Quark mixing in weak interactions

N. G. Ural'tsev and V. A. Khoze

B. P. Konstantinov Leningrad Institute of Nuclear Physics, Academy of Sciences of the USSR
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CONTENTS

Introduction 617
1. Quark mixing in the $SU(2)_L \times U(1)_Y$ theory 618
2. What is known about the mixing angles 619
3. Prospects of a more accurate determination of the mixing angles 622
4. Quark mixing and the physics of K^0 mesons 624
   a) Mass difference between the K_L and K_S mesons b) CP nonconservation in K^0 mesons
Conclusions 629
References 629

INTRODUCTION

The discovery in 1974 of the first heavy quark (the charmed c quark), which finally conferred the status of citizenship on the quark model, was a momentous landmark in the history of particle physics. It stimulated further important progress in both theory and experiment. An important step in this direction was the discovery in 1977 of the b quark (and the third lepton τ). It led, in particular, to the replacement of the well-known four-quark scheme of Glashow, Iliopoulos, and Maiani[1] by the six-quark model of Kobayashi and Maskawa,[2] which had already been proposed in 1973 in order to include CP violation in the weak interactions of quarks in a natural way.[1)]

As is well known, the weak interaction is generated by the exchange of the vector $W^\pm$ and Z^0 bosons, whose source is the charged and neutral currents. The currently known quarks and leptons can be grouped in a natural way into three generations:

ν_e	ν_μ	ν_τ
e	μ	τ
u	c	t
d	s	b

Whereas in the case of emission of neutral Z^0 bosons there is no change in the species (flavor) of these fermions, the interaction involving charged bosons necessarily leads to a change in the fermion flavor. In the lepton sector, the charged current couples only particles belonging to the same generation—each of the leptons is converted into its corresponding neutrino when a W boson is emitted. The charged quark current contains not only terms describing transitions within a single generation, but also nondiagonal terms corresponding to transitions $s \to u$, $c \to d$, etc., and is thus characterized by nine coefficients V_{ij} ($i =$ u, c, t and $j =$ d, s, b). The coefficients V_{ij} form a unitary 3×3 matrix, which determines the mixing of the quarks in the weak charged current.

In recent years a qualitative change has occurred in the experimental investigation of the parameters of quark mixing. Besides further refinement of the "old" mixing angles referring to the first two generations, vital information has been obtained on the decays of particles containing b quarks (see, for example, Refs. 6 and 7). Despite its large mass ($m_b \approx 4.8$ GeV), the b quark was found to be surprisingly long-lived: $\tau_b \sim 10^{-12}$ sec. The decays of the b quark are dominated by the transition $b \to c + W^-$, and the transition $b \to u + W^-$ is strongly suppressed. These results express an exceptionally important fact: the quarks of the first and second generations are mixed more strongly than the quarks of the second and third generations, and the mixing of the first and third generations is quite small.

At the present time, the six-quark model makes it possible to give a successful description of a large set of data, including the decays of the heavy quarks—the quarks with charm and beauty.[2)] (According to communications from CERN,[8] the last missing fundamental object of this model—the t quark—has been found.) It is now becoming the order of the day to make a more detailed investigation of the parameters of the Kobayashi-Maskawa scheme and to seek methods of making a critical test of its most characteristic predictions.

Here we shall discuss the contemporary phenomenological status of the quark mixing parameters and the closely related problem of the physics of neutral K mesons. The point is that the mass difference between the neutral K_L^0 and K_S^0 mesons is very sensitive to the strength of the mixing of the third generation of quarks with the first two generations, and CP nonconservation in the decays of K^0 mesons in the Kobayashi-Maskawa model appears only because of the mixing of the t and b quarks. Therefore at the present time the neutral K mesons represent a natural laboratory in which one can, on the one hand, study the effects of quark mixing and, on the other, test the Kobayashi-Maskawa model as a whole.

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In Sec. 1 we recall briefly the mechanism which gives rise to quark mixing in the framework of gauge theories of the electroweak interaction. In Sec. 2 we present the existing direct experimental information about the mixing angles, and in Sec. 3 we describe briefly the prospects for their further refinement. Section 4 is devoted to the physics of the neutral K mesons—the mass difference between the K_L^0 and K_S^0 mesons and the violation of CP invariance in their decays. Here it is explained that the experimental value of the K_L^0–K_S^0 mass difference cannot be used directly for a determination of the mixing angles. At the same time, the CP nonconservation observed in K^0-meson decays imposes substantial constraints on the parameters of the Kobayashi-Maskawa scheme. The value which this scheme predicts for the ratio ε'/ε characterizing the deviation from the superweak model of CP violation is close to the contemporary experimental limit, and, thus, a more accurate experimental determination of the value of ε'/ε may serve as an indirect test of the validity of the Kobayashi-Maskawa model.

1. QUARK MIXING IN THE $SU(2)_L \times U(1)_Y$ THEORY

As is well known, in the $SU(2)_L \times U(1)_Y$ theory of Glashow, Weinberg, and Salam the left-handed quarks and leptons are combined into doublets of the group SU(2), whereas the right-handed quarks and leptons are SU(2) singlets:

$$\begin{pmatrix} \nu'_e \\ e' \end{pmatrix}_L, \ e'_R; \quad \begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix}_L, \ \mu'_R; \quad \begin{pmatrix} \nu'_\tau \\ \tau' \end{pmatrix}_L, \ \tau'_R; \ \ldots,$$
$$\begin{pmatrix} u' \\ d' \end{pmatrix}_L, \ u'_R, \ d'_R; \quad \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \ c'_R, \ s'_R; \quad \begin{pmatrix} t' \\ b' \end{pmatrix}_L, \ t'_R, \ b'_R; \ \ldots \tag{1}$$
$$\left(\Psi_{L,R} = \frac{1 \mp \gamma_5}{2}\Psi\right).$$

The primes indicate that we are dealing with "current" states, which are present initially in the theory and are components of SU(2) × U(1) multiplets. In terms of these "current" fields, the interaction of the gauge bosons has a diagonal character with respect to the generations. Thus, the W^+ converts e' only into ν'_e and μ' only into ν'_μ, d' → u', and, for example, the W bosons do not interact with the current $\bar{s}'_L \gamma_\mu u'_L$:

$$\begin{aligned} j_{\mu\,\mathrm{lept}}^{(+)} &= \bar{\nu}'_e \gamma_\mu e'_L + \bar{\nu}'_\mu \gamma_\mu \mu'_L + \ldots, \\ j_{\mu\,\mathrm{hadr}}^{(+)} &= \bar{u}'_L \gamma_\mu d'_L + \bar{c}'_L \gamma_\mu s'_L + \ldots, \end{aligned} \tag{2}$$

where j_μ is the weak charged current of the quarks and leptons.

The current fermions (1), which are introduced initially into the theory, are massless particles. In fact, the fermion mass term in the Lagrangian has the form

$$-\mathcal{L}_{\mathrm{mass}} = m\bar{\Psi}\Psi = m(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L). \tag{3}$$

Since the left-handed fields are SU(2) doublets and the right-handed fields are singlets, the mass term (3) would violate SU(2) × U(1) symmetry. Therefore the quarks and leptons can acquire mass only as a result of spontaneous breakdown of this symmetry.

In the standard theory, the gauge $SU(2)_L \times U(1)_Y$ symmetry is broken down to $U(1)_{em}$ on account of the development of a nonzero vacuum expectation value of the neutral component of the doublet scalar (Higgs) field $\varphi = (\varphi^{(+)}/\varphi^{(0)})$:

$$\langle\varphi^0\rangle = \frac{v}{\sqrt{2}}, \quad v = (G_F\sqrt{2})^{-1/2} = 246\ \mathrm{GeV}. \tag{4}$$

Here the fermions can also acquire mass. In fact, let us consider the Yukawa interaction

$$h(\bar{Q}'_{L\alpha}\varphi^\alpha d'_R + \bar{d}'_R\varphi^+_\alpha Q'^{(\alpha)}_L), \quad Q'_\alpha = \begin{pmatrix} u' \\ d' \end{pmatrix}, \tag{5}$$

where Q'_L denotes the doublet of left-handed quarks, and α is the index of the group SU(2). Unlike the mass term (3), this interaction does not explicitly violate SU(2) × U(1) invariance and can therefore be present in the Lagrangian. It is easy to see that the development of a nonzero vacuum expectation value (4) of the Higgs field as a result of the interaction (5) leads to a nonzero mass of the d quark: replacing the components of the scalar field φ in (5) by their vacuum expectation values, we obtain

$$h(\bar{Q}'_{L\alpha}\varphi^\alpha d'_R + \bar{d}'_R\varphi^+_\alpha Q'^{(\alpha)}_L) \to \frac{hv}{\sqrt{2}}\bar{d}'d',$$

and, thus,

$$m_d = \frac{hv}{\sqrt{2}}.$$

In order to ensure that the u quark would have a mass, we must use a Yukawa coupling of the form

$$h'\varepsilon_{\alpha\beta}(\bar{Q}'_{L\alpha}\varphi^+_\beta u'_R + \bar{u}'_R\varphi_\beta Q'^{(\alpha)}_L), \quad \varepsilon_{\alpha\beta} = i(\tau^2)_{\alpha\beta}, \tag{5a}$$

which leads to the mass term $(h'v/\sqrt{2})\bar{u}u$, and the mass of the electron appears as a result of the interaction

$$h''(\bar{l}'_{L\alpha}\varphi^\alpha e'_R + \bar{e}'_R\varphi^+_\alpha l'^{(\alpha)}_L), \quad l'_\alpha = \begin{pmatrix} \nu'_e \\ e' \end{pmatrix}. \tag{5b}$$

The explicit form of the interactions (5), (5a), and (5b) refers to the case in which there is only one generation of fermions—say, leptons e^-, ν_e and quarks u, d. In the realistic case of several fermion generations, SU(2) × U(1) symmetry requires only that the Yukawa interaction have the structure (5)–(5b) but allows, for example, the right-handed d quark d_R in (5) to be replaced by the s quark s_R, or the first doublet $(u', d')_L$ of left-handed quarks to be replaced by the doublet $(c', s')_L$, and so forth. The Yukawa constants then become matrices with respect to the generations, and the most general Yukawa couplings can be written in the form

$$-\mathcal{L}_Y = h^{(1)}_{ik}\bar{Q}'^i_L\varphi d'^k_R + h^{(2)}_{ik}\bar{Q}'^i_L\varphi^c u'^k_R + h^{(3)}_{ik}\bar{l}'^i_L\varphi e'^k_R + \mathrm{H.c.},$$
$$\varphi^c = \begin{pmatrix} \varphi^{(0)} \\ \varphi^{(-)} \end{pmatrix} = \begin{pmatrix} \varphi^{(0)} \\ -\varphi^{(+)*} \end{pmatrix}; \tag{6}$$

here φ^c is the Higgs doublet charge-conjugate to φ; Q'^i_L, u'^i_R, d'^i_R, l'^i_L, e'^i_R are the "current" quarks and leptons of the ith generation, and h_{ik} are arbitrary $n \times n$ matrices, where n is the number of generations.

