# Low-frequency current noise with a $1/f$ spectrum in solids

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The problem of low-frequency current noise with a $1/f$ spectrum in solids (flicker noise) is reviewed. The fundamental general properties of this noise are described: frequency-dependence of the spectral density, time-dependence of the correlation function, form of the distribution function of the fluctuations, dependence of the spectral density on the potential applied to the specimen, anisotropy of the fluctuations of the resistivity tensor, correlation length of the fluctuations, dependence of the noise on the dimensions of the specimen and on the concentration of current carriers, the empirical Hooge relationship, etc. A model of $1/f$ noise is presented and discussed that associates its spectrum with the presence in solids of an extensive hierarchy of relaxation times. Concrete models of systems having an exponentially broad spectrum of relaxation times are described (the McWhorter model, two-level tunneling systems, the disordered Ising kinetic model). The theory and experimental data on current noise caused by temperature fluctuations are analyzed. The problem is treated of surface noise in semiconductors and $1/f$ noise in metal films.

## TABLE OF CONTENTS

1. Introduction ..... 170
2. Some general properties of $1/f$ noise ..... 172
   2.1. Frequency-dependence of the spectral density. 2.2. Variance of the noise and the correlation function of the fluctuations in time. 2.3. Measurements of the distribution function of the fluctuations and of the quantity $\langle x(t)|x(0)=x_0\rangle$. 2.4. Stationarity of $1/f$ noise. 2.5. Dependence of $1/f$ noise on the mean voltage or current. 2.6. Anisotropy of conductivity fluctuations. 2.7. Correlation length of electric-conductivity fluctuations and dependence of the noise on the dimensions of the conductor. 2.8. Dependence of $1/f$ noise on the number of free current carriers in the semiconductor. Empirical Hooge relationship. 2.9. Observations of $1/f$ noise at low temperatures. 2.10. Possible influence of infralow-frequency fluctuations on measurements of $1/f$ noise.
3. The model of an exponentially broad distribution of relaxation times ..... 181
   3.1. General idea. 3.2. Some models of systems with an exponentially broad distribution of relaxation times.
4. Current noise caused by temperature fluctuations ..... 185
   4.1. Idea of the mechanism and first experiments. 4.2. Spectrum of temperature fluctuations. 4.3. Spatial correlation of fluctuations.
5. Low-frequency current noise in semiconductors ..... 188
   5.1. Surface noise. 5.2. Other problems.
6. Current noise in metals and semimetals ..... 190
   6.1. Temperature-dependence of the SDF in metals. 6.2. Low-frequency current noise and internal friction in metals.
7. Fluctuations with a $1/f$ spectrum in other systems ..... 193
8. Conclusion ..... 193
References ..... 194

## 1. INTRODUCTION

The noise spectrum, or dependence of the spectral density of fluctuations (SDF) on the frequency, is one of the most important characteristics of a fluctuational process. The SDF equals the ratio of the mean square of the noise signal passed by a filter with a passband of sufficiently small width $\Delta f$ near the frequency $f$ to the width $\Delta f$ of the band. According to the Wiener-Khintchine theorem, the spectral density (SD) of stationary noise equals twice the Fourier transform of the correlation function of the fluctuations $x(t)$ in time:

$$S_x(f)=2\int_{-\infty}^{+\infty} d(t_1-t_2)\, e^{i\omega(t_1-t_2)}\,\overline{x(t_1)\,x(t_2)} \tag{1.1}$$

$$=4\int_0^{\infty} d(t_1-t_2)\cos\omega(t_1-t_2)\,\overline{x(t_1)\,x(t_2)}.$$

Thus the spectrum $S(f)$ reflects the kinetics of the processes

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that govern the observed noise.

In a short-circuited conductor in a state of thermodynamic equilibrium (mean current $I = 0$) current fluctuations $\delta I(t)$ arise, while voltage fluctuations $\delta U(t)$ arise at the ends of an open-circuit conductor. The corresponding SDFs are (Nyquist theorem):

$$S_U(f) = 4kT \operatorname{Re} Z(f), \quad S_I(f) = 4kT \operatorname{Re} Z^{-1}(f). \tag{1.2}$$

Here $T$ is the absolute temperature of the conductor, $k$ is the Boltzmann constant, and $Z(f)$ is the impedance of the conductor at the frequency $f$ at which the SDF is measured.

When current is passed through the conductor, the SD of electric fluctuations increases above the equilibrium value given in (1.2). This increase in $S_U$ or $S_I$ is called current noise. Usually current noise arises from fluctuations of the resistance $\delta R(t)$, which are caused by fluctuations of the number of current carriers in the conductor, in their mobility, etc. If a noisy specimen with a mean resistance $R$ is connected in series with the load resistance $R_L$ (Fig. 1), then the SD of the current noise is

$$\begin{aligned} S_U &= U^2 \left[1 + \left(\frac{R}{R_L}\right)\right]^{-2} \frac{S_R}{R^2}, \\ S_I &= I^2 \left[1 + \left(\frac{R_L}{R}\right)\right]^{-2} \frac{S_R}{R^2}. \end{aligned} \tag{1.3}$$

Here $U$ is the mean voltage applied to the specimen, $I$ is the mean current, and $S_R(f)$ is the SDF of the resistance of the specimen.

Generally current noise is not just an equilibrium Nyquist noise that has been modified (by the current passing through the specimen): different mechanisms of fluctuations contribute to the two types of noise.[1)] For example, generation-recombination fluctuations in semiconductors, which amount to fluctuations in the number of current carriers, give rise to fluctuations in the resistance, and correspondingly, to current noise, but they contribute practically nothing to the equilibrium noise of (1.2). Thus, such fluctuational mechanisms in a conductor are often manifested in current noise, and are practically not at all detected in the equilibrium noise. The frequency-dependence of the SD of the current noise is generally completely different than for equilibrium noise.

In 1925 Johnson[1] studied the current fluctuations of thermoelectric emission, and found a noise whose SD increases with decreasing frequency $f$, alongside the shot noise, whose SD does not depend on the frequency in the low-frequency region. Schottky[2] suggested that this noise arises from slow fluctuational changes in the surface of the thermocathode, and proposed for it the name "flicker effect." Since an increase in the SD of current noise with decreasing frequency was found in the 1930's in carbon microphones (granular conductor) and metal films, and also in various

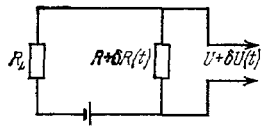


FIG. 1. Diagram of measurement of current noise.

semiconductors and semiconductor devices in the 1940's and 1950's, it has become evident that flicker noise is a highly widespread (if not universal) phenomenon in conductors.

Up to the present, measurements of the spectra of current noise have been performed on a vast number of the most varied semiconductors, semimetals, electronic and other devices, etc. One observes in practically all cases an increase in the spectral density of the current noise with decreasing frequency $f$ approximately proportional to $1/f$ down to the very lowest frequencies at which SD measurements have been performed. This current noise is usually called $1/f$ noise (or of $1/f$ type), and more rarely flicker noise, or finally excess noise.

The part of the problem of $1/f$ noise that deals with the statistical properties of this noise has been mainly solved. The frequency-dependence and the order of magnitude of the SD of the noise are known for many classes of conductors and electronic devices (Sec. 2.1). Also there have been many measurements of the distribution function of the fluctuations to find whether this random process is Gaussian (Sec. 2.3). A number of experimental studies has been devoted to finding whether this process is stationary (Sec. 2.4). Although usually the observed quadratic dependence of the SD on the applied potential in homogeneous ohmic conductors indicates that the noise is caused by equilibrium fluctuations of the resistance, while the current only "reveals" these fluctuations, this fact has been proved also by special experiments (Sec. 2.5). A new line of studies has arisen in the past several years—study of the anisotropy of fluctuations of the conductivity. It has been possible to establish that they are anisotropic in a number of conductors (Sec. 2.6). In almost all the cases in which the spatial correlation of the conductivity fluctuations giving rise to $1/f$ noise has been measured experimentally, it could not be detected. That is, the correlation radius proved to be very small (Sec. 2.7). Finally, an empirical formula (of Hooge) has been found that enables one to estimate the order of magnitude of the SD of $1/f$ noise in homogeneous conductors (Sec. 2.8). Thus answers to the question "in what way is it making noise?" have mainly been obtained.

The other part of the problem—"what precisely is making noise and causing the observed current noise?" has been solved to a much lesser degree, and the search for mechanisms of $1/f$ noise is continuing intensively even now.

The hypothesis was advanced about 10 years ago that $1/f$ noise is caused by temperature fluctuations. It seemed that certain experiments favor precisely this mechanism. However, it has been shown in the past several years, both experimentally and theoretically, that temperature fluctuations cannot be responsible for the observed $1/f$ noise (Sec. 4).

[1)] A special case is the change in the noise caused by overheating of the current carriers in a strong electric field: a part of the change in the SD as compared with the equilibrium value involves simply the change in the energy distribution function of the charge carriers (with increase in the "electron temperature").

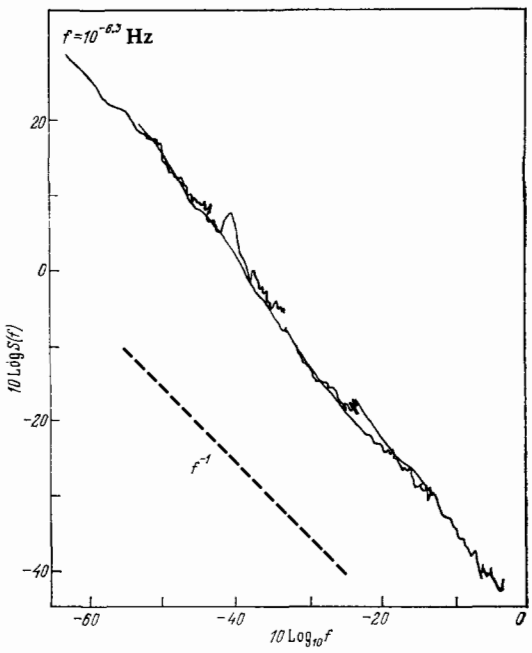


FIG. 2. Frequency-dependence of the spectral density of current noise of an operational amplifier.[12] The broken line corresponds to a $f^{-1}$ relation.

In the only general approach to explaining $1/f$ noise, against which there are no direct objections at present, one assumes that varied relaxation processes occur in conductors that exhibit this noise, with a broad spectrum of relaxation times $\tau$ that encompasses many orders of magnitude of variation of $\tau$ (Sec. 3). For example, these processes involve the kinetics of defects, i.e., ultimately—the disorder of solids. The connection of $1/f$ noise with defects of solids (including uncontrolled defects) is indicated by the fact that specimens prepared by the same technology and having similar electrophysical parameters often show a different $1/f$ noise: the spectral densities can differ by an order of magnitude or more (Sec. 2.8). In evaluating the reality of such a physical picture of the noise, one must also bear in mind the fact that a broad distribution of relaxation times has been adduced to explain also other physical phenomena in disordered solids—dielectric and magnetic relaxation and internal friction. Unfortunately, satisfactory microscopic models of the relaxation processes that give rise to $1/f$ noise in real conductors and devices have been developed only for several systems (the situation is analogous also in the other fields cited above of the kinetics of disordered systems: the concrete mechanisms of the broad spread of relaxation times are most often unknown). From this standpoint the problem of $1/f$ noise seems to be a problem of the low-frequency kinetics of disordered solids.

The great interest in studies of $1/f$ noise arises from the fact that this problem is very general and to a considerable degree unsolved. However, the large practical importance of these studies also plays a large role, since $1/f$ noise is an interference that limits the parameters of many electronic devices at low frequencies. It also affects the operation of such high-frequency devices as quartz generators and atomic frequency standards: their working frequency undergoes random "drift," the spectrum of these fluctuations being of $1/f$ type. For this reason, in particular, the relative error of time measurements cannot be reduced below a certain value as one increases the measured time interval without limit (see Sec. 7).

This review is far from complete: $1/f$ noise in devices is practically not treated, and many theories of $1/f$ noise that seem unconvincing are not analyzed and even not mentioned. Yet we can hope that this review will supplement those already existing in the literature.[3-11]

## 2. SOME GENERAL PROPERTIES OF $1/f$ NOISE

### 2.1. Frequency-dependence of the spectral density

The principal feature of the noise being discussed is that its SD $S(f)$ increases monotonically with decreasing frequency $f$. In practically all cases this increase is observed down to the lowest frequencies to which one can conduct measurements, and no leveling of $S(f)$ onto a plateau can be seen. In Ref. 12 the measurements of the excess-noise spectrum (of operational amplifiers) have been taken to the record low frequency of $5\times10^{-7}$ Hz (Fig. 2). Reference 9 cites an unpublished study in which the spectrum of $1/f$ noise has been followed to $10^{-7}$ Hz. Such measurements require times of averaging of the order of a month, and are technically extremely difficult.

At high frequencies the $1/f$ noise "drowns" in the equilibrium generation-recombination or shot noise, whose SD at the same frequencies does not depend on $f$. The frequency at which the $1/f$ noise ceases to be appreciable depends on the intensity of this noise, and in various systems ranges from $\sim10^2$ to $\sim10^6$ Hz.

Often the experimental noise spectrum is approximated by a power function $f^{-\gamma}$. Figure 3 shows a typical spectrum of current noise in a $VO_2$ specimen.[13] To a high accuracy it is

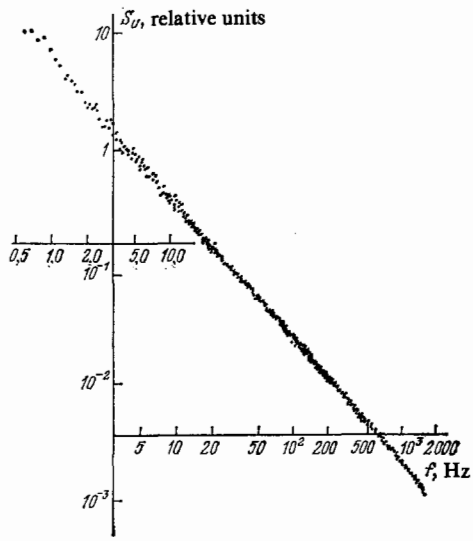


FIG. 3. Typical spectrum of current noise in $VO_2$.[13] Exponent $\gamma = 1.188 \pm 0.002$.

TABLE I. Dependence of the spectral density of $1/f$ noise on the frequency.

| Noise source | Measured range of frequencies, Hz | $\gamma = -\mathrm{d}\ln S/\mathrm{d}\ln f$ | Remarks |
|---|---|---|---|
| Ag on sapphire[17] | 0.25–50 | 0.9–1.15 | Accuracy of determination of $\gamma$ is[17] $\pm 0.06$; $\gamma$ declines with increasing temperature (Fig. 4). |
| Cu on sapphire | 0.25–50 | 1.1–1.3 | |
| Au on sapphire | 0.25–50 | 0.9–1.45 | |
| Ni on sapphire | 0.25–50 | 0.85–1.2 | |
| Sn on glass[18] | 0.1–50 | 1 | |
| Pb on glass | 1–$10^3$ | 1.1 | |
| Cr on glass[19] | 0.1–50 | 1.1 | |
| Bi on glass | 0.1–50 | 1.0 | |
| Au on sapphire[20] | 1–100 | 1.0–1.1 | |
| Bi, whiskers[21] | 1–100 | 0.9–1.2 | $\gamma$ depends on the crystallographic orientation of the whisker. |
| Cr, island films[22] | 1–2 · $10^5$ | 0.9–1.3 | |
| Pt, island films[15] | 30–3 · $10^4$ | 0.8–1.2 | |
| Ge[23] | 3 · $10^{-2}$–1 | 1 | $\rho_{300} = 50$ ohm · cm |
| Ge in a 0.5 T magnetic field[23] | 3 · $10^{-2}$–1 | 1.1–1.2 | |
| Inversion layers on Si[16] | 0.2–500 | 0.8–1 | Hopping conduction |
| $VO_2$ [13] | 2–2 · $10^4$ | 1.188 | Accuracy $\pm 0.002$ |
| $VO_2$ [13] | 2–2 · $10^4$ | 1.05 | Accuracy $\pm 0.01$ |
| Carbon resistors[24] | 2.5 · $10^{-4}$–10 | 1 | |
| Plane Ge photodiodes[25] | 2 · $10^{-3}$–2 · $10^4$ | 1–1.28 | |
| Bipolar transistor[26] | $10^{-4}$–$10^{-1}$ | 0.86 | Accuracy $\pm 0.02$ |
| Operational amplifiers[12] | $10^{-6.3}$–1 | 1–1.3 | In the greater part of the range $\gamma = 1.3$ (Fig. 2) |
| MOS-field-effect transistors[27] (Si, n-channel) | 75–5 · $10^4$ | 1 | |
| MO-p-Si system | 10–$10^3$ | 1 | |

described by a power law with $\gamma = 1.188 \pm 0.002$. Usually the exponent $\gamma$ is very close to unity, which justifies the name $1/f$ noise. For example, in continuous metal films (of thickness $\sim 10^{-5}$ cm) at room temperature, the values of $\gamma$ averaged over many specimens are[14]: $1.19 \pm 0.07$ (Ag), $1.17 \pm 0.09$ (Cu), $1.08 \pm 0.08$ (Au), $1.20 \pm 0.08$ ($Au_x Ag_{1-x}$), $1.14 \pm 0.08$ (In), $1.16 \pm 0.11$ (Sn), $1.15 \pm 0.10$ (Pb), $1.15 \pm 0.07$ (Pt). The exponent $\gamma$ is very close to unity in carbon resistors and in many other cases. However, very often $\gamma$ differs appreciably from unity, reaching $\sim 0.8$ (island films of Pt,[15] inversion layers on silicon, etc.) and $\sim 1.45$ (Au film on sapphire at $\sim 150$ K[17]). Table I gives the values of $\gamma$ in different systems.

The spectra of low-frequency current noise depend on the temperature. In films of Ag, Au, Ni, and Cu the exponent $\gamma$ declines to several tenths upon increasing the temperature from 200 to 600 K, while passing through (in the case of the first three metals) the value $\gamma = 1$ (Fig. 4). We note that the inaccuracy in determining $\gamma$ in Ref. 17 ($\pm 0.06$) is considerably smaller than the observed variations of $\gamma$. The values $\gamma = 1.188 \pm 0.002$ and $1.05 \pm 0.01$ have been measured in specimens of $VO_2$: the difference between them is also larger than the inaccuracy of the measurements.[13]

In many noise spectra one also sees a small "ripple" and even "waviness." Such a nonmonotonicity of the derivative $\mathrm{d}S/\mathrm{d}f$ most often is found to be within the limits of error of the measurements, but it sometimes exceeds these limits. Figure 5 shows the current-noise spectra of n-InSb specimens having similar electrophysical parameters, while Fig. 6 shows the spectra of the same specimens in a magnetic field. Despite the similarity of the electrophysical parameters, the noise spectra of the different specimens differ not only in magnitude, but even in form. For some specimens the spectra do not fit a power-law dependence on the frequency. A magnetic field alters not only the magnitude but also the very form of the noise spectrum.[29]

Thus the experimental data indicate rather that there is no universal spectrum of low-frequency current noise. Apparently we can conclude from this that there is no unitary mechanism of $1/f$ noise.

