdisplacements, we multiply the operator $\mathscr{V}(x_0,\rho,\theta_0,\bar{\beta})$ from the left by, say, $\exp(-i\overline{Q\varepsilon})$ and, using the commutation relations for the generators, we reduce the result of this multiplication to a redefinition of the initial parameters:

$$\exp(-i\bar{Q}\bar{\varepsilon})\,\mathscr{V}(x_0, \rho, \theta_0, \bar{\beta}) = \mathscr{V}(x_0', \rho', \theta_0', \bar{\beta}')\,F, \tag{3.9}$$

where F is some transformation from the stationary group. We note that since the expansion of the exponential function with respect to the Grassmann numbers contains a finite number of terms, the transformations of the coordinates also have an algebraic character.

Thus, we can establish the law of transformation of the collective coordinates under superdisplacements with the parameters ε and $\bar{\varepsilon}$:

$$\delta(x_0)_{\alpha\dot{\alpha}} = -4i(\theta_0)_\alpha\bar{\varepsilon}_{\dot{\alpha}}, \quad \delta\rho^2 = -4i(\bar{\varepsilon}\bar{\beta})\rho^2, \tag{3.10}$$
$$\delta(\theta_0)_\alpha = \varepsilon_\alpha, \quad \delta\bar{\beta} = -4i\bar{\beta}_{\dot{\alpha}}(\bar{\varepsilon}\bar{\beta}).$$

The only thing that is used in the derivation is the commutation relations of the generators of the group of superconformal transformations.

It is somewhat more complicated to find the explicit dependence of the superfields associated with the instanton from the collective coordinates introduced in this way. To solve this problem, we must above all bear in mind that the superfield as a function of the coordinates x_L and θ and of the collective coordinates is an invariant with respect to the simultaneous supertransformation of both of them. It is therefore convenient as a preliminary step to construct the invariants from (x_L, θ) and $(x_0, \rho, \theta_0, \bar{\beta})$. Similarly, if we follow the Poincaré invariance, the corresponding combination is the difference $(x - x_0)$, which is obviously invariant with respect to the simultaneous displacement of x and x_0. It is equally obvious that the instanton field depends on $(x - x_0)$.

It is easy to see that the combination $\tilde{\theta}_\alpha$, where

$$\tilde{\theta}_\alpha = (\theta - \theta_0)_\alpha + (x - x_0)_{\alpha\dot{\alpha}} \bar{\beta}^{\dot{\alpha}}, \tag{3.11}$$

transforms relatively simply,

$$\delta\tilde{\theta}_\alpha = -4i(\bar{\varepsilon}\bar{\beta})\tilde{\theta}_\alpha, \tag{3.12}$$

and the quantity

$$\frac{\tilde{\theta}_\alpha}{\rho^2}$$

is an invariant of the supertransformations.

It is now clear that the instanton superfield W^2 of the intensities has the form

$$\mathrm{Tr}\, W^2 = \mathrm{const} \cdot \frac{\tilde{\theta}^2 \rho^4}{[(x_L - x_0)^2 + \rho^2]^4}. \tag{3.13}$$

In fact, let us first consider the case of collective Grassmann coordinates equal to zero: $\theta_0 = \bar{\beta} = 0$. Then we are dealing with the purely boson solution (3.1). In the superfield W^2, the boson term appears as the coefficient of θ^2 [see (2.4)]. From this we know the result for W^2 when $\theta_0 = \bar{\beta} = 0$. All that is necessary to obtain the complete result (3.13) is to extend θ/ρ^2 to the invariant $\tilde{\theta}/\rho^2$. The skeptically inclined reader can see, for example, that the expression (3.13) correctly reproduces the zero fermion modes which appear in the coefficients of the zeroth and first powers of θ. (We note that the transformations of $x_L - x_0$ are discussed below in a more general context.)

For explicit calculations, it is also necessary to know the measure of the integration with respect to the collective coordinates:

$$d\mu(\rho, x_0, \theta_0, \bar{\beta})\, e^{-S} = C\Lambda^6\, d^4x_0\, d\rho^2\, d^2\theta_0\, d^2\bar{\beta}. \tag{3.14}$$

The measure of the integration is of course invariant with respect to the supersymmetry transformations.

Suppose that we now add *matter fields with one flavor*. The instanton field (3.1) remains as before. We can also find the solution of the equations for the scalar field φ, $\mathscr{D}^2\varphi = 0$, in a given external vector field:

$$\varphi^\alpha_{\dot{f}} = \bar{\varphi}^\alpha_{\dot{f}} = x^\alpha_{\dot{f}} \frac{v}{\sqrt{x^2 + \rho^2}}, \tag{3.15}$$

where α is the color index, $\dot{f}$ is the flavor index, and v is the vacuum expectation value of the field φ (see Sec. 2).

It should be noted that the fields (3.1) and (3.15) are not exact solutions of the classical equations, since in the equations for the vector field in the presence of a scalar field there is a source which is not taken into account. At large distances, this source is equivalent to a vector-field mass and leads to an exponential decrease of the field A_μ. Accordingly, there is an addition to the instanton action, which is now given by[26]

$$S_{\text{bos}} = \frac{8\pi^2}{g^2} + 4\pi^2 v^2 \rho^2. \tag{3.16}$$

It can be seen that for $\rho \neq 0$ we are strictly speaking not dealing with an extremal of the action. Nevertheless, the instanton contribution can be important—a detailed discussion of this problem can be found in the original papers.[20,26] We shall be interested in the generalization to the case of supersymmetry.

The novelty of the situation with regard to the scalar field consists in the fact that the field (3.15) is noninvariant with respect to the displacements generated by $(\overline{Q\varepsilon})$, as can be seen at once if we make the transformation (2.2) of the coordinates. Therefore we cannot now include $\bar{Q}_{\dot{\alpha}}$ in the stationary subgroup, but we must introduce a new collective coordinate $\bar{\theta}_0$.

