Cosmology, primordial black holes, and supermassive particles

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Analysis of astrophysical restrictions on the spectrum of primordial black holes (PBH) makes it possible to obtain indirect information about the physical conditions in the very early universe. These restrictions are compared with the probability of PBH production in early dust stages as predicted on the basis of modern models of quantum field theory. As a result of such comparison, restrictions are obtained on the parameters of various models corresponding to different values of the parameters of the spectrum of initial small-scale inhomogeneities.

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INTRODUCTION

The fundamental connection between elementary-particle physics and the evolution of the early universe attracts more and more attention from both elementary-particle physicists as well as astrophysicists interested in problems of cosmology. Comprehensive analysis of this connection makes it possible to construct a physically justified cosmology of the early universe. On the other hand, analysis of the cosmological consequences of elementary-particle theories provides a possibility to obtain the unique information about the properties of elementary particles and their interactions that contemporary laboratory methods are incapable of giving.[1] Indeed, taken together, the observational data on the expansion dynamics of the universe, the isotropy and spectrum of the background radiation, data on the abundance of light elements, and the observed large-scale inhomogeneity structure of the universe have made it possible to construct a quite definite qualitative picture of the evolution of the universe after the first second from the beginning of expansion.[2,3] This picture is based on physical laws reliably tested under laboratory conditions. The construction of a picture of the earlier evolution necessarily requires the modern theory of elementary particles. In the framework of such theories, which unify different types of interaction of the elementary particles (see, for example, Ref. 4), it has proved possible to develop (as yet only qualitatively) a number of nontrivial scenarios for the evolution of the early universe, these making it possible to explain the observed entropy and baryon asymmetry of the universe.[1] The mechanisms of spontaneous symmetry breaking in such theories give rise to phase transitions[5] accompanied by the production of supermassive magnetic monopoles,[6–9] and, in a number of variants of the theory, superdense walls[7,10] or strings.[7] Besides magnetic monopoles, the theories predict numerous new particles. Some of the particles are stable, and in the framework of the scenarios of the early universe one can estimate their residual concentration. Analysis of the effect of these particles on processes occurring in the universe after 1 sec such as nucleosynthesis, the formation of the spectrum of the micro-

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wave background, the formation of the observed large-scale structure, and the overall dynamics of the universe has made it possible to obtain restrictions on the numbers of species of particles,[11] their masses,[1,12-22] and residual concentrations,[1,13-22] and this has, in its turn, made it possible to obtain restrictions on the parameters of the theories themselves, i.e., the theories that predict these particles. We mention here particularly the possible connection revealed by the analysis of nonrelativistic stable particles with the development of inhomogeneities in the universe. The effects of stable particles in the development of the contemporary large-scale structure have been widely discussed[16,17,23-30] in connection with the possibility that the neutrino has a rest mass.[31] Theoretical investigation of the spatial distribution of such particles in the observed inhomogeneities of the universe at the present epoch has made it possible to obtain restrictions[14,32] on the parameters of these particles that are stronger than those obtained by analyzing their possible effect on the evolution of the universe as a whole.[18,20]

However, most of the new particles predicted by the theory are unstable, and their lifetimes are appreciably less than 1 sec. Moreover, for many of these particles the masses are predicted to be so large that their production in the late universe, i.e., later than 1 sec from the beginning of the expansion, is effectively impossible for energy reasons, and therefore they could have existed only in the very early universe.

In the epoch when the thermal energy kT of the particles appreciably exceeded their rest energy mc^2, these particles in no way differed from other species of relativistic particles as regards their effect on the dynamics of the early universe as a whole and the development of inhomogeneities in that epoch. If the lifetime of the particles appreciably exceeded the cosmological time at the period when $kT \sim mc^2$, then the role of such "long-lived" particles in the development of the inhomogeneity of the early universe could have been similar to the part played by stable particles in the development of inhomogeneities in the late universe. It is natural to consider whether, using all the astrophysical data, one can obtain information about the small-scale inhomogeneities and about the influence of the long-lived particles on the development of these inhomogeneities and, thus, make an indirect test of the theories that predict the existence of particles with given parameters (such a theory could be any of the unified gauge theories or a theory based on some as yet unknown principles). To answer this question, we consider the following scenario for the evolution of the early universe.

Suppose that in the early stages in the expansion of the universe there existed supermassive particles whose residual concentration is fairly high, so that these particles, having become nonrelativistic, begin to be dominant in the early universe. This means that in the course of its evolution the universe could have entered an early dust stage of expansion (Sec. 1). In addition, a dust stage could have been realized at the end of the inflationary stage in the course of oscillations of the classical scalar field.[33-35,22] In the early dust stages, there could be growth of initial inhomogeneities, this leading in the nonlinear stage of the development of inhomogeneity to the formation of configurations detached from the general expansion of the universe. In the course of the subsequent contraction, a small fraction of the configurations could be transformed into primordial black holes (PBH), the possible production of which in the early stages of expansion was first pointed out in Ref. 36 in connection with an analysis of the possibility of collapse of low-mass bodies.[37] A key aspect of this scenario is that the spectrum of such primordial black holes preserves a certain information about the spectrum of the initial small-scale inhomogeneities and about the properties of the supermassive metastable particles for a long time after these particles have themselves ceased to exist.

Specifying the spectrum of initial inhomogeneities, we can estimate the minimal probability of PBH production (Sec. 2). In Sec. 3, this minimal probability is compared with the observed astrophysical upper bounds on the PBH spectrum, and this leads to a number of nontrivial restrictions on the properties of the supermassive metastable particles corresponding to the choices for the initial perturbation spectrum. In particular (Sec. 4), it is possible to restrict the parameters of definite variants of unified gauge theories.

1. SUPERMASSIVE METASTABLE PARTICLES AND EARLY DUST STAGES

a) Metastable particles

As noted in the Introduction, we are interested in only the long-lived particles, i.e., the metastable particles. Metastable particles (or even particles absolutely stable with respect to decay) can appear in theories by virtue of a new approximately or strictly conserved quantum number. Magnetic monopoles provide one of the best examples.[6,7] Their absolute stability against decay follows from the strict conservation of magnetic charge. The appearance in the theory of this conserved quantum number is due to the topological structure of the gauge group that unifies electromagnetism with the remaining interactions.[6,7,38] The existence of a conserved quantum number (a generalized charge[1)]) must lead to absolute stability of the lightest of the supermassive particles which possess such a charge, as also occurs in the case of the magnetic monopoles. However, theories of such kind necessarily predict the existence of the corresponding antiparticles. Pair annihilation of the particles and antiparticles leads to a "freezing" (or "quenching") of their concentration (considered for the first time for antiprotons in Ref. 40 and for quarks in Ref. 41) and can subsequently lead to a strong decrease in the concentration of the absolutely stable particles in the inhomogeneities (see below).

Approximate conservation of a quantum number has the consequence that the supermassive particles possessing the corresponding charge are metastable with respect to decays into lighter particles that do not possess the charge. An analogous situation, albeit on different time and energy scales, arises in the framework of the proposal that the proton is unstable (or rather, metastable) due to nonrigorous conservation of baryon number.

b) Residual concentration of metastable supermassive particles

Depending on the particular properties of the particles, they could be produced under both equilibrium and nonequilibrium conditions. However, at later times, when $kT < mc^2$, the concentration of the metastable particles in the expanding universe necessarily exceeds the equilibrium concentration (corresponding to the given temperature T). In other words, sooner or later there is a freezing of the concentration of the supermassive metastable particles.

The frozen concentration n_m of the metastable particles can be conveniently characterized by the relative concentration ν, which is equal to

$$\nu = \frac{n_m}{n_r} = \frac{n_m}{\varkappa n_\gamma}, \tag{1.1}$$

where n_r and $\varkappa$ are the total concentration and number of species of relativistic particles, and n_γ is the photon concentration. The concentration ν is determined by the number $\varkappa$ of species of relativistic particles, by the cross section of pair annihilation of the metastable particles with their antiparticles, by the nature of their interaction with the remaining particles, and by the nature of their production, if the metastable particles are produced in a nonequilibrium manner.

If in the course of the production of the particles there were effects of symmetry breaking between the particles and antiparticles, ν can be determined by the excess of the particles over the antiparticles (or vice versa). According to modern ideas about the generation of baryon charge (see the review of Ref. 1), an analog of such an excess of particles over antiparticles is the baryon concentration in the contemporary universe.

c) Stages of dominance of the supermassive particles

After the freezing of the supermassive particles with mass m, the ratio of the density $\varepsilon_m = mc^2 \nu n_r$ of such particles to the density $\varepsilon_r \sim kT \cdot n_r$ of the relativistic particles increases during the subsequent expansion of the universe as mc^2/kT. Indeed, the particle number density of both relativistic and nonrelativistic particles decreases during the expansion as a^{-3}, where a is the scale factor. At the same time, the mass of a nonrelativistic particle does not change, so that their mass density decreases as a^{-3}. But the energy of each ultrarelativistic particle decreases as a^{-1} due to the red shift associated with the expansion of the universe. Therefore, the relative contribution of the nonrelativistic particles to the mean density of the universe increases as a, i.e., in inverse proportion to the temperature. At the time

$$t_0 \sim \left(\frac{m_{\rm Pl}}{\nu m}\right)^2 t_{\rm Pl} \tag{1.2}$$

(where $t_{\rm Pl} = \sqrt{G\hbar/c^5}$ and $m_{\rm Pl} = \sqrt{\hbar c/G}$ are the Planck time and Planck mass), when the temperature has fallen to

$$T_0 = \frac{\nu mc^2}{k}, \tag{1.3}$$

the stage in which the supermassive particles are dominant commences. In this stage, the effective equation of state of the universe becomes close to $p = 0$. Thus, we conclude that one of the cosmological consequences of models that predict the existence of metastable, sufficiently long-lived particles is that they have early dust stages of expansion of the universe. Posing the problem of clarifying the basic connection between these elementary-particle models and the cosmology of the early universe, we restrict ourselves to the consideration of a very approximate but simple scenario.

For simplicity, we assume that there is just one species of supermassive metastable particles of mass m with frozen concentration ν. When $t < t_0$, their contribution to the cosmological density is small, and the expansion dynamics is determined by the relativistic particles, which make the main contribution to the cosmological density. It is well known[2,3] that in this case the equation of state of the matter (i.e., the connection between the pressure and the density) has the form $p = \varepsilon/3$. At $t \sim t_0$, the contribution of the nonrelativistic particles to the cosmological density becomes comparable to the contribution of the relativistic particles, and the transition to the dust stage of expansion begins. The analogous transition to the stage in which massive neutrinos are dominant, at a much later epoch in the expansion, was investigated in detail in Ref. 24. In the process of such a transition, the pressure of the relativistic particles has less and less influence on the expansion dynamics, and to an ever greater accuracy the equation of state of the matter approaches that of a dust medium: $p = 0$. Finally, at $t \sim t_e$ the early dust stage terminates, either because of decays of the metastable particles or because of pair annihilation of such particles in inhomogeneities formed during the $p = 0$ stage. In either case, the termination of the $p = 0$ stage is accompanied by the gradual transition of the rest energy density of the nonrelativistic supermassive particles into much lighter ultrarelativistic particles. Therefore, for $t > t_e$ the matter equation of state again takes the form $p = \varepsilon/3$. Although neither the beginning nor the end of the early dust stage is instantaneous, for subsequent order-of-magnitude estimates we adopt the following simple dependence of the matter equation of state on the time:

$$p = \begin{cases} \frac{\varepsilon}{3} & \text{for} \quad t < t_0, \\ 0 & \text{for} \quad t_0 < t < t_e, \\ \frac{\varepsilon}{3} & \text{for} \quad t > t_0. \end{cases} \tag{1.4}$$

We note that if the termination of the dust stage is due to the annihilation of supermassive particles formed in the $p = 0$ stage in gravitationally bound structures, alternations of dust stages can be expected. The point is that a certain fraction of the particles does not enter such configurations and persists in the universe at the termination of the early dust stage. Despite the fact that these particles initially form a small fraction of the annihilated particles, they can again begin to be dominant in the cosmological density in the subsequent expansion of the universe. An alternation of dust stages is also possible in the case of several species of supermassive metastable particles with different νm.

A very important consequence of the existence of sufficiently long early dust stages is the growth of small initial perturbations, as a result of which primordial black holes can be formed. Their properties and the conditions of their formation from small initial perturbations are discussed in the following section.

2. PRIMORDIAL BLACK HOLES AND INHOMOGENEITY OF THE EARLY UNIVERSE

a) PBH properties

As long ago as 1794, Laplace[42] pointed out the possible existence in the universe of objects with such a strong gravitational field that the parabolic velocity at the surface of such objects exceeds the velocity of light, so that even light cannot overcome their gravitational attraction. However, prior to the creation by Einstein of the general theory of relativity this prediction of Laplace remained only an inspired guess. The present history of the theoretical investigation of black holes begins with the work of Schwarzschild,[43] who, immediately after the publication by Einstein of the gravitational field equations in 1916, obtained the first exact solutions of these equations in the spherically symmetric case. The solution contained a free parameter, subsequently interpreted as the mass of a "black hole." We shall not dwell here on the properties of the Schwarzschild solution, nor its generalizations to the cases of charged[44] and rotating[45] black holes but recommend to the interested reader the books and the detailed review of Ref. 46. We merely mention that in all the solutions that describe the gravitational field of a black hole the mass of the hole itself is arbitrary, i.e., it is, as we have already said, a free parameter of the solution. An important stage in the development of the theory of black holes was the paper of Oppenheimer and Volkoff[47] and also Oppenheimer and Snyder,[48] who obtained a theoretical bound on the masses of black holes formed in the final stage of stellar evolution. It was found that black holes can be formed only from stars whose masses exceed $(2.5–3)M_{\odot}$. However, in 1962 Zel'dovich[37] pointed out that a gravitationally bound configuration of arbitrary mass is metastable with respect to the formation of a black hole, i.e., in principle black holes can be formed from objects of any mass. For this, it is necessary to overcome the energy barrier due to the matter pressure gradients. The height of this barrier for an object of a given mass depends strongly on its initial density. The limiting mass obtained in Refs. 47 and 48 corresponded to the disappearance of this barrier at the nuclear densities attained during stellar evolution.

If a black hole is to be formed from a less massive body, it must be compressed to a higher density by applying an external pressure. To each preassigned mass there corresponds a characteristic radius

$$r_g = \frac{2GM}{c^2}, \tag{2.1}$$

the mass contracting spontaneously under the influence of self-gravitation forces once this is reached. In principle, if a fluctuation were to occur resulting in a body of mass M contracting to density

$$\rho(M) \sim \frac{M}{r_g^3}, \tag{2.2}$$

this would result in the subsequent formation of a black hole. However, in the contemporary universe the probability of such a fluctuation is exponentially small.

