Supersymmetric models of elementary particles—the physics for new-generation accelerators?

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A review of elementary-particle models based on low-energy supersymmetry is given. Supersymmetry is used in these models to compensate the quadratic divergence in the radiative correction to the mass of the Higgs boson in the Weinberg-Salam model and to solve the hierarchy problem in grand unification theories. The standard low-energy $SU(3) \otimes SU(2)_L \otimes U(1)$ theory is replaced in this approach with the supersymmetric theory. A large number of new particles with masses M_W is predicted. The properties of these particles and possible experimental searches for them are discussed. Models based on different ways of supersymmetry breaking are considered. The central place is given to models with supersymmetry breaking due to supergravity, which have been particularly actively discussed in the last two years. Proton decay in supersymmetric GUT models is analyzed. Introductory ideas on supersymmetry, necessary for the understanding of most of this review, are presented in Section 1 and in the Appendix.

CONTENTS

1. INTRODUCTION

Different symmetries have always played a major role in the development of elementary-particle physics. It appears that it all started with the good old isotopic symmetry generalized by Gell-Mann and Ne'eman to SU(3) symmetry—the so-called eightfold way. The next to appear was chiral $SU(2)_L \times SU(2)_R$ symmetry, which lies at the basis of current algebra. Local symmetries developed at the same time: non-Abelian Yang-Mills theories grew out of the quantum-electrodynamic U(1) theory, namely, the $SU(2)_L \times U(1)$ theory of weak interactions and the colored SU(3) theory of strong interactions. The internal symmetries enumerated above stand under the umbrella of the Poincaré symmetry of space-time: all reasonable models of particle interaction must be Poincaré-invariant.

A. The Kaluza-Klein program

There has long been a tendency to erase the boundary between internal and spatial symmetries. The first step in this direction was made by Kaluza and Klein,[1] who noted that the local Poincaré invariance of the extended (5-dimensional) space-time can be implemented in the form of the general covariance of the 4-dimensional manifold + electrodynamics [local U(1) invariance] in the same 4-dimensional space. In the Kaluza-Klein approach, the "extra" dimensions were looked upon as compact, with a small radius of the order of the reciprocal of Planck's mass. The symmetries of these compact manifolds show themselves, at distances much greater than their radius, as internal gauge symmetries (in the case of five dimensions, one of the "extra" dimensions closes into a ring and leads to U(1) electrody-

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namics; $U(1)\times U(1)$ symmetry can be realized in 6-dimensional space and, possibly, $SU(2)$ symmetry can be realized by closing two extra coordinates into a sphere, and so on). At Planck distances, dynamics is complicated. In particular, states corresponding to motion in the inner space begin to be excited and have masses of the order of Planck's mass (the spectrum of these states is infinite).

The idea of geometrization of internal symmetries looks particularly attractive at the present time, since additional local symmetries arise naturally in the Kaluza-Klein approach and we know that gauge $SU(3)\times SU(2)\times U(1)$ invariance lies at the basis of low-energy (<100 GeV) dynamics. At first sight, rigorous geometrization à la Kaluza-Klein is impossible for the following simple reason: only boson fields can be obtained from the metric, while all matter that surrounds us consists of fermions. This shortencoming of the entire approach was pointed out by Pauli in his footnote to the English-language edition of his book.[2] But it is precisely at this point that supersymmetry[3] (SUSY) comes to our aid. According to SUSY, each boson found in nature (or, more precisely, in the fundamental Lagrangian of all fields and interactions) has a corresponding fermion. Supersymmetry extends the Poincaré group and is, in this sense, spatial. Its structure is most clearly defined in superspace which, in addition to the boson coordinates x_μ, contains anticommuting (Grassman) fermion coordinates θ_α. We thus have the basic possibility of constructing all fundamental particles out of the extended superspace metric. This path leads to considerable difficulties (in particular, it is difficult to obtain chiral fermions[4]). Nevertheless, the program as a whole looks very tempting and—this is an important point for us—SUSY is the necessary component of modern attempts to construct unified field theories according to the Kaluza-Klein recipe.

To avoid misunderstanding, we emphasize that the unified SUSY theory ($N=8$ supergravity in 4-dimensional space?) may turn out to have no relation to the Kaluza-Klein program.

B. The problem of hierarchies

At the end of 1982 and the beginning of 1983, experiments performed on the CERN collider resulted in the discovery of the W and Z bosons. These are intermediate vector bosons responsible for weak interactions, and their masses ($M_W = 80.9 \pm 1.5$ GeV, $M_Z = 95.6 + 1.4$ GeV) are equal, to within experimental error, to the values predicted by the $SU(2)_L\times U(1)$ theory of Glashow, Weinberg, and Salam. In addition to the vector W and Z bosons, the $SU(2)_L\times U(1)$ theory of weak interactions predicts the existence of a further type of particle, namely, the scalar Higgs bosons (in the minimal variant, there is only one neutral Higgs boson). The discovery of these particles is awaited with even greater impatience than the discovery of the vector bosons. Scalar fields are essential for $SU(2)\times U(1)$ symmetry breaking and the assignment of nonzero mass to the vector bosons in a renormalizable manner. At the same time, elementary scalar fields lead to internal difficulties for the theory. When radiative corrections are taken into account, all the constants of the fundamental $SU(3)\times SU(2)_L\times U(1)$-symmetric Lagrangians are renormalized logarithmically, which means that they are not very sensitive to details of the dynamics at ultrahigh energies (of the order of the grand unification mass $M_{GUM}\approx 10^{15}$ GeV or the Planck mass $M_P \approx 10^{19}$ GeV). Actually, the constants change only by a factor of a few units in the mass range 100–10^{19} GeV. The so-called "decoupling" of low-energy and high-energy parts of the theory takes place. The only exception is the mass of the scalar field, which is renormalized quadratically, so that the nucleating mass (~ 100 GeV) acquires the increment $\sim gM_P$ and is pulled up into the high-energy scale. At the same time, we recall that it is precisely the negative quadratic mass of the Higgs field, $M_H^2 \sim -(100\ \text{GeV})^2$, that gives rise to the $SU(2)_L\times U(1)$ symmetry breaking and determines the order of magnitude of the W and Z boson masses. The fundamental Higgs field of the Weinberg-Salam model disrupts the scale decoupling and implicates the weak interaction in the Planck mass scale (or the grand unification mass scale which is equally inadmissible). This is the essence of the so-called "problem of hierarchies."[5] In the theory involving the fundamental scalar particles, mass hierarchy at the

$$g^2 \sim 10^{-2} < M_1^2/M_2^2 < g^{-2} \sim 10^2$$

level is natural, but the required hierarchy is $M_P^2/M_W^2 \sim 10^{34}$.

The masses of the spinorial particles are renormalized logarithmically (the simple example is for the renormalization of the electron mass in quantum electrodynamics). In SUSY theory, the boson and fermion masses are degenerate and there is no quadratic boson mass renormalization. This is the supersymmetric version of the solution of the hierarchy problem.[6] It has been under intensive investigation for the last three years, and the present review is devoted to this development. The cancellation of quadratic divergences in SUSY theory occurs when the contributions of loops in which bosons and fermions propagate are combined. SUSY ensures that the moduli of the corresponding integrals are equal and the fermion loops have the additional negative sign because of Fermi statistics.

As already noted, in SUSY models, a boson corresponds to each fermion, and vice versa. Each supermultiplet contains an equal number of bosons and fermions. Unfortunately, the known bosons (intermediate, vector, or Higgs) and fermions (leptons and quarks) cannot all be fitted into a single supermultiplet at the present stage of development of the theory. The number of fundamental particles must be doubled by associating a superpartner with each known particle.

Experiment shows no trace of the Fermi-Bose degeneracy, and this means that realistic models must contain a SUSY-breaking mechanism. Supersymmetry is introduced to achieve a cancellation of quadratic divergences, and is broken in order to describe the observed particle spectrum. This should not be too worrying because the symmetry-breaking idea has turned out to be very fruitful in particle physics, the last example being the discovery of the W and Z bosons. However, the following point is perplexing: while the previously-introduced symmetries predicted the existence of one or two new particles, the discovery of which was regarded as a triumph, SUSY models involve, at a stroke, as many new particles as there are old particles that we already know. Nevertheless, we shall try to provide some justification for the study of SUSY models. First, as we have already said, SUSY extends the group of space-time transformations and is therefore much more radical than internal symmetries. Second, SUSY is mathematically beautiful (it is sometimes compared with the beauty of non-Abelian gauge theories, which have completely changed the physics of elementary particles during the last decade). Third, the locally implemented SUSY (and since we have the principle that all fundamental symmetries must be local, it is natural to apply it to supersymmetry as well) is the only internally consistent theory of the interaction between point particles with spin 3/2 (gravitino) and is the most promising candidate for the final theory of gravity (other theories of gravity lead to uncontrollable divergences in loop corrections). Fourth, the ultraviolet behavior of global SUSY theories is much "softer" than that of ordinary field theories. We have already encountered one example of this: the absence of quadratic divergences in the mass renormalization of scalar particles. Another is the nullification in all loops of the Gell-Mann–Low β-function in a large proportion of SUSY theories.[7] Fifth, we recall the unification program for all particles and interactions according to the Kaluza-Klein mechanism with which we began this introduction. These arguments are sufficient to lead us to a systematic study of SUSY models of elementary particles and their predictions.

C. SUSY breaking

A SUSY-breaking theory is essential for the description of the real world. It is expected of any breaking scheme that the basic advantage of supersymmetry, namely, cancellation of quadratic divergences between boson and fermion loops, must remain in force. This cancellation must operate for masses ~ 100 GeV, so that in all the SUSY models that will be described, new particles will have masses characterized by this scale. Some new particles occasionally turn out to be much lighter, and have masses of the order of a few GeV.

Existing models employ three ways of supersymmetry breaking: explicit soft breaking, spontaneous breaking of global SUSY, and spontaneous breaking of local SUSY (supergravity). In the first of these, the globally supersymmetric Lagrangian is augmented with terms that explicitly break supersymmetry but do not lead to quadratic divergences. In the second method, the Lagrangian has global supersymmetry but the ground state is not supersymmetric, and bosons and fermions belonging to the same supermultiplets have different masses. The third method is based on the locally supersymmetric Lagrangian that includes both supermultiplets of matter and a supermultiplet of mutually compensating fields (graviton with spin 2 and gravitino with spin 3/2). The ground state is not supersymmetric and the effective low-energy ($\ll M_P$) renormalizable Lagrangian includes terms that produce soft supersymmetry breaking. A detailed discussion of models using these methods will be given below but, for the moment, we note that the first requires a very artificial choice of parameters in the Lagrangian, the second involves light scalar particles, not observed experimentally, and the third method enables us to construct models that satisfy all the phenomenological requirements. It is precisely models based on SUSY breaking by supergravity (SUGRA) that are the most popular.

D. New particles

We shall now list the new particles introduced in SUSY models, the experimental limits on their masses, and certain possible searches for them. The superpartners of known particles have the same quantum numbers with respect to internal $SU(3)\times SU(2)_L\times U(1)$ symmetries and differ from them only in their spins. As yet, there is no established terminology. We shall identify them by adding the ending "ino" and indicate them by the same letter as ordinary particles but with an added tilda. Quarkino is occasionally referred to as a squark, and leptino and higgsino as a slepton and shiggs, respectively.

We begin with quarks. The left-handed quark q_L and the complex scalar quarkino $\tilde{q}_L$ are members of the same supermultiplet $(q_L, \tilde{q}_L)$. This is referred to as a chiral supermultiplet. The field $\tilde{q}_L$ describes two degrees of freedom: a left-handed quark and an associated right-handed quark: $\overline{(q_L)} = (\bar{q})_R$. Two degrees of freedom also correspond to the complex field $\tilde{q}_L$ (the superscript L is indicated on the scalar field for convenience). The field $\tilde{q}_L$ is a member of a weak isodoublet and has a triplet index with respect to the color group and the same electrical charge as the spinorial quark. A further chiral multiplet consists of the right-handed quark q_R and the right-handed quarkino $\tilde{q}_R$: $(q_R, \tilde{q}_R)$. The chiral multiplet is indicated by an upper case letter, for example, $U_R\equiv(u_R, \tilde{u}_R)$, $Q_L\equiv[(u_L, \tilde{u}_L), (d_L, \tilde{d}_L)]$.

Chiral multiplets containing leptons have an analogous appearance: $E_R\equiv(e_R, \tilde{e}_R)$, $L_L\equiv[(\nu_L, \tilde{\nu}_L), (e_L, \tilde{e}_L)]$. Scalar leptons are collectively referred to as leptinos.

Gauge vector bosons are members of vector supermultiplets, for example, $(\gamma, \tilde{\gamma})$. The photino $\tilde{\gamma}$ is a Majorana (real) spinor and has two degrees of freedom ($\tilde{\gamma}_L$ and $\tilde{\gamma}_R$) in the same way as the massless photon. Analogous vector supermultiplets form the octet of gluons g^a and an octet of Majorana gluinos $\tilde{g}^a$ $(g^a, \tilde{g}_a)$, and the intermediate weak-interaction bosons $(W^\pm, \tilde{W}^\pm)$, $(Z, \tilde{Z})$, where $\tilde{W}^\pm$ is the $\tilde{W}$-bosino and $\tilde{Z}$ the $\tilde{Z}$-bosino. We shall use the designation "gaugino" for the superpartners of gauge bosons. Everything that we have said above in relation to gluons and gluinos is absolutely correct but, in relation to $\tilde{W}^\pm$, $\tilde{Z}$, and $\tilde{\gamma}$, we have oversimplified the situation to some degree. The point is that the Lagrangian contains as nucleating fields three vector supermultiplets of the SU(2) group $(W_i, \tilde{W}_i)$ and the gauge hypercharge multiplet $(B, \tilde{B})$. These multiplets form an irreducible representation of the massless superalgebra. The Higgs effect then takes place, and the field must be classified in accordance with the representations of massive superalgebra, which are formed by merging vector supermultiplets with chiral Higgs supermultiplets. We now proceed to the description of the latter.

Scalar Higgs bosons and spinorial higgsinos form chiral multiplets. In the Weinberg-Salam model with a minimal Higgs sector, there is one weak Higgs isodoublet, namely, (H^0, H^-). The corresponding chiral isodoublet is $[(\tilde{H}^0, H^0), (\tilde{H}^-, H^-)]$. A single chiral Higgs doublet is insufficient for a generalization of the $SU(3)\times SU(2)\times U(1)$ theory. This is most simply reflected in the way the charged W-bosinos obtain their mass. In the Higgs effect, a massive charged field W_μ^+ (six degrees of freedom) is obtained from two massless vector particles $W_{1\mu}$ and $W_{2\mu}$ (four degrees of freedom), and this occurs at the expense of one complex Higgs field. Two Majorana particles $\tilde{W}_1$ and $\tilde{W}_2$ (four degrees of freedom) form two massive Dirac particles ($\tilde{W}_I^+$ and $\tilde{W}_{II}^+$) (eight degrees of freedom), for which they must merge with two charged higgsinos. Thus, the minimum number of Higgs multiplets is two: H and $H' = [(\tilde{H}^{+\prime}, H^{+\prime}), (\tilde{H}^{0\prime}, H^{0\prime})]$ (the charges of the H′ are determined by the requirement of the absence of triangular anomaly in the hypercharge Y). The remaining charged scalar particle closes the representation of the massive superalgebra: $(\tilde{W}_\mu^+, \tilde{W}_I^+, \tilde{W}_{II}^+, \mathscr{H}^+)$. Eight boson degrees of freedom correspond to the eight fermion degrees of freedom. In the limit of exact SUSY, the masses of all these particles are equal. Neutral particles form three supermultiplets in the limit of exact SUSY: the massless vector supermultiplet (γ, $\tilde{\gamma}$), the massive vector supermultiplet (Z, $\tilde{Z}$, H) (H is one neutral scalar field), and the chiral supermultiplet ($\tilde{\mathscr{H}}, \mathscr{H}$).

We have enumerated all the particles forming part of the SUSY generalization of the standard $SU(3)\times SU(2)\times U(1)$ model. As noted above, the number of particles has been doubled, and there are new physical Higgs scalars as compared with the minimal set in the Weinberg-Salam model. We now proceed to the experimental limits on the masses of the new particles.

Charged scalar leptons must be created in pairs in e^+e^- annihilation, with subsequent decay into leptons and photinos (Fig. 1). Experimentally, this event looks like an e^+e^- annihilation into a pair of charged leptons (e^+e^- or $\mu^+\mu^-$) with uncompensated total momentum. The absence of such events leads to the lower bound for the leptino mass. For example, analysis of the experimental data obtained on the PETRA rings has yielded[8] the following lower bound: $M_{\tilde{l}} > 17.8$ GeV (assuming zero photino mass). The analogous creation and subsequent decay of the quarkino leads to the appearance of hadron jets with uncompensated $p_\perp$, which has produced the lower bound $M_{\tilde{q}} > 20$ GeV for the quarkino mass.

The question is: can these limitations on the leptino and quarkino masses be improved without raising the energy of the e^+e^- beams? It turns out[9a] that the answer is that this is indeed possible. Figure 2 illustrates the creation of a single

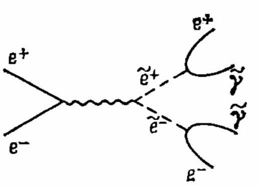

FIG. 1. Creation and subsequent decay of the leptino.

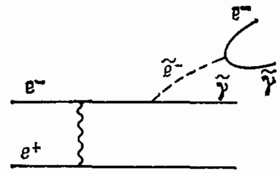

FIG. 2. Single creation of the electrino.

electrino. Such events were looked for on the PEP accelerator at a total beam energy of $\sqrt{s_{e^+e^-}} = 29$ GeV. Assuming that the photino rest mass was zero, it was found that[9b] $M_{\tilde{l}} > 22.4$ GeV. The imminent commissioning of the SLC accelerator at Stanford in the United States is expected to result in the discovery of superpartners of ordinary particles in e^+e^- annihilation.

There are at present no experimental limitations on the mass of the photino, which is not surprising because $\tilde{\gamma}$ is, in many ways, similar to the neutrino: it has no electric charge and a weak interaction (since it excites heavy intermediate states). Thus, superparticles decaying into $\tilde{\gamma}$ have not as yet been discovered, and experimental limits on its mass are still awaited. There are only indirect limitations that follow from cosmology and are analogous to the limitations on the neutrino mass. They are: $m_{\tilde{\gamma}} < 100$ eV or $m_{\tilde{\gamma}}$ greater[10] than a few GeV. If the mass of the photino were to lie in the forbidden window, there would be too many photinos after the Big Bang, which would have an inadmissible effect on the dynamics of the expansion of the Universe (it would be too young). As far as theoretical predictions are concerned, models can be divided into two classes, namely, those with light photinos with masses of a few GeV, and those with heavy photinos with masses of about 100 GeV.

Gluinos interact strongly and should therefore be created at a high rate in hadron-hadron collisions. Thus, in pp-collisions, the principal source of gluinos is the gluon-gluon annihilation (Fig. 3) where, because of color enhancement, gluinos should be created ten times as frequently as quarks of the same mass. Figure 4 shows the cross section for the production of gluinos in pp-collisions. If the photino is lighter than the gluino, the decay of the gluino (or, more precisely, of the new hadrons containing the gluino) should produce a quark-antiquark pair and a photino (Fig. 5). The lighter hadrons containing the gluinos are the fermions ($\tilde{g}g$) and ($\tilde{g}q\bar{q}$). If we neglect the photino mass, we find that the lifetime of the lightest gluinoball is

$$\tau \approx 10^{-11}\,(1\ \mathrm{GeV}/m_{\tilde{g}})^5 \times (M_{\tilde{q}}/100\ \mathrm{GeV}).^4$$

Photinos formed as a result of the decay of the gluino produce an excess of neutral currents in beam-dump experiments. These experiments show that[11] $m_{\tilde{g}} > 2$–3 GeV. Further progress in obtaining the limitations on $m_{\tilde{g}}$ (and, possibly, the discovery of the gluino) may come as a result of the analysis of jet creation in the CERN $p\bar{p}$-collider, or in other new-generation hadron machines. Another possible source of the gluino is the decay of the $^3P_1$ state of ($b\bar{b}$) quarkonium into the $\tilde{g}\tilde{g}$ pair and a gluon.[12] The theoretical predictions for $m_{\tilde{g}}$ are also very uncertain and range from a few (10?) GeV to 100 GeV.

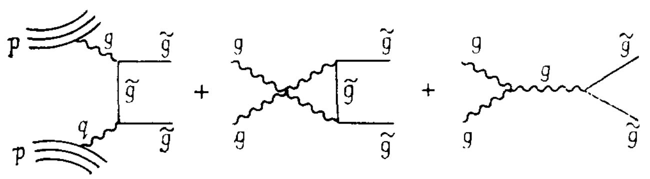

FIG. 3. Creation of the gluino in pp-collisions.

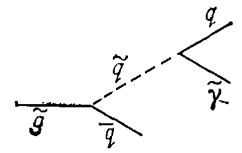

FIG. 5. $\tilde{g}\to p\bar{p}\tilde{\gamma}$ decay.