The transformation law for the new set of collective coordinates can be found as usual by introducing the operator

$$\mathscr{V}(x_0, \rho, \theta_0, \bar{\theta}_0, \bar{\beta}) = e^{iPx_0} e^{-iQ\theta_0} e^{-i\bar{S}\bar{\beta}} e^{-i\bar{Q}\bar{\theta}_0} e^{iD\ln\rho} \tag{3.17}$$

and multiplying it by $\exp(-i\,\overline{Q\varepsilon})$ or $\exp(-iQ\varepsilon)$. Without dwelling on the details of the calculations, we indicate that

$$\delta\bar{\theta}_0 = \bar{\varepsilon} - 4i\bar{\beta}(\bar{\theta}_0\bar{\varepsilon}),$$

and the transformations of the other collective coordinates were found earlier [see (3.10)].

Further, proceeding with the construction of the invariants, we note that, apart from $\tilde{\theta}$ introduced in (3.11), there is a simple transformation law for the quantity $\tilde{x}$, where

$$\tilde{x}_{\alpha\dot{\alpha}} = (x_L - x_0)_{\alpha\dot{\alpha}} + 4i\tilde{\theta}_\alpha(\bar{\theta}_0)_{\dot{\alpha}}.$$

In fact,

$$\delta\tilde{x}_{\alpha\dot{\alpha}} = 4i\tilde{x}_{\alpha\dot{\gamma}}\bar{\beta}^{\dot{\gamma}}\bar{\varepsilon}_{\dot{\alpha}}, \quad \delta\tilde{x}^2 = -4i(\bar{\varepsilon}\bar{\beta})\tilde{x}^2. \tag{3.18}$$

The list of superinvariants takes the form

$$\frac{\tilde{x}^2}{\rho^2}, \quad \frac{\tilde{\theta}_\alpha}{\rho^2}, \quad \rho^2[1 + 4i(\bar{\beta}\bar{\theta}_0)]. \tag{3.19}$$

More precisely, our discussion refers to *chiral* fields. An analogous technique can be developed for antichiral fields and for superfields of a general form.

From the expressions (3.19) it is clear how to introduce collective coordinates into the chiral superfield. If for $\theta_0 = \bar{\theta}_0 = \bar{\beta} = 0$ the field depends on x_L and θ, in the general case we must make the substitution

$$\frac{(x_L - x_0)^2}{\rho^2} \to \frac{\tilde{x}^2}{\rho^2}, \quad \frac{\theta_\alpha}{\rho^2} \to \frac{\tilde{\theta}_\alpha}{\rho^2}. \tag{3.20}$$

In particular, for the square of the superfield $S^{\alpha f}$ this prescription leads to the expression

$$(S^2)_{\text{inst}} \equiv (S^{\alpha\dot{f}} S_{\alpha\dot{f}})_{\text{inst}} = 2\frac{v^2\tilde{x}^2}{\tilde{x}^2 + \rho^2}, \tag{3.21}$$

where α is the color index and f is the index of global SU(2).

The expression (3.13) for W^2 remains as before, since the difference between $x_L - x_0$ and $\tilde{x}$ in this case is unimportant because of the factor $\bar{\theta}^2$, which is equivalent for the Grassmann numbers $\delta^2(\bar{\theta})$.

In order to elucidate the physical meaning of $\bar{\theta}_0$, we expand (3.21) with respect to $\bar{\theta}_0$ for $\theta_0 = \bar{\beta} = 0$, $x_0 = 0$ in the form[20]

$$(S^2)_{\text{inst}} = 2v^2\left[\frac{x^2}{x^2+\rho^2} + 4i\theta^\alpha x_{\alpha\dot{\alpha}}\bar{\theta}_0^{\dot{\alpha}}\frac{\rho^2}{(x^2+\rho^2)^2} - 8\frac{\rho^4\theta^2\bar{\theta}_0^2}{(x^2+\rho^2)^3}\right] \tag{3.22}$$

and, comparing with the general expression

$$S^2 = \varphi^{\alpha\dot{f}}\varphi_{\alpha\dot{f}} + 2\sqrt{2}(\theta\psi^{\alpha\dot{f}})\varphi_{\alpha\dot{f}} + [2\varphi_{\alpha\dot{f}}F^{\alpha\dot{f}} - (\psi^{\alpha\dot{f}}\psi_{\alpha\dot{f}})]\theta^2$$

observe that the spinor matter field is

$$\psi_\gamma^{\alpha\dot{f}} = 2\sqrt{2}\, iv(\bar{\theta}_0)^{\dot{f}}\delta_\gamma^\alpha\frac{\rho}{(x^2+\rho^2)^{3/2}} \tag{3.23}$$

(γ is the Lorentz index, α is the color index, and $\dot{f}$ is the index of global SU(2); $F^{\alpha\dot{f}} = 0$ according to the equations of motion).

Apart from a factor, the spinor field (3.23) is identical to the ordinary fermion zero mode.[25] The Grassmann parameter $\bar{\theta}_0$ (more precisely, the two parameters $\bar{\theta}_0^{\dot{1}}$ and $\bar{\theta}_0^{\dot{2}}$) is proportional to the coefficients of the expansion with respect to these zero modes.

Thus, supersymmetry gives a geometrical meaning to the zero mode of matter.

Now that we have introduced the collective coordinates and constructed superinvariant expressions for the instanton superfields, we come to the most nontrivial aspect of models with matter, one of the key aspects of the formalism which has been developed—the measure of the integration with respect to the collective coordinates. In the purely boson case, this problem—the measure of the integration in the theory with scalar particles and spontaneous breakdown of the gauge symmetry—was solved in the pioneering work of Ref. 26. In the SU(2) model with two doublets of Higgs scalars, the result[26] is proportional (in the Bogomol'nyĭ limit) to

$$(\Lambda\rho)^b\frac{d\rho}{\rho^5}d^4x_0 e^{-4\pi^2v^2\rho^2}, \tag{3.24}$$

where b is the first coefficient of the Gell-Mann–Low function, and $v^2 = \langle\varphi_f^2\rangle$, $f = 1, 2$. The exponential function $\exp(-4\pi^2v^2\rho^2)$ reflects the fact that in the classical approximation the instanton action already depends on one of the collective coordinates, ρ. This dependence is in turn explained as follows: the instanton in the presence of scalar fields with nonzero v is not an exact extremal but gives only an approximate solution of the classical equations, so that the complete classical action varies slowly as a function of ρ. In other words, in the integration in the functional space we use the saddle-point method in all directions along which the action varies rapidly, and we study explicitly the dynamics of one of the directions, on which the action depends weakly.