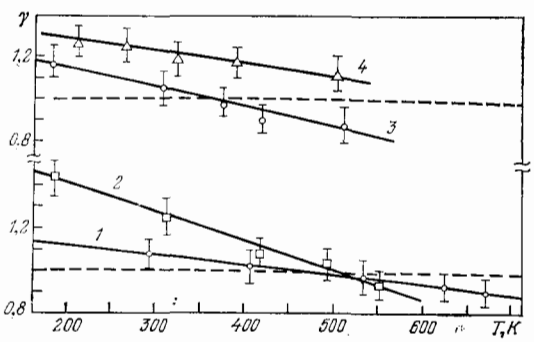


FIG. 4. Temperature-dependence of the exponent $\gamma$ in the spectrum of current noise ($\gamma = -\mathrm{d}\ln S/\mathrm{d}\ln f$) of metal films.[17] *1*—Ag, *2*—Au, *3*—Ni, *4*—Cu.

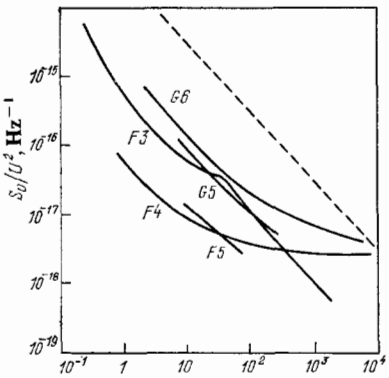


FIG. 5. Spectra of the relative voltage fluctuations in several specimens of highly pure $n$-InSb ($n = 7\times10^{13}$ cm$^{-3}$), $T = 75$ K.[29] The broken line corresponds to the empirical Hooge relationship (2.8.1); the abscissa is the frequency in Hz.

### 2.2. Variance of the noise and the correlation function of the fluctuations in time

As soon as one knows the SD $S_x(f)$ of the stationary noise $x(t)$, one can use the Wiener-Khintchine theorem (1.1) to find the correlation function in time $\psi_{xx}(t_1 - t_2) = \overline{x(t_1)x(t_2)}$ as a function of the time difference $t_1 - t_2$ (we assume that $\bar{x} = 0$):

$$\psi_{xx}(t) = \int_0^\infty df \cos \omega t\, S_x(f). \tag{2.2.1}$$

In particular, when $t_1 = t_2$, the correlation function equals the mean square of the fluctuations, i.e., the variance of the noise:

$$\overline{x^2} = \int_0^\infty df\, S_x(f). \tag{2.2.2}$$

If we substitute here the SD in the form usual for $1/f$ noise, i.e., $S(f) \propto f^{-\gamma}$, then when $\gamma > 1$ the integral in (2.2.2) diverges at the lower limit, while if $\gamma < 1$, then it diverges at the upper limit. If $\gamma = 1$, it diverges at both limits, but only logarithmically (slowly). The divergence of the mean square of the fluctuations of $1/f$ noise is usually treated as a paradox—the best known paradox in this problem.

Of course, no infinite variance is observed in measurements of the statistical characteristics of noise, if only because the frequency band is bounded both below and above in such measurements. It is bounded below either by a filter that hinders input of dc potential to the analyzer, or simply by the finite duration of each realization of the random process. If this time equals $t_m$, then the measurable frequencies of the random process are $\omega \gtrsim t_m^{-1}$. At high frequencies the $1/f$ noise "drowns" in the noise of any other origin. For measurement of the characteristics of the $1/f$ noise itself, one is restricted to the range of frequencies in which it predominates, and the higher frequencies are cut off.

Let us denote by $f_1$ and $f_2$ the minimum and maximum frequencies passed by the measuring system, and let us study the properties of the noise that is actually measured, i.e., with allowance for filtration. We can assume approximately that the SD is generally equal to zero outside the range from $f_1$ to $f_2$, while in this range it is proportional to $1/f$:

$$S(f) = \begin{cases} C_{1/f} f^{-1}, & f_1 \leqslant f \leqslant f_2, \\ 0, & <f_1,\ f>f_2. \end{cases} \tag{2.2.3}$$

Here $C_{1/f}$ is a coefficient that determines the intensity of the noise.

According to (2.2.2), the variance of this noise is

$$\overline{x^2} = C_{1/f} \ln \frac{f_2}{f_1}. \tag{2.2.4}$$

If the lower frequency $f_1 \sim t_m^{-1}$ (see above), then the variance increases with increasing $t_m$.

In the most interesting case we have $f_2 \gg f_1$. As Eq. (2.2.1) implies, the correlation function $\psi_{xx}(t)$ of noise having the SD of (2.2.3) declines with increasing $t$ from the initial value $\psi_{xx}(0) = x^2$ parabolically at first (when $t \ll \omega_2^{-1}$, where $\omega_2 = 2f_2$). Then it declines (when $\omega_2^{-1} \ll t \ll \omega_1^{-1}$) by the logarithmic law.

$$\frac{\psi_{xx}(t)}{\overline{x^2}} \approx 1 - \frac{1}{\ln(f_2/f_1)}(C + \ln \omega_2 t), \tag{2.2.5}$$

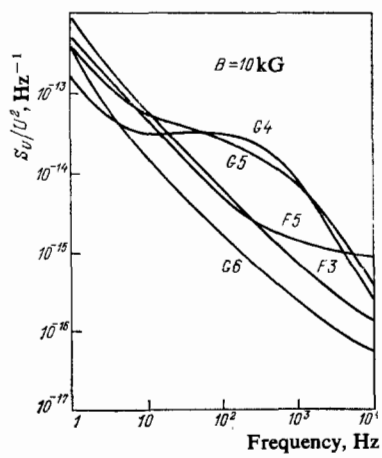


FIG. 6. Spectra of the relative voltage fluctuations in several specimens of highly pure $n$-InSb ($n = 7\times10^{13}$ cm$^{-3}$) in a magnetic field $B = 1$ T, $T = 75$ K.[29] The spectra of the same specimens at $B = 0$ are shown in Fig. 5.

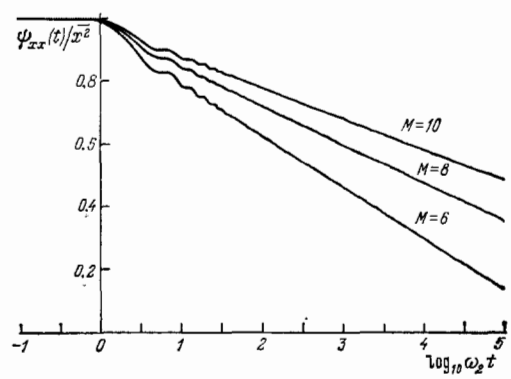


FIG. 7. Autocorrelation function of a stationary random process whose SD is proportional to $1/f$ in the range from $f_1$ to $f_2$ and equals zero outside this range. The numbers on the curves give the values of $M = \log_{10}(f_2/f_1)$.

Here $C = 0.577\ldots$ is Euler's constant. Finally, when $t \gg \omega_1^{-1}$, it has the form of damped oscillations that arise from the sharp cutoff of the SD in (2.2.3) at $f = f_1$. Figure 7 shows graphs of $\psi_{xx}(t)$ for several values of $f_2/f_1$.

Thus, in the fundamental region from $\approx\omega_2^{-1}$ to $\sim\omega_1^{-1}$, the correlation function declines logarithmically with time, i.e., very slowly. The experimentally determined[30] time-dependences of the quantity which, in the case of Gaussian fluctuations is, apart from a coefficient, equal to the correlation function, are presented below in Sec. 2.3. We shall be able to convince ourselves that the experimental dependences are actually logarithmic in the case of a Gaussian distribution function.

### 2.3. Measurements of the distribution function of the fluctuations and of the quantity $\langle x(t)|x(0) = x_0\rangle$

The Gaussian character of noise indicates that it is composed of a large number of random processes. Therefore, to elucidate the nature of $1/f$ noise, it is important to know whether it is a Gaussian random process. Strictly speaking, in order to establish this, one should measure arbitrarily high moments of the fluctuating quantity and compare them with the second moment. Since this is impossible, we should content ourselves with testing the Gaussian character of the first several moments. Of course, one must measure the first distribution function of the fluctuations $x(t)$ and find whether it has a Gaussian form:

$$w_1(x) = \frac{1}{\sqrt{2\pi\overline{x^2}}}\, e^{-x^2/2\overline{x^2}}. \qquad (2.3.1)$$

Such measurements have been made repeatedly (see, e.g., Refs. 30–33). As a rule, whenever the $1/f$ noise is observed in sufficiently pure form (e.g., not mixed with the so-called burst, or pulsed noise), $w_1(x)$ is a Gaussian function.

In Ref. 30 the measurements were performed on five different sources of $1/f$ noise: $A$) a Mosfet; $B$) a carbon resistor; $C$) a reverse-biased collector-base junction of an Si n-p-n transistor (current fluctuations); $D$) a Si n-p-n transistor configured as a common-emitter amplifier (fluctuations of output potential); $E$) a reverse-biased p-n junction (current fluctuations). In contrast to the rest of the sources, the latter showed considerable burst noise. The noise spectra in the studied frequency range (0.03 Hz–5 kHz) were all of $1/f$ type. Figure 8 shows the corresponding distribution functions of the fluctuations $w_1(x)$. We see that in the cases $A$, $B$, and $C$ it is a Gaussian function, while in case $D$ and especially in $E$ the deviations from (2.3.1) are considerable.

In Ref. 30 the quantity $\langle x(t)|x(0) = x_0\rangle$ was measured for each of the five sources, i.e., the mean of the fluctuations at the instant $t$ of time under the condition that the fluctuation was equal to a given value $x_0$ at the initial instant. In each series the magnitude of the fluctuation of $x(t)$ was measured at $2N + 1$ successive instants ($N = 1024$) with an interval $\Delta t = 0.2$ ms, i.e., at instants of time $t_n = n\Delta t$, where the number $n$ took on integral values from $-N$ to $N$. These data were recorded, and then those realizations of the noise (or those series) were selected with a computer for which the magnitude of the fluctuations in the middle of the series ($n = 0$) lay in a small interval around the given value $x_0$. The quantity $\langle x(t)|x(0) = x_0\rangle$ was obtained by averaging over a large number of series of measurements that satisfied the given condition.

The idea of Voss[30] consisted in the following: if the kinetics of the fluctuations in the $1/f$ noise is linear, then the ratio

$$\varphi(x_0, t) = \langle x(t)|\, x(0) = x_0\rangle\, x_0^{-1} \qquad (2.3.2)$$

should not depend on the quantity $x_0$. That is, it should be an identical function of the time $t$ for all $x_0$. The results of the measurements are shown in Fig. 9. For the sources $A$ and $B$ we note that the spectra are closest of all to the $1/f$ law. Their values of $\varphi(x_0, t)$ lie on a single curve for all $x_0$. In the cases $C$ and $D$, the relaxation curves for different $x_0$ differ appreciably, while for source $E$ the dependence on $x_0$ is very great. The conclusion was drawn[30] that the kinetics of different

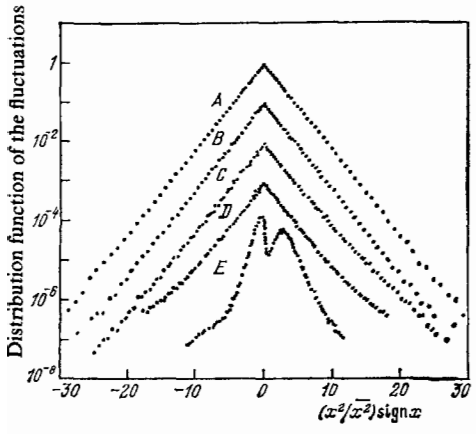


FIG. 8. Distribution functions of the fluctuations $w_1$ of five different sources of current noise described in the text.[30] The curves are shifted with respect to one another for ease of examination.

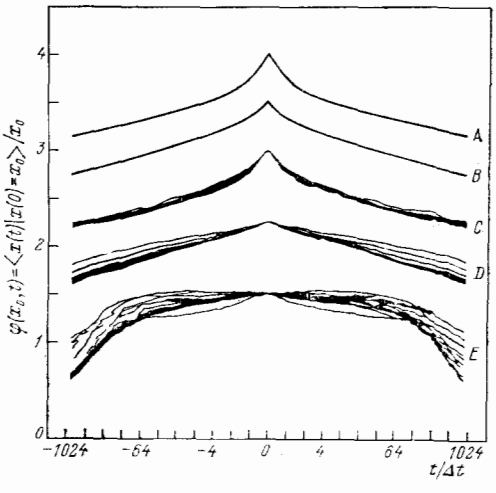


FIG. 9. Time-dependence of the quantity $\varphi(x_0, t)$ [see (2.3.2)] for different values of $x_0$ (see the text).[30] $A$, $B$, $C$, $D$, and $E$ are the different sources of current noise described in the text. The ratio $t/\Delta t$, where $\Delta t = 0.2$ ms, is plotted along the horizontal axis.

systems exhibiting $1/f$ noise can differ—it can be either linear or nonlinear, and hence, no unitary mechanism of $1/f$ noise exists.

One must make this interpretation more precise. As was noted in Ref. 34, Voss's conclusion does not refer to the linearity or nonlinearity of the dynamics of the microscopic processes responsible for the $1/f$ noise. One can speak only of the linearity or nonlinearity of the phenomenological stochastic equations for the fluctuating quantity. Moreover, if the stationary noise amounts to a Gaussian random process, then, as we know, the following equation holds ($\psi_{xx}(t)$ is the autocorrelation function of the fluctuations):

$$\varphi(x_0, t) = \psi_{xx}(t)\,\overline{x^2}^{-1}. \tag{2.3.3}$$

Consequently $\varphi(x_0, t)$ does not depend on $x_0$. Therefore one can interpret the result of Ref. 30 as follows: whenever the distribution function $w_1(x)$ is Gaussian, the $1/f$ noise also satisfies another necessary condition for Gaussian character (qualitative)—the lack of dependence of $\varphi(x_0, t)$ on $x_0$. A quantitative test of (2.3.3) on $x_0$ has not been performed.

Attempts have been made in a number of studies[35–37] to associate the appearance of $1/f$ noise with nonlinearity of the kinetics of the fluctuations. No substantial results have been obtained along this line.

### 2.4 Stationarity of $1/f$ noise

If the noise is stationary, i.e., does not vary with time on the average (for a rigorous definition of a stationary random process, see Ref. 38), the distribution function $w_1(x)$ does not depend on the time, while the correlation function $\psi_{xx}$ depends only on the time difference $t_1 - t_2$. The SD $S(f)$ in the case of stationary noise depends only on the single frequency $f$. Some authors[3,39] have tried to explain the paradoxical properties of $1/f$ noise (the absence of a minimum frequency below which the SD does not rise with decreasing frequency) by its nonstationarity. A mathematical model has been proposed[39] of a nonstationary random process whose mean SD is proportional to $1/f$. However, one can see no connection between this model and any physical processes.

The following idea is the basis of several experiments to reveal a possible nonstationarity of $1/f$ noise.[31–33,40–42] One records a large number of realizations of the noise, each over the course of the time interval $T$. One can find the variance $\overline{x_T^2}$ for each of the obtained realizations. One can find their mean value $x^2$ and also the mean square $\langle(\overline{x_T^2} - \overline{x^2})\rangle$—the variance of the magnitudes of the variance. Nonstationarity of the noise must affect the magnitude of the dimensionless ratio $\langle(\overline{x_T^2} - \overline{x^2})^2\rangle/\langle\overline{x^2}^2\rangle$. With a sufficiently large nonstationarity, this ratio will prove to be too large to be explained within the framework of the assumption of a stationary character of the noise. We shall present only the last of these experiments.[42]

Let $x(t)$ be a stationary random process with the mean value $\bar{x} = 0$, while the correlation function equals $\psi_{xx}(t_1 - t_2)$. The quantity

$$y(t) = \frac{1}{T}\int_0^\infty dt'\, x^2(t - t')\, e^{-t'/T} \tag{2.4.1}$$

amounts to the square of the fluctuations averaged over a time interval such that the end of this interval coincides with the instant $t$ of time, while its effective duration is $T$. The quantity $y(t)$ is random. Equation (2.4.1) implies that its mean value is $\bar{y} = \overline{x^2}$. Under the assumption that $x(t)$ is a stationary Gaussian process, one can express the correlation function of the fluctuations of $y(t) - \bar{y}$ in terms of $\psi_{xx}$:

$$\overline{(y(t) - \bar{y})(y(t+\tau) - \bar{y})} = \int_0^\infty du e^{-u}[\psi_{xx}^2(uT + \tau) + \psi_{xx}^2(uT - \tau)]. \tag{2.4.2}$$

This implies (when $\tau = 0$) that the "variance of the variance" is

$$\overline{[y(t) - \bar{y}]^2} = 2\int_0^\infty du e^{-u}\psi_{xx}^2(uT). \tag{2.4.3}$$

In Ref. 31 the measurements were performed with noise whose SD was proportional to $1/f$ in a certain frequency range from $f_1$ to $f_2$, but zero outside this range [see (2.2.3)]. For such a noise one can easily find $\psi_{xx}(t)$ and $x^2$ and use (2.4.3) to calculate the ratio

$$\eta = \overline{(y - \bar{y})^2}\,(\overline{x^2})^{-2} \tag{2.4.4}$$

as a function of the quantities $\omega_2 T$ and $f_2/f_1$, which can be varied in the experiment, and then to compare the calculation with a direct measurement of $\eta$. In Ref. 42 the experiments were performed on carbon resistors and bipolar transistors. As we see from Fig. 10, the agreement between experiment and calculation is good. If the noise were nonstationary, the experimental points would lie appreciably higher than the calculated points and could even exceed the maximum value of $\eta$, which is 2. No manifestation of nonstationarity of the noise was noted.