We now turn to the W-bosino. Weinberg[13] has noted that, under some very wide assumptions with regard to the SUSY-breaking mechanism, one charged Dirac particle turns out to be lighter than the W boson. If, in addition, the $\tilde{\gamma}$ contributes a few GeV, then $W^{\pm}\to\tilde{W}^{\pm}\tilde{\gamma}$ decay becomes allowed. Weinberg has found[13] that $\Gamma(W\to\tilde{W}\tilde{\gamma})/\Gamma(W\to e\nu)=0.6$. Since about 100 decays of the form $W\to e\nu$ have been seen in the CERN collider, the discovery of the $W\to\tilde{W}\tilde{\gamma}$ decay seems possible. Unfortunately, the signature of this decay is much poorer than that of $W\to e\nu$ because $\tilde{W}$ decays into $e\nu\tilde{\gamma}$ and the electrons are found to be relatively soft. If the phase space is available, the relative decay probabilities $\Gamma(W^{\pm}\to\tilde{W}^{\pm}\tilde{Z}^0)/\Gamma(W^{\pm}\to e^{\pm}\nu)$ and $\Gamma(Z^0\to\tilde{W}^+\tilde{W}^-)/\Gamma(Z^0\to e^+e^-)$ turn out to be of the order of unity.[14] The only question is whether the Z-bosino and the W-bosino are light enough.

E. Decay of the proton and the EDM of the neutron

Apart from a direct experimental confirmation of low-energy SUSY, i.e., the discovery of one (or, better still, several) of the above particles, indirect evidence for it is also possible. We shall consider two sources of such evidence, namely, the decay of the proton and the electric dipole moment (EDM) of the neutron. Standard grand unification schemes are dominated by the proton decay mode $p\to e^+\pi^0$. This type of decay has not been seen experimenally at a level corresponding to $\tau_p<10^{32}$ y, which is an argument against the simple grand unification scheme which predicts that $\tau_{p\to e^+\pi^0}=10^{28\pm 2}$. In supersymmetric grand unification models (SUSY GUT's), the predicted proton decay is rather different. The unification mass is greater because of the presence of a large number of new light particles ($M_{\rm GUM}\sim 10^{14}$ GeV, $M_{\rm SUM}\sim 10^{16}$ GeV), which leads to the lifetime $\tau_{p\to e^+\pi^0}\approx 10^{34}$ y, i.e., a figure that lies outside the limits of experimental possibilities. At the same time, a new proton decay mechanism[15,16] appears and is due to 5-dimensional operators. This decay occurs as a result of single-loop graphs in which the higgsino propagates. The decay of the proton through 5-dimensional operators was investigated in Refs. 17–20. It was found that the dominant mode was $p\to K^+\nu$, and the corresponding lifetime could be $\sim 10^{30}$ y for a reasonable choice of the parameters of the theory. This mode is difficult to detect experimentally and it is easier to look for $p\to K^0\mu^+$ decays. The 5-dimensional operators also lead to the decay to $K^0\mu^+$, but

$$\Gamma(p\to K^0\mu^+)/\Gamma(p\to K^+\nu)\sim 10^{-3},$$

i.e., the discovery of the $p\to K^0\mu^+$ decay and the absence of the $p\to K^+\nu$ decay, which is more likely by a factor of about 1000, would signify that the operators with $d=5$ bear no relation at all to this decay. In the standard model, there is a source for the $p\to K^0\mu^+$ decay, namely, a tree graph involving the exchange of the triplet of colored Higgs bosons. If, for some reason, the triplet Higgses were to turn out to be light ($M_{H_3}\sim 10^{11}$ GeV), they should induce the decay of the proton into $K^0\mu^+$, $K^+\nu_\mu$ in a time of $\sim 10^{30}$ y. The difference as compared with decays due to the $d=5$ operator is that, in the nonsupersymmetric scenario, we have

$$\Gamma(p\to K^0\mu^+)\approx\Gamma(p\to K^+\nu_\mu).$$

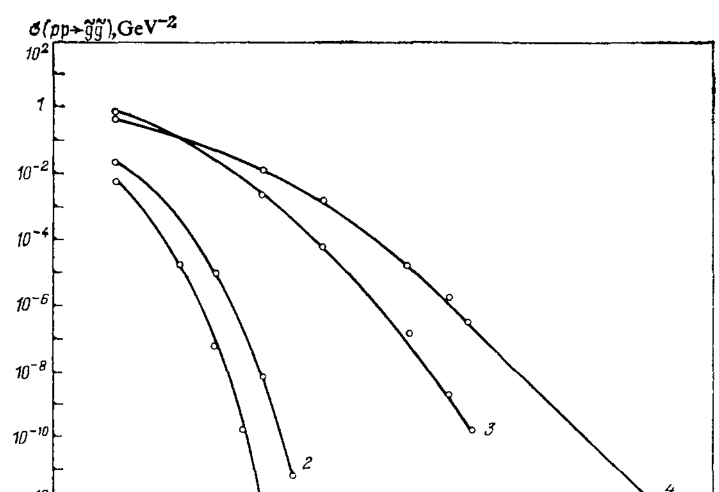

FIG. 4. Cross section for gluino production in pp-collisions as a function of their mass for different proton energies. 1—SPS, $\sqrt{s}=28$ GeV; 2—Tevatron, $\sqrt{s}=45$ GeV; 3—SPS-collider, $\sqrt{s}=540$ GeV; 4—Tevatron, collider, $\sqrt{s}=2000$ GeV. The $q\bar{q}$-annihilation must be taken into account in the case of the $p\bar{p}$-collider, so that curves 3 and 4 must be regarded as the lower bounds for the cross section.

In the standard SU(3)×SU(2)×U(1) model, there are two sources of the neutron EDM, namely, the CP-odd phase in the Kobayashi-Maskawa matrix and the θ-term. We shall assume that the effect of the θ-term is annulled by the presence of an axion, or for some other reason. The Kobayashi-Maskawa phase leads to[21] $d_n \sim 10^{-32}\, e\cdot$cm, whereas the modern experimental limit is[22] $d_n < 6\times 10^{-25}\, e\cdot$cm. In models with low-energy symmetry, there are new sources of the CP-odd phase that lead to EDM values at the level of the experimental limit.[23–26] Of course, the experimental confirmation of a nonzero EDM for the neutron would not be an unambiguous confirmation of SUSY: it can always be said that $\theta \sim 10^{-9}$ (for which $d_n \sim 10^{-25}\, e\cdot$cm).[27] However, such a low value of θ is not natural, and since $\theta \sim 1$ is natural but does not occur, there must be some θ-screening mechanism.

Our plan of presentation from now on is as follows. Section 2 gives general information about supersymmetry. An attempt has been made to make this review accessible to the reader who has not studied supersymmetry previously. The same purpose is served by the Appendix, to which all technical details have been relegated (this Appendix should be studied before reading the main part of this paper, contained in Sections 3–5). Section 3 discusses the mechanisms responsible for global SUSY breaking and models based upon them. The conclusion that follows from this section is mostly negative: it has not been possible to construct a phenomenologically acceptable model. Nevertheless, this material is presented not for historical reasons but because (a) spontaneous global SUSY breaking is of fundamental interest and deserves special examination and (b) in the only reasonable models with spontaneous breaking, based on geometric hierarchy, one can see the necessity for taking into account supergravity effects (SUGRA). On the other hand, when we present the SUGRA-based models in Section 4, we shall see how to solve problems that arise in models with spontaneous or explicit breaking. Finally, Section 5 discusses grand unification and proton decay in SUSY models. The SUSY breaking mechanism due to SUGRA, which is presented in Section 4, can be generalized readily to unified models, but a "natural" unified theory, whose Lagrangian does not contain the two scales $M_{\rm GUT}$ and $M_{\rm W}$, is still to appear.

There are two lines of research discussed intensively in the literature that touch upon the theme of this review which we shall mention but will not discuss in detail. These are, first, the attempts to construct a realistic SU(3)×SU(2)×U(1) model starting with $N=1$ supergravity and using spontaneous compactification from spaces of higher dimensionality (cf. the review of Ref. 28) and, second, the use of supersymmetry in constructing light composite quarks and leptons.[29]

We conclude this introduction by listing the authors of reviews that will be helpful in studying supersymmetry: V. I. Ogievetskiĭ and L. Mezincescu; P. Fayet and S. Ferrara; and A. Salam and J. Strathdee.[30] These reviews are concerned with global supersymmetry. Supergravity is reviewed by Peter van Nieuwenhuizen[31] (see also the popular paper by D. Friedman and van Nieuwenhuizen[32]). Finally, there is a number of books that can be read by beginners.[33]

2. $N=1$ SUPERSYMMETRY

A. Uniqueness of SUSY; algebra

The symmetry of different models of elementary-particle interaction always takes the form of a direct product of a group of internal symmetries and the Poincaré group. A theory with a simple symmetry group, in which internal symmetry is "built into" space groups, would be more attractive. The geometry of space-time would then dictate the dynamics, imposing stringent restrictions on the form of the theory. Attempts to construct theories of this type have led to the proof of the so-called no-go theorems,[34] which state that, in theories involving interactions in 4-dimensional space, the Poincaré group can be extended only in the form of the direct product with the group of internal symmetries. The point is that the Poincaré group guarantees the conservation of the total momentum and the angular momentum in the interaction process. Its extension by tensorial generators would require the conservation of tensorial quantities that are different from the sum of angular momenta and momenta. The corresponding conservation laws can be satisfied only by requiring the conservation of the momenta of individual particles in the scattering process, and this means the absence of scattering, i.e., the absence of interaction in the system. The only exception from the no-go theorems is the extension of the Poincaré group by generators with Lorentz spin 1/2. If, at the same time, the (anti)commutator of these generators is proportional to the 4-momentum, the realization of this symmetry does not require additional conserved tensorial quantities, and the theory can be self-consistent and can include interaction terms. This type of construction is called $N=1$ supersymmetry.[3] If there are N spinorial generators, we have extended supersymmetry.

The algebra of extended supersymmetry is specified by commutators of the generators P_μ, $M_{\mu\nu}$ and Q^i_α (Q^i_α are the Majorana spinors and $\alpha = 1, \ldots, 4$, $i = 1, \ldots, N$). The commutators of P with P, M with M, and P with M are the same as in the ordinary Poincaré group, so that we reproduce only the commutators that include the spinorial charges Q:

$$[M_{\mu\nu},\ Q^i_\alpha] = \sigma^{\alpha\beta}_{\mu\nu} Q^i_\beta, \tag{2.1}$$

where $\sigma^{\alpha\beta}_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)_{\alpha\beta}/2$. This commutator shows that the spin Q^i_α is 1/2 and

$$[P_\mu,\ Q^i_\alpha] = 0, \tag{2.2}$$

which means that Q^i_α is constant in space and time. Moreover,

$$\{\overline{Q^i_\alpha},\ Q^j_\beta\} = -\frac{1}{4}\delta_{ij}\gamma^{\alpha\beta}_\mu P_\mu, \tag{2.3}$$

where $\overline{Q^i_\alpha} = (Q^i\gamma_0)_\alpha$. The appearance of the anticommutator in (2.3) is due to the fact that the Q^i_α are spinors.

In mathematics, the above construction is called a Lie algebra with Z_2 grading or a Lie superalgebra. The mathematics of superalgebras is considered in Berezin's monograph.[35] For $N>1$, the right-hand side of (2.3) may contain scalar operators that are antisymmetric in i, j. These are the so-called central charges.[36]

B. Representations of superalgebras

To construct supersymmetric models, we must know the irreducible representations of superalgebras. We shall systematically examine the $N=1$ case and then give two examples of representations of extended superalgebras. The representations of superalgebras are discussed in greater detail in the review by Fayet and Ferarra.[30]

We begin by considering a massive superparticle. Let us take a coordinate frame in which it is at rest: $P_\mu = (M, 0)$. The anticommutator (2.3) is written for the Majorana 4-component real spinor Q. We now transform to the two-component complex Weyl spinors:

$$Q_\alpha = \left(\frac{1+i\gamma_5}{2} Q\right)_\alpha, \quad \bar{Q}_{\dot\alpha} = \left(\frac{1-i\gamma_5}{2} Q\right)_{\dot\alpha} \quad (\alpha, \dot\alpha = 1, 2). \tag{2.4}$$

With suitable normalization, the commutation relations for these quantities have the form

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0, \quad \{Q_\alpha, \bar{Q}_{\dot\beta}\} = \delta_{\alpha\dot\beta}. \tag{2.5}$$

The algebra defined by (2.5) is the same as the algebra of creation and annihilation operators in quantum-field theory, and the irreducible representations are constructed in the usual way. The "vacuum" is taken to be the state of one particle at rest: $|\Omega\rangle = |J, J_3, M\rangle$. We then have

$$\bar{Q}_{\dot\alpha} |\Omega\rangle = 0. \tag{2.6}$$

The irreducible representation has the form

$$|\tilde{J}, \tilde{J}_3, n_1, n_2, M\rangle = Q_1^{n_1} Q_2^{n_2} |\Omega\rangle. \tag{2.7}$$

Possible values of (n_1, n_2) are: $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$. This representation has the dimensionality of $4(2J+1)$. Its decomposition over states with definite spin has the form

$$\left(J - \frac{1}{2}, M\right) \times 2(J, M) \times \left(J + \frac{1}{2}, M\right). \tag{2.8}$$

As expected, the number of boson and fermion degrees of freedom is the same.

Consider two examples. The vacuum state has $J=0$. The superparticle consists of the scalar complex particle and a particle with spin 1/2. The next possible case is $J = 1/2$. The superparticle consists of two particles of spin 1/2 (one vector and one scalar particle).

We now turn to massless particles. In the coordinate frame $P_\mu = (E, 0, -E, 0)$, we arrive at the following algebra:

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0, \quad \{Q_1, \bar{Q}_{\dot 1}\} = 1,$$
$$\{Q_2, \bar{Q}_{\dot 2}\} = \{Q_2, \bar{Q}_{\dot 1}\} = \{Q_1, \bar{Q}_{\dot 2}\} = 0. \tag{2.9}$$

If we take as the "vacuum" the state of the particle with particular helicity $|\Omega\rangle = |J, \lambda\rangle$, we obtain from it a single state with nonzero norm: $Q_1|\Omega\rangle$. The states $Q_2|\Omega\rangle$ and $Q_2 Q_1|\Omega\rangle$ have the norm 0. In CPT-symmetric theory, the irreducible representation of the superalgebra for the massless particle contains the four states $|\pm\lambda\rangle$ and $|\pm(\lambda + 1/2)\rangle$.

In realistic models, there are massless gauge bosons which form a (massless) supermultiplet with a spin 1/2 particle (gaugino). The vector boson acquires a mass as a result of the Higgs-Kibble effect, and should transform in accordance with the representation of the massive algebra. This is achieved by combining the massless vector multiplet with the chiral Higgs multiplet (containing both the complex scalar particle and the spinorial particle). The supermultiplet contains one vector, one Dirac particle, and one real scalar particle. In the limit of unbroken supersymmetry, all these particles have the same mass.

Let us now consider examples of representations of extended supersymmetries. The $N=4$ Yang-Mills theory (maximum spin 1) is widely discussed in the literature. We start with one vector meson. Generalizing (2.9) with the aid of (2.3) in the case of extended supersymmetry, we find that the multiplicity of states with helicity S is equal to the number of combinations of N elements $2(J-S)$ at a time:

$$\binom{2(J-S)}{N}.$$

Thus, in this theory, there is one particle with spin 1, four particles with spin 1/2, and six real scalar particles. We note that we shall obtain a CPT self-adjoint multiplet. Each particle is in the associated representation of the gauge group, which in fact determines the dynamics of the model. Apart from the $N=4$ Yang-Mills theory, the self-adjoint multiplet is also obtained in the $N=8$ theory with maximum spin 2. The multiplicity of states in the $N=8$ theory is: 1 particle with spin 2, 8 with spin 3/2, 28 vector particles, 56 spinorial particles, and 70 real scalars. This theory thus actually claims to be a unified theory of all particles and interactions, including gravity. Theories with $N>8$ have not been examined because a self-consistent description of interacting point masses with spin 5/2 or above does not exist.

In models based on $N=1$ SUSY, examined in Sections 3–5, we use mostly two types of representation, namely, the chiral multiplet that includes the spin 1/2 Weyl particle and a complex scalar, and a vector multiplet consisting of a vector particle and a spin 1/2 Majorana particle. In the models discussed in Section 4, based on SUSY breaking by supergravity, we use the further gravity supermultiplet containing a spin 2 graviton and a spin 3/2 gravitino.

C. Special role of $N = 1$ theory

There are experimental indications that, if low-energy (~ 100 GeV) physics is described by a supersymmetric theory, it must be based on $N=1$ supersymmetry.[37] Let us suppose that $N \geqslant 2$. In that case, we have a supermultiplet containing left-handed quarks (or leptons) and right-handed quarks (or leptons) or vector particles. All the members of the supermultiplet have the same quantum numbers with respect to internal $SU(3) \times SU(2) \times U(1)$ symmetries and, as we know, left-handed particles are doublets in SU(2), right-handed particles are singlets, and vector particles are either triplets or singlets, so that we must confine our attention to $N=1$ SUSY. The question is whether the following scenario is possible: there is extended supersymmetry at high energies, which breaks down at a certain intermediate mass scale μ ($M_W < \mu < M_P$) to $N=1$ SUSY. It turns out that the answer to this question is "no," and this follows[37] from the anticommutator (2.3). Let us multiply the right- and left-hand sides of (2.3) by γ_0, take the trace, and consider elements diagonal in i, j:

$$P_0 = Q_1^2 = Q_2^2 = \ldots \quad . \tag{2.10}$$

If we take the vacuum average, we find that, if a particular supersymmetry is broken $\langle 0|Q_i^2|0\rangle \neq 0$, all other supersymmetries will also be broken. When gravity is taken into account, this ceases to be valid and the following (and very attractive) variant becomes possible: extended (say, $N=8$) supergravity breaks down to $N=1$ at the Planck scale, and $N=1$ SUGRA breaks down at the intermediate scale $\sqrt{M_P M_W}$, inducing low-energy softly extended $N=1$ global SUSY, which does not allow the ~ 100 GeV scale to be "pulled-up" as a result of radiative corrections.

D. Cancellation of quadratic divergences in SUSY QED

We have already frequently mentioned that the basic reason for turning to SUSY models is that they produce a cancellation of quadratic divergences in radiative corrections to the masses of scalar particles. Let us consider this cancellation in the simple example of supersymmetric Gol'fand-Likhtman electrodynamics (which, historically, was the first SUSY theory considered in the literature).[3]

Single-loop electrodynamic corrections to the mass of a scalar particle are shown in Fig. 6a and b; they have a quadratic divergence. The question is: what happens in the supersymmetric generalization of the theory? The vector multiplet then contains both the spin 1/2 photon and the photino: $(\gamma, \tilde{\gamma})$. The superpartner of the complex scalar particle H is the spin 1/2 higgsino $\tilde{\mathrm{H}}$. Together, they form the chiral multiplet $(\mathrm{H}, \tilde{\mathrm{H}})$. The SUSY QED Lagrangian is[1)]

$$\mathcal{L} = \frac{1}{4}\int d^4\theta\,[H^+ e^{2gV} H] + \left\{\frac{1}{8}\int d^2\theta_+\,[W_\alpha W^\alpha] + \text{c.c.}\right\}. \tag{2.11}$$

(This theory is not, strictly speaking, self-consistent: it contains a triangular Adler-Bell-Jackiw anomaly. The anomaly can be obviated by introducing a further chiral multiplet of opposite charge, but this is not necessary in the present context.) The diagrams shown in Fig. 6 c and d appear in addition. We now reproduce the vertices that are contained in (2.11) and are necessary for the evaluation of these diagrams:

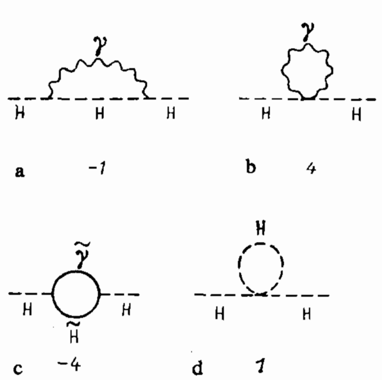

FIG. 6. Diagrams describing the renormalization of the mass of a scalar particle in SUSY QED. The coefficients of the term $(g^2/16\pi^2)\Lambda^2 H^* H$, obtained when the diagrams are evaluated, are shown under each diagram.

$$-\sqrt{2}\, gH\left(\bar{\tilde{\gamma}}\,\frac{1+i\gamma_5}{2}\,\tilde{H}\right). \tag{2.12}$$

Just as the photon transforms H into H (and $\tilde{\mathrm{H}}$ into $\tilde{\mathrm{H}}$) with a gauge constant g, so, by analogy, a photino with the same constant transforms H into $\tilde{\mathrm{H}}$. The Yukawa interaction with a gauge coupling constant appears in the theory.

$$-\frac{1}{2} g^2 (H^* H)^2. \tag{2.13}$$

This interaction appears when we solve the Lagrange equation of motion for the auxiliary field of the vector multiplet D:

$$\mathcal{L}_D = \frac{1}{2} D^2 + gH^* DH \to D = -gH^* H,$$

$$\Delta\mathcal{L}_H = -\frac{1}{2} g^2 (H^* H)^2. \tag{2.14}$$

The universal four-boson interaction with coupling constant g^2 appears in the theory.