As a result of the development of the nonzero vacuum expectation value of the field $\varphi^{(0)}$, $\langle\varphi^0\rangle = v/\sqrt{2}$, the interaction (6) leads to the following terms in the Lagrangian:

$$-\mathcal{L}_{\mathrm{mass}} = m^{(d)}_{ik}\bar{d}'^i_L d'^k_R + m^{(u)}_{ik}\bar{u}'^i_L u'^k_R + m^{(e)}_{ik}\bar{e}'^i_L e'^k_R + \mathrm{H.c.} \tag{7}$$

It is easy to see that (7) represents the usual mass terms for the quarks and charged leptons, except that here, in contrast to the simplest case of one generation, we are combining not d'_L with d'_R, u'_L with u'_R, and e'_L with e'_R directly into a single massive Dirac particle, but certain combinations: $(d'_L, s'_L, \ldots)$ with $(d'_R, s'_R, \ldots)$, etc. In fact (see, for example, Refs. 5 and 9), it is always possible to define new physical fermion fields

$$\begin{array}{lll} u_{Li} = (U_L)_{ik} u'_{Lk}, & d_{Li} = (V_L)_{ik} d'_{Lk}, & e_{Li} = (E_L)_{ik} e'_{Lk}, \\ u_{Ri} = (U_R)_{ik} u'_{Rk}, & d_{Ri} = (V_R)_{ik} d'_{Rk}, & e_{Ri} = (E_R)_{ik} e'_{Rk}, \end{array} \tag{8}$$

obtained from the corresponding "current" quarks and leptons by unitary rotations $U_{L,R}$, $V_{L,R}$, $E_{L,R}$, in such a way that in terms of the new fields the mass term (7) has the standard form

$$\begin{aligned} -\mathscr{L}_{\text{mass}} = m_d (\bar{d}_L d_R + \bar{d}_R d_L) + m_s (\bar{s}_L s_R + \bar{s}_R s_L) + \ldots \\ + m_u (\bar{u}_L u_R + \bar{u}_R u_L) \\ + m_c (\bar{c}_L c_R + \bar{c}_R c_L) + \ldots + m_e (\bar{e}_L e_R + \bar{e}_R e_L) \\ + m_\mu (\bar{\mu}_L \mu_R + \bar{\mu}_R \mu_L) + \ldots \end{aligned} \tag{9}$$

(here, by definition, $d \equiv d_1, s \equiv d_2, u \equiv u_1, c \equiv u_2, e \equiv e_1, \mu \equiv e_2$, etc.). This means that the Dirac fermions defined in (8) are massive physical particles.

It now remains only to rewrite the interaction of the quarks with the W bosons in terms of the physical fields:

$$\bar{u}'_{Li} \gamma_\mu d'_{Li} \cdot W^+_\mu = \bar{u}_{Li} \gamma_\mu (U_L V_L^+)_{ik} d_{Lk} W^+_\mu \equiv \bar{u}'_{Li} \gamma_\mu (V)_{ik} d_{Lk} W^+_\mu. \tag{10}$$

The unitary matrix $V = U_L V_L^+$ defined here is in fact the quark mixing matrix which describes the interaction of the quarks with the W bosons. We note that the interaction of the fermions with the Z^0 boson and the photon remains diagonal with respect to the flavors, since the corresponding neutral currents are combinations of the type $\bar{u}'_{Li} \gamma_\mu u'_{Li}$, $\bar{u}'_{Ri} \gamma_\mu u'_{Ri}$, etc., which are invariant with respect to the transformations (8).

As regards the charged lepton current, in the case of strictly massless neutrinos their physical states are determined not by diagonalization of the mass matrix, but by "diagonalization" of their interactions. The physical neutrinos ν_i are then the same combinations of ν' as for the charged leptons [cf. (8)],

$$\nu_i = (E_L)_{ik} \nu'_k,$$

and therefore in the lepton sector transitions can in fact occur only within each of the generations. If, however, the neutrinos are massive, the charged lepton current, like the hadron current, is described by some $n \times n$ unitary matrix (n is the number of generations; see, for example, Ref. 10).

We note that the diagonalization of the quark mass matrix leaves a certain arbitrariness in the definition of the fields corresponding to the physical states, since the diagonal mass terms (9) permit multiplication of the field of any of the Dirac fermions Ψ by a phase factor: $\psi = (\psi_L, \psi_R) \to e^{i\varphi}\psi = (e^{i\varphi}\psi_L, e^{i\varphi}\psi_R)$. In the language of the quark mixing matrix V, this corresponds to the freedom to multiply any row or column by an arbitrary phase factor. This ambiguity can be used to choose the most convenient representation for the matrix V.

If there were only four quarks in nature, the charged-current matrix would have the form[1]

$$\begin{pmatrix} \cos\theta_C & -\sin\theta_C \\ \sin\theta_C & \cos\theta_C \end{pmatrix},$$

where θ_C is the Cabibbo angle. In the six-quark case with three doublets of quarks, the mixing matrix is characterized by three angles and one complex phase δ.[2] We shall use the following parametrization of the Kobayashi-Maskawa mixing matrix[2,11]:

$$V = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} \quad d \quad & \quad s \quad & \quad b \quad \end{array} \\ \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \end{array}, \tag{11}$$

where $c_i = \cos\theta_i \geqslant 0$, $s_i = \sin\theta_i \geqslant 0$, $0 \leqslant \delta < 2\pi$. The main difference between this case and the four-quark case is the fact that here for $\delta \neq 0$ there is a violation of CP invariance in the interaction of the quarks with the W bosons.[2]

In the literature, use is sometimes made of a different parametrization of the six-quark mixing matrix (11)—the so-called Maiani parametrization[12]:

$$V = \begin{pmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta \\ -s_\gamma c_\theta s_\beta e^{i\delta'} - s_\theta c_\gamma & c_\gamma c_\theta - s_\gamma s_\beta s_\theta e^{i\delta'} & s_\gamma c_\beta e^{i\delta'} \\ -s_\beta c_\gamma c_\theta + s_\gamma s_\theta e^{-i\delta'} & -c_\gamma s_\beta s_\theta - s_\gamma c_\theta e^{-i\delta'} & c_\gamma c_\beta \end{pmatrix}; \tag{12}$$

here the independent parameters are the angles β, γ, θ and the phase δ'. This parametrization is in many cases more convenient, since in it the decays of b quarks fix directly the values of s_γ and s_β, and not a combination of angles, as in the parametrization (11).

As is well known, cosmological arguments based on consideration of processes involving the synthesis of helium in the early Universe impose a limit on the number of massless or light neutrinos.[3)] For example, the calculations of the standard model of the hot Universe show that their number must not be greater than three.[13] More conservative estimates which take into account the chemical evolution of matter in galaxies admit the existence of 8–10 generations.[14] However, these restrictions may be lifted if, for example, the neutrinos of the next generations are sufficiently heavy.

In the following sections, we shall discuss the six-quark scheme of Kobayashi and Maskawa, and in passing we shall enumerate only briefly the restrictions on the mixing parameters for the case of a larger number of generations.

2. WHAT IS KNOWN ABOUT THE MIXING ANGLES

In this section, we list the available information about the various matrix elements V_{ik} which follows directly from the experimental data that have so far been obtained. In principle, there exists a completely sufficient number of measurable quantities for a unique determination of the four parameters of the matrix V. Unfortunately, contemporary experiments often make it possible to establish only bounds on some quantities. Moreover, there are a number of theoretical uncertainties associated, in particular, with the fact that the experiments measure not the purely quark characteristics, but only hadron characteristics.

We begin with a formulation of the general features of the determination of the mixing angles θ_i, which makes it possible to visualize clearly the structure of the matrix V.

All the angles are found to be small: $s_1 \approx \sin\theta_C \approx 9.22$, $s_2 \sim 0.05$, $s_3 \lesssim 0.03$, i.e., $s_2, s_3 \sim \sin^2\theta_C$. Consequently, the structure of the mixing matrix is such that only transitions within the same generation are in fact "allowed." Transitions between generations are "forbidden"—they are proportional to the small mixing angles, and transitions across a generation are "superforbidden." Moreover, whereas transitions between the first and second generations are proportional to $\sin\theta_C$, transitions between the second and third are suppressed by a factor $\sim \sin^2\theta_C$, and transitions between the first and third are of the order of $\sin^3\theta_C$ in the amplitude.

As regards the CP-violating phase δ, if the Kobayashi-Maskawa mechanism is the only source of the observed CP nonconservation in K^0 mesons, for a mass of the t quark which is not too large ($m_t \leqslant 80$ GeV) $\sin\delta$ cannot be much less than unity: $\sin\delta \gtrsim \frac{1}{4}$. More detailed estimates of the phase δ are discussed in Sec. 4.

1. $V_{ud} = c_1 \equiv \cos\theta_C$. The quantity V_{ud} is measured in β decays of nonstrange particles. A comparison of the data on superallowed nuclear $0^+ \to 0^+$ Fermi transitions (which are most convenient for a theoretical analysis) with the muon decay probability gives the value[15]

$$c_1 = 0.9735 \pm 0.0015, \quad s_1 = 0.229 \pm 0.006. \qquad (13)$$

It should be stressed that the strictly experimental uncertainty is not more than one third of the error given here; the main error in (13) arises from the need to take into account radiative corrections, which violate isotopic invariance in nuclei.[15]

An accurate measurement at the meson factory LAMPF of the probability of β decay of the π^+ meson, $\pi^+ \to \pi^0 e^+ \nu_e$, also makes it possible to determine the quantity V_{ud} (Ref. 16): $c_1 = 0.937 \pm 0.035$, which agrees with the value (13). Unfortunately, this theoretically "pure" decay cannot at present compete in accuracy with nuclear transitions.

2. $V_{us} = s_1 c_3$. This matrix element is studied directly in semileptonic decays of K mesons and hyperons. In 1983 the WA2 group ($Sp\bar{p}S$, CERN) presented results of measurements of five different semileptonic decays of hyperons, $B_i \to B_f + e + \bar{\nu}_e$, in a single experiment with high statistics. An analysis of the data of that experiment, including results on the neutron lifetime, led to the value[17]

$$|V_{us}| = 0.231 \pm 0.003. \qquad (14)$$

(For a review of the data on hyperon decays, see also Ref. 18). It should be stressed, however, that this analysis was based on Cabibbo's theory,[19] including the assumption of SU(3) symmetry of the form factors (at zero momentum transfer) which determine the matrix elements of the hadronic V–A current with respect to the hyperon states. In reality, this symmetry may be quite strongly broken, particularly for the axial form factors, for which the theorem of Ademollo and Gatto is not applicable. A fit to these data without the assumption of SU(3) of the axial form factors led to the value[18]

$$|V_{us}| = 0.231 \pm 0.005. \qquad (15)$$

However, a violation of SU(3) symmetry for the vector form factors as well may be sufficient to introduce a relative error in the determination of V_{us} at the level of several percent[20-22]; but a realistic estimate of these effects is highly problematic.