Since Eq. (2.4.3) was derived under the assumption that

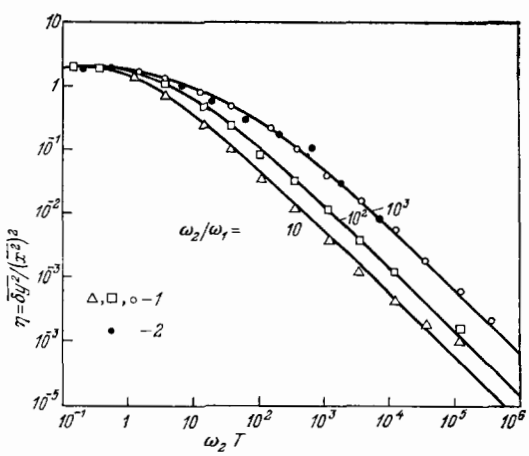


FIG. 10. Dependence of the quantity $\eta$ [Eq. (2.4.4)] on the duration of the time interval $T$ (see text).[42] $\omega_1$ and $\omega_2$ are the minimum and maximum frequencies in the spectrum of $1/f$ noise. Solid curves—calculation. *1*—measurements on carbon and metallic resistors, *2*—measurements on an *n-p-n* transistor.

the noise is Gaussian, one can treat it as a necessary condition for the noise to be actually Gaussian. The agreement of the results of experiment and calculations in Ref. 42 means that this condition is actually fulfilled.[34]

Stationarity of noise has also been tested indirectly: the intensity of the noise in a resistor prepared by ion implantation remained invariant (within the accuracy of measurements of $\pm$ 10%) over 2.5 years, while the intensity of noise in a semiconductor stabilitron remained invariant for 4.5 years (to an accuracy of $\pm$ 20%).[39]

### 2.5. Dependence of 1/f noise on the mean voltage or current

As a rule, the SD of current noise is proportional to the square of the mean voltage $U^2$ or the current $I^2$ throughout the voltage region in which Ohm's law is obeyed. The relationship (1.3) $S_U(f) \propto U^2 \propto I^2$ has been tested on metal films, e.g., in Ref. 43. It breaks down at very high currents at which either the entire specimen is strongly overheated, or the current carriers become hot. It is not surprising that the dependence of the SD of the noise on the bias in semiconductor structures, devices, etc., is more complex: an increase in the voltage alters the barriers for the current carriers, etc.

An ohmic relationship between the mean current and voltage does not always correspond to a linear dependence of the SD of the noise of $U^2$. In island Pt films one finds $S_U \propto U^\beta$, where the exponent $\beta$ in different specimens varies from 1 to 4.[15] In granular composites consisting of Ni particles in an $Al_2O_3$ matrix we have $S_U \propto U^2$ at small $U$ ($\lesssim$ 2.5 V), while at higher $U$ (up to 35 V) $S_U \propto U$, although the deviations from Ohm's law are inappreciable throughout this voltage region.[44] Another example is the contact noise in specimens of n-InSb.[45] Although the resistance of these contacts is very small, while rectification is completely inappreciable at them, the contact $1/f$ noise varies severalfold (from 6 to 10 times) upon reversing the direction of the current (the noise is larger when the potential of the contact is negative with respect to the specimen).

The problem of the dependence of $S_U(f)$ on $U$ involves the question: does the electric current cause the observed $1/f$ noise, or does it only "manifest" the fluctuations (of resistance) that occur also in the absence of a current, i.e., in a state of thermodynamic equilibrium? The opinion has been expressed in the literature that $1/f$ noise stems from the instability of the gas of current carriers caused by passage of a current. This mechanism contradicts not only the usually observed relationship $S_U \propto U^2$, but also a specially designed experiment of Voss and Clarke.[43] The idea of the experiment consists in the following. The SD of the equilibrium voltage fluctuations depends on the resistance $R$ of the specimen [see (1.2)]. If the resistance fluctuates, then the equilibrium noise itself fluctuates at the same frequencies. By measuring the spectrum of low-frequency fluctuations of the equilibrium noise ("noise of the noise"), one can find the SDF of the resistance $S_R(f)$, and convince oneself that it is proportional to $1/f$ at low frequencies, as is implied by other experiments—by measuring the SD of the voltage fluctuations $S_U(f)$ when a current is flowing. The important point is that, in contrast to the experiments to measure $S_U(f)$, no voltage was applied to the specimen in the experiment of Voss and Clarke ($\overline{U} = 0$, $\overline{I} = 0$), and only fluctuational currents flow in it.

This idea is realized as follows. The fluctuational voltage $\delta U(t)$ taken from the conductor passes through a filter with a passband from some frequency $\nu_1$ to the frequency $\nu_2$. The signal $\delta U_{\Delta\nu}(t)$ obtained after filtration is squared. The quantity $P(t) = \delta U^2_{\Delta\nu}(t)$ fluctuates about the mean value given by the Nyquist formula (1.2):

$$\overline{P} = \int_{\nu_1}^{\nu_2} d\nu \cdot 4kT \operatorname{Re} Z(\nu). \qquad (2.5.1)$$

Here we have ($C$ is the capacitance of the conductor, and $\tau = RC$):

$$\operatorname{Re} Z(\nu) = R(1 + 4\pi^2\nu^2\tau^2)^{-1}.$$

Experimentally one measures the SD $S_P(f)$ of the fluctuations $\delta P(t) = P(t) - \overline{P}$ at low frequencies $f \ll \nu_1$.

The low-frequency fluctuations of $P(t)$ arise from two sources: fluctuations of the temperature $T$ and the resistance $R$, which enter into the expression for $\overline{P}$ (2.5.1), and also from the random character of the movement and scattering of the current carriers in the conductor. The second mechanism would give rise to fluctuations of $\delta P(t)$ [we shall denote these collateral fluctuations as $\delta P_{\text{ext}}(t)$], even if $T$ and $R$ do not fluctuate. Thus we have

$$\delta P(t) = \frac{\partial \overline{P}}{\partial T}\delta T(t) + \frac{\partial \overline{P}}{\partial R}\delta R(t) + \delta P_{\text{ext}}(t). \qquad (2.5.2)$$

In the second term on the right-hand side, $\delta R$ amounts to the part of the fluctuations of the resistance that does not involve the fluctuations $\delta T$.

Since the individual terms on the right-hand side of (2.5.2) are uncorrelated, their SDs add:

$$S_P(f) = \left(\frac{\partial \overline{P}}{\partial T}\right)^2 S_T(f) + \left(\frac{\partial \overline{P}}{\partial R}\right)^2 S_R(f) + S_{P_{\text{ext}}}(f). \qquad (2.5.3)$$

One can show[46] that, if the equilibrium fluctuations are Gaussian for fixed $T$ and $R$, then we have the following relationship at frequencies $f \ll \nu_1$:

$$S_{P_{\text{ext}}} = 2\int_{\nu_1}^{\nu_2} d\nu S_U^2(\nu) \qquad (2.5.4)$$

This does not depend on the frequency $f$. Therefore, if $S_T$ or $S_R$ contains a component proportional to $1/f$, then it specifically becomes predominant in the SD $S_P(f)$ at low enough frequencies.

In Ref. 43 the measurements were performed on an InSb specimen and an Nb film, while in Ref. 47 they were performed on carbon resistors according to the same scheme. As we see from Fig. 11, which was taken from Ref. 47, we actually find $S_P \propto 1/f$ at low frequencies. The magnitude of $S_R$ found from these measurements by using (2.5.3) agrees with the magnitude of $S_R$ found in the usual way from measurements of the current noise by (1.3). For an analysis of experiments of the type of Refs. 43 and 47, see Ref. 48.

Thus the experiments to observe low-frequency fluctu-

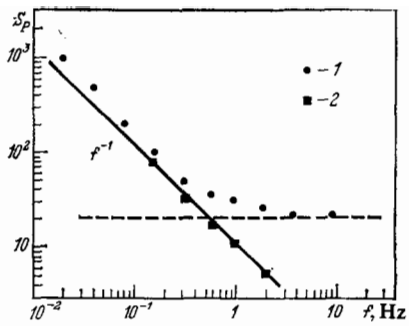


FIG. 11. Spectrum of the fluctuations of the square of the Johnson (Nyquist) fluctuations in a carbon resistor.[47] *1*—experiment, *2*—after subtracting the white noise, which is shown by the dotted line; solid line—$1/f$ law.

ations of equilibrium electric noise in the absence of current confirm that $1/f$ noise is caused by fluctuations of the resistance of the conductor, and the current usually serves only to detect these fluctuations.

A set of other experimental data confirming this conclusion has been obtained in Ref. 49.

### 2.6. Anisotropy of conductivity fluctuations

In spatially homogeneous isotropic media and crystals of the cubic system in the absence of a magnetic field, the electric-conductivity tensor is $\sigma_{ij} = \sigma\delta_{ij}$. That is, it reduces to the scalar quantity $\sigma$. However, fluctuations of the local electric conductivity $\delta\sigma_{ij}(\mathbf{r}, t)$ can be a tensor quantity even in an isotropic medium. It reduces to a scalar quantity, e.g., in the case when it arises from fluctuations of the temperature $\delta T$ or of the concentration $\delta n$ of current carriers. If the noise source is temperature fluctuations, then we have

$$\delta\sigma_{ij} = \frac{\partial\sigma_{ij}}{\partial T}\delta T = \delta_{ij}\frac{\partial\sigma}{\partial T}\delta T.$$

We should expect that spontaneous displacement of defects in the crystal is a source of anisotropic conductivity fluctuations. Therefore, in order to elucidate the mechanism of bulk current noise, it is useful to determine whether the corresponding electric-conductivity fluctuations are isotropic or anisotropic.

If the correlation radius of the electric-conductivity fluctuations is small in comparison with the dimensions of the region of the specimen that governs the measurable current fluctuations, one can consider the correlation function of the electric-conductivity fluctuations at two points $\mathbf{r}_1$ and $\mathbf{r}_2$ to be proportional to $\delta(\mathbf{r}_1 - \mathbf{r}_2)$ (for spatial correlation of conductivity fluctuations, see Sec. 2.7):

$$\langle\delta\sigma_{ij}(\mathbf{r}_1, t_1)\,\delta\sigma_{kl}(\mathbf{r}_2, t_2)\rangle = F_{ijkl}(t_1 - t_2)\,\delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (2.6.1)$$

The symmetry of the tensor $F_{ijkl}$ introduced here reflects the anisotropy of the conductivity fluctuations. In particular, in the case of isotropic fluctuations we have $F_{ijkl} = F\delta_{ij}\delta_{kl}$, where $F$ is a certain scalar function.

In the general case one can represent the fluctuation of the electric-conductivity tensor as the sum of three terms:

$$\delta\sigma_{ij} = \delta_{ij}\delta\sigma + \delta\sigma_{ij}^{(s)} + \delta\sigma_{ij}^{(a)},$$
$$\delta\sigma = \frac{1}{3}\operatorname{Sp}\widehat{\delta\sigma}, \quad \delta\sigma_{ij}^{(a)} = \frac{1}{2}(\delta\sigma_{ij} - \delta\sigma_{ji}), \quad (2.6.2)$$
$$\delta\sigma_{ij}^{(s)} = \frac{1}{2}(\delta\sigma_{ij} + \delta\sigma_{ji}) - \frac{1}{3}\delta_{ij}\operatorname{Sp}\widehat{\delta\sigma}.$$

The first term on the right-hand side of (2.6.2) is the scalar (isotropic) component of the fluctuation; $\delta\sigma_{ij}^{(s)}$ forms an irreducible second-order tensor; $\delta\sigma_{ij}^{(a)}$ is an antisymmetric tensor. This subdivision of the conductivity fluctuations into components of definite tensor symmetry is analogous to the subdivision of the fluctuations of the dielectric permittivity in the theory of scattering of electromagnetic waves.[50] In an isotropic medium only fluctuations of the same tensor symmetry are correlated. Therefore we have

$$F_{ijkl} = F_0\delta_{ij}\delta_{kl} + F_s\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right)$$
$$+ F_a(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}). \quad (2.6.3)$$

In studying $1/f$ noise, the quantities of interest are the frequencies small in comparison with the reciprocal kinetic times of the current carriers and phonons, and also the lengths that exceed the kinetic lengths. We should expect that the state of a system containing fluctuations of the electric conductivity is quasiequilibrium in this case, while the fluctuation satisfies Onsager's symmetry principle $\delta\sigma_{ij}(\mathbf{r}, t; \mathbf{B}) = \delta\sigma_{ji}(\mathbf{r}, t; -\mathbf{B})$, where $\mathbf{B}$ is the magnetic field. This implies that when $\mathbf{B} = 0$ the function $F_{ijkl}$ is symmetric, not only with respect to transposition of the first and second pairs of indices, but also with respect to transposition of indices within each pair. Here we also have $\delta\sigma_{ij}^{(a)} = 0$, $F_a = 0$, and in an isotropic medium the correlation function (and also the SD) of the fluctuations of electric conductivity is described by the two functions: $F_0$ and $F_s$ (by three functions in cubic crystals). The magnitude of the ratio of Fourier transforms $F_s(\omega)/F_0(\omega)$ is a measure of the anisotropy of the fluctuations of electric conductivity at the given frequency $f = \omega/2\pi$.

References 51 and 52 have proposed studying the anisotropy of fluctuations of conductivity by using specimens in the form of a "Maltese cross" (Fig. 12) with four arms. Each arm has two mutually separated contacts: one current

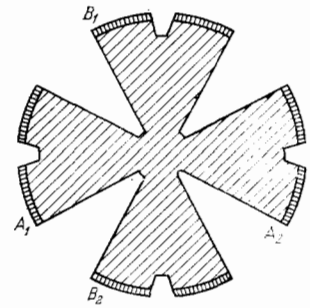


FIG. 12. Shape of specimens used for studying the anisotropy of electric-conductivity fluctuations.[51]

contact, the other potentiometric. The voltage fluctuations at the potentiometric contacts upon passing a current are governed by the fluctuations of the electric conductivity in the central (narrow) region of the cross. Under the condition that the currents $I_A$ (between the contacts $A_1$ and $A_2$) and $I_B$ (between $B_1$ and $B_2$) are fixed, the corresponding voltage fluctuations are equal to

$$\delta U_A = I_A \delta R_{AA} + I_B \delta R_{AB}, \quad \delta U_B = I_A \delta R_{AB} + I_B \delta R_{BB}. \tag{2.6.4}$$

One can find by using the phenomenological theory[53] that

$$\delta R_{AB}(t) = \int \mathrm{d}\mathbf{r}\, e_{Ai}(\mathbf{r})\, \delta\sigma_{ij}(\mathbf{r}, t)\, e_{Bj}(\mathbf{r}). \tag{2.6.5}$$

Here the vector $\mathbf{e}_A(\mathbf{r})$ is equal to the ratio of the ohmic field $\mathbf{E}(\mathbf{r})$ at the point $\mathbf{r}$ caused by the passage of the current $I_A$ to the magnitude of this current. One can easily write expressions analogous to (2.6.5) for $\delta R_{AA}$ and $\delta R_{BB}$. Equations (2.6.1) and (2.6.5) imply that the SD of the fluctuations $\delta R_{AB}$ equals

$$\langle \delta R_{AB} \delta R_{AB} \rangle_f = 2F_{ijkl}(\omega) \int \mathrm{d}\mathbf{r} e_{Ai} e_{Bj} e_{Ak} e_{Bl}. \tag{2.6.6}$$

By measuring the SD of the voltages $S_{UA}$ and $S_{UB}$ and the cross-SD $S_{UAB}$ for different $I_A$ and $I_B$ values, one can find the SDs $\langle \delta R_{AA} \delta R_{BB} \rangle_f$ and $\langle \delta R_{AB} \delta R_{AB} \rangle_f$, and find the components of the tensor $F_{ijkl}$ from them by using (2.6.6). Here one can either calculate the distribution of the quantities $\mathbf{e}_A(\mathbf{r})$ and $\mathbf{e}_B(\mathbf{r})$ or measure it on an analog model.

Let us substitute the expression (2.6.3) for the tensor $F_{ijkl}$ of an isotropic medium into (2.6.6) and into an analogous formula for $\langle \delta R_{AA} \delta R_{BB} \rangle_f$. One can find that

$$\frac{F_s}{F_0} = \frac{G+1-Q}{\left(G+\frac{4}{3}\right)Q - \frac{2}{3}(2-G)}. \tag{2.6.7}$$

Here we have

$$Q = \frac{\langle \delta R_{AA} \delta R_{BB} \rangle_f}{\langle \delta R_{AB} \delta R_{AB} \rangle_f}, \quad G = \frac{\int d\mathbf{r} ([\mathbf{e}_A \mathbf{e}_B])^2}{\int d\mathbf{r} (\mathbf{e}_A \mathbf{e}_B)^2}. \tag{2.6.8}$$

The specimens studied in Ref. 52 had $G = 1.35$. If one substitutes the values of $Q$ measured in Ref. 52 into (2.6.7), then one obtains for the two carbon films $F_s/F_0 = 0.43$ and 0.56, for the Au film 0.65, and for the two Cr films 1.18 and 3.42. An even more appreciable anisotropy of conductivity fluctuations has been observed in Bi.[54]

The considerable anisotropy of the low-frequency fluctuations of the conductivity indicates that they arise from some mechanism other than temperature fluctuations.

References 55 and 97 have treated conductivity fluctuations caused by spontaneous hopping of defects whose symmetry is lower than the point symmetry of the crystal between several positions of identical energy but with different orientation in the lattice. In this case we have $\mathrm{Sp}\,\hat{\delta}\sigma = 0$, $F_0 = 0$, and the fluctuations are purely anisotropic.

### 2.7. Correlation length of electric-conductivity fluctuations and dependence of the noise on the dimensions of the conductor

Studies of the correlation length of electric-conductivity fluctuations have been performed mainly in connection with testing the model in which the $1/f$ noise arises from temperature fluctuations (Sec. 4). It has been found[20] that the fluctuations with a $1/f$ spectrum in two Au films isolated from one another with a SiO layer 0.6-$\mu$m thick are uncorrelated. There is no correlation between the fluctuations in two adjacent regions of a thin and narrow metal film lying at a distance $\sim 1$ mm.[19] No correlation of the current fluctuations was found in two adjacent transistors of an integrated circuit (the distance between the transistors was several tens of $\mu$m).[56] There is no correlation between the conductivity fluctuations of different regions of an inversion channel on p-Si (distance $\sim 0.1$ mm).[57] This means that the correlation length of the fluctuations causing noise was in any case smaller than the stated lengths.

If the conductor is homogeneous, the conductivity fluctuations are of bulk type (rather than involving, say, the surface or a contact) and the correlation radius of these fluctuations is small in comparison with the dimensions of the conductor [i.e., Eq. (2.6.1) holds], then the SD of the relative voltage or current fluctuations is inversely proportional to the volume $V$ of the conductor: $S_U/U^2 \propto V^{-1}$. Actually one can express the resistance of the conductor in terms of the quantity $\mathbf{e}(\mathbf{r})$ introduced above:

$$R = \sigma \int \mathrm{d}\mathbf{r} \mid \mathbf{e}(\mathbf{r}) \mid^2.$$

Equations (1.3) and (2.6.6) imply that

$$\frac{S_U(f)}{U^2} = \frac{S_R(f)}{R^2} = 2F_{ijkl}(\omega) \frac{\int \mathrm{d}\mathbf{r} e_i e_j e_k e_l}{\left(\sigma \int \mathrm{d}\mathbf{r} \mid \mathbf{e}(\mathbf{r}) \mid^2\right)^2}. \tag{2.7.1}$$

When all the dimensions of the conductor vary in the same way, or when one dimension varies in a direction in which $\mathbf{e}(\mathbf{r})$ does not vary, the expression on the right-hand side of (2.7.1) varies as $V^{-1}$.