The quadratic divergence cancels out when the graphs of Fig. 6 are summed.

3. MODELS BASED ON SOFT AND SPONTANEOUS SUSY BREAKING

A. Necessity for Higgs multiplets

The particles that appear in SU(3)×SU(2)×U(1)-SUSY models were listed in the Introduction. The SU(2) composition of the first family is

$$\begin{pmatrix} \mathrm{U} \\ \mathrm{D} \end{pmatrix}_L, \quad \mathrm{U}_R^*, \; \mathrm{D}_R^*, \quad \begin{pmatrix} \nu \\ \mathrm{E} \end{pmatrix}_L, \quad \mathrm{E}_R^*. \tag{3.1}$$

The upper-case letters represent chiral multiplets. Instead of right-handed particles, we use the complex conjugate multiplets because it is convenient to use multiplets of a particular chirality (see Appendix) when we write down the SUSY Lagrangians.

At first sign, the most economical approach would be to make the scalar components of the chiral multiplets of quarks or leptons play the role of Higgs bosons. Actually, the SU(3)×SU(2)×U(1) quantum numbers of the doublet $(\tilde{\nu}, \tilde{e})_L$ are the same as the quantum numbers of the Higgs doublet in the Weinberg-Salam model. The arguments against the Higgs interpretation of the doublet $(\tilde{\nu}, \tilde{e})_L$ are as follows. The vacuum average $\langle\tilde{\nu}\rangle$ is not sufficient to give a mass to the up and down quarks in a supersymmetric manner. We recall that, in the Weinberg-Salam model, it has been possible to use only one Higgs doublet by exploiting the C-conjugate doublet:

$$\Delta\mathcal{L} = \frac{m_u}{\langle H^0\rangle}(ud)_L^* u_R \begin{pmatrix} H^0 \\ H^- \end{pmatrix} + \frac{m_d}{\langle H^0\rangle}(ud)_L^* d_R \varepsilon \begin{pmatrix} H^0 \\ H^- \end{pmatrix}^*, \tag{3.2}$$

where

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The vacuum average $\langle\tilde{\nu}\rangle$ in the supersymmetric model can be used to generate the masses of the down quarks:

$$\Delta\mathcal{L} = \frac{m_d}{\langle\tilde{v}\rangle}\,[L_L \varepsilon Q_L D_R^*]_F. \tag{3.3}$$

However, the rule for writing down the supersymmetric Lagrangians (F component of the product of chiral superfields of a given chirality) forbids the use of the C-conjugate doublet $L_L^* \equiv L_R^*$. The presence in the Lagrangian of terms such as (3.3) implies a number of possible pitfalls.[16] For example, if we admit the further term

$$\Delta\mathcal{L} = f\,[U_R^* D_R^* D_R^*]_F, \tag{3.4}$$

the interactions (3.3)–(3.4) will lead to proton decay due to the operators

$$\frac{1}{m_{\tilde{d}}^2}\, f\, \frac{m_d}{\langle\tilde{v}\rangle}\left(u^T\gamma_0 \frac{1-i\gamma_5}{2}\, d\right) \times\left[d^T\gamma_0\frac{1+i\gamma_5}{2}\,\nu + u^T\gamma_0\frac{1-i\gamma_5}{2}\, e\right]. \tag{3.5}$$

The masses of the superpartners of ordinary particles (including $\tilde{d}$) are ~ 100 GeV (this is necessary for the cancellation of the quadratic divergences at momenta greater than $1/\sqrt{G_F}$), the constant f is $f \sim m_q/\langle\tilde{v}\rangle$, and the proton decay turns out to be too rapid.

The foregoing discussion shows the necessity for special Higgs supermultiplets. Supersymmetry then serves only as a moral justification for the introduction of scalar particles. At the present level of development of the theory, it is not possible to consider the Higgs scalars as the superpartners of known particles.

The minimum set of Higgs particles consists of two chiral isodoublets

$$H_L = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}_L, \quad H'_L = \begin{pmatrix} H'^+ \\ H'^0 \end{pmatrix}_L, \tag{3.6}$$

where, for example, $H_L^0 = (\tilde{H}_L^0, H_L^0)$, $\tilde{H}_L^0$ is the Weyl fermion, and H_L^0 is the complex scalar field. The vacuum averages $\langle H_L^0\rangle$ and $\langle H_L'^0\rangle$ generate the masses of the W and Z bosons, the quarks, and leptons. Models with 4-doublets of higgses are occasionally considered. The chiral Higgs doublets are introduced in pairs with opposite hypercharges in order to cancel triangular Adler-Bell-Jackiw anomalies in the U(1) hypercharge group.

The Lagrangian in the SU(3)×SU(2)×U(1) SUSY model contains kinetic terms for the vector (g, W, and B) supermultiplets and chiral supermultiplets of matter [extended in a superinvariant manner to ensure SU(3) ×SU(2)×U(1) invariance]. Moreover, there are supergeneralizations of the Yukawa couplings which generate the quark and lepton masses. For the first generation, they are

$$\Delta\mathcal{L} = \frac{m_u}{\langle H'^0\rangle}[H'_L\varepsilon Q_L U_R^*]_F + \frac{m_d}{\langle H^0\rangle}[H_L\varepsilon Q_L D_R^*]_F + \frac{m_e}{\langle H^0\rangle}[H_L\varepsilon L_L E_R^*]_F + \text{c.c.} \tag{3.7}$$

It is also necessary to write down the supersymmetric interaction between the Higgs fields that ensures nonzero values of $\langle H^0\rangle$ and $\langle H'^0\rangle$ at the minimum of the scalar-field potential. We shall not pause to consider this problem (it can be solved) but, instead, turn to the examination of SUSY breaking. The appearance of the terms (3.3) and (3.4) in the Lagrangian can be avoided by demanding invariance under sign reversal in front of all the quark and lepton superfields. Unfortunately, this discrete symmetry must be elevated into a fundamental principle when the SUSY generalization of the SU(3)×SU(2)×U(1) theory is constructed.

B. Operators producing soft SUSY breaking

The model described at the end of the last section has one important deficiency: it suffers from the Bose-Fermi degeneracy of the spectrum, which is in clear contradiction with experimental data. We thus arrive at the following question which is the most crucial for all SUSY models: how can supersymmetry be broken? The simplest way is explicitly to introduce further terms into the Lagrangian, which produce heavier superpartners of ordinary particles. The authors of Refs. 38 and 39 chose this particular path. It is then important to check that the terms introduced into the Lagrangian do not spoil the cancellation of quadratic divergences (this type of SUSY breaking is referred to as "soft"). Terms whose explicit introduction into the SUSY Lagrangian does not result in quadratic divergences are listed in Ref. 40.

According to the results reported in Ref. 40, the introduction of operators of dimensionality 2 (scalar-particle mass terms) into the SUSY Lagrangian is soft. Caution has to be exercised with operators of dimensionality 3: the mass terms corresponding to spinorial particles from vector multiplets are soft, whereas the mass terms corresponding to spinorial particles from chiral multiplets generate quadratic divergences. Let us decompose the scalar field from the chiral multiplet into real and imaginary parts: $Z = A + iB$. The operator $Z^3 + Z^{*3}$ is then also soft, i.e., the term $W(Z_i) + W^*(Z_i^*)$, where W is the superpotential, can be added to the Lagrangian. Let us illustrate the foregoing by considering the example of a nonsupersymmetric addition to the mass of a spinorial particle from the chiral multiplet. This operator is hard, which can be seen in the appearance of the quadratic divergence in the sum of graphs of Fig. 7. In the supersymmetric theory, the masses of the virtual boson and fermion are equal to M. The three-boson vertex of Fig. 7b is proportional to the same mass M. The quadratic divergences in the sum of graphs of Figs. 7a and b cancel out. The introduction of the nonsupersymmetric addition Δm to the mass of the fermion prevents the cancellation of the quadratic divergence. By inverting the foregoing, we can demonstrate that the operator Z^*Z^2, which generates the three-boson vertex of Fig. 7b, is hard.

C. Arguments against models with explicit breaking

There are three arguments against models with explicit breaking. First, there is the esthetic argument: explicitly nonsymmetric terms in the SUSY Lagrangian are repug-

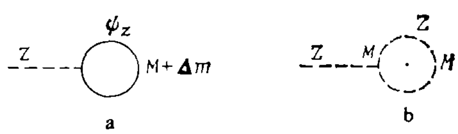

FIG. 7. Diagrams demonstrating the hardness of the nonsupersymmetric operator $\Delta m\bar{\psi}\psi$.

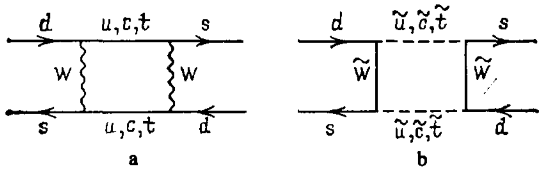

FIG. 8. Graphs for the $K^0 - \bar{K}^0$ transition in the standard theory (a) and additional graphs that appear in SUSY models (b).

nant. Second, such models have low predictive power. The masses of all the new particles introduced in SUSY models with explicit breaking (quarkino, leptino, gaugino) are free parameters. The third argument relates to neutral currents with a change in strangeness.[41] In the standard theory, the $K^0 - \bar{K}_0$ transition, which determines the mass difference between the K_L and K_S mesons, is described by the graph of Fig.8a. As a result of GIM cancellation, the amplitude for the $K^0 - \bar{K}^0$ transition is of the order of $(m_c^2 - m_u^2)/M_W^2$ (the contribution of the heavy t-quark is suppressed by the small Kobayashi-Maskawa angles and is of the same order as for the c-quark). The additional diagram of Fig. 8b appears in the SUSY theory. To achieve agreement with experiment, we must have a SUSY GIM cancellation, i.e., $m_{\tilde{c}}^2 - m_{\tilde{u}}^2 \approx m_c^2$. Since $m_{\tilde{u}}^2, m_{\tilde{c}}^2 > (20\ \text{GeV})^2$, such a small difference between the scalar quark masses, introduced into the Lagrangian "by hand," looks very unnatural. On the other hand, it is natural to introduce terms such as $\tilde{u}_R^* \tilde{c}_R$, which are off-diagonal in flavor. The quarkino mass terms are then diagonalized by matrices that differ from the matrices that diagonalize the quark mass terms, and off-diagonal neutral currents with gluino emission are found to appear. In the diagram of Fig. 8b, gluinos can propagate along the inner fermion lines, and down quarkinos along the boson lines. The strong coupling constants g_S appear at the vertices, and this leads to still more stringent conditions on the degeneracy of quarkinos belonging to different generations.

We thus arrive at the following requirement: the mechanism responsible for the generation of quarkino masses in a realistic SUSY model must be "flavor-blind," i.e., it must produce equal quarkino masses for equal quantum numbers that differ only by the flavor.

D. Spontaneous SUSY breaking[2)]

Spontaneous SUSY breaking can be analyzed with the aid of the scheme that is well known in the case of the spontaneous breaking of internal symmetries. If the vacuum of the theory is supersymmetric, then $\hat{Q}|0\rangle = 0$ and SUSY is unbroken. If, on the other hand, $Q|0\rangle \neq 0$, SUSY is broken. Since $\hat{H} = Q^2$ [this is a key equation in the theory of spontaneous SUSY breaking and is obtained by multiplying both parts of the anticommutator (2.3) by γ_0 and evaluating the trace], the order parameter of the globally supersymmetric system is the total energy. If the ground-state energy is zero, SUSY is unbroken and, when it is greater than zero, $(\hat{H} = Q^2 \geqslant 0)$, SUSY is broken.

The scalar-particle potential in a system of chiral multiplets interacting with one another and with the vector multiplets in a gauge manner has the form

$$V = \sum_i |F_i(A_K)|^2 + \sum_i \operatorname{tr} D_i^2(A_K), \tag{3.8}$$

where, in the case of U(1) symmetry, we must replace $\operatorname{tr} D_i^2(A_K)$ with $(1/2) D_{U(1)}^2(A_K)$. There are two mechanisms for spontaneous SUSY breaking: $\langle F_j(A_K)\rangle \neq 0$ (O'Raifeartaigh-Fayet mechanism) and $D_j(A_K) \neq 0$ (Iliopoulos-Fayet mechanism). SUSY breaking due to a nonzero D-term requires the presence in the theory of gauge U(1) symmetry. Let us consider two examples illustrating spontaneous SUSY breaking.

(1) SUSY *breaking by the Iliopoulos-Fayet* U(1) *D-term*

Consider the simple example of the supersymmetric Gol'fand-Likhtman electrodynamics. The composition of the field is: $V \equiv (\gamma, \tilde{\gamma}, D)$—vector multiplet, $S_L \equiv (S, \psi_S, F_S)$—chiral multiplet of charge $Q = +1$, $T_L \equiv (T, \psi_T, F_T)$—chiral multiplet of charge $Q = -1$. The two multiplets with opposite charges are necessary for the cancellation of the U(1) anomaly. The supersymmetric Lagrangian is

$$\mathcal{L} = \frac{1}{8}[W_\alpha W^\alpha]_F + \xi[V]_D + \frac{1}{4}[S^* e^{2gV} S]_D + \frac{1}{4}[T^* e^{-2gV} T]_D + m[ST]_F + \text{c.c.}; \tag{3.9}$$

where ξ is a parameter with the dimensions of GeV^2, and the term $\xi[V]_D$ is added to generate supersymmetry breaking. Let us rewrite the Lagrangian (3.9) in terms of the components:

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\bar{\tilde{\gamma}}\hat{\partial}\tilde{\gamma} + \frac{1}{2}D^2 + \xi D + |D_\mu S|^2 \\ & + \bar{\psi}_{S_L}\hat{D}\psi_{S_L} + |F_S|^2 + gS^* DS \\ & - \sqrt{2}\, g\bar{\psi}_{S_L}\tilde{\gamma}S + |D_\mu T|^2 + \bar{\psi}_{T_L}\hat{D}\psi_{T_L} - gT^* DT \\ & + \sqrt{2}\, g\bar{\psi}_{T_L}\tilde{\gamma}T + |F_T|^2 \\ & + m\psi_{S_L}^T \gamma_0 \psi_{T_L} + mF_S T + mF_T S + \text{c.c.}\end{aligned} \tag{3.10}$$

Solving the algebraic Lagrangian equations for the auxiliary fields D, F_S, and F_T, we obtain the following expression for the potential energy of the scalar fields:

$$V(S, T) = \frac{1}{2}D^2(S, T) + |F_S(S, T)|^2 + |F_T(S, T)|^2. \tag{3.11}$$

According to the foregoing, the case $V = 0$ corresponds to unbroken supersymmetry and $V > 0$ to broken supersymmetry. For supersymmetry breaking, the set of equations

$$\begin{cases} D(S, T) = 0, \\ F_S(S, T) = 0, \\ F_T(S, T) = 0 \end{cases} \tag{3.12}$$

must be incompatible. Substituting in (3.11) the expressions for the auxiliary fields in terms of physical fields, we obtain

$$V = \frac{1}{2}[\xi + g(S^*S - T^*T)]^2 + m^2(S^*S + T^*T). \tag{3.13}$$

When $\xi \neq 0$ and $m \neq 0$, the set of equations given by (3.12) is incompatible, i.e., supersymmetry is broken. Let us examine the spectrum of the model. Let $g\xi < m^2$, in which case, $\langle S\rangle = \langle T\rangle = 0$ and the gauge U(1) symmetry is not broken: $M_\gamma = 0, \langle F_S\rangle = \langle F_T\rangle = 0, \langle D\rangle = \xi$. Spontaneous breaking of Bose symmetry can be produced with the aid of the massless spin zero Goldstone particle and, similarly, in the SUSY case, there is the spin 1/2 massless goldstino. In the model we are considering, the photino $\tilde{\gamma}$ plays the role of the goldstino, $M_{\tilde{\gamma}} = 0$. The fields ψ_{S_L} and ψ_{T_L} form the Dirac spinor $\chi = \psi_{S_L} + \psi_{T_L}, M_\chi = m$. The two complex scalar particles S and T have the following masses:

$$m_S^2 = m^2 + g\xi,\ m_T^2 = m^2 - g\xi.$$

Since S and T have charges of different sign (this is the necessary condition for the cancellation of anomalies), their masses diverge in opposite directions from the mass of the spinor field χ.

Let us now consider the case where $g\xi > m^2$. We then have $\langle S\rangle = 0, \langle T\rangle = \sqrt{(g\xi - m^2)/g^2}$. In this case, both supersymmetry and gauge U(1) symmetry are broken. $\langle F_T\rangle = 0,\ \langle F_S\rangle = -m\sqrt{(g\xi - m^2)/g^2}, \langle D\rangle = m^2 g$. The spectrum contains the massless goldstino

$$\psi_G = \frac{1+i\gamma_5}{2}\left(\frac{m}{\sqrt{2g\xi - m^2}}\tilde{\gamma} - \frac{\sqrt{2g\xi - 2m^2}}{\sqrt{2g\xi - m^2}}\psi_S\right). \quad (3.14)$$

The combination of $\tilde{\gamma}$ and $\tilde{\psi}_S$ that is orthogonal to the goldstino forms together with ψ_T the Dirac spinor χ of mass $m_\chi = \sqrt{2g\xi - m^2}$. The mass of the vector meson is $M_{A_\mu} = \sqrt{2g\xi - 2m^2}$. The complex scalar field S has the mass $M_S = \sqrt{2}m$, and the mass of the physically real field T is $M_T = \sqrt{2g\xi - 2m^2}$.

Let $m = 0$. Supersymmetry is then unbroken and U(1) symmetry is spontaneously broken. The two massless supermultiplets (chiral T and vector V) form one massive supermultiplet: (A_μ, χ, T). The masses of all the supermultiplet members are equal to $\sqrt{2g\xi}$.

(2) SUSY *breaking by the F-term*

Consider the set of interacting chiral fields without gauge interactions. The scalar field potential is then the sum of the squares of the S terms, and the energetically most convenient supersymmetric vacuum state is present if the set of n algebraic equations with n unknowns, $F_i(A_K) = 0$, has a solution. The only way to produce SUSY breaking is to obtain a degenerate set of equations. The minimum condition for this is that at least one combination of the vacuum average of the scalar fields remains undetermined at the tree level. The minimum number of chiral multiplets necessary to achieve spontaneous breaking is three.

Consider the set of chiral fields $\hat{A}, \hat{B}$ and $\hat{X}$, described by the following superpotential:

$$W(\hat{A},\ \hat{B},\ \hat{X}) = M\hat{A}\hat{B} + \lambda\hat{X}(\hat{A}^2 - \mu^2). \quad (3.15)$$

The equations

$$F_B = -MA = 0,\ F_X = -\lambda(A^2 - \mu^2) = 0 \quad (3.16)$$

are incompatible: supersymmetry is spontaneously broken. From (3.15), we obtain the potential energy of the scalar fields:

$$V(A, B, X) = M^2A^2 + \lambda^2(A^2 - \mu^2)^2 + (MB + 2\lambda XA)^2. \quad (3.17)$$

Let $M^2 > 2\lambda^2\mu^2$. We then have $\langle A\rangle = 0, \langle B\rangle = 0$, and $\langle X\rangle$ is undetermined in the tree approximation. $F_A = F_B = 0$, $\langle F_X\rangle = -\lambda\mu^2$ and ψ_X is the goldstino. In the tree approximation, the model contains the massless scalar field X. If $M^2 < 2\lambda^2\mu^2$, then $\langle A\rangle = \sqrt{\mu^2 - (M^2/2\lambda^2)}$. The vacuum averages of the fields B and X are separately undetermined and only their ratio is determined:

$$\frac{\langle B\rangle}{\langle X\rangle} = -\frac{2\lambda}{M}\langle A\rangle,\quad \langle F_B\rangle = -M\langle A\rangle,$$
$$\langle F_X\rangle = \frac{M^2}{2\lambda},\quad \langle F_A\rangle = 0. \quad (3.18)$$

The mass of the field $2\lambda\langle A\rangle B - MX$ is zero in the tree approximation. The wave function of the goldstino has the form

$$\psi_G = 2\lambda\langle A\rangle\psi_B - M\psi_X. \quad (3.19)$$

Thus, when supersymmetry is broken by the O'Raifeartaigh mechanism, the spectrum contains both the massless fermion (goldstino) and the scalar field—the superpartner of the goldstino—which is massless in the tree approximation.[3)] Its vacuum average is also undetermined by the potential. Radiative corrections determine the vacuum average of this field and its mass.