In 1966, Zel'dovich and Novikov[36] (and then, in 1971, Hawking[49]) pointed out the possible formation of so-called primordial black holes of arbitrary mass in the early universe.

The process of PBH formation in the early universe can be represented as follows. Taking as the zero-order approximation a homogeneous and isotropic Friedmann model,[2] one can describe the space-time of the universe by the metric tensor $g_{ik} = g_{ik}^{(0)} + h_{ik}$, where $g_{ik}^{(0)}$ is the unperturbed tensor corresponding to the Friedmann model, and h_{ik} are the perturbations of the metric. Suppose that in some region of space a perturbation of the metric is initially of order 1. At sufficiently early times, this region is not causally connected; for only a region whose diameter is less than ct can be causally connected at the time t after the beginning of the expansion of the universe. The region with such a characteristic diameter is called an horizon in cosmology. At a certain time t_h, a perturbation is within the horizon, and, beginning at this time, we are justified in regarding the region encompassed by the perturbation as an object with respect to an external observer (it is only when $t > t_h$ that a signal can reach an observer outside this region). The perturbation of the metric in the region under consideration means essentially that the rate of the expansion of the region differs from the expansion rate of the remaining universe. If the amplitude of the perturbation of the metric is of order 1, then during time $t \sim t_h$ the expansion of the matter in the perturbed region is replaced by contraction. Can pressure forces prevent this contraction? To answer this question, it is necessary to compare the density in the given configuration at the time $t \sim t_h$ with the characteristic density of gravitational self-closure of an object, which is given by (2.2). In the course of the expansion of the universe, the matter density varies in accordance with the law[2]

$$\rho(t) \propto (Gt^2)^{-1} \tag{2.3}$$

(only the numerical factor, which we do not give here explicitly,[2] depends on the equation of state). The mass of the matter enclosed in the considered region at the time t_h is

$$M = \rho(t_h)(ct_h)^3 = \frac{c^3 t_h}{G}. \tag{2.4}$$

Then the gravitational radius r_g in the region is

$$r_g \sim \frac{GM}{c^2} \sim ct_h \tag{2.5}$$

and it follows from (2.3)–(2.5) that $\rho(t_h) \sim M/r_g^3$. This means that no pressure gradients can halt the contraction. In other words, the region comes within the horizon already in the form of a black hole. But if the perturbations of the metric are much less than 1, then the replacement of expansion by contraction cannot occur earlier than a time $t \gg t_h$. During this time, the pressure gradients could prevent contraction.

As is well known from Lifshitz's paper[50] on the evolution of small perturbations of the metric in a homogeneous isotropic universe, perturbations of the metric remain constant in a stage of expansion with matter equation of state $p = \varepsilon/3$. Therefore, in this stage the formation of a black hole, which presupposes large perturbations $h \sim 1$ of the metric, is possible only if the perturbations of the metric on this scale were initially large. The situation is different in a $p = 0$ stage, in which, as was shown in the same Ref. 50, it is possible for there to be growth of perturbations of the metric and the density which has the consequence that the density needed for the formation of a black hole of mass M is attained appreciably later than the time t_h,[51–54] in fact, already in the nonlinear stage of the growth of the perturbations. We shall consider in detail the possible formation of black holes in the dust stage of expansion in Sec. 2. Here, we already consider two important aspects of the subsequent fate of the primordial black holes formed in the $p = \varepsilon/3$ stage. First, the mass of an already formed black hole can increase through accretion, i.e., through surrounding matter and radiation falling into it. Since at the time of its formation (in the $p = \varepsilon/3$ stage) the characteristic diameter of the black hole is comparable with the scale of the cosmological horizon, the question naturally arises of whether it is possible to have an accretion regime (a regime of catastrophic accretion) in which the mass of the black hole is always of the order of the mass within the cosmological horizon. As is noted in the book of Ref. 55, to answer this question it is not sufficient to use order-of-magnitude estimates, since the answer to this question depends critically on how close the mass of the black hole that is formed is to the mass enclosed within the cosmological horizon at this time. Therefore, to clarify the possibility of catastrophic accretion, attempts were made,[56] on the one hand, to find an exact self-preserving solution to the problem and, on the other, to make numerical calculations[57] of the formation and subsequent evolution of the primordial black holes in the $p = \varepsilon/3$ stage with allowance for the pressure gradients that then occur. Both approaches led to the conclusion that there is no catastrophic accretion, and this means that the PBH mass cannot increase significantly during the expansion of the universe. We mention in passing that the problem of catastrophic accretion does not arise in the case of PBH formation in a dust stage,[53] since the mass of such black holes is from the beginning, i.e., from the time of their formation, much less than the mass within the cosmological horizon.[2)]

However, even if the accretion of matter to the PBH is slight, their relative contribution to the cosmological density increases during the expansion of the universe. Indeed, as for all nonrelativistic particles (see above), the PBH number density decreases during the expansion as $1/a^3$, where a is the scale factor. Their contribution to the cosmological density decreases in accordance with the same law, since the mass of each primordial black hole remains unchanged. At the same time, the number density of the ultrarelativistic particles, which are dominant in the $p = \varepsilon/3$ stage, also decreases as $1/a^3$, but because of the red shift the energy of each particle decreases as a^{-1}. Therefore, the relative contribution of the primordial black holes to the cosmological density increases with the time as a. This makes it possible to put strong restrictions on the fraction $\beta(M)$ of matter in the primordial black holes in the early stages of the expansion of the universe by using the fact that in the contemporary universe there are no direct astrophysical manifestations of such black holes. Indeed, regarding them as one of the forms of the so-called hidden mass, one can use a simple universal restriction on the contemporary density of the hidden mass, this restriction being independent of its physical nature. The obvious physical argument given in Ref. 12 leads to the following: The cosmological density depends on the time that has elapsed since the beginning of the expansion in accordance with the law (2.3). The age of the universe certainly exceeds the age of the Earth. And this means that we have a restriction on the total contemporary density of the universe and, therefore, on the density of the hidden mass. It is found that this density cannot exceed by more than 100 times the density of the visible matter. And since as we have already said, the relative PBH contribution to the cosmological density in the $p = \varepsilon/3$ stage increased, this restriction bounds more strongly the fracion β of matter than went into the primordial black holes at the time of their formation the earlier this occurred. It is clear that the PBH are formed earlier, the lower their mass. Therefore, the restrictions on $\beta(M)$ are stronger the lower the PBH mass.

In 1974, Hawking[61] discovered theoretically the process of black hole evaporation, the essence of which is that in the strong gravitational field of a black hole there is creation from the vacuum of particles which carry away the mass of the black hole to infinity (for more details, see the review of Ref. 46).

We should here mention two important circumstances that make it possible to understand the Hawking effect.[61] First, it follows from the energy conservation law that for the production of particles with energy ε from the vacuum in a gravitational field it is necessary to have the existence of two points in space for which the difference of the gravitational potentials between them satisfies the inequality

$$\frac{\varepsilon}{c^2}\Delta\varphi \geqslant \varepsilon \tag{2.6}$$

(this should be compared with the analogous condition for particle production in an electric field: $e\Delta\varphi_{el} \geqslant \varepsilon$). It follows from (2.6) that $\Delta\varphi \geqslant c^2$. But this condition, in its turn, presupposes the presence of an horizon: either an event horizon in the field of a black hole or a cosmological horizon. (For processes of particle production by the gravitational field in cosmology, see Ref. 62 and for "horizon evaporation" see Ref. 63.)

On the other hand, the gravitational field within the event horizon of a black hole is essentially nonstationary.[64] Because of the nonstationarity of the gravitational field below the event horizon and the fact that the particle production process is not localized, a consequence of the uncertainty principle, particles are produced near the black hole horizon to which wavelengths of order r_g correspond. One of the particles escapes to infinity, carrying the energy of the black hole. The rate of energy loss due to particles escaping to infinity is determined by the gravitational radius of the black hole and corresponds to the radiation of a black body with a temperature $T \propto 1/r_g$. The characteristic time of total mass loss by the black hole, i.e., its evaporation time, is in accordance with Ref. 61

$$t_{ev} \sim \left(\frac{M}{m_{Pl}}\right)^3 t_{Pl}. \tag{2.7}$$

It can be seen from (2.7) that for black holes formed during stellar evolution the characteristic time of evaporation is appreciably greater than the age of the universe, and it is only for primordial black holes of very low masses that this evaporation process is important. During the lifetime of the universe, a PBH with mass $M < 10^{15}$ g has sufficient time to evaporate. Such low-mass black holes cannot contribute to the hidden mass, and we cannot use the above restriction on the fraction of matter that went into these PBH at the time of their formation. However, restrictions on the PBH spectrum (and this is the fraction of matter that went into the PBH at the time of their formation) can be obtained by taking into account the influence of the evaporation products of these PBH on various astrophysical processes (see the reviews of Refs. 65 and 66). Restrictions on the PBH spectrum give important information about the spectrum of the initial perturbations of the metric and density in the early universe. If, as is generally assumed, the rms perturbations of the metric are small in the $p = \varepsilon/3$ stage (radiation-dominated stage), primordial black holes arise from metric perturbations ~ 1, which correspond to the exponential tails of the Gaussian amplitude distribution.[59,60,65,67] If one assumes, which is very natural,[30,68] that the spectrum of the initial perturbations of the metric is flat, i.e., the perturbations of the metric are the same in all mass scales, then from the restrictions obtained from the observed isotropy of the microwave background (see, for example, Refs. 30, 69, and 70), the perturbations of the metric and the density perturbations $\delta\rho/\rho$ associated with them are small at all scales ($h \sim \delta\rho/\rho \lesssim 10^{-4}$). At the same time, as was noted above, the time of formation of a primordial black hole is uniquely determined[36] by its mass M:

$$t_{\text{for}} \sim t_{\text{h}} \sim \frac{M}{m_{\text{Pl}}} t_{\text{Pl}}. \tag{2.8}$$

From the time t_{for} and to the time of evaporation t_{ev} [see (2.7) and (2.8)],

$$t_{\text{ev}} = t_{\text{for}} \left(\frac{t_{\text{for}}}{t_{\text{Pl}}}\right)^2, \tag{2.9}$$

if the PBH of mass M is evaporated during the $p = \varepsilon/3$ stage, or to the end t_* of the radiation-dominated stage, the PBH contribution to the cosmological density increases as $a \propto t^{1/2}$.[2] Therefore, the connection between the fraction $\beta(M)$ of the matter that went into primordial black holes of mass M at the time of their formation and the contribution $\alpha(M)$ of such PBH to the total density of the universe at their evaporation, if $M < 10^{15}$ g, or to the contemporary cosmological density if $M > 10^{15}$ g, has the form

$$\frac{\alpha}{\beta} = \sqrt{\min\{t_{\text{ev}}, t_*\} t_{\text{for}}^{-1}}. \tag{2.10}$$

Hence, taking into account (2.7)–(2.9), we obtain an explicit dependence of the ratio α/β on the PBH mass M (see curve A in Fig. 1):

$$\frac{\alpha(M)}{\beta(M)} = \begin{cases} \dfrac{M}{m_{\text{Pl}}} \min\left\{1, \left(\dfrac{M_{**}}{M}\right)^{3/2}\right\} & \text{for } M < M_*, \\ 1 & \text{for } M > M_*, \end{cases} \tag{2.11}$$

where $M_* = m_{\text{Pl}}(t_*/t_{\text{Pl}}) \sim 10^{15} M_\odot$, and M_{**} is the mass of the primordial black holes that evaporate at the time t_* of the end of the radiation-dominated stage:

$$M_{**} \sim m_{\text{Pl}} \left(\frac{t_*}{t_{\text{Pl}}}\right)^{1/3} \sim 10^{13}\ \text{h}. \tag{2.12}$$

It is obvious that for $M > M_*$, i.e., for primordial black holes formed at the end of the radiation-dominated stage, the relative contribution to the total density does not increase, i.e., $\alpha/\beta = 1$.

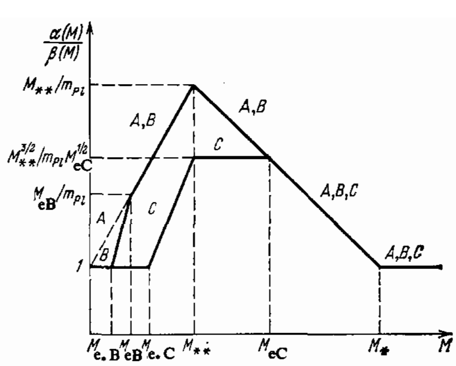

FIG. 1. Connection between $\alpha(M)$ and $\beta(M)$ (in a logarithmic scale). The line A is without allowance for early dust stages; the lines B and C are with allowance for such stages, B corresponding to a stage with $t_e < 10$ sec, C to a stage with $t_e > 10^{-25}$ sec. $M_* \sim 10^{15} M_\odot$ is the mass below the horizon at the time t_* at which the radiation-dominated stage ends; $M_{**} \sim 10^{13}$ g, which is equal to the mass within the cosmological horizon at the time $t \sim 10^{-25}$ sec, the mass of a primordial black hole that evaporates at the time t_*; M_{eB} and M_{eC} are the masses below the horizon at the time at which the early dust stage ends in cases B and C; $M_{e.B}$ and $M_{e.C}$ are the masses of PBH evaporate by the time t_e in cases B and C, respectively.

The relation (2.11) must be modified in the presence of early dust stages. Such stages [see (1.3)] can arise because metastable supermassive particles are dominant in the universe. In principle, dust stages could also arise through dominance of previously formed low-mass primordial black holes,[71,72] for example, through large-amplitude random deviations[67,72] in a spatial distribution of inhomogeneities with low dispersion.

b) PBH production and early dust stages

We now consider the qualitative Fig. 2, which illustrates the connection between t_{for} and the PBH mass M in the presence of a dust stage. We recall that to each mass M there corresponds a time t_{h} at which the mass M_{h} within the cosmological horizon is equal to M (line 1 in Fig. 2). This time t_{h} is a lower bound for t_{for}: $t_{\text{for}} > t_{\text{h}}$, since the concept of a black hole is meaningless when $M > M_{\text{h}}$. On the other hand, the PBH mass at the time of formation of the PBH must exceed the so-called Jeans mass $M_J \sim \rho(c_{\text{ac}} t)^3$, where c_{ac} is the speed of sound; M_J is the mass contained within the region through which a sound wave can pass during the cosmological time, and therefore the physical meaning of the Jeans mass is that it is the minimal mass for which pressure forces do not yet prevent the development of gravitational instability. In a $p = \varepsilon/3$ stage, M_J is somewhat less (since $c_{\text{av}} = c/\sqrt{3}$) but of the same order as M_{h} (line 2 in Fig. 2): $M_J = 3^{-3/2} M_{\text{h}}$, and PBH production is possible only in the small region I bounded by lines 1 and 2, so that in this case the relation (2.8) can be used for order-of-magnitude estimates.