At first sight, it appears that the only change introduced into 't Hooft's result by the supersymmetrization reduces to the addition of differentials of the fermion collective coordinates, $d^2\theta_0 d^2\bar{\theta}_0 d^2\bar{\beta}$. It is easy to see, however, that the measure of the integration becomes noninvariant with respect to the supertransformations. More precisely, using the transformation laws (3.10) and (3.17), we find that it is the exponential function $\exp(-4\pi^2v^2\rho^2)$ which spoils the superinvariance.

Fortunately, this unpleasant feature does not in any way indicate an incurable defect of the formalism, but it merely reveals an error in our naive discussion. The point is that in supersymmetric theories the concept of the radius is generalized, and the instanton amplitudes contain a quantity which we have called the invariant radius. In SQCD with one flavor,

$$\rho_{\text{inv}}^2 = \rho^2[1 + 4i(\bar{\theta}_0\bar{\beta})] \tag{3.25}$$

[cf. (3.19)]. In terms of ρ_{inv}^2, the measure of the integration with respect to the collective coordinates reduces to[20]

$$d\mu e^{-S} = \text{const}\cdot\frac{\Lambda^5}{v^2}d^4x_0 d^2\theta_0 d^2\bar{\theta}_0 d^2\bar{\beta}\frac{d\rho^2}{\rho^2}e^{-4\pi^2v^2\rho_{\text{inv}}^2}, \tag{3.26}$$

in which the numerical constant, which is nonzero, can in principle also be readily calculated. The difference between $\exp(-4\pi^2v^2\rho^2)$ and $\exp(-4\pi^2v^2\rho_{\text{inv}}^2)$ is related to the fact that the SQCD Lagrangian contains Yukawa vertices of the type $g\bar{\varphi}\lambda\psi$. If for λ and ψ we substitute the zero modes of the gluinos and matter, and for $\bar{\varphi}$ we substitute the solution (3.15), it can be verified that the corresponding contribution to the action is actually contained in ρ_{inv}^2. For further details, see Ref. 20.

Using the elements of the instanton calculus outlined above, it is easy to calculate explicitly the effective superpotential which arises in SQCD with one flavor in the single-instanton approximation. It is convenient to formulate the problem as the problem of finding the energy associated with an instanton having the center (x_0, θ_0) and "living" in the external field $S^{\alpha f}(x_L, \theta)$.

For a constant external field which does not depend on x and θ at all ($S^{\alpha f}S_{\alpha f} = 2v^2$), the result is in fact given in (3.26). All that we must do is to generalize it to the case of weakly varying external fields. Since we are interested in the low-energy limit, the derivatives with respect to x and θ of the external (super)field can be consistently neglected. Then the effective superpotential will obviously depend only on $S^{\alpha f}(x_0, \theta_0)S_{\alpha f}(x_0, \theta_0)$, and to find this dependence it is sufficient to make the substitution

$$2v^2 \to S^{\alpha f}(x_0, \theta_0)S_{\alpha f}(x_0, \theta_0) \tag{3.27}$$

in Eq. (3.26).

Expanding the exponential function in (3.26) with respect to $(\bar{\theta}_0\bar{\beta})$ and performing the integrations with respect to $d\rho^2$, $d^2\bar{\theta}_0$, and $d^2\bar{\beta}$, we arrive at the following effective action:

$$A_{\text{eff}} = \text{const}\cdot\int d^4x_0 d^2\theta_0\frac{\Lambda^5}{S^{\alpha f}(x_0, \theta_0)S_{\alpha f}(x_0, \theta_0)}. \tag{3.28}$$

This same expression was obtained in Sec. 2 on the basis of general arguments. A concise diagrammatic representation of the problem of the effective superpotential is given in Fig. 3.

Some other examples of instanton calculations, both in this simplest model and in more complicated cases (for ex-

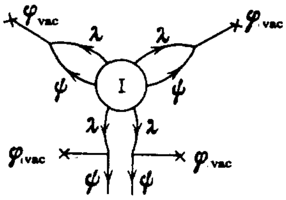

FIG. 3. Diagram for the effective superpotential induced by the instanton in the SU(2) model with one flavor.

ample, SQCD with two flavors), are given in the original paper of Ref. 20.

4. CLASSIFICATION OF THE MODELS. SPONTANEOUS BREAKDOWN OF SUSY

We hope that an acquaintance with the simplest example, supersymmetric quantum chromodynamics, has given the reader a general idea of how dynamical symmetry breaking takes place as a result of instantons and how the weak-coupling regime arises. In the model discussed in Sec. 2, the color symmetry is spontaneously broken, while SUSY remains unbroken.

This circumstance—the survival of supersymmetry—is a consequence of the index theorem.[24] In theories with a nonzero Witten index (and in the studied model with the gauge group SU(2) and nonchiral matter the Witten index is equal to two), spontaneous breakdown of the supersymmetry is impossible.

One of the most striking aspects of the studied phenomenon is its universality. In almost any model with matter, there is a classical degeneracy of the vacuum in the form of troughs, and together with them there arises a potential possibility for destabilization of the "trivial" vacuum corresponding to zero values of the scalar fields—a destabilization triggered by instanton effects.

The schemes according to which large vacuum condensates can develop are extremely diverse. By introducing some kind of multiplets of matter in a definite set, we realize a specific scheme from the very rich spectrum of dynamical scenarios which include (as particular cases) the spontaneous breakdown of color symmetry discussed in Sec. 2 and the spontaneous breakdown of SUSY.