E. Attempts to construct models with spontaneous SUSY breaking at the scale of 100 GeV

Let us now construct a model based on spontaneous SUSY breaking. We shall suppose that, in the ground state, the F-term of one chiral field (we shall denote it by the letter A) is different from zero. We shall consider which nonsupersymmetric additions to the spectrum will produce $\langle F_A\rangle \neq 0$. Suppose that the superpotential contains the term $\Delta W = AB^2$. The field B will then contain the nonsupersymmetric mass terms $\Delta\mathcal{L} = \langle F_A\rangle[(\mathrm{Re}\ B)^2 - (\mathrm{Im}\ B)^2]$. Thus, the complex scalar field B splits into two real components. The mass of one of them is greater than the supersymmetric value, and the mass of the other is smaller by the same amount. The mass of the spinorial field ψ_B remains unaltered and is equal to the supersymmetric value. It is clear that this mechanism is unsuitable for a realistic model. Actually, if an attempt is made to make the electrino $\tilde{e}$ heavier by introducing a nonzero F-term of some auxiliary field, a scalar charged particle with a negative squared mass is found to appear, i.e., the condensate of this field destroys electric charge. Thus, spontaneous SUSY breaking by the O'Raifeartaigh-Fayet mechanism does not provide us with a way of constructing a realistic model.

Another SUSY-breaking mechanism at our disposal is the Iliopoulos-Fayet mechanism. This requires local U(1) symmetry in the theory. The standard model does have this symmetry—it is the U(1) group of hypercharge. We shall show that it cannot be used to construct a realistic model. The vacuum charge $\langle D_Y\rangle \neq 0$ leads to nonsupersymmetric mass terms: $\Delta\mathcal{L} = \Sigma_i g_{A_i} A_i^* A_i \langle D\rangle$, where g_{A_i} is the hypercharge of the fields A_i. The fields $\tilde{u}_R^*$ and $\tilde{d}_R^*$ have different hypercharge signs, i.e., $M^2_{\tilde{u}_R^*} < -2\,(20\ \text{GeV})^2$ is obtained simultaneously with $M^2_{\tilde{d}_R^*} > (20\ \text{GeV})^2$, the field $\tilde{u}_R^*$ falls out into the condensate, and the electric charge and color are destroyed. The conclusion that, in the standard SU(3) ×SU(2)×U(1) model, spontaneous SUSY breaking necessarily leads to the existence of scalar particles that are lighter than the spinorial superpartners, is referred to in the literature as the Fayet-Dimopoulos-Georgi theorem.[42,39]

To avoid the appearance of light scalars, Fayet has extended the standard model and introduced the additional U(1) group[42] in which all quark and lepton superfields have positive charges. In a series of papers, Fayet investigated the phenomenology of the low-energy SU(3)×SU(2) ×U(1)×U′(1) symmetry and, in particular, the properties of the additional neutral vector boson and neutral currents transported by it. Weinberg[16] tried to remove the triangular anomalies from the Fayet model. In standard SU(3)×SU(2)×U(1) theory, the quark and lepton contributions to the triangular anomalies are found to cancel out. The absence of anomalies is necessary for the renormalization of the theory. This is the theoretical justification of quark-lepton symmetry observed experimentally. It is therefore natural to demand the cancellation of anomalies in the SU(3)×SU(2)×U(1)×U′(1) scheme, as well. The introduction of the extra U′(1) symmetry leads to a large number of new anomalous amplitudes. To achieve their cancellation, Weinberg was forced to introduce a large number of new particles into the model. Subsequent studies have shown that, although the triangular anomalies are not present in the Weinberg variant of the SU(3)×SU(2)×U(1) ×U′(1) model, the model does contain the supersymmetric (and, therefore, energetically most convenient) vacuum, in which the vacuum averages of the electrically charged fields are nonzero (see the note in Ref. 16).

All this gives rise to pessimism about the possibility of a realistic model based on spontaneous SUSY breaking. The source of the trouble is the direct (at the tree level) interaction between superfields, whose auxiliary components develop vacuum averages (thus, breaking SUSY) with quark and lepton superfields. A fruitful approach is that in which the quark and lepton superfields "recognize" SUSY breaking by radiative corrections.[43–48] The scalar superpartners are then automatically heavier than the spinors.

F. Models with geometric hierarchy

These are models in which SUSY breaking occurs in two stages. The first stage involves the introduction of a world of heavy particles, characterized by mass M, in which spontaneous SUSY breaking occurs at the scale $\mu \ll M$. The superfield containing the goldstino does not then interact at the tree level with the supermultiplets containing quarks, leptons, Higgs bosons, and vector fields that gauge the SU(3)×SU(2)×U(1) group. SUSY breaking penetrates the light-particle sector during the second stage, and is due to loop diagrams in which heavy particles of mass M propagate. The SUSY-breaking scale in the light-particle sector is therefore $m \approx \mu^2/M$, which explains the phrase "geometric hierarchy"; the scale of SUSY breaking is the geometric average of the large and small scales.

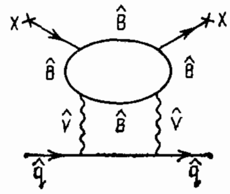

FIG. 9. SUSY breaking in the light-particle sector in models with geometric hierarchy.

As an example, consider the diagram of Fig. 9, in which $\hat{X}$ and $\hat{B}$ are the chiral superfields. SUSY is broken: $\langle F_X \rangle \neq 0$. At the tree level, the field $\hat{X}$ containing the goldstino interacts with the heavy field $\hat{B}$, which, in turn, has a common gauge interaction with the quark (lepton, Higgs) superfields. According to the well-known theorem enunciated in Ref. 49, F-terms due to loop graphs do not arise in perturbation theory. The diagram of Fig. 9 generates the following operator:

$$O = \frac{1}{M^2}\,[X^*Xq^*q]_D = \frac{\langle F_X \rangle^2}{M^2}\,(\tilde{q}^*\tilde{q}), \tag{3.20}$$

and the quarkino acquires the SUSY-breaking addition to the mass $\Delta M_{\tilde{q}} = \mu^2/M$. This is a universal addition: quarkinos belonging to different families with the same quantum numbers become heavier to the same extent, i.e., the super-GIM cancellation appears naturally. Diagrams analogous to Fig. 9 give mass to the leptino and Higgs bosons. Since the mass terms of Higgs bosons determine the scale at which electroweak SU(2)×U(1) symmetry is broken, the low-energy scale m in models with geometric hierarchy turns out to be of the order of the mass M_W of the intermediate bosons. The mass scale of the heavy particles is naturally taken to be the grand unification scale M_{GUM}. In the grand unification SUSY models, $M_{GUM} \sim 10^{16}$ GeV (see Section 5). The SUSY breaking scale is then

$$\mu = \sqrt{M_W M_{GUM}} \approx 10^9\ \text{GeV}. \tag{3.21}$$

In particular models,[45–47] the quantity μ depends on dimensional parameters as well as the gauge and Yukawa coupling constants, which leads to the somewhat greater value $\mu = 10^{10} - 10^{12}$ GeV.

The key point for the ensuing discussion is that, when the SUSY-breaking parameter is so high, we cannot neglect supergravity effects. The SUSY-breaking SUGRA corrections are characterized in the low-energy sector by the scale $\mu^2/M_P = 10 - 10^5$ GeV (where M_P is the Planck mass, $M_P = 10^{19}$ GeV), i.e., they are 100% important for models considered in this section. Here, we encounter an exceedingly promising situation: for the first time in physics, gravity effects are playing an important role in elementary-particle physics at the low energy scale of 100 GeV.

To summarize, we note that, first, the only phenomenologically acceptable $N = 1$ SUSY model requires SUSY breaking at the intermediate mass scale $\mu = \sqrt{M_W M}$. By identifying M with the grand unification scale M_{GUM}, we arrive at a model in which gravity corrections are 100% important.

4. SUSY BREAKING WITH ALLOWANCE FOR SUPERGRAVITY

A. ($N=1$)-matter + supergravity

In this section, we shall investigate the system "($N=1$)-supergravity + matter." We shall examine why local supersymmetry leads to supergravity and, what is the origin of the spin 3/2 field. The Poincaré supergroup has as its generators the shift P_a, rotation M_{ab}, and supertransformations Q_α. The local realization of P_a and M_{ab} leads to GTR with P_a gauging graviton e^a_μ, as the physical field (because of the presence of spinorial fields, we have to use the tetrad representation of gravity); the M_{ab} gauging connectedness of ω^{ab}_μ is expressed through the e^a_μ tetrad and its derivative. The localization of the $\varepsilon_\alpha \to \varepsilon_\alpha(x)$ supertransformation parameter requires the introduction of the gauge field with the transformation law $\delta\psi^a_\mu \sim \partial_\mu \varepsilon_\alpha(x)$, i.e., the spin of the ψ^a_μ is 3/2.

The physical degrees of freedom of the gravity multiplet consists of the massless graviton e^a_μ with spin 2 (two degrees of freedom on the mass surface) and the gravitino ψ_μ with spin 3/2 (again, two degrees of freedom on the mass surface). Outside the mass surface, the graviton has six degrees of freedom (10 parameters of the symmetric 4×4 tensor after the subtraction of the four degrees of freedom corresponding to the shift of the coordinate frame gauged by the graviton) and the gravitino has 12 degrees of freedom (16 components of the Rarita-Schwinger field after subtraction of four supertransformations performed by the Majorana spinor—the supersymmetry generator Q—which gauges the gravitino). Hence, auxiliary fields are essential for the supersymmetric description outside the mass surface and the linear realization of supersymmetry, by analogy with the case of chiral and vector multiplets. Different sets of auxiliary fields are available for ($N=1$)-supergravity. The simplest set consists of two scalar fields and one vector field (six additional boson degrees of freedom), and this corresponds to the so-called "minimal SUGRA."

We shall now renormalize the Lagrangian of the system ($N=1$)-supergravity + matter. Divergences in the loop diagrams lead to uncontrollable infinities. Extended supersymmetries with $N>1$ offer some hope for a finite theory. In such theories, matter is confined to a single multiplet with a graviton and a gravitino, and this improves the ultraviolet behavior. In particular, the $N=2$ theory unifies gravity and electromagnetism, and $\gamma\gamma$-scattering in this theory turns out to be finite in one loop while, without the gravitino, the sum of one-loop diagrams diverges.[33] The ($N=8$)-SUGRA is the most promising in this connection. The much simpler system ($N=1$)-SUGRA + matter will be examined below. It may be considered that it is realized as a result of the spontaneous $(N=8)_{\rm loc} \to (N=1)_{\rm loc}$ breaking at energies $\sim M_{\rm P}$. The responsibility for the divergences for $E>M_{\rm P}$ is then taken up by the $N=8$ theory, and the Lagrangian of the $N=1$ theory must be looked upon as the effective low-energy Lagrangian valid for $E<M_{\rm P}$. As already noted, the effective Lagrangian will be renormalized, which will enable us to introduce nonpolynomial functions of superfields into it. We can then readily obtain a theory with spontaneous SUSY breaking: one chiral multiplet of matter will suffice for this. Moreover, since energy ceases to play the role of the order parameter when gravity is taken into account, spontaneous SUSY breaking in the ($N=1$)-SUGRA + matter system is an example of the general situation. This involves the so-called "superhiggs" effect:[50] the gravitino absorbs the massless goldstino and a massive spin 3/2 particle is formed (by analogy with the usual Higgs effect, the gauge vector field absorbs the scalar Goldstone particle and becomes massive).

B. Supersymmetric Higgs effect

The supersymmetric Higgs effect can be considered in a very general way without specifying the type of interaction between the supermultiplets of matter.[51] Suppose that spontaneous global SUSY breaking occurs in the sector of material fields, giving rise to the appearance of the cosmologic term

$$\Delta\mathscr{L}_{\rm matter} = \int d^4x \sqrt{-g}\,(-M^4_{\rm SB}). \tag{4.1}$$

The supergravity Lagrangian consisting of the Hilbert action for the graviton and the Rarita-Schwinger action for the gravitino may also contain the "supercosmologic" term. An addition of the form (4.1) will clearly violate supersymmetry since $\sqrt{-g}$ transforms under the supertransformation through $\bar\varepsilon\psi$. It turns out that the superinvariant cosmologic term has the form[52]

$$\Delta\mathscr{L}_{\rm SA} = \int d^4x \sqrt{-g}\left(\frac{3m^2_{3/2}}{k^2} + m_{3/2}\bar\psi_\mu\sigma_{\mu\nu}\psi_\nu\right), \tag{4.2}$$

where $k=\sqrt{8\pi}/M_{\rm P}$. The presence of the "gravitino mass" in (4.2) does not actually signify that this particle is massive. The point is that the equations for the metric that include the cosmologic term from (4.2) lead to anti-de-Sitter space-time, in which the problem of the particle mass is not solved as simply as in the flat Minkowski space-time. However, we need not consider the anti-de-Sitter world because phenomenologically we accept only the Minkowski world which corresponds to zero cosmologic constant. The cosmologic constant cancels out in the sum of the matter (4.1) and gravity (4.2) Lagrangians provided

$$m_{3/2} = \sqrt{\frac{8\pi}{3}}\,\frac{M^2_{\rm SB}}{M_{\rm P}}. \tag{4.3}$$

The gravitino obtained in this way in flat space can have mass equal to $m_{3/2}$.

The locally supersymmetric Lagrangian of interacting chiral superfields, including gauge interactions with vector superfields, was first obtained in Ref. 53. Let us introduce the notation $s_i \equiv (z_i, \chi_i, F_i)$ for chiral superfields. The interaction between the fields s_i is then determined by a single arbitrary real function $G(z_i, z_i^*)$, called the Kahler potential. The potential energy of the scalar fields has the form (here and below, we substitute $\sqrt{8\pi}/M_{\rm P}=1$)

$$V(z_i) = e^G[G^i (G^i_j)^{-1} G_j - 3], \tag{4.4}$$

where

$$G^i \equiv \frac{\partial G}{\partial z_i}, \qquad G_j \equiv \frac{\partial G}{\partial z_j^*}, \qquad G^i_j = \frac{\partial^2 G}{\partial z_i\,\partial z_j^*}.$$

The kinetic energy of the field z_i is given by

$$T(z_i) = G^i_j(\partial_\mu z_i)(\partial_\mu z^*_j). \quad (4.5)$$

The Lagrangian thus acquires a "mass term" for the gravitino:

$$\Delta\mathcal{L}_{\psi\mu} = e^{G/2}\bar{\psi}_\mu\sigma_{\mu\nu}\psi_\nu. \quad (4.6)$$

If the minimum of the potential (4.4) is reached for z_{0i} such that $V(z_{0i}) = 0$, the supersymmetric Higgs effect takes place and the gravitino acquires the mass $m_{3/2} = e^{G(z_{0i})/2}$. The goldstino field, absorbed by the gravitino, has the form

$$\eta = \langle e^{G/2}G_i\rangle\chi_i. \quad (4.7)$$

To establish a relationship with the previously-investigated global SUSY, we write G in the form

$$G(z_i, z^*_i) = d(z_i, z^*_i) + \ln[f(z_i)f^*(z^*_i)]. \quad (4.8)$$

Substituting this in (4.4) and (4.5), we obtain

$$T = d^j_i(\partial_\mu z_i)(\partial_\mu z^*_j), \quad (4.9)$$

$$V = e^d\{(d^i_j)^{-1}(f^i + d^i f)(f^*_j + d_j f^*) - 3|f|^2\}, \quad (4.10)$$

where

$$d^i \equiv \frac{\partial d}{\partial z_i}, \quad d_j \equiv \frac{\partial d}{\partial z^*_j}, \quad f^i \equiv \frac{\partial f}{\partial z_i}, \quad f_j \equiv \frac{\partial f}{\partial z^*_j},$$

$$d^i_j \equiv \frac{\partial^2 d}{\partial z_i\,\partial z^*_j}. \quad (4.11)$$

To transform to the flat case, we bring back in (4.10) the dimensional factor $M \equiv M_P/\sqrt{8\pi}$:

$$V = e^{d/M^2}\Big[\Big(\frac{\partial^2 d}{\partial z_i\,\partial z^*_j}\Big)^{-1}\Big(\frac{\partial f}{\partial z_i} + \frac{1}{M^2}\frac{\partial d}{\partial z_i}f\Big) \times \Big(\frac{\partial f^*}{\partial z^*_j} + \frac{1}{M^2}\frac{\partial d}{\partial z^*_j}f^*\Big) - \frac{3|f|^2}{M^2}\Big]. \quad (4.12)$$

In the limit as $M \to \infty$, we obtain

$$T = \frac{\partial^2 d}{\partial z_i\,\partial z^*_j}\partial_\mu z_i\,\partial_\mu z^*_j, \quad V = \Big(\frac{\partial^2 d}{\partial z_i\,\partial z^*_j}\Big)^{-1}\frac{\partial f}{\partial z_i}\frac{\partial f^*}{\partial z^*_j}. \quad (4.13)$$

These expressions for T and V are obtained from the following expression for the action, written in terms of the superfields:

$$S = \int d^4x\,d^2\theta\,d^2\bar{\theta}\,d(s_i, \bar{s}_i) + \int d^4x\,d^2\theta f(s_i) + \text{c.c.}, \quad (4.14)$$

where S is the most general action for the set of chiral superfields in flat space. It is the starting point for the generalization to the case of local symmetry. The dependence on the two functions d and f can be reduced to a single function G because of the presence in (4.8) of invariance under the transformation

$$d \to d + g(z_i) + g^*(z^*_i), \qquad f \to e^{-g(z_i)}f. \quad (4.15)$$

In the global case (4.14), this invariant reduces to the transformation

$$d(s_i, s^*_j) \to d + g(s_i) + g^*(\bar{s}_i), \quad (4.16)$$

under which the action (4.14) remains unaltered. Using (4.16), we rewrite (4.14) in the form

$$S = \int d^4x\,d^2\theta\,d^2\bar{\theta}\frac{\partial^2 V}{\partial s_i\,\partial\bar{s}_j}s_i\bar{s}_j + \int d^4x\,d^2\theta f(s_i) + \text{c.c.} \quad (4.17)$$

The first term in this can be interpreted geometrically. The scalar components of the fields s_i form the complex N-dimensional (in the number of chiral fields N) Kahler manifold (or $2N$-dimensional Riemann manifold). The quantity V is the Kahler potential of the manifold; $\partial^2V/\partial z_i\partial z^*_j$ defines the metric on it. The metric is invariant under the Kahler transformation (4.16) (with d replaced with V). The inclusion of gravity in the nonlinear supersymmetric models described by the Lagrangian (4.7) is discussed from the point of view of the Kahler geometry in Refs. 54–56.

The analogy between the Kahler transformation (4.16) and the gauge U(1) transformation of the vector multiplet $V \to V + \Lambda + \bar{\Lambda}$ explains why, by analogy with the field V, the Lagrangian of the gauge noninvariant supermatter has the factor d as an exponent in (4.10). The expression given by (4.10) is obtained by solving the equations for the auxiliary fields. The negative contribution to the potential of the scalar fields, which is specific to supergravity, is due to the auxiliary fields of the gravity supermultiplet.

When the interaction with the vector superfields is taken into account, there is a change in the argument of the function d in (4.14): $d(s_i, \bar{s}_i e^{2gV})$. It is also necessary to demand the invariance of d and f under gauge transformations. The supersymmetric Yang-Mills field in flat space is generalized as follows when the requirement of renormalization is removed:

$$\int d^4x\,d^2\theta W_\alpha W^\alpha \to \int d^2x\,d^2\theta\varphi^\beta_\alpha(s_i)\,W^\alpha W_\beta; \quad (4.18)$$

where α, β are group symbols and $\varphi_{\alpha\beta}(s_i)$ is an arbitrary function of chiral superfields. Higher powers of W_α in the action (4.18) also do not lead to higher-order derivatives in the Lagrangian but, for simplicity, we confine our attention to the action (4.18). The auxiliary fields of vector multiplets induce the following addition to the scalar-field potential (4.4):

$$\Delta V_{\text{gauge}}(z_i) = \frac{1}{2}g^2\varphi^{-1}_{\alpha\beta}[G^i(T^\alpha)^j_i z_j][G^m(T^\beta)^n_m z_n]. \quad (4.19)$$

The "minimal coupling" between the set of chiral and vector fields with ($N = 1$)-supergravity is the coupling that does not affect the canonical form of the kinetic terms of matter and Yang-Mills fields:

$$d(z_i, z^*_i)_{\min} = \sum_i |z_i|^2, \quad \varphi_{\alpha\beta}(s_i)_{\min} = \delta_{\alpha\beta}. \quad (4.20)$$

In the case of minimal coupling, of the three arbitrary functions, d, f, φ only f remains, and this simplifies very considerably the analysis of models. Most of the models considered in the literature (and in this section) are therefore based on the minimal coupling to supergravity (not to be confused with "minimal supergravity" which has a minimum number of auxiliary fields in the gravity multiplet).