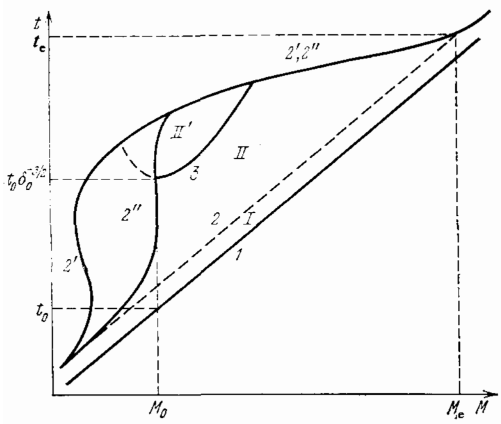

FIG. 2. Relationship between the time of formation of a primordial black hole and its mass: 1) relation between the mass M of a configuration and the time $t_h(M)$ at which it comes within the horizon; 2), 2') and 2") the relationship between the Jeans mass M_J and the cosmological time: 2, in the absence of an early dust stage; 2', with allowance for an early dust stage in which supermassive particles that do not interact with ultrarelativistic matter are dominant; and 2", with allowance for an early dust stage of dominance of supermassive particles that interact effectively (are coupled) to the ultrarelativistic particles. 3) The beginning of the nonlinear stage in the evolution of inhomogeneities of mass M. Region I corresponds to formation of PBH from metric perturbations $h \sim 1$; regions II and II' correspond to the formation of PBH with $h < 1$ (see the text).

We recall that the formation of a black hole automatically presupposes that the perturbation h of the metric in the configuration from which the PBH is formed is of order 1. It is also known that for $t < t_h$ a metric perturbation on the scale M remains constant.[50] It is from this that one can conclude that PBH production in the $p = \varepsilon/3$ stage (region 1 in Fig. 2) takes place in the exponential tail of the Gaussian distribution of the metric perturbations with respect to the amplitude h.[65] If the dispersion of the metric perturbations satisfies $\delta(M) < 1$, then the probability of realization of a configuration with $h \sim 1$ is

$$P(h) \sim \delta^{-1}(M) \exp\left(-\frac{h^2}{\delta^2(M)}\right). \tag{2.13}$$

For a detailed discussion, see Refs. 59, 60, 67, and 72. (For the hydrodynamics of PBH formation with $h \sim 1$ in the $p = \varepsilon/3$ stage, see Ref. 57.)

The situation is quite different in the $p = 0$ stage.[51–54] Beginning from the time t_0 and until t_e, the Jeans mass M_J is much less than the mass within the cosmological horizon (curves 2' and 2" in Fig. 2). For $t > t_0$, M_J is determined by the properties of the particles dominant in the $p = 0$ stage. If the supermassive particles dominant when $t > t_0$ do not interact with the matter, then the variation of M_J is shown schematically by curve 2'. But if the supermassive particles interact with the relativistic particles and the radiation, then in the stage $p = 0$ the dependence of M_J on the time is represented by curve 2". Because $M_J \ll M_h$, PBH formation from configurations with $h < 1$ is possible when $p = 0$ (regions II and II' in Fig. 2). Indeed, a configuration with initial metric perturbation $h < 1$ at the time when its diameter is comparable with the horizon has, as in the case $p = \varepsilon/3$, density contrast $\delta\rho/\rho \sim h$. But $\delta\rho/\rho$ increases in accordance with the law $\delta\rho/\rho \propto (t/t_0)^{2/3}$. At the time

$$t_1 \sim t_0 h^{-3/2}, \tag{2.14}$$

when $\delta\rho/\rho \sim 1$, such a configuration enters the nonlinear stage in the development of perturbations, and the configuration becomes detached from the cosmological expansion. In the course of the subsequent contraction, such a configuration becomes with a certain probability a PBH at a time $t_{for} \gtrsim t_1(M, h)$.

Following Refs. 51–54, we estimate the minimal probability of PBH formation in the nonlinear stage in the development of inhomogeneities in the epoch in which the supermassive particles are dominant.

At the time t_1 of the beginning of the contraction, the density distribution of the configuration and its shape are characterized by the following quantities: a) the mean density ρ_1, equal in order of magnitude to the mean density of the universe at the time t_1; b) the characteristic radius r_1 of the configuration: $M = (4\pi/3)\rho_1 r_1^3$; c) the degree of deviation of the configuration from spherical symmetry s: $s = \max\{|\gamma_1 - \gamma_2|, |\gamma_1 - \gamma_3|, |\gamma_2 - \gamma_3|\}$, where γ_1, γ_2, and γ_3 correspond to the principal values of the deformation tensor of the configuration; d) the inhomogeneity u of the density distribution within the configuration itself: $u \approx (\rho_{c1} - \rho_1)/\rho_1$, where ρ_{c1} is the density in the center of the configuration at the time t_1.

The formation of a PBH means that in the course of the nonlinear stage the configuration has contracted at the time t_{for} to $\sim r_g$, where r_g is the gravitational radius of the configuration. The mean density ρ_{PBH} of the configuration at the time t_{for} must be

$$\rho_{PBH} \sim \frac{M}{(4\pi/3)\, r_g^3} \sim \rho_1 x^{-3}, \tag{2.15}$$

where

$$x = \frac{r_g}{r_1}. \tag{2.16}$$

On the other hand, the maximal density attained in contraction in which the configuration remains nearly spherical in shape is

$$\rho_{max} \sim \rho_1 s^{-3}. \tag{2.17}$$

The estimate (2.17) follows from the fact that the minimal configuration radius is $r_{min} > \min\{t_{con}, \Delta v, \Delta r\}$, where t_{con} is the characteristic contraction time of the configuration, $\Delta v \lesssim s r_1/t_{con}$ is the characteristic difference between the initial contraction velocities along different axes, and $\Delta r \lesssim s r_1$ is the characteristic initial oblateness of the configuration.

It can be concluded from (2.16) and (2.17) that for PBH formation the configuration must be almost spherically symmetric:

$$s \leqslant x \lessdot 1. \tag{2.18}$$

For $s \gtrsim x$, i.e., for the absolute majority of the configurations, primordial black holes are not formed but instead structures with small gravitational potentials (for example, strongly nonspherical "pancakes"[2,30]).

At the time when the density ρ_{PHB} is reached, the equation of state in the configuration may again become $p = \varepsilon/3$. But if the inhomogeneity in the configuration at the time t_{for} is sufficiently small,

$$\frac{\delta\rho_{PBH}}{\rho_{PBH}} < 1, \tag{2.19}$$

then the pressure forces acting on unit volume of the matter, $f_p \sim \text{grad}\, p \lesssim (\delta\rho/r_g)c^2$, do not exceed the corresponding gravitational forces, $f_h \sim (GM/r_g^2)\rho \sim \rho c^2/r_g$ and certainly cannot prevent PBH formation.

If the particles do not interact with one another and the radiation, then a change in the equation of state need not occur on the contraction to the density ρ_{PBH}. In this case, a primordial black hole is certainly formed if the configuration can contract to r_g earlier than the density at the center formally becomes infinite due to the self-intersection of the layers of radially freely falling particles (formation of a so-called caustic):

$$t_{for} < t_{cau}. \tag{2.20}$$

Otherwise, passing through the center, the particles may again fly apart from the central region, thus preventing PBH formation.

If by the time t_{for} the configuration has succeeded in fragmenting into weakly interacting "clumps" with characteristic mass $\Delta M < \delta \cdot M$, then even with allowance for the velocity dispersion of such clumps the condition certainly sufficient for PBH formation again reduces to the relation (2.20).

The contraction of an almost spherical dust ($p = 0$) configuration can be described approximately by the Tolman solution (see, for example, Ref. 73). It is shown in the Appendix that the conditions (2.19) and (2.20) reduce to one and the same restriction on the inhomogeneity of the configuration at the time of commencement t_1 of contraction:

$$u \lesssim x^{3/2}. \tag{2.21}$$

We estimate the probability W_u that the configuration has a degree of inhomogeneity u corresponding to the inequality (2.21). In accordance with Ref. 2, the quantity r_*, which characterizes in Eq. (A5) of the Appendix the inhomogeneity of the density distribution within the configuration, is uniquely determined by the actual spectrum of the initial perturbations.

We suppose that up to a numerical coefficient of order unity $\sqrt{\langle r_*^2 \rangle} \sim r_1$. Then for practically any law of distribution with respect to u the probability of formation of configurations with anomalously small u satisfying (2.21) is in order of magnitude

$$W_u \sim x^{3/2}. \tag{2.22}$$

(For example, for the normal distribution law with dispersion $\sigma \sim 1$, which follows from the normalization of u, $W_u \sim \int_0^{x^{3/2}} e^{-u^2}\, du \sim x^{3/2}$.)

The assumption that $\sqrt{\langle r_*^2 \rangle} \sim r_1$ is entirely natural for configurations on the average (after averaging over scales $r < r_1$). In other words, the influence of the effects of the "clumpiness" of the configuration are not analyzed explicitly. But configurations satisfying the condition (2.21) evidently cannot contain clumps. The question of the connection between the criterion (2.21) and the nature of the small-scale inhomogeneity within a configuration is of independent interest and requires the use of rigorous mathematical methods.

We now estimate the probability W_s that a configuration has a high degree of sphericity, satisfying the inequality (2.18). This rather subtle question was investigated by Doroshkevich.[74] Although the random components of the deformation tensor are distributed in accordance with a random law, the actual procedure of reducing the deformation tensor to principal axes is an essentially nonlinear procedure. Therefore, the law of distribution of the principal values γ_1, γ_2, γ_3 introduced previously differs from a normal law by the factor $(\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)(\gamma_1 - \gamma_3)$. With allowance for all the *numerical factors, taken from Ref. 74, we find that* (see Refs. 52–54)

$$W_s \approx 2 \cdot 10^{-2} x^5. \tag{2.23}$$

Thus, it follows from (2.22) and (2.23) that a primordial black hole is certainly formed with probability not less than

$$W_{PBH} = W_s W_u \gtrsim 2 \cdot 10^{-2} x^{13/2}. \tag{2.24}$$

Equation (2.21) determines the minimal fraction of matter that passes into PBH of mass M at the time of PBH formation.

In (2.24), the quantity x can be readily related to h:

$$x \sim \frac{GM}{c^2}\left(\frac{\rho_1}{M}\right)^{1/3} \sim \frac{G}{c^2} M^{2/3}\rho_0^{1/3}\left(\frac{t_0}{t_1}\right)^{2/3}.$$

But in accordance with (2.14),

$$x \sim \frac{GM_0^{2/3}\rho_0^{1/3}}{c^2}\, h\left(\frac{M}{M_0}\right)^{2/3},$$

where

$$M_0 = \rho_0 (ct_0)^3 \tag{2.25}$$

is the mass within the cosmological horizon at the time t_0. Since $\rho_0 \sim 1/Gt_0^2$, we finally have

$$x(M) \sim h(M)\left(\frac{M}{M_0}\right)^{2/3}. \tag{2.26}$$

The (2.24) can be rewritten as

$$W_{PBH} \gtrsim W(M, h) \approx 2 \cdot 10^{-2} h^{13/2}\left(\frac{M}{M_0}\right)^{13/3}. \tag{2.27}$$

With allowance for (2.13), (2.26), and (2.27), the fraction of matter that goes into a PBH formed from configurations of mass $M > M_0$ is

$$\beta(M) \approx 2 \cdot 10^{-2} \delta^{13/2}\left(\frac{M}{M_0}\right)^{13/3}. \tag{2.28}$$

If $t_1(M, \delta(M))$ (curve 3 in Fig. 2) is less than t_e, then the overwhelming majority of configurations with mass M enter the nonlinear stage (region II′ in Fig. 2). Therefore, the main contribution to $\beta(M)$ is made by $h \sim \delta$, so that the exponential smallness in the probability of PBH formation characteristic of the radiation-dominated stage is absent. The region II′ is bounded on the side of small masses, $M < M_0$, by curve 4, which takes into account the partial or complete damping of low-mass configurations that come through the cosmological horizon before the radiation-dominated stage.

Note also that in the $p = 0$ stage primordial black holes can be formed as a result of the nonlinear evolution of more massive configurations (for example, as a result of fragmentation), or during clustering of less massive isolated configurations (or already formed PBH of lesser mass).

c) Influence of the dust stages on the connection between $\alpha(M)$ and $\beta(M)$

If in the early universe when $t < t_*$ (i.e., before the end of the radiation-dominated stage in the standard hot model) there is realized a dust stage, which begins at t_0 and ends at t_e, the relative PBH contribution to the cosmological density does not increase during the period $t_0 < t < t_e$. In the presence of such a stage, the dependence of t_{for} on M is no longer so simple as (2.8). However, in the case of an arbitrary but single-valued connection between t_{for} and M the relation (2.10) can be generalized as follows[53]:

$$\frac{\alpha(M)}{\beta(M)} = \sqrt{\frac{t_0}{\min\{t_{for},\ t_0\}}} \times \begin{cases} 1 & \text{for } t_{ev} < t_e, \\ \sqrt{\min\{t_{ev},\ t_*\}/\max\{t_e,\ t_{for}\}} & \text{for } t_{ev} > t_e. \end{cases} \tag{2.29}$$

The relation (2.29) shows that for PBH with $t_{for} < t_e$ early dust stages, $p = 0$, have a strong influence on the ratio of α to β. For $t_{for} > t_e$, the relation (2.29) goes over into (2.10).

For PBH formed in an early dust stage and evaporating after its end (and it is only such PBH that are of particular interest to us), i.e., for $t_0 < t_{for} < t_e$ and $t_{ev} > t_e$, we find that

$$\beta(M) = \alpha(M) \sqrt{\frac{t_e}{\min\{t_{ev},\ t_*\}}}. \tag{2.30}$$

Taking into account (2.10), which expresses the analogous connection between $\beta(M)$ and $\alpha(M)$ in the absence of early dust stages, and also (2.8), we can rewrite (2.30) in the following form, which is convenient for discussion:

$$\beta(M) = \beta_0(M)\left(\frac{t_e}{t_{Pl}}\right)^{1/2}\left(\frac{M}{M_{Pl}}\right)^{-1/2}, \tag{2.31}$$

where $\beta_0(M)$ is the fraction of the matter that goes over into PBH of mass M for given $\alpha(M)$ in the absence of an early dust stage.

In Sec. 2b we saw that $\beta(M)$, defined earlier as the fraction of matter that goes into PBH with mass M at the time of their formation, loses its meaning in the presence of an early dust stage, since, in contrast to the case of a continuous radiation-dominated stage, $\beta(M)$ depends now not only on the PBH mass but also on the initial characteristics of the configurations from which the PBH were formed, and also on the actual mechanism of their formation. However, as follows from (2.29), for PBH formed in the early dust stage the time t_{for} does not occur explicitly in the relationship between $\alpha(M)$ and $\beta(\mu)$. Therefore, it is expedient to redefine $\beta(M)$ as follows: For $M < M_e$, i.e., the horizon mass at the time t_e, $\beta(M)$ is the fraction of matter in PBH with mass M at the time t_e of the ending of the early dust stage, while for $M > M_e$ the quantity $\beta(M)$ has its previous meaning, i.e., $\beta(M)$ is the fraction of the matter that goes into the PBH with mass M at the time of their formation.