A systematic study of the various versions was undertaken in Ref. 19. We shall list here the basic, most typical situations, and then dwell on the simplest model which involves spontaneous breakdown of the supersymmetry in the weak-coupling regime.

a) Catalog of dynamical scenarios

First of all, we describe the general structure of the supersymmetric models to which the present review is devoted. The gauge sector includes gluons and gluinos, which transform according to the adjoint representation of the gauge group G. The following groups have been investigated in detail in the literature:

$$G = \mathrm{SU}(N), \quad G = \mathrm{SO}(N).$$

Some examples with a gauge group in the form of a direct product of the type $G = \mathrm{SU}(N) \times \mathrm{SU}(M)$ or $G = \mathrm{SU}(N) \times \mathrm{U}(1)$ have also been analyzed.

As regards the structure of the "material" sector, by combining multiplets of matter in various representations and in various numbers, it is possible to obtain a great number of variants, which can be divided into two fundamentally different classes: nonchiral and chiral matter.

In the first case, a mass term can be attributed to all the matter fields without breaking the gauge symmetry (and SUSY). Supersymmetric QCD belongs to this class, and, as we have already noted, spontaneous breakdown of SUSY cannot occur in these models.

Chiral matter is matter for which the introduction of a mass term is forbidden by gauge invariance. The Witten index for chiral matter has not been calculated, so that the possibility of a zero value of the index has not been excluded, and searches for spontaneous breakdown of supersymmetry have had to be restricted in advance to this class of models.

We note that whereas nonchiral matter admits a very large arbitrariness in the choice of the multiplets of fields, for chiral matter the arbitrariness is much smaller, since there exists the very stringent requirement of cancellation of the "intrinsic" axial anomalies in the theory. For example, for the gauge group SU(5) the theory is anomaly-free only if the material sector contains *the same number* of quintets and antidecuplets of chiral superfields.

After fixing the set of multiplets of matter, the next thing to do is to investigate the problem of troughs. This investigation in turn is divided into several stages, which are conveniently formulated as an algorithm.

Step 1. We consider the classical Lagrangian with the Yukawa and mass terms switched off in the superpotential. (In models with nonchiral matter the superpotential can in general contain both types of terms, while in models with chiral matter only Yukawa terms are admissible.) If the Yukawa and mass terms are neglected, there remain in the Lagrangian only the gauge interactions

$$\int d^2\theta W^2, \quad \int d^2\theta\, d^2\bar{\theta} \bar{S} e^{V} S,$$

where S is a generalized notation for the matter superfields.

Step 2. We write the expression for the D terms:

$$D^a = -\sum_f g^2 \bar{\varphi}^f T^a \varphi^f; \tag{4.1}$$

here φ^f are scalar fields, which are the lower components of the superfields S^f, f is the flavor index, the summation in (4.1) being over all the multiplets of matter, and T^a are the generators of the gauge group in the corresponding representation. Further, two variants are logically possible.

The system of equations $D^a = 0$ has only the trivial solution with zero values of all the scalar fields. In other words, there are no troughs. An example of this situation is provided by the SU(5) model with one quintet and one antidecuplet of matter.[15] In this case, the gauge symmetry remains unbroken, a strong-coupling regime is realized, and the problem of spontaneous (nonperturbative) breakdown of SUSY can be investigated only by means of indirect methods, which will not be discussed in the present review. We note in passing that the arguments of Refs. 14 and 15 favor spontaneous breakdown of supersymmetry in the abovementioned model with one quintet and one (anti)decuplet.

We shall concentrate below on another, richer variant, which potentially leads to spontaneous breakdown of the color symmetry and to a weak-coupling regime. We shall assume that the system of equations $D^a = 0$ has nontrivial solutions in the space of the scalar fields. If we represent the energy of the self-action of the scalar fields

$$V_{\text{pot}} = \frac{1}{2g^2} D^a D^a = \frac{1}{2} g^2 \sum_a \Big(\sum_f \overline{\varphi}^f T^a \varphi^f \Big)^2 \tag{4.2}$$

in the form of a profile, we can in this case pictorially represent the system of equations as a network of "ravines" or "valleys," whose bottoms in the approximation under consideration are completely flat and correspond to the zero level $V_{\text{pot}} = 0$, and whose walls have a steepness determined by the gauge coupling constant g. (In fact, the solutions of the equations $D^a = 0$ give a set of points lying at the bottom of the ravines.) In the simplest example of SQCD with one flavor (see Sec. 2), a ravine extended as a straight line from the coordinate origin $\varphi_1 = \varphi_2 = 0$ to infinity; in more complicated models, ravines, starting from the origin, bend and branch out, forming a ramified network analogous to what exists in real mountain ranges.

As far as we know, a general method of solving the equations $D^a = 0$ has not been developed. Certain techniques which facilitate the determination of the valleys in frequently encountered cases are described in Refs. 19 and 26. We stress that the task of finding the valleys, particularly in models with chiral matter, is a rather complex technical problem, and the solutions frequently have a most whimsical appearance.

In order to avoid unsubstantiated statements, we reproduce here one of the families of valleys which we found in the SU(5) model with two quintets $V^{(1)\alpha}$ and $V^{(2)\alpha}$ and two (anti)decuplets $X^{(1)}_{\alpha\beta}$ and $X^{(2)}_{\alpha\beta}$ (this model was discussed in Refs. 15 and 19).

It is most convenient to represent the solution in a polar parametrization in which it depends on three numbers—a radius r and two angles α and θ. In this parametrization, the following values of the scalar fields from the quintets and antidecuplets correspond to a vacuum trough:

$$V^{(1)} = \begin{bmatrix} a_1 \\ 0 \\ 0 \\ a_4 \\ 0 \end{bmatrix}, \quad V^{(2)} = \begin{bmatrix} 0 \\ a_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{4.3a}$$

$$X^{(1)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & s & 0 \\ 0 & 0 & b & 0 & 0 \\ 0 & -b & 0 & 0 & f \\ -s & 0 & 0 & 0 & 0 \\ 0 & 0 & -f & 0 & 0 \end{bmatrix}, \tag{4.3b}$$

$$X^{(2)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & d & 0 & 0 & g \\ -d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h \\ -g & 0 & 0 & -h & 0 \end{bmatrix}, \tag{4.3c}$$

where

$$a_1 = r\cos\theta, \quad a_4 = r\cos\alpha \cdot \operatorname{tg}\theta, \quad a_2 = r \operatorname{tg}\theta \sqrt{\frac{\cos^2\theta - \cos^2\alpha}{\sin^2\theta + \cos^2\alpha}}, \tag{4.4a}$$

$$\left.\begin{aligned} s &= \frac{r\cos\alpha}{\cos\theta\sqrt{\sin^2\theta + \cos^2\alpha}}, \\ b &= r\sqrt{\sin^2\theta + \cos^2\alpha}, \\ f &= r\frac{\cos\alpha}{\cos\theta}\sqrt{\frac{\cos^2\theta - \cos^2\alpha}{\sin^2\theta + \cos^2\alpha}}, \\ d &= \frac{r}{\cos\theta}\sqrt{\cos^2\theta - \cos^2\alpha}, \\ g &= r\cos\alpha, \quad h = r\sin\theta. \end{aligned}\right\} \tag{4.4b}$$