Independent information about the quantity V_{us} is provided by the K_{e3} decays $K^+ \to \pi^0 e^+ \nu_e$ and $K_L^0 \to \pi^- e^+ \nu_e$, in which only the vector part of the hadron current is operative, and the effects of symmetry breaking are small and are to some extent amenable to theoretical calculation. The careful analysis of K_{e3} decays performed in Ref. 20 (an earlier analysis was made in Refs. 21 and 22) gave the value

$$|V_{us}| = 0.2196 \pm 0.0023, \qquad (16)$$

where the error takes into account both the experimental and theoretical uncertainties. The inclusion of (16) as an additional constraint in the analysis of the hyperon data leads to the value[20]

$$|V_{us}| = 0.221 \pm 0.002, \qquad (17)$$

and the quality of the fit remains quite good.[18,20] We adopt the estimate (17) for the quantity V_{us}.

A comparison of the matrix elements V_{ud} and V_{us} makes it possible to obtain an upper estimate of the possible value of $\sin\theta_3$:

$$\cos\theta_3 = |V_{us}|(1 - |V_{ud}|^2)^{-1/2} = 0.966 \pm 0.035,$$

or $\sin\theta_3 < 0.36$.

As we shall show below, the decays of b quarks indicate that in the six-quark scheme $\sin\theta_3$ should be about an order of magnitude smaller than this quantity. Nevertheless, the foregoing estimate is important in case it turns out that the number of quarks is actually greater than six:

$$(|V_{ub}|^2 + |V_{ub'}|^2 + \dots)^{1/2}$$

$$= (1 - |V_{us}|^2 - |V_{ud}|^2)^{1/2} < 0.36 \sin\theta_1 \leqslant 0.085. \qquad (18)$$

3. $V_{cd} = -s_1 c_2$. The quantity $|V_{cd}|$ is determined directly from the data on production of charm in the neutrino reactions

$$\overset{(-)}{\nu}_\mu + N \to \mu^- + c + \text{all}.$$

In the cross-section difference

$$\sigma(\nu_\mu + N \to \mu^- + c + \text{all}) - \sigma(\bar{\nu}_\mu + N \to \mu^+ + \bar{c} + \text{all})$$

there is a cancellation of the contribution to the production of c quarks from the sea s quarks, which is proportional to $|V_{cs}|^2 \approx 1$, and allowance is made only for the process

$$\nu_\mu + d \to \mu^- + c.$$

The CDHS group, having measured these cross sections, obtained in this way the value[23] (see also Ref. 24)

$$|V_{cd}| = 0.24 \pm 0.03. \quad (19)$$

(Analyses of these same data performed in Refs. 22 and 25 gave the similar values $|V_{cd}| = 0.25 \pm 0.04$ and $|V_{cd}| = 0.20 \pm 0.03$, respectively.)

Using the values $|V_{cd}|$ (19) and $|V_{ud}|$ (13), it is possible to find upper bounds on the angle θ_2 and on the mixing of the $Q = 2/3$ quarks of the next generations:

$$\sin\theta_2 < 0.45, \quad \sqrt{|V_{td}|^2 + \ldots} < 0.45 \sin\theta_1 = 0.105. \quad (20)$$

If, however, there exist only three generations of quarks, then by using the stringent contraints on θ_2 and θ_3 from the decays of b quarks (see below) it is possible to deduce that

$$V_{cd} \approx -V_{us}, \quad \text{or} \quad V_{cd} = -0.221^{+0.002}_{-0.003}.$$

(We recall that the sign of V_{cd} is dictated by the parametrization which is used for the matrix V.)

4. $V_{cs} = c_1c_2c_3 + s_2s_3e^{i\delta}$. This matrix element enters into the amplitudes of the Cabibbo-allowed decays of the charmed quark. Its precise measurement seems a rather complex problem. At the same time, for small θ_2 and θ_3 the difference between $|V_{cs}|$ and $|V_{ud}|$ is quadratic in θ_2 and θ_3. Therefore the experimental determination of $|V_{cs}|$ does not give significant information about the mixing angles θ_1, θ_2, and θ_3.

The most direct method of determining V_{cs} is to measure the semileptonic decays $D \to K^* e\nu_e, Ke\nu_e$. The width of the decay $D^+ \to \overline{K}^0 e\nu_e$ can be expressed in terms of the vector form factor $f_+^{D\to K}$ of the transition $D^+ \to K^0$ (see, for example, Refs. 4 and 26):

$$\Gamma(D^+ \to \overline{K}^0 e^+ \nu_e) \approx 1.5 \cdot 10^{11} c^{-1} \cdot |f_+^{D\to K}(0)|^2 \, |V_{cs}|^2, \quad (21)$$

for which the value $f_+^{D\to K}(0) = 0.6 \pm 0.1$ was obtained in Ref. 26 by means of QCD sum rules. Using these relations, the value of the $D^+ \to \overline{K}^0 e^+ \nu_e$ decay width, $\Gamma_{exp} = (1 \pm 0.5) \cdot 10^{11}\,\text{sec}^{-1}$, makes it possible to conclude that[26]

$$|V_{cs}| > 0.9. \quad (22)$$

The value of $|V_{cs}|$ can also be estimated from the data on production of charm in neutrino reactions. However, the extracted value has a substantial dependence on the proportion and distribution of strange sea quarks in the nucleon wave function and is thus subject to appreciable uncertainties. Thus, using the results of the CDHS group,[23] various authors obtained lower bounds on $|V_{cs}|$ which are somewhat weaker than (22): $|V_{cs}| > 0.81$,[22] > 0.66,[25] > 0.59.[24,27]

From the unitarity of the matrix V it is easy to find an upper bound on $|V_{cs}|$:

$$|V_{cs}| \leqslant \sqrt{1 - |V_{us}|^2} \leqslant 0.976.$$

In the case of three generations of quarks, $|V_{cs}|^2 = 1 - |V_{cd}|^2 - |V_{cb}|^2$, $|V_{cd}| \simeq |V_{us}|$; using the value of V_{cb} given below, we have

$$|V_{cs}| = 0.9742 \pm 0.0010.$$

5. $V_{cb} = c_1c_2s_3 - s_2c_3e^{i\delta}$, $V_{ub} = s_1s_3$. These elements of the mixing matrix are determined from the decays of B mesons ($B^0 = b\overline{d}$, $B^- = b\overline{u}$). The existing data on these decays are in good agreement with the theoretical expectation (see, for example, Ref. 4) that they should take place mainly as a result of quasifree decay of the b quark, in which the light antiquark plays the role of a spectator (Fig. 1). The various preasymptotic effects (the annihilation mechanism, "quark interference," etc.), which are quite substantial in the decays of D and F mesons,[28] should be small for B mesons.[4,6,7,29]

The analysis of the electron spectrum in the semileptonic decays of mesons with beauty agrees with the assumption that the b quark is converted only into the c quark and admits only a small relative proportion of $b \to ue\overline{\nu}$ decays[30]:

$$\frac{\Gamma(b \to ue\overline{\nu})}{\Gamma(b \to ce\overline{\nu})} < 0.03 \quad (90\%\ \text{c.l.}). \quad (23)$$

The widths of the semileptonic decays themselves can be represented in the form

$$\Gamma_{sl}(b \to c) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 z_c,$$
$$\Gamma_{sl}(b \to u) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 z_u, \quad (24)$$

where the factors z_c and z_u take into account in the standard way the effects of phase space and the QCD corrections; in the calculation of z_u and z_c, allowance is also made for the effects of the bound state of the quarks in the B meson. According to the calculations of Ref. 29,

$$0.73 \leqslant z_u \leqslant 0.94, \quad 0.33 \leqslant z_c \leqslant 0.46 \quad (25)$$

(the main uncertainty in z_u and z_c is associated with the mean value of the momentum of the Fermi motion of the quarks in the meson, which in the model of Ref. 29 describes the effects of the bound state, and the ratio $z_u/z_c \approx 2.1$ is quite stable here).[4)] From this there follows the bound

$$\frac{|V_{ub}|}{|V_{cb}|} < 0.12. \quad (26)$$

This indicates that the decay of the b quark "skipping a generation" is strongly suppressed in comparison with the decay into the adjacent generation.

Estimates of the mixing parameters V_{ub} and V_{cb} themselves can be made by using the B-meson lifetime. The data which have so far been obtained indicate that the B mesons live surprisingly long—$\tau_B \sim 10^{-12}$ sec, apparently even longer than the D mesons (we recall that the average values are $\tau_{D^+} \approx 8.9 \cdot 10^{-13}$ sec and $\tau_{D^0} \approx 3.7 \cdot 10^{-13}$ sec), although for the quark Q we have $\tau_Q \sim m_Q^{-5}$:

$$\Gamma_b \sim \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \times (\text{number of open channels}).$$

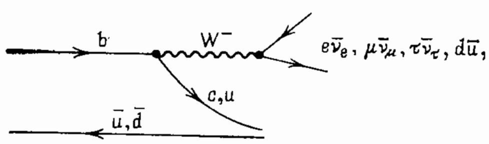

FIG. 1. Diagram describing quasifree decay of a heavy quark in a hadron.

The large lifetime of the B particles, which is in itself undoubtedly a pleasant surprise, makes it possible to conclude that θ_2 and θ_3 are small.

The angles θ_2 and θ_3 can be determined both on the basis of the total width $\Gamma_B = \tau_B^{-1}$ and by using the width of the semileptonic decays of the B mesons, $\Gamma_{sl} = B_{sl}/\tau_B$. These two approaches give approximately the same result, since the experimental value of the fraction of semileptonic decays, B_{sl}, is close to the theoretical expectation $B_{sl} \approx 13$–15%.[6,7,29] We shall follow the second method, which is more reliable from the point of view of theoretical estimates.

The lifetime of the B mesons has now been measured by the MAC, MARK II, DELCO, and JADE groups. Their results give the value[31]

$$\langle\tau_B\rangle_{av} = (1.26 \pm 0.19)\cdot 10^{-12}\ \text{sec}.$$

As regards B_{sl}, its average value over all the experiments is $B_{sl} = 11.7 \pm 0.5\%$ (see, for example, Ref. 7). From this, using the theoretical predictions (24) and (25), it is possible to deduce that

$$\sqrt{|V_{cb}|^2 + 2.1|V_{ub}|^2} = 0.046\sqrt{\frac{1.5\cdot 10^{-12}\ \text{sec}}{\tau_B}}\sqrt{\frac{B(b\to e\bar{\nu}X)}{0.117}}\sqrt{\frac{0.38}{z_c}}.$$

In order to estimate the limits of variation of V_{cb} and V_{ub}, we take

$$10^{-12}\text{sec} \leqslant \tau_B \leqslant 2\cdot 10^{-12}\text{sec},$$
$$0.33 \leqslant z_c \leqslant 0.46,$$
$$0.11 \leqslant B(b\to e\bar{\nu}X) \leqslant 0.13.$$

Then

$$\sqrt{|V_{cb}|^2 + 2.1|V_{ub}|^2} \approx 0.046^{+0.017}_{-0.011}.$$

Bearing in mind the bound $|V_{ub}/V_{cb}| < 0.12$, it is easy to obtain

$$|V_{ub}| = s_1 s_3 < 0.0055^{+0.0020}_{-0.0015},\quad s_3 < 0.024^{+0.010}_{-0.006}.$$

Since $V_{cb} = c_1c_2s_3 - s_2c_3e^{i\delta} \approx c_1s_3 - s_2e^{i\delta}$, for the angle θ_2 it is possible to deduce the bounds

$$|V_{cb}| - c_1s_3 \leqslant s_2 \leqslant |V_{cb}| + c_1s_3,$$
$$0.021^{+0.009}_{-0.005} \leqslant s_2 \leqslant 0.069^{+0.026}_{-0.016}.$$

Thus, from the analysis of the data on B mesons we have

$$|V_{cb}| \approx 0.046^{+0.017}_{-0.012},\quad |V_{ub}| < 0.12|V_{cb}|, \tag{27}$$

and for the angles θ_2 and θ_3, taking the largest interval, we can give the limits

$$\sin\theta_3 < 0.034,\quad 0.016 < \sin\theta_2 < 0.095.$$

In the parametrization (12), the values obtained for V_{cb} and V_{ub} imply that

$$\sin\beta < 0.0075,\quad \sin\gamma = 0.046^{+0.017}_{-0.012},$$
$$\sin\theta = 0.221 \pm 0.002.$$

It should be noted that in connection with the measurement of the lifetime of the B mesons there have appeared a large number of papers in which bounds are derived for the quark mixing angles in the Kobayashi-Maskawa model, for example, Refs. 6, 7, 27, and 31–36. Both the schemes of the arguments and the bounds obtained in these studies are more or less in agreement with each other and with those given above. Some differences between the numerical values reflect, as a rule, only the different degrees of confidence which the authors have in the results of the individual experiments and the different accuracies ascribed to the theoretical calculations.

6. $V_{td} = -s_1s_2$, $V_{ts} = c_1s_2c_3 - c_2s_3e^{i\delta}$, $V_{tb} = c_1s_2s_3 + c_2c_3e^{i\delta}$. Direct experimental information about these matrix elements can be obtained only after analyzing the decays of the t quark. However, preliminary information about them can be obtained by using the values of the mixing angles θ_1, θ_2, and θ_3 given above if it is assumed that there exist only three quark generations:

$$|V_{td}| = s_1s_2 = (0.10 - 0.34)|V_{cb}| = (0.35 - 2.2)\cdot 10^{-2},$$
$$|V_{ts}| = |c_1s_2c_3 - c_2s_3e^{i\delta}| \approx c_1|V_{cb}|\left(1 + \left|\frac{V_{ub}}{V_{cb}}\right|^2\right)$$
$$= (0.975 - 0.99)|V_{cb}|, \tag{28}$$
$$|V_{tb}| = |c_1s_2s_3 + c_2c_3e^{i\delta}| = \sqrt{1 - |V_{cb}|^2 - |V_{ub}|^2}$$
$$\approx 0.9990^{+0.0004}_{-0.0010}.$$

If the number of quarks is greater than six, the unitarity of V leads to the much weaker conditions

$$|V_{td}| < 0.105,\quad |V_{ts}| < 0.38,\quad |V_{tb}| < 0.9994.$$

Certain constraints on the mixing of the t (t') quarks in this case are provided by the decay $K_L \to \mu^+\mu^-$,[37,25,38,39] where the contribution of sufficiently heavy quarks, which appears in the second order in the weak interaction, might be inadmissibly large. These constraints become significant for $m_{t(t')} \gtrsim 40$ GeV and impose an upper limit on the product $|V_{t(t')d}V^*_{t(t')s}|$ at the level 0.04, 0.02, and 0.01 if the mass of the heavy quark is 40 GeV, 60 GeV, and 100 GeV, respectively.

In Table I we present a summary of the values of the elements of the mixing matrix. In the third column we give the values of the moduli of the corresponding elements measured directly in various experiments or the upper bounds obtained by using the unitarity property of V (here we have taken $\tau_B = 1.5\cdot 10^{-12}$ sec and have disregarded the uncertainties in z_u and z_c described above). In the fourth column we give the values of $|V_{ij}|$ in the six-quark scheme.

3. PROSPECTS OF A MORE ACCURATE DETERMINATION OF THE MIXING ANGLES

1. Turning to the prospects of a more accurate determination of the quark mixing parameters, it should be stressed, first of all, that in the framework of the standard six-quark scheme the decays of the t quark can hardly provide new information about the angles θ_i and the phase δ. In fact, its principal decay must proceed through the channel $t\to b$ with almost unit probability; the decay $t \to s$ is expect-

TABLE I.

Matrix element V_{ij}	Kobayashi-Maskawa parametrization (Ref. 11)	$\|V_{ij}\|$	$\|V_{ij}\|$, three generations
V_{ud}	$c_1 \equiv \cos\theta_C$	0.9735 ± 0.0015	0.9753 ± 0.0005
V_{us}	s_1c_3	0.221 ± 0.002	0.221 ± 0.002
V_{ub}	s_1s_3	< 0.0055	< 0.0055
V_{cd}	$-s_1c_2$	0.24 ± 0.03	0.221 ± 0.002
V_{cs}	$c_1c_2c_3 + s_2s_3e^{i\delta}$	$0.9 - 0.976$	0.9742 ± 0.0010
V_{cb}	$c_1c_2s_3 - s_2c_3e^{i\delta}$	0.046	0.046
V_{td}	$-s_1s_2$	< 0.105	$0.0045 - 0.015$
V_{ts}	$c_1s_2c_3 - c_2s_3e^{i\delta}$	< 0.38	0.045
V_{tb}	$c_1s_2s_3 + c_2c_3e^{i\delta}$	< 0.999	0.999

ed at the level $2\cdot 10^{-3}$, although, apart from an overall phase, the corresponding element V_{ts} is practically identical to V_{cb}, whose accurate measurement is undoubtedly simpler and will be carried out in the near future. It would be interesting to measure the width of the decay $t \to d$ in order to make an unambiguous determination of the angle θ_2, although the fraction of such decays should be only of the order of 10^{-4}, and this is also complicated by the absence of clear triggers for these decays. Nevertheless, an experimental determination of V_{tb}, V_{ts}, and V_{td} seems essential, since it may provide indirect evidence for the existence of the next quark generations.

2. A much more accurate determination of V_{ud} and V_{us}, and hence of the angle θ_1, seems improbable. Since the principal error in the determination of V_{ud} and V_{us} is due to the theoretical uncertainties, a reduction of the error in θ_1 requires above all a detailed calculation of the radiative corrections and of the effects of symmetry breaking.

3. At the present time, it is planned to make a direct determination of the ratio $|V_{cd}/V_{cs}|$ on the basis of a measurement of the decay-width ratios $\Gamma(D \to \pi e\nu)/\Gamma(D \to Ke\nu)$ and $\Gamma(D \to \rho e\nu)/\Gamma(D \to K^* e\nu)$. An accurate measurement of $|V_{cd}/V_{cs}|$ seems interesting as one of the "probes" of new generations of quarks.

4. Future experiments to investigate the decays of B mesons should play a major role in reducing the admissible intervals of the angles θ_2 and θ_3. It is necessary above all to determine their lifetimes more accurately by directly measuring the path lengths. The fraction of semileptonic decays of charged and neutral B mesons will also be determined more accurately (the contemporary experimental value of B_{sl} is somewhat smaller than the theoretical expectation; see, for example, Refs. 6, 7, and 29). Moreover, the currently available data have been averaged over the charged and neutral B mesons. An independent measurement of $B_{sl}(B^\pm)$ and $B_{sl}(B^0)$ will also make it possible to determine the lifetimes of both mesons:

$$\frac{\tau(B^-)}{\tau(B^0)} = \frac{B(B^- \to e^-\nu X)}{B(B^0 \to e^-\nu X)}.$$

5. It seems very important to determine more accurately the proportion of decays $b \to u$ in relation to the decays $b \to c$ and thereby to determine (or to establish more stringent constraints on) the angle θ_3. Experiments planned at DORIS and CESR will be able to measure the ratio $\Gamma(b \to u)/\Gamma(b \to c)$ with an accuracy sufficient for a determination of the proportion of decays $b \to u$ at the level of one percent. Here, however, there arises the question of whether it is possible to give a correct theoretical description of such an accurate value of this ratio.

6. For a measurement of the matrix element V_{ub} and a determination of the angle θ_3, it seems promising to consider the decay of charged B mesons into a $\tau\nu_\tau$ pair (Fig. 2). The probability of this process can be expressed directly in terms of $|V_{ub}|$ and f_B—the axial decay constant of the B meson, the analog of f_π and f_K:

$$\Gamma(B^- \to \tau^-\bar{\nu}_\tau) = \frac{G_F^2 f_B^2 m_\tau^2 M_B}{8\pi} |V_{ub}|^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2.$$

For $f_B = 130$ MeV (Ref. 40) and $|V_{ub}/V_{cb}| \approx 0.12$, the value of $B(B^- \to \tau\bar{\nu}_\tau)$ is approximately 10^{-4}. The constant f_B can evidently be calculated from QCD sum rules with an accuracy of about 15%.[40] It is proposed to make an experimental determination of the relative probability of the decays $B^\pm \to \tau\nu_\tau$ using the facility CESR. There is also interest in the analogous decay of the charmed B meson, $B_c^- \sim b\bar{c}$, for which the $\tau\nu_\tau$ mode is not Cabibbo-suppressed and is expected at the level of 1%. The value of the corresponding axial constant f_{B_c} can also be estimated comparatively reliably by means of QCD sum rules. A measurement of the fraction of decays $B_c \to \tau\nu_\tau$ would make it possible to reduce the theoretical uncertainties in the determination of $|V_{cb}|$, although it represents a rather complex experimental problem.

7. There are also discussions in the literature of the possibility of obtaining additional information about the quark mixing angles from the strength of the mixing of the neutral B mesons, primarily[41] on the basis of a study of the yield of singly charged dileptons in the decays of $B^0\bar{B}^0$ pairs produced in e^+e^- annihilation (see, for example, the review of Ref. 5 and the more recent papers of Refs. 25, 32, 34, 35, and 42–46). The amplitude of the transition $B^0 \to \bar{B}^0$ is determined here by the contribution of the t quark, which is pro-

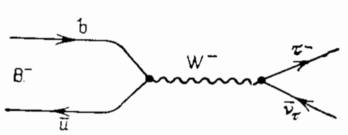

FIG. 2. Diagram corresponding to the decay $B^- \to \tau\bar{\nu}_\tau$.

portional to $(V_{tb}V_{td}^*)^2$. The ratio $x_B = \Delta m_B/\Gamma_B$, which characterizes the effects of the mixing of B^0 and $\bar{B}^0$, is then proportional to $|V_{tb}V_{td}^*|^2/|V_{cb}|^2$:

$$x_B \approx \beta |V_{tb}V_{td}^*|^2 |V_{cb}|^{-2}, \quad \beta \approx 1.5 \quad \text{for} \quad m_t = 40 \text{ GeV}.$$

Unfortunately, in the framework of the standard scheme $|V_{tb}V_{td}^*|^2/|V_{cb}|^2 \lesssim 0.1$, and therefore the yield of singly charged dileptons in relation to that of oppositely charged dileptons, $r_B = (n_{++} + n_{--})/n_{+-} = x_B^2/(2 + x_B^2)$,[41] does not exceed 1%. For B_s^0 and $\bar{B}_s^0$ mesons, $(b\bar{s}, b\bar{s})x_{B_s} \approx \beta |V_{tb}V_{ts}^*|^2/|V_{cb}|^2$, and this quantity is of the order of unity. However, in the six-quark scheme the ratio $|V_{tb}V_{ts}^*|^2/|V_{cb}|^2$ is practically equal to c_1^2 [see (28)] and thus cannot provide new information about the mixing angles. For a larger number of generations, the experimental limit on the yield of singly charged dileptons in the decays of $B^0\bar{B}^0$ pairs, $r_B \lesssim 0.3$,[7,47] gives a certain upper bound on the product $V_{t(t')b}V_{t(t')d}$ at the level 0.04–0.02 for m_t between 40 and 80 GeV.

4. QUARK MIXING AND THE PHYSICS OF K⁰ MESONS

For a long time, very important information about the quark mixing angles has been extracted from the study of the K^0–$\bar{K}^0$ system (see, for example, Ref. 5). The point is that CP nonconservation, which is described in the Kobayashi-Maskawa model by the phase δ, has so far been observed only in the decays of neutral K mesons. In addition, the mass difference Δm_{LS} between the K_L and K_S mesons is quite sensitive to the mixing of the t quark with the u and c quarks. A detailed analysis of the K_L–K_S system is contained in the review of Ref. 5; here we shall recall only the most basic information.

In the absence of weak interactions, strangeness is a strictly conserved quantum number, and we have a stable particle $K^0 \sim \bar{s}d$ with strangeness $S = 1$ and a corresponding antiparticle $\bar{K}^0 \sim s\bar{d}$ with $CP|K^0\rangle = -|\bar{K}^0\rangle$.[5] If the weak interaction is included, strangeness-changing transitions become possible, and these lead to mixing of the K^0 and $\bar{K}^0$ in the physical states possessing definite mass. If there is no violation of CP invariance, the states with definite mass are the K_1^0 and K_2^0 mesons,

$$K_1 = \frac{K^0 - \bar{K}^0}{\sqrt{2}}, \qquad K_2 = \frac{K^0 + \bar{K}^0}{\sqrt{2}},$$

possessing definite CP parity:

$$CP\,(K_1) = 1, \; CP\,(K_2) = -1.$$

In the case of CP nonconservation, the physical states are the K_L and K_S mesons,

$$K_S = \frac{K_1 + \bar{\varepsilon} K_2}{\sqrt{1 + |\bar{\varepsilon}|^2}}, \qquad K_L = \frac{K_2 + \bar{\varepsilon} K_1}{\sqrt{1 + |\bar{\varepsilon}|^2}},$$

and the value of Re $\bar{\varepsilon}$ is a measure of the CP nonconservation.

a) Mass difference between the K_L and K_S mesons

Since experimentally the CP nonconservation for the K^0 mesons is small, in this section we neglect the distinction between the K_S and K_L mesons and the K_1 and K_2 mesons. The mass difference between the K^0 mesons is determined by the matrix element for the transition $K^0 \to \bar{K}^0$ (a relativistic normalization of the wave functions is assumed):

$$\Delta m_{LS} = m_L - m_S = m_K^{-1}\,\mathrm{Re}\,\langle \bar{K}^0 | H_{eff}\,(\Delta S = 2) | K^0 \rangle,$$

where $H_{eff}(\Delta S = 2)$ is the effective Hamiltonian for the transitions with a change of strangeness equal to 2. The quantity $H_{eff}(\Delta S = 2)$ arises in the second order in the weak interaction and has the order of magnitude G_F^2. From the time of the earliest studies[48,49] in which Δm_{LS} was estimated in the framework of the GIM four-quark model,[1] it has been assumed that the main contribution to $H_{eff}(\Delta S = 2)$ comes from the box diagrams of Fig. 3. In the six-quark scheme, the structure of the effective Hamiltonian remains the same as before, but the overall coefficient is changed, and $H_{eff}(\Delta S = 2)$ takes the form

$$H_{box} = \frac{G_F^2}{16\pi^2} (\bar{s}\gamma_\mu (1 - \gamma_5) d)(\bar{s}\gamma_\mu (1 - \gamma_5) d) \times \left[m_c^2 V_{cd}^2 V_{cs}^{*2} + m_t^2 V_{td}^2 V_{ts}^{*2} + 2m_c^2 \ln \frac{m_t^2}{m_c^2} V_{td}V_{ts}^* V_{cd} V_{cs}^* \right] \quad (29)$$

(here it is assumed that $m_t^2 \ll m_W^2$). In the four-quark scheme, the second and third terms, associated with the $t\bar{t}$ and $c\bar{t}$, $t\bar{c}$ quarks in the intermediate state, are absent, and the expression in the square brackets reduces simply to $m_c^2 \sin^2\theta_C \cos^2\theta_C$.

Taking for the matrix element $\langle \bar{K}^0 | (\bar{s}\gamma_\mu (1 - \gamma_5) d)^2 | K^0 \rangle$ the value $-8/3 m_K^2 f_K^2$ which follows from the vacuum-insertion hypothesis, we have

$$\Delta m_{LS} = \Delta m_{box} = \frac{4}{3\pi} \frac{\Gamma(K^+ \to \mu^+ \nu)}{m_\mu^2 s_1^2 c_3^2} \frac{1}{[1 - (m_\mu^2/m_K^2)]^2} (A_c + A_t + A_{ct}), \quad (30)$$

where A_c, A_t, and A_{ct} are the corresponding terms in the square brackets in (29). In the four-quark model, the value of Δm_{LS} calculated in this way agrees with the experimental value for $m_c = 1.58$ GeV. The discovery of the c quark, whose mass was at that time assigned a value of about 1.5 GeV, was regarded as a triumph of the GIM four-quark scheme. However, this estimate of Δm_{LS} was subsequently reconsidered from several points of view. Inclusion of the strong interactions at small distances leads to a certain sup-

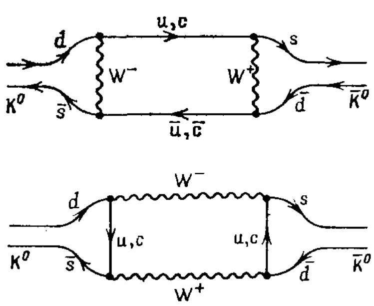

FIG. 3. Box diagrams corresponding to the transition $K^0 \to \bar{K}^0$.

pression of Δm_{LS} in comparison with (30) (Refs. 50 and 51):

$$A_c \to 0.7A_c,\ A_t \to 0.6\ A_t,\ A_{ct} \to 0.4\ A_{ct}.$$

The mass of the c quark, which enters into the expression for A_c, is 1.3–1.4 GeV according to contemporary estimates.[4] These two facts have the consequence that the contribution of the c quark can provide not more than 40–45% of the experimental value of Δm_{LS}.

We note that some uncertainty in the calculation of Δm_{LS} comes from the matrix element $\langle \overline{K}^0 | (\bar{s}\gamma_\mu(1-\gamma_5)d)^2 | K^0\rangle$. In Ref. 50 it was argued that the vacuum insertion, which gives the expression $-8/3m_K^2 f_K^2$, has here an accuracy not worse than 10%. Nevertheless, in many studies (see Ref. 5) the result of the vacuum insertion is multiplied by some constant B, as an estimate of which, for example, in the bag model[52,53] or using SU(3) symmetry and PCAC,[54] a value $B \approx 0.3$–0.5 is usually given.[5] However, both these approaches have well-known drawbacks,[50] and it is doubtful whether they have greater accuracy than the vacuum insertion (see also the discussion of Ref. 55, in which a value of B somewhat greater than unity was obtained).

Thus, even for $B = 1$ the contribution of the c quarks to Δm_{LS} was found to be much smaller than the experimental value, and this fact was used to obtain lower bounds on the mixing angles θ_2 and θ_3 of the t quark.

The estimates of the parameters of the six-quark scheme given in Sec. 2 make it possible to assert, however, that the contribution of the t quarks [the terms A_t and A_{ct} in (29) and (30)] to Δm_{LS} is small, at least if m_t is assigned reasonable values[6)]: for $\tau_B \gtrsim 10^{-12}$ sec with $m_t = 40$ GeV the contribution of the t quark is not more than 2% of $(\Delta m_{LS})_{exp}$, and for $m_t = 80$ GeV it is not more than 5% of $(\Delta m_{LS})_{exp}$.

Thus, in terms of the conventional logic, the standard six-quark model could not explain the observed mass difference Δm_{LS}. The missing mass difference might then be explained as follows.

a) There exist four (or more) generations of quarks. In this case, the bounds on the mixings of the t(t') quarks are much weaker, $|V_{t(t')d}| < 0.105$, $|V_{t(t')s}| < 0.38$ (see Table I), and their contribution to Δm_{LS} may be quite large.

b) There is a large contribution to Δm_{LS} from "supersymmetric" particles. Namely, the diagrams analogous to those of Fig. 3, in which the $W^\pm$ bosons are replaced by gluinos (superpartners of the gluons) and the intermediate u, c, and t quarks are replaced by scalar quarks with charge $-1/3$ (superpartners of the d, s, and b quarks) can give, for a gluino mass up to 50–100 GeV, a contribution comparable with the experimental value of Δm_{LS} (Refs. 56 and 57) [we note that the direct experimental limit on the gluino mass is only $m_{\tilde{g}} > 3$–5 GeV (Ref. 58); moreover, the data of the UA1 group evidently exclude the existence of gluinos with mass less than 40 GeV (Ref. 59)].

c) A practically arbitrary contribution to Δm_{LS} might come from strangeness-violating neutral currents induced, for example, by "horizontal" or "right-handed" gauge bosons or by superheavy Higgs particles. The most stringent constraints on the masses of these exotic objects follow in fact from the analysis of the K_L–K_S system.[60–62]

In reality, however, we do not at present have at our disposal strong arguments for the existence of such nonstandard contributions to Δm_{LS}. The point is that the calculation given above, which is the ordinary calculation of the mass difference between the neutral K mesons described by the effective Hamiltonian (29), takes into account correctly the contribution of only sufficiently massive intermediate states. A naive estimate of the contribution of virtualities of the order of the characteristic hadron mass μ from the diagrams of Fig. 3 gives a small factor $\sim\mu^2/m_c^2$. In fact, the contribution of large distances to the transition $K^0 \to \overline{K}^0$ is numerically much larger, this being due to a dynamical enhancement of the $\Delta T = \frac{1}{2}$ decay amplitudes of strange particles. This is indicated, in particular, by the experimental fact that

$$\Delta m_{LS} \approx -\frac{\Delta\Gamma_{LS}}{2} \approx \frac{\Gamma_S}{2},$$

which means that the imaginary (absorptive) part of the amplitude $M_{K^0\overline{K}^0}$ for the transition $K^0 \to \overline{K}^0$ is practically equal to its real (more precisely, dispersive) part. If in the expansion of the amplitude $M_{K^0\to\overline{K}^0}$ with respect to intermediate states,

$$M_{K^0\overline{K}^0} = \frac{1}{2m_K}\sum_n \frac{\langle \overline{K}^0 \mid H_W(\Delta S=1) \mid n\rangle \langle n \mid H_W(\Delta S=1) \mid K^0\rangle}{m_K - E_n + i0},$$
$$\mathrm{Re}\, M_{K^0\overline{K}^0} = \frac{\Delta m_{LS}}{2},\quad \mathrm{Im}\, M_{K^0\overline{K}^0} = -\frac{\Delta\Gamma_{LS}}{4}, \tag{31}$$

we retain only the contribution of the two-pion state, which determines $\mathrm{Im} M_{K^0\overline{K}^0}$, the quantity Δm_{LS} is given by the diagram of Fig. 4, in which the points at the vertices denote $H_W(\Delta S = 1)$, the total weak Hamiltonian for the transitions with $\Delta S = 1$. In order to estimate this contribution to Δm_{LS}, we can assume that the phenomenological $K^0 \to 2\pi$ vertex does not depend on the momenta, calculate it from the $K_S \to 2\pi$ decay width, and truncate the integral with respect to the pion momenta at some scale Λ. Then for $\Lambda \approx 700$–800 MeV the real part of this amplitude is already comparable with the imaginary part and thus gives the required value of Δm_{LS}.

It should be stressed that if we add to the standard $K^0 \to \overline{K}^0$ transition amplitude M_{box} given by the box diagrams of Fig. 3 the contribution of, say, a two-pion state, then there will be no double counting of the contributions already contained in Δm_{box}. Thus, Δm_{box} is proportional to m_c^2, whereas the width of the decay $K_S \to 2\pi$, which depends on m_c through the coefficients in the expansion of

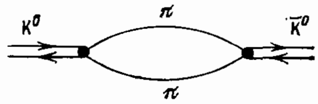

FIG. 4. Diagram corresponding to the contribution of the two-pion intermediate state to the transition $K^0 \to \overline{K}^0$.

$H_W(\Delta S=1)$ with respect to the local operators o_1–o_6 (see Refs. 3 and 63), varies weakly with m_c.[63] Moreover, in the standard calculation, even when the strong interactions are included, the K^0–$\bar{K}^0$ mixing occurs as a result of the operators containing only the left-handed quark fields, whereas the relatively large $K_S \to 2\pi$ decay width evidently arises from the operators involving right-handed currents.[63] More detailed arguments that the low-lying intermediate states give a contribution which is not taken into account by the ordinary diagrams of Fig. 3 were given in Ref. 64 (see also the earlier paper of Ref. 65). Special mention should be made of the fact that the arguments outlined above confirm the fact that the factor B introduced for the parametrization of the matrix element $\langle \bar{K}^0 | (\bar{s}\gamma_\mu(1-\gamma_5)d)^2 | K^0\rangle$ in the framework of the standard calculation bears no relation to the large-distance contribution discussed above, which is an independent additive contribution to Δm_{box}.[64,65]

Unfortunately, it is not possible at the present time to make a more or less definite calculation of the contribution from the states with small virtuality. Below, we shall consider this problem briefly.

It is easy to establish the signs of the contributions of the various states to Δm_{LS}. In the representation (31) of the amplitude $M_{K^0\bar{K}^0}$, the intermediate states can be classified according to their CP parity. Since K_L can be converted only into CP-odd states, and K_S only into CP-even states, the states with CP $= -1$ having mass greater than m_K will decrease, and those lighter than the K meson will increase, m_{K_L} in comparison with m_{K_S},[7)] and for the CP-even states the contribution will have the opposite sign. It is clear from this that the π^0 meson in the intermediate state will increase Δm_{LS}, while the η^0 meson will decrease it. The two-pion states lead to contributions of either sign, depending on the pion invariant mass, but for reasonable values of the scale of the energy cutoff the total displacement is positive. In the chiral limit, i.e., for parametrically small masses of the π and K mesons, the largest contributions are the π^0 and η^0 pole contributions. They were calculated in Ref. 66 using the PCAC hypothesis:

$$\tau_S \cdot \Delta m_{LS}|_{\pi^0,\,\eta^0} = -\frac{32}{9}\pi f_\pi^2 \frac{m_K}{\sqrt{m_K^2-4m_\pi^2}} \frac{4m_K^2-3m_\eta^2-m_\pi^2}{(m_K^2-m_\pi^2)\,(m_\eta^2-m_K^2)}$$

$$= -0.67\ (\text{cf. } 0.48 \pm 0.02 \text{ exper. }). \qquad (32)$$

The numerical value given here corresponds to the experimental values of the meson masses. However, in accordance with the formula of Gell-Mann and Okubo, the numerator in (32) must vanish in the limit of SU(3) symmetry, which was assumed in the derivation of (32), so that in this approach there is a complete cancellation of the π^0 and η^0 contributions. We note that the individual π^0 and η^0 contributions calculated in this way are approximately equal to $2\Gamma_S$ in magnitude. The contribution of the two-pion state depends to some extent on the scale of the ultraviolet cutoff and, as we have already noted, is approximately $1/2\Gamma_S$. In Ref. 67 it was emphasized that states with intermediate virtuality, for example, the η' meson, can also give an appreciable contribution to Δm_{LS}.

Thus, among the contributions of the low-lying states there are terms of both signs, and the individual terms in the sum can be three or four times as large as the experimental value of Δm_{LS}. Since it is not possible to calculate these contributions with sufficient accuracy, the absolute accuracy of the various theoretical estimates of Δm_{LS} is probably even worse than the value of Δm_{LS} itself. It is interesting to note that similar calculations of the mass difference between the K^0 mesons on the basis of the low-lying states were performed even before gauge theories of the electroweak and strong interactions were developed.[66,68–71] At that time, however, it was not possible to take into account correctly the small-distance contribution Δm_{box}, which is parametrically dominant in the case of a heavy c quark. In reality, the mass of the c quark proved to be insufficiently large, and these two mechanisms appear to give approximately the same contribution to Δm_{LS}.

b) CP nonconservation in K^0 mesons

One of the remarkable properties of the six-quark scheme of Kobayashi and Maskawa is the fact that it contains a natural possibility of violation of CP symmetry.[2] Since experimentally the violation of CP invariance has been observed only in the K_L–K_S system, the neutral K mesons represent the only source of information about the CP-violating parameter $e^{i\delta}$ in this model.

In the Kobayashi-Maskawa matrix (11), none of the elements which determine the interaction of the u and d quarks with the W bosons contain the CP-odd phase $e^{i\delta}$, so that in the four-fermion interaction describing the decay of K mesons the complex coefficients leading to CP nonconservation appear only in the operators containing pairs of heavy c or t quarks. In one of the earliest papers[72] in which the effects of CP violation in the six-quark scheme were analyzed systematically, the amplitudes for the direct CP-odd decays $K_2 \to 2\pi$ generated by these operators were estimated to be very small, and the observed CP nonconservation was due solely to the CP-odd mixing of the K_1 and K_2 mesons resulting from the complex character of the mass difference Δm_{LS} given by the diagrams of Fig. 3. This led to the conclusion that within the framework of the K_L–K_S system the Kobayashi-Maskawa model simulates the superweak mechanism of CP violation (Ref. 73),[8)] and the deviation from it was predicted to be very small. The quantity ε itself is proportional to $2s_2s_3\sin\delta$, and from a comparison with the observed values of

$$\eta_{+-} = \frac{\langle \pi^+\pi^- \mid H_W \mid K_L\rangle}{\langle \pi^+\pi^- \mid H_W \mid K_S\rangle}, \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 \mid H_W \mid K_L\rangle}{\langle \pi^0\pi^0 \mid H_W \mid K_S\rangle},$$
$$\varepsilon \approx \frac{2\eta_{+-}+\eta_{00}}{3}, \qquad \varepsilon' \approx \frac{\eta_{+-}-\eta_{00}}{3} \qquad (33)$$

the product $s_2s_3\sin\delta$, which enters as a factor into all the CP-violating amplitudes for the K^0 mesons, was expected to be at the level of $0.5\cdot 10^{-3}$. Subsequently, however, it was shown[63] that because of the "annihilation" diagrams of Fig. 5 the contribution of the virtual heavy quarks is very important in the decays of K^0 mesons and may explain the $\Delta T = \frac{1}{2}$

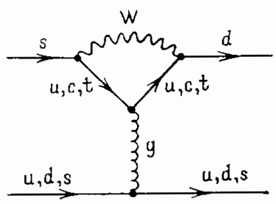

FIG. 5. "Annihilation" diagram for decays of strange particles.

rule and the relatively small lifetime of the K_S meson. In the framework of the Kobayashi-Maskawa scheme, this fact leads to the conclusion that the observed CP nonconservation may also have an appreciable contribution from the direct decays $K_2 \to 2\pi$, since here the amplitudes for the weak decays have a CP-odd phase ξ which is in general of the same order of magnitude $s_2 s_3 \sin\delta$ as the CP-odd phase of Δm_{LS}.[76,79] It is important, however, that the "annihilation" diagrams lead only to operators with isotopic spin $\frac{1}{2}$, so that the violation of the prediction $\eta_{+-} = \eta_{00}$ of the superweak model[73] induced by the direct decays arises only as a result of the violation of the $\Delta T = \frac{1}{2}$ rule in the ordinary CP-even decays $K_1 \to 2\pi$ (see, for example, Refs. 80 and 81) and is therefore expected to be at the percentage level.[76–79,82]

In the theoretical estimation of the effects of CP nonconservation for K^0 mesons in the Kobayashi-Maskawa model, there arises above all the question of the validity of the calculation of the CP violating imaginary part[9)] of the amplitude $\langle \overline{K}^0|T|K^0\rangle \mathrm{Im} M_{K^0\overline{K}^0}$ by means of the ordinary box diagrams of Fig. 3. As was explained in the preceding subsection, in the calculation of the K_L–K_S mass difference, i.e., $\mathrm{Re}\, M_{K^0\overline{K}^0}$, this procedure can admit almost uncontrollable errors associated with the effects of large distances, and the magnitude of these contributions can exceed the standard quantity Δm_{box}. However, in Refs. 64 and 65 it was shown that under certain assumptions the large-distance effects in the CP-odd transitions $K^0 \to \overline{K}^0$ can be taken into account. Thus, if the effective $\Delta S = 1$ Hamiltonian for the weak nonleptonic transitions is dominated by only one operator with $\Delta T = \frac{1}{2}$, the CP-odd phase of the transition of $K^0(\overline{K}^0)$ into any intermediate state will be determined by the complex phase factor $e^{i\xi}$ of the coefficient of this operator, and therefore the contribution of these states to the $K^0 \to \overline{K}^0$ transition amplitude $\langle \overline{K}^0|T|K^0\rangle$ will have the phase $e^{2i\xi}$. In this case, the large-distance contribution to $\mathrm{Im} M_{K^0\overline{K}^0}$ can be expressed in terms of the quantity ξ and the contribution of the low-lying states to the K_L–K_S mass difference.

Roughly this situation is realized in the decays of K^0 mesons, where it is possible that about 70–80% of the $K_S \to 2\pi$ amplitude is provided by the operator $O_5 + (3/16)O_6$,[63] which arises as a result of the "annihilation" diagrams of Fig. 5 (see also Ref. 3). If we adopt the hypothesis of dominance of the operators O_5 and O_6 the CP-nonconservation parameter ε can be readily calculated.

Making use of the smallness of CP nonconservation for K mesons and the experimental equality $\Delta\Gamma \simeq \Gamma_S \approx 2\Delta m_{LS}$, the expression for ε can be written in the form (see, for example, Ref. 5)

$$\varepsilon \approx \frac{1}{1+i}\left(-\frac{i\,\mathrm{Im}\, M_{K^0\overline{K}^0}}{2\,\mathrm{Re}\, M_{K^0\overline{K}^0}} + \xi\right)$$
$$+ i\xi = -\frac{e^{i\pi/4}}{2\sqrt{2}}\left(\frac{\mathrm{Im}\, M_{K^0\overline{K}^0}}{\mathrm{Re}\, M_{K^0\overline{K}^0}} - 2\xi\right),$$
$$\mathrm{Re}\, M_{K^0\overline{K}^0} \approx \frac{1}{2}\Delta m_{LS}, \quad M_{K^0\overline{K}^0} = \langle \overline{K}^0|T|K^0\rangle, \tag{34}$$

where ξ is the CP-odd phase of the amplitude for decay of the K^0 meson into two pions with zero isotopic spin, from which the pion scattering phase is separated:

$$\langle(\pi\pi)_{I=0}|T|K^0\rangle = iA_0 e^{i\delta_0}, \quad \xi = \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \sim s_2 s_3 \sin\delta. \tag{35}$$

If we now represent the $K^0 \to \overline{K}^0$ transition amplitude $M_{K^0\overline{K}^0}$ in the form of the sum of the small-distance contribution M_{box} and the large-distance contribution M_{long}, then, as we explained above, $\mathrm{Im}\, M_{long} \approx 2\xi\ \mathrm{Re}\, M_{long}$ and hence

$$\varepsilon \approx -\frac{e^{i\pi/4}}{2\sqrt{2}}\left(\frac{\mathrm{Im}\, M_{box} + 2\xi\,\mathrm{Re}\, M_{long}}{\mathrm{Re}\, M_{box} + \mathrm{Re}\, M_{long}} - 2\xi\right)$$
$$= \frac{e^{i\pi/4}}{\sqrt{2}}\ \frac{\mathrm{Im}\, M_{box} - 2\xi\,\mathrm{Re}\, M_{box}}{\Delta m_{LS}}. \tag{36}$$

Thus, inclusion of the direct decays and the effect of large distances reduces to the addition to $\mathrm{Im}\, M_{box}$ of the quantity $-2\xi\ \mathrm{Re}\, M_{box}$.[64,65]

It should be stressed, however, that this statement has limited accuracy. In reality, the dominance of the operators O_5 and O_6 in the ordinary CP-even decays of K^0 mesons is not absolute. Moreover, although it is probable that these operators, which contain the right-handed quark fields, are indeed dominant in the transitions $K^0 \to \pi^0, \eta^0$, which are evidently most important in the large-distance contribution, there are no special reasons to expect that O_5 and O_6, which contain small coefficients in comparison with the operators O_1–O_4,[63] will be decisive in transitions into states with intermediate energy. If such states give an appreciable contribution to $M_{K^0\overline{K}^0}$, there may be significant corrections to the relation (36).

Bearing in mind this remark, we shall nevertheless adopt the estimate (36) for ε. Further, it is convenient to introduce the notation $z = \mathrm{Re}\, M_{box}|\mathrm{Re}\, M_{K^0\overline{K}^0} = 2\ \mathrm{Re}\, M_{box}|\Delta m_{LS}$. Then

$$|\varepsilon| \approx \frac{1}{2\sqrt{2}}|z|\left|\frac{\mathrm{Im}\, M_{box}}{\mathrm{Re}\, M_{box}} - 2\xi\right|. \tag{37}$$

The expression for M_{box} in the six-quark scheme is well known (see, for example, Refs. 50 and 51). For $s_1^2, s_2^2, s_3^2 \ll 1$,

$$\mathrm{Im}\, M_{box} \approx \langle \overline{K}^0|(\bar{s}\gamma_\mu(1-\gamma_5)d)^2|K^0\rangle$$
$$\times \frac{G_F^2}{16\pi^2} s_1^2 m_c^2 \cdot 2 s_2 s_3 \sin\delta\left[-\eta_1\right.$$
$$\left. + \eta_3 \ln\frac{m_t^2}{m_c^2} + \frac{m_t^2}{m_c^2}(\eta_2 x + \tilde{\eta}_2 y)\, s_2(s_2 - s_3\cos\delta)\right],$$
$$\mathrm{Re}\, M_{box} \approx \langle \overline{K}^0|(\bar{s}\gamma_\mu(1-\gamma_5)d)^2|K^0\rangle \frac{G_F^2}{16\pi^2} s_1^2 m_c^2 \left\{\eta_1\right.$$
$$+ 2s_2(s_2 - s_3\cos\delta)\,\eta_3 \ln\frac{m_t^2}{m_c^2}$$
$$\left. + s_2^2\left[(s_2 - s_3\cos\delta)^2 - s_3^2\sin^2\delta\right]\frac{m_t^2}{m_c^2}(\eta_2 x + \tilde{\eta}_2 y)\right\}. \tag{38}$$

Here $\eta_1, \eta_2, \tilde{\eta}_2, \eta_3$ are small-distance gluon-renormalization factors, and the factors x and y take into account the finiteness of the mass of the W boson in the diagrams involving two t quarks[50] (see Fig. 3); for $m_t^2 \ll M_W^2$ we have $x \approx 1, y \approx 0$, while for $m_t^2 = M_W^2$ we have $x = 1/3, y = 5/12$. As we noted in the preceding subsection, for the existing constraints on the mixing angles θ_2 and θ_3 the terms corresponding to the contribution of the t quark to Re $M_{\rm box}$ are small, and therefore

$$\frac{{\rm Im}\, M_{\rm box}}{{\rm Re}\, M_{\rm box}} \simeq 2s_2s_3 \sin\delta \left[\frac{\eta_3}{\eta_1} \ln\frac{m_t^2}{m_c^2} - 1 + \frac{m_t^2}{m_c^2} s_2(s_2 - s_3\cos\delta) \times \left(\frac{\eta_2}{\eta_1} x + \frac{\tilde{\eta}_2}{\eta_1} y\right)\right]. \quad (39)$$

Numerically, $\eta_3/\eta_1 \approx 0.6$–$0.55$ and $\eta_2/\eta_1 \approx \tilde{\eta}_2/\eta_1 = 0.9$, and it is obvious that these quantities do not depend on the normalization point of the operators.

As regards the phase ξ in (37), its precise value is determined by the relative contribution of the operators o_5 and o_6 to the various strangeness-changing amplitudes. According to the estimates of Ref. 82, in the decays $K_S \to 2\pi$ we have $\xi \simeq (0.8$–$1.2)\, s_2s_3 \sin\delta$; for complete dominance of the operators o_5 and o_6, we have $\xi \simeq (1.3$–$1.5) s_2s_3 \sin\delta$.[82] The quantity z, which determines the proportion of the contribution of $\Delta m_{\rm box}$ to the total mass difference $\Delta m_{\rm LS}$, has a substantial dependence on the value of the matrix element $\langle \bar{K}^0 | (\bar{s}\gamma_\mu(1-\gamma_5)d)^2 | K^0 \rangle$. Using for it the vacuum-insertion estimate, z is found to be approximately 0.5, as was shown in the preceding subsection.

Since the angles θ_2 and θ_3 are small, the CP nonconservation observed for the K^0 mesons, $|\varepsilon| \approx |\eta_{+-}| \approx 2.27 \cdot 10^{-3}$, leads to rather stringent constraints on the parameters of the Kobayashi-Maskawa matrix. Thus, if we adopt the value $|V_{cb}| = 0.051$, which corresponds to $\tau_B \approx 1.2 \cdot 10^{-12}$ sec, and $\sin\theta_3 < 0.028$ $(\Gamma(b\to u)/\Gamma(b \to c) < 3\%)$, then to obtain the indicated value of ε the mass of the t quark must be greater than 40–50 GeV; for all t-quark masses not significantly greater than the mass of the W boson, the parameter region $\cos\delta < 0$ is rejected. As to the phase δ itself, we have (for $m_t \lesssim m_W$) $\sin\delta \gtrsim 1/4$. Thus, if CP nonconservation is ensured only as a result of the Kobayashi-Maskawa mechanism, we are dealing with practically the maximum possible violation of CP invariance.

A lower bound on the mass of the t quark as a function of the B-meson lifetime was derived in Ref. 36 on the basis of analogous arguments. However, in the numerical estimates of that paper use was made of the value of the matrix element $\langle \bar{K}^0 | (\bar{s}\gamma_\mu(1-\gamma_5)d)^2 | K^0 \rangle$, modified in comparison with the case of the vacuum insertion by a factor $B \approx 0.37$ (at the normalization point, $\alpha_s(\mu^2) = 1$), which corresponds to a decrease of z down to about 0.2. For this reason, more stringent constraints on the mass of the t quark were obtained in Ref. 36. In addition, no allowance was made in that paper for the contribution of the direct decays to the quantity ε, which is opposite in sign to the CP violation originating from the $K_1^0 - K_2^0$ mixing. However, this contribution cannot be particularly important, since it would lead to an inadmissibly large value of the quantity ε'/ε (see below).

The last problem on which we shall dwell is the deviation from the prediction $\varepsilon'/\varepsilon = 0$ of the superweak theory of CP violation (Refs. 76–79, 82, 64, 65, and 32). In the Kobayashi-Maskawa model, $\varepsilon' \neq 0$ on account of the direct CP-odd decays $K_2^0 \to 2\pi$ and, at the same time, the violation of the $\Delta T = \frac{1}{2}$ rule in the ordinary weak decays of K^0 mesons (see, for example, Ref. 5):

$$\varepsilon' \approx -\frac{i}{\sqrt{2}}\,\omega\xi e^{i(\delta_2-\delta_0)}, \quad \omega = \left|\frac{A_2}{A_0}\right| \approx \frac{1}{20}, \quad \xi = \frac{{\rm Im}\, A_0}{{\rm Re}\, A_0} \quad (40)$$

[δ_0 and δ_2 are the pion scattering phases in the corresponding isotopic states, and $\delta_2 - \delta_0 \approx -45°$ (Ref. 83)]. Thus, for the quantity ε'/ε we obtain the relation

$$\frac{\varepsilon'}{\varepsilon} \approx \frac{\omega}{z}\,\frac{2\xi}{({\rm Im}\, M_{\rm box}/{\rm Re}\, M_{\rm box}) - 2\xi}\, e^{i[(\delta_2-\delta_0)+\pi/4]}. \quad (41)$$

If we adopt for ξ the estimate[82]

$$\xi = H s_2 s_3 \sin\delta, \quad H \approx 1, \quad (42)$$

then ε'/ε is roughly positive ($\arg \varepsilon'/\varepsilon \approx 0$), and its absolute value $|\varepsilon'/\varepsilon|$ is about 2.5% for $m_t = 40$ GeV, 1.5% for $m_t = 55$ GeV, and 1% for $m_t = 80$ GeV. The experimental limit $|\varepsilon'/\varepsilon| \lesssim 1/50$ (Ref. 84) thus lies very close to the value predicted in the Kobayashi-Maskawa model.[10)] However, it should be stressed that the literal theoretical values of ε'/ε given above include substantial uncertainties. They are associated primarily with the estimate of the amplitude for the direct CP-odd decays, which is proportional to the matrix element $\langle \pi\pi | o_{5,6} | K^0 \rangle$ [calculations according to the bag model have given for H the values 0.25–0.5 (Ref. 79), 0.54 (Ref. 38), and 0.4 (Ref. 86)]. Moreover, appreciable uncertainties arise from the possible deviation of the model-dependent factor B from unity: $\varepsilon'/\varepsilon \sim 1/z \sim 1/B$. Finally, as we noted above, some uncertainty is associated with the inclusion of the large-distance contribution to the CP-odd amplitude for the transition $K^0 \to \bar{K}^0$. For these reasons, one often encounters in the literature the more cautious statement that $|\varepsilon'/\varepsilon| > 0.5\%$ in the Kobayashi-Maskawa model (for example, Refs. 87 and 88). Nevertheless, it can be asserted that $\varepsilon'/\varepsilon > 0$.[38,87,88] We note also that for a large t-quark mass ε'/ε depends to some extent on the specific values of the angles θ_2 and θ_3, and for a reduction of $\sin\theta_3$ in comparison with $|V_{cb}|$ [i.e., a reduction of $\Gamma(b\to u)/\Gamma(b\to c)$] the possible interval of variation of ε'/ε is shifted into the region of larger values (the estimates of ε'/ε presented above are the minimum possible values).

In Ref. 87 values of ε'/ε for various values of the parameters B, m_t, $\Gamma(b\to u)/\Gamma(b\to c)$, and τ_B, calculated in an analogous manner, were given [for H the value 0.54 (Ref. 38) was used]. We note that according to the estimates of Ref. 89 the contribution of the electromagnetic interaction to the quantity $|\varepsilon'/\varepsilon|$ could reach tenths of a percent.

Summarizing the content of this section, it should be stressed that the mass difference between the neutral K mesons can probably not be used directly to extract additional information about the quark mixing parameters, in view of the difficulty of taking into account the large-distance contribution, which evidently has a magnitude of the order of the mass difference $\Delta m_{\rm LS}$ itself. As regards the CP violation which is observed for the K^0 mesons, the situation here is somewhat better. Thus, in the framework of the explanation proposed in Ref. 63 for the $\Delta T = \frac{1}{2}$ rule in the ordinary nonleptonic decays, the large-distance effects in the CP-odd $K_1 \leftrightarrow K_2$ mixing can be taken into account, and a comparison of the theoretical estimate for the quantities η_{+-} and η_{00} with their experimental values indicates that for $\tau_B \gtrsim 10^{-12}$ sec the mass of the t quark must exceed 40–50 GeV, the phase δ must lie in the first quadrant ($\cos\delta > 0$), and $\sin\delta$ must be of the order of unity. Moreover, the predicted value of ε'/ε is close to the contemporary experimental limit. For this reason, it seems very interesting to determine more accurately, as is planned in future experiments, the quantity ε'/ε and the ratio $\Gamma(b \to u)/\Gamma(b \to c)$ on the one hand, and the mass of the t quark on the other. A reduction of the experimental uncertainties in ε'/ε and $\Gamma(b \to u)/\Gamma(b \to c)$ by a factor of several fold would provide forceful arguments in favor of the mechanism of CP nonconservation discussed here or would present the Kobayashi-Maskawa model with very serious difficulties.

As in the case of the K_L-K_S mass difference Δm_{LS}, the CP-odd effects may be strongly modified if there exist more than three generations of quarks, and also as a result of a contribution from supersymmetric particles or neutral currents with $\Delta S \neq 0$ associated with superheavy "horizontal" or "right-handed" gauge bosons and/or neutral Higgs particles having an interaction with the quarks which is nondiagonal with respect to the generations (see the preceding subsection). In the first case, in view of the weaker constraints on the mixing of the t(t') quarks, their contribution to the transition $K^0 \to \bar{K}^0$ may be much larger than in the six-quark scheme, and the observed value of ε is attainable for a relatively weak complex character of the mixing parameters. Moreover, in the case of four generations of quarks the CP violation in the decays of K^0 mesons is already described by two phases in the quark mixing matrix. It can be shown that in this case the ratio ε'/ε can have either sign, and this does not require a substantial increase of the mixing angles $V_{t(t')d}$ and $V_{t(t')s}$ in comparison with the six-quark scheme. In the absence of special cancellations in the magnitude of the CP-odd mixing ε, the ratio $|\varepsilon'/\varepsilon|$ should not exceed several percent, as before.

In supersymmetric theories, the box diagrams involving gluino exchange can give a large contribution to ε and can ensure the observed violation of CP invariance.[90-93] The value of ε'/ε predicted in these theories is, as a rule, somewhat smaller than in the standard scheme.[92,93] Such models can lead to a neutron dipole moment close to the experimental limit.[90]

Finally, CP violation with an arbitrary scale can be introduced in the K_L–K_S system by means of "horizontal" gauge bosons or superheavy scalar particles. This would simulate the superweak model of CP nonconservation[73]; in particular, $\varepsilon' \approx 0$.

CONCLUSIONS

To conclude the review, we note that measurement of the lifetime and investigation of the semileptonic decays of B mesons, together with the previously performed analysis of the decays of K mesons and hyperons as well as nuclear β decays, makes it possible to determine quite accurately the absolute values of all the elements of the quark mixing matrix in the case of three generations. Estimates of the phase δ require the additional assumption that the only source of CP nonconservation in the decays of K^0 mesons is the interaction of the W bosons with the quarks (the Kobayashi-Maskawa mechanism). In this case, δ must lie in the first quadrant, and $\sin\delta$ is found to be of the order of unity. In the framework of the six-quark scheme, the investigation of the decays of the t quark will probably give no additional information about the mixing angles. From the point of view of a more accurate determination of the angles θ_2 and θ_3, it seems most promising to perform further experiments to measure the B-meson lifetime, the fraction of semileptonic decays of the b quark into the u quark, $\Gamma_{sl}(b \to u)/\Gamma_{sl}(b \to c)$, and also to investigate the decays of charged B mesons into a $\tau\nu_\tau$ pair. It is also important to make an independent determination of all the elements of the mixing matrix, since this may give evidence for the existence of additional quark generations.

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[1)] The necessary information about the theory of the electroweak interactions, which we assume is known to the reader, is contained in the book of Ref. 3. Useful information concerning quark mixing can be found in the reviews of Refs. 4–7.

[2)] The problem of quark mixing and problems related to it are of vital importance at the present time, and the literature devoted to them is exceptionally extensive. Without aiming to give an exhaustive list of references, we quote in the references only a number of reviews and basic papers of which we have made direct use.

[3)] The authors are grateful to A. D. Dolgov and M. Yu. Khlopov for a discussion of this class of problems.

[4)] A recent direct analysis of the lepton spectrum in the semileptonic decays of B mesons reduced considerably the uncertainties in the value of z_c [$z_c(m_b/5\,\text{GeV})^5 = 0.39 \pm 0.025$ (Ref. 31)] in the framework of the model of Ref. 29. However, the actual accuracy is evidently not so high, owing to the corrections to the model itself.

[5)] We use the convention $C|K^0\rangle = |\bar{K}^0\rangle$, and therefore $CP|K^0(\mathbf{p})\rangle = -|\bar{K}^0(-\mathbf{p})\rangle$. This determines certain differences between our formulas and those of Refs. 3 and 5.

[6)] According to communications from CERN,[8] the UA1 group at the Sp$\bar{\text{p}}$S collider found the t quark with mass $m_t = 40 \pm 10$ GeV.

[7)] This fact is analogous to the well-known statement from quantum mechanics that the second-order correction to the energy of the ground state is negative.

[8)] The dipole moment of the neutron in the Kobayashi-Maskawa model is also extremely small—$D_n \sim 10^{-32}$ e · cm.[74,75]

[9)] To avoid misunderstandings, we emphasize that in this subsection the "imaginary part" will refer to the CP-odd part of the amplitude, and not the absorptive part, as in the preceding subsection, in which we neglected the violation of CP invariance.

[10)] New results on the measurement of ε'/ε were presented at the International Conference at Leipzig[85] (July 1984): $\varepsilon'/\varepsilon = -0.0046 \pm 0.0053 \pm 0.0024$ (Chicago-Stanford-Saclay Collaboration, FNAL) and $\varepsilon'/\varepsilon = 0.0017 \pm 0.0084$ (Brookhaven-Yale, BNL).

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