Supersymmetry: Kaluza-Klein theory, anomalies, and superstrings

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Progress in the search for a unified theory of elementary particles is reviewed. The supersymmetrical Kaluza-Klein theories are described: 11-, 10-, and 6-dimensional models of supergravity. The methods of spontaneous compactification, with whose help the four-dimensional theories are obtained, are described. The properties of the massless sector—zero modes in the Kaluza-Klein theories—and the question of the stability of vacuum solutions are discussed. An important criterion for the selection of a self-consistent theory is the absence of anomalies. The basic formulas for multidimensional chiral and gravitational anomalies are presented. The mechanism of the cancellation of the anomaly for Green and Schwarz's 10-dimensional effective field theory of superstrings with the gauge groups SO(32) and $E_8 \times E_8$ is described. The basic concepts and the results of the theory of superstrings are presented. This theory has no divergences and is at the present time a very attractive candidate for a unified theory of elementary particles.

CONTENTS

INTRODUCTION

The theory of supersymmetry[1] is the next step in the realization of the hopes of theoretical physics to construct a unified theory of elementary particles.[1)] The theory of supersymmetry has not yet predicted experimentally confirmed effects, but it has given a number of remarkable "theoretical effects," including a result which is fundamental to the quantum theory of fields[2]: the construction of a four-dimensional model without ultraviolet divergences.[2)]

We shall briefly describe below some recent results regarding supersymmetry. For convenience, we have divided the presentation into the following sections: the Kaluza-Klein approach, anomalies, and superstrings. It should be kept in mind, however, that the most interesting results have been obtained at the frontier separating different fields with the use of diverse mathematical and physical ideas, which clearly demonstrates the striving toward unification and complex simplicity. This refers primarily to the Green and Schwarz superstring,[5,6] which we shall discuss below, with the gauge group SO(32) or $E_8 \times E_8$—one of the present candidates for a unified theory of all known interactions: weak, electromagnetic, strong, and gravitational.

1. THE KALUZA-KLEIN APPROACH

a) The problems of grand unification theories

Kaluza's fantastic idea[7] that space-time has more than four dimensions has been revived in recent years in the theory of elementary particles.[8] The present development of this approach started with the work of Scherk and Schwarz[9] and Cremmer and Scherk[10] in connection with the dual string models. They propose that the extra dimensions should be studied as physical dimensions equivalent to the observable four dimensions. It was proposed that the obvious difference between the four observed and the extra microscopic dimensions be interpreted as a spontaneous breaking of the symmetry of the vacuum or, in other words, the result of spontaneous compactification of the extra dimensions.

Here we shall review the basic ideas which have led to

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the conclusion that candidates for a realistic unified theory must be sought amongst—at first glance—strange models in space-time with more than four dimensions, and in addition gravity must be taken into account.

As is well known, the grand unification theories,[11,12] which have been very successful in explaining the properties of the standard $SU(3)\times SU(2)\times U(1)$ model of electroweak and strong interactions, encounter a number of problems.

1. The problem of the hierarchy of interactions, i.e., the necessity of explaining the enormous difference in the mass scales of the electroweak interaction $M_W\sim 10^2$ GeV and the grand unification mass $M_X\sim 10^{15}$ GeV.

2. The existence of three generations of fermions is not explained.

3. The problem of chiral fermions, i.e., it is necessary to explain why right and left fermions enter into the Lagrangian unsymmetrically.

4. The choice of the gauge group is largely arbitrary.

5. A large number of parameters is required to describe the Higgs bosons.

6. Grand unification theories do not include gravity, although the characteristic mass in these theories $M_X\sim 10^{15}$ GeV is not very different from the Planck mass $M_P\sim 10^{19}$ GeV.

The solution of the problem of the hierarchy of interactions must be stable with respect to radiation corrections. This led to the use of global supersymmetry[13] in order to ensure that the value of the ratio M_W/M_X is fixed, because some of the usual divergences in quantum field theory do not occur in supersymmetrical models, which enables one to make the ratio M_W/M_X constant. To obtain a realistic particle spectrum in this approach, spontaneous breaking of supersymmetry must occur. This is possible only if supergravity is included,[14] namely, in a theory including $(N=1)$-supergravity, $(N=1)$-supersymmetrical Yang-Mills theory and a number of $N=1$ chiral scalar fields (the latter ensure breaking of gauge and supersymmetry). It is remarkable that in the past it was hoped[15,16] that gravity would be important in the theory of elementary particles, whereas now it has become clear that it is in fact impossible to construct a phenomenologically satisfactory theory without gravity. But even this theory has its problems: it is not renormalizable, it has a large number of free parameters, and it does not solve the problems 2–5 listed above. Therefore, such a theory cannot be fundamental and is only an effective theory. If a field theory is regarded as being fundamental, then such a theory must have a higher-order symmetry than $(N=1)$-supergravity. The symmetry can be raised by two methods:

the *first* method consists of studying theories with extended N-supersymmetry in $(d=4)$-dimensional space-time[17];

the *second* method, which dates back to Kaluza and Klein's idea, consists of studying theories in which the symmetry is the group of local $(N=1)$-supertransformations of a d-dimensional space[18] $(d>4)$.

A phenomenological theory in the first approach requires a mechanism for breaking $(N>1)$-supersymmetry down to $(N=1)$-supersymmetry—spontaneous breaking of supersymmetry, while the second approach requires a mechanism for breaking the general-coordinate symmetry of a d-dimensional space down to the symmetry of the space $M^4\times B^{d-4}$. The second mechanism is customarily called spontaneous compactification. However, difficulties linked to the nonrenormalizability of these theories make it necessary to regard them (we have in mind theories in dimensions $d\leqslant 10$), in their turn, as effective theories, which are obtained from a more fundamental theory—superstrings in a 10-dimensional space-time.[19] Divergences apparently do not occur in the theory of superstrings.[6,19] It is possible that extended supergravity also has a hidden symmetry which will make the theory renormalizable.[20] [3)]

b) Extended supergravity in ($d=4$)-dimensional space-time

N-extended supergravities with $N=1,\ldots,8$ (N indicates the number of supercharges in the theory) are possible in $d=4$ dimensions. The limit on the number N is associated with the fact that for $N>8$ there arise fields which correspond to particles with spin greater than 2 and a systematic theory of such fields interacting with the gravitational field does not exist.[118] The maximally symmetrical model of supergravity is $(N=8)$-supergravity.[17] There exist several variants of this theory,[21,22] but in all these theories the elementary gauge fields transform according to a group which cannot include $SU(3)\times SU(2)\times U(1)$. Therefore, without appealing to the idea of using composite gauge fields.[17,23] $(N=8, d=4)$-supergravity cannot be regarded as realistic.

Attempts at constructing a realistic theory based on N-supergravity with $2\leqslant N\leqslant 4$ interacting with matter fields have not produced a satisfactory model, because for these models it is difficult to provide a mechanism for breaking supersymmetry down to $N=1$.

Thus a realistic theory with extended supersymmetry in a four-dimensional space-time does not exist at present. Theories in multidimensional $(d>4)$ space-time are more promising. The basic idea here is to construct a geometrical model with $d>4$, from which a more complicated realistic theory is obtained for $d=4$ with the help of spontaneous compactification. We note, incidentally, that the four-dimensional theories with extended supersymmetry were constructed precisely with the help of the simplest dimensional reduction from multidimensional theories.

c) Supergravity in ($d>4$)-dimensional space-time

The maximum dimensionality of space-time with which supergravity can be constructed is equal to 11.[121] The maximum number $(N=8)$ of extended supersymmetries in $d=4$ dimensions is related to the maximum dimensionality 11, since in the dimensional reduction (see below) the 32-component spinor-generators of the simple supersymmetry $Q_{\mathscr{A}}$, $\mathscr{A}=1,\ldots,32$, separate in the 11-dimensional theory into eight groups $(N=8)$ of four-component spinor-generators with the extended supersymmetry—$Q_{\alpha i}$, $\alpha=1,2,3,4$; $i=1,\ldots,8$—in four-dimensional space. For $d=11$ there are no multiplets containing fields solely with spin J, $J\leqslant 1$

TABLE I.

	Composition of multiplets	Boson part of the Lagrangian
$d=11$, $N=1$	Gravitational multiplet (e^A_M,ψ_M,A_{MNP}); ψ_M—a Rarita-Schwinger field A_{MNP}—an antisymmetrical tensor M, N, $P=0,1,..,10$	$S=\int d^{11}xe\Big[\frac{1}{2}R-\frac{1}{48}F_{MNPQ}F^{MNPQ}$ $+\frac{2e}{6(4!)^2}\varepsilon^{M_1\ldots M_{11}}$ $\times F_{M_1M_2M_3M_4}F_{M_5M_6M_7M_8}A_{M_9M_{10}M_{11}}\Big]$, $F_{MNPQ}=4!\partial_{[M}A_{NPQ]}$
$d=10$, $N=1$	Gravitational multiplet $(e^A_M,\psi_M,B_{MP},\lambda,\varphi)$; Vector multiplet (A_M,ψ); ψ_M—a Rarita-Schwinger field B_{MP}—an antisymmetrical tensor λ—a Dirac field φ—a scalar A_M—a Yang-Mills field $M, N, P=0,1,..,9$	$S=\int d^{10}xe\Big(\frac{1}{2}R-\nabla_M\varphi\nabla^M\varphi$ $-\frac{e^{-\varphi}}{4g^2}F^a_{MN}F^{aMN}$ $-\frac{3}{2g^4}e^{-2\varphi}H_{MNK}H^{MNK}\Big)$; $H_{MNK}=\partial_{[M}B_{NL]}$ $+\mathrm{tr}\Big(A_{[M}F_{NL]}-\frac{1}{3}A_{[M}A_NA_{L]}\Big)$
$d=6$, $N=2$	Gravitational multiplet (e^A_M,ψ_M,B^+_{MP}); Antisymmetrical tensor multiplet (B^-_{MP},χ,φ); Gauged multiplet (A_M,λ); (Hypermultiplet) (ψ^Δ,ϕ^i); $B^{+(-)}$—Self (anti)-dual intensity ψ_M and λ—Left fermions χ—Right fermions ϕ^i—Scalars, parametrizing $HP(n-1,1)$, ϕ—Scalar, ψ^Δ—$Sp(n)$ Spinors $M,P=0,1,...,5$; $a=1,...n(2n+1)$—Isotopic index, $i=1,...,4n$, $\Delta=1,...,2n$	$S=\int d^6xe\Big[\frac{1}{2}R-\frac{1}{2}\nabla_M\varphi\nabla^M\varphi$ $-\frac{e^{2\varphi}}{6g^4}H_{MNK}H^{MNK}-\frac{e^{\varphi}}{4g^2}F^a_{MN}F^{aMN}$ $-\frac{1}{2}G_{ij}D_M\varphi^iD^M\phi^j-e^{-\varphi}V(\phi^i)\Big]$; G_{ij}—Kähler metric

The gravitational constant $\varkappa$ is equal to 1; g is the Yang-Mills constant; e^A_M is the field of coordinate frames; F_{MN} is the Yang-Mills intensity; H_{MNK} is the intensity of the field B_{MP} up to Chern-Simons terms of the gauge field A_M; $e=\sqrt{\det|g_{MN}|}$.

(multiplets of matter fields),[119] and there is thus only one possible unification theory.[18] This theory contains an 11-dimensional field of coordinate frames e^A_M (44 boson degrees of freedom), the gravitino field $\psi_{\mu\alpha}$ (128 fermion degrees of freedom), and an antisymmetrical tensor of rank three A_{MNK} (84 boson degrees of freedom).

Table I shows the components of the models of supergravity, including the possible interaction with matter multiplets, depending on the dimensionality of the space-time; only the most realistic candidates for a unified theory are shown. Multiplets of matter fields exist only for $d \leqslant 10$.[119] In the 10-dimensional case the only possible candidate is the $(N=1)$-Yang-Mills multiplet.[120] This multiplet contains only spin-1/2 fields and gauge fields and does not contain scalar fields. For $d=10$ there are three theories of supergravity: $(N=1)$-supergravity,[24] $(N=2)$-supergravity,[25] and $(N=2)$-chiral supergravity.[26] $(N=1,\ d=10)$-supergravity interacts with a multiplet containing gauge fields (see Table I); in addition, the gravitational constant is the only independent constant.[111]

The $(N=2,\ d=6)$-theory contains several multiplets: a gravitational multiplet, an antisymmetrical tensor multiplet, a gauge multiplet, and a hypermultiplet. The hypermultiplet includes scalar fields ϕ^i, on which additional restrictions are imposed: they are parameters of the noncompact quaternion Kähler manifold $HP(n=1,n)$ (the number n is related to the dimensionality of the gauge group).

The study of the physical properties of these theories is based on the mechanism of spontaneous compactification.

d) Spontaneous compactification: vacuum solutions

The mechanism of spontaneous compactification in theories of the Kaluza-Klein type consists of the following.[8-10] Equations describing the gravitational field, possibly interacting with matter fields, are studied in a d-dimensional space-time. A solution of special form (vacuum solution) to these equations, corresponding to the representation of the d-dimensional manifold in the form $M^d = M^4 \times B^{d-4}$, where M^4 is a four-dimensional space-time (preferably Minkowski space, but often the anti-de Sitter space is also studied) and B^{d-4} is the compact "inner" space, is sought. The vacuum metric $\mathring{g}_{MN}$ has the block form:

$$\mathring{g}_{MN} = \begin{pmatrix} \mathring{g}_{\mu\nu}(x) & 0 \\ 0 & \mathring{g}_{mn}(y) \end{pmatrix};$$

here x^μ are coordinates on M^4 and y^m are coordinates on B^{d-4}. Einstein spaces[29] (i.e., the Ricci tensor is proportional to the metric) are usually considered for M^4 and B^{d-4}. It is natural to suppose that the vacuum solution must be stable in some sense and must exhibit a symmetry. Then all fields, which we denote by $\Phi(x,y)$, are expanded in terms of harmonics (eigenfunctions of the mass operators M^2) on B^{d-4}:

$$\Phi(x,\ y) = \sum_k \varphi_k(x)\,\chi_k(y), \quad M^2\chi_k(y) = m_k^2\chi_k(y);$$

the expansion coefficients $\varphi_k(x)$ are interpreted as physical fields. The mass operator is given by some differential operator on B^{d-4}, determined by expanding the action up to second order in the fluctuations with respect to the vacuum solution. The equations for the effective four-dimensional theory are obtained by integrating over y and describe an infinite set of mass states with masses of the order of $1/L$, where L is the characteristic length of B^{d-4}, and a finite set of massless states—the zero modes. If L is chosen to be of the order of the Planck length, then mass states with an extremely large Planck mass, far removed from experimentally accessible limits, are obtained.[4)] In this approach only the massless particles are observable. Thus the extra dimensions are manifested at low energies only in the special (depending on the model) form of the interactions of massless particles.

The equations of motion of the gravitational field in a d-dimensional space have the form

$$R_{MN} - \frac{1}{2} g_{MN} R = T_{MN} - \Lambda g_{MN}, \tag{1.1}$$

where T_{MN} is the energy-momentum tensor of the matter fields, and Λ is the lambda term. When $\Lambda = 0$, the vacuum with $T_{MN} = 0$ corresponds to the compactification $M^d = M^r \times B^{d-r}$, where M^r is a Minkowski space and B is a Ricci-flat compact space. The space B can be chosen as the $(d-r)$-dimensional torus T^{d-r}. This theory, however, has only abelian gauge fields (see below), and it is desirable to have a more nontrivial compactification because in the vacuum the tensor T_{MN} differs from zero.

The block representation of the metric g_{MN} is compatible with the equations if it is assumed that

$$T_{\mu\nu} = -\gamma_1 g_{\mu\nu}(x), \quad T_{mn} = \gamma_2 g_{mn}(y). \tag{1.2}$$

From (1.1) and (1.2) it follows that $M^d = M^4 \times B^{d-4}$, where M^4 and B^{d-4} are Einstein spaces, i.e.,

$$R_{\mu\nu} = -\frac{(d-4)(\gamma_1+\gamma_2) - 2(\gamma_1+\Lambda)}{d-2} g_{\mu\nu}, \tag{1.3a}$$

$$R_{mn} = \frac{4\gamma_1 + 2\gamma_2 + 2\Lambda}{d-2} g_{mn}. \tag{1.3b}$$

The vacuum solutions for the fields can be of two types:

1. solutions which do not break the maximal symmetry of the four-dimensional space-time, i.e., the components of the vector and tensor fields differ from zero only in the inner space (the rank-four tensor can be proportional to $\varepsilon_{\mu\nu\lambda\sigma}$—the totally antisymmetrical tensor of rank four), and

2. solutions admitting breaking of the maximal symmetry of the four-dimensional space, the so-called cosmological solutions.

If the interaction of gravitation with the vector, tensor, and scalar fields without the potential term, i.e., the term in T_{MN} proportional to $g_{MN}v(\varphi)$, is studied, then in the first case it turns out that $\gamma_2 = \tilde{\gamma}_1 - \gamma_1$, where $\tilde{\gamma}_1$ is a constant which is of the same order of magnitude but several times greater than γ_1. Because the energy is positive, $T_{00} > 0$, the constant γ_1 is positive. It then follows from (1.3) that if $\Lambda = 0$, then M^4 is an anti-de Sitter space (adS^4), while B^{d-4} is a compact space, and it is impossible to obtain a vacuum solution in the form of a flat four-dimensional space with $T_{MN} \neq 0$. If, on the other hand, T_{MN} includes a potential scalar term, then the solution $\varphi = \text{const}$ is equivalent to the introduction of a Λ term and a flat space M^4 is possible with a curved space B^{d-4}, as in fact happens in the ($d=6$, $N=2$)-theory.

The following possibilities exist for constructing a solution corresponding to a Minkowski space for M^4 and $T_{MN}\neq 0$ without the effective Λ term:

include terms with higher-order derivatives,[32,33] radiation corrections,[86] or ghost fields,[30 5)] which appear for hypermultiplets containing noncompact spaces,[34] and

adopt the hypothesis[31] that there are several time dimensions, i.e., use, for example, the signature $(-+++-+\dots+)$.[30]

e) Mechanisms for spontaneous compactification

The basic problem in constructing the compactification $M^d=M^4\times B^{d-4}$ is to find the solutions of the classical field equations whose energy-momentum tensor satisfies the conditions (1.3a) and (1.3b). We shall study the vacuum solutions in which only the boson fields are different from zero, i.e., it is necessary to find the solution of the equations following from the Lagrangians presented in Table I which are consistent with the splitting (1.3a) and (1.3b).

The following mechanisms of spontaneous compactification are known:

1) the mechanism proposed by Freund and Rubin[35] which uses a special ansatz for antisymmetrical tensors, as well as Englert's mechanism;

2) embedding the connection for the group of internal symmetries into the Lorentz connection[37,50];

3) monopole or instanton mechanism[38];

4) compactification induced by scalar chiral fields[39,40];

5) compactification induced by radiation corrections.[86]

The Freund-Rubin mechanism

This mechanism is usually used in ($N=1$, $d=11$)-supergravity. The solutions of the equations of motion for the boson fields—the Einstein equations with $T_{MN}=F_{MKPL}F_N^{KPL}-\frac{1}{8}g_{MN}F^2$ and the equations for the antisymmetrical tensor field

$$\nabla_M F^{MNPQ}=-\frac{1}{e(x)}\varepsilon^{M_1\dots M_4NPQ}F_{M_1\dots M_4}F_{M_5\dots M_8},\qquad(1.4)$$

consistent with the block form of g_{MN}, are sought in the form

$$F_{\mu\nu\rho\lambda}=e(x)f\varepsilon_{\mu\nu\rho\lambda};\qquad(1.5)$$

f is a constant and the remaining components of F_{MNPQ} are equal to zero. The equations (1.3a) and (1.3b) are obtained for $R_{\mu\nu}$ and R_{mn}.

Englert's mechanism

Equation (1.4) also has more complicated solutions, leading to an energy-momentum tensor of the form (1.2). For solutions of Englert's type,[36] the tensor F_{MNPQ} also has nonvanishing components in the inner space. A solution of the Englert type was first constructed on a sphere S^7 with torsion. It can be constructed[83,84] on any Einstein space B^7, on which the metric satisfies the equation $R_{mn}=6\lambda^2g_{mn}$ and which admits at least one Killing spinor, satisfying the equation $[\nabla_m-(\lambda/2)\gamma_m]\eta=0$. Then the solution is given by the following ansatz:

$$F_{\mu\nu\lambda\sigma}=-2\lambda e(x)\,\varepsilon_{\mu\nu\lambda\sigma},\qquad F_{mnkl}=-\lambda\bar{\eta}\Gamma_{mnkl}\eta;$$

in this case $R_{\mu\nu}=-10\lambda^2g_{\mu\nu}$, γ_m are the Dirac matrices, and $\Gamma_{mnkl}=\gamma_{[m}\gamma_n\gamma_k\gamma_{k]}$.

Mechanism for embedding the Yang-Mills connection into the Lorentz connection

This mechanism[37,50] is based on the fact that with a given metric a Yang-Mills connection satisfying the condition of parallelizability

$$D_PF^{MN}=0;\qquad(1.6)$$

where D is the Yang-Mills derivative, can be constructed in a uniform Riemannian space.

For the case when the space B^{d-4} is a sphere S^{d-4} and the gauge group includes the group $O(d=3)$ as a subgroup, the intensity of the gauge field, satisfying (1.6) and the Bianchi identities, has the form

$$F_{mn}^{ab}=r\,(e_m^ae_n^b-e_m^be_n^a);\qquad(1.7)$$

r is the radius of the sphere and e_m^a is the field of coordinate frames. The formulas for F_{mn}^{ab} in the case when B^{d-4} is a uniform space B/H can be found in Ref. 37.

The monopolar mechanism

In the particular case of an abelian gauge group, the monopolar mechanism coincides with the Freund-Rubin mechanism, for which the antisymmetrical tensor of rank two—the intensity of the electromagnetic field $F_{mn}=ce(y)\varepsilon_{mn}$, where ε_{mn} is the totally antisymmetrical tensor of rank two—differs from zero. This mechanism of compactification is used in ($d=6$, $N=2$)-supergravity.[28]

Compactification with the help of chiral fields

The equations have the form

$$R_{MN}=\frac{2}{\lambda^2}h_{mn}(\phi)\,\partial_M\phi^m\,\partial_N\phi^n,$$

$$\frac{2}{e}\partial_M\,(eg^{MN}h_{mn}\partial_N\phi^n)=g^{MN}\frac{\partial h_{pq}}{\partial\phi^m}\partial_M\phi^p\partial_N\phi^q.$$

The following ansatz is used[39]: $\phi^m(x,\ y)=y^m$, $g_{mn}=-(2/\lambda^2)h_{mn}$; $g_{\mu\nu}$ is the Minkowski metric; and, h_{mn} is a metric of constant positive curvature.

Gell-Mann and Zwiebach[40] also studied the case when h_{mn} is the metric of constant negative curvature. In this case they constructed the spontaneous dimensional reduction onto a noncompact surface of finite area.

f) Zero modes

1) Gauge fields

In the foregoing discussion of vacuum solutions it was pointed out that only the massless states (the zero modes of the mass operator, i.e., the Beltrami-Laplace, Dirac, and Lichnerowicz operators) are observable at low energies in the Kaluza-Klein theory. To obtain an effective low-energy theory for massless gauge fields, the following standard "Kaluza-Klein ansatz" is used:

$$g_{\mu\nu} = \overset{\bullet}{g}_{\mu\nu}(x) + A^{\alpha}_{\mu}(x)\, A^{\beta}_{\nu}(x)\, K^{m\alpha}(y)\, K^{n\beta}(y)\, \overset{\circ}{g}_{mn}(y),$$
$$g_{\mu n} = A^{\alpha}_{\mu}(x)\, K^{m\alpha}(y)\, \overset{\bullet}{g}_{mn}(y), \quad g_{mn} = \overset{\circ}{g}_{mn}(y); \qquad (1.8)$$

where $(\mathring{g}_{\mu\nu}(x), \dot{g}_{mn}(y))$ is the vacuum solution on $M^4 \times B^{d-4}$. It is presumed that B has a group of isometries G with the Killing vectors $K^{m\alpha}(y)$, where α runs through values equal in number to the dimensionality of the group G. Under infinitesimal transformations of the coordinates $y^m \to y^m + \varepsilon^{\alpha}(x) K^{m\alpha}(y)$, the quantities $A^{\alpha}_{\mu}(x)$ transform like gauge fields:

$$A^{\alpha}_{\mu} \to A^{\alpha}_{\mu} + (D_{\mu}\varepsilon)^{\alpha}.$$

Thus Yang-Mills fields with the gauge group G arise in the Kaluza-Klein theory as components of metrics. Within the framework of a purely multidimensional gravity, a metric of the form (1.8), generally speaking, does not satisfy the equations of motion,[41] and the standard Yang-Mills Lagrangian is not obtained for the fields A^{α}_{μ}.[8,43] Compactification on a torus (in this case the group G is abelian) and the case $B = H$, where H is a compact semisimple Lie group, are important exceptions. In this case, the left-invariant vector fields are the Killing vectors $K^{m\alpha'}$ for the biinvariant metric on H and the relation (Killing-Cartan)

$$K^{\alpha'}_{m} K^{m\beta'} = \delta^{\alpha'\beta'}$$

holds. In this case the ansatz (1.8) satisfies the equations of motion. This is also valid for the case when B is a uniform space. In both cases the gauge group is reduced to some subgroup of the group G. The compatibility of the ansatz (1.3) with the equations of motion can also require that scalar fields be included (as in Kaluza's initial work). In the presence of matter fields the ansatz (1.8) can be consistent with the equations of motion (as, for example, in 11-dimensional supergravity[41]), but each specific model must be analyzed separately.

2) Scalar fields

Scalar fields play a very important role in modern gauge field theories,[12] because they provide the Higgs mechanism of spontaneous symmetry breaking and a large number of parameters is required to describe them. The geometrical interpretation of scalar fields therefore becomes important. Scalar fields as components of a metric appear in Kaluza-Klein theories,[7,8,42] but the results depend strongly on the model and their interaction in this case is described by exotic Lagrangians, which have little in common with the standard Yang-Mills-Higgs Lagrangian.

It was proposed in Ref. 43 that the Kaluza-Klein theory with dynamic torsion be studied and definite components of the torsion be interpreted as scalar fields. If linear connection coefficients Γ^{P}_{MN} are given, then the torsion tensor is defined as $T^{P}_{MN} = \Gamma^{P}_{MN} - \Gamma^{P}_{NM}$. We shall assume that there is no torsion in the flat Minkowski space, and we shall set $T^{\nu}_{\mu m} = \delta^{\nu}_{\mu} \Phi_m(x)$. Then a Yang-Mills-Higgs Lagrangian is obtained for the scalar fields Φ_m (they transform according to some gauge group).

We note that torsion also appears in Weinberg's approach[44] to the Kaluza-Klein theory and in 11-dimensional supergravity.[36,45]

3) Fermion fields

The massless fermion with spin 1/2 is described in the Kaluza-Klein theory by the Dirac equation in d-dimensional space:

$$\gamma^M \nabla_M \psi = 0,$$

or

$$\gamma^{\mu} \nabla_{\mu} \psi = -\gamma^m \nabla_m \psi; \qquad (1.9)$$

here ∇_M is the covariant derivative, $\gamma^{\mu}\nabla_{\mu}$ is the usual four-dimensional Dirac operator, and $\gamma^m \nabla_m$ is the Dirac operator in the space B. It is evident from (1.9) that the eigenvalues of the operator $\gamma^m \nabla_m$ will play the role of mass for the four-dimensional theory. In particular, the low-energy observables will be the zero modes of $\gamma^m \nabla_m$, corresponding to quarks and leptons.[46] The number of zero modes (i.e., the number of quarks and leptons) depends on the geometry of the manifold B. For example, on manifolds with positive curvature, which often arise with compactification, the Dirac operator does not have zero modes at all (Lichnerowicz's theorem[47]). Fortunately, theories of supergravity also contain Rarita-Schwinger spin-$\frac{3}{2}$ fields, some of whose components appear in four-dimensional space-time as spin-$\frac{1}{2}$ fields. The Rarita-Schwinger operator can have zero modes on manifolds with positive curvature also.

In the Kaluza-Klein theories with fermions, however, the following basic problem arises.[46,48] As is well known, the observed right and left fermions transform according to different (complex) representations of the gauge group. This symmetry cannot, however, be obtained with the help of multidimensional gravity. There is a theorem, due to Atiyah and Hirzebruch,[49,50] stating that for any continuous symmetry group the zero modes of the Dirac operator form a real representation. Witten[50] showed that the same is true for the Rarita-Schwinger operator on compact uniform manifolds.

Two methods for solving this problem of chiral fermions were proposed. The first method is to introduce additional gauge fields[50,51] (aside from those which appear as components of the metric). Such fields always exist in the theory of superstrings.[19] When a nontrivial configuration of these fields exists in the inner space, the problem of chiral fermions can be solved.[50,51] The other method is to study noncompact inner spaces with finite volume.[52] This method may be useful in 11-dimensional supergravity.[53]

The problem of chiral fermions is related to the problem of anomalies (see below).

At the present time most examples of spontaneous compactification have been developed in detail for the case when the space M^d is represented as a direct product $M^d = M^4 \times B^{d-4}$, i.e., it is a trivial fibering over M^4. In the more general case, spontaneous compactification can be accompanied by the appearance of a structure with nontrivial fibering on M^d with M^4 as the basis. A model example of this type is studied in Ref. 123.

Aside from the method of spontaneous compactification, the method of dimensional reduction is also used to obtain a four-dimensional theory from a multidimensional theory.[61] In this case, the condition of G invariance is imposed on the metric on $M^4 \times G/H$ instead of the requirement that it satisfy the equations of motion. This metric is described in terms of fields on M^4.

g) Cosmology—a window into higher dimensions[54]

The spontaneously broken symmetry is restored at high temperatures.[55] From the viewpoint of spontaneous compactification, therefore, under the extreme conditions of the early stages of evolution of the universe space-time has its own "true" 10 (or 11) dimensions. Compactification then occurs in principle into all possible vacuum solutions. As a result, "islands," in which space-time can have a different topology,[122] different dimensionality,[56] and different signature[30,31 6)] can form. The existence of tunneling transitions and the formation of bubbles (compare Ref. 57) of one vacuum into another vacuum has not been excluded.

The process of inflation of the universe could be related to the collapse of inner dimensions.[58]

The Kaluza-Klein theory can in principle be checked experimentally, since in this approach the fine-structure constant and the gravitational constant are functions of time.[59]

Since the theory of superstrings does not contain divergences,[6] this theory could shed light on the nature of the cosmological singularity. It is proposed in Ref. 60 that the big-bang phase was preceded (more precisely replaced) by a nonlocal string phase with $d = 10$, after which spontaneous compactification and phase localization (i.e., the strings transformed into the usual particles), followed by expansion, occurred.

h) Spontaneous compactification of ($d = 11$)-supergravity

In this section we shall describe the vacuum solutions of 11-dimensional supergravity. In principle, all vacuum solutions must be studied, because tunneling transitions can occur and different solutions can be realized in different islands of the universe.

The simplest compactification is obtained when M^4 is a Minkowski space. while $B^7 = T^7$ is a 7-dimensional torus. The massless sector gives Cremmer and Julia's ($d = 4$, $N = 8$)-theory,[12] which has (global E_7) $\times$ [hidden local SU(8)]-symmetry (the hidden gauge symmetry is realized on composite gauge fields, formed from scalars and their derivatives).

The other solution with ($N = 8$)-supersymmetry[62] has the form $M^{11} = adS^4 \times S^7$, where adS^4 is the anti-de Sitter space and $S^7 = \mathrm{SO}(8)/\mathrm{SO}(7)$ is a 7-dimensional sphere with the standard metric. This theory has [local SO(8)] $\times$ [hidden local SO(8)]-symmetry. There are no other solutions with ($N = 8$) supersymmetry.[63] It turns out that SO(3, 2) (anti-de Sitter symmetry)-boson symmetry together with ($N = 8$) fermion symmetry can be extended up to the OSp(4|8) supersymmetry.[64] The massless sector in the linear approximation[65] is given by the de Witt and Nicolai[21] ($d = 4, N = 8$) gauge supergravity, which was obtained from the theory[17] by introducing gauge fields for the SO(8) subgroup of E_7. In the nonlinear theory, however, this coincidence may not occur.[66] Other[22] gauge versions of ($d = 4$, $N = 8$)-supergravity, corresponding to noncompact subgroups of E_7, also exist.

The sphere S^7 admits, aside from the standard metric, a second Einstein metric[67] (squashed metric), which gives a theory with $\mathrm{SO}(5) \times \mathrm{SU}(2)$-gauge symmetry and ($N = 1$ or $N = 0$)-supersymmetry, depending on the orientation of S^7. The 7-dimensional uniform spaces $N^{pqr} = \mathrm{SU}(3) \times \mathrm{U}(1)/\mathrm{U}(1) \times \mathrm{U}(1)$, where p, q, and r describe different embeddings of $\mathrm{U}(1) \times \mathrm{U}(1)$, each admit two Einstein metrics also. These solutions have $\mathrm{SU}(3) \times \mathrm{SU}(2)$-gauge symmetry and ($N = 1$) supersymmetry.[68]

The gauge group SO(8) for elementary gauge fields does not contain the phenomenological $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ subgroup. More complicated vacuum solutions must therefore be studied in order to obtain a realistic theory. As pointed out by Witten,[77] it is remarkable that $n = 7$ is the minimum dimensionality of the space B^n containing the $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ group of isometries.

Each compact uniform space G/H, where H is the maximal subgroup, admits an Einstein metric,[69] and therefore such 7-dimensional spaces are solutions of the equations of ($d = 11$)-supergravity. The group of isometries of this metric coincides with G. For the more interesting case of the gauge group $G = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ introduced by Witten,[77] the corresponding Einstein space has the form

$$M^{pqr} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)/\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1),$$

where the integers p, q, and r characterize the nature of the embedding of the denominator into the numerator. The remaining 7-dimensional Einstein spaces of the form G/H are described in Ref. 77.

For the spaces M^{pqr} with $p \neq q$, the holonomy group is SO(7) and supersymmetry is completely broken. For $p = q$ the holonomy group is SO(3) and ($N = 2$)-supersymmetry exists.

The complete particle spectrum has been obtained for solutions of the Freund-Rubin type both on the sphere S^7 and on arbitrary uniform spaces.[72] In addition to the massless gauge multiplets there also exist quasimassless multiplets, in which the scalar does not have a mass, but all other states with spin J, $J \leqslant \frac{1}{2}$ do. Such multiplets could play the role of matter multiplets interacting with gauge supergravity, and they can participate in the super-Higgs mechanism. The effective potential of the theory in this case changes, and it has not been excluded that the problem of the cosmological constant can be solved in this manner.

All models constructed based on the compact spaces B^7, however, suffer from the following drawbacks:

1. they do not solve the problem of chiral fermions;

2. supersymmetry is broken down to $N = 0$ or $N = 2$ (but not to $N = 1$) in those cases when the gauge symmetry group is $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$; and,

3. in $d = 4$ dimensions an anti-de Sitter space is obtained rather than a flat space, and the relation between the gauge coupling constant and the cosmological constant is such that one of them is necessarily unrealistic.

It is proposed in Refs. 52 and 53 that the difficulties with the chiral fermions be overcome by considering for B a noncompact space with finite volume.

If it is possible to construct a dynamic theory of composite gauge fields, in spite of the appearance of ghosts[103] for $d>4$, then in this case the hidden SU(8) gauge symmetry of the de Witt–Nicolai theory would be sufficient for phenomenology, and it would even be possible to obtain chiral fermions.[23,42] In the 11-dimensional program, however, even if it is possible to overcome the difficulties enumerated above, the problem of the nonrenormalizability of the 11-dimensional supergravity remains.

i) Spontaneous compactification of 10-dimensional models

Ten-dimensional supersymmetrical field models play a special role, since they are obtained in the low-energy limit from superstrings.[19]

The construction of the vacuum solution for this theory encounters a difficulty[73] related to the equation for the scalar field

$$\nabla_M\nabla^M\varphi+\frac{1}{8g^2}e^{-\varphi}F^2+\frac{3}{2g^4}e^{-2\varphi}H^2=0. \qquad (1.10)$$

It is evident from here that if $\varphi=$ const and the components F and H are nonzero only for the inner closed space with positive signature, then Eq. (1.10) is not satisfied, because the left side of (1.10) contains sums of positive terms.

Thus under the usual assumptions spontaneous compactification of the form $M^4\times M^6$ does not occur in this theory, if M^4 is a space with the maximal group symmetry. This difficulty can be circumvented in the following manner. It is hypothesized in Ref. 31 that there exist regions of the universe in which the metric has a different signature. The solutions of the effective field theory of superstrings[5] and a theory[27] of the form $M^4\times adS^3\times B^3$, where M^4 is a Minkowski space, adS^3 is a three-dimensional anti-de Sitter space (containing a time variable), and B^3 is a compact space (in this situation one must apparently not talk about spontaneous compactification, since adS^3 is noncompact, but rather about spontaneous reduction), were found in Ref. 30. Thus the hypothesis[31] that several time dimensions exist in the dynamic Kaluza-Klein theory makes possible the solution of the problem of the cosmological constant. A solution of the problem of ghosts in theories with additional time dimensions is proposed in Ref. 130.

The vacuum configuration for the effective Green-Schwarz field theory is constructed in Ref. 32. This configuration has $(N=1)$-supersymmetry and a four-dimensional Minkowski space and compactifies into a Ricci-flat (the requirement of $(N=1)$-supersymmetry in the Minkowski space leads to a Ricci-flat inner space in an analogous manner in 11-dimensional supergravity also[104]) Kähler manifold and produces four generations of chiral fermions. This configuration does not satisfy Eqs. (1.10) of the Green-Schwarz theory, but is rather a solution of a modified theory with an additional term containing higher-order derivatives, which is obtained from an $E_8\times E_8$ superstring.[74]

j) Stability of vacuum solutions

The vacuum solution must be stable with respect to classical or quantum perturbations.

One criterion for stability is the absence of ghosts and tachyons in the spectrum calculated in the linear approximation against the background formed by the vacuum solution.[75] Another criterion is that any perturbation must have a higher energy than the vacuum.[76] If the solution exhibits supersymmetry, then it is stable.[77,78] This follows formally from the fact that the Hamiltonian is equal to the sum of the squares of supercharges and vanishes only in the supersymmetrical state. In particular, supersymmetrical solutions in $(N>4)$-supergravity are stable, though here the scalar potential is not bounded from below.[76]

The nonsupersymmetrical solutions can also be stable, for example, for topological reasons.[79] This occurs, for example, for the Einstein-Maxwell $(d=6)$-system with compactification on the sphere S^2 with the help of the Dirac monopole[80] and for the $(d=8)$- theory of instanton compactification on the sphere S^4.[81]

The necessary and sufficient condition for classical stability[82] of a solution of the Freund-Rubin type, compacting onto $B^7(R_{mn}=\gamma g_{mn})$, is that all eigenvalues λ of Lichnerowicz's operator Δ_L, acting on the symmetrical transverse traceless tensors h_{mn},

$$\Delta_L h_{mn}=-\Delta h_{mn}-2R_{mnpq}h^{pq}+2R_{(m}{}^{p}h_{n)p}=\lambda h_{mn}$$

(here Δ is the Beltrami-Laplace operator), must satisfy the inequality $\lambda>\gamma/2$. In particular, the solution (squashed S^7) is stable. The spaces M^{pqr} give stable solutions only for

$$\frac{7}{27}\sqrt{6}<\left|\frac{p}{q}\right|<\frac{17}{117}\sqrt{66},$$

i.e., for p and q in some neighborhood of the symmetrical solution with $p/q=1$.

Solutions of Englert's type are unstable, if B^7 admits at least two Killing spinors.[84] Englert's solution on a deformed (squashed) sphere S^7 is also unstable.[85]

Even the stable classical solutions can be unstable in the quantum theory. Single-loop corrections were studied in Refs. 86.

Aside from the conditions of vacuum stability, the theory must also satisfy the condition that there be no anomalies.

2. ANOMALIES

The chiral anomalies associated with the destruction of invariance and appearing in the triangle diagram with massless fermions are well known.[12,87] The anomalies enable explaining the decay $\pi^0\to2\gamma$ on the basis of current algebra, and also solving the U(1) problem in quantum chromodynamics.

An anomaly also appears when the Yang-Mills field interacts with a linear combination of vector and axial currents, as, for example, in the standard SU(2)×U(1) theory of electroweak interactions. Since in the presence of anomalies gauge invariance and, therefore, unitarity break down in the gauge theory, the theory with anomalies is inconsistent. In particular, the usual SU(2)×U(1) theory makes sense only for interaction with quarks, when the anomalies cancel. An anomaly cannot be cancelled by adding local counter terms to the Lagrangian.

We shall present the basic formulas for four-dimensional anomalies.[88-91]

Abelian anomalies arise as a result of the breaking, at the quantum level, of the symmetry $\psi(x)\to e^{iu(x)\gamma^5}\psi(x)$ of the classical Lagrangian describing the interaction of massless fermions with a Yang-Mills field. In the quantum case the divergence of the axial current $j^A_\mu=\bar\psi\gamma_\mu\gamma_5\psi$ is equal to

$$\partial_\mu j^{\mu A}=-\frac{1}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}\operatorname{tr}F_{\mu\nu}F_{\rho\sigma}.$$

The appearance of abelian anomalies can be interpreted as noninvariance of the fermion measure in the path integral[92] under a chiral transformation.

Nonabelian anomalies also arise in the triangle diagram with massless fermion lines and are associated with the breaking of the nonabelian symmetry $\psi(x)\to e^{iu(x)\gamma^5}\psi(x)$, $u(x)=u^a(x)\lambda^a$ at the quantum level; λ^a are the generators of the algebra of the gauge group G.

We shall study the interaction of the Yang-Mills fields $A_\mu(x)$ with chiral (left) fermions:

$$\exp[i\Gamma(A)]=\int d\psi\, d\bar\psi \exp\left[i\int\bar\psi\left(\tilde\partial+\tilde A\frac{1+\gamma_5}{2}\right)\psi\, d^4x\right],$$

$$\tilde\partial=i\gamma^\mu\partial_\mu,\quad \tilde A=i\gamma^\mu A_\mu,\quad A_\mu=\lambda^a A^a_\mu,\quad \bar\psi\lambda^a\psi=\bar\psi^i\lambda^a_{ij}\psi^j. \tag{2.1}$$

Under the gauge transformation $A_\mu\to A_\mu+D_\mu u$ (D_μ is a covariant derivative) the functional $\Gamma(A)$ transforms as

$$\Gamma(A)\to\Gamma(A+Du)=\Gamma(A)+\mathfrak{A}(A,u), \tag{2.2}$$

$$\mathfrak{A}(A,u)\equiv\delta_u\Gamma(A)=-i\int d^dx\operatorname{tr}\left(u(x)D_\mu\frac{\delta\Gamma}{\delta A_\mu(x)}\right), \tag{2.3}$$

i.e., the generator of the gauge transformations on functionals of A is $-iD_\mu\delta/\delta A^a_\mu$. If $\Gamma(A)$ were gauge invariant, as follows from formal transformations, then we would have $D_\mu\delta\Gamma/\delta A^a_\mu(x)=0$. But, a careful calculation taking into account regularization gives the following result:

$$D_\mu\frac{\delta\Gamma}{\delta A^a_\mu(x)}=-\frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}\partial_\mu\operatorname{tr}\left[\lambda^a\left(A_\nu\partial_\rho A_\sigma+\frac{1}{2}A_\nu A_\rho A_\sigma\right)\right]. \tag{2.4}$$

The formula (2.4) leads to a violation of covariant current conservation in the presence of an external gauge field:

$$D_\mu j^{\mu a}=D_\mu\left\langle\bar\psi\gamma^\mu\lambda^a\left(\frac{1+\gamma^5}{2}\right)\psi\right\rangle=D_\mu\frac{\delta}{\delta A^a_\mu}\Gamma\neq0.$$

The formula (2.4) shows that there is no invariance relative to infinitesimal gauge transformations. With respect to the gauge transformations $A_\mu\to A^g_\mu=g^{-1}(x)(A_\mu-i\partial_\mu)g(x)$ $\Gamma(A)$ transforms as follows:

$$\Gamma(A^g)-\Gamma(A)=-2\pi\int_{\mathbb{R}^{d+1}_+}d^{d+1}x\,[\omega_{d+1}(A^g)-\omega_{d+1}(A)]; \tag{2.5}$$

where $d=4$ and the integration is performed over $\mathbb{R}^5_+=\{(x^\mu,x^5)|x^5\geqslant0\}$; $\omega_{d+1}(A)$ is the density of the Wess-Zumino action (it is the $(d+1)$-dimensional Chern-Simons class); for $d=4$ the following formula holds:

$$\omega_5(A)=-\frac{i}{192\pi^3}\varepsilon^{pqrst}\operatorname{tr}\left(F_{pq}F_{rs}A_t-F_{pq}A_rA_sA_t+\frac{2}{5}A_pA_qA_rA_sA_t\right). \tag{2.6}$$

The formula (2.5) contains gauge fields which depend on the points of a $(d+1)$-dimensional space, and any continuation of the fields $A_\mu(x)$ into the $(d+1)$-dimensional space can be used (if there are no topological obstacles, which can lead to quantization of physical parameters; see Refs. 89-91, 93 for a more detailed discussion).

The appearance of geometrical characteristics in the formula (2.5) is not accidental. The appearance of anomalies is itself a result of the nontrivial topology of the space of orbits of the gauge group. The normalization of a nonabelian anomaly is fixed by the normalization of an abelian anomaly, which represents a local variant of the Atiyah-Singer index theorem.[96,91] An interpretation of anomalies in terms in cocycles is given in Ref. 94.

a) Chiral anomalies in higher dimensions

Theories of the Kaluza-Klein type, which lead to chiral fermions, are potentially anomalous. They are comprehensible only if the anomalies cancel. In order to follow the cancellation of anomalies, their explicit form must be known, i.e., formulas of the type (2.4) must exist. Gauge anomalies in a multidimensional space have recently been calculated.[88-91] We shall present below the derivation of the formulas for nonabelian anomalies in a $2n$-dimensional space, using the language of differential forms.[7)] Since $\delta_u\Gamma(A)=\mathfrak{A}(u,A)$ corresponds to an infinitesimal transformation of the gauge group, the following condition must hold (the Wess-Zumino conditions[95]):

$$\delta_u\mathfrak{A}(A,v)-\delta_v\mathfrak{A}(A,u)=\mathfrak{A}(A,[u,v]). \tag{2.6'}$$

The condition (2.6) imposes nontrivial restrictions on the possible form of the anomalies. We shall show that the following functional of A and u satisfies Eq. (2.6):

$$\mathfrak{A}(A,u)=\int_{M^{2n}}d^{-1}\delta_u d^{-1}\Omega_{2n+2}(A). \tag{2.7}$$

Let us clarify the notation[8)] used in (2.7): the $\Omega_{2n+2}(A)-(2n+2)$ form in the $(2n+2)$-dimensional space (two auxiliary variables are introduced) belongs to the Chern-Pontryagin class,

$$\Omega_{2n+2}(A)=\operatorname{tr}F^{n+1},\qquad F=\frac{1}{2}F_{MN}\,dx^M\wedge dx^N; \tag{2.8}$$

$d^{-1}\Omega_{2n+2}(A)$ is a $(2n+1)$-form $\omega^0_{2n+1}(A)$ such that

$$\Omega_{2n+2}(A)=d\omega^0_{2n+1}(A). \tag{2.9}$$

The form $\omega^0_{2n+1}(A)$ exists, since the condition of local solvability $d\Omega_{2n+2}=0$ evidently holds.

The expression $\delta_u\omega^0_{2n+1}(A)$ denotes variation by an infinitesimal gauge transformation. In order for the application of the operator d^{-1} to $\delta_u\omega^0_{2n+1}(A)$ to make sense, we verify that $d\delta_u\omega^0_{2n+1}=0$. Indeed,

$$d\delta_u\omega^0_{2n+1}=\delta_u\, d\omega^0_{2n+1}=\delta_u\Omega_{2n+2}=0$$

because of the gauge invariance of $\Omega_{2n+2}(A)$. Therefore there exists a $2n$-form $\omega^1_{2n}(A, u)$, such that

$$d\omega^1_{2n}(A, u) = \delta_u \omega^0_{2n+1}(A). \tag{2.10}$$

We shall show that (2.7) solves (2.6). We have

$$\delta_u\delta_v\omega^0_{2n+1}(A) = (\text{due to (2.10)}) = \delta_u\, d\omega^1_{2n}(A, v)$$
$$= (\delta_u \text{ and } d \text{ commutate}) = d\delta_u\omega^1_{2n}(A, v),$$

and therefore the identity

$$(\delta_u\delta_v - \delta_v\delta_u - \delta_{[u,v]})\,\omega^0_{2n+1}(A) = 0$$

can be written in the form

$$d\delta_u\omega^1_{2n}(A, v) - d\delta_v\omega^1_{2n}(A, u) - d\omega^1_{2n}(A, [u, v]) = 0. \tag{2.11}$$

Integrating the relation (2.11) over the $(2n+1)$-dimensional manifold whose boundary is M^{2n}, because of Stokes formula we obtain the relation

$$\int_{M^{2n}} \delta_u\omega^1_{2n}(A, v) - \int_{M^{2n}} \delta_v\omega^1_{2n}(A, u) = \int_{M^{2n}} \omega^1_{2n}(A, [u, v]),$$

which proves that (2.7) solves (2.6).

The formula (2.6) is convenient for following the possible cancellation of the anomalies, since the expression (2.7) for the form Ω_{2n+2} has a simple form. Thus in order to reveal the cancellation of anomalies in the $2n$-dimensional theory, because of (2.6) it is sufficient to perform the calculations in a $(2n+2)$-dimensional space. We note that the trace in (2.8) is calculated in a representation corresponding to fermion fields. In supersymmetrical theories fermions acquire significance in the adjoint representation, and for these theories tr_{adj} enters in (2.8). On the other hand, the contribution of gauge fields to the Lagrangian of supersymmetrical theories is written as a trace in the fundamental representation, so that in the formula for Ω_{2n+2} it is convenient to recalculate tr_{adj} as the trace in the fundamental representation. In particular, for ($N = 1$, $d = 10$) Yang-Mills gauge theory we have

$$\Omega^{\mathrm{YM}}_{12} = 15\,\mathrm{tr}\,F^2\,\mathrm{tr}\,F^4 + (n-32)\,\mathrm{tr}\,F^6, \qquad G = \mathrm{SO}(n), \tag{2.12a}$$

$$\Omega^{\mathrm{YM}}_{12} = \frac{1}{7200}(\mathrm{tr}\,F^2)^3, \qquad G = \mathrm{E}_8 \times \mathrm{E}_8. \tag{2.12b}$$

We note that Eq. (2.7) determines $\mathfrak{A}(A, u)$ to within a coefficient, which can be found directly from the Feynman diagrams or with the help of Fujikawa's method.[92] The normalization of abelian anomalies, which is most simply calculated using the Atiya-Singer index theorem, also fixes the normalization of the nonabelian anomalies.[96,91]

b) Gravitational anomalies

Alvarez-Gaume and Witten discovered in spaces with more than four dimensions a new type of anomaly with purely gravitational vertices, corresponding to the breakdown of general-coordinate invariance.[97] Purely gravitational anomalies appear in spin-1/2 and spin-3/2 Weyl spinors in spaces with dimensions $d = 4k + 2$, $k = 0, 1, 2, \ldots$ .The boson antisymmetrical field B_{MN} also contributes to the gravitational anomaly.

We shall present the answer for the form $\Omega^{\mathrm{Gr}}_{12}$, corresponding to the 10-dimensional purely gravitational anomaly in ($N = 1$)-supergravity, interacting with matter:

$$\Omega^{\mathrm{Gr}}_{12} = -\left[\frac{1}{32} + (n-496)/13\,824\right](\mathrm{tr}\,R^2)^3$$
$$- \left[\frac{1}{8} + (n-496)/5760\right]\mathrm{tr}\,R^2\,\mathrm{tr}\,R^4$$
$$- [(n-496)/7560]\,\mathrm{tr}\,R^6; \tag{2.13}$$

where n is the number of left spin-1/2 fermions from the matter multiplet.

Mixed anomalies, corresponding to diagrams in which gauge fields and the energy-momentum tensor appear at the vertices, also arise.

As shown in Ref. 97, for ($d = 10$, $N = 2$) chiral supergravity the anomalies are immediately cancelled.

c) Cancellation of anomalies

Remarkably, for ($N + 1$, $d = 10$)-supergravity interacting with an SO(32)- or $\mathrm{E}_8 \times \mathrm{E}_8$ gauge field (these models are obtained in the low-energy limit of a type I string) some of the terms in (2.12) and (2.13) vanish. The last term in (2.12a) vanishes for $n = 32$ and the last term in (2.13) vanishes for $G = \mathrm{SO}(32)$ and $E_8 \times E_8$, since $n = 496$ is the dimensionality of the adjoint representations of SO(32) and $\mathrm{E}_8 \times \mathrm{E}_8$. As shown in Ref. 5, the remaining anomalous terms can be compensated by the addition of local terms to the starting action. These terms contain higher-order derivatives and can be obtained from the low-energy expansion of the superstring.

Cancellation of anomalies also occurs for ($N = 2$, $d = 6$)-supergravity interacting with multiplets of matter fields (see Table I) for the gauge group $\mathrm{E}_6 \times \mathrm{E}_7 \times \mathrm{U}(1)$.[125]

3. SUPERSTRINGS

The theory of superstrings has a paradoxical history. This theory arose in the study of hadron phenomenology, but it literally forced itself to be considered as a fundamental unified theory of elementary particles. At the end of the 1960s, the study of hadron interactions with the help of dispersion relations and Regge dynamics led to Veneziano's dual resonance model, which satisfactorily describes the phenomenology of mesons.[105]

The dual resonance amplitude can be represented with the help of a set of oscillators, and it was shown that these oscillators can be viewed as normal excitations of a relativistic string. The theory contains states with negative norm (ghosts), which are absent only if the space-time has a critical dimensionality $d = 26$ (it is possible that the inclusion of conformal anomalies[106] and the correct choice of the ground state[107,128] can lower this dimensionality). The appearance of an unphysical dimensionality led later to a revival of the Kaluza-Klein idea. The fermion degrees of freedom were included in the Raymond-Neveu-Schwarz model,[108] for which the critical dimensionality is $d = 10$. This model was one of the sources of the modern theory of supersymmetry. However, the string model of hadrons, though it was intuitively attractive (the idea of quarks connected by a string is very convenient), encountered the following problems. First of all, to eliminate ghosts the parameter of the model (the slope coefficient in the Regge trajectory α') must have an unphysical value. Second, particles with imaginary mass—tachyons—remain in the theory even with critical dimensionalities. Finally, the theory contains spin-2 massless hadrons, which do not exist in nature.

A radically new interpretation enabling the solution (or at least circumvention) of these problems of string models was made by Scherk and Schwarz,[9,109] who showed that the spin-2 particles can be viewed as gravitons, and the theory of strings can be viewed as a unified theory of elementary particles, including gravity. In this case, the characteristic length scale of the theory $L\sim(\alpha')^{-1/2}$ can be naturally assumed to be of the order of the Planck length, $L\sim 10^{-33}$ cm (while for the string model of hadrons $L\sim 10^{-13}$ cm). This program was developed by Schwarz and Green in the theory of superstrings.[19] They constructed two types of explicitly supersymmetrical theories of superstrings (differing by the values of the supercharges) without ghosts and tachyons with $d=10$. Each theory of superstrings has a massless sector plus an infinite set of massive excitations with arbitrarily large spins. The massless sector has the same set of states as some supersymmetrical field model (with $d=10$) into which the superstring transforms in the low-energy limit ($\alpha'\to 0$). Superstrings of the first kind can be open (with free ends) or closed. The interaction of the massless excitations of open strings in the low-energy limit is described by the supersymmetrical Yang-Mills theory and that of closed strings is described by ($N=1$, $d=10$)-supergravity. Each of these field models has chiral anomalies. However, in the low-energy field theory, describing both closed and open superstrings of type I, the anomalies cancel for the gauge groups SO(32) and $E_8\times E_8$.[5] Type-II superstrings can only be closed. The massless sector in the low-energy limit is described by ($N=2$, $d=10$)-supergravity. The anomalies here also cancel.[97]

An important property of superstring theories is the absence of ultraviolet divergences. These theories are finite at the one-loop level and there are grounds for expecting finiteness in higher-order loops also.[6] The absence of divergences has formally a simple explanation. A superstring has two dimensional constants—the gravitational constant $\varkappa$ and the Regge parameter α'. The parameter α' essentially plays the role of a cutoff parameter, and for $\alpha'>0$ we have a finite string theory. In the limit $\alpha'\to 0$ a renormalizable field theory which includes gravity is obtained. The physical reason for the absence of divergences lies in the fact that we are dealing with an extended object.

The theory of superstrings, as it is now understood, must be studied in the spirit of the Kaluza-Klein approach and is not directly related to hadronic strings, from which it originated.[9)] A satisfactory relationship can be imagined as follows. Quantum chromodynamics must be obtained from the superstring, and a hadronic string must be obtained from QCD in the limit of a large number of colors.[112]

It is well known that in order to satisfy the condition of causality, the interaction of two extended objects must be local.[16] Vacuum fluctuations prevent the interactions from being local and, generally speaking, lead to a breakdown of causality. If, however, the model is supersymmetrical, then the vacuum fluctuations are cancelled, and interactions can be taken into account systematically.

a) Free strings

1) Boson string

In d-dimensional space-time M^d with the coordinates x^M, $M=0,1,\ldots,d-1$, and the metric $g_{MN}(x)$ the world surface of the string is given by the equation $x^M=x^M(\sigma,\tau)$, where σ and τ are space- and time-like coordinates, respectively. The action for the string has the form

$$S=-\frac{1}{2}T\int_0^\pi d\sigma\int_{\tau_i}^{\tau_f} d\tau e\eta^{\alpha\beta}(\zeta)\,\partial_\alpha x^M(\zeta)\,\partial_\beta x^N(\zeta)g_{MN}(x(\zeta)); \tag{3.1}$$

where $\zeta=(\sigma,\tau)$, $\eta^{\alpha\beta}(\zeta)$ is the metric on the world surface of the string, $\partial_\alpha=(\partial/\partial\sigma,\partial/\partial\tau)$, and $T\sim 1/\alpha'$ is a dimensional parameter (the tension of the string). The metric g_{MN} is, generally speaking, nontrivial; it should be determined dynamically from the extremum of the effective action.[19,107,115,116] In the theory of superstrings, however, only the case when M^d is a Minkowski space with the metric $g_{MN}=\mathrm{diag}(-1,+1,\ldots,+1)$ (as well as the case of compactification on a 6-dimensional torus[19]), to which we shall adhere below, has been analyzed in detail.

$$S=-\frac{1}{2}T\int_0^\pi d\sigma\int d\tau\eta^{\alpha\beta}\partial_\alpha x^i(\zeta)\,\partial_\beta x^i(\zeta). \tag{3.2}$$

The solution of the classical equations following from the action (3.2) for an open string ($\partial_\sigma x^i=0$ with $\sigma=0,\pi$) can be written in the form

$$x^i(\sigma,\tau)=x^i+p^i\tau+i\sum_{n\neq 0}\frac{1}{n}\alpha_n^i\cos n\sigma e^{-in\tau},$$

$$x^+(\sigma,\tau)=\frac{1}{\sqrt{2}}(x^0+x^9)=x^++p^+\tau;$$

where x^i, p^i, α_n^i, x^+, p^+ are constants.

Canonical quantization leads to the commutation relations

$$[\alpha_m^i,\alpha_n^j]=m\delta_{m+n,0}\delta^{ij}.$$

The mass operator is

$$M^2=2\sum_{n=1}^\infty\alpha_{-n}^i\alpha_n^i$$

and has a tachyon as the lowest eigenstate. The Poincaré algebra is closed only in $d=26$ dimensions. The appearance of this critical dimensionality could be related to the quantization procedure in the light-like gauge, and the possibility that it will somehow be possible to quantize the string directly in four-dimensional space-time has not been excluded.

2) The free superstring

The action for a superstring in a light-like gauge has the form

$$S = -\frac{1}{2}\int_0^{\pi} d\sigma \int d\tau \left(\eta^{\alpha\beta}\partial_\alpha x^i \partial_\beta x^i + \frac{i}{2p^+}\lambda^a \rho^\alpha \partial_\alpha \lambda^a\right); \quad (3.3)$$

where p^+ is a constant.

We shall study a superstring in the critical number of dimensions $d = 10$. The action (3.3) is SO(8) invariant. We recall that the SO(8) algebra has three nonequivalent 8-dimensional representations, one vector and two spinor. We shall use the indices i and j for the vector representation, a and b for one spinor representation, and $\dot{a}$ and $\dot{b}$ for the other spinor representation. In the expression (3.3) ρ^α are 2×2 Dirac matrices (Pauli matrices), the Fermi field λ^{aA} has a two-component spinor index A and an eight-component spinor index a. The solution of the equations of motion for λ^{aA} has the form

$$\lambda^{a1} = \sum_{n=-\infty}^{\infty} Q_n^a e^{-in(\tau-\sigma)}, \quad \lambda^{a2} = \sum_{n=-\infty}^{\infty} Q_n^a e^{-in(\tau+\sigma)}, \quad (3.4)$$

and the anticommutation relations have the form

$$\{Q_m^a, Q_n^b\} = 2p^+\delta_{m+n,\,0}\delta^{ab}.$$

In this case there is no tachyon, the lowest states of the mass operator form a massless supermultiplet consisting for open type-I strings of a vector A^i and a spinor $\psi^{\dot a}$, which coincides with the Yang-Mills multiplet in a space with $d = 10$. It may be assumed that these states transform according to some representation of an arbitrary gauge group.

For closed strings with periodic boundary conditions the solutions of the equations of motion are written in the form

$$x^i(\sigma,\tau) = x^i + p^i\tau + \frac{1}{2}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^i e^{-2in(\tau-\sigma)} + \tilde{\alpha}_n^i e^{-2in(\tau+\sigma)}\right), \quad (3.5)$$

$$\lambda^{a1}(\sigma,\tau) = \sum_{n=-\infty}^{\infty} Q_n^{1a} e^{-i2n(\tau-\sigma)},$$

$$\lambda^{a2}(\sigma,\tau) = \sum_{n=-\infty}^{\infty} Q_n^{2a} e^{-i2n(\tau+\sigma)}.$$

There are two supersymmetry generators (supercharges), which in this case coincide with Q_0^{1a} and Q_0^{2a}. If both supercharges are conserved and belong to the same representation of the SO(8) group, then such a (closed) string is called a type IIb string. This theory is a chiral theory (mirror antisymmetrical). The superstring is said to be of type IIa if the supercharges belong to different spinor representations of SO(8). Finally, if only one of the supercharges is conserved, then such a (closed) superstring is of type I.

The massless spectrum of a type-I closed superstring has the following composition (with $d = 10$): one graviton g^{ij}, one antisymmetrical tensor B^{ij}, one scalar φ, one spinor ψ^a, and one Rarita-Schwinger state ψ^{ai}. The answer to the question "What is the spectrum of the massless states in the four-dimensional space-time?" depends on the compactification procedure. For the simplest compactification onto a 6-dimensional torus an extensive set of massless particles, not observed in nature, is obtained.

b) Interacting strings

Interaction between strings is introduced very simply. Two strings can either join at their ends, forming a third string, or a string can separate into two strings. The orientation of the strings is taken into account. The orientation is related to the direction of the wave traveling along the closed string. The substitution $\sigma\to\pi-\sigma$ corresponds in the formulas (3.5) to the interchange $\alpha_n \leftrightarrow \tilde{\alpha}_n$, $Q_n^1 \leftrightarrow Q_n^2$. At the same time, for type-I strings nothing changes, and they turn out to be nonorientable, while for type-II strings the orientation changes. For an open string (type I) it is possible to introduce an internal isotopic symmetry group, if a "quark" and "antiquark" are placed on its ends. Joining the ends of such a string, we obtain a closed string; it will be oriented if "quarks" differ from "antiquarks," as, for example, for the group SU(n). On the other hand, type-I strings are nonorientable, and the isotopic group SU(n) is therefore forbidden. A theory of superstrings using only the gauge groups SO(n) and Sp($2n$) has been developed.[19,114]

However a theory of closed superstrings, which in the low-energy limit is described by ($d = 10$, $N = 1$)-supergravity interacting with a supersymmetrical Yang-Mills theory with the gauge group $E_8\times E_8$ or Spin $(32)/\mathbb{Z}_2$, was recently constructed.[74] This theory was obtained as a hybrid of a ($d = 10$) fermionic string and a ($d = 26$)-bosonic string with the help of the compactification of 16 extra bosonic coordinates onto the maximal torus with a definite radius ($26 - 10 = 16 = 8 + 8!$). This string does not have ghosts and tachyons and is free of anomalies.

Interacting superstrings are described by introducing field operators $\Psi[x^i(\sigma), \vartheta^s(\sigma), x^+, p^+]$ (for closed strings) and $\Phi^{ab}[x^i(\sigma), \vartheta^s(\sigma), x^+, p^+]$ (for open strings) in a manner analogous to the way the interaction of point particles is described by transforming from the primary quantum variables x, p to second-quantized field operators $\varphi(x)$; here $\vartheta^s(\sigma)$ are anticommuting functions, $s = 1,\ldots,8$; a and b are isotopic indices. Unlike the usual quantum fields, which depend on the point x, here Ψ and Φ are operator functionals. (Functionals of anticommuting functions can be defined mathematically in the spirit of Ref. 113.) The Hamiltonian corresponding to the dynamics in the variable x^+ [see (3.2)] is the sum of a free Hamiltonian and the interaction Hamiltonian $\mathscr{H}_{\rm int}$. For a type-II theory the Hamiltonian $\mathscr{H}_{\rm int}$ has an interaction of the form $\varkappa\Psi^3$:

$$\mathscr{H}_{\rm int} = \varkappa\int\left(\prod_{k=1}^{3} dp_k^+ \mathscr{D}^{16} z_k\right) \times \delta\left(\sum_k p_k^+\right)\Delta^{16}[z_1(\sigma)+z_2(\sigma)-z_3(\sigma)] \times K(\sigma)\Psi(1)\Psi(2)\Psi(3); \quad (3.6)$$

where $\varkappa$ is the gravitational constant,

$$z_k = (x_k^i(\sigma), \vartheta_k^s(\sigma)),$$

Δ^{16} is a δ function, the operator K contains variational derivatives, and $\Psi(1)$, $\Psi(2)$, $\Psi(3)$ are field operators of the corresponding arguments.

For open strings there are also terms of the type Φ^3, $\Phi\Psi$, Ψ^3, $\Phi^2\Psi$ and Φ^4.

The perturbation theory for superstrings is in many ways reminiscent of the old perturbation theory in the quantum theory of fields before the introduction of Feynman diagrams. A covariant formalism does not exist at the present time.

It is interesting that in contrast to quantum supergravity, where the interaction is a nonpolynomial function of $\varkappa$, in the theory of superstrings the interaction has the simple form (3.6).[10)] There are no ultraviolet divergences (at least in the one-loop theory). The transition from gravity to the superstring can be compared to the transition from the unrenormalizable four-fermion weak interaction to the modern theory of the electroweak interaction.

Spontaneous compactification for the low-energy field theory of superstrings is discussed above in Sec. 1i. The solutions of the equations of the effective Green-Schwarz field theory of the form $adS^3 \times S^1 \times S^3 \times S^3$ are obtained in Ref. 129. Solutions of the form $M^4 \times \Pi^3 \times B^3$ where M^4 is Minkowski space, Π^3 is a compact manifold with constant negative curvature, and B^3 is a manifold with positive curvature, are constructed in this theory in Ref. 130. It is assumed that the six extra dimensions are time-like. The conditions on compactification, with which a satisfactory four-dimensional theory of massless fields without ghosts is obtained, are also discussed in Ref. 130.

The possible phenomenological consequences of the vacuum configuration proposed in Ref. 32 with the Ricci-flat Kähler manifold with SU(3) holonomy group, for which the starting $E_8 \times E_8$ gauge group is broken down to $E_6 \times E_8$, are examined in Ref. 131. It is shown that further breaking of gauge invariance by the Higgs mechanism and breaking of the supersymmetry with the help of the gluon condensate can lead to models which are in qualitative agreement with ($d = 4$, $N = 1$) low-energy theories of supergravity.[14]

Low-temperature field theories of superstrings contain terms with higher-order derivatives, which, generally speaking, can destroy unitarity. It is shown in Ref. 132 that for a certain ratio of the constants in front of these terms there are no ghosts.

CONCLUSIONS

In conclusion, returning to the problems formulated in Sec. 1 which the fundamental theory must solve, we shall list once again the models which seem to deserve the greatest attention at the present time.

1) Maximally symmetrical models of supergravity: $d = 11, N = 1$ and $d = 10, N = 2$. The problem of chiral fermions and of the cosmological constant remain unsolved.

2) The gauge chiral ($d = 6$, $N = 2$) model. Here there is a compactification on a Minkowski space, chiral fermions can be obtained, and cancellation of anomalies has been shown for some gauge groups. However, classical stability of the vacuum solution is obtained only for two generations of fermions.

3) ($d = 10$, $N = 1$)-supergravity, interacting with SO(32)- or $E_8 \times E_8$-gauge fields, free of anomalies. This theory could be realistic in the sense that it could lead to an effective four-dimensional SU(5) gauge theory with an arbitrary number of generations.

All these theories have one other drawback in common: they are formally nonrenormalizable. It can be stated with some degree of confidence that this problem is solved in the theory of superstrings.

4) The theory of superstrings makes possible the solution of problems associated with quantum numbers—the problem of chiral fermions and the problem of generations—and eliminates divergences in quantum gravity. It should be emphasized that a gauge group ($E_8 \times E_8$ or SO(32)) has been distinguished for the first time on the basis of theoretical considerations.

Important dynamic questions such as spontaneous compactification, spontaneous breaking of gauge symmetry, calculation of a realistic mass spectrum, and other problems remain unsolved.

At the present time, however, the theory of superstrings appears to be very attractive as a possible candidate for a unified theory of elementary particles. This theory is capable of providing qualitative answers to many questions in the physics of elementary particles. Only the future can tell whether or not a quantitative theory agreeing with experiment can be constructed.

[1)]In this short paper, we restrict the citations largely to recent publications and reviews, without considering the history of the problem.

[2)]We have in mind the ($N = 4$) supersymmetrical Yang-Mills theory (see Ref. 3) and its generalizations of the soft-inclusion of mass type.[4]

[3)]The method of the inverse problem, hidden symmetry, and higher conservation laws in superspace for a supergravity-like ($N = 4$) supersymmetrical Yang-Mills theory are studied in Refs. 110, 98, 20, 99, and 117. The supersymmetrical formulation of this theory is given in Ref. 100. The geometrical formulation of ($N = 1$) supergravity is given in Refs. 101 and 102.

[4)]More precisely: if $-\lambda_k, k = 1, 2, \ldots$, are the eigenvalues of the Laplace operator with the Dirichlet or Neumann boundary conditions on an n-dimensional compact Riemannian manifold with volume v, then the estimate $\lambda_k \geqslant c_n (k/v)^{2/n}$ holds for all k.[127]

[5)]Compact spaces with negative curvature appear in the spontaneous reduction of the theory with ghost fields.[30]

[6)]We note here that the existence of additional space-time dimensions in the Kaluza-Klein approach into which an observer can be placed even in an imaginary way, apparently opens up the possibility for a new interpretation of quantum theory, different from the Copenhagen interpretation and from Everett's interpretation.[124]

[7)]As an illustration we present the Maxwell equations in the language of differential forms. We introduce the one-form $A = A_\mu dx^\mu$ and the two-form $F = (1/2)F_{\mu\nu} dx^\mu \wedge dx^\nu$. Then the relation $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is written in the form $f = dA$. The equation $\partial_\mu F_{\nu\lambda} + \partial_\lambda F_{\mu\nu} + \partial_\nu F_{\lambda\mu} = 0$ will have the form $dF = 0$, and the equation $\partial_\mu F^{\mu\nu} = 0$ is written in the form $d^*F = 0$, where $^*F$ is the two-form dual to F. The Yang-Mills equations can be written in an analogous manner. The exterior product of differential forms with matrix coefficients is formed according to natural rules.

[8)]We shall use the notation of S. P. Novikov[93]; it is natural to denote by d^{-1} the operator inverse to the operator of exterior differentiation d.

[9)]Freund[126] proposed that the 10-dimensional superstring can be obtained from the 26-dimensional boson string.

[10)]It has been hypothesized that the theory of interacting superstrings may turn out to be exactly solvable in some sense[110] and can have a hidden symmetry.[6]

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