By varying r in the interval $(0, \infty)$ with fixed α and θ, we move along the bottom of the trough, from the coordinate origin in the space of the scalar fields to infinity.

Step 3. Each point at the bottom of the trough corresponds to $V_{\text{pot}} = 0$. Thus, there is an infinite degeneracy of the vacuum (which is preserved also in any order of perturbation theory). Further, the orderly structure of our catalog is broken, since we are again faced with the need to set forth several alternative possibilities.

An effective superpotential which removes the degeneracy of the vacuum energy may or may not occur in the single-instanton approximation. An example of the situation in which the degeneracy is not removed and the bottom of the trough remains flat is the gauge theory with extended ($N = 2$) supersymmetry[28] or SQCD with $N_f \geqslant N_c$ and with strictly massless quarks[16] (N_f and N_c are the numbers of flavors and colors, respectively). Further analysis is required in this case, since the vacuum state of the theory is not determined uniquely. In fact, the same Lagrangian describes a set of theories, which differ by the values of the condensates and by completely different physics in different phases (for example, the phase with unbroken gauge symmetry and confinement and the phase with broken color symmetry). A more detailed discussion can be found in Ref. 28; here we merely note that, apart from the dependence on which of the phases is realized, such models necessarily contain a strictly massless scalar meson (dilaton). According to the supersymmetry, this implies the presence of a whole supermultiplet of strictly massless particles.

From the point of view of the present review, another situation is of greater interest—the instanton generates a nonzero superpotential. In all the models with an instanton superpotential analyzed in the literature, the effective superpotential leads to the expulsion of the theory from the coordinate origin (see Fig. 1), which seems natural. In fact, the larger the value of the scalar field, the more strongly suppressed is the instanton contribution to the vacuum energy, and as $\langle \varphi \rangle$ tends to infinity along the bottom of the trough the instantons are no longer effective and $E_{\text{vac}} \to 0$.[1)] If no further measures are taken, the theory is unprotected against the formation of infinitely large condensates, since this is advantageous energetically. In this variant, there is no vacuum state at all.

Thus, there arises the problem of stabilization of the theory at large φ by means of the introduction of mass or Yukawa terms in the superpotential at the classical level. In other words, if the theory is to have a true vacuum state, it is necessary to close the exits from all the troughs at large values of the scalar fields by raising their bottoms slightly. In theories with nonchiral matter, the problem is readily solved by adding mass terms (see the discussion of SQCD with one flavor in Sec. 2). In the case of chiral matter, we have at our disposal only Yukawa terms. If we consider only renormalizable theories, the set of Yukawa terms (cubic in the matter superfields) is restricted by symmetry considerations, and stabilization of the theory for large values of $\langle \varphi \rangle$ is not always possible.

As an example, we mention the SU(N) model (N is even, $N>6$), in which there is one supermultiplet of matter, which transforms with respect to color as an antisymmetric tensor $X_{[ij]}$, and $N-4$ supermultiplets in the fundamental representation, V^*.[19] The general form of the Yukawa classical superpotential is

$$W_{\rm cl} = h^{(fg)} X_{[ij]} V^i_{(f)} V^j_{(g)}, \quad (4.5)$$

where $f, g = 1,2,\ldots,(N-4)$ are flavor indices, and the matrix h of Yukawa constants satisfies the conditions $h^T, = h$, $|h| \ll g$. The instanton generates a superpotential which is repulsive from the coordinate origin, and the interaction (4.5) does not guarantee stabilization in all directions. As a result, this model evidently has no vacuum state.

Our catalog is completed by the most interesting dynamical scheme with spontaneous breakdown of supersymmetry in the weak-coupling regime. If in a model with chiral matter there are troughs, if an instanton generates a repulsive superpotential, if the theory can be completely stabilized by Yukawa terms in such a way that the escape to $\langle \varphi \rangle = \infty$ is impossible—if all these "ifs" are satisfied, then in such a model dynamical breakdown of SUSY will almost certainly occur. It is here, along this path, that we must apparently concentrate our efforts in the search for a realistic theory of quarks and leptons, provided, of course, that the theory is indeed constructed on the basis of supersymmetry. Historically, the first "toy" model with spontaneous breakdown of SUSY in the (controllable) weak-coupling regime was the SU(5) model with two quintets and two antidecuplets.[15,14]

The conditions enumerated above are necessary but in general not sufficient.

Two criteria are known from the literature,[18,19] each of which guarantees spontaneous breakdown of SUSY.

Criterion 1. Suppose that the Yukawa terms in the superpotential, introduced for the sake of stabilization at large φ, do not contain any matter supermultiplet S (or a linearly independent combination of supermultiplets). In this case, the gluino condensate is an order parameter—the nonzero vacuum expectation value

$$\langle \lambda\lambda \rangle \neq 0$$

implies spontaneous breakdown of the supersymmetry. In fact, if the supermultiplet S appears in the Lagrangian only in the form $\bar{S}e^V S|_D$, then the anomalous Konishi identity[25] takes the form

$$\bar{D}^2 (\bar{S}e^V S) = (\text{numerical constant}) \times W^2, \quad (4.6)$$

where $\bar{D}$ is the spinor derivative. If this is the case, then the vacuum expectation value of W^2 is equivalent to the vacuum expectation value of the operator $\bar{D}^2 \bar{S} e^V S$. On the other hand, for unbroken supersymmetry it is obvious that $\langle \bar{D}^2 \bar{S} e^V S \rangle = 0$.