We can now express explicitly the ratio of $\alpha(M)$ to the redefined $\beta(M)$ as a function of M and a unique parameter M_e of the dust stage (curve B or C in Fig. 1). We note right away that $\alpha(M)/\beta(M) \sim 1$ for PBH with $t_{ev} < t_e$, i.e., with $M < M_{e*} \equiv m_{Pl}(t_e/t_{Pl})^{1/3}$, since even if such PBH existed at time t_e they would evaporate so rapidly that their relative contribution to the density of the universe when $t > t_e$ would not be able to increase appreciably. For more massive PBH, as follows from (2.29),

$$\frac{\alpha(M)}{\beta(M)} = \begin{cases} 1 & \text{for } M < M_{e*}, \\ \frac{M}{m_{Pl}} \min\left\{1,\ \left(\frac{M_{**}}{M}\right)^{3/2}\right\} \min\left\{1,\ \left(\frac{M}{M_e}\right)^{1/2}\right\} & \text{for } M_{e*} < M < M_*, \\ 1 & \text{for } M > M_*. \end{cases} \tag{2.32}$$

It is obvious that for $M > M_*$, as before, $\alpha(M)/\beta(M) = 1$.

d) Astrophysical restrictions on the PBH spectrum

The PBH formed in the very early universe persist to later stages of the expansion; moreover, their contribution to the cosmological density in the subsequent $p = \varepsilon/3$ stage increases, and their effect on the physical processes in the universe is accordingly strengthened. As noted earlier, PBH with masses less than 10^{15}g will have evaporated by the present epoch through quantum processes in accordance with Hawking's results.[61] A correct treatment of PBH evaporation is possible only for PBH masses exceeding the Planck mass of 10^{-5} g, since for PBH with smaller masses the width with respect to evaporation is comparable to the magnitude of their mass (for the possibility of stable maximons, i.e., PBH with mass of 10^{-5} g, see Ref. 75; for PBH with mass less than 10^{-5} g, see Ref. 76). Primordial black holes with $M < 10^9$ g evaporate before 1 sec, and restrictions on such PBH can be obtained by analyzing the effect of their evaporation on the entropy of the universe.[66] In such an analysis, it is necessary to take into account not only the generation of baryon charge[77] but also the generation of additional entropy in PBH evaporation processes.[66] The evaporation of PBH with $M > 10^9$ g takes place in the period after 1 sec. If the PBH masses are in the range 10^9–10^{10} g, which corresponds to evaporation time in the interval 1–10^3 sec, the evaporation products of such black holes influence the relationship between the frozen concentrations of the protons and neutrons and, therefore, the abundance of primordial helium, so that from the observed helium abundance one can put restrictions[78,79] on the admissible value of $\alpha(M)$ in this interval of masses. And, using the observed deuterium abundance, one can restrict[79,80] the value of $\alpha(M)$ for PBH with masses 10^{10}–10^{13} g, which evaporate in the period 10^3–10^{12} sec. Analysis of recombination dynamics makes it possible to restrict $\alpha(M)$ for $M \sim 10^{13}$–10^{14} g,[81] i.e., for PBH evaporating in the epoch of the decoupling of radiation from matter. The strongest restrictions on $\alpha(M)$ are obtained for PBH

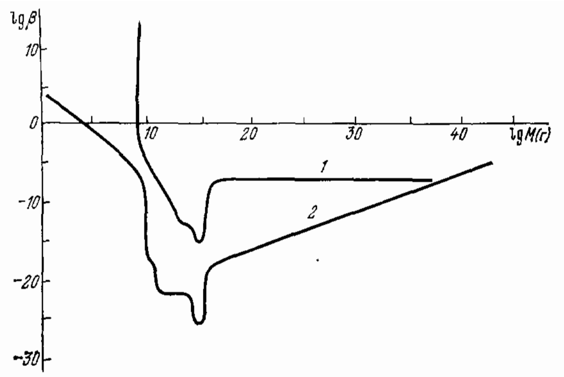

FIG. 3. Restriction on the value of $\beta(M)$: 1) with allowance for the possibility of a dust stage with $t_e \leqslant 1$ sec, 2) under the assumption that there are no early dust stages.

with masses 10^{14}–10^{15} g[82,83] from observations of the γ background and γ bursts, see Refs. 82–84, and also from analysis of synchrotron radio radiation.[85]

Primordial black holes with $M > 10^{15}$ g persist to the present time, and restrictions on their contemporary concentration can be obtained from their contribution to the cosmological density[36] or from their effect on the dynamics of superclusters[66] (for $M > 10^{15} M_\odot$) and on the large-scale structure of the universe.

The astrophysical restrictions on $\alpha(M)$ make it possible to obtain an upper bound for $\beta(M)$. In the case of a continuous radiation-dominated stage from the Planck time to the epoch near hydrogen recombination the relationship between $\alpha(M)$ and $\beta(M)$ is expressed by (2.10), and the observational restrictions on $\alpha(M)$ lead to the restrictions on $\beta(M)$ shown in Fig. 3 (curve 2). In the presence of early dust stages, the restrictions on $\beta(M)$ determined in the previous section depend strongly on the time t_e at which the early dust stage terminates. The value of t_e may depend not only on the parameters of the specific grand unification models but also on many details in the evolution of the nonlinear structures formed in the early dust stage. However, as was shown in Refs. 15 and 53, one can give an admittedly crude but quite reliable upper bound on t_e:

$$t_e \leqslant 1 \text{ Sec.} \tag{2.33}$$

This restriction is obtained from the condition that the early dust stages must end earlier than 1 sec after the beginning of the expansion of the universe in order to avoid a contradiction with the observed abundances of the light elements and the spectrum of the microwave background.

Although the restrictions on the PBH spectrum (curve 1 in Fig. 3) obtained on the basis of (2.32) and (2.33) are much weaker than the ones obtained earlier (see, for example, Refs. 66 and curve 2 in Fig. 3), they are more reliable, since they do not use the implicit assumption that early dust stages are absent.[3)]

In addition, we do not rule out the possibility that at the termination of an early $p = 0$ dust stage a baryon charge excess is generated in the universe together with additional entropy. In this case, the initial entropy and the contribution to it of the low-mass PBH that evaporate before 1 sec are "forgotten." This in fact lifts any astrophysical restrictions on $\alpha(M)$ and, therefore, on $\beta(M)$ for $M < 10^9$ g.

Thus, in the presence of early dust stages it is no longer possible in principle to eliminate a to any extent inhomogeneous expansion of the universe on scales $M < 10^9$ g.

In the framework of specific grand unification models that predict the parameters of early dust stages with $t_e \ll 1$ sec, the restrictions on the PBH spectrum are greatly strengthened compared with those given by curve 1 in Fig. 3, though they remain weaker than those corresponding to curve 2 in Fig. 3. Such models establish a connection between the parameters of the violation of CP invariance, the possible magnitude of the baryon charge excess generated in nonequilibrium processes in the early universe, and the parameters of the supermassive particles. This imposes additional restrictions on the duration of the dust stages. Indeed, if such a stage was determined by the dominance of particles of mass m with frozen relative concentration $\nu = (n_m/n_\gamma)_0$ and lasted from the time t_0 to the time t_e, and at its end the initial baryon charge excess $(n_B/n_\gamma)_0 = \nu (n_B/n_m)_0$ was augmented by the further baryon charge excess $\Delta n_B/n_m = \Delta B$, then the final ratio of the number of baryons to the number of photons is

$$\left(\frac{n_B}{n_\gamma}\right)_e = \left[\left(\frac{n_B}{n_\gamma}\right)_0 + \nu\Delta B\right]\sqrt{\frac{t_0}{t_e}}. \tag{2.34}$$

To obtain (2.34), it is sufficient to note that

$$(n_B)_e = (n_B)_0 + \Delta n_B, \quad (n_\gamma)_e = (n_\gamma)_0 + \frac{m\nu\,(n_\gamma)_0}{T_e},$$

$$T_e = T_0\sqrt{\frac{t_0}{t_e}}, \quad T_0 \sim \nu m.$$

The quantities that occur on the right-hand side of (2.34) are determined by the parameters of the specific models. It can be seen from this that if specific models are not specified it is impossible to obtain restrictions on the dust stages solely on the basis of the observed entropy of the universe.

In the framework of the specific model of asymptotically free SU(5) theory,[86] analysis of the cosmological consequences of the existence of metastable supermassive G fermions (including their effect on the entropy of the universe) makes it possible to give stringent restrictions[87] on the free parameter k_D of the theory, as we shall discuss in Sec. 4b.

In the presence of several early dust stages, we shall, since we do not have a detailed picture of their alternation with the radiation-dominated stages, understand by t_e in the relationship (2.32) obtained earlier between $\alpha(M)$ and $\beta(M)$ the time at which the final $p = 0$ stage ends.

We note also that allowance for the possibility of early dust stages can lead to a weakening of the restrictions obtained without allowance for such stages on the concentrations of not only the PBH but also of a number of particles[14,15] (for example, gravitinos[20]).

3. RESTRICTIONS ON THE PARAMETERS OF SUPERMASSIVE PARTICLES AND INHOMOGENEITY OF THE EARLY UNIVERSE

a) Connection between the PBH spectrum and the parameters of supermassive particles

In the previous chapter, we estimated the minimal probability $W_{\rm PBH}(M)$ of formation of PBH of mass M in an early dust stage. This probability depends strongly on the initial perturbation spectrum $\delta(M)$ of the metric. Suppose the perturbation spectrum of the metric has the form

$$\delta(M) = \delta_0 \left(\frac{M}{M_0}\right)^{-n}, \tag{3.1}$$

where M_0 is taken to be the mass within the cosmological horizon at the time at which the early dust stage commences. The corresponding perturbations of the density at the beginning of the dust stage have the form (see, for example, Ref. 2)

$$\frac{\delta\rho}{\rho}(M,\ t_0) = \delta(M)\begin{cases} 1, & M_{\rm min} < M < M_0, \\ \left(\frac{M}{M_0}\right)^{-2/3}, & M > M_0, \end{cases} \tag{3.2}$$

where $M_{\rm min}$ is the characteristic scale of the damping of the density perturbations. To make the results of the following restrictions on the parameters of the supermassive particles independent of the specific perturbation damping mechanisms in the $p = \varepsilon/3$ stage when $t < t_0$ (in particular, independent of the quantity $M_{\rm min}$ itself, which, however, is certainly less than M_0), we restrict the consideration to scales exceeding M_0. Then from (2.27), we have[51–54]

$$W_{\rm PBH}(M) > 2\cdot 10^{-2}\delta_0^{13/2}\left(\frac{M}{M_0}\right)^{-13n/2}. \tag{3.3}$$

This minimal estimate is valid for not too large masses $M_0 < M < M_{\rm max}$, where $M_{\rm max}$ is the maximal mass of the configurations that succeed in detaching themselves from the general cosmological expansion before the termination of the early dust stage, i.e., for which the characteristic time t_1 of the commencement of the nonlinear stage, determined by the condition

$$\frac{\delta\rho}{\rho}(M_{\rm max},\ t_0)\left(\frac{t_1}{t_0}\right)^{2/3} \sim 1, \tag{3.4}$$

is in order of magnitude equal to the time of termination of the early dust stage: $t_e \sim \tau$. From (3.1), (3.2), (3.4), (2.25), and (1.2) we obtain

$$M_{\rm max} \approx m_{\rm Pl}\left(\frac{\tau}{t_{\rm Pl}}\right)^{2/(2+3n)}\left(\frac{m_{\rm Pl}}{\nu m}\right)^{6n/(2+3n)}\delta_0^{3/(2+n)}. \tag{3.5}$$

Thus, the interval of PBH masses $(M_0, M_{\rm max})$ in which the minimal probability of their formation in the early dust stages is estimated depends strongly on the parameters τ and νm of the supermassive particles, and also on the amplitude δ_0 of the spectrum and the exponent n in the spectrum of the initial perturbations of the metric.

b) Comparison of minimal probability of PBH production with astrophysical restrictions

The essence of the restrictions on τ and νm, whose values depend on the specific elementary-particle model and are parameters of the supermassive particles, is as follows: A concrete variant of the model certainly contradicts the astrophysical restrictions on the PBH spectrum, $\beta(M)$, if

$$W_{\rm PBH}(M) > \beta(M), \tag{3.6}$$

for at least one value of M in the interval $(M_0, M_{\rm max})$ (see the schematic Fig. 4).

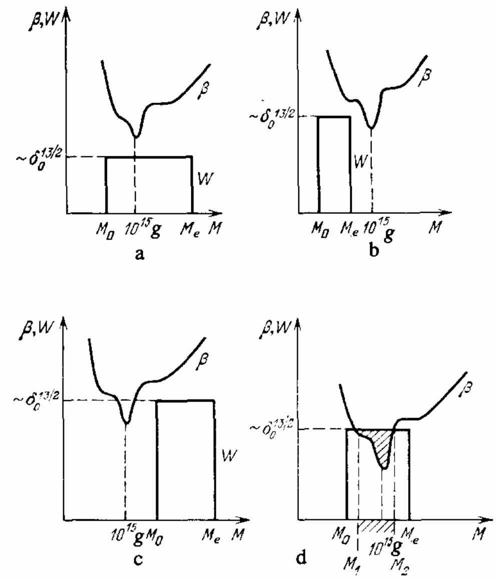

FIG. 4. Comparison of the observational upper bounds on β, the fraction of matter that passes into primordial black holes, and the minimal probability of PBH formation in early dust stages. The figure shows schematically four possible relationships between β and W. The existence of an early dust stage does not contradict the observational data if the metric perturbations are sufficiently small (case a). In this case, the beginning and end of the stage can be arbitrary. In the case of large initial perturbations of the metric, the early dust stage must either end early (so that M_e is sufficiently small—case b), or begin sufficiently late (so that M_0 is sufficiently large—case c). Thus, the restriction on the parameters of the theory of elementary particles is dictated by the condition that the situation shown in Fig. 4d, when for a certain interval (M_1, M_2) of PBH masses $W > \beta$, should not arise.

We illustrate what was said above in more detail for the case of a flat spectrum of the initial metric perturbations, i.e., by setting $n = 0$.