The author of Ref. 58 measured the SD of the $1/f$ noise of complex resistors put together by connecting $m$ nominally identical carbon resistors in series and parallel such that the total resistance matched the resistance of a single resistor. It turned out that the SD of the relative fluctuations of the voltage was proportional to $1/m$, i.e., the reciprocal volume of the complex resistor.

In thin Pt films and filaments, the SD of the noise obeys the law $S_U f/U^2 \propto N^{-1}$, where $N$ is the number of Pt atoms in the specimen, over the vast interval of variation of $N$ from $\sim 10^7$ to $\sim 10^{14}$.[59]

If the correlation radius of the conductivity fluctuations were larger than the dimensions of the specimen, the SD of the relative voltage fluctuations with a current flowing would not vary at all upon changing these dimensions. For example, this would happen if the source of $1/f$ noise were any geophysical or cosmophysical phenomenon (such hypotheses have also been advanced). The experimentally observed small correlation length of the current $1/f$ noise enables one to rule out such explanations of this noise.

### 2.8. Dependence of $1/f$ noise on the number of free current carriers in the conductor. Empirical Hooge relationship

In the cases most often encountered in which the SD of the current noise is $S_U \propto U^2$ and the exponent $\gamma$ in the fre-

quency-dependence of the SD($f^{-\gamma}$) is close to unity, one can represent the SD of the relative fluctuations in the form $S_U/U^2 = C/f$, where $C$ is a dimensionless coefficient. Hooge[60] correlated a large bulk of experimental data obtained by different authors on the magnitude of $1/f$ noise in semiconductors and metal films. He plotted these data on the $C$–$N_c$ plane, where $N_c = nV$ is the total number of free current carriers in the specimen, $n$ is their concentration, and $V$ is the volume. Each point pertained to a certain semiconductor or metal. The distribution of the points revealed a qualitative rule, which Hooge approximated with the formula $C = \alpha/N$. He assumed that $\alpha = 2\times10^{-3}$ is a universal coefficient. The following empirical relationship has become termed the Hooge relationship in the literature:

$$\frac{S_U(f)}{U^2} = \frac{\alpha}{N_c f}. \qquad (2.8.1)$$

The constant $\alpha$ was called the Hooge constant and was denoted as $\alpha_H$.

The qualitative tendency toward decrease in the $1/f$ noise with decreasing resistivity $\rho$ of the specimen (while $\rho \propto n^{-1}$) was noted as early as the fifties in Ge specimens.[61] The known fact pertains here also that the $1/f$ noise in metals is smaller by orders of magnitude than in semiconductors. Hooge was the first to try to impart the form of a quantitative relationship to this qualitative tendency.

If the Hooge relationship were exact, and this implies universal, it would be a convincing argument favoring a unitary mechanism of $1/f$ noise. However, even a superficial analysis of the experimental data shows that this relationship is not exact, and hence cannot be universal. For example, the noise in metallic films is known to depend strongly (varying by two orders of magnitude or more) on the temperature and the type of substrate, although the number of free carriers does not vary in all these changes. In Bi specimens the $1/f$ noise is approximately the same as in metallic films of the same dimensions, despite the fact that the concentration of free carriers in the semimetal Bi is smaller by several orders of magnitude.

The conclusion drawn in Ref. 60 that the mechanism of $1/f$ noise is always of bulk type contradicts the known effects of strong variation in the noise upon changing the conditions at the surface (Sec. 5). The decrease in noise in semiconductors with increasing $n$ can also be explained within the framework of a surface mechanism of noise: with increasing $n$ the thickness decreases of the region of space charge where the conductivity is altered by the charge fluctuations in surface states.

Table II presents the values of

$$\alpha = \frac{S_U(f)\, fnV}{U^2}$$

for a number of semiconductors and metals. It is interesting to compare them with the Hooge constant $\alpha_H = 2\times10^{-3}$. If the experimental values of $\beta$ always deviated from $\alpha_H$ on the high side, one could assume a universal noise described by the Hooge relationship ("$\alpha$-noise"), accompanied by some sort of other noise with the same spectrum. However, often we have $\alpha < \alpha_H$. For example, in certain specimens of n-InSb, whose spectra are shown in Fig. 5, $\alpha$ is smaller by two orders of magnitude than $\alpha_H$ (but there are specimens of n-

TABLE II. Values of $\alpha = S_U f nV/U^2$ in different conductors.

| Material | $T$, K | $n$, cm$^{-3}$ | $V$, cm$^3$ | $C$ | $\alpha$ | References |
|---|---|---|---|---|---|---|
| n-Ge | 300 | $0{,}8\cdot10^{15}$ | $2{,}1\cdot10^{-3}$ | $6\cdot10^{-16}$ | $1\cdot10^{-3}$ | 62 |
| n-Ge | 300 | $1{,}5\cdot10^{14}$ | $1{,}8\cdot10^{-3}$ | $1{,}3\cdot10^{-14}$ | $3{,}5\cdot10^{-3}$ | 61 |
| n-Ge | 300 | $1{,}3\cdot10^{14}$ | $1{,}8\cdot10^{-3}$ | $7{,}7\cdot10^{-16}$ | $1{,}8\cdot10^{-4}$ | 61 |
| n-Ge | 300 | $5\cdot10^{13}$ | $1{,}8\cdot10^{-5}$ | $6\cdot10^{-13}$ | $6\cdot10^{-4}$ | 63 |
| p-Ge | 300 | $8{,}7\cdot10^{15}$ | $1{,}8\cdot10^{-3}$ | $3{,}2\cdot10^{-16}$ | $5\cdot10^{-3}$ | 61 |
| InSb | 300 | $2{,}8\cdot10^{16}$ | $2{,}7\cdot10^{-5}$ | $6{,}9\cdot10^{-14}$ | $5{,}1\cdot10^{-2}$ | 64 |
| InSb | 300 | $4{,}6\cdot10^{16}$ | $2{,}3\cdot10^{-5}$ | $5{,}5\cdot10^{-14}$ | $5{,}5\cdot10^{-2}$ | 64 |
| InAs | 300 | $1{,}2\cdot10^{18}$ | $2{,}5\cdot10^{-6}$ | $5{,}6\cdot10^{-15}$ | $1{,}8\cdot10^{-3}$ | 64 |
| n-InSb | 76 | $1\cdot10^{14}$ | $2{,}1\cdot10^{-4}$ | $2\cdot10^{-14}$ | $4\cdot10^{-4}$ | 65 |
| n-InSb | 76 | $1\cdot10^{14}$ | $1{,}1\cdot10^{-3}$ | $3\cdot10^{-16}$ | $3{,}4\cdot10^{-5}$ | 65 |
| n-InSb | 77 | $1{,}6\cdot10^{14}$ | $3{,}2\cdot10^{-5}$ | $2\cdot10^{-13}$ | $1\cdot10^{-3}$ | 66 |
| n-InSb | 295 | $1{,}6\cdot10^{16}$ | $3{,}2\cdot10^{-5}$ | $8\cdot10^{-15}$ | $4\cdot10^{-3}$ | 66 |
| n-InSb | 77 | $1{,}6\cdot10^{14}$ | $1{,}1\cdot10^{-4}$ | $9\cdot10^{-14}$ | $1{,}6\cdot10^{-3}$ | 66 |
| n-InSb | 295 | $1{,}6\cdot10^{16}$ | $1{,}1\cdot10^{-4}$ | $1{,}6\cdot10^{-15}$ | $2{,}7\cdot10^{-3}$ | 66 |
| p-InSb | 77 | $1{,}2\cdot10^{16}$ | $1{,}4\cdot10^{-4}$ | $4\cdot10^{-15}$ | $7\cdot10^{-3}$ | 66 |
| p-InSb | 295 | $1{,}6\cdot10^{16}$ | $1{,}4\cdot10^{-4}$ | $1{,}6\cdot10^{-15}$ | $3{,}7\cdot10^{-3}$ | 66 |
| n-GaAs | 295 | $2{,}3\cdot10^{16}$ | $8\cdot10^{-4}$ | $3\cdot10^{-16}$ | $6\cdot10^{-3}$ | 66 |
| p-GaAs | 295 | $2{,}3\cdot10^{16}$ | $1\cdot10^{-3}$ | $7{,}5\cdot10^{-17}$ | $1{,}7\cdot10^{-3}$ | 66 |
| p-GaAs | 295 | $2{,}3\cdot10^{16}$ | $5{,}1\cdot10^{-4}$ | $5\cdot10^{-16}$ | $5\cdot10^{-3}$ | 66 |
| n-GaP | 295 | $2{,}9\cdot10^{16}$ | $1{,}1\cdot10^{-4}$ | $3\cdot10^{-16}$ | $9\cdot10^{-3}$ | 66 |
| Au | 300 | $5{,}9\cdot10^{22}$ | — | — | $2{,}4\cdot10^{-3}$ | 67 |
| Au | 308 | $5{,}9\cdot10^{22}$ | — | — | $4\cdot10^{-3}$ | 17 *) |
| Cu | 490 | $8{,}4\cdot10^{22}$ | — | — | $4{,}8\cdot10^{-2}$ | 17 **) |
| Ag | 410 | $5{,}8\cdot10^{22}$ | — | — | $6{,}8\cdot10^{-2}$ | 17 |
| Bi | 300 | $2{,}8\cdot10^{18}$ | $1{,}1\cdot10^{-10}$ | $1{,}3\cdot10^{-14}$ | $4{,}3\cdot10^{-6}$ | 43 |
| Sn | 300 | — | — | — | $0{,}7\cdot10^{-3}$ | 68 ***) |
| Sn | 300 | — | — | — | $5{,}4\cdot10^{-3}$ | 68 ****) |

*$\alpha$ increases by a factor of 50 with increase of $T$ from 100 to 500 K.
**$\alpha$ varies by almost three orders of magnitude with increase of $T$ from 100 to 410 K.
***Film on sapphire with an intermediate substrate (Sn oxide).
****Film on glass.

InSb in which $\alpha$ is an order of magnitude larger than $\alpha_H$).

As a rule, films made of the same metal, prepared by the very same technique and having approximately the same resistivity $\rho$ strongly differ in the magnitude of $1/f$ noise (the difference can exceed an order of magnitude).[14] It is completely unknown what factors are responsible for this. However, if one takes for each metal a specimen having the minimum value of $S_U(f)N/U^2$, where $N$ is the number of atoms in the film, and plots these quantities as a function of $\rho$, then one obtains an empirical relationship[14] that makes the Hooge relationship (2.8.1) more precise:

$$\left[\frac{S_U(f)\,N}{U^2}\right]_{\min} = \frac{\rho_0}{\rho}\,\frac{1}{f^\gamma}.$$

Here the value of $\rho_0$ is $6\times 10^{-9}$ ohm·cm for $\gamma \approx 1$.

It would also be incorrect to undervalue the Hooge relationship. It is the sole relationship that allows one, however crudely, to estimate the magnitude of the SD of $1/f$ noise in homogeneous conductors with an accuracy that most often is no poorer than one or two orders of magnitude.

### 2.9. Observations of $1/f$ noise at low temperatures

Many studies of $1/f$ noise at liquid-nitrogen temperature exist (see Table II). The question is natural whether this noise disappears at very low temperatures.

In Ref. 16 $1/f$ noise was studied in inversion layers on Si at 4.2 K under conditions in which the conduction is of hopping type. According to Ref. 57, the noise in an inversion layer on p-Si even increases upon lowering the temperature from 4.2 to 1.5 K.

Also, $1/f$ noise constitutes a substantial interference in the operation of SQUIDs (employing the Josephson effect) at low frequencies.[69]

### 2.10. Possible influence of infralow-frequency fluctuations on measurements of $1/f$ noise

Shul'man[70] has expressed an interesting idea. Let us assume that a mechanism exists in a conductor of resistance fluctuations with a very long relaxation time $\tau$ such that $\tau^{-1} \ll f_1$, where $f_1$ is the lower frequency of the range in which one is measuring the spectrum of low-frequency noise. Such a fluctuation process was called an infralow-frequency (ILF) process in Ref. 70. One can treat the spectrum of a fluctuational process with a definite relaxation time $\tau$ as a Lorentzian line centered on zero frequency, and the part of the spectrum corresponding to frequencies $f \gg \tau^{-1}$ as the outer tail of this line. As is known, measurement of the shape of a line requires that the bandwidth $\Delta f$ of the spectrum analyzer must be small in comparison with the line width $\tau^{-1}$. Usually in measuring the spectrum of low-frequency noise one can ensure fulfillment only of the condition of a sufficiently good $Q$-factor $f \gg \Delta f$. Yet the breakdown of the requirement $\Delta f \ll \tau^{-1}$ can cause the response spectrum of the analyzer to prove to be proportional, not to the true spectrum of the ILF noise, but to some integral of this spectrum over a region of frequencies $f$ considerably smaller than the central frequency $f_0$ of the analyzer (for each given $f_0 > f_1$). Although the response of the analyzer is small in this region, actually it is not zero. It was shown[70] that, if the measurements are performed with a constant $Q$-factor: $f_0/\Delta f = \text{const}$, the response of the analyzer to the ILF noise varies with $f_0$ as $1/$ Thus attention is called in Ref. 70 to the possibility that the $1/f$ noise being measured results from an inadequate measurement of ILF noise, which in actuality has some other spectrum.

This remark[70] is correct in principle, but one can advance a number of arguments favoring the idea that the $1/f$ noise usually measured is genuine. First, if the measured $1/f$ spectrum were the result of incorrect filtration, its intensity and the very shape of the spectrum would depend on the spectral apparatus employed, and this has never been noted by anyone. Moreover, the results would depend on the characteristics of the filters at frequencies $f$ smaller by orders of magnitude than the central frequency $f_0$, i.e., in the frequency region where the response of the analyzer is nominally considered negligibly small. Second, studies are known (e.g., Ref. 42) that have obtained excellent agreement of the quantities being measured (in Ref. 42—the variance of the magnitudes of the variance of the fluctuations: see Sec. 2.4) with the same quantities calculated under the assumption that the noise spectrum in the frequency range being studied actually has a $1/f$ form. Yet a direct experimental test of the possibility pointed out in Ref. 70 is desirable as applied to different types of systems that exhibit $1/f$ noise.

## 3. THE MODEL OF AN EXPONENTIALLY BROAD DISTRIBUTION OF RELAXATION TIMES

### 3.1. General idea

Let $x(t)$ be a fluctuating quantity with zero mean. In the simplest case in which the kinetics of the fluctuations is characterized by a single relaxation time $\tau$, the correlation function is $\psi_{xx} = \overline{x^2}\exp(-|t_1 - t_2|/\tau)$, while the SD of the noise has the form of a Lorentz function:

$$S_x(f) = 4\int_0^\infty dt\,\psi_{xx}(t)\cos\omega t = \overline{x^2}\,\frac{4\tau}{1+\omega^2\tau^2}. \qquad (3.1.1)$$

In more complex cases one can describe the kinetics of the fluctuations with several relaxation times. In the general case a continuous distribution of relaxation times can exist with the distribution function $p(\tau)$. Then we have

$$S_x(f) = \int_0^\infty d\tau\,p(\tau)\,\frac{4\tau}{1+\omega^2\tau^2}. \qquad (3.1.2)$$

Since the mean square (variance) of the fluctuations is

$$\overline{x^2} = \int_0^\infty df\,S_x(f) = \int_0^\infty d\tau\,p(\tau), \qquad (3.1.3)$$

the quantity $p(\tau)d\tau$ is the contribution to the variance of the processes whose relaxation times lie in the interval from $\tau$ to $\tau + d\tau$.

If $p(\tau) \propto 1/\tau$ in some interval from $\tau_1$ to $\tau_2 \gg \tau_1$, but is zero outside this interval, then according to (3.1.2), we have

$S_x \propto 1/f$ in the frequency range $\tau_2^{-1} \ll f \ll \tau_1^{-1}$.[71] One obtains the required distribution $p(\tau)$ if the processes responsible for the noise are activational, i.e., $\tau = \tau_0 \exp(E/kT)$, where $E$ is the activation energy, $\tau_0^{-1}$ is the frequency of attempts to overcome the activation barrier, and the distribution of activation energies $F(E)$ depends weakly on $E$ over some sufficiently broad interval.[72,73] Actually we have $p(\tau) = (kT/\tau) F(E)$. When $F(E) = \text{const}$, $p(\tau) \propto 1/\tau$. An analogous situation arises whenever the relaxation is governed by the tunneling of particles: in this case $\tau$ depends exponentially on the distance through which the particles must tunnel.[74]

For the sake of definiteness, let us study the noise arising from activational processes with a certain distribution of activation energies $F(E, T)$. For example, a dependence of $F$ on the temperature $T$ can arise in the case in which the mean concentration of defects whose fluctuations in number or orientation create the noise is not fixed, but increases with $T$. The SD of the noise is

$$S(f) = \int_0^\infty dE\, F(E, T) \frac{4\tau_0 e^{E/kT}}{1+\omega^2\tau_0^2 e^{2E/kT}} = \frac{1}{\pi f} \int_0^\infty dE \frac{F(E, T)}{\cosh[(E-E_\omega)/kT]}. \quad (3.1.4)$$

In the last integral we have introduced the notation $E_\omega = kT \ln(\omega\tau_0)^{-1}$. In condensed media we have $\tau_0^{-1} \sim 10^{12}$–$10^{14} s^{-1}$. At the frequencies at which one observes $1/f$ noise, the magnitude of $(\omega\tau_0)^{-1}$ is $\gg 1$, and even $\ln(\omega\tau_0)^{-1} \gg 1$. Therefore the energy $E_\omega$ can be of the order of ordinary activation energies ($\sim 1$ eV).

In the integral of (3.1.4), the coefficient of $F(E, T)$ as a function of $E$ constitutes a narrow peak of width $\sim kT$ with a maximum at $E = E_\omega$. As $E$ moves away from $E_\omega$ on either side by several $kT$, this function declines exponentially with increasing $|E - E_\omega|$. If the width of the distribution of activation energies $F(E)$ is considerably smaller than $kT$, naturally, the SD of the noise reduces to the Lorentz function (3.1.1). However, if we take into account the fact that the energy $kT$ is small in comparison with ordinary activation energies even when $T$ is several hundred kelvins, then the converse case seems real, in which the width of the distribution $F(E)$ is far greater than $kT$. Then one can remove $F(E)$ for $E = E_\omega$ from within the integral in (3.1.4). The SD of the noise equals[72,75]

$$S(f) = kTF(E_\omega, T)\frac{1}{f}. \quad (3.1.5)$$

Since $F(E_\omega, T)$ depends weakly on $E_\omega$, while $E_\omega$ depends only logarithmically on the frequency, the deviation of $S(f)$ from the $1/f$ law is small. Let us recall (Sec. 2.1) that the noise spectrum being measured approximates a $f^{-\gamma}$ relationship, and $\gamma$ differs from unity only within the range $\sim \pm 0.2$. Since $F(E, T)$ can either increase with increasing $E$ near $E = E_\omega$, the SD $S(f)$ can deviate from $1/f$ either toward steeper or gentler decline with increasing $f$.