Criterion 2. Suppose that we are considering a theory in which either troughs are completely absent or they are "blocked" by Yukawa or mass terms in the superpotential (in such a way that their bottoms are raised slightly by an increase of the scalar fields). If in such a theory some *exact continuous global* symmetry (for example, axial symmetry) is spontaneously broken, then SUSY is also spontaneously broken.

We shall outline the proof[31] in its general features, omitting certain subtle details. (A similar discussion in the particular case of extended ($N-2$) supersymmetry is contained in Ref. 28.) If a continuous global invariance is spontaneously broken, there is a massless Goldstone boson π. Suppose that the supersymmetry of the theory is not broken. Then π must be accompanied by massless superpartners, in particular, a scalar particle σ with spin 0. Since the field π, being a Goldstone field, appears in the Lagrangian with a zero potential $V_{\rm pot}(\pi) = 0$, the potential for the field σ must also vanish and, as a consequence, the vacuum expectation value $\langle \sigma \rangle$ is not fixed. In other words, the field σ appears in the role of a dilaton which connects the various vacua with the same—zero—energy.[2)] However, this conclusion contradicts the initial assumption that there are no absolutely flat directions in the Lagrangian. The only possibility of getting rid of the contradiction is to conclude that the symmetry is spontaneously broken.

We note that the two criteria given above are not completely independent. In fact, if some superfield does not enter into the classical superpotential (see Criterion 1), then there exists an axial current—a linear combination of the matter current S and the R current—which is strictly conserved. Further, the operator $\lambda^{a\alpha} \lambda^a_\alpha$ is obviously noninvariant with respect to the transformations generated by this current. Therefore the $\lambda\lambda$ condensation automatically implies spontaneous breakdown of the corresponding axial symmetry.

In conclusion, we present Table I which summarizes the situation for a number of models considered in the literature (see Ref. 19, which also gives a list of earlier references). The next subsection is a mini-review of the *simplest* model in which there is spontaneous breakdown of supersymmetry in the weak-coupling regime.

b) Dynamical breakdown of SUSY: the SU(3)×SU(2) model[19]

If we set ourselves the task of constructing a model with chiral matter and troughs, the result may be suggested by the scheme of Glashow, Weinberg, and Salam. Since only the pedagogical aspect is important for us here, we shall go even further, following Ref. 19, and simplify this scheme: we shall retain only one generation of matter, eliminate the hypercharge, and discard the $\bar{e}_L$ particle, which is a singlet with respect to both color and the weak isospin and interacts only with the boson which gauges the hypercharge.

Proceeding in this way, we in fact obtain SQCD with three colors and two flavors, u and d. In other words, the matter sector includes the following chiral (left-handed) superfields:

$$\{u^\alpha, d^\alpha\} \equiv Q^{\alpha f} \quad (\alpha = 1, 2, 3;\ f = 1, 2), \quad \bar{u}_\alpha, \bar{d}_\alpha. \quad (4.7)$$

TABLE I. Dynamical scenarios realized in certain models with matter discussed in the literature [N_c is the number of colors, N_f is the number of flavors, G is the gauge group, m is the mass (of the supermultiplet of matter), and Λ is the scale parameter which determines the variation of the gauge coupling constant].

1) Model	SQCD	SQCD	SQCD	SQCD
Matter sector	$m \equiv 0$	$N_f < N_c - 1$, $m_1 = m_2 = \ldots = m_{N_f} \ll \Lambda$	$N_f = N_c - 1$, $m_i \ll \Lambda$, $i = 1,\ldots,N_f$	$N_f > N_c - 1$, $m_1 = \ldots = m_{N_f}$
Scheme	No vacuum state	$SU(N_c)$ is broken down to $SU(N_c - N_f)$, and SUSY is not broken	The color symmetry is completely broken (weak-coupling regime), and SUSY is not broken	An instanton superpotential is not generated, and the color symmetry and SUSY are not broken
2) Model	SQCD	$G = SU(4)$	$G = SU(N)$, N is even, $N > 6$	$G = SU(5)$
Matter sector	$N_f > N_c - 1$, $m_1 \ll m_2 \ll \ldots \ll m_{N_f} \ll \Lambda$	One antisymmetric tensor $X_{[ij]}$, $m \neq 0$	One antisymmetric tensor $X_{[ij]}$ and $N - 4$ (anti)multiplets in the fundamental representation	One quintet and one antidecuplet
Scheme	The color symmetry is completely broken (weak-coupling regime), and SUSY is not broken	$SU(4) = O(6)$ is broken down to $Sp(4) = O(5)$, and SUSY is not broken	No vacuum state	The color symmetry is not broken (strong-coupling regime), and SUSY is apparently broken
3) Model	$G = SU(N)$, N is odd, $N > 7$	$G = SU(N)$	$G = SU(5)$	$G = SU(3) \times SU(2)$
Matter sector	One antisymmetric tensor $X_{[ij]}$ and $N - 4$ (anti)multiplets in the fundamental representation	One symmetric tensor S_{ij} and $N + 4$ (anti)multiplets in the fundamental representation	Two quintets and two antidecuplets	u, d quarks and ν, e_L
Scheme	No vacuum state	An instanton superpotential is not generated, and the vacuum degeneracy is not removed	The color symmetry is broken (weak-coupling regime), and SUSY is spontaneously broken	The color symmetry is completely broken, and SUSY is spontaneously broken (Sec. 4b)

Further, the flavor interaction of the left-handed particles (u^α, d^α) (but not the antiparticles $\bar{u}_\alpha, \bar{d}_\alpha$!) is gauged. The corresponding gauge bosons W^+, W^-, and W^0 and their superpartners obviously transform according to the adjoint representation of the group SU(2) of the weak isospin. It is here that the asymmetry appears between right- and left-handed matter.