With allowance for (2.25), the condition $M_0 < M$ reduces to an inequality for the parameter νm of the supermassive particle:

$$\frac{\nu m}{m_{\rm Pl}} > \left(\frac{M}{m_{\rm Pl}}\right)^{-1/2}. \tag{3.7}$$

The condition $M < M_{\rm max}$ as follows from (3.5) if we set $n = 0$, can be rewritten in the form of an inequality for the other parameter τ, which characterizes the duration of the dust stage ($\tau \sim t_e$):

$$\frac{\tau}{t_{\rm Pl}} > \frac{M}{m_{\rm Pl}}\delta_0^{-3/2}. \tag{3.8}$$

As follows from (2.31), the inequality (3.6) can itself be rewritten in the form of a restriction on the parameter τ itself:

$$\frac{\tau}{t_{\rm Pl}} < \frac{M}{m_{\rm Pl}}\left[\frac{W_{\rm PBH}(M)}{\beta_0(M)}\right]^2. \tag{3.9}$$

Thus, if the theory predicts values of the parameters νm and τ of the supermassive particles for which there exists an interval of masses (M_1, M_2) $(M_1 \geqslant M_0,\ M_2 \leqslant M_e)$ in which the

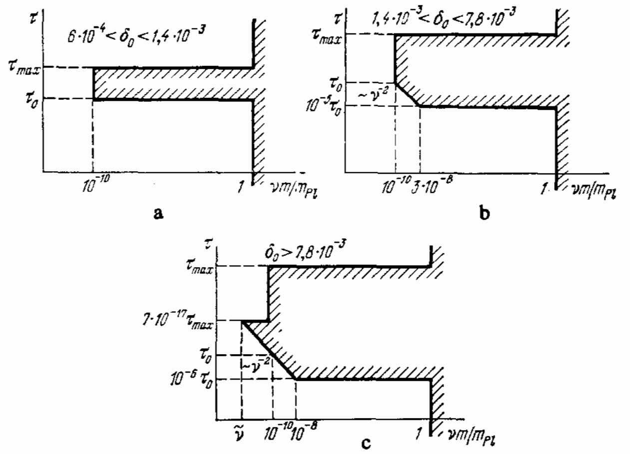

FIG. 5. Restrictions on the parameters of supermassive metastable particles (τ, νm) with allowance for the influence of early dust stages on the connection between α and β under the assumption of a flat spectrum of the initial metric perturbations. The hatched region is forbidden by the observational restrictions on the PBH spectrum.[53]

inequalities (3.7)–(3.9) are simultaneously satisfied, then such a theory contradicts the observational astrophysical restrictions on the PBH spectrum under the assumption $n = 0$.

The obtained restrictions on τ and νm are given in Fig. 5.[53]

Note that for sufficiently small δ_0 there are no restrictions on τ and νm (a corresponding mass interval does not exist, since the inequalities (3.8) and (3.9) are incompatible). As can be seen from (3.8), (3.9), and curve 2 in Fig. 3, this occurs when

$$\delta_0^{3/4} \leqslant \frac{\min \beta_0(M)}{\max W_{\mathrm{PBH}}(M)}, \tag{3.10}$$

i.e., when

$$\delta_0 \leqslant 6 \cdot 10^{-4}. \tag{3.11}$$

For $6\times10^{-4} < \delta_0 < 1.5\times10^{-3}$, the interval (M_1, M_2) reduces to a narrow region near $M \sim 10^{15}$ g ($M_1 \sim M_2 \sim 10^{15}$ g) and the forbidden (τ, νm) region is shown in Fig. 5a.

For $1.4\times10^{-3} \lesssim \delta_0 < 8\times10^{-3}$, $M_1 \sim 10^{10}$ g, and $M_2 \sim 10^{15}$ g (see Fig. 5b).

Finally, for $8\times10^{-3} \lesssim \delta_0 < 1$, $M_1 \sim 10^9$ g and $M_2 \sim 10^{15}$ g ($\delta_0 > 7.8\cdot10^{-3}$).[29,2] The corresponding forbidden region of the parameters (τ, νm) is shown in Fig. 5c.

The restrictions on νm and τ shown in Fig. 5 have a completely general nature and must be satisfied by all elementary-particle theories, in particular various forms of unified gauge theories. Therefore, the obtained restrictions not only demonstrate the basic possibility of establishing connections between the parameters of the supermassive particles and the astrophysical restrictions on the PBH spectrum but also serve as a kind of prescription for comparing elementary-particle models with astrophysical observations. Indeed, for all types of metastable particles it is in principle possible to estimate τ and ν in the framework of the corresponding unified gauge theory model, as will be demonstrated in the following section for the case of magnetic monopoles, heavy quarks, and leptons. The direct comparison of such estimates of τ and νm with the restrictions shown in Fig. 5 makes it possible to obtain nontrivial restrictions on the properties of such particles, for example, their mass.

4. RESTRICTIONS ON THE PARAMETERS OF UNIFIED GAUGE THEORIES

The principles of local gauge invariance have made it possible to describe within the framework of a unified theory the weak and electromagnetic interaction of elementary particles.[4,88,89] Such an approach has also proved fruitful for the construction of a theory of the strong interaction—quantum chromodynamics (QCD).[4,90] The theory that combines the electromagnetic and weak interaction predicts a weak logarithmic growth of the electromagnetic coupling "constant" with increasing energy, and also of the parameter $\sin^2\theta_W$ of the theory (see Ref. 4). Quantum chromodynamics predicts a logarithmic decrease, in agreement with accelerator experiments, of the fundamental strong coupling "constant" with the energy. Extrapolation of these dependences to high energies, inaccessible under contemporary laboratory conditions, identifies a characteristic energy of order $\Lambda \sim 10^{14}$–10^{16} GeV, at which the "constants" of QCD and the electromagnetic interaction become equal. It was this circumstance that laid the foundation of the present models that unify the strong, weak, and electromagnetic interactions.[4,91] Such models are in complete agreement with the experiments as regards the measurement of the parameter $\sin^2\theta_W$ of the electroweak theory (see, for example, Ref. 4). Experiments are being made to test other predictions of such models. Such experiments are the search for instability of the proton[92–94] and neutron-antineutron oscillations.[95–96] It must be emphasized that all these effects, which occur in the region of energies low compared with the unification scale, have a very low probability, being due to virtual transitions with the participation of supermassive particles with masses $m \sim \Lambda$.[4)] The characteristic lifetime τ_1 of such particles predicted by the theory is determined by the probability W_1 of direct decay of a boson into two light fermions or of a fermion into a boson and a fermion and is very small. In a system of units with $\hbar = c = 1$, in which the probability is measured in units of mass, W_1 has the form

$$W_1 = \frac{1}{\tau_1} = k_1 m, \tag{4.1}$$

where the numerical factor k_1 depends on the properties of the specific particles. However, besides short-lived particles, for which $k_1 \sim \alpha$–α^2, the theories predict several possible families of metastable particles, whose lifetimes appreciably exceed the above lifetimes of the "normal" short-lived particles (i.e., $k_1 \ll \alpha^2$). All the currently existing models are based on simple group symmetries of unified interaction and inescapably predict the existence of absolutely stable magnetic monopoles.[5)]

In addition, in a theory there may be particles for which two-particle transitions to lighter particles are forbidden for some reason. Decays of such particles can take place only through intermediate (virtual) bosons with masses $m \sim \Lambda$. Using dimensional arguments, it is easy to estimate the probability (characteristic time) of such three-particle decay:

$$W_2 = \frac{1}{\tau_2} = k_2 \frac{1}{\Lambda^4} m^5, \qquad (4.2)$$

where k_2 is a numerical factor that depends on the particular parameters of the model, for example, the statistical weight of the particles, the magnitudes of the corresponding mixing factors, the number of possible decay channels, etc. Of this kind are the following particles: a) supermassive neutral fermions, in particular the right-handed neutrinos in the O(10) models[97,99]; b) supermassive quarks, of the so-called B multiplet in the SU(5) model with asymptotic freedom. These quarks have the color properties of gluons, and for this reason their direct transitions into ordinary quarks are forbidden.

Supergravity models predict massive fermions with spin 3/2—partners of the graviton in a supersymmetric multiplet; they are called gravitinos.[39] The gravitino mass m_G is determined by the scale of the supersymmetry breaking. Estimates[20] of the gravitino decay probability lead to the value

$$W_3 = \frac{1}{\tau_3} = k_3 \frac{m_G^3}{m_{Pl}^2} \qquad (4.3)$$

with numerical factor $k_3 \lesssim 1$. An analogous dependence can also hold for stages in which a scalar field is dominant; for example, in the case of scalarons[100] one must take m_G to be the scalaron mass.

We now turn to an analysis of the cosmological consequences of the existence of these metastable particles, using the restrictions given in Sec. 3 and particularizing them for each of the particle species. For this, we take into account the interaction of such particles with one another (pair annihilation) and with ultrarelativistic, much less massive particles, and estimate in each particular case the residual concentration of the supermassive particles. This, in its turn, makes it possible to relate the parameters of the early dust stage in the expansion of the universe (t_0 and t_e) to the mass of the particles and, thus, to impose on it restrictions that depend on the amplitude of the initial perturbations of the metric.

a) Restrictions on the mass of magnetic monopoles

Unified gauge theories predict[6,38] the existence of monopoles and antimonopoles with magnetic charges $g = \pm \hbar c/2e$. An estimate of the residual concentration of the monopoles and their lifetime with respect to annihilation can be obtained[8,9] on the basis of the following physical considerations:

1) In a relativistic plasma with temperature $kT < mc^2$, the main mechanism of interaction of the monopoles with the ambient medium is multiple scattering of a monopole by particles with effective cross section (in units for which $\hbar = c = k = 1$)

$$\sigma_s \sim \frac{1}{Tm}. \qquad (4.4)$$

2) The annihilation of monopoles and antimonopoles is determined by their magnetic "Coulomb" attraction to each other.

3) In a rarefied plasma, in which the effective deceleration of the monopoles by scattering by light charged particles is unimportant, the rate of annihilation is determined by the capture of a monopole by an antimonopole with the emission of dipole radiation[8]:

$$\langle \sigma v \rangle \sim \pi v^{-9/5} \frac{g^4}{m^2}. \qquad (4.5)$$

4) In a dense plasma, the annihilation rate is determined by the rate of diffusion of a monopole to an antimonopole in the course of multiple scattering of monopoles by particles of the medium.

5) For the calculation of monopole annihilation, the diffusion approximation is valid provided the stopping length for the monopoles (antimonopoles) due to multiple scattering, $\lambda = 1/n_r\sigma_s$, is much less than the characteristic "Coulomb" radius of the monopole (antimonopole) $r_C = g^2/T$.

6) In the early universe, monopoles (and antimonopoles) are produced in the course of the phase transition from the symmetric vacuum of the unified theory to the antisymmetric vacuum with electromagnetism separated from the remaining interactions. The concentration of the produced monopoles depends strongly on the duration and nature of the phase transition.

It should be said immediately that in inflationary models such a low monopole concentration at the end of the phase transition is predicted, in accordance with Refs. 101–103, that a stage in which they are dominant is ruled out. However, in Ref. 104 there is a discussion of the possible production of an appreciable number of monopoles after the inflationary phase as well. In addition attempts have been made to solve the monopole problem without recourse to the inflationary scenario, according to which the monopole concentration is suppressed from the very beginning. For example, in Ref. 105 there is a discussion of the possibility of annihilation of monopoles in the stage of their confinement in a comparatively late epoch in the expansion of the universe corresponding to $10^{13} \gtrsim T \gtrsim 10^2$ GeV. According to Ref. 105, the monopole annihilation in this period must take place in two stages, an appreciable fraction of the monopoles being annihilated only in the second stage, at $T \sim 10^2$ GeV, after the phase transition with the breaking of the Weinberg-Salam symmetry. Thus, this mechanism[105] ensures the absence of an appreciable number of monopoles in the universe when $T < 10^2$ GeV but does not rule out a high concentration of them in the period corresponding to $T > 10^2$ GeV. It is therefore of interest to obtain an independent restriction on the mass of the monopoles under the assumption that their initial concentration is fairly high and the mechanism of diffusion annihilation fairly effective. In this case, the frozen concentration does not depend on the magnitude of the initial concentration and is determined solely by the mass and charge of the monopole.

With allowance for all that was said above under points 1)–6), the residual concentration is in order of magnitude[8,9]

$$\nu \sim \frac{m}{m_{Pl} g^5} \sim 10^{-9} \frac{m}{10^{16}\ \text{GeV}} \left(\frac{137}{g}\right)^5, \qquad (4.6)$$

so that

$$\nu m \approx 10^7 \left(\frac{m}{10^{16}\ \text{GeV}}\right)^2 \text{GeV}. \qquad (4.7)$$

The monopoles are absolutely stable with respect to decay, and the ending of the early dust stage of their dominance can be due solely to their annihilation with antimonopoles in

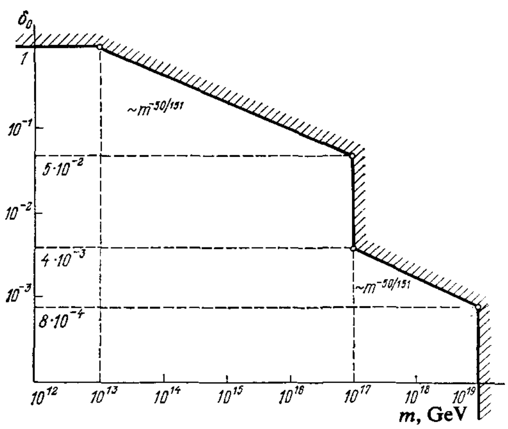

FIG. 6. Restrictions on the mass of magnetic monopoles for different values of δ_0.

configurations that become isolated in this stage. The characteristic time of such annihilation is

$$\tau \approx (n_m \langle \sigma v \rangle)^{-1}, \tag{4.8}$$

where n_m is the characteristic density of the monopoles in such configurations, and $\langle \sigma v \rangle$ is the mean rate of annihilation. In order of magnitude, n_m corresponds to the cosmological density at the time t_1 of commencement of the nonlinear stage for configurations of mass M_0. With allowance for (4.6) and (4.7) we have

$$n_m \sim \frac{\rho}{m}\Big|_{t=t_1} \sim \delta_0^3 \cdot 10^{53} \left(\frac{m}{10^{16}\ \text{GeV}}\right)^7. \tag{4.9}$$

We assume that the characteristic velocity of the monopoles in a configuration is of the order of the virial velocity, $v \sim \sqrt{GM/r_1}$. Then with allowance for (2.26) with $M = M_0$ and $h = \delta_0$ we have

$$v \sim c\delta_0^{1/2}. \tag{4.10}$$

With allowance for (4.10), the monopole annihilation rate (4.5) is in order of magnitude

$$\langle \sigma v \rangle \sim \frac{g^4}{m^2} \delta_0^{-9/10}. \tag{4.11}$$

From (4.9)–(4.11), we obtain

$$\tau \sim 10^{-8} \sec \left(\frac{10^{16}\ \text{GeV}}{m}\right)^5 \delta_0^{-21/10}. \tag{4.12}$$

The restrictions on the monopole mass obtained from Fig. 5 with allowance for (4.7) and (4.12) for different values of δ_0 rule out the possibility of existence of monopoles with $m > 10^{17}$ GeV for $\delta_0 > 2.5 \times 10^{-3}$ (Fig. 6). This restriction is weaker than the one obtained in Refs. 51 and 52 without allowance for the weakening of the restrictions on $\beta(M)$ in the presence of early dust stages. It should however be noted that in principle one could strengthen this restriction by additionally taking into account PBH production during the subsequent evolution of the isolated configurations that do not belong to the considered class of specially spherically symmetric and homogeneous configurations. A quantitative analysis of such formation of primordial black holes has not as yet been made; for a qualitative discussion, see Refs. 106 and 87.

b) Restrictions on the parameters of asymptotically free SU(5) theory

When one speaks of asymptotic freedom in quantum field theory, one has in mind logarithmic decrease of the coupling constants at high energies. In models of unified gauge theories, asymptotic freedom is realized only if one requires a definite symmetry between the fermion and bosom multiplets, and also the fulfillment of numerous algebraic relations between the constants of the interactions that ensure the spontaneous breaking of the gauge symmetry of the unified gauge theory. Thus, the condition of asymptotic freedom restricts the arbitrariness in the choice of the parameters of the spontaneous symmetry breaking of the unified gauge theories. The conditions of asymptotic freedom in the simplest unified gauge theory, which possesses SU(5) gauge symmetry, require the introduction into the theory of a fermion 24-plet (the so-called B multiplet[86]), which besides other particles necessarily contains neutral G fermions, supermassive metastable quarks with the color properties of gluons. Such supermassive G quarks of mass m can interact with the light quarks q and the gluons g and be annihilated with their antiparticles $\bar{G}$. The thermodynamic equilibrium of the supermassive quarks with the remaining particles is realized by reactions of the type $G\bar{G} \rightleftarrows q\bar{q}$, gg. In QCD calculations, wide use is made of the analogy with quantum electrodynamics. In this sense, the considered color interaction reactions are analogous to the annihilation processes $e^+e^- \to \mu^+\mu^-$, $\gamma\gamma$, whose rate is known.[107] This analogy makes it possible to give a rough estimate of the rate of color interaction reactions when $T \ll m$:

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{m^2} N_c, \tag{4.13}$$

where $N_c \sim 10$ is the number of possible channels of such reactions, and $\alpha \sim 1/50$.

A detailed analysis of the kinetics of the freezing of the G and $\bar{G}$ concentrations (see the review of Ref. 1) gives, the rate of their pair annihilation being taken to be (4.13), the frozen concentration of the supermassive quarks,

$$\nu \approx \frac{m}{m_{\text{Pl}}} \cdot \frac{10}{\alpha^2 N_c} A, \tag{4.14}$$

where in the system of units with $\hbar = c = 1$ the numerical coefficient A is given by

$$A = \frac{1}{\varkappa^2}\left[42 + \ln(\langle \sigma v \rangle m_p m) - \frac{1}{2} \ln \varkappa\right] \sim \frac{1}{10};$$

m_p is the proton mass, and $\langle \sigma v \rangle$ is determined by (4.13).

In the theory[86] there is predicted to be a very small value of the parameter k_2, which characterizes the decay of the supermassive quarks and determines their lifetime τ in accordance with (4.2), $k_2 < 10^{-26}$, so that the lifetime of the supermassive quarks is fairly large, and already when $\delta_0 > 10^{-7}$ the duration of the dust stage of their dominance is determined by annihilation in configurations that have become detached from the cosmological expansion. An estimate of the characteristic annihilation time, made in the same way as in the monopole case, for the annihilation rate

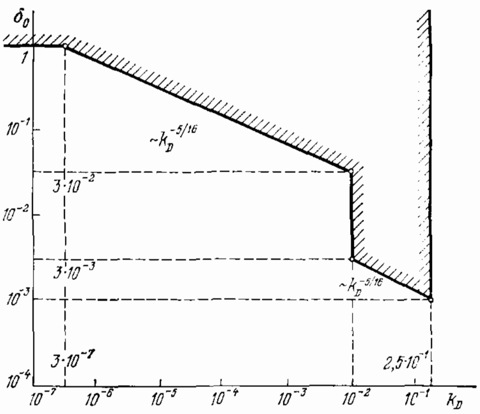

FIG. 7. Supermassive quark restrictions on the parameter k_D in an asymptotically free SU(5) theory for different values of δ_0. The right-hand boundary of the hatched region arises because for sufficiently large k_D supermassive quarks are unstable and decay before they begin to be dominant in the density of the universe.

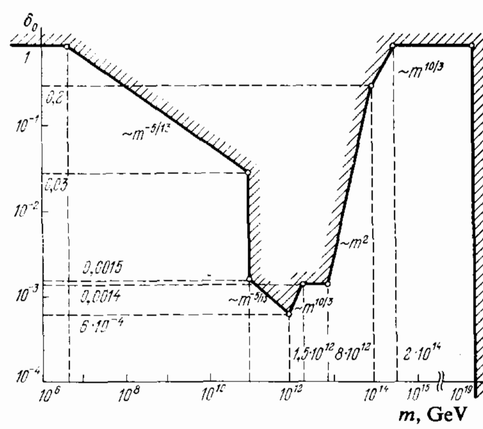

FIG. 8. Restrictions on the mass of the supermassive neutrino for different values of δ_0.

(4.13) gives

$$\tau \approx 3\cdot 10^{-25}\ \mathrm{sec}\left(\frac{10^{15}\ \mathrm{GeV}}{m}\right)^5 \delta_0^{-8}. \qquad (4.15)$$

The mass m of the supermassive quarks, in terms of which we have expressed τ and ν, can be expressed in terms of the mass of the X boson, $m_X \sim 10^{15}$ GeV, which is related to the proton lifetime and the free parameter k_D of the theory:

$$m = 2k_D m_X. \qquad (4.16)$$

In the framework of our approach, there is a unique possibility for restricting the parameter k_D. This parameter does not occur in the predictions of the theories in the region of lower energies. At the same time, it is very important for the theory itself, since it is this parameter that determines the masses of all particles of the B multiplet, both unstable and metastable. In addition, depending on the particular value of k_D, different particles in the B multiplet will be the metastable ones. Thus, in our view, astrophysical restrictions on the parameter k_D have a more general nature than restrictions on the mass of the supermassive quarks alone.

Using (4.14)–(4.16), we express τ and νm in terms of k_D:

$$\tau \approx 10^{-27}\ \mathrm{sec}\ k_D^{-5}\delta_0^{-8}, \qquad (4.17)$$

$$\nu m \approx 10^{14}\ \mathrm{GeV}\ k_D^2. \qquad (4.18)$$

From Fig. 5 and the relations (4.17) and (4.18) we obtain restrictions on k_D for different values of δ_0; these are shown in Fig. 7. The restrictions in Fig. 7 on the parameter k_D make it possible to restrict considerably the class of asymptotically free SU(5) models.

c) Restrictions on the parameters of supermassive neutral fermions

In unified theory models that assume a higher gauge symmetry than SU(5), there are supermassive neutral leptons, which can be regarded as right-handed neutrinos. As several models predict,[97,99] the interaction of these particles with one another and with the remaining particles takes place only through exchange of the X bosons, whose mass has the order of magnitude Λ and appreciably exceeds the mass of the particles under consideration. In this case, the characteristic time of such interaction becomes greater than the cosmological time shortly after the temperature T becomes comparable with Λ ($T \sim \Lambda \gg m$). This means that the right-handed neutrinos come out of thermodynamic equilibrium with the remaining particles, and they remain relativistic. Therefore, their relative residual concentration is

$$\nu \sim \frac{1}{\varkappa} \sim \frac{1}{100}, \qquad (4.19)$$

where $\varkappa$ is the number of species of all the relativistic particles in thermodynamic equilibrium when $T \sim \Lambda$.

In contrast to the monopoles and supermassive quarks, the lifetime of the heavy neutrinos and, thus, the duration of the corresponding dust stage are determined by their decay.

If we regard the mass m of these particles as a free parameter of the theory, then the restrictions on m for different δ_0 obtained in Fig. 5 and the relations (4.2) are given in Fig. 8.

A number of models predict a relationship[97,99] between the mass m of the heavy neutrinos and the mass m_ν of the light neutrinos:

$$m_\nu = \frac{\lambda^2}{m}, \qquad (4.20)$$

where $\lambda \sim 100$ GeV is the characteristic scale of the energies of the unification of the electromagnetic and weak interaction. Our method enables us in principle to obtain as well restrictions on the mass of the light neutrino for different values of the initial inhomogeneity δ_0. It should however be noted that the interaction of the supermassive neutrinos with the comparatively light Higgs bosons (with mass of order $\lambda \ll m$) may lead to a too rapid (i.e., occurring before an early dust stage can commence) decay of the heavy neutrinos into light fermions and Higgs bosons.[108] This is just the situation in the currently existing models, and this makes it impossible to obtain nontrivial restrictions on m_ν. However, the actual idea of obtaining restrictions on the mass of the light particles from restrictions associated with the dust stages of dominance of the corresponding supermassive particles can be of interest in the analysis of models in which a relationship of the type (4.20) is not associated with rapid decays of supermassive particles. A relationship analogous to (4.20) is realized in supergravity models,[39] in which the gravitino mass is related to the Planck mass by[39,20]

$$m=\frac{d}{m_{\mathrm{Pl}}}, \tag{4.21}$$

where d, the scale of the supersymmetry breaking, is a free parameter of the theory (with dimensions of m^2 in units for which $\hbar=c=1$).

If the amplitude of the initial perturbations of the metric is specified, then for large d our method makes it possible to obtain restrictions on the gravitino mass for different values of the free parameter d. These restrictions are shown for different δ_0 in Fig. 9.

In locally supersymmetric models with $N=1$ appreciably smaller gravitino masses are predicted, ~ 100 GeV, and the method we have described does not make it possible to obtain restrictions on their concentration directly. However, to suppress the residual concentration of such "light gravitinos" one usually requires that at the termination of the inflationary stage there be a sufficiently long dust stage of oscillations of the scalar field.[22] Our method makes it possible to give restrictions on the admissible inhomogeneity in this stage. Therefore, in this case too the method does in principle make it possible, albeit indirectly, to obtain restrictions on the parameters of supergravity models.

d) Origin of the spectrum of initial perturbations in the expanding universe; dust stages

Hitherto, we have regarded the amplitude δ_0 of the initial perturbations as a free parameter of the cosmological model.

Recently, ways have been suggested by which the spectrum of the initial perturbations can be related to quantum density fluctuations in the very early universe (near the singularity) through the existence of a de Sitter stage of exponential expansion of the universe—the so-called inflation stage (the stage in which the "false" vacuum is dominant). Such a stage may arise because of vacuum polarization near the singularity[100] or because of the vacuum phase transitions predicted by unified gauge theories[101–103] (for a detailed review, see Ref. 109). The existence of a stage of exponential expansion of the universe—exponential growth of the perturbation scale—relates the global structure of the universe today to physical processes near the singularity. On the transition from the de Sitter to the Friedmann stage of expansion there arise perturbations of the metric and density due to nonsimultaneity of the phase transition in different regions.[110–113] In addition, a spectrum of initial perturbations can be formed as a result of so-called parametric enhancement of small initial fluctuations, regarded as phonons in a hydrodynamic medium[114–116] (see also Refs. 117 and 118). This mechanism is analogous to the parametric amplification of gravitational waves.[119] For an analysis of the evolution of quantum fluctuations that does not treat them as phonons in a hydrodynamic medium in the de Sitter stage, see Ref. 120. In one way or another, when the exponential stage ends theory predicts some perturbation spectrum, which, in particular, must explain the large-scale structure

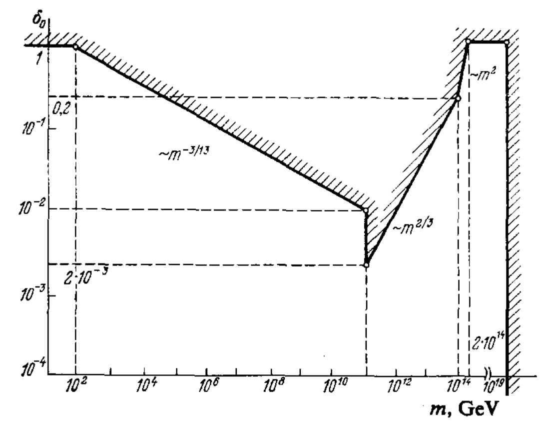

FIG. 9. Restrictions on the mass of supermassive gravitinos for different values of δ_0.

of the universe.

It is here important to emphasize that the parameters of this spectrum of initial perturbations depends strongly, first, on the parameters of the de Sitter stage (its commencement and duration) and, second, on the manner in which the "false" vacuum decays on the transition to the subsequent Friedmann stage of expansion.

In the light of all that we have said above, a program for constructing a self-consistent picture of the evolution of the very early universe in the framework of some unified gauge theory model is of particular interest. Such a scenario must include the beginning and end of the de Sitter stage and, in particular, must relate the parameters of the metric perturbations at the end of this stage to the parameters of the corresponding unified gauge model. Although such a program has not yet been completed, certain successes have already been achieved in this direction. Thus, calculations[110–113] show that the perturbation spectrum is almost flat ($n=0$). A flat spectrum is also generated by the supermassive strings predicted by certain unified gauge models.[121,122]

On the other hand, we have already said more than once in this review that the parameters of the same unified gauge models also determine the parameters of the early dust stages.

It is very important to emphasize that such dust stages, in which the supermassive metastable particles are dominant, are realized much later than the stages of exponential expansion, so that the nature of the physical processes in the exponential stage does not influence the frozen concentration of the supermassive particles, but fairly early, so that the formation of primordial black holes is determined by the small-scale part of the spectrum that subsequently undergoes complete damping, i.e., the only information about this part of the spectrum is the PBH spectrum in the corresponding mass scales. (For various aspects of PBH formation during the phase transition, see Refs. 123–125 and, directly after it, Refs. 126 and 127.)

Besides the early dust stages in which the supermassive particles are dominant, certain variants of the inflationary models predict "superearly" dust stages of oscillations of a classical scalar field (see, for example, Refs. 22 and 100) and subsequent dominance of nonrelativistic scalar particles. Such superearly stages follow directly after the inflationary stages and can be fairly long, so that in these stages the growth of the perturbations can reach the nonlinear stage. The comparison made above between the probability of PBH formation and the existing observational restrictions on the PBH spectrum make it possible to draw the following important conclusion: Irrespective of the duration, the superearly dust stages do not contradict the restrictions on the primordial black holes if the amplitude of the flat spectrum of the metric perturbations does not exceed 6×10^{-4}. But if it is found that the actual probability of PBH formation significantly exceeds the lower bound obtained above, then comparison of the predicted PBH spectrum with the astrophysical restrictions can, in principle, give a restriction on the duration of the superearly dust stages and, therefore, on the allowed variants of the inflationary scenario.