The fact that a $1/f$ spectrum arises as the result of superposition of random processes having the most varied relaxation times, and correspondingly different energies of activation, has apparently been demonstrated rather pictorially in recent experiments[141] with field-effect transistors based on a metal-dielectric-semiconductor (Si) structure having extremely small dimensions ($1 \times 0.1\ \mu m^2$). It turned out that in each such transistor the current fluctuations in the inversion channel amount to a random sequence of switchings between two states having resistances that differ by several tens of percent (but were definite under the given conditions). The mean time of stay of the device in a state of higher resistance ($\langle\tau_{on}\rangle$) and the mean time of stay in a state of lesser resistance ($\langle\tau_{off}\rangle$) depend exponentially, both on the reciprocal temperature (with certain activation energies $E_{on}$ and $E_{off}$) and on the voltage $V_g$ applied to the gate, i.e., between the metal and the semiconductor. In different devices the magnitudes of $E_{on}$ and $E_{off}$ differ.

In certain transistors the noise amounts to a superposition of two types of switching with different characteristic times. Interestingly, in transistors of larger dimensions ($10 \times 20\ \mu m^2$) grown in a single process with submicron transistors, one cannot resolve the individual switchings, and one sees an ordinary $1/f$ noise.

The obtained results are interpreted in Ref. 141 as follows. The current noise is caused by capture of electrons in traps and emission of electrons from the traps (it was found in Ref. 141 that the traps lie in the oxide layer). In the submicron specimens, owing to their small area, the noise is governed by one or two traps, while with larger dimensions it is governed by a superposition of effects from a large number of traps with the most varied characteristic times and activation energies, which leads to $1/f$ noise.

A general idea was advanced in Ref. 148 that the difference in relaxation in a disordered material from simple exponential relaxation with one time $\tau$ involves the existence of a series of kinetic processes that, however, do not act in parallel (as is usually assumed). So to speak, they act sequentially: each successive process can take place after the system arrives in a certain state owing to a faster preceding process. Thus the nonexponential character of the relaxation as a whole is a consequence of the coupling between the different kinetic processes.

### 3.2. Some models of systems with an exponentially broad distribution of relaxation times

#### *a) The McWhorter model* [74]

One assumes that the fluctuations in the number of current carriers in the surface layer of a semiconductor (and correspondingly, conductivity fluctuations) arise from exchange of electrons between the surface layer and traps lying in the oxide layer covering the surface, or on the outer surface of the oxide. Transfer of an electron into or out of a trap occurs by tunneling. Therefore the characteristic reciprocal relaxation time of such a process declines exponentially with increasing distance $x$ from the surface of the semiconductor to the trap: $\tau^{-1} = \tau_0^{-1} \exp(-x/\lambda)$. Here $\lambda \sim 10^{-8}$ cm, while $\tau_0^{-1}$ is a preexponential factor that depends only weak-

ly on $x$. Since the distances $x$ to different traps differ (with a scatter $\gtrsim\lambda$), a distribution of relaxation times of the number of current carriers in the semiconductor arises that encompasses many orders of magnitude.

We should expect that the exponential increase of $\tau$ with $x$ extends only to distances $x$ of the order of the mean distance between traps in the dielectric: exchange with the traps most remote from the surface occurs in such a way that the electron tunnels first to a trap closer to the surface, and then reaches the remote traps by hopping motion via the traps.

The model has been employed repeatedly for explaining $1/f$ current noise in MOSFETS (see Sec. 5.1).

*b) Fluctuations in hopping conduction in a semiconductor*[76,77]

The conduction is of hopping type in a weakly doped, compensated semiconductor at sufficiently low temperatures. The frequency of tunneling hops of charge carriers between two impurity centers $i$ and $j$ depends exponentially on the distance $r_{ij}$ between them: $\nu(r_{ij}) = \nu_0 \exp(-2r_{ij}/a)$, where $a$ is the effective Bohr radius of the ground state of the impurity, and $\nu_0$ is a coefficient. Since the distance between the impurity centers is a random quantity, the semiconductor under conditions of hopping conduction constitutes an example of a disordered medium having an exponentially broad distribution of local conductivities and relaxation times. As is known,[78] the hopping conduction of a semiconductor is governed by the conductivity of the so-called critical network, which is constructed as follows. A pair of impurity centers is considered bound if $r_{ij} < r$, where $r$ is a given distance. The mutually bound centers form a cluster. For small $r \ll N^{-1/3}$, where $N$ is the concentration of impurity centers, only randomly close centers prove to be bound, and form small clusters. With increasing $r$, clusters arise with an ever larger number of centers. When $r = r_c = 0.865\, N^{-1/3}$, an infinite cluster (IC) first arises and penetrates the entire macroscopic specimen. Hopping conduction is determined by the IC with $r_{ij} < r_c + ma$, where $m \sim 1$. This IC amounts to a rather rarefied network, which is called the critical net; it is precisely what governs the conductivity. Low-frequency conductivity fluctuations with frequencies $f \ll \nu(r_c)$ arise from fluctuations in the number of carriers in the critical net, while they in turn arise from the random character of the exchange of charge carriers between the critical net and the finite clusters lying in its pores.

Decrease in the frequency $f$ increases for greatest distance between centers $r(f) = (a/2)\ln(\nu_0/2\pi f)$ for which the reciprocal time of transfers between them of current carriers is greater than $f$. At the same time, the IC grows (including the critical net as a part of it) within which all $\nu(r_{ij}) > f$. The SDF of the number of charge carriers in the critical net and the conductivity increase with decreasing $f$, owing to the inclusion in the process of new finite clusters and individual impurity centers. Those centers are linked by the latter to the IC (when $r(f_1) \approx 2r_c$) that fortuitously lie in "empty" cavities, so that the closest neighboring centers lie at distances considerably closer than $r_c$. Since there are exponentially few of these centers, the SD below $f_1$ ceases to increase with decreasing $f$. The character of the decline of the SD when $f > f_1$ depends on the value of $Na^3$. In the limit of very small impurity concentrations ($Na^3 \to 0$), the SD declines according to a law approximating $1/f$. However, the peculiarity of the system being treated is that the width of the distribution of relaxation times involves the magnitude of the mean resistance: at very small $Na^3$ the resistance is enormous, and observation of either the current noise or of the conductivity is extremely difficult. In Ge and Si with shallow impurities, one observes hopping conduction only when $Na^3 \gtrsim 10^{-4}$. The numerical calculations performed in Ref. 77 for such concentrations of impurity that are not too small have shown that (upon allowing for the Coulomb interaction between the charge carriers) $S(f) \propto f^{-\gamma}$ with $\gamma \approx 0.6$. That is, it declines according to a considerably more gentle law than $1/f$.

This result indicates that modulation mechanisms of $1/f$ noise are more likely: relaxation processes (e. g., atomic) modulate the electronic conduction, so that the scatter in relaxation times is not directly connected with the magnitude of the mean conductivity.

*c) Two-level tunneling systems*

A number of disordered systems—first of all dielectric glasses—exhibit an anomalous temperature-dependence of the heat capacity and the heat conductivity at low temperatures, as well as specific phenomena in the absorption of sound.[79,80] All these phenomena were successfully explained on the basis of a model of two-level tunneling systems.[81] According to this model atoms or groups of atoms exist in amorphous structures that can occupy two positions, so one can represent their energy as a function of the configuration in the form of two potential wells separated by a barrier of height $V$ (Fig. 13). In the general case the wells are asymmetric and the energy of their minima differ by $\varepsilon$. Atoms (or groups of atoms) can tunnel from one well to the other. The quantum-mechanical transparency of the barrier between them is of the order of $e^{-\lambda}$, where $\lambda = d\sqrt{2MV}/\hbar$, $d$ is the thickness of the barrier, and $M$ is the mass of the tunneling particles.

The systems being studied possess two low-lying energy levels, the distance between which is $E = \sqrt{\varepsilon^2 + \Delta^2}$, where $\Delta = \hbar\omega_0 e^{-\lambda}$, and $\omega_0$ is of the order of the vibration frequency of a particle in an individual potential well. It is precisely the transition of the atoms between these levels and the

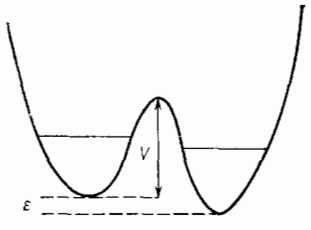


FIG. 13. Two-well potential of a two-level tunneling system.

change in the relative occupancy of these levels upon varying temperature or when acted on by acoustic vibrations that is responsible for the low-temperature effects observed in the glasses. The same type of effects, which are associated with the existence of two-level tunneling systems (TLTS), has also been found in amorphous metals and ionic conductors.[80]

Owing to the disorder, the TLTSs have different values of $\varepsilon$ and $\lambda$. Simple physical considerations lead one to assume that the distribution function of the TLTSs in $\varepsilon$ and $\lambda$ is almost constant at the values of $\lambda \gg 1$ and $\varepsilon \ll \hbar\omega_0$ that govern the observed effects, i.e., in this region we have $P(\lambda, \varepsilon) \approx \overline{P}$. One finds the magnitude of $\overline{P}$ by comparing theory with experiment.

The rate of relaxation of a TLTS is determined by its interaction, either with phonons (in dielectric glasses) or with electrons (in metals). The corresponding reciprocal relaxation times equal[79,80]

$$\tau_{ph}^{-1} = aE\Delta^2 \coth \frac{E}{2kT}, \quad \tau_e^{-1} = b\frac{\Delta^2}{\hbar E} \coth \frac{E}{2kT}. \quad (3.2.1)$$

Here $a$ is a coefficient that depends on the parameters of the interaction of the TLTS with acoustic phonons and on the velocity of sound, and $b$ is a dimensionless coefficient that depends on the interaction of the TLTS with the electrons of the metal and on the density of electronic states at the Fermi surface ($b$ can be $\sim 1$). Equation (3.2.1) implies that the distribution function with respect to the relaxation times $\tau$ and energies $E$ equals $p(\tau, E) = (E/2\varepsilon\tau)P(\lambda, \varepsilon)$, and that larger values of $\tau$ exist in systems having $\Delta \ll \varepsilon \simeq E$. Thus for large $\tau$ we find that the function $p(\tau, E) = P(\lambda, \varepsilon)/2\tau \simeq \overline{P}/2\tau$. Thus, owing to the exponential dependence of $\tau$ on the tunneling parameter ($\tau \propto e^{2\lambda}$), the TLTS is characterized by an exponentially broad distribution of relaxation times (hierarchy of times), and the distribution function with respect to $\tau$ is approximately inversely proportional to $\tau$, as is characteristic of systems exhibiting $1/f$ noise (see Sec. 3.1).

Spontaneous transitions between the levels of a TLTS can lead to fluctuations in the macroscopic quantities (resistance of disordered metals,[97,98] density of electronic states at the surfaces of semiconductors and in MOS structures, tunneling current through dielectric interstices between metals,[144] etc.). According to what we have said, this noise has a $1/f$ spectrum.

Let $\delta x_i$ be the variation in the quantity $x$ (e.g., the resistance) when the $i$th TLTS goes from the lower to the upper level. Then the fluctuation is $\delta x(t) = \Sigma_i \delta x_i \delta n_i(t)$, where $\delta n_i$ is the fluctuation of the occupation number of the upper state. Since the mean square $\overline{\delta n_i^2} = [4\cosh^2(E_i/2kT)]^{-1}$, and the transitions in the individual TLTSs are uncorrelated, the SDF of the quantity $x$ equals

$$S_x(f) = V \int d\tau\, dE\, (\overline{\delta x^2})_{\lambda,\varepsilon} \frac{p(\tau, E)}{\cosh^2(E/2kT)} \frac{\tau}{1+\omega^2\tau^2}. \quad (3.2.2)$$

Here $(\overline{\delta x^2})_{\lambda,\varepsilon}$ is the square of $\delta x_i$ averaged over the TLTSs having similar values of $\lambda$ and $\varepsilon$, and $V$ is the volume over which the fluctuation $\delta x$ is averaged.

Let $\tau_T$ be the relaxation time of those TLTSs for which $\Delta \simeq kT$, $E \approx kT$, while $\tau_{max}$ is the relaxation time corresponding to the maximum $\lambda$ at which we still have $P(\lambda, \varepsilon) \simeq \overline{P} = \text{const}$ (we shall denote this value as $\lambda_{max}$). In the frequency range $\tau_{max}^{-1} \ll f \ll \tau_T^{-1}$, the fundamental contribution to the SD of (3.2.2) comes from the values of $\tau$ and $E$ for which $p(\tau, E) \approx \overline{P}/2\tau$. Usually one can consider the quantity $(\overline{\delta x^2})_{\lambda,\varepsilon}$ in the corresponding interval of $\lambda$ and $\varepsilon$ in any case not to depend strongly on $\lambda$ and $\varepsilon$. Then (3.2.2) acquires the form

$$S_x(f) \approx \frac{1}{4}\overline{\delta x^2} V \overline{P} kT \frac{1}{f}, \quad \tau_{max}^{-1} \ll f \ll \tau_T^{-1}, \quad kT \ll \hbar\omega_0. \quad (3.2.3)$$

Owing to the exponential dependence of the relaxation time of the TLTSs on the tunneling parameter $\lambda$ and the uniform distribution of $\lambda$, spontaneous transitions in the TLTSs lead to fluctuations in the physical quantities (which are sensitive to these transitions) with a $1/f$ spectrum over an extensive range of $f$. Owing to the uniform density of the excitation energies $E$ of the TLTSs, the noise spectrum increases linearly with the temperature (just like the contribution of the TLTSs to the heat capacity).

The lower frequency bound of the $1/f$ spectrum is determined either by $\lambda_{max}$ or by the rate of the activational processes. According to Ref. 142 we have $\lambda_{max} \sim 10^2$. Therefore the corresponding value of $\tau_{max} \propto e_{max}^{-2\lambda}$ is in any case smaller than any frequencies technically accessible for measuring the noise spectrum. The rate of the activational processes is $\sim \omega_0 \exp(-\hbar\omega_0/kT)$ and is also very small at low temperatures $kT \ll \hbar\omega_0$. Moreover, as $f \to 0$, $S_x(f)$ does not diverge, and the paradox noted in Sec. 2.2 does not arise.

An interesting example are the fluctuations in the tunneling resistance of a metal-dielectric-metal junction used as a Josephson element in cryoelectronics. A tunneling-thin layer of a dielectric (oxide) is usually strongly disordered.[143] We can naturally assume that TLTSs exist in it. Upon spontaneous transitions in the TLTSs, fluctuations $\delta G(t)$ arise in the tunneling conductivity $G$ of the dielectric layer. In correspondence with (3.2.3), the SD of the relative fluctuations of $G$ equals[144]

$$S_G(f) G^{-2} = \frac{d}{4} \frac{\overline{\delta G^2}}{(G/A)^2} \overline{P} kT \frac{1}{Af}.$$

Here $A$ is the contact area, and $d = V/A$ is the thickness of the dielectric.

The fluctuations in the tunneling conductivity in the Josephson S–I–S junction lead to fluctuations in the critical current $I_c$ that affect the sensitivity of devices (e.g., SQUIDs) at low frequencies.[69] Interestingly, the observed SD of the $1/f$ noise in Josephson junctions actually increases linearly with the temperature.[145,146] If we assume (as in Ref. 144) that $\overline{\delta G^2}/(G/A)^2 \sim a_0^4$, where $a_0 \sim 1$ Å is the interatomic distance, while we assume $\overline{P}$ equal to a value typical for amorphous materials,[80] then the estimate of the noise proves to be of the same order of magnitude as the noise measured in Ref. 146, but larger than that measured in Ref. 145. In the TLTS model the noise depends on the technique of preparation of the dielectric layer (via the quantity $\overline{P}$).

*d) Disordered Ising kinetic model. The 1/f noise and logarithmic relaxation of the order parameter*

A characteristic feature of a number of magnetics is the extremely slow relaxation of the magnetization $M(t)$ after turning off the magnetic field (magnetic viscosity). The course of $M(t)$ after a certain relatively small time $t_1$ can be approximated by a logarithmic function of the time $t$:

$$M(t) = M_1 - s \ln \frac{t}{t_1}. \qquad (3.2.4)$$

Here $M_1 = M(t_1)$ is usually several times smaller than the initial magnetization $M(0)$, and $s$ is a coefficient. The logarithmic law (3.2.4) can hold only up to some maximum time $t_m$ that does not exceed $t_1 \exp(M_1/s)$. In particular, logarithmic relaxation is characteristic of spin glasses,[82] i.e., dilute solid solutions of a transition metal (Fe, Mn) in a noble metal (Au, Cu). The dielectric polarization in disordered dielectrics behaves analogously.[83]

In line with the fluctuation-dissipation relationship (see Ref. 84), the spectral density (SD) of the equilibrium fluctuations of the magnetization (in the absence of a magnetic field) is related to the relaxation of $M(t)$ with time after turning off (at $t = 0$) of the small magnetic field $H_0$:

$$S_M(f) = \frac{2kT}{\pi f} \int_0^\infty dt \sin(2\pi f t) \left[ -\frac{d}{dt} \frac{M(t)}{H_0} \right]. \qquad (3.2.5)$$

In the case of the logarithmic relaxation of (3.2.4), Eq. (3.2.5) implies that[85]

$$S_M(f) = \frac{kTs}{H_0} \frac{1}{f}, \quad t_m^{-1} \ll 2\pi f \ll t_1^{-1}. \qquad (3.2.6)$$

Thus the logarithmic relaxation of a physical quantity and proportionality of the SD of its fluctuations to $1/f$, as well as the logarithmic decline of the correlation function of the $1/f$ noise (see Sec. 2.2) are phenomena closely associated with one another.[85]

Studies of the kinetic disordered Ising model, which serves as a model for a spin glass, have yielded much toward understanding the nature of logarithmic relaxation.[86–90] From the standpoint that we have adopted, this is a model of a system with a continuous distribution of activation energies.

The disordered Ising model amounts to a system of spins interacting with one another while lying at the nodes of a lattice. Each spin $S_i$ can acquire two values (e.g., $+1$), while the energy of the system is $\Sigma_{ij} J_{ij} S_i S_j$. Here $J_{ij}$ is the interaction constant of the spins $i$ and $j$, while the summation is performed over all pairs of spins (one usually assumes that only the spins lying at adjacent lattice nodes interact). In the model the $J_{ij}$ take on random values—both positive (ferromagnetic interaction) and negative (antiferromagnetic interaction)—and the character of the disorder is determined by the distribution function $f(J)$ of these energies. One assumes in studying the kinetics of the model that the spins interact not only with one another, but also with the thermostat (electrons of the metal, phonons). This latter interaction leads to reversal of the spins ($S_i \to -S_i$). The probability $W(E_i, T)$ of reversal of a spin per unit time depends on the change $E_i$ in the energy of the system upon reversing the $i$th spin: $E_i = S_i \Sigma_j J_{ij} S_j$. The probabilities of the direct and inverse processes must satisfy the condition

$$\frac{W(E_i, T)}{W(-E_i, T)} = e^{-E_i/kT}.$$

Here $T$ is the temperature of the thermostat.