Finally, since an odd number of left-handed doublets (u^α, d^α), $\alpha = 1,2,3$, in the group SU(2) is forbidden by the anomaly,[32] it is necessary to add one more doublet of chiral superfields, a lepton doublet

$$L^f = \{\nu, e\}. \qquad (4.8)$$

We obtain altogether 14 left-handed Weyl spinors plus their superpartners; the fifteenth left-handed spinor $\bar{e}_L$ is excluded from the discussion, as we have already mentioned.

It is easy to verify that there exists no mass term which is invariant with respect to the gauge group $G = SU(3) \times SU(2)$. If we confine ourselves to the class of renormalizable theories, the only admissible interaction, apart from a (super)gauge interaction, is given by Yukawa terms in the classical superpotential, which can be chosen in the form

$$W_{cl} = hQ^{\alpha f}\bar{d}_\alpha L^{f'}\varepsilon_{ff'} \qquad (f, f' = 1, 2), \qquad (4.9)$$

where h is the Yukawa constant. We call attention to the fact that the field $\bar{u}_\alpha$ does not appear in W_{cl}; we shall make use of this fact later.

As in the usual scheme of Glashow, Weinberg, and Salam, we assume that the SU(3) gauge constant g_3 is much greater than the SU(2) gauge constant g_2 and, in addition, that

$$h \ll g_2 \ll g_3. \qquad (4.10)$$

In the limit $h \to 0$, the model possesses troughs, the determination of which does not present any problems. Namely, if

$$u^{\alpha}=\begin{pmatrix}a\\0\\0\end{pmatrix},\quad d^{\alpha}=\begin{pmatrix}0\\b\\0\end{pmatrix},\quad \bar{u}_{\alpha}=\begin{pmatrix}a\\0\\0\end{pmatrix},\quad \bar{d}_{\alpha}=\begin{pmatrix}0\\b\\0\end{pmatrix},$$
$$L^{f}=(0,\ \sqrt{|a|^2-|b|^2}), \tag{4.11}$$

where a and b are arbitrary complex parameters, then all the D terms, with respect to both SU(3) and SU(2), vanish. The Yukawa term (4.9) guarantees stabilization by raising slightly the bottom of the troughs for $|a|, |b|\to\infty$.

For $a\neq 0$ and $b\neq 0$, the whole gauge symmetry is completely broken, since 11 (= 8 + 3) chiral superfields are eaten by the super-Higgs mechanism, giving a mass to all 11 vector supermultiplets present in the model. Three light chiral superfields remain massless in this approximation ($h=0$).

We now include nonperturbative effects, taking into account only the instantons with respect to the gauge group SU(3), but neglecting the SU(2) instantons by virtue of the smallness of g_2. For the reader who is acquainted with the preceding material, it will evidently not be difficult to write at once the effective superpotential induced by the instanton:

$$W_{\text{eff. inst}}=\frac{2\Lambda_3^7}{\det\{\bar{Q}Q\}}, \tag{4.12}$$

where Λ_3 is a scale factor which determines g_3, the factor 2 is introduced for convenience, and

$$\det\{\bar{Q}Q\}=(u^{\alpha}\bar{u}_{\alpha}d^{\beta}\bar{d}_{\beta}-u^{\alpha}\bar{d}_{\alpha}d^{\beta}\bar{u}_{\beta}). \tag{4.13}$$

This superpotential "expels" the scalar field from the coordinate origin (i.e., from the region $a=0$ and/or $b=0$) and generates spontaneous breakdown of the gauge invariance and the supersymmetry. Under the condition (4.10), this symmetry breaking takes place in the weak-coupling regime.

To prove the breakdown of SUSY, we can use Criterion 1 (see Sec. 4a), since $\bar{u}_{\alpha}$ does not appear in W_{cl}. Figure 4 demonstrates that the model actually has a nonzero condensate $\langle\lambda\lambda\rangle$, where λ is the octet of SU(3) gluinos.

To obtain more detailed information, it is necessary to investigate the superpotential ($W_{\text{cl}}+W_{\text{eff. inst}}$). The procedure is standard: first, combining (4.12) and (4.9), we find all the F terms as functions of the scalar fields; next, we substitute the scalar fields lying at the bottom of the troughs [see (4.11)]; then, we minimize the potential $V_{\text{pot}}=\Sigma|F|^2$ with respect to a and b, thus fixing the vacuum values of the scalar fields and the vacuum energy. The following values were obtained in Ref. 19:

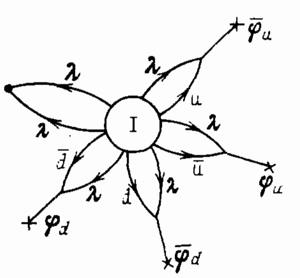

FIG. 4. Single-instanton contribution to the gluino condensate $\langle\lambda\lambda\rangle$ in the SU(3)×SU(2) model with two flavors. The SU(3) instanton contains six zero gluino modes and four matter modes. The crosses label the vacuum scalar fields, which are determined by the parameters a and b. The heavy dot labels the operator $\lambda\lambda$.

$$a_{\text{vac}}\approx 1.29\frac{\Lambda_3}{h^{1/7}},\qquad b_{\text{vac}}\approx 1.25\frac{\Lambda_3}{h^{1/7}},$$
$$E_{\text{vac}}\approx 3.59h^{10/7}\Lambda_3^4. \tag{4.14}$$

In the limit $h\to 0$ the parameters a and b tend to infinity as expected, and this justifies the assertion about the weak-coupling regime (the masses of the gauge bosons are $m\sim g\Lambda_3 h^{-1/7}\to\infty$).

The sector of light particles contains a Goldstone (strictly massless) fermion, the "electron," whose existence can be deduced from 't Hooft's self-consistency condition[33] for the anomalous triangle induced by the hypercharge. It also contains a neutral fermion with mass $\approx 11.3h^{6/7}\Lambda_3$. Among the spinless bosons there are one strictly massless Goldstone boson, corresponding to spontaneous breakdown of a certain axial invariance, and one charged and three neutral scalars with masses $h^{6/7}\Lambda_3$.