Thus, the PBH spectrum is a link between all the astrophysical data that have the nature of restrictions on the PBH spectrum and the unified gauge theory parameters that determine the spectrum of the initial perturbations and the parameters of the dust stages. Therefore, looking to the future, we can expect that when this program of constructing a self-consistent picture of the early universe in the framework of elementary-particle theory has been completed the method considered in this review will serve as a test of such a theory.

CONCLUSIONS

Thus, elementary-particle theories in conjunction with the theory of the hot universe can predict the existence of dust stages of expansion in the early universe, an unavoidable consequence of which is the mechanism considered above for formation of primordial black holes from small initial perturbations of the metric. The connection between the parameters of the theory and the initial inhomogeneity of the universe, which, in its turn, is related to the PBH spectrum in the framework of the considered mechanism, makes it possible to obtain a number of observational astrophysical restrictions on the parameters of the theory depending on the initial inhomogeneity of the universe. All the estimates given above must still be confirmed by detailed calculations, and in this sense the quantitative results given in the review cannot be regarded as final.

It is still necessary to construct a more detailed picture of the evolution of the perturbations in the dust stages; for example, to take into account the smooth beginning and end of the stage, the residual pressure of the relativistic particles, the part played by the "clumpiness" of the configurations, and, possibly, much else. In the case of particles that interact with radiation, allowance for concrete mechanisms of entropy reduction in the nonlinear stage of evolution of the perturbations will make it possible to raise the lower bound for the probability of PBH formation and thus strengthen the restrictions on the parameters of a theory which predicts the existence of such particles. But already the estimates given above demonstrate the basic possibility of obtaining information about the parameters of elementary-particle theory by comparing the astrophysical consequences of the existence of early dust stages with observational restrictions on the PBH spectrum.

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APPENDIX

Allowance for inhomogeneity of the configuration in the framework of the Tolman solution

In a synchronous and simultaneously comoving frame, the spherically symmetric line element describing the behavior of dust matter has the form (see the books of Ref. 46)

$$ds^2 = d\tau^2 - e^{\lambda(\tau,R)}dR^2 - r^2(\tau,R)(d\theta^2+\sin^2\theta\, d\varphi^2); \qquad (A1)$$

where τ is the proper time, R is the Lagrangian radius, and r is the Eulerian radius, chosen such that $2\pi r$ is the circumference of a circle with center at the origin.

We assume for simplicity (although this does not greatly affect the estimates made below) that expansion is replaced by contraction simultaneously in the complete configuration at the time t_1. Then integration of Einstein's equations for these initial conditions makes it possible to obtain the dependence $r(\tau, H)$ in the parametric form (see Ref. 46)

$$r=\frac{1}{2}r_1(R)(1+\cos\eta), \qquad (A2)$$

$$\tau=t_1+\frac{1}{2}\frac{r_1^{3/2}(R)}{r_g^{1/2}(R)}(\eta+\sin\eta); \qquad (A3)$$

where $0\leqslant\eta\leqslant\pi$, r_g is the gravitational radius of the mass within the Lagrangian radius R, and $r_1(R)$ is determined by the density distribution $\rho_1(R)$ at the time t_1. The further evolution of the density is determined from the same Einstein equations by the relation

$$\rho(\tau, R)=\rho_1(R)\frac{r_1^2(R)\,r_1'(R)}{r^2(R,\tau)\,r'(R,\tau)}; \qquad (A4)$$

here, the prime denotes the partial derivative with respect to R. Suppose that at the time t_1 the density $\rho_1(R)$ is distributed in accordance with the law

$$\rho_1(R)=\rho_{c1}\left[1-\frac{r_1^2(R)}{r_*^2}\right], \qquad (A5)$$

where r_* characterizes the degree of inhomogeneity of the configuration and is determined by the second derivative of the distribution of the density with respect to the Eulerian radius at the center of the configuration. The degree of inhomogeneity of the configuration defined earlier is $u=(r_1/r_*)^2$, where r_1 is the previously introduced characteristic radius of the configuration. It follows directly from (A5) that

$$r_g(R)=\frac{8\pi G\rho_{c1}}{3}r_1^3(R)\left[1-\frac{3r_1^2(R)}{5r_*^2}\right]. \qquad (A6)$$

The time of formation of the caustic at the center of the configuration occurs in accordance with (A2), (A3), and (A6) at the time t_{caus}, which is equal to

$$t_{caus}=t_1+\left(\frac{3\pi}{32G\rho_{c1}}\right)^{1/2}. \qquad (A7)$$

The time at which the entire configuration is below its gravitational radius, t_{PBH}, is determined by

$$t_{PBH} \approx t_1 + \left(\frac{3\pi}{32G\rho_{c1}}\right)^{1/2}\left(1-\frac{4}{3\pi}x^{3/2}\right)\left(1-\frac{3}{5}u\right)^{-1/2}, \quad (A8)$$

where $x = r_g/r_1$.

As follows from Eqs. (A7) and (A8), the condition (2.20) does indeed reduce to the inequality (2.21). It follows from (A2)–(A6) that

$$\rho(\tau,R) \approx \rho_1(R)\left(\frac{2}{1+\cos\eta}\right)^3\left[1+\frac{3}{5}\frac{\sin\eta\,(\eta+\sin\eta)\,r_1^2(R)}{(1+\cos\eta)^2\,r_*^2}\right]^{-1}. \quad (A9)$$

At the time t_{PBH}, the parameter η at the center of the configuration takes on the value

$$\eta_c = \pi - 2x^{1/2}(1-\xi)^{1/3}, \quad (A10)$$

where

$$\xi = \frac{9\pi}{40}\frac{u}{x^{3/2}}. \quad (A11)$$

At the same time, the parameter η on the outer boundary of the configuration is equal to

$$\eta_b = \pi - 2x^{1/2}. \quad (A12)$$

Then, with allowance for (A9)–(A12), the condition (2.19) is satisfied for $\xi \leqslant 1$, which again reduces to the inequality (2.21).

[1)]Note that the generalized charge is not necessarily a scalar. In the case of supersymmetric theories,[39] for example, there are spinor charges.

[2)]For exotic (in our view) cases when catastrophic accretion can still occur, see Refs. 58–60.

[3)]In principle, relatively short dust stages in the period 1–10^4 sec are not ruled out.[13–15] However, precisely because such stages have a relatively short duration they cannot lead to a significant weakening of the restrictions on $\beta(M)$.

[4)]Analogous virtual transitions can ensure in some models a small neutrino mass and a low probability of neutrino oscillations.[29,97,98]

[5)]Besides monopoles, the theories predict the existence of antimonopoles. See Ref. 8 and Section 4a of the present review for a discussion of pair annihilation of monopoles and antimonopoles.

[1]A. D. Dolgov and Ya. B. Zel'dovich, Usp. Fiz. Nauk **130**, 559 (1980) [Revised Engl. Transl., Rev. Mod. Phys. **53**, 1 (1981)].
[2]Ya. B. Zel'dovich and I. D. Novikov, Stroenie i évolyutsiya Vselennoĭ (Structure and Evolution of the Universe), Nauka, Moscow (1975).
[3]S. Weinberg, The First Three Minutes, Fontana:Collins (1978); [Russ. Transl., Énergoatomizdat, M., 1981].
[4]L. B. Okun', Leptony i kvarki (Leptons and Quarks), Nauka, Moscow (1980); Usp. Fiz. Nauk **134**, 3 (1981) [Sov. Phys. Usp. **24**, 341 (1981)].
[5]A. D. Linde, Rep. Prog. Phys. **42**, 390 (1979).
[6]A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 430 (1974) [JETP Lett. **20**, 194 (1974)]; G 't Hooft, Nucl. Phys. **B79**, 276 (1974).
[7]T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[8]Ya. B. Zeldovich and M. Yu. Khlopov, Phys. Lett. **B79**, 239 (1978).
[9]J. P. Preskill, Phys. Rev. Lett. **43**, 1365 (1979).
[10]Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun', Zh. Eksp. Teor. Fiz. **67**, 3 (1974) [Sov. Phys. JETP **40**, 1 (1975)].
[11]V. F. Shvartsman, Pis'ma Zh. Eksp. Teor. Fiz. **9**, 315 (1969) [JETP Lett. **9**, 184 (1969)]; G. Steigman, D. N. Schramm, and J. E. Gunn, Phys. Lett. **B66**, 202 (1977); S. M. Austin, Prog. Part. Nucl. Phys. **7**, 1 (1981); R. Wagoner, W. A. Fowler, and F. Hoyle, Astrophys. J. **148**, 3 (1967); K. A. Olive, D. H. Schramm, and G. Steigman, in: Proc. of the Seminar on Proton Stability, University of Wisconsin (1978); S. L. Shapiro, S. A. Teucolsky, and I. Wasserman, Phys. Rev. Lett. **45**, 665 (1980); M. Yu. Khlopov and S. T. Petcov, Phys. Lett. **B99**, 117 (1981); A. D. Dolgov, Yad. Fiz. **33**, 1309 (1981) [Sov. J. Nucl. Phys. **33**, 700 (1981)].
[12]Ya. B. Zel'dovich and Ya. A. Smorodinskiĭ, Zh. Eksp. Teor. Fiz. **41**, 907 (1961) [Sov. Phys. JETP **14**, 647 (1962)]; S. S. Gershteĭn and Ya. B. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. **4**, 174 (1966) [JETP Lett. **4**, 120 (1966)].
[13]A. G. Doroshkevich and M. Yu. Khlopov, Pis'ma Astron. Zh. **9**, 323 (1983) [Sov. Astron. Lett. **9**, 171 (1983)].
[14]A. G. Doroshkevich and M. Yu. Khlopov, Yad. Fiz. **39**, 869 (1984) [Sov. J. Nucl. Phys. **39**, 551 (1984)].
[15]A. G. Polnarev and M. Yu. Khlopov, Astron. Zh. **59**, 15 (1982) [Sov. Astron. **26**, 27 (1982)].
[16]A. S. Szalay and G. Marx, Astron. Astrophys. **49**, 437 (1976).
[17]R. Cowsic and J. McClelland, Phys. Rev. Lett. **29**, 669 (1972); S. Tremaine and J. E. Gunn, Phys. Rev. Lett. **42**, 407 (1979); J. E. Gunn, B. W. Lee, I. Lerche, D. N. Schramm, and G. Steigman, Astrophys. J. **223**, 1015 (1978).
[18]M. I. Vysotskiĭ, A. D. Dolgov, and Ya. B. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. **26**, 200 (1977) [JETP Lett. **26**, 188 (1977)]; B. W. Lee and S. Weinberg, Phys. Rev. Lett. **39**, 165 (1977).
[19]M. Dine and W. Fishler, Phys. Lett. **B120**, 137 (1983); L. Abott and P. Sikivie, Phys. Lett. **B120**, 133 (1983); J. Preskill, M. Wise, and F. Wilczek, Phys. Lett. **B120**, 127 (1983); J. Ipser and P. Sikivie, Phys. Rev. Lett. **50**, 925 (1983); F. W. Stecker and O. Shafi, Phys. Rev. Lett. **50**, 928 (1983).
[20]S. Weinberg, Phys. Rev. Lett. **48**, 1303 (1982).
[21]F. Balestra, G. Piragino, G. Pontecorvo, M. G. Sapozhnikov, I. V. Falomkin, and M. Yu. Khlopov, Yad. Fiz. **39**, 990 (1984) [Sov. J. Nucl. Phys. **39**, 626 (1984)]; Nuovo Cimento **A79**, 193 (1984).
[22]M. Yu. Khlopov and A. D. Linde, Phys. Lett. **B138**, 265 (1984).
[23]P. J. E. Peebles, The Large Scale Structure of the Universe, Princeton University Press, Princeton (1980) [Russ. Transl., Mir, M., (1983)].
[24]A. G. Doroshkevich, Ya. B. Zel'dovich, R. A. Syunyaev, and M. Yu. Khlopov, Pis'ma Astron. Zh. **6**, 457, 465 (1980) [Sov. Astron. Lett. **6**, 257 (1980)]; A. G. Doroshkevich and M. Yu. Khlopov, Astron. Zh. **58**, 913 (1981) [Sov. Astron. **25**, 521 (1981)]; A. G. Doroshkevich, M. Yu. Khlopov, R. A. Sunyaev, A. S. Szalay, and Ya. B. Zeldovich, Ann. N. Y. Acad. Sci. **375**, 32 (1981).
[25]G. S. Bisnovatyĭ-Kogan and I. D. Novikov, Astron. Zh. **57**, 899 (1980) [Sov. Astron. **24**, 516 (1980)]; G. S. Bisnovatyi-Kogan, G. S. Lukash, and I. D. Novikov, in: Variability in Stars and Galaxies: Proc. of Fifth European Regional Meeting in Astronomy, Liẽge (1980).
[26]J. R. Bond, G. Efstathiou, and J. Silk, Phys. Rev. Lett. **45**, 1980 (1980).
[27]H. Sato and F. Takahara, Prog. Theor. Phys. **64**, 2029 (1980).
[28]D. N. Schramm and G. Steigman, Astrophys. J. **243**, 1 (1981).
[29]Ya. B. Zel'dovich and M. Yu. Khlopov, Usp. Fiz. Nauk **135**, 45 (1981) [Sov. Phys. Usp. **24**, 755 (1981)].
[30]S. F. Shandarin, A. G. Doroshkevich, and Ya. B. Zel'dovich, Usp. Fiz. Nauk **139**, 83 (1981) [Sov. Phys. Usp. **26**, 46 (1983)].
[31]V. A. Lyubimov, E. G. Novikov, V. Z. Nozik, E. F. Tret'yakov, and V. S. Nozik, Yad. Fiz. **32**, 301 (1980) [Sov. J. Nucl. Phys. **32**, 154 (1980)]; V. A. Lyubimov, E. G. Novikov, V. Z. Nozik, E. F. Tret'yakov, V. S. Nozik, and N. F. Myasoedov, Zh. Eksp. Teor. Fiz. **81**, 1158 (1981) [Sov. Phys. JETP **54**, 616 (1981)].
[32]Ya. B. Zel'dovich, A. A. Klypin, M. Yu. Khlopov, and V. M. Chechetkin, Yad. Fiz. **31**, 1286 (1980) [Sov. J. Nucl. Phys. **31**, 664 (1980)].
[33]A. D. Linde, Rep. Prog. Phys. **47**, No. 8 (1984).
[34]G. D. Coughlan, W. Fishler, E. W. Kolb, S. Raby, and G. G. Ross, Phys. Lett. **B131**, 59 (1983); A. S. Goncharov, A. D. Linde, and M. I. Vysotsky, Preprint ITEP-109, Moscow (1984).
[35]M. S. Turner, Phys. Rev. D **28**, 1243 (1983).
[36]Ya. B. Zel'dovich and I. D. Novikov, Astron. Zh. **43**, 758 (1966) [Sov. Astron. **10**, 602 (1967)].
[37]Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **42**, 641 (1962) [Sov. Phys. JETP **15**, 446 (1962)].
[38]A. S. Schwartz, Nucl. Phys. **B112**, 358 (1976).
[39]D. Z. Freedman and P. van Nieuwenhuizen, Sci. Am. **238(2)**. 126 (1978) [Russ. Transl., Usp. Fiz. Nauk **128**, 137 (1979)]; P. Fayet, in: Unification of Fundamental Particle Interactions (eds. S. Ferrara *et al.*) (1980), p. 587; S. Deser and B. Zumino, Phys. Rev. Lett. **38**, 1433 (1977); E. Cremer, B. Julia, J. Sherk, S. Ferrara, L. Girardello, and P. van Nieuwenhuizen, Nucl. Phys. **B147**, 105 (1979); E. Cremer, S. Ferrara, L. Girardello, and A. van Proyen, Nucl. Phys. **B212**, 413 (1983).
[40]Ya. B. Zel'dovich, Adv. Astron. Astrophys. **3**, 241 (1965).
[41]Ya. B. Zel'dovich, L. B. Okun', and S. B. Pikel'ner, Usp. Fiz. Nauk **87**, 113 (1965) [Sov. Phys. Usp. **8**, 702 (1966)].
[42]P. S. Laplace, Le Systeme de Mond., Vol. 2, Paris (1795); The system of the World (ed. W. Flint), London (1809).
[43]K. Schwarzschild, Sitzungsber. Dtsch. Akad. Wiss. Berlin, Kl. Math.

Phys. Tech. p. **189** (1916).
[44] H. Reissner, Ann. Phys. (Leipzig) **50**, 106 (1916); G. Nordstrøm, Proc. K. Ned. Akad. Wet. **20**, 1238 (1918).
[45] R. P. Kerr, Phys. Rev. Lett. **11**, 237 (1963).
[46] Ya. B. Zel'dovich and I. D. Novikov, Teoriya tyagoteniya i évolyutsiya zvezd (Theory of Gravitation and Evolution of Stars), Nauka, Moscow (1971); C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation, W. H. Freeman, San Francisco (1973) (Russ. Transl., Mir, M., (1977)); L. D. Landau and E. M. Lifshitz, Teoriya polya, Nauka, Moscow (1973) [Engl. Transl., The Classical Theory of Fields, 4th ed., Pergamon Press, Oxford (1975)]; V. P. Frolov, Usp. Fiz. Nauk **118**, 473 (1978) [Sov. Phys. Usp. **19**, 244 (1976)].
[47] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1938).
[48] J. R. Oppenheimer and H. Snyder, Phys. Rev. **56**, 455 (1939).
[49] S. W. Hawking, Mon. Not. R. Astron. Soc. **152**, 75 (1971).
[50] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **16**, 587 (1946).
[51] M. Yu. Khlopov and A. G. Polnarev, Phys. Lett. **B97**, 383 (1980).
[52] A. G. Polnarev and M. Yu. Khlopov, Astron. Zh. **58**, 706 (1981) [Sov. Astron. **25**, 406 (1981)].
[53] A. G. Polnarev and M. Yu. Khlopov, Astron. Zh. **59**, 639 (1982) [Sov. Astron. **26**, 391 (1982)].
[54] M. Yu. Khlopov and A. G. Polnarev, in: The Very Early Universe (eds. G. W. Gibbons, S. W. Hawking, and S. Siklos), Cambridge Univ. Press, Cambridge (1983), p. 407.
[55] Ya. B. Zel'dovich and I. D. Novikov, Relyativistskaya astrofizika, Nauka, Moscow (1967). [Revised Engl. Transl. Relativistic Astrophysics, University of Chicago Press, 1971].
[56] B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. **168**, 399 (1974).
[57] D. K. Nadezhin, I. D. Novikov, and A. G. Polnarev, Astron. Zh. **55**, 216 (1978) [Sov. Astron. **22**, 129 (1978)]; I. D. Novikov and A. G. Polnarev, Astron. Zh. **57**, 250 (1980) [Sov. Astron. **24**, 147 (1980)].
[58] D. N. C. Lin, B. J. Carr, and S. M. Fall, Mon. Not. R. Astron. Soc. **177**, 51 (1976); G. V. Bicknell and R. N. Henriksen, Astrophys. J. **219**, 1043 (1978); **232**, 670 (1979).
[59] N. A. Zabotin and P. D. Nasel'skiĭ, Astron. Zh. **59**, 647 (1982) [Sov. Astron. **26**, 395 (1982)].
[60] N. A. Zabotin and P. D. Nasel'skiĭ, Pis'ma Astron. Zh. **6**, 14 (1980) [Sov. Astron. Lett. **6**, 7 (1980)]; Astrofizika, **18**, 310 (1982).
[61] S. W. Hawking, Nature, **43**, 199 (1974); **248**, 30; Commun. Math. Phys. **43**, 199 (1975).
[62] Ya. B. Zel'dovich and A. A. Starobinskiĭ, Zh. Eksp. Teor. Fiz. **61**, 2161 (1971) [Sov. Phys. JETP **34**, 1159 (1972)]; Pis'ma Zh. Eksp. Teor. Fiz. **26**, 373 (1977) [JETP Lett. **26**, 252 (1977)].
[63] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976); G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
[64] I. D. Novikov, Astron. Zh. **38**, 564 (1961) [Sov. Astron. **5**, 423 (1961)].
[65] B. J. Carr, Astrophys. J. **201**, 1 (1975).
[66] I. D. Novikov, A. G. Polnarev, A. A. Starobinsky, and Ya. B. Zel'dovich, Astron. Astrophys. **80**, 104 (1979); B. J. Carr, in: Trudy vtorogo seminara "Kvantovya gravitatsiya" (Proc. Second Seminar "Quantum Gravity"), Moscow, October 13–15, 1981, Institute of Nuclear Research, USSR Academy of Sciences, Moscow (1982), p. 195.
[67] N. A. Zabotin, L. S. Marochnik, and P. D. Nasel'skiĭ, Astrofizika **18**, 161 (1982).
[68] Ya. B. Zeldovich, Mon. Not. R. Astron. Soc. **160**, 1 (1972).
[69] Yu. N. Pariĭskiĭ, Z. N. Petukhov, and A. N. Chernov, Pis'ma Astron. Zh. **3**, 483 (1977) [Sov. Astron. Lett. **3**, 263 (1977)]. R. B. Partridge, Phys. Scr. **21**, 624 (1980); C. Ceccarelli, F. Melchiorri, L. Pietranera, G. Dall'Oglio, and B. Melchiori-Olivo, Astrophys. J. Lett. **269**, 27 (1983); D. J. Finen, E. S. Cheng, and D. T. Wilkinson, Phys. Rev. Lett. **50**, 620 (1983).
[70] A. A. Starobinskiĭ, Pis'ma Astron. Zh. **9**, 573 (1983) [Sov. Astron. Lett. **9**, 300 (1983)].
[71] J. D. Barrow, Mon. Not. R. Astron. Soc. **192**, 457 (1980).
[72] N. A. Zabotin and P. D. Nasel'skiĭ, Astrofizika **16**, 337 (1980).
[73] A. G. Polnarev, Astrofizika **13**, 375 (1977).
[74] A. G. Doroshkevich, Astrofizika **6**, 581 (1970).
[75] M. A. Markov, Prog. Theor. Phys. Suppl. Ext. **85** (1965); Zh. Eksp. Teor. Fiz. **51**, 878 (1966) [Sov. Phys. JETP **24**, 584 (1967)].
[76] Ya. B. Zel'dovich, Quoted in the Collection in Ref. 66, p. 146.
[77] Ya. B. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 29 (1976) [JETP Lett. **24**, 25 (1976)].
[78] B. V. Baĭer and P. D. Nasel'skiĭ, Pis'ma Astron. Zh. **3**, 147 (1977) [Sov. Astron. Lett. **3**].
[79] Ya. B. Zel'dovich, A. A. Starobinskiĭ, M. Yu. Khlopov, and V. M. Chechetkin, Pis'ma Astron. Zh. **3**, 208 (1977) [Sov. Astron. Lett. **3**, 110 (1977)].
[80] V. M. Chechetkin, M. Yu. Khlopov, M. G. Sapozhnikov, and Ya. B. Zel'dovich, Phys. Lett. **B118**, 329 (1982); V. M. Chechetkin, M. Yu. Khlopov, and M. G. Sapozhnikov, Riv. Nuovo Cimento **10**, 1 (1982).
[81] N. D. Nasel'skiĭ, Pis'ma Astron. Zh. **4**, 387 (1978) [Sov. Astron. Lett. **4**, 209 (1978)].
[82] D. N. Page and S. W. Hawking, Astrophys. J. **206**, 1 (1976).
[83] B. J. Carr, Astrophys. J. **206**, 8 (1976).
[84] M. J. Rees, Nature, **268**, 333 (1977); R. D. Blanford, Mon. Not. R. Astron. Soc. **181**, 489 (1977).
[85] P. D. Nasel'skiĭ and N. V. Pelikhov, Astron. Zh. **56**, 714 (1979) [Sov. Astron. **23**, 402 (1979)].
[86] O. K. Kalashnikov, Preprint-79, Lebedev Inst., Moscow (1980).
[87] O. K. Kalashnikov and M. Yu. Khlopov, Phys. Lett. **B127**, 407 (1983).
[88] A. Salam, in: Eighth Nobel Symposium (ed. N. Svartholm), Stockholm (1968).
[89] S. W. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
[90] H. Politzer, Phys. Rev. Lett. **30**, 1364 (1973); D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).
[91] H. Georgy and S. L. Glashow, Phys. Rev. Lett. **32**, 483 (1974).
[92] J. Learned, F. Reines, and A. Soni, Phys. Rev. Lett. **43**, 907 (1979).
[93] M. L. Cherry, M. Deakyne, K. Lande, C. K. Lee, R. I. Steinberg, and B. Cleveland, Phys. Rev. Lett. **47**, 1507 (1981).
[94] Proc. of 1982 Summer Workshop on Proton Decay Experiment (ed. D. S. Ayres), ANL (1982).
[95] V. A. Kuz'min, Pis'ma Zh. Eksp. Teor. Fiz. **12**, 335 (1970) [JETP Lett. **12**, 238 (1970)].
[96] In the Review: J. Fidecaro, Preprint CERN-EP/83-102 (1983).
[97] E. Witten, Phys. Lett. **B91**, 81 (1979).
[98] M. Gell-Mann, P. Ramond, and R. Slansky, Rev. Mod. Phys. **50**, 721 (1978) [Russ. Transl., Usp. Fiz. Nauk **130**, 459 (1980)].
[99] C. Wetterich, Nucl. Phys. **B187**, 343 (1981).
[100] A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980).
[101] A. Guth, Phys. Rev. D **23**, 347 (1981).
[102] A. D. Linde, Phys. Lett. **B108**, 389 (1982).
[103] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[104] M. Moss, Phys. Lett. **B128**, 385 (1983).
[105] A. D. Linde, Phys. Lett. **B96**, 293 (1980).
[106] T. Goldman, E. W. Kolb, and D. Toussaint, Phys. Rev. D **23**, 867 (1981).
[107] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika, Fizmatgiz, Moscow (1959). [Engl. Transl., Quantum Electrodynamics, Interscience, New York (1965)].
[108] F. R. Klinkhamer, G. Branko, J. P. Deredinger. P. Hut, and A. Masiero, Astron. Astrophys. **94**, L19 (1981).
[109] A. D. Linde, Usp. Fiz. Nauk **144**, 177 (1984) [Expanded Engl. Transl. in Rep. Prog. Phys. **47**, 925 (1984)].
[110] S. W. Hawking, Phys. Lett. **B117**, 175 (1982).
[111] A. Guth and S. -Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982).
[112] A. A. Starobinsky, Phys. Lett. **B117**, 175 (1981).
[113] J. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **29**, 679 (1983).
[114] V. N. Lukash, Zh. Eksp. Teor. Fiz. **79**, 1601 (1980) [Sov. Phys. JETP **52**, 807 (1980)].
[115] A. D. Kompaneets, V. N. Lukash, and I. D. Novikov, Astron. Zh. **59**, 424 (1982) [Sov. Astron. **26**, 259 (1982)].
[116] V. N. Lucash and I. D. Novikov, in: 10th Intern. Conference on General Relativity and Gravitation, Padova, Italy, July 4–19 (1983).
[117] G. V. Chibisov and V. F. Mukhanov, Mon. Not. R. Astron. Soc. **200**, 535 (1982).
[118] D. A. Kompaneets and V. N. Lukash, Astron. Zh. **58**, 482 (1981) [Sov. Astron. **25**, 272 (1981)].
[119] L. P. Grishchuk, Zh. Eksp. Teor. Fiz. **67**, 825 (1974) [Sov. Phys. JETP **40**, 409 (1975)].
[120] V. F. Mukhanov and G. V. Chibisov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 549 (1981) [JETP Lett. **33**, 532 (1981)].
[121] A. S. Schwartz, Nucl. Phys. **B208**, 100 (1982); A. Everett and A. Vilenkin, Nucl. Phys. **B207**, 43 (1982); A Everett, Phys. Rev. D **24**, 858 (1981); T. Kibble, . Lazarides, and Q. Shafi, Phys. Rev. D **26**, 435 (1982); Q. Shafi, Quoted in the Collection in Ref. 54, p. 147; A. Vilenkin, Quoted in the Collection in Ref. 54, p. 163.
[122] Ya. B. Zeldovich, Mon. Not. R. Astron. Soc. **190**, 48 (1980).
[123] S. W. Hawking, I. G. Moss, and J. M. Stewart, Phys. Rev. D **26**, 2681 (1982).
[124] V. A. Berezin, V. A. Kuzmin, and I. I. Tkachev, Phys. Lett. **B120**, 91 (1983); in: Proc. of Intern. Symposium "Phenomenology of Unified Theory," World Scientific Publ., Singapore (1984).

[125]K. Sato, M. Sasaki, M. Kodama, and K. Maeda, Prog. Theor. Phys. **65**, 1443 (1981); Phys. Lett. **B108**, 98 (1982).
[126]N. S. Kardashev and I. D. Novikov, in: Proc. of IAU Symp. No. 104, Crete, Greece (1982).
[127]P. D. Nasel'skiĭ and A. G. Polnarev, Preprint PR-913 [in Russian], Institute of Cosmic Research, USSR Academy of Sciences, Moscow (1984); Astron. Zh. **62** (1985) [Sov. Astron. **29** (1985)].

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