The scalar-field potential for minimal coupling has the following form:

$$V = \exp\Big(\sum_i\frac{|z_i|^2}{M^2}\Big)\Big(\Big|\frac{\partial f}{\partial z_i} + \frac{z^*_i f}{M^2}\Big|^2 - 3\frac{|f|^2}{M^2}\Big) + \frac{g^2}{2}|z^*_i T z_i|^2, \quad (4.21)$$

and the gravitino mass is

$$m_{3/2}=e^{\langle |z_i|^2\rangle/2M^2}\,\frac{\langle f\rangle}{M^2}. \quad (4.22)$$

Consider the simplest example, namely, the coupling between one chiral field and SUGRA. We take the superpotential $f(z)$ in the form[57]

$$f(z) = M^2(z+\beta). \quad (4.23)$$

From (4.21), we then obtain $V_{\min}=0$ when $\beta=(2-\sqrt{3})M$, $z_{\min}=(\sqrt{3}-1)M$. The mass of the gravitino is given by

$$m_{3/2} = Me^{(1/2)(\sqrt{3}-1)^2}. \quad (4.24)$$

It is clear from this example that a fine adjustment of the parameters of the superpotential (β) is necessary if we are to obtain the zero cosmologic term.

Without writing out the general form of the Lagrangian for the spinorial fields of matter, we note that the supertrace $M_J^2[\Sigma_J(2J+1)(-)^{2J}M_J^2]$ is proportional to the gravitino mass[53] $m_{3/2}^2$. To construct phenomenologic models, we must therefore have $m_{3/2}^2\sim(M_W)^2$ because greater values of $m_{3/2}^2$ will not ensure a SUSY solution of the hierarchy problem (low cutoff of quadratic divergences in the masses of Higgs particles), while lower values are experimentally forbidden (by the experimental absence of superpartners of ordinary particles). The scalar-particle potential (4.21) contains a natural scale, namely, the Planck mass, which leads to $m_{3/2}\sim M_P$ [see, for example, (4.24)]. The small scale M_W can be introduced into the scalar-particle potential through the superpotential $f(z)$. Taking

$$f(z) = M_W\varphi(z), \quad (4.25)$$

we obtain from (4.21) and (4.22)

$$\left\langle \frac{\partial f}{\partial z}\right\rangle \equiv M_{SB}^2\sim M_W M_P, \quad m_{3/2}\sim M_W. \quad (4.26)$$

Thus, the realistic models that we shall now consider suffer from two "inherent" defects: the parameters must be finely adjusted to annul the cosmologic term, and a small scale must be artificially introduced into the superpotential. It may be that the transition to extended SUSY theories will help us in a natural manner to avoid these maladies. A possible "therapy" within the framework of $N=1$ theories will be considered at the end of this Section.

C. Low-energy Lagrangian

In addition to quark, lepton, and Higgs superfields (denoted by Y_i), we introduce the hidden sector,[58,59] i.e., the chiral superfields z_i. The super-Higgs effect will occur in the hidden sector. We shall take the superpotential in the form[58,59]

$$f(y_i, z_i) = g(y_i) + h(z_i). \quad (4.27)$$

In the case of global SUSY, the superpotential (4.27) leads to two noninteracting sectors of the theory. Gravity effects will mix these sectors.

For simplicity, we confine our attention to one gauge interaction singlet of the chiral superfield z in the hidden sector. Substituting (4.27) in (4.21), and proceeding to the limit as $M\to\infty$, we obtain the following scalar-particle potential:

$$V(y_i, z) = V(z) + V(y_i), \quad (4.28a)$$

$$V(z)=\exp\left(\frac{|z|^2}{M^2}\right)\left(\left|\frac{\partial h}{\partial z}+\frac{z^*h}{M^2}\right|^2-3\,\frac{|h|^2}{M^2}\right), \quad (4.28b)$$

$$V(y_i)=\exp\left(\frac{|z|^2}{M^2}\right)\left\{\left|\frac{\partial g}{\partial y_i}+\frac{y_i^*h}{M^2}\right|^2 + \frac{g}{M^2}\left[z^*\left(\frac{\partial h}{\partial z}+\frac{z^*h}{M^2}\right)-3h\right]+\text{c.c.}\right\} + \frac{g^2}{2}|y_i^*\mathrm{T}y_i|^2. \quad (4.28c)$$

If we take h in the form of (4.25), we find that $\langle z\rangle\sim M_P$, $\langle h\rangle\sim M_W M_P^2$ at the minimum of (4.28b). The parameters in h must be chosen so that $V(z)$ is zero at the minimum (this cancels the cosmologic term $\Lambda\sim M_W^2M_P^2$). When $V(y_i)$ (4.28c) is minimized, z and $h(z)$ must be replaced with their vacuum values. The effective potential of the low-energy sector is found to be

$$V(y_i)=\left|\frac{\partial g}{\partial y_i}+m_{3/2}y_i^*\right|^2+(A-3)\,m_{3/2}(g+g^*) + \frac{1}{2}\sum_a D_a^2(y_i), \quad (4.29)$$

where

$$A=\left\langle \exp\left(\frac{|z|^2}{2M^2}\right)z^*\left(\frac{\partial h}{\partial z}+\frac{z^*h}{M^2}\right)h^{-1}\right\rangle\sim\frac{z_{\min}}{M}.$$

In deriving (4.29), we used the replacement $g(y_i)\to\exp(-\langle z\rangle^2/2M^2)\times g(y_i)$; D_a represents the D-term of the vector multiplet V_a, i.e., $D_a\equiv gy_i^*T^ay_i$. Having begun with a set of fields y_i and z that interact because of supergravity, we have arrived at the potential (4.29) in which the hidden sector appears in only two parameters, namely, the gravitino mass $m_{3/2}$, which fixes the scale of the theory, and the number $A\sim1$. The spinorial fields ψ_{y_i} interact as in global SUSY theory with superpotential $g(y_i)$. We thus arrive at a global SUSY theory that is explicitly broken in the sector of interacting scalar fields. The SUSY-breaking operators are soft and so do not lead to quadratic divergences (see Section 3).

The superpotential of the low-energy sector $g(y_i)$ contains two types of term: first, there is the interaction between quark and lepton chiral multiplets with the Higgs multiplets, which gives mass to quarks and leptons and, second, there is the selfinteraction of the Higgs multiplets. In the simplest case of two Higgs multiplets, we obtain

$$g=h_u H_L'Q_LU_R^*+h_dH_LQ_LD_R^*+h_lH_LL_LE_R^*+\mu H_L'H_L. \quad (4.30)$$

The conformally invariant form of g with $\mu=0$ is esthetically the more attactive. The mass scale of the fields y_i is then wholly determined by SUGRA effects.

D. The masses of the quarkino, leptino, and gaugino

The quarkino and the leptino receive a universal mass contribution equal to the gravitino mass $m_{3/2}$ [see (4.29)]. The problem of heavier scalar partners of quarks and leptons, which prevents the construction of a realistic model based on spontaneous SUSY breaking (see Section 3), is thus solved at a stroke. If the vacuum averages of the Higgs doublets H and H' are not equal, the term proportional to $D^2(y_i)$ will also contribute to the quarkino and leptino masses. This contribution depends on the particle charges with respect to the gauge U(1) group, but is independent of the generation number. (We shall assume below that $\langle H^0\rangle = \langle H'^0\rangle$.) Finally, the terms $m_{3/2} y_i \partial g/\partial y_i$ and $(A-3)m_{3/2}(g+g^*)$ lead to contributions to the mass matrix that are proportional to the masses of the corresponding quarks and leptons and mix right-handed and left-handed quarkinos and leptinos. The quarkino (leptino) eigenvalues of the mass matrices are

$$m_{\tilde{q}(\tilde{l})} = m_{3/2} + \alpha m_{q(l)}, \qquad (4.31)$$

where α is a number of the order of unity. It follows from (4.31) that the quarkino and leptino masses in the first two generations are practically degenerate.

We begin our analysis of gaugino masses with the simplest case of the gluino. In the case of minimal coupling between vector multiplets and supergravity [see (4.20)], the gluino is massless in the tree approximation. At the one-loop level, the graphs of Fig. 10 ensure that the gluino assumes the mass

$$m_{\tilde{g}} \approx \frac{\alpha_s}{\pi} \frac{1}{m_{3/2}} \sum_q m_q^2.$$

Substituting $m_{3/2} = 100$ GeV, $\alpha_s = 0.1$ and $m_q = 30$ GeV, we obtain $m_{\tilde{g}} \sim 300$ MeV. In grand unification theories, the transmission of heavy particles must be taken into account in the loop of Fig. 10. The gluino mass is then given by $m_g \sim c(\alpha_s/\pi) m_{3/2}$, where the factor $m_{3/2}$ appears because of the transformation of the "right-handed" scalar particle into a "left-handed" particle, which clearly violates SUSY (c is the Casimir operator). On the SU(5) 24-plet of Higgses, $c = 5$ and $m_g \sim 15$ GeV. The SU(2) and U(1) gauginos receive an analogous diagonal mass with $\alpha_s(\alpha_3)$ replaced with α_2 and α_1, respectively.

In the case of nonminimal coupling to SUGRA, the gauginos acquire diagonal mass in the tree approximation. Taking $\varphi_{\alpha\beta}(s_i)$ in (4.18) in the form $\delta_{\alpha\beta}\hat{z}$, we obtain the universal diagonal mass of the gauginos:

$$m_{\tilde{g}_i \tilde{g}_i} = \frac{F_z}{M_P} (\tilde{g}_i \tilde{g}_i) = m_{3/2} (\tilde{g}_i \tilde{g}_i). \qquad (4.32)$$

Thus, in the case of nonminimal coupling, the gauginos acquire a large universal diagonal mass $\sim m_{3/2}$. When the SU(2)×U(1) gaugino masses are examined, one has to deal with mass matrices. We begin with the charged bosinos $\tilde{W}^{\pm}$. The most general interaction between the four Weyl fields $\tilde{W}_R^{\pm}$, $\tilde{H}_L'^{+}$, and $\tilde{H}_L^{-}$ is

$$\hat{V} = \mu_1 \bar{\tilde{W}}_R^{+} \tilde{H}_L'^{+} + \mu_2 \bar{\tilde{W}}_R^{-} \tilde{H}_L^{-} + M_{\tilde{W}\tilde{W}} \tilde{W}_R^{+} \tilde{W}_R^{-} + M_{\tilde{H}\tilde{H}'} \tilde{H}_L'^{+} \tilde{H}_L^{-} + \text{c.c.} \qquad (4.33)$$

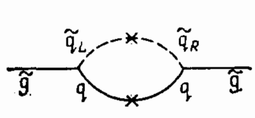

FIG. 10. Emergence of the gluino mass at the one-loop level.

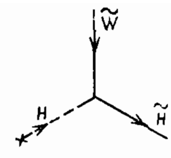

FIG. 11. Emergence of the supersymmetric off-diagonal mass of the W-bosino as a result of the Higgs effect.

The first two terms which mix $\tilde{W}$ and $\tilde{H}$ arise when the vacuum averages appear in the scalar Higgs fields (Fig. 11); $\mu_1 = g\langle H'^0\rangle$, $\mu_2 = g\langle H^0\rangle$. The third term is SUSY-breaking. Its origin and magnitude in SUGRA-based models have just been discussed. Finally, the fourth term is due to the term $M_{\hat{H}H'} \hat{H}\hat{H}'$ in the superpotential $g(y_i)$.

If the superpotential does not contain the term $M_{\tilde{H}\tilde{H}'} \hat{H}\hat{H}'$, the $\tilde{H}_L'^{+} \tilde{H}_L^{-}$ transition will occur as a result of the radiative corrections (Fig. 12a). The transition matrix element is then of the order of

$$M_{\tilde{H}'^{+}\tilde{H}^{-}} \approx \frac{1}{16\pi^2} \frac{m_t m_b}{\langle H'^0\rangle \langle H^0\rangle} \frac{m_t m_b m_{3/2}}{m_{\tilde{t}}^2} \approx 60\ \text{keV}, \qquad (4.34)$$

where, for the purposes of numerical estimates, we have substituted $\langle H^0\rangle = \langle H'^0\rangle$, $m_t = 30$ GeV, $m_{3/2} = m_{\tilde{t}} = 100$ GeV.

Let us rewrite (4.33) as the matrix

$$(\bar{\tilde{W}}_R^{+} \tilde{H}_L^{-} \tilde{W}_R^{-} \bar{\tilde{H}}_L'^{+}) \begin{pmatrix} 0 & 0 & M_{\tilde{W}\tilde{W}} & \mu_1 \\ 0 & 0 & \mu_2 & M_{\tilde{H}\tilde{H}} \\ M_{\tilde{W}\tilde{W}} & \mu_2 & 0 & 0 \\ \mu_1 & M_{\tilde{H}\tilde{H}} & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{W}_R^{+} \\ \bar{\tilde{H}}_L^{-} \\ \bar{\tilde{W}}_R^{-} \\ \tilde{H}_L'^{+} \end{pmatrix}$$

and find its eigenvalue. From the four nucleating fields (two Weyl $\tilde{H}_L'^{+}$ and H_L^{-} and two Majorana $\tilde{W}_1$ and $\tilde{W}_2$), we form the two Dirac particles $\tilde{w}_{1,2}$:

$$M_{\tilde{w}_{1,2}}^2 = \frac{1}{2} \Big(\mu_1^2 + \mu_2^2 + M_{\tilde{W}\tilde{W}}^2 + M_{\tilde{H}\tilde{H}}^2 \pm \sqrt{(\mu_1^2 + \mu_2^2 + M_{\tilde{H}\tilde{H}}^2 + M_{\tilde{W}\tilde{W}}^2)^2 - 4(\mu_1\mu_2 - M_{\tilde{H}\tilde{H}} M_{\tilde{W}\tilde{W}})^2} \Big). \qquad (4.35)$$

The mass of the W-boson is $M_W^2 = (\mu_1^2 + \mu_2^2)/2$. As first noted by Weinberg,[13] one of these new particles is lighter than the W-boson in a wide range of models. In fact, if $M_{\tilde{H}\tilde{H}}$ and/or $M_{\tilde{W}\tilde{W}} \ll \mu_{1,2}$, theory predicts the existence of a Dirac particle of mass lower than that of the W. The decay $W^{\pm} \to \tilde{w}^{\pm} + \tilde{\gamma}$ can then occur with appreciable probability (see Introduction).

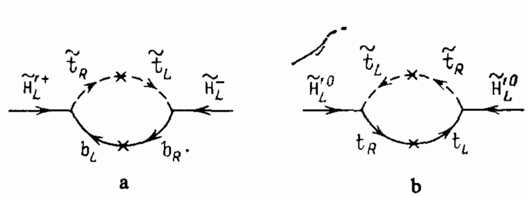

FIG. 12. Emergence of the higgsino masses due to radiative corrections (the greatest contribution is due to third-generation quarks).

The quadratic part of the interaction of the neutral W_3 and B bosinos and the $\tilde{H}^0$- and $\tilde{H}'^0$-higgsinos has the form

$$\hat{V} = \frac{\eta}{\sqrt{2}}(g\bar{\tilde{W}}_3 - g'\bar{\tilde{B}})_R\tilde{H}^0_L + \frac{\eta'}{\sqrt{2}}(g\bar{\tilde{W}}_3 - g'\bar{\tilde{B}})_R\tilde{H}'^0_L$$
$$+ M_{\tilde{W}\tilde{W}}(\tilde{W}_{3R}\tilde{W}_{3R}) + M_{\tilde{B}\tilde{B}}(\tilde{B}_R\tilde{B}_R) + M_{\tilde{H}\tilde{H}'}(\tilde{H}^0_L\tilde{H}'^0_L), \tag{4.36}$$

where $\eta/\sqrt{2} \equiv \langle H^0 \rangle$, $\eta'/\sqrt{2} \equiv \langle H'^0 \rangle$. In the general case, there are four massive neutral Majorana fermions. If the diagonal masses of the gauginos are small (minimal coupling to SUGRA), $M_{\tilde{B}\tilde{B}}$, $M_{\tilde{W}\tilde{W}} < g\eta, g\eta'$, we obtain a light photino $\tilde{\gamma} = (g'\tilde{W}_3 + g\tilde{B})/\sqrt{g^2 + g'^2}$. If the superpotential does not contain the term $M_{\hat{H}\hat{H}'}\hat{H}\hat{H}'$, the spectrum contains the light higgsino $\chi = (\eta'\tilde{H}^0_L - \eta H'^0_L)/\sqrt{\eta^2 + \eta'^2}$. The mass of this light higgsino is determined by the graphs of Fig. 12b. An estimate analogous to (4.34) yields $M\chi \sim 2$ MeV. The light higgsino is the superpartner of the axion, which appears in the absence of the term $M_{\hat{H}\hat{H}'}\hat{H}\hat{H}'$ in the superpotential. If the light higgsino is the lightest superparticle, then it is stable and is the final product of the decay of SUSY particles. The creation of gauginos in $W^{\pm}$ and Z boson decays, which is important from the practical point of view, is discussed in detail in a recent paper by Fayet.[60]

E. SU(2)×U(1) symmetry breaking

The Higgs bosons form part of the composition of chiral multiplets and, by analogy with the quarkinos and leptinos, acquire a universal positive squared mass equal to $m^2_{3/2}$. However, a *negative* squared mass of Higgs bosons is necessary for SU(2)×U(1) breaking. The question can be formulated in a general form: how do the Higgs chiral multiplets differ from quark and lepton multiplets, and what leads to different signs of the squared masses of the bottom components of chiral multiplets?

The simplest way of producing SU(2)×U(1) breaking is to extend the Higgs sector by introducing the singlet chiral field Y. Let us take the conformally invariant superpotential

$$g = \frac{\sigma}{3}Y^3 + \lambda YH'H \tag{4.37}$$

and, for simplicity, substitute $A = 3$ in (4.29) [there is a wide class of models which lead to the scalar-particle potential (4.29) with $A = 3$].[61] For the scalar-particle potential, we then obtain

$$V(Y, H', H) = |\sigma Y^2 + m_{3/2}Y + \lambda H'\varepsilon H|^2 + |\lambda YH + mH'|^2$$
$$+ |\lambda YH' + mH|^2 + \frac{g^2 + g'^2}{8}|H^+H - H'^+H'|^2. \tag{4.38}$$

$V(Y, H', H)$ can have a minimum corresponding to broken SU(2)×U(1) symmetry:

$$\langle H^0 \rangle = \langle H'^0 \rangle = (m/\lambda)\sqrt{1 - (\sigma/\lambda)}, \; Y = -m/\lambda.$$

The introduction of the singlet field Y is acceptable within the framework of the SU(3)×SU(2)×U(1) model, but this cannot be done in the unified SUSY theory. This field has been given a special name: "the sliding singlet."[47,62] It leads to the low-energy scale M_W being pulled in the upward direction. The graph of Fig. 13 appears in SU(5)

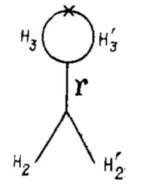

FIG. 13. Generation of a large transition mass between the doublets H_2 and H'_2 due to the singlet Y.

theory. The cross represents the supersymmetry-breaking mass insert $m_{3/2}MH_5H'_5$, where M is the mass of the colored triplets H_3, H'_3, $M \sim M_{GUT}$. The diagram of Fig. 12 leads to an inadmissibly large mass insert $m_{3/2}M_{GUT}H_2H'_2$, i.e., $M_W \sim \sqrt{m_{3/2}M_{GUT}}$. Since it follows from the experimental absence of charged quarkinos and leptinos that $m_{3/2} > 20$ GeV, the singlet field Y cannot be used without SU(2)×U(1) breaking. Extension of the Higgs sector by nonsinglet fields violates the relation $M^2_W = M^2_Z \cos^2\theta_W$ which has been confirmed experimentally with good precision. Can SU(2)×U(1) symmetry-breaking be achieved without extending the Higgs sector? An answer to this question will be given in the next section.

F. SU(2)×U(1) symmetry breaking by radiative corrections

In the most general form, the potential of two Higgs doublets is

$$V(H, H') = \frac{m_1^2}{2}|H|^2 + \frac{m_2^2}{2}|H'|^2 - m_3^2H'H$$
$$+ \frac{g^2 + g'^2}{8}(|H|^2 - |H'|^2)^2, \tag{4.39}$$

where the cross interaction $m_3^2H'H$ arises when the term $M\hat{H}'\hat{H}$ is present in the superpotential. The requirement that the potential must have a lower bound along the line $|H| = |H'|$ is

$$m_1^2 + m_2^2 - 2m_3^2 > 0. \tag{4.40}$$

The condition that $H = H' = 0$ is a saddle point [this is necessary for the presence of a minimum with broken SU(2)×U(1) symmetry] is

$$m_1^2m_2^2 < m_3^4. \tag{4.41}$$

When $m_1 = m_2$, inequalities (4.40) and (4.41) are incompatible: if the coupling between H and H' is small, the potential has a single SU(2)×U(1) symmetric minimum with $\langle H \rangle = \langle H' \rangle = 0$ and, as soon as m_3 is large enough for the minimum $\langle H \rangle = \langle H' \rangle = 0$ to transform into a saddle, the absolute minimum of the potential departs to infinity and the stability of the scale M_W (for which ($N = 1$)-SUSY is introduced) is absent even in the tree approximation. The root of this evil must be sought in the symmetry of the problem under the replacement of H with H'. As was first noted in Ref. 63, the equation $m_1 = m_2$ is violated when radiative corrections are taken into account. The point is that H and H' have different interactions with quarks: the doublet H' gives mass to the up quarks and H to the down quarks, so that the quark loops produce a different renormalization of the masses m_1 and m_2.

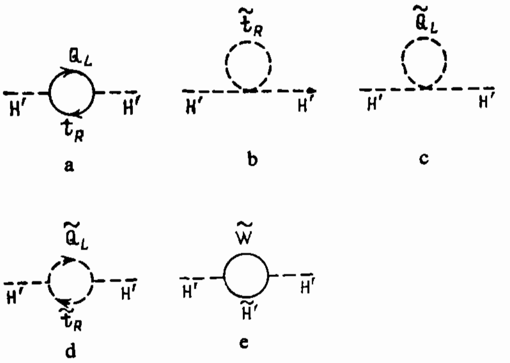

FIG. 14. Renormalization of the mass of the Higgs boson H'.

The largest contribution to the renormalization is provided by loops with heavy quarks. Let us therefore confine our attention to the t-quark. The renormalization of M_2^2 is determined by the graphs of Fig. 14. The individual graphs contain quadratically divergent contributions to m_2^2 but, since SUSY is softly broken, only the logarithmic divergence remains in the sum of the diagrams of Fig. 14. Let us examine the origin of the individual diagrams in Figs. 14. In the SUSY part of the theory, the diagrams of Figs. 14 a–c are due to the term $h_t H'_L \tilde{Q}_L \tilde{t}_R^*$ in the superpotential $g(y_i)$, while the diagram of Fig. 14d is due to the term

$$(A-3)\, m_{3/2}(g+g^*) + m_{3/2}\left(y_i \frac{\partial g}{\partial y_i} + y_i^* \frac{\partial g}{\partial y_i^*}\right)$$

in the scalar-particle potential (4.29). Finally, the diagram of Fig. 14e is due to the gauge interaction of the Higgs chiral multiplets. If we evaluate the diagrams of Fig. 14, we obtain the following equation for the renormalization group:

$$Q\frac{dm_{H'}^2}{dQ} = \frac{h_t^2}{8\pi^2}\, 3\,(m_{H'}^2 + m_{\tilde{t}_R}^2 + m_{\tilde{Q}_L}^2 + A^2 m_{3/2}^2) - \frac{\alpha_2}{2\pi}\, 3M_2^2, \tag{4.42a}$$

where Q is the running momentum, M_2 represents $M_{\tilde{w}\tilde{w}}$, and the factor 3 in front of the parentheses is due to the sum over the four colors of quarks in the loop. Before (4.42a) can be solved, it must be augmented by the renormalization-group equations for the parameters on the right-hand side and by the boundary conditions. Let us consider the boundary conditions first. The derivation of the potential (4.29) shows that the parameters of this equation are normalized for virtualities $\sim M_P$, so that the boundary condition for (4.42a) is $m_{H'}^2(Q^2 = M_P^2) = m_{3/2}^2$ (when the term $M_{H'H}$ is present in the superpotential, it is replaced with $m_{H'}^2 = m_{3/2}^2 + M^2$). The first term on the right-hand side of (4.42a) facilitates the reduction in $m_{H'}^2$ with decreasing virtuality, and the second increases $m_{H'}^2$. The renormalization-group equations for $m_{\tilde{t}_R}^2$ and $m_{\tilde{Q}_L}^2$ are obtained from diagrams analogous to those shown in Fig. 14 and identical with (4.42a) except for two replacements: the diagram of Fig. 14e is replaced with the analogous diagram in which the gluino is in the intermediate state and the factor in front of the parentheses in (4.42a) is replaced with 2 for t_R [summation in the loop over the two isospins states of $SU(2)_L$] and with 1 for Q_L (all the group indices leave the loop in the outward direction):

$$Q\frac{dm_{\tilde{t}_R}^2}{dQ} = \frac{h_t^2}{8\pi^2}\, 2\,(m_{H'}^2 + m_{\tilde{t}_R}^2 + m_{\tilde{Q}_L}^2 + A^2 m_{3/2}^2) - \frac{8}{3}\frac{\alpha_3}{\pi} M_3^2, \tag{4.42b}$$

$$Q\frac{dm_{\tilde{Q}_L}^2}{dQ} = \frac{h_t^2}{8\pi^2}(m_{H'}^2 + m_{\tilde{t}_R}^2 + m_{\tilde{Q}_L}^2 + A^2 m_{3/2}^2) - \frac{8}{3}\frac{\alpha_3}{\pi} M_3^2. \tag{4.42c}$$

A numerical analysis of the complete set of renormalization-group equations is given in Refs. 64–66. Here, we shall confine our attention to a qualitative analysis. We begin with the case of small M_2, M_3 (minimal coupling between vector multiplets and gravity). The right-hand sides of (4.42a)–(4.42c) are positive, so that, as Q decreases from M_P to $m_{3/2}$ (the region of logarithmic loops in Fig. 14, for which (4.42) are valid), the squares of the Higgs and quarkino masses decrease. Because of the presence of the coefficient 3, the most rapidly decreasing is the $m_{H'}^2$, so that $m_{H'}^2$ may become negative while $m_{\tilde{t}_R}^2$ and $m_{\tilde{Q}_L}^2$ remain positive. Thus, SU(3) remains unbroken in SU(3)×SU(2)×U(1) theory and SU(2)×U(1) is broken because 3 > 2 and 3 > 1. An increase in M_2, M_3 leads to a slower fall in m_i^2 with Q^2. When $M_2, M_3 \sim m_{3/2}$ (nonminimal coupling with gravity), the main effect is the slowing down of the fall in $m_{\tilde{t}_R}^2$, $m_{\tilde{Q}_L}^2$ and, since their values appear in the parentheses of (4.42a), large M_i stimulate a more rapid reduction in $m_{H'}^2$. When $m_3^2 < m_1^2$, m_2^2 and the t-quark mass is $m_t > 60$ GeV, the inequalities given by (4.40)–(4.41) are satisifed for $Q^2 \sim m_{3/2}^2$ and SU(2)×U(1) symmetry is broken for nonzero vacuum averages $\langle H'^0\rangle$, $\langle H^0\rangle$, $\langle H'^0\rangle > \langle H^0\rangle$. When $m_{3/2} \sim 100$ GeV and $A \sim 1$, it is possible to obtain the vacuum averages of Higgs fields that are necessary[64] for the generation of $M_W \sim 85$ GeV. The question is: what will happen if experiment shows that the t-quark is light, say, $m_t = 30$ GeV? The most straightforward escape from this dilemma is to introduce the fourth generation of matter with sufficiently heavy quarks. Another path was proposed in Ref. 66, where it was noted that, if $m_3^2(M_P^2)$ is somewhat less than $m_2^2(M_P^2)$, the inequalities (4.40)–(4.41) will be satisfied for quite minor evolution of m_2^2. The t-quark can then be light, and the only limitation on its mass follows from the absence of the creation of new hadrons in e^+e^- annihilation, i.e., $m_t > 20$ GeV.

An interesting variant of SU(2)×U(1) symmetry-breaking by radiative corrections was examined in Ref. 65, where a model with superpotential $g(y_i)$ not containing the term $M_{H'H}$ was considered. The superpotential $g(y_i)$ does not contain parameters with the dimensions of mass, and the mass scale in the low-energy sector is wholly determined by the gravitino mass $m_{3/2}$, which makes the model particularly attractive. In the potential (4.39), $m_3^2 = 0$, which is inadmissible at first sight since, even if m_2^2 changes sign in the course of evolution, $\langle H'\rangle \neq 0$, $\langle H\rangle = 0$ at the minimum of the potential (4.39), and the down quarks and charged leptons are massless. As noted in Ref. 65, if the sum $m_1^2(Q_0^2) + m_2^2(Q_0^2)$ vanishes for some Q_0^2, the potential (4.39) will be unbounded from below along the line $|\langle H^0\rangle| = |\langle H'^0\rangle|$ as Q^2 is reduced further. Formally, the minimum is reached for $|\langle H^0\rangle| = |\langle H'^0\rangle| = \infty$ and, in reality, $|\langle H^0\rangle| = |\langle H'^0\rangle| \approx Q_0$. The model demands a heavy t-quark with $m_t \geqslant 65$ GeV and a heavy gaugino (nonminimal coupling to gravity) for sufficient evolution of m_2^2.

G. Renormalization of gaugino and light quarkino masses

When the quarkino and leptino masses were discussed, it was noted that, in the tree approximation, the scalar partners of fermions in the first two generations are practically degenerate. We must now consider how this conclusion is affected when radiative corrections are taken into account. The renormalization of the quarkino mass by the strong interaction is the most important. From (4.42b) and (4.42c), we find that

$$Q\frac{dm_{\tilde Q}^2}{dQ} = -\frac{8}{3}\frac{\alpha_3}{\pi}M_3^2. \tag{4.43a}$$

This equation must be augmented with the equation describing the renormalization of the coupling constant α_3:

$$Q\frac{d\alpha_3}{dQ} = -\frac{b_3}{2\pi}\alpha_3^2. \tag{4.43b}$$

In the Yang-Mills theory, the Gell-Mann-Low coefficient b_3 is equal to $(11/3)(c_V) - (4/3)N_f/2$, where $c_V = N$ for SU(N) and N_f is the number of quark flavors. In the SUSY theory, the Majorana fields are added to the vector fields, and $(11/3)c_V$ is replaced with $(11/3)c_V - (2/3)c_V = 3c_V$. The contribution of quark multiplets is altered by the presence of the quarkinos, and $(4/3)(1/2)N_f$ becomes $[(4/3) + (2/3)]N_f/2 = N_f$. Combining all these together, we find that $b_3 = 3c_V - N_f$. For SU(3) and six quark flavors, we have $b_3 = 3$. Finally, we need the evolution equation for M_3:

$$Q\frac{dM_3}{dQ} = -\frac{\alpha_3}{2\pi}b_3M_3. \tag{4.43c}$$

Solution of (4.43) yields

$$\frac{M_3(Q^2)}{M_3(\mu^2)} = \frac{\alpha_3(Q^2)}{\alpha_3(\mu^2)},$$

$$m_{\tilde Q}^2(\mu^2) = m_{\tilde Q}^2(Q^2) + \frac{8}{9}[M_3^2(\mu^2) - M_3^2(Q^2)]. \tag{4.44}$$

The parameters of the effective Lagrangian are normalized to $Q^2 = M_P^2$, and for μ^2 we take the low-energy point $\mu^2 = (100\text{ GeV})^2$. If we choose $\alpha_3(\mu^2) = 0.1$, we find from (4.43b) that $\alpha_3(M_P^2) = 0.035$, i.e., the "experimental" gluino is heavier by a factor of 3 as compared with the normalization point. As regards the quarkino, we have two possibilities: (a) the light gluino $M_3 \ll m_{3/2}$ (minimal coupling to gravity) leads to slight renormalization of $m_{\tilde Q}^2$, and the quarkinos and leptinos in the first two generations are practically degenerate and (b) the heavy gluino $M_3(M_P^2) = m_{3/2}$ (nonminimal coupling to gravity) leads to $m_{\tilde Q}(100\text{ GeV}) \approx 2.5m_{3/2}$, and the quarkino is much heavier than the leptino.

H. Off-diagonal neutral currents

A new and potentially significant source of off-diagonal neutral currents appears in SUSY models, namely, gluino exchange.[67–69] Let us introduce the matrices $U_{L(R)}$ and $D_{L(R)}$ that diagonalize the mass matrices of the up and down (left and right) quarks, respectively, and the matrices $\tilde U_{L(R)}, \tilde D_{L(R)}$ that diagonalize the quarkino mass matrices. We shall confine our attention to left-handed particles since they lead to large off-diagonal neutral currents. The coupling between the gluino and the down particles is

$$g_3\tilde{\bar g}D_Ld_L\tilde d_L^*\tilde D_L^+, \tag{4.45}$$

where g_3 is the SU(3) coupling constant and d_L is the column of left-handed down quarks and $\tilde d_L^*$ is a row of left-handed down quarkinos. In the models considered in this section, the quarkino mass matrix consists of three parts: the flavor-independent diagonal contribution $m_{3/2}^2$, the supersymmetric contribution $\tilde d_L^* M_d^+ M_d \tilde d_L$, and the SUSY breaking contribution $Am_{3/2}\tilde d_L M_d \tilde d_R^*$, where M_d is the mass matrix of the down quarks. It is clear that the rotation of the field $\tilde d_L$ by the matrix D_L (and the rotation of $\tilde d_R$ by the matrix D_R) takes out transitions in the mass matrix that are off-diagonal in the flavor and, at the same time, it diagonalizes the interaction (4.45) because D_L is unitary. Gluino exchange does not lead to off-diagonal currents in the tree approximation. The situation changes when radiative corrections are taken into account. Diagrams analogous to those shown in Fig. 14 lead to a correction to the mass matrix of down quarkinos that is proportional to the mass matrix of the up quarkinos $M_u^+M_u$ (the inverse effect is numerically less important). If we suppose that this correction exceeds the Yukawa tree contribution to the mass of the down quarkinos, the mass matrix is diagonalized by the matrix that diagonalizes the masses of the up quarks. The emission of the gluino during the transition of down quarks to up quarkinos is described by the matrix $U_L^+D_L$, equal to the Kobayashi-Maskawa matrix. The evaluation of the graphs of Fig. 15 and their comparison with the experimental mass difference between the K_L- and K_S-mesons has led to the following lower bound for the gluino mass:[67] $m_{\tilde g} > 37$ GeV. This number was obtained on the assumption that $m_{\tilde g} \approx m_q$. However, light gluinos are not excluded by Δm_{LS} if the quarkinos are heavy enough.

I. Neutron EDM

In the standard SU(3)×SU(2)×U(1) model, there are two sources for the neutron dipole moment that violates CP-invariance, namely, the θ-term and the CP-odd phase of the Kobayashi-Maskawa matrix. We shall assume that θ is negligible. The phase δ leads to $d_n \sim 10^{-32}$ e·cm,[21] whereas the modern experimental limit is $d_n < 6\times10^{-25}$ e·cm.[22] The predictions of SUSY models are exceedingly interesting in this situation. The point is that additional sources of CP-odd phase, which contributes to d_n, appear in these models.[23–26]

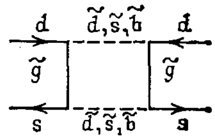

FIG. 15. Contribution of gluino exchanges to the mass difference between the K_L- and K_S-mesons.

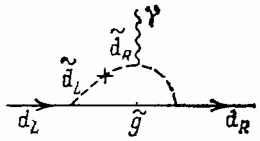

FIG. 16. Appearance of the neutron dipole moment in SUSY theories.

The maximum contribution to d_n is given by the diagram with gluino exchange (Fig. 16):

$$d_n = e\frac{\alpha_3}{\pi}\frac{m_d m_{\tilde{g}}}{M^3_{\tilde{d}}}\,\mathrm{Im}\,A \approx 10^{-22}\ \mathrm{Im}\,A\ e\cdot\text{см}, \qquad (4.46)$$

where, in the numerical estimate, we have substituted $\alpha_3 = 0.1$, $m_d = 10$ MeV, $m_{\tilde{g}} = M_{\tilde{d}} = 100$ GeV. Assuming that $\mathrm{Im}\,A \sim 1$, we find that d_n is greater by two or three orders of magnitude than the experimental upper bound. The question is: how can we suppress the theoretical prediction (4.46)? The first thing we can think of is to reduce the gluino mass $m_{\tilde{g}}$. In the limit of the massless gluino, the CP-odd phase may be "deflected" from zero. A strictly massless gluino would be in conflict with the observed spectrum of pseudoscalar mesons[7] but the light gluino with a mass of about 5 GeV is, at present, experimentally allowed. It is possible that the low experimental value of d_n is indirect evidence for minimal SUGRA, which predicts a relatively light gluino. Methods of reducing d_n^{SUSY} that use different mechanisms for the suppression of the CP-odd phase are discussed in Ref. 26.

J. Toward a "natural" theory

The models considered in this section have two flaws: first, the superpotential parameters are artificially chosen so that the cosmologic term at the minimum of the scalar particle potential is zero and, second, in addition to the natural mass scale equal to the Planck mass M_P, another scale M_W (lower by 17 orders of magnitude) is introduced into the theory "by hand." In this section, we shall consider recent attempts to construct a scheme that is free from these defects.

Cremmer[71] has noted an interesting property of theories with the Kahler potential of the chiral field Z of the form

$$G(Z, Z^*) = -\frac{3}{2}\ln[f(Z)+f^*(Z^*)]^2, \qquad (4.47)$$

where $f(z)$ is an arbitrary function. The potential of the field Z determined from (4.4) is identically zero, i.e., $\langle z\rangle$ is undetermined in the tree approximation. On the other hand, the gravitino mass is found to be

$$m_{3/2} = e^{G/2} = \frac{1}{[f(Z)+f^*(Z^*)]^{3/2}}. \qquad (4.48)$$

We thus have a theory with a cosmologic term that is identically zero, while the SUGRA-breaking scale is undetermined. It is interesting that, in extended ($N=4$)-SUGRA, the scalar sector is described[72] by a potential analogous to (4.47). If we use the field Z with the Kahler potential (4.47) as the hidden sector, the first of the defects listed at the beginning of this section will be removed.

An attempt to generate dynamically the scale M_W in the theory with the hidden sector (4.47) and to construct a natural theory was undertaken in Refs. 73 and 74. The Kahler potential was taken in the form

$$G(y_i, Z) = -\frac{3}{2}\ln[f(Z)+f^*(Z^*)]^2 + \sum_i |y_i|^2 + h(y_i) + h^*_i(y_i^*), \qquad (4.49)$$

where y_i was the observed sector, which has led to a potential for the scalar field y_i of the form given by (4.29), with $m_{3/2}$ left undetermined. The quantity $m_{3/2}$ is determined when radiative corrections to the potential (4.29) are taken into account, and turns out to be of the order of

$$m_{3/2} \sim \exp\left(-\frac{1}{h_t^2}\right) M_P, \qquad (4.50)$$

where h_t is the Yukawa coupling constant between t-quarks and the Higgs field. It appears in (4.50) because heavy-quark loops play a dominant role in the radiative corrections that determine the minimum of the scalar-particle potential in this case.

5. SUSY GUTs AND PROTON DECAY

A. The hierarchy problem

One of the factors that have stimulated the intensive development of theories with low-energy SUSY in recent years has been the hope that the hierarchy problem might be solved in grand unification theories. Two scales arise in the theory, namely, the grand unification scale $M_{\mathrm{GUM}} \sim 10^{15}$ GeV and the electroweak unification scale $M_W \sim 100$ GeV, when nonzero vacuum averages appear in Higgs fields. The hierarchy problem then has two aspects: (a) unnatural tree Lagrangian containing mass parameters differing by 13 orders of magnitude and (b) the mixing of these parameters when radiative corrections are taken into account.[5] Let H denote the Higgs doublet ensuring the breaking of $SU(2)\times U(1)$ invariance, and Σ the SU(5)-violating 24-plet. The SU(5)-GUT tree potential is chosen so that $\langle H\rangle \sim 100$ GeV and $\langle\Sigma\rangle \sim 10^{15}$ GeV. The loop graphs of Fig. 17 lead to the appearance of the term $(c_1 g^4 + c_2 g^6 + c_3 g^8 + \dots)\ln \Lambda^2 H^2 \Sigma^2$ in the effective potential. To avoid "pulling up" $\langle H\rangle$ onto the scale of 10^{15} GeV, we must introduce a counterterm into the Lagrangian that exactly cancels the radiative correction. The loop graphs of Fig. 17 are exactly cancelled in SUSY-SU(5) by analogous graphs with spinorial partners of gauge and Higgs particles in an intermediate state.[75] This is the essence of the SUSY scenario for the solution of the technical aspect (b) of the GUT hierarchy problem.

The vanishing of scale mixing in SUSY GUT models can be illustrated by the following formula, which describes

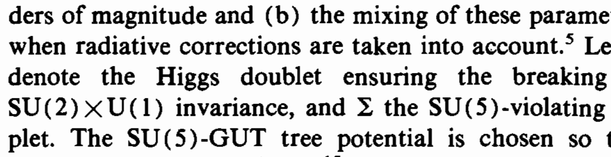

FIG. 17. Graphs leading to the mixing of GUT scales due to radiative corrections. Wavy lines represent gauge bosons.

the effective potential of light scalars in the single-loop approximation:

$$V_{\text{eff}}(H)=V_{\text{cl}}(H)+g^2H^2 \times\left(\int\frac{d^4P}{P^2-M^2-g^2\Sigma^2}-\int\frac{d^4P}{P^2-M^2-\Delta m^2-g^2\Sigma^2}\right).$$

The first term in parentheses corresponds to the nonsupersymmetric theory. It generates the quadratically divergent contribution to the mass of the field H and, at the same time, leads to the mixing of the fields H and Σ. The second term is due to the fermion loop. Taking into account SUSY breaking in the masses of the virtual particles $\sim\Delta m^2$, we see that the quadratic divergence and the mixing of H and Σ has gone, and the remaining contribution to the effective potential is $\sim g^2H^2\Delta m^2\ln\Lambda^2$.

Let us describe SUSY SU(5)-GUT in greater detail.[39] The chiral doublets $\varepsilon H'$ and H are members of the antiquintuplet H_5' and the quintuplet H_5, respectively. The model contains the chiral 24-plet Σ. The most general superpotential corresponding to the renormalized theory is

$$W=\text{tr}\left(\frac{\lambda_1}{3}\hat{\Sigma}^3+\frac{M}{2}\hat{\Sigma}^2\right)+\lambda_2H'\hat{\Sigma}H+M'H'H, \quad (5.1)$$

where $\hat{\Sigma}\equiv\Sigma_a t_a$, t_a are the SU(5) generators, and $\text{Sp}\, t_a t_b=\frac{1}{2}\delta_{ab}$.

From (5.1), we find that the scalar-field potential is

$$V=2\,\text{tr}\left[\lambda_1\left(\hat{\Sigma}^2-\frac{1}{5}\text{tr}\,\hat{\Sigma}^2\right)+M\hat{\Sigma}+2\lambda_2(H'tH)t\right]^2 +|(\lambda_2\hat{\Sigma}+M')H|^2+|H'(\lambda_2\hat{\Sigma}+M')|^2+D\text{-terms}. \quad (5.2)$$

The $V=0$ minimum of (5.2) lies at zero vacuum values of the fields H and H' and the following configurations of the field Σ:

$$\langle\Sigma\rangle_1=0,\quad \langle\Sigma\rangle_2=\frac{M}{\lambda_1}\begin{pmatrix}2&&&&\\&2&&&\\&&2&&\\&&&-3&\\&&&&-3\end{pmatrix},\quad \langle\Sigma\rangle_3=\frac{M}{3\lambda_1}\begin{pmatrix}1&&&&\\&1&&&\\&&1&&\\&&&1&\\&&&&-4\end{pmatrix},$$

which correspond to the residual SU(5), SU(3) ×SU(2)×U(1) and SU(4)×U(1) symmetries. For obvious reasons, we shall work in the neighborhood of the second minimum. Taking M' from the condition

$$-3\frac{M}{\lambda_1}\lambda_2+M'=\mu',\quad \mu'\sim 100\,\text{GeV},$$

we find that the potential of the Higgs doublets is

$$V(H,H')=\mu'^2(|H|^2+|H'|^2)+\frac{g'^2}{8}(|H|^2-|H'|^2)^2 +\frac{g^2}{8}(H^+\sigma H+H'^+\sigma H')^2, \quad (5.3)$$

where σ are the Pauli matrices. [The quaternary term $\lambda_2^2(H'tH)^2$ in the potential (5.3) cancels out when the Born graph with exchange of the scalar Σ is taken into account.] The potential (5.3) does not contain parameters $\sim10^{15}$ GeV because of the "fine adjustment" of M. Because the theory is supersymmetric, this adjustment need only be introduced once, since radiative corrections do not destroy it.

The question now is: how is the foregoing discussion modified as we pass from SUGRA to SU(5)-GUT? Substituting (5.1) in (4.29), we find that the scalar-particle potential is given by

$$\begin{aligned}V=&2\,\text{tr}\left[\lambda_1\left(\hat{\Sigma}^2-\frac{1}{5}\text{tr}\,\hat{\Sigma}^2\right)+M\hat{\Sigma}+2m_{3/2}\hat{\Sigma} +2\lambda_2(H't^aH)t^a\right]^2\\ &+|(\lambda_2\hat{\Sigma}+M')H+m_{3/2}H|^2\\ &+[H'(\lambda_2\hat{\Sigma}+M')+m_{3/2}H']^2\\ &+(A-3)m_{3/2}\left[\text{tr}\left(\frac{\lambda_1}{3}\hat{\Sigma}^3+\frac{M}{2}\hat{\Sigma}^2\right)+\lambda_2H'\Sigma H\right.\\ &\left.+M'H'H+\text{c.c.}\right]+D\text{-terms}.\end{aligned} \quad (5.2')$$

The position of the minimum corresponding to SU(5) → SU(3)×SU(2)×U(1) breaking is shifted:

$$\langle\Sigma\rangle=\left(\frac{M+2m}{\lambda_1}+\frac{A-3}{\lambda_1}2\frac{m^2}{M}\right)\begin{pmatrix}2&&&&\\&2&&&\\&&2&&\\&&&-3&\\&&&&-3\end{pmatrix}$$

The value of the potential at the minimum (cosmologic constant) is now different from zero. A corresponding constant must be added to the superpotential (5.1) to annul the cosmologic constant. Finally, we reproduce the formula describing the interaction between the Higgs doublets at energies below the unification energy. Taking M' from the condition $(\lambda_2\Sigma+M')^{-3}=\mu'$, $\mu'\sim100$ GeV, we obtain:

$$\begin{aligned}V(H,H')&\\=&(\mu'^2+m_{3/2}^2)(H^+H+H'^+H')\\&+\left[(A-1)m_{3/2}\mu'+3(A-3)\frac{\lambda_2}{\lambda_1}m_{3/2}^2\right]\left[(H'\varepsilon H)+\text{c.c.}\right.\\&\left.+\frac{g^2}{8}(H'^+\sigma H'+H^+\sigma H)^2+\frac{g'^2}{8}(H^+H-H'^+H')^2\right].\end{aligned} \quad (5.3')$$

We thus see that the unification scale M_{GUM} does not penetrate the low-energy sector. This conclusion was proved in Ref. 76 without referring to any particular model.

B. Evaluation of M_{GUM} and $\sin^2\theta_W$

In this section, we shall determine why, in SUSY unified models, the unification mass M_{GUM} (which determines the proton lifetime) and $\sin^2\theta_W$ (whose successful evaluation is the most striking result of GUT) are equal. The mass M_{GUM} is equal to the virtuality at which the running SU(3), SU(2), and U(1) coupling constants $\alpha_3(Q^2)$, $\alpha_2(Q^2)$, and $\alpha_1(Q^2)$ become equal. The evolution of the coupling constants between M_W and M_{GUM} is determined by the light particles. A large number of new particles with masses M_W is introduced into the model, and this is responsible for the change in M_{GUM} as compared with the predictions of standard unification models. The gauge coupling constants of SU(N) are given by the following celebrated formula:

$$\alpha_N(M_{\text{GUM}}^2)=\frac{\alpha_N(M_W^2)}{1-(b_N/2\pi)\alpha_N(M_W^2)\ln(M_{\text{GUM}}/M_W)}. \quad (5.4)$$

The conversion of the coefficients of the Gell-Mann-Low

TABLE I.

SU (5) (1 doublet of Higgses)	SUSY SU (5) (2 doublets of Higgses)
$-11+\frac{4}{3}n_{\Phi}$	$-9+2n_{\Phi}$
$-\frac{22}{3}+\frac{4}{3}n_{\Phi}+\frac{1}{3}$	$-6+2n_{\Phi}+1$
$\frac{4}{3}n_{\Phi}+\frac{1}{10}$	$2n_{\Phi}+\frac{3}{5}$

function b_N to the SUSY theory is described at the end of the last section. The table lists the values of b_3, b_2, and b_1 for standard SU(5) with one light Higgs doublet and SUSY-SU(5) with two light Higgs doublets (n_Φ is the number of generations of matter).

The main reasons for the change of the unification point $M_{\rm GUM}$ in SUSY theories is the reduction in b_3 due to the presence of the gluino octet which leads to a slowing down of the evolution of $\alpha_3(Q^2)$ and an increase in $M_{\rm GUM}$. From the two equations for equal constants α_1, α_2, and α_3 at the unification point, we find that

$$\ln\frac{M_{\rm GUT}}{M_{\rm W}}=\frac{6\pi}{-8b_3+3b_2+5b_1}\left[\frac{1}{\alpha\,(M_{\rm W})}-\frac{8}{3\alpha_3\,(M_{\rm W})}\right],$$
$$\sin^2\theta_{\rm W}\,(M_{\rm W})=\frac{3\,(b_2-b_3)+5\,(b_1-b_2)\,\alpha\,(M_{\rm W})/\alpha_3\,(M_{\rm W})}{-8b_3+3b_2+5b_1},\tag{5.5}$$

where α is the electromagnetic coupling constant. In numerical estimates, we put $\alpha(M_{\rm W})=1/128, \alpha_3(M_{\rm W})=0.1$. From (5.5), we then have

$$M_{\rm GUM}=1.6\cdot 10^{14}\,{\rm GeV},\qquad M_{\rm GUM}^{\rm SUSY}=5.3\cdot 10^{15}\,{\rm GeV},$$
$$\sin^2\theta_{\rm W}=0.219,\qquad \sin^2\theta_{\rm W}^{\rm SUSY}=0.236.\tag{5.6}$$

We recall that the experimental result is $\sin^2\theta_{\rm W}=0.226\pm 0.015$. The coupling constants at the unification point are

$$\frac{g^2}{4\pi}=0.024,\quad \left(\frac{g^2}{4\pi}\right)_{\rm SUSY}=0.040.\tag{5.7}$$

The numerical estimates in (5.6) illustrate how a radical change in the low-energy composition of the theory can appreciably increase the proton lifetime ($\tau_{\rm p}\sim J^4_{\rm GUM}$) without essentially changing the successful prediction for $\sin^2\theta_{\rm W}$.

2. C. Proton decay

In the standard SU(5) model, the proton decays mostly according to the $\mathrm{p}\to \mathrm{e}^+\pi^0$ scheme with the lifetime[77] $\tau_{\rm p}\simeq 10^{28\pm 2}$ y, so that the contemporary experimental lower bound $\tau_{\mathrm{p}\to e\pi}>10^{32}$ y overlaps the standard model. In SUSY generalizations, the unification mass is greater, as was shown in the last section. The proton lifetime is then

$$\tau_{\rm p}^{\rm SUSY}\approx\left(\frac{M_{\rm GUM}^{\rm SUSY}}{M_{\rm GUM}}\right)^4\tau_{\rm p}^{\rm stand}\approx 10^{34}\ {\rm y}.$$

From the experimental point of view, this prediction cannot be regarded as satisfactory because the cosmic neutrino background will, in all probability, prevent the successful measurement of the decay of the proton with a lifetime in excess of 10^{33} y. So far, we have been concerned with decay due to propagators of dimension 6 ($d=6$), generated by the graph of Fig. 18:

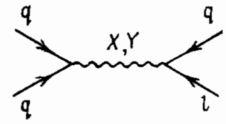

FIG. 18. Proton decay due to the exchange of superheavy vector bosons.

$$O_6=\frac{1}{M^2_{\rm GUM}}\,(qq)\,(ql).\tag{5.8}$$

On the other hand, it was first noted in Refs. 15 and 16 that operators with $d=5$ appear in SUSY models and lead to proton decay (Fig. 19):

$$O_5=\frac{1}{M_{\rm GUM}}\,(\tilde{q}\tilde{q})\,(qe).\tag{5.9}$$

The proton decay amplitude due to $d=5$ operators (5.9) is enhanced by the factor $M_{\rm GUM}/M_{\tilde{\rm q}}\approx 10^{14}$ as compared with (5.8), and therefore requires careful examination.

SUSY is not broken at energies $\sim M_{\rm GUM}$ SUSY, and the $d=5$ operators can be written in the supersymmetric form. There are two $d=5$ operators that violate baryon charge conservation:

$$O_5^{\rm L}=\varepsilon_{abc}\,[Q_{\rm L}^aQ_{\rm L}^bQ_{\rm L}^cL_{\rm L}]_{\rm F},\quad O_5^{\rm R}=[u_{\rm R}^{*a}u_{\rm R}^{*b}D_{\rm R}^{*c}E_{\rm R}^*]_{\rm F}\varepsilon_{abc},\tag{5.10}$$

where we have explicitly indicated the antisymmetry in color. The scalar quarks and/or leptons present in (5.9)–(5.10) should be "converted" into spinorial quantities in order to produce the 4-fermion amplitude describing proton decay. This is accompanied by the appearance of single-loop graphs. The "conversion" of the quarkino and leptino into spinorial particles is performed by the gaugino. The antisymmetry in color requires the presence in $O_5^{\rm R}$ of different chiral superfields of up quarks, say, $\mathrm{u_R}$ and $\mathrm{c_R}$ and, since the interaction between the gaugino and the superfields of right-handed particles does not affect the flavor, the c-quark will leave the loop in the outward direction and a 4-fermion operator that does not annihilate the proton is obtained. Thus, the operator $O_5^{\rm R}$ does not lead to proton decay.[17] Analysis of the proton decay due to the operator $O_5^{\rm L}$ must begin with graphs involving gluino exchange because the gluino interacts most strongly with the quark. The three diagrams of Fig. 20 must be evaluated (the operator $O_5^{\rm L}$ with the charged lepton does not lead to the decay for the same reason as $O_5^{\rm R}$). Because the quarkino masses are very similar, the result of the evaluation of the diagrams of Fig. 20 will be written in the form

$$M=K\varepsilon_{abc}\,[(u_{\rm L}^{a\rm T}\gamma_0\nu_{\mu\rm L})\,(s_{\rm L}^{c\rm T}\gamma_0d_{\rm L}^b)+(s_{\rm L}^{c\rm T}\gamma_0\nu_{\mu\rm L})\,(u_{\rm L}^{a\rm T}\gamma_0d_{\rm L}^b)+(d_{\rm L}^{b\rm T}\gamma_0\nu_{\mu\rm L})\,(u_{\rm L}^{a\rm T}\gamma_0s_{\rm L}^c)],\tag{5.11}$$

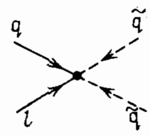

FIG. 19. Diagram corresponding to the $d=5$ operator.

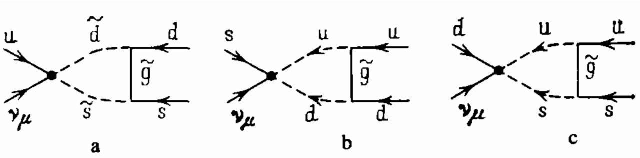

FIG. 20. Proton decay due to gluino exchange. Sum of diagrams is zero.

where K includes the coupling constants and the result of the integration over the loop. It is then readily verified with the aid of the Fierz identities that the bracket in (5.11) is equal to zero. Gluino exchange does not lead to proton decay.[19,20]

It will now be necessary to examine the black circle in Figs. 19 and 20. Baryon charge conservation is violated during the exchange of the gauge X and Y bosons (bosinos) or higgses (higgsinos). In the operator O_5^L (5.10), the quark fields are part of the SU(5) decuplet, and the lepton fields are part of the antiquintuplet. Hence, it is clear that this operator can arise only during higgsino exchange because the gaugino interacts with matter as a result of the (super) extension of the derivative in the kinetic energy, which would lead to an even number of quintuplets and decuplets in O_5^L.

The following superpotential terms lead to the operator O_5^L:

$$g = h_u \varepsilon_{iklmn} Q^{ik} Q^{lm} H'^n + h_d Q^{ik} Q_i H_k + M H'^i H_i, \qquad (5.12)$$

where the symbols $i, \ldots$ refer to SU(5). The first two terms give mass to the fermions, and are present in any variant of SUSY SU(5). The last term gives mass to the colored Higgs triplet H_3, $M \sim M_{GUM}$, and is also present in simpler models. Figure 21 shows how the interacton (5.12) leads to the operator O_5^L. The heavy colored higgsino propagates along the inner fermion line. Since loops analogous to those of Fig. 20 converge for $q^2 \sim M_{\tilde q}^2 \ll M_{GUM}^2$, the fermion propagator in Fig. 21 can be replaced with the factor $1/M_{GUM}$. The Higgs vertices in Fig. 21 are proportional to the quark masses (up m_u and down m_s), so that it is advantageous to let through heavy particles in the intermediate state. The effective 4-fermion Lagrangian for the proton decay, corresponding to the graphs of Fig. 22, was evaluated in Ref. 19 (third-generation particles were taken into account in the intermediate state; according to recent results, the angles θ_2 and θ_3 are small and $0.015 < s_2 < 0.09$, $s_3 < 0.04$ (Ref. 78), so that we reproduce the formula for second-generation particles in the intermediate state). For the two limiting cases (a) $M_{\tilde W\tilde W}^2 \ll M_{\tilde q}^2$ and (b) $M_{\tilde W\tilde W}^2 \gg M_{\tilde q}^2$, it was shown in Ref. 19 that

$$\mathscr{L}_{\tilde W} = \frac{4\sqrt{2}G_F}{16\pi^4 M_{GUM}} m_s m_c s_1^2 K g_2^2 \varepsilon_{abc} A \, [2\,(s_L^{aT}\gamma_0\nu_L)\,(d_L^{cT}\gamma_0 u_L^b)$$

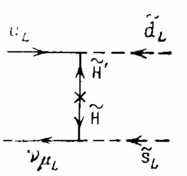

FIG. 21. The origin of the operator O_5^L.

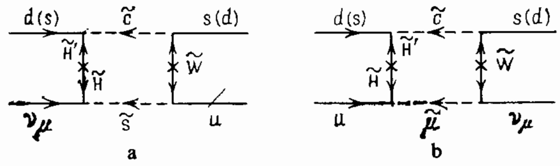

FIG. 22. Graphs describing the $p \to K^+\nu$ decay.

$$+2\,(d_L^{aT}\gamma_0\nu_L)\,(s_L^{cT}\gamma_0 u_L^b)]\left[\frac{M_{\tilde W\tilde W}}{3M_{\tilde q,\tilde l}^2}\,(a);\right.$$

$$\left.\frac{1}{M_{\tilde W\tilde W}} \ln \frac{M_{\tilde W\tilde W}^2}{M_{\tilde q,\tilde l}^2}\,(b)\right], \qquad (5.13)$$

where

$$K = \frac{1}{2}\left(\frac{\eta}{\eta'} + \frac{\eta'}{\eta}\right) \geqslant 1, \qquad \frac{g_2^2}{4\pi} = \frac{1}{30},$$

and the factor A represents the anomalous dimensionality of the $d = 5$ operator: $A = 1/3$ (Ref. 18). The matrix element $\langle p|\mathscr{L}_{\tilde W}|k\rangle$ was evaluated with the aid of the QCD sum rules.[79] Using the results reported in Ref. 19, we find that the proton lifetime is

$$\tau_p = 3\cdot 10^{21}\left[\frac{3M_{\tilde q,\tilde l}^2}{M_{\tilde W\tilde W}}\,(\mathrm{GeV})\ (a);\quad \frac{M_{\tilde W\tilde W}\,(\mathrm{GeV})}{\ln(M_{\tilde W\tilde W}^2/M_{\tilde q,\tilde l}^2)}\,(b)\right]^2 \mathrm{y}. \qquad (5.14)$$

Using the experimental lower bound $\pi_{p\to K^+\nu} > 3\times 10^{30}$ y and substituting $M_{\tilde q,\tilde l} = 100$ GeV, we obtain $M_{\tilde W\tilde W} < 1$ GeV or $M_{\tilde W\tilde W} > 10^3$ TeV. In low-energy SUSY models, the value $M_{\tilde W\tilde W} > 10^3$ TeV must be acknowledged as inadmissible, and we have the upper bound

$$M_{\tilde W\tilde W} < 1\,\mathrm{GeV}. \qquad (5.15)$$

In minimal SUGRA, the gaugino masses are actually small. From (5.15), we find that the gluino mass is $m_{\tilde g} = (\alpha_3/\alpha_2)M_{\tilde W\tilde W} < 3$ GeV, which is in the region of the experimental lower bound.[11] How are we to understand the limit set by (5.15)? If $M_{\tilde q,\tilde l} = 200$ GeV then, instead of (5.15), we obtain $M_{\tilde W\tilde W} < 4$ GeV. Another approach is to consider heavier triplet higgses. Let us suppose that, for $M_{H_3} = 10^{18}$ GeV, the mass $M_{\tilde W\tilde W}$ can be ~ 100 GeV, i.e., models with nonminimal inclusion of vector multiplets in SUGRA[20] are admissible.

Since the discovery of the $d = 5$ operators, searches have been in progress for ways of eliminating them from the theory.[16] All the variants proposed so far can be said to be extremely unappealing.

6. CONCLUSIONS

We have reviewed low-energy ($N = 1$)-supersymmetry—a branch of elementary-particle physics—using the mathematically elegant SUSY theories to cancel quadratic divergences in the Glashow-Weinberg-Salam electroweak model, and to solve the problem of hierarchies in grand unification theories. The basic physical predictions of SUSY models are: each known fundamental particle (quark, lepton, γ, $W^\pm$, Z, gluon, Higgs boson) has a spin partner (superpartner) of mass $M_s \sim 0.1$–1 TeV (the mass scale is determined by the requirement that quadratic divergences must cancel at high energies; some superparticles may turn out to be relatively light, with masses $\sim$1–10 GeV). The superpartner mass difference arises because of supersymmetry breaking. Various ways of SUSY breaking have been examined. The most popular models used at present are those based on SUSY breaking by supergravity effects.[58,59,64–66] It is important to emphasize that, whatever the model, the low-energy $SU(3)\times SU(2)\times U(1)$ symmetric Lagrangian has an exceedingly simple form: it consists of supersymmetrized kinetic terms and Yukawa interactions with the Higgs bosons, plus mass terms corresponding to the leptino, quarkino, higgsino, and gaugino, plus certain cubic interactions of scalar particles.

The SUSY generalization of unified theories leads to a change in the predicted lifetime and decay mode of the proton. We note that, in several models, the dominant process involves the emission of the K meson:[17–20] $p \to K^1 \nu$.

How can the models described above be developed further? The most hopeful expectations relate to the experimental discovery of superparticles. So far, these discoveries have not been made, and we must therefore conclude with possible theoretical developments. The obvious way forward is to proceed to SUSY theories with $N > 1$. Models with low-energy $N = 2$ SUSY have already been investigated.[80] As noted in Section 2, such theories have left-right symmetry. The number of quarks and leptons is doubled in Ref. 80. The undesirable $SU(2)_W$ right-handed doublets and left-handed singlets (mirror fermions) are given a large mass. It is possible that the main result of Ref. 80 is the demonstration of the fact that a direct transition to $N > 1$ leads to excessively complicated models.

Another development involves dynamic supersymmetry breaking (DSB). In 1981, Witten proposed a scenario for the solution of the hierarchy problem in grand unification theories.[37] Suppose that, in the tree approximation, the theory has fields of mass M_{GUT} and massless fields. If the theory is supersymmetric, loop corrections to massless scalar fields do not produce an increase in mass, which is all to the good because the necessary hierarchy is of the order of $(M_{GUT}/M_W)^2 = 10^{28}$ and not $1/g^2 \sim 100$. Mass-generation can occur outside the framework of perturbation theory. This phenomenon was referred to by Witten as dynamic supersymmetry breaking. The natural mass scale due to, say, instanton effects, is $M^2_{GUT} e^{-1/g^2}$. The exponential can ensure the necessary hierarchy of 28 orders of magnitude. Witten has reported[37] a number of examples of DSB, including a supersymmetric quantum mechanics. However, he has not succeeded in constructing a field theory in 4-dimensional space with DSB. Searches for such theories have continued for three years and have recently culminated in success.[81,82] It has been found that DSB occurs in the supersymmetric Yang-Mills theory with matter in the chiral representation of the gauge group.[4)] The construction of a realistic theory based on DSB is a matter for the future. The following surprising coincidence is immediately apparent: chiral filling is necessary both for DSB and the natural confinement of massless (on the scale of unified theories) quarks and leptons.

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APPENDIX

How to write down supersymmetric Lagrangians

In this review, we use the real γ-matrices

$$\left.\begin{aligned}
&\gamma_0=\begin{pmatrix}0 & -\sigma_3\\ \sigma_3 & 0\end{pmatrix},\quad \gamma_1=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix},\ \gamma_2=\begin{pmatrix}0 & -i\sigma_2\\ i\sigma_2 & 0\end{pmatrix}\\
&\gamma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\ \gamma_5\equiv\gamma_0\gamma_1\gamma_2\gamma_3=\begin{pmatrix}0 & -\sigma_1\\ \sigma_1 & 0\end{pmatrix},\\
&\sigma_1=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\ \sigma_3=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},
\end{aligned}\right\}\quad (A.1)$$

$$\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=-2g_{\mu\nu},\quad \gamma_5^2=-1,\quad \gamma_0^T=-\gamma_0,\quad \gamma_i^T=\gamma_i.$$

The definition of the Majorana spinor is $\psi^+ = \psi^T$, $\bar\psi = \psi^T\gamma_0$.

The following equations are available for the two Majorana spinors α_1 and α_2:

$$\left.\begin{aligned}
&\bar\alpha_1\Gamma\alpha_2=\bar\alpha_2\tilde\Gamma\alpha_1,\quad \tilde\Gamma=-\gamma_0\Gamma^T\gamma_0,\\
&\tilde\gamma_A=\gamma_A\quad\text{for}\quad 1,\ \gamma_5,\ \gamma_5\gamma_\mu,\\
&\tilde\gamma_A=-\gamma_A\quad\text{for}\quad \gamma_\mu,\ \sigma_{\mu\nu}.
\end{aligned}\right\}\quad (A.2)$$

Fierz identities. Consider the 16 matrices $O_i = \{1, i\gamma_5, i\gamma_\mu\gamma_5, i\gamma_\mu, i\sigma_{\mu\nu}\}$, where $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$. For any matrices M, N and spinors λ, χ, ψ, and φ, we then have the following identity:

$$(\bar\lambda M\chi)(\bar\psi N\varphi)=-\frac{1}{4}\sum_{i=1}^{16}(\bar\lambda MO_iN\varphi)(\bar\psi O_i\chi).\quad (A.3)$$

The following useful equations can be obtained from the Fierz identity:

$$\begin{aligned}
&(\bar\lambda\lambda)\lambda_\alpha=(\bar\lambda\gamma_5\lambda)(\gamma_5\lambda)_\alpha=\frac{1}{4}(\bar\lambda\gamma_5\gamma_\mu\lambda)(\gamma_5\gamma_\mu\lambda)_\alpha,\\
&(\bar\lambda\gamma_5\lambda)\lambda_\alpha=-(\bar\lambda\lambda)(\gamma_5\lambda)_\alpha,\quad (\bar\lambda\gamma_5\gamma_\mu\lambda)\lambda=-(\bar\lambda\lambda)(\gamma_5\gamma_\mu\lambda)_\alpha.
\end{aligned}\quad (A.4)$$

Boson fields commute with one another, $\Phi_1\Phi_2 = \Phi_2\Phi_1$, while fermion fields anticommute: $\psi_1\psi_2 = -\psi_2\psi_1$, so that it is difficult to understand how fermion-boson symmetry can obtain. Introducing the Grassman (anticommuting) variables θ, we can rewrite the fermion anticommutator in the form of the commutator $\theta_1\psi_1\theta_2\psi_2 - \theta_2\psi_2\theta_1\psi_1 = 0$, and construct a symmetric theory.

The supersymmetry algebra is most simply realized in the superspace containing four boson variables x_μ and the Majorana (real 4-component) spinor θ. The supersymmetry transformation operates in this space as follows:

$$\begin{aligned}
&\theta\to\theta+\varepsilon,\\
&x_\mu\to x_\mu+\frac{1}{2}\bar\varepsilon\gamma_\mu\theta,
\end{aligned}\quad (A.5)$$

where ε is the constant Majorana spinor: $\bar\varepsilon = \varepsilon^T\gamma_0$. The commutator of the two supertransformations shifts x_μ by the constant vector $a_\mu = \bar\varepsilon_1\gamma_\mu\varepsilon_2$ in accordance with (2.3). In addition to the Majorana spinor θ, we shall need the Weyl spinor $\theta_{\pm}$:

$$\theta_{\pm}=\frac{1\pm i\gamma_5}{2}\theta,\ \theta_{+}=\theta_{-}^{*}. \qquad (A.6)$$

We now introduce the complex coordinate z_μ:

$$z_\mu=x_\mu+\frac{i}{4}\bar{\theta}\gamma_\mu\gamma_5\theta. \qquad (A.7)$$

Under the supertransformation we obtain

$$z_\mu\to z_\mu+\bar{\varepsilon}\gamma_\mu\theta_+,\quad \theta_+\to\theta_++\varepsilon_+. \qquad (A.8)$$

The simplest supermultiplet forms the chiral superfield $S(z,\theta_+)$. Let us expand it into a Taylor series in θ_+ and use the fact that, since θ anticommutes, the product of three or more spinors θ_+ must be zero:

$$S_{z,\,\theta_+}=A(z)+\theta_+^T\gamma_0\psi(z)+\frac{1}{4}(\theta_+^T\gamma_0\theta_+)F(z), \qquad (A.9)$$

where A and F are complex fields, and ψ is a Majorana spinor. The product and sum of the chiral fields is again a chiral field. Using (A.8), we can define the transformation law for the components A, ψ, and F under the supertransformation:

$$\left.\begin{aligned}&A(z)\to A(z)+\bar{\varepsilon}\psi_+(z),\\&\psi_+(z)\to\psi_+(z)-\frac{1+i\gamma_5}{2}\hat{\partial}A(z)\varepsilon+F(z)\varepsilon_+,\\&F(z)\to F(z)-\bar{\varepsilon}\hat{\partial}\psi_+(z).\end{aligned}\right\} \qquad (A.10)$$

From this, it follows that the F-component of the chiral superfield will change under the supertransformation by the total derivative. Consequently, it can be used as the density of the supersymmetric Lagrangian:

$$\mathscr{L}_V=\frac{1}{2}M[S^2]_F+\frac{\lambda}{3}[S^3]_F+\text{c.c.}, \qquad (A.11)$$

where $[\ldots]_F$ represents the F-component of the corresponding superfield, and powers of S in excess of three lead to a renormalized theory. $\int d^2\theta_+$ is sometimes used instead of $[\ldots]_F$:

$$[\ldots]_F\equiv\int d^2\theta_+. \qquad (A.12)$$

The validity of (A.12) follows from the rules for integration with respect to the Grassman variables:

$$\int d\theta=\frac{\partial}{\partial\theta},\quad \int\theta\, d\theta=1,\ \int 1\, d\theta=0. \qquad (A.13)$$

We have used the concept of the superpotential, which is a function of the scalar components z_i of chiral superfields $\hat{z}_i$: $w\equiv w(z_i)$. The Lagrangian in global supersymmetric theory has the form

$$\mathscr{L}_V=\int d^2\theta_+w(z_i)+\text{c.c.}\equiv[w(\hat{z}_i)]_F. \qquad (A.14)$$

For (A.11),

$$w(A)=\frac{1}{2}MA^2+\frac{\lambda}{3}A^3.$$

Let us write out the terms in (A.11) in terms of the components:

$$\frac{1}{2}M[S^2]_F+\text{c.c.}=MAF+MA^*F^*-\frac{M}{2}\bar{\psi}\psi, \qquad (A.15)$$

$$\frac{1}{3}\lambda[S^3]_F+\text{c.c.}$$

$$=\lambda A^2F+\lambda A^{*2}F^*-\lambda\frac{A}{2}\bar{\psi}(1+i\gamma_5)\psi-\lambda\frac{A^*}{2}\bar{\psi}(1-i\gamma_5)\psi. \qquad (A.16)$$

It follows from (A.16) that Re A interacts as a scalar field and Im A as a pseudoscalar field. (A.11) does not contain terms with derivatives. To introduce the kinetic terms in a supersymmetric manner, we must become familiar with the vector supermultiplet

$$\begin{aligned}V(x,\theta)&=A(x)+\bar{\psi}(x)\theta+\frac{1}{4}(\bar{\theta}\theta)F(x)\\&+\frac{1}{4}\bar{\theta}\gamma_5\theta G(x)+\frac{1}{4}\bar{\theta}\gamma_\nu\gamma_5\theta V_\nu(x)\\&+\frac{1}{4}(\bar{\theta}\theta)(\bar{\theta}\chi(x))+\frac{1}{32}(\bar{\theta}\theta)^2D(x)\\V^*&=V.\end{aligned} \qquad (A.17)$$

$V(x,\theta)$ contains the two Majorana spinors ψ and χ (eight fermion degrees of freedom outside the mass shell), four real scalars A, F, G, and D, and the vector field V_ν (eight boson degrees of freedom outside the mass shell). The product and sum of two vector superfields V_1 and V_2 give the vector superfield V_3.[5] The last component of $V(x,\theta)$ will change by the total derivative under the supertransformation, so that $\int[\ldots]_D d^4x$ is a superinvariant.

Let us expand (A.9) into a series in terms of θ:

$$\begin{aligned}S(z,\theta_+)&=A(x)+\bar{\theta}\psi_+(x)+\frac{1}{2}(\bar{\theta}\theta_+)F(x)\\&+\frac{1}{2}(\bar{\theta}\gamma_\mu\theta_+)\partial_\mu A(x)-\frac{1}{4}(\bar{\theta}\theta)(\bar{\theta}\hat{\partial}\psi_+(x))\\&-\frac{1}{32}(\bar{\theta}\theta)^2\Box A(x).\end{aligned} \qquad (A.18)$$

From (A.17) and (A.18), we have

$$\frac{1}{4}[S^*S]_D=|\partial_\mu A|^2+\frac{1}{2}\bar{\psi}\hat{\partial}\psi+|F|^2\equiv\mathscr{L}_T. \qquad (A.19)$$

A word now about dimensions: A is a scalar field, $[A]$ = GeV, ψ is a spinor field, $[\psi]=\text{GeV}^{3/2}$, F is an auxiliary field, $[F]=\text{GeV}^2$. It follows from (A.9) that $[\theta]=\text{GeV}^{-1/2}$.

The Lagrangian in the Wess-Zumino model (simplest supersymmetric model) has the following form:

$$\mathscr{L}_{W\text{-}Z}=\frac{1}{4}[S^*S]_D+\frac{M}{2}[S^2]_F+\frac{\lambda}{3}[S^3]_F+\text{c.c.} \qquad (A.20)$$

Under the supertransformation, $\mathscr{L}$ changes by the total derivative, and this once again demonstrates that supersymmetry is a spatial (and not internal) symmetry.

In terms of components:

$$\begin{aligned}\mathscr{L}_{W\text{-}Z}&=|\partial_\mu A|^2+\frac{1}{2}\bar{\psi}\hat{\partial}\psi+|F|^2+MAF+MA^*F^*\\&-\frac{M}{2}\bar{\psi}\psi+\lambda A^2F+\lambda A^{*2}F^*\\&-\lambda\frac{A}{2}\bar{\psi}(1+i\gamma_5)\bar{\psi}-\lambda\frac{A^*}{2}\bar{\psi}(1-i\gamma_5)\psi.\end{aligned} \qquad (A.21)$$

The model contains two physical fields: namely, the complex scalar A and the Majorana spinor ψ. The field F is not dynamic and $\mathscr{L}$ does not contain terms with derivatives of F. The Lagrange equations for the field F are the algebraic equations:

$$F=-\lambda A^{*2}-MA^{*}. \quad (A.22)$$

Substituting this in (A.21), we obtain the expression for $\mathscr{L}_{W-Z}$ in terms of physical fields. The potential energy of the field A is

$$V(A)=|F(A)|^2. \quad (A.23)$$

In addition to chiral superfields, we have used vector superfields that are necessary for the description of gauge fields. We begin with Abelian U(1) symmetry. The chiral superfield Φ transforms with the aid of the chiral superfield Λ:

$$\Phi\to e^{-i\Lambda}\Phi. \quad (A.24)$$

The kinetic energy is not an invariant:

$$\Phi^+\Phi\to\Phi^+e^{i\Lambda^*}e^{-i\Lambda}\Phi. \quad (A.25)$$

To recover U(1) symmetry, we introduce the vector superfield V, and the expression for the kinetic energy assumes the new form

$$\Phi^+e^{2gV}\Phi\to\Phi^+e^{i\Lambda^*}e^{2gV'}e^{-iV}\Phi, \quad (A.26)$$

$$V\to V-\frac{i}{2g}(\Lambda^*-\Lambda). \quad (A.27)$$

Let us now determine the effect of the U(1) transformation (A.27) on the components of the vector supermultiplet $V(C,\chi,M,N,V_\nu,\lambda,D)$. Using (A.17) and (A.18), we obtain

$$V+\tilde{\Lambda}+\tilde{\Lambda}^+=C+A+A^*+\bar{\theta}(\chi+\psi)+\frac{1}{4}(\bar{\theta}\theta)(M+F+F^*)$$
$$+\frac{1}{4}(\bar{\theta}\gamma_5\theta)[N+i(F-F^*)]$$
$$+\frac{1}{4}(\bar{\theta}\gamma_\mu\gamma_5\theta)[V_\mu+i\partial_\mu A-i\partial_\mu A^*]+\frac{1}{4}(\bar{\theta}\theta)\bar{\theta}(\lambda-\hat{\partial}\psi)$$
$$+\frac{1}{32}(\bar{\theta}\theta)^2[D-\Box\ A+A^*)]. \quad (A.28)$$

The components A, ψ, and F of the multiplet of the gauge transformation can be selected so that the lowest components of the vector superfield C, χ, M, and N are annulled. This leaves the ordinary gauge transformation $V_\mu\to V_\mu^+i(\partial_\mu A-\partial_\mu A^*)$. This gauge of the superfield V is called the Wess-Zumino gauge, in which the field V can be written in the form

$$V_{WZ}=\frac{1}{4}(\bar{\theta}\gamma_\mu\gamma_5\theta)V_\mu(x)+\frac{1}{2\sqrt{2}}(\bar{\theta}\theta)(\bar{\theta}\lambda(x))+\frac{1}{16}(\bar{\theta}\theta)^2D(x). \quad (A.29)$$

The convenience of the Wess-Zumino gauge is that the expansion of $\exp(2gV)$ into a series stops with the second term.

For the kinetic energy of the chiral superfield in the Wess-Zumino gauge, we obtain

$$\left[\frac{1}{4}\Phi^+e^{2gV}\Phi\right]_D=|(\partial_\mu-igV_\mu)A|^2$$
$$+\left[\frac{1}{2}\bar{\psi}(\hat{\partial}-ig\hat{V})\frac{1+i\gamma_5}{2}\psi-\sqrt{2}gA^*\bar{\lambda}\frac{1+i\gamma_5}{2}\psi+\text{c.c.}\right]$$
$$+gDA^*A+|F|^2. \quad (A.30)$$

We shall now obtain the expression for the kinetic energy of the vector supermultiplet. The analog of the gauge invariant intensity tensor of the vector field $F_{\mu\nu}$ is the chiral multiplet of intensities W_α. To construct it, we introduce the covariant derivative D:

$$D\equiv\frac{\partial}{\partial\bar{\theta}}-\frac{1}{2}\hat{\partial}\theta,\quad \{D_\alpha,\bar{D}_\beta\}=(\hat{\partial})_{\alpha\beta},$$
$$\bar{D}\equiv\frac{\partial}{\partial\theta}+\frac{1}{2}\bar{\theta}\hat{\partial},\quad D_\pm=\frac{1\pm i\gamma_5}{2}D. \quad (A.31)$$

The following equations can readily be verified:

$$D_-\Phi_+(\theta_+,z)=D_+\Phi_-(\theta_-,z^*)=0. \quad (A.32)$$

They can be used as definitions of chiral superfields. The definition of W_α that is invariant under (A.27) is

$$W_\alpha=(\bar{D}_+D_-)(D_+)_\alpha V, \quad (A.33)$$

where W_α is a chiral field and $D_-W=0$. The evaluation of W_α from (A.33) may be recommended as a useful exercise in θ and γ gymnastics (it need only be remembered that the spinors anticommute). Using the Fierz identities, we obtain

$$-W_\alpha=\sqrt{2}\lambda_+(z)+\left[D(z)-\frac{i}{2}\sigma_{\mu\nu}F_{\mu\nu}(z)\right]\theta_++\frac{1}{\sqrt{2}}\hat{\partial}\lambda_-(z)(\bar{\theta}\theta_+). \quad (A.34)$$

The expression for the kinetic energy of the vector supermultiplet is

$$\mathscr{L}=\frac{1}{8}\int d^2\theta_+W_\alpha^T\gamma_0W_\alpha+\text{к. c.}=\frac{1}{2}\bar{\lambda}\hat{\partial}\lambda+\frac{1}{2}D^2-\frac{1}{4}F_{\mu\nu}^2. \quad (A.35)$$

The superinvariant term $\xi|V|_D=\xi D$ can be added to the Lagrangian. In the case of non-Abelian symmetry of the fields, λ, D, and V_μ must be chosen in the associated representation: $V_u\equiv V_\mu^a t^a$, $\lambda\equiv\lambda^a t^a$, $D\equiv D^a t^a$. Equations (A.30) and (A.35) then remain unaltered, and only the trace of the matrices of the gauge group must be taken, for example,

$$\mathscr{L}_{\text{non-Ab}}=2\operatorname{tr}\left(\frac{1}{2}\bar{\lambda}\hat{\partial}\lambda+\frac{1}{2}D^2-\frac{1}{4}F_{\mu\nu}^2\right],$$

since $\operatorname{tr} t_a t_b=(1/2)\delta_{ab}$.

[1)]See Appendix.

[2)]A remarkably clear presentation of problems relating to spontaneous SUSY breaking has been given by Witten.[37]

[3)]In the case of spontaneous SUSY breaking, one of the supermultiplets containing the goldstino is always found to contain a physical field that is massless in the tree approximation. When supersymmetry is due to the F-term, this is a scalar field and, if it is due to the D-term, it is a vector field (photon).

[4)]A chiral representation (in contrast to a vector representation) is one in which a gauge-invariant mass cannot be written for the fermions of matter. For example, for SU(5), $\bar{5}$, 10 is a chiral set, and the quark set 3, $\bar{3}$ is a vector set for SU(3).

[5)]A useful exercise is to obtain the field $V_3=V_1V_2$ in terms of the components of V_1 and V_2.

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