5. CONCLUSIONS

The problem of dynamical breakdown of supersymmetry by nonperturbative effects in four-dimensional gauge theories was raised in Ref. 12. The protracted search for such a scheme has finally been crowned with success. We may consider that we have received an extra and very valuable gift: when SUSY is spontaneously broken in theories with matter, the gauge symmetry is as a rule also broken, so that we have a weak-coupling regime, which is completely controllable theoretically.

Now that the fundamental possibility of the phenomenon has been proved, priority is being given to the search for a realistic scheme based on the mechanism which we have described above. The first steps in this direction have been taken in Refs. 14 and 19. We do not discuss the corresponding results in this review, referring the reader to the original literature, since the progress in this direction is clearly incomplete in character. The proposed schemes do not seem to us completely satisfactory, for either esthetic or purely phenomenological reasons. At best, they have the status of "semirealistic models," which, however, is recognized by their authors. Thus, the main work lies ahead of us, and its result is not known.

At the same time, there is now little doubt that instantons in supersymmetric theories, whose analysis represents an extremely fascinating theoretical problem, will play a key role in the solution of practical problems facing those investigating these models.

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[1)] In Ref. 19 it was shown that under certain conditions the instanton superpotential might, on the contrary, raise the bottom of the trough slightly at large φ, tending to "slide" the theory to the coordinate origin. However, models in which such a regime is realized for all the troughs are as yet unknown.

[2)] The field σ cannot describe a Goldstone boson corresponding to spontaneous breakdown of any other global symmetry, since in this case the values of $\langle\sigma\rangle$ would have to form some compact manifold. In Ref. 9 arguments were put forward to show that the manifold $\{\sigma\}$ is noncompact!

[1]Yu. A. Gol'fand and E. P. Likhtman, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 452 (1971) [JETP Lett. **13**, 323 (1971)].
[2]D. V. Volkov and V. P. Akulov, Phys. Lett. **B46**, 109 (1973).
[3]J. Wess and B. Zumino, Nucl. Phys. **B70**, 39 (1974).
[4]V. I. Ogievetskiĭ and L. Mezincescu, Usp. Fiz. Nauk **117**, 637 (1975) [Sov. Phys. Usp. **18**, 960 (1975)].
[5]P. Fayet and S. Ferrara, Phys. Rep. **32**, 249 (1977).
[6]A. Salam and J. Strathdee, Fortschr. Phys. **26**, 57 (1978).
[7]J. Bagger and J. Wess, *Supersymmetry and Supergravity*, Princeton University Press, Princeton (1983).
[8]S. J. Gates, M. T. Grisaru, M. Roček, and W. Siegel, *Superspace, or One Thousand and One Lessons in Supersymmetry*, Benjamin/Cummings, Reading, MA (1983).
[9]M. I. Vysotskiĭ, Usp. Fiz. Nauk **146**, 591 (1985) (in this issue).
[10]H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985).
[11]H. P. Nilles, Phys. Rep. **110**, 3 (1984).
[12]E. Witten, Nucl. Phys. **B188**, 513 (1981).
[13]M. T. Grisaru, M. Siegel, and M. Roček, Nucl. Phys. **B159**, 429 (1979).
[14]I. Affleck, M. Dine, and N. Seiberg, Phys. Lett. **B137**, 187 (1984); Phys. Rev. Lett. **52**, 1677 (1984).
[15]Y. Meurice and G. Veneziano, Phys. Lett. **B141**, 69 (1984).
[16]I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B241**, 493 (1984).
[17]A. I. Vaĭnshteĭn, V. 1. Zakharov, V. A. Novikov, and M. A. Shifman, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 494 (1984) [JETP Lett. **39**, 601 (1984)].
[18]D. Amati, G. C. Rossi, and C. Veneziano, Nucl. Phys. **B249**, 1 (1985).
[19]I. Affleck, M. Dine, and N. Seiberg, Phys. Lett. **B140**, 59 (1984); "Dynamical SUSY breaking in four dimensions and its phenomenological implications," Preprint, Princeton (1984).
[20]V. A. Novikov, M. A. Shifman, A. I. Vaĭnshteĭn, and V. I. Zakharov, Preprint ITEP-31, Moscow (1985); Nucl. Phys. (in press).
[21]A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Yu. S. Tyupkin, Phys. Lett. **B59**, 85 (1975).
[22]A. I. Vaĭnshteĭn, V. I. Zakharov, V. A. Novikov, and M. A. Shifman, Usp. Fiz. Nauk **136**, 553 (1982) [Sov. Phys. Usp. **25**, 195 (1982)].
[23]L. B. Okun', Leptony i kvarki, Nauka, M., 1981; [Engl. Transl. Leptons and Quarks, North-Holland, Amsterdam (1982)]; M. B. Voloshin and K. A. Ter-Martirosyan, Teoriya kalibrovochnykh vzaimodeĭstviĭ (Theory of Gauge Interactions), Énergoatomizdat, Moscow (1984).
[24]E. Witten, Nucl. Phys. **B202**, 253 (1982).
[25]T. E. Clark, O. Piguet, and K. Sibold, Nucl. Phys. **B159**, 1 (1979); K. Konishi, Phys. Lett. **B135**, 439 (1984).
[26]G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).
[27]F. Buccella, J. P. Derendinger, C. A. Savoy, and S. Ferrara, Preprint CERN TH. 3212, Geneva (1983).
[28]M. A. Shifman, A. I. Vaĭnshteĭn, and M. I. Vysotsky, Nucl. Phys. **B254**, 619 (1985).
[29]V. A. Novikov, M. A. Shifman, A. I. Vaĭnshteĭn, and V. I. Zakharov, Nucl. Phys. **B229**, 407 (1983).
[30]G. C. Rossi and G. Veneziano, Phys. Lett. **B138**, 195 (1984).
[31]I. Affleck, M. Dine, and N. Seiberg, Phys. Lett. **B137**, 187 (1984).
[32]E. Witten, Phys. Lett. **B117**, 324 (1982).
[33]G. 't Hooft, in: *Recent Developments in Gauge Theories* (eds. G. 't Hooft *et al.*), Plenum Press, New York (1980).

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