An important feature of the model, which stems from its high degree of disorder, is the great density of low-lying metastable states, i.e., valleys in the state space of the system, which are separated from one another and from the ground state by barriers of varying height $E$. Within each large valley there are many shallow valleys separated from one another by low barriers. At low temperatures at which the width of the distribution $f(J)$ is large in comparison with the energy $kT$, the system in its thermal motion relatively rapidly overcomes these low barriers, but it very seldom succeeds in hopping from one metastable state to the next, since the characteristic time for overcoming a barrier $E$ is proportional to $\exp(E/kT)$. In line with this picture, the relaxation from a nonequilibrium state is composed of a fast phase involving transitions within the metastable states, and a slow phase involving hopping between the metastable states. This slow relaxation can be approximated by a logarithmic function of the time like (3.2.4).

The essential point is that there are many metastable states in disordered systems whose energy differs very little from the energy of the ground state. On the one hand, this causes such a system, even at low temperatures, to spend a considerable fraction of the time in metastable states, while jumping from one such state to another, as is revealed in low-frequency fluctuations. On the other hand, it causes weak perturbation to convert the system into metastable states, from which it slowly "extricates itself" after the external perturbation has been turned off.

Apparently, in most cases the observed $1/f$ noise does not involve fluctuations of an order parameter that could be altered by an external field (as in the case of fluctuations of the magnetization). However, the structure of the energy spectrum of disordered systems described above (the presence of low-lying metastable states) and the pattern of thermal motion of such a system remain valid also in the more general case.

It is an important point for the model of $1/f$ noise that the upper bound of the hierarchy of relaxation times should be large enough. In the simplest model with a distribution of activation energies—a one-dimensional disordered Ising model with interaction of nearest neighbors—the largest activation energy is approximately equal to the variance of the interaction energies of adjacent spins. That is, this proves to be the microscopic single-particle energy. However, in the general case the activation energies in disordered spin models can attain considerable values that correspond to "geological" times.

## 4. CURRENT NOISE CAUSED BY TEMPERATURE FLUCTUATIONS

### 4.1. Idea of the mechanism and first experiments

The temperature of any object existing in thermal equilibrium with surrounding objects that play the role of a ther-

mostat fluctuates about the mean value, which equals the overall temperature $T$ of the entire system. The mean square of these fluctuations is[84]

$$\overline{(\delta T)^2} = \int_0^\infty df S_T(f) = kT^2C^{-1}, \tag{4.1.1}$$

Here $C$ is the heat capacity of the object, and $S_T(f)$ is the SDF of its temperature. Thus the integral of $S_T$ over all frequencies is a thermodynamic quantity that does not depend on the conditions of thermal contact of the given object with the objects surrounding it [in contrast to the function $S_T(f)$ itself].

A model that associates the low-frequency current noise with the equilibrium temperature fluctuations of the conductor has been proposed.[43] According to this model the current fluctuations arise from the temperature-dependence of the resistance: $\delta R(t) = (dR/dT)\delta T(t)$. The SD of the relative fluctuations of the voltage in a fixed-current regime is

$$\frac{S_U(f)}{U^2} = \frac{S_R(f)}{R^2} = \left(\frac{d\ln R}{dT}\right)^2 S_T(f). \tag{4.1.2}$$

The voltage fluctuations were measured in Ref. 43 at the ends of thin (250–2000 Å) continuous films of Au, Ag, Cu, Sn, Bi, and the alloy manganin at room temperature. In all cases but one, they observed noise with an SD $S_U(f)$ approximately proportional to $1/f$. The remarkable exception was manganin, whose resistance at room temperature is almost independent of the temperature: $|\beta| = |d\ln R/dT| < 10^{-4}K^{-1}$. This fact was treated in Ref. 43 as indicating that $1/f$ noise in metals arises from temperature fluctuations. Since the authors of Ref. 43 had no data indicating that the spectrum of temperature fluctuations $S_T(f)$ could have a $1/f$ form under any conditions of heat transport, they simply postulated that $S_T \propto 1/f$ in the range of frequencies $f$ from $f_1$ to $f_2$, and constructed a model continuous function $S_T(f)$ whose integral over all frequencies coincides with (4.1.1):

$$S_T(f) = \frac{kT^2}{cV[3+\ln(f_2/f_1)]} \begin{cases} \dfrac{1}{f_1}, & f < f_1, \\ \dfrac{1}{f}, & f_1 \leqslant f \leqslant f_2, \\ \dfrac{f_2^{1/2}}{f^{3/2}}, & f > f_2. \end{cases} \tag{4.1.3}$$

Here $V$ is the volume of the specimen, and $c$ is the specific heat capacity of the metal. The authors[43] associated the frequencies $f_1$ and $f_2$ with the reciprocal times of heat conduction respectively through the length $l$ and width $w$ of the specimen: $f_1 = D/\pi l^2$, $f_2 = D/\pi w^2$, where $D$ is the heat conductivity of the metal. In the three-dimensional case we have $S_T(0) = \text{const}$, while at frequencies exceeding the reciprocal time of heat propagation over the minimum dimension of the film, $S_T(f)$ must fall off in accordance with $f^{-3/2}$. This has been taken into account in (4.1.3).

The SD of the current noise measured in Ref. 43 proved to be close to the value given by (4.1.2) and (4.1.3), i.e., close to

$$S_U(f) = U^2\left(\frac{d\ln R}{d\ln T}\right)^2 \frac{k}{cV[3+\ln(f_2/f_1)]}\frac{1}{f}. \tag{4.1.4}$$

This was treated as another argument favoring the idea that current $1/f$ noise arises from temperature fluctuations.

Analogous studies of the noise spectrum have been performed in Ref. 91 on Cr films. The spectra regularly exhibited a break at a frequency of the order of $D/2\pi w^2$.

Yet another method was pointed out in Ref. 43 for testing whether the observed current noise is caused by temperature fluctuations. The idea consists in measuring the correlation of the potential fluctuations $\delta U_1(t)$ and $\delta U_2(t)$ at the ends of two portions of the same film lying at a distance $L$ from one another. If the potential fluctuations are caused by temperature fluctuations, they must be correlated at low frequencies, when both portions of the film have a common temperature and $\delta T_1 = \delta T_2$, and uncorrelated at high frequencies, starting with a frequency $\sim D/\pi L^2$. An experiment performed on two bismuth films found that the correlation actually declines with increasing $f$ and vanishes approximately at $D/\pi L^2$, in agreement with the temperature mechanism of current noise. However, in one of the specimens the correlation became negative after passing through zero. The authors[43] explain this by errors of measurement. They conclude that the measured correlation of fluctuations also confirms the temperature mechanism of current noise in metal films.

A considerable correlation of voltage fluctuations in adjacent regions of metal films (distance $\sim 0.2$ mm) has also been observed in Ref. 91. However, it declined with decreasing current, and became rather small at the lowest current density employed in the measurements.

In subsequent measurements,[17] $S_U(f)$ was measured in metal films over the broad temperature range 100–600 K. Although at room temperature $S_U$ is close to the value obtained in Ref. 43, and hence to the value given by (4.1.4), at other temperatures $S_U(f)$ was found to be very far from (4.1.4). Principally, the temperature-dependence of the noise is not at all described by this model expression. Hence the agreement of $S_U(f)$ in Ref. 43 with (4.1.4) seems fortuitous. Measurements have also been performed on the correlation of the voltage fluctuations in thermally coupled metal films,[19,20] but no correlation was found, in contrast to the result[43] with Bi films (for more details see Sec. 4.3).

### 4.2. Spectrum of temperature fluctuations

One can calculate the spectral density of temperature fluctuations $S_T(f)$ by two equivalent methods. First, one can solve the heat conduction equation for the temperature fluctuations $\delta T(t)$ with the lateral Langevin fluxes, whose correlation function is known.[92] Second, one can calculate the response $\Delta T(\omega)e^{-i\omega t}$ to the alternating power $P(\omega)e^{-i\omega t}$ released in the specimen. The component of the response that is in phase with the alternating power is associated by the dissipation-fluctuation relationship with $S_T(f)$ (see, e.g., Refs. 93 and 94):

$$S_T(f) = 2kT^2 \operatorname{Re}\frac{\Delta T(\omega)}{P(\omega)}. \tag{4.2.1}$$

If a specimen of finite dimensions is in contact with a three-dimensional infinite or semiinfinite medium, then at sufficiently low frequencies $f$ the propagation of heat from it into the medium is three-dimensional in character and $S_T \to$ const as $f \to 0$. The frequency below which $S_T$ practically does not vary depends on the dimensions of the specimen, the heat conductivity of the specimen and the medium, and the character of the thermal contact between them.

The most interesting situation from the standpoint of the mechanism of low-frequency noise arises when the process of heat propagation can be considered one-dimensional. Calculations of $S_T(f)$ in such systems have been performed in Ref. 93 (a film on a substrate of the same area) and Ref. 95 (specimen in contact with a uniform wire or an external circuit). It turned out that, when the thickness of the substrate or the length of the external circuit is infinitely large, one has $S_T(f) \to f^{-1/2}$ as $f \to 0$. If these lengths are large but finite, then at a sufficiently low frequency the increase in $S_T(f)$ with decreasing $f$ is replaced by leveling to a constant value. According to Ref. 93 a spectrum $S_T$ of $1/f$ type can arise whenever there is a purely surface fluctuational energy flux (owing to fluctuations of thermal radiation). However, if such a spectrum can be manifested, this can happen only in a very narrow frequency range: a $f^{-1/2}$ spectrum dominates.

An analogous overall conclusion has been drawn in Ref. 95. The component of $S_T(f)$ involving lateral heat fluxes in the specimen itself actually behaves like $1/f$ in some frequency range under conditions in which heat exchange with the external circuit is difficult. However, under these conditions the predominant component of $S_T(f)$ is the one associated with lateral heat fluxes in the external circuit, and it has a different frequency-dependence. It was shown in Ref. 95 that this conclusion remains valid also for a three-dimensional system, which effectively reduces to a one-dimensional system in the range of low frequencies.

Thus the theoretical calculations[93,95] show that the SDF of the temperature does not have a $1/f$ form in any broad frequency range.

The fluctuation-dissipation relationship (4.2.1) not only allows one to calculate, but also to measure $S_T(f)$. By using a current pulse, one introduces into the specimen a certain amount $Q$ of heat, and measures the temperature relaxation $\Delta T_{\text{im}}(t)$ from the change in the resistance $R(t)$ of the specimen. We can easily see that

$$\operatorname{Re}\frac{\Delta T(\omega)}{P(\omega)} = \int_0^\infty dt \cos \omega t \, \frac{\Delta T_{\text{im}}(t)}{Q}. \tag{4.2.2}$$

The measurements performed in Ref. 43 on metal films showed that the magnitude of the right-hand side of (4.2.2), and this implies also $S_T$, does not depend on the frequency at low frequencies (in Ref. 43—for $f < \sim 10$ Hz).

It was proposed in Ref. 96 to average the temperature fluctuations over a volume having dimensions equal to the distance to which heat propagates in a time $1/f$, rather than over the volume of the specimen (which would be correct). This dimension $\propto f^{-1/2}$ as $f \to 0$. On the basis of this assumption, a spectrum was obtained $S_T \propto 1/f$, which contradicts the thermodynamic relationship (4.1.1), the rigorous calculations (in particular, Refs. 93 and 95), and experiment.

### 4.3. Spatial correlation of fluctuations

The idea of Voss and Clarke[43] on measuring the correlations of the current fluctuations in different specimens that are thermally coupled with one another has proved fruitful. A number of experiments[19,20,56] has been performed, but in contrast to the first experiments,[43,91] no spatial correlation was found.

The system studied in Ref. 20 consisted of two Au films (of thickness $\sim 600$ Å) galvanically isolated from one another by a layer of SiO dielectric $\sim 0.6\,\mu$m thick. Measurements of two types were performed. First, a measurement was made of the SD $S_{T12}$—the Fourier transform of the correlation function of the temperature fluctuations $\delta T_1$ and $\delta T_2$ in the different films, and also the SDs $S_{T1}$ and $S_{T2}$ of the temperature fluctuations in each of the films. Here a direct measurement was made the component of the temperature response in one film to an alternating heating of the other film with a current that was in phase with the alternating power. From a fluctuation-dissipation relationship of the type of (4.2.1) $S_{T12}$ was determined ($S_{T1}$ and $S_{T2}$ were determined analogously). The measure of the correlation of the temperature fluctuations in the different specimens was the quantity

$$\gamma_T(f) = \frac{S_{T12}(f)}{\sqrt{S_{T1}(f)\,S_{T2}(f)}}. \tag{4.3.1}$$

At frequencies less than the reciprocal time of propagation of heat from one film to the other, their temperature fluctuates "in the same way," and $\gamma_T(f) \approx 1$. The measurements confirmed this.

On the other hand, one can measure the correlation of the voltage fluctuations in both films upon passing a current, i.e., $S_{U12}(f)$, and also the SDs $S_{U1}$ and $S_{U2}$, and find the dimensionless ratio $\gamma_U$, analogous to $\gamma_T$. If the conductivity fluctuations of the films were completely associated with the temperature fluctuations, the following equation would hold:

$$\gamma_U^2 = \gamma_T^2 \left\{\left[1 + \frac{S_{B1}}{S_{U1}(f)}\right]\left[1 + \frac{S_{B2}}{S_{U2}(f)}\right]\right\}^{-1}. \tag{4.3.2}$$

Here $S_{B1}$ and $S_{B2}$ are the "background" SDs of the fluctuations, which in the two films are known to be uncorrelated [hence a correction has been introduced into (4.3.2)]. The $\gamma_U^2(f)$ relationship determined from (4.3.2) on the basis of the measured $\gamma_T$ (thermal measurements) is shown in Fig. 14 (curve *1*).

However, direct measurement of $S_{U12}$ has shown no correlation of the voltage fluctuations: to an accuracy of $5 \times 10^{-4}$, $\gamma_U^2$ is equal to zero (line *2* in Fig. 14). Moreover, an estimate by Eq. (4.1.2) showed that the temperature fluctuations are responsible for the current noise (with a spectrum independent of the frequency for $f < \sim 10$ Hz), which is more than three orders of magnitude smaller than the observed current noise at the frequency 10 Hz.

Similar experiments have been performed on transistors in an Si-integrated circuit. Within the limits of accuracy

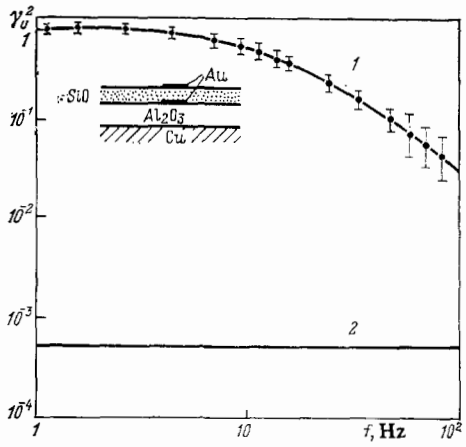


FIG. 14. Frequency-dependence of the square of the correlation coefficient of the current fluctuations in two Au films separated by a dielectric.[20] *1*—the dependence calculated by Eq. (4.3.2) using the measured values of $\gamma_T$; *2*—straight line corresponding to the direct measurement of the cross-correlation performed at the level of the experimental error. Inset: schematic cross-section of the specimen.

of the experiment ($\sim 2\%$), no correlation of the voltage fluctuations in different transistors was found, although the temperature fluctuations in them at the same frequencies are practically completely correlated.[56] There is also no correlation of the voltage fluctuations at the ends of the two halves of the same metal film $\sim 1$ mm long.[19]

Despite the fact that the reasons have not been elucidated why an appreciable spatial correlation was observed[43,91] in the current fluctuations having a $1/f$ spectrum, we can infer from the experiments and theoretical calculations presented here that $1/f$ noise is usually not caused by temperature fluctuations, but by some other mechanisms. Apparently temperature fluctuations actually govern the current noise in a state in transition from the normal to the superconductive state,[18] when the temperature-dependence of the resistance is enormous, but the current noise observed here does not have a clearly marked $1/f$ spectrum.

## 5. LOW-FREQUENCY CURRENT NOISE IN SEMICONDUCTORS

### 5.1. Surface noise

Intensive studies of the mechanisms of low-frequency current noise in semiconductors and semiconductor devices have been conducted from the beginning of the fifties, i.e., from the onset of vigorous growth of the physics of semiconductors and semiconductor electronics, which followed the creation of the technology of pure semiconductor materials and the invention of the transistor. In particular, much attention has been drawn to the question: where are the sources of $1/f$ noise—in the bulk or at the surface of the semiconductor?

As is known, the electric conductivity of the near-surface space-charge region (SCR) depends on the magnitude of the charge in the surface states of the semiconductor. Fluctuations of this charge must lead to fluctuations of the electric conductivity of the SCR and of the entire specimen, which are manifested in the form of current noise. In order to establish what role this surface mechanism of current noise plays, we must elucidate how it is affected by changes in the surface that are not accompanied by a change in the bulk properties of the semiconductor—application of an electric field perpendicular to the surface, through a dielectric layer (as in the field effect) and chemosorption or desorption of different substances by changing the medium in which the specimen is situated.

The effect of an electric field perpendicular to the surface on $1/f$ noise has been studied[99] on $n$- and $p$-type Ge specimens 50–100 $\mu$m thick. The noise increases as the type of conduction in the SCR becomes opposite to the type of conduction in the bulk, i.e., as the inversion layer at the surface increases. The greatest increase in noise is approximately by a factor of 5 or 6. In $p$-type specimens, the noise increases only when the surface potential $\varphi_s$ increases in the positive direction, but conversely in $n$-type specimens—only when $\varphi_s$ changes toward the negative side. A change in $\varphi_s$ in the opposite direction caused no appreciable change in the noise. Interestingly, upon changing the field the noise relaxes in time by a very slow (logarithmic) law, but the conductivity (field effect) relaxes far more rapidly.

An effect of a gaseous medium on $1/f$ noise has been observed[100] in $n$- and $p$-type Ge specimens with a transverse cross-section of $0.5\times 0.5$ mm$^2$. The relative change in the SD of the noise in going from dry to moist nitrogen in different specimens was from $-0.5$ to $+5.5$. A tenfold increase in the noise was also observed in going from dry nitrogen to liquid $CCl_4$. Here also the exponent $\gamma$ in the frequency dependence of the spectral density ($f^{-\gamma}$) changed from 1 to 1.2.

Another observation of a strong dependence of $1/f$ noise on the composition of the gaseous medium has been made[101] on a Si photodiode under reverse-biasing conditions. Upon replacing the dry atmosphere with a moist one with unchanged current, the noise increased by $\sim 10^3$ times at the frequency of $10^4$ Hz and by $\sim 5\times 10^5$ times at the frequency of 100 Hz.

The changes in conductivity caused by a change in the surface charge are restricted to the thin SCR, whose thickness in Si and Ge is usually $\sim 10^{-5}$–$10^{-4}$ cm. Most often the thickness $d$ of the specimens is far larger than the thickness of the SCR, and we should expect that the influence of the surface on the noise will be greater with thinner specimens. In order to increase the sensitivity of the noise to the conditions at the surface, the measurements in Ref. 63 were performed on rather thin specimens of $n$-Ge($d\sim 2$–$6$ $\mu$m). The composition of the gaseous medium was cyclically changed from wet nitrogen to ozone. This cycle enables one to change the potential of the surface from positive values (the SCR is enriched in electrons) to negative values that cause the type of conduction at the surface to invert. As we should expect,[102] the conductivity of the specimens passed through a minimum $G_m$. The SD of the noise $S_G$ was directly measured as a function of the excess of the conductivity $G$ over $G_m$ (Fig. 15). Interestingly, the noise is minimal at the same com-

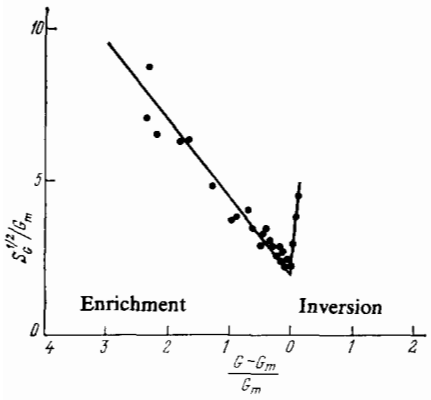


FIG. 15. Dependence of the SD of the relative conductivity fluctuations of an $n$-Ge specimen on the relative variation of the conductivity in the cycle of changing the gaseous medium above the surface.[63] Thickness of the specimen was 6 $\mu$m, $\rho = 32$ ohm·cm; frequency 20 Hz. The axis of ordinates is plotted in units of $10^{-7}$ Hz$^{-1/2}$.

position of the gaseous medium as the conductivity is. At the boundary points of the cycle the noise is larger than at the minimum by a factor of 25 to 40. In contrast to Ref. 99, an increase in noise was observed in Ref. 63, not only upon varying the potential in the direction toward formation of an inversion layer, but also upon the opposite variation in $\varphi_s$. However, the increase in the noise as a function of $G - G_m$ was far steeper in the former case.

The effect of surface treatment (etching) on the magnitude of the SD of the $1/f$ noise in $n$-InSb specimens is shown in Fig. 16.

One of the fundamental devices of semiconductor electronics—the field-effect triode based on a metal-dielectric-semiconductor structure—exhibits a considerable $1/f$ current noise. One is dealing with the fluctuations of the current (in the semiconductor) of the dielectric-semiconductor junction parallel to the surface. A large number of studies[103–107,141] has been devoted to studying the noise in these devices. A number of the factors found here have served as the basis for deducing a surface mechanism of the noise. In Ref. 103, measurements of the dependence of the SD of the noise on the voltage $V_g$ applied to the gate (i.e., between the semiconductor and the metal) have been compared with the dependence of the density of surface states $\rho_{ss}(V_g)$ measured on the same structure. These relationships proved similar at low frequencies $f < 1$ kHz (Fig. 17). The relationships $S_U(V_g)$ and the transverse conductivity $G_{ss}(V_g)$ involving transfer of charge into the surface states also proved similar. An analogous comparison of $S_U$ and $\rho_{ss}$ has been made in Ref. 104. It was found that all the points of the $S_U(\rho_{ss})$ relationship lie on a single straight line on a log-log plot, independently of the temperature and the crystallographic orientation of the Si surface.

In the fluctuations in conductivity of the semiconductor arise from fluctuations in the concentration of current carriers in the SCR, and ultimately from fluctuations of the surface potential $\varphi_s$, then the SD of the voltage fluctuations equals

$$\frac{S_U(f)}{U^2} = \left(\frac{dG_s}{d\varphi_s}\right)^2 \frac{S_\varphi(f)}{(G_V + G_s)^2}. \qquad (5.1.1)$$

Here $G_V$ is the conductivity of the specimen in the absence of a charge on the surface, $G_s$ is the conductivity of the SCR, and $S_\varphi$ is the SDF of the surface potential.

We see that the surface current noise must vanish at the same surface potential at which the surface conductivity $G_s$ is minimal. This agrees with experiment,[63] but only in the sense that experimentally the noise passes through a minimum simultaneously with $G$. However, the noise does not vanish at $G = G_m$, although it is relatively small at this point. We can explain its nonzero value either by inhomogeneity of the surface potential, or by fluctuations in the mobility of the current carriers, or by the presence of bulk sources of noise.

One can also understand in rather general form the de-

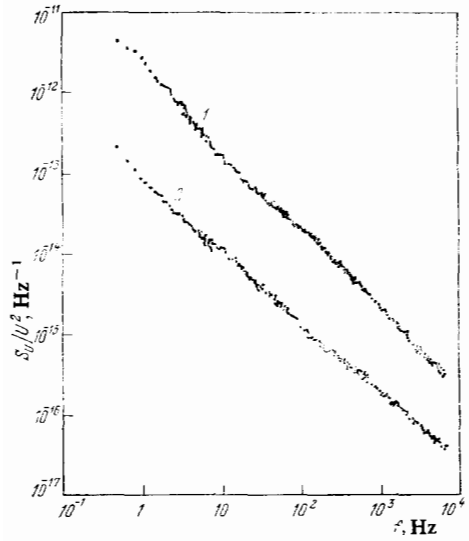


FIG. 16. Frequency-dependence of the SD of the relative voltage fluctuations in an $n$-InSb specimen.[65] $n = 10^{14}$ cm$^{-3}$, dimensions $7 \times 0.4 \times 0.4$ mm$^3$; *1*—before etching; *2*—after etching in CP-4.

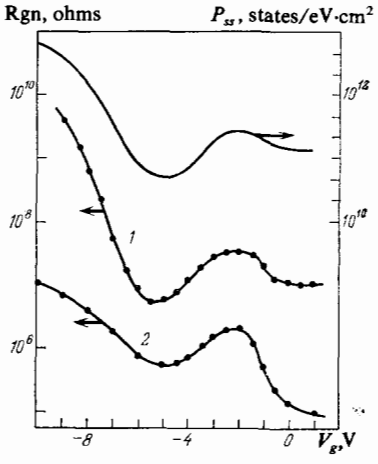


FIG. 17. Equivalent noise resistance of a field-effect transistor (left-hand scale) and density of surface states (right-hand scale) as functions of the voltage applied to the gate.[103] *1*—frequency 20 Hz; *2*—frequency 1 kHz.

pendence of the surface current noise on the dimensions of the specimen. If the surface is homogeneous and the correlation radius of the charge fluctuations in the surface states is small in comparison with the dimensions of the surface, then we have $S \propto A^{-1}$, where $A$ is the area of the surface. Equation (5.1.1) implies that, when $G_V \gg G_s$, the noise $S_U/U^2 \propto (Vd)^{-1}$, where $d$ is the thickness of the specimen and $V$ is its volume. When $G_V \propto G_s$, the noise obeys $S_U/U^2 \propto A^{-1}$. The variation of the SD of the relative voltage fluctuations $S_U/U^2 \propto (Vd)^{-1}$ is given in Ref. 108. It was confirmed experimentally[108] on $n^+$-$n$-$n^+$ GaAs resistors. This enabled the authors to deduce a surface mechanism of the $1/f$ noise in these resistors (in the case of pure bulk noise sources we have $S_U/U^2 \propto V^{-1}$; see Sec. 2.7).

The dependence of the surface current noise on the concentration of majority current carriers $n$ in the bulk of the semiconductor is generally not unambiguous, since in going from one specimen with one concentration $n$ to another with a different concentration $n$ (or even the same concentration), all the characteristics of the surface can change. However, a spectrum of surface states in the forbidden band of the semiconductor is known to arise that fixes the position of the Fermi level $E_F$ with respect to the edges of the bands $E_c(0)$ and $E_v(0)$ at the surface, independently of $n$, with a typical treatment of the surfaces of Ge and Si[109] and also GaAs.[110] In particular, in Ge and Si we have $E_c(0) - E_F \approx E_g/2$, where $E_g$ is the width of the forbidden band.

A large density of surface states leads not only to a lack of dependence on $n$ of the Fermi level at the surface, but also to the fact that the capacity of the surface states is large in comparison with the capacity of the SCR. For these reasons, we should expect that the SD $S_\varphi$ does not depend on the bulk concentration $n$ of carriers. Moreover, whenever the Fermi level at the surface is fixed far from the edges of the bands in materials in which the concentration of carriers in the bulk is large in comparison with the intrinsic concentration $n_i$, a depletion layer is formed in the SCR. By using (5.1.1) and the expressions for $dG_s/d\varphi_s$ from Ref. 102, one can easily find that the following relationship holds in the case of a depletion layer for not very thin specimens ($G_V \gg G_s$):

$$\frac{S_U(f)}{U^2} = \frac{\varkappa e^2}{8\pi kTnVd(|Y|-1)} AS_\varphi. \qquad (5.1.2)$$

In this equation $\varkappa$ is the dielectric permittivity of the crystal, we have

$$|Y| = \frac{e|\varphi_s|}{kT} = [E_c(0) - E_F](kT)^{-1} + \ln\frac{n}{N_c},$$

$N_c$ is the effective density of states in the band of the majority carriers.

Thus, in the rather typical conditions being treated, the SD of the current noise $S_U/U^2$ is proportional to $1/n$ apart from a weak logarithmic dependence of $|Y|$ on $n$ and the expected weak dependence of $S_\varphi$ on $n$. This rule (proportionality of the SD of the $1/f$ noise to the resistivity of the material) has been noted[61] in experiments on $n$- and $p$-type Ge. It also corresponds to the empirical Hooge relationship (2.8.1), which, however, Hooge viewed as favoring a bulk mechanism of the noise.

Concrete mechanisms of charge fluctuations in surface states with a $1/f$ spectrum are clearly insufficiently known at present. However, the general conclusions of the model of surface current noise (dependence on the potential, the dimensions of the specimen, and concentration of current carriers in the bulk) agree with experiment. What we have said does not mean that in semiconductors the $1/f$ noise always involves the surface or contacts. We have not treated here the bulk mechanisms of $1/f$ noise because, if one may so express it, they are even less known than the surface noise mechanisms.

### 5.2. Other problems

Much attention has been paid to the question of what causes the $1/f$ noise in semiconductors—fluctuations in the number of current carriers or fluctuations in their mobility?[10] In Ref. 147 the SD of $1/f$ noise was measured in an epitaxial $n$-GaAs film under conditions of strong geometric magnetoresistance, in which the length of the specimen in the direction of the current is small in comparison with the lineal dimensions of the contact. If the noise were caused by fluctuations in the mobility $\mu$, the measured SDF of the voltage $S_U$ would vanish when $\mu B = 1$, where $B$ is the magnetic induction. Actually the SD does not vary with $B$ at all, which implies that the noise is certainly not caused by mobility fluctuations.

Owing to lack of space, we cannot discuss a number of interesting problems of noise in semiconductors: the effect of a magnetic field on the noise,[23,29,45,61,111] fluctuations of the Hall potential,[112,113] fluctuations of the thermo-emf,[114] the effect of a strong electric field,[115,116] and deformation.[62] Noise in semiconductor devices has been treated in the reviews of Refs. 6, 117. Measurements of the intensity of low-frequency noise have been used for rejection of unreliable semiconductor devices: the greater the noise, the more probable is failure of the device during operation.[118]

## 6. CURRENT NOISE IN METALS AND SEMIMETALS

### 6.1. Temperature-dependence of the SDF in metals

We have already presented above a set of data on $1/f$ noise in continuous metal films ($\sim 10^{-5}$ cm thick): on the frequency-dependence $S_U(f)$ (Sec. 2.1), and on the absolute magnitude of $S_U/U^2$ and its agreement with the Hooge relationship (Sec. 2.8). In this section we shall present the experimental data on the temperature-dependence of the noise.

Apparently $1/f$ noise in continuous metal films (Au) was first measured in Ref. 67 (previously the measurements had been performed on island films in which the $1/f$ noise is far greater). The authors found that the SDF of Au on glass substrates is $S_U/U^2 = \alpha/nVf$, where $n$ is the concentration of electrons in Au, $V$ is the volume of the specimen, and $\alpha = 2.4 \times 10^{-3}$. That is, the SDF agrees with the Hooge formula (2.8.1). They also found that $S_U(f)$ depends weakly on the temperature.

A completely different temperature-dependence of the noise is observed in continuous films of Ag, Au, Cu, and Ni on sapphire substrates.[17,119–121] It was measured over a

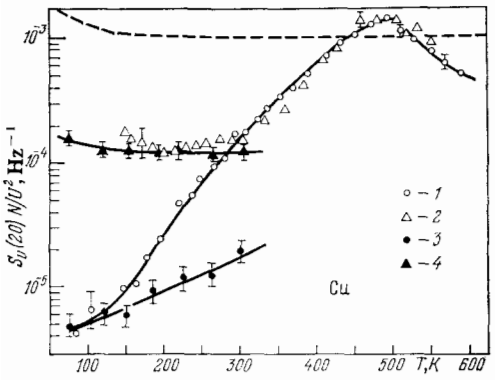


FIG. 18. Temperature-dependence of the SD of the relative voltage fluctuations in Cu films.[120] $N$ is the total number of atoms in the film, $f = 20$ Hz. Direct noise measurements: substrate on sapphire (*1*) and quartz (*2*). Calculation based on measuring the response to a stepped input of power into the film according to Voss and Clarke: substrate on sapphire (*3*) and quartz (*4*). Broken line—calculation from the model of Voss and Clarke.

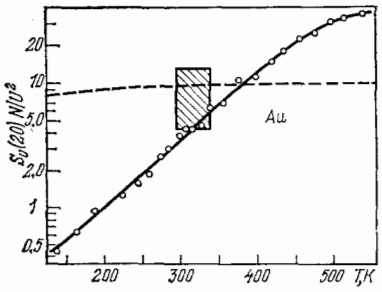


FIG. 20. Temperature-dependence of the SD of the relative voltage fluctuations in an Au film 800-Å thick on a sapphire substrate.[17] $N$—total number of atoms in the film, $f = 20$ Hz. Broken curve—calculation by the temperature-fluctuation model.[43] Hatched rectangle—region in which the experimental data lie.[43]

broad temperature range from 100 to ~600 K, mainly in order to test the model of $1/f$ noise previously proposed[43] (see Sec. 4.1). The data that were obtained are shown in Figs. 18–21.

According to Refs. 9 and 17, the noise consists of two components: A) noise almost independent of the temperature, but strongly dependent on the nature of the substrate, and B) noise strongly dependent on the temperature, but very weakly dependent on the substrate. For $T \lesssim 350$ K, the type-B noise is proportional to $\exp(-E_g/kT)$, with the value of $E_g/k$ equal to 1750 K (Ag), 1400 K (Au), and 1250 K (Cu). As a rule, type-B noise predominates at room temperature. References 9, 17, and 75 discuss possible mechanisms of the origin of such a temperature-dependence of the noise. We note only that the resistance $\rho$ of the films depends on the temperature in a way, characteristic of metals, such that an exponential dependence is not observed.

In Figs. 18–21 the dotted lines show the dependence of $S_U/U^2$ on the temperature in the case where the SDF follows Eq. (4.1.4) of Voss and Clarke. We see that the observed noise below room temperature is far smaller. That is, the model formula (4.1.4) does not describe the temperature-dependence of $1/f$ noise in metals.

The noise in Sn films strongly depends on the nature of the substrate (glass or sapphire) and on the bond of the film to the substrate.[68] In films deposited on sapphire the noise is smaller. The noise is diminished further if the tin is deposited on an intermediate layer (~50 Å thick) of tin oxide prepared with a glow discharge.

The numerous experimental data obtained by Zhigal'skiĭ and his associates[122–125] on Al and Cr films indicate that the current noise increases with increasing concentration of defects in the metal.

From the standpoint of the model of temperature fluctuations (see Sec. 4), the observed strong effect of the substrate material on the noise has been explained by the differing heat conductivity at the film-substrate boundary, depending on the material of the substrate and the quality of the bond of the film to it. At present the temperature model

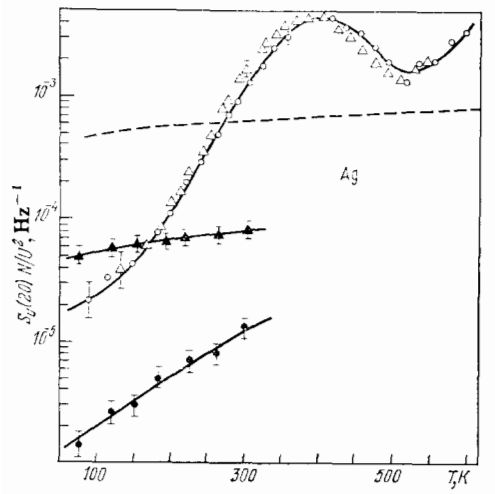


FIG. 19. Temperature-dependence of the SD of relative voltage fluctuations in Ag films.[120] The symbols are the same as in Fig. 18.

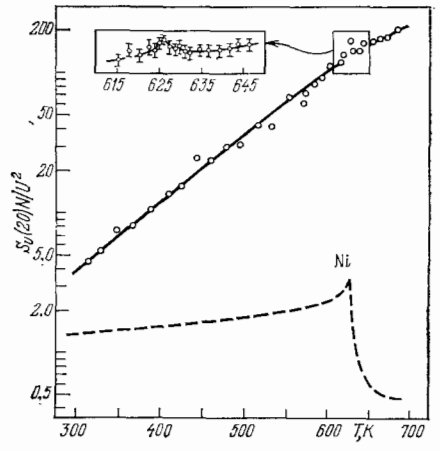


FIG. 21. Temperature-dependence of the SD of the relative voltage fluctuations in an Ni film 800-Å thick on a sapphire substrate.[17] The inset shows in greater detail the region near the Curie point. Broken curve—calculation by the temperature-fluctuation model of Voss and Clarke.[43]

seems unconvincing (Sec. 4), and perhaps one can ascribe the effect of the substrate on the noise only to the differing defect structure of films prepared on different substrates.

Island metal films exhibit a far greater current noise than continuous films of the same metals (studies of noise in island films were performed, in particular, in Ref. 22). The authors of Ref. 21 note that bismuth whiskers also generate far stronger noise than film specimens.

### 6.2. Low-frequency current noise and internal friction in metals

The idea seems rather natural of associating low-frequency current $1/f$ noise with other relaxation phenomena in solids; the reciprocal relaxation times of these phenomena are of the same order of magnitude as the frequencies at which one observes $1/f$ noise. One of these phenomena is internal friction, the mechanisms of which are rather well studied.[126] At low frequencies internal friction arises from various movements of defects—reorientation, migration, etc. The contribution of these well established mechanisms of internal friction to current noise has been studied in Refs. 55 and 97.

Many defects have a lower symmetry than the point-group symmetry of the crystal, and they can exist in several positions whose energies are identical. An example is inclusion impurities in the octahedral interstices of a bcc crystal (C and N in $\alpha$-Fe). Each impurity atom has a fourfold axis along one of the three cubic axes of the crystal. In this concrete case the number of possible positions of a defect is $s = 3$. When it hops to an adjacent interstice its orientation changes. In a state of thermodynamic equilibrium and in the absence of deformation, all $s$ possible orientations of the defect are equally probable. Upon imposing the elastic stresses $\sigma_{ij}$, the free energies corresponding to the different orientations $\alpha (\alpha = 1, \ldots, s)$ are nonidentical, and the orientations are not equally probable. If the frequency $\omega$ of deformation is not too small, and not too large in comparison with the reciprocal time $\tau^{-1}$ for hopping of a defect from one position to another, a phase difference arises between $\sigma_{ij}(\omega)$ and the deformation $u_{ij}(\omega)$, and elastic energy is dissipated. The reciprocal $Q$-factor characterizing the magnitude of the internal friction can be written in the form[126]

$$Q^{-1}(\omega, T) = B \frac{n_d}{kT} \frac{\omega\tau}{1+\omega^2\tau^2}. \qquad (6.2.1)$$

Here $n_d$ is the concentration of defects, and the coefficient $B$ is $\sim 10^{-34}$–$10^{-33}$ erg·cm$^3$.

Owing to the random nature of the reorientations, the concentration $n_\alpha$ of defects having each given orientation fluctuates about the mean value $n_d/s$. Since the defects whose symmetry is lower than the point-group symmetry of the crystal, and which contribute to internal friction, scatter the electrons of the conductor differently in their different positions, the fluctuations $\delta n_\alpha$ give rise to fluctuations of the electric-resistivity tensor:

$$\delta\rho_{ij}(\mathbf{r}, t) = \sum_\alpha \frac{\widetilde{\partial\rho_{ij}}}{\partial n_\alpha} \delta n_\alpha(\mathbf{r}, t). \qquad (6.2.2)$$

Here the tilde denotes that the component has been subtracted from the derivative that does not depend on the orientation $\alpha$ and has the same symmetry as the mean resistance of the crystal. This implies that $\Sigma_\alpha \widetilde{\partial\rho_{ij}}/\partial n_\alpha = 0$.

The quantity $\Sigma_i \widetilde{\partial\rho_{ii}}/\partial n_\alpha$ (trace of the tensor), being a scalar, is the same for the different orientations $\alpha$. Therefore it is equal to $(1/s)\,\Sigma_\alpha \Sigma_i \widetilde{\partial\rho_{ii}}/\partial n_\alpha$. Since the summation over $\alpha$ equals zero, the trace of the tensor $\widetilde{\partial\rho_{ij}}/\partial n_\alpha$ also equals zero. This implies that the fluctuation tensor of the resistivity (6.2.2) in the model being treated is characterized by a zero trace.

The spectral density of the fluctuations of the resistivity at the points $\mathbf{r}_1$ and $\mathbf{r}_2$ is proportional to $\delta(\mathbf{r}_1 - \mathbf{r}_2)$ [see (2.6.1)], while the proportionality coefficient equals[55,97]

$$F_{ijkl}(\omega) = \frac{4n_d\tau}{s(1+\omega^2\tau^2)} \sum_{\alpha\beta} \frac{\widetilde{\partial\rho_{ij}}}{\partial n_\alpha} \frac{\widetilde{\partial\rho_{kl}}}{\partial n_\beta} \left(\delta_{\alpha\beta} - \frac{1}{s}\right). \qquad (6.2.3)$$

An estimate gives $|\widetilde{\partial\rho_{ij}}/\partial n_\alpha|/\rho \sim \sigma_s l$, where $\sigma_s \sim a_0^2$ is the scattering cross-section for electrons by a defect, $a_0$ is the interatomic distance (dimension of a point defect), and $l$ is the mean free path of the electrons. Hence we have

$$\frac{S_U}{U^2} \approx \frac{n_d(\sigma_s l)^2}{V} \frac{\tau}{1+\omega^2\tau^2}. \qquad (6.2.4)$$

Since the relaxation time $\tau$ of the defects usually exceeds by many orders of magnitude both the time of free flight of the electrons and the relaxation time of the temperature of the film, the current noise caused by displacements of defects at low frequencies $\omega \lesssim \tau^{-1}$ can exceed both the Nyquist noise (at the current densities usually employed for measuring current noise in a metal $j \sim 10^6$ A/cm$^2$) and the current noise caused by temperature fluctuations.

If there is a smooth distribution $n(E)$ of defects with respect to the activation energies $E$ ($\tau = \tau_0 \exp(E/kT)$), then, on the one hand, current noise arises with a spectrum close to $1/f$ (see Sec. 3.1):

$$\frac{S_U(f)}{U^2} \approx \frac{n(E_\omega)[l\sigma_s(E_\omega)]^2 kT}{Vf}, \qquad E_\omega = kT \ln(\omega\tau_0)^{-1}, \qquad (6.2.5)$$

and on the other hand, internal friction, which depends only weakly on the frequency $f$ and the temperature $T$ (background in the internal-friction spectrum):

$$Q^{-1}_{\text{bkgd}} = \frac{\pi}{2} n(E_\omega) B(E_\omega). \qquad (6.2.6)$$

Upon using (6.2.5) and (6.2.6), one can associate the magnitude of the $1/f$ noise with the magnitude of the background in the internal-friction spectrum[55,97]:

$$\frac{S_U(f)}{U^2} = \frac{2kT[l\sigma_s(E_\omega)]^2 Q^{-1}_{\text{bkgd}}}{\pi B(E_\omega)} \cdot \frac{1}{Vf}. \qquad (6.2.7)$$

The background in the $Q^{-1}(T)$ is actually observed and is especially high ($Q^{-1}_{\text{bkgd}} \sim 10^{-4}$–$10^{-3}$) in metals (Fig. 22). For values of the parameters typical of metals, the coefficient of $1/Vf$ in (6.2.7) is of the order of $10^{-26}$–$10^{-25}$ cm$^3$, i.e., of the order of $\alpha_H/n$, where $n \sim 10^{22}$–$10^{23}$ is the concentration of electrons, and $\alpha_H = 2\times 10^{-3}$ is the Hooge constant. Thus the mechanism presented here of conductivity fluctuations enables one to explain not only their anisotropy,

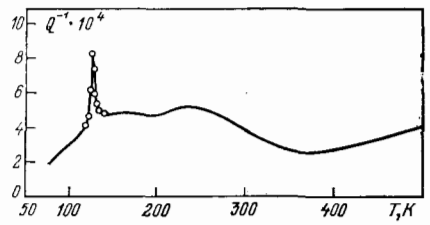


FIG. 22. Spectrum of internal friction in chromium at a frequency $\sim 1.7$ Hz—dependence of the reciprocal $Q$-factor on the temperature.[127] A Debye peak at 126 K and the background of internal friction are seen.

but also the characteristic magnitude of the SD of the $1/f$ noise in metal films. However, it remains unclear what is involved in the broad scatter of activation energies of the defects.

## 7. FLUCTUATIONS WITH A $1/f$ SPECTRUM IN OTHER SYSTEMS

Electric fluctuations with a $1/f$ spectrum are observed not only in solids, but also in liquids: in liquid gallium[128] and in an electrolyte.[129] The SD of the voltage fluctuations at the membrane of a living, i.e., nonequilibrium, but resting nerve also follows a $1/f$ law.[130]

A number of studies exists on observation of magnetic noise with a $1/f$ spectrum in ferromagnets.[131–134]

The frequency $\nu$ of the signal generated by frequency standards undergoes slow "shifts" (along with "drift"). The SD of the relative fluctuations of the frequency $S_y(f) = \nu_0^{-2} S_\nu(f)$ contains not only a component independent of the frequency $f$ of the fluctuations ("white" noise), but also a component inversely proportional to the frequency $f$. One can write it in the form $C/f$, where $C$ is a certain dimensionless coefficient. A measure of the spread (inaccuracy) of the frequency $\nu$ is $\sigma^2(t_m)$, the mean square of the relative deviations of the frequency $\nu$ as a function of the time $t_m$ of measurement. One can show[135] that the contribution of the white noise to $\sigma^2(t_m)$ is proportional to $t_m^{-1}$, while the contribution of the $1/f$ noise equals $2C \ln 2$, i.e., is independent of the time of measurement $t_m$. Thus the $1/f$ noise fixes the minimum attainable error of measurement of frequency and time.

Measurements on many quartz frequency standards have shown that the magnitude of the $1/f$ noise in the spectrum of frequency shifts (i.e., the coefficient $C$) depends on the $Q$-factor of the quartz resonator: $C = 62\, Q^{-4.3}$ (an empirical analog of the fluctuation-dissipation relationship).[136]

A number of geophysical (velocities of currents, floods of rivers, etc.) and astrophysical processes (power of various radiation sources) fluctuate. In a number of cases the corresponding SDF is of $1/f$ type (see the review of Ref. 5).

The authors of Ref. 137 have observed fluctuations in the number of cars passing per unit time on a high-speed highway. Correlation analysis showed that the SDF is proportional to $1/f$ at low frequencies (shot noise occurs at high frequencies).

The amount of insulin required by a diabetic patient to maintain a constant amount of sugar in the blood fluctuates (with unvarying diet). Correlation analysis of the fluctuations showed the SDF to follow a $1/f$ law over a broad frequency range (the measurements were performed for more than eight years).[138]

Quite probably, each of the cited random processes is composed of a large number of "elementary" processes with extemely strongly differing characteristic relaxation times.

Spectra of $1/f$ type can arise in *dynamic* (deterministic) nonlinear systems (see, e.g., Refs. 139, 140). It is unclear at present how these results are related to the $1/f$ noise discussed above in *kinetic* systems.

## 8. CONCLUSION

We shall try to assess the state of the problem of $1/f$ noise. The fundamental properties of this noise have been established up to now by numerous experiments (some of them are very ingenious and have become classical), and also theoretical analysis of these experiments has established the fundamental properties of this noise. A general property of noise of this type is the monotonic (as a rule) increase in the spectral density (SD) with decreasing frequency and the visible absence of leveling of the SD onto a plateau, down to record low frequencies ($\sim 10^{-7}$ Hz in the case of electrical noise). However, it has also been reliably established that, despite the similarity of the noise spectra, their form nevertheless differs in different systems, and even in a single specimen the spectrum depends on the temperature and other factors (Sec. 2.1). This compels one to doubt the existence of a universal mechanism of $1/f$ noise. In typical cases the distribution function of the fluctuations is Gaussian (2.3), and the noise amounts to a stationary random process (Sec. 2.4) arising from equilibrium fluctuations of the conductivity (Sec. 2.5) whose correlation radius is so small that, in the experiments in which the spatial correlation of the fluctuations has been measured, only an upper bound of the correlation length was found (Sec. 2.7). Direct theoretical calculations (Sec. 4.2), the anisotropy of the conductivity fluc-tuations in a number of solids (Sec. 2.6), and the absence of spatial correlation of the conductivity fluctuations in "thermal scales" (Secs. 2.7 and 4.3) give us grounds for thinking that $1/f$ noise does not arise from temperature fluctuations, but from other mechanisms. Apparently varied processes of appearance and displacement of defects in conductors can claim the role of such mechanisms at present. This idea, which was expressed in essence by Schottky immediately after the discovery of $1/f$ noise (as applied to a thermal-emission cathode) seems even now to be the most likely explanation of this noise. As we have already mentioned in the Introduction, the association of $1/f$ noise with the defect structure of solids is indicated, besides others, by the strong dependence of the noise on the technology of preparation of the specimens, and also by the fact that, even in specimens obtained by a single technique and having similar electrophysical parameters, the intensity of the $1/f$ noise often is completely different (Sec. 2.8).

Despite the fact that the fundamental properties of $1/f$ noise are known, the problem is far from a satisfactory solution, since many effects involving this noise are not under-

stood, the noise mechanisms for many systems have not been found, and even localization of the noise sources remains indefinite. A difficult problem is $1/f$ noise in liquids[128,129]: do the noise sources exist within the liquid or at the contact with a solid?

Most often the $1/f$ noise cannot be associated with other phenomena in the same systems. One can explain this, first, with the fact that measurement of the current-noise spectrum (fluctuation spectroscopy) is far more sensitive than other methods to the low-frequency kinetics of defects, so that the relaxation phenomena observable from the noise spectrum cannot be simultaneously observed (on the same specimens) from other physical effects. Second, one can explain it with the fact that the kinetics of defects has not at all been sufficiently studied. On the other hand, the exponentially broad distribution of relaxation times (or smooth distribution of activation energies and tunneling parameters) by which one explains the $1/f$ noise spectrum (Sec. 3) enables one also to explain a number of other phenomena of dielectric and magnetic relaxation and internal friction, so that this picture of the low-frequency kinetics of real solids cannot be deemed merely a hypothesis devised to explain a single phenomenon—$1/f$ noise.

We must bear in mind the fact that $1/f$ noise is a serious interference in many electronic devices. There is hope of eliminating it, at least partially, if one can solve the problem more fully, and in particular, elucidate the concrete mechanisms of low-frequency current noise.

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[1]J. B. Johnson, Phys. Rev. **26**, 71 (1925).
[2]W. Schottky, *ibid.* **28**, 74 (1926).
[3]A. N. Malakhov, Radiotekh. Elektron. **4**, 54 (1959).
[4]F. N. Hooge, Physica (Utrecht) B **83**, 14 (1976).
[5]W. H. Press, Commun. Astrophys. Space Phys. **7**, 103 (1978).
[6]A. Van der Ziel, Adv. Electron. Electron Phys. **49**, 225 (1979).
[7]D. A. Bell, J. Phys. C **13**, 4425 (1980).
[8]M. B. Weissman, in: Proc. 6th Intern. Conference on Noise in Physical Systems, Washington, D. C., U. S. Department of Commerce, 1981, p. 133.
[9]P. Dutta and P. M. Horn, Rev. Mod. Phys. **53**, 497 (1981).
[10]F. N. Hooge, T. G. M. Kleinpenning, and L. K. J. Vandamme, Rep. Prog. Phys. **44**, 479 (1981).
[11]M. S. Keshner, Proc. IEEE **70**, 212 (1982).
[12]M. A. Calloyanides, J. Appl. Phys. **45**, 307 (1974).
[13]V. N. Andreev, B. P. Zakharchenya, Yu. S. Kapshin, V. A. Noskin, and F. A. Chudnovskiĭ, Zh. Eksp. Teor. Fiz. **79**, 1353 (1980) [Sov. Phys. JETP **52**, 684 (1980)].
[14]D. M. Fleetwood and N. Giordano, Phys. Rev. B **27**, 667 (1983).
[15]J. L. Williams and I. L. Stone, J. Phys. C **5**, 2105 (1972).
[16]R. F. Voss, *ibid.* **11**, L923 (1978).
[17]J. W. Eberhard and P. M. Horn, Phys. Rev. B **18**, 6681 (1978).
[18]J. Clarke and T. Hsiang, *ibid.* **13**, 4790 (1976).
[19]R. D. Black, M. B. Weissman, and F. M. Fliegel, *ibid.* **24**, 7454 (1981).
[20]J. H. Scofield, D. H. Darling, and W. W. Webb, *ibid.*, p. 7450.
[21]C. Leemann, M. J. Scove, and E. P. Stillwell, Solid State Commun. **35**, 97 (1980).
[22]V. P. Palenskis, Abstract of candidate's dissertation, Vil'nyus, 1976.
[23]T. Dilmi, A. Chovet, and P. Viktorovich, J. Appl. Phys. **50**, 5348 (1979).
[24]B. V. Rollin and I. M. Templeton, Proc. Phys. Soc. London B **66**, 259 (1953).
[25]T. M. Lifshitz and L. Ya. Perova, Radiotekh. Elektron. **4**, 1543 (1959).
[26]M. Stoisiek, D. Wolf, and W. Werner, Electron. Lett. **16**, 372 (1980).
[27]M. Aoki, H. Katto, and E. Yamada, J. Appl. Phys. **48**, 5135 (1977).
[28]K. Amberiadis, A. van der Ziel, and L. M. Rucker, *ibid.* **52**, 6989 (1981).
[29]P. Van de Voorde and W. F. Love, Phys. Rev. B **24**, 4781 (1981).
[30]R. F. Voss, Phys. Rev. Lett. **40**, 913 (1978).
[31]J. J. Brophy, Phys. Rev. **166**, 827 (1968); L. J. Greenstein and J. J. Brophy, J. Appl. Phys. **40**, 682 (1969).
[32]W. J. Moore, J. Appl. Phys. **45**, 1986 (1974).
[33]P. J. Restle, M. B. Weissman, and R. D. Black, *ibid.* **54**, 5844 (1983).
[34]M. Nelkin and A. M. S. Tremblay, J. Stat. Phys. **25**, 253 (1981).
[35]S. Putterman, Phys. Rev. Lett. **39**, 585 (1977).
[36]S. Teitler and M. F. M. Osborne, Phys. Rev. Lett. **27**, 912 (1971).
[37]M. Mikulinsky and S. Starobinets, Phys. Rev. B **21**, 5558 (1980).
[38]S. M. Rytov, Vvedenie v statisticheskuyu radiofiziku. Ch. 1: Sluchaĭnye protsessy (Introduction to Statistical Radiophysics, Part 1: Random Processes), Nauka, M., 1976.
[39]J. L. Tandon and H. R. Bilger, J. Appl. Phys. **47**, 1697 (1976).
[40]W. E. Purcell, *ibid.* **43**, 2890 (1972).
[41]R. A. Dell, *ibid.* **44**, 472 (1973).
[42]M. Stoisiek and D. Wolf, *ibid.* **47**, 362 (1976).
[43]R. F. Voss and J. Clarke, Phys. Rev. B **13**, 556 (1976).
[44]J. V. Mantese, W. I. Goldburg, D. H. Darling, H. G. Craighead, U. J. Gibson, R. A. Buhrman, and W. W. Webb, Solid State Commun. **37**, 353 (1981).
[45]P. Van de Voorde and W. F. Love, Appl. Phys. Lett. **39**, 575 (1981).
[46]Sh. M. Kogan, Usp. Fiz. Nauk **123**, 131 (1977) [Sov. Phys. Usp. **20**, 763 (1977)].
[47]H. G. E. Beck and W. P. Spruit, J. Appl. Phys. **49**, 3384 (1978).
[48]A. M. S. Tremblay and M. Nelkin, Phys. Rev. B **24**, 2551 (1981).
[49]V. P. Palenskis, G. E. Leont'ev, and G. S. Mikolaĭtis, Radiotekh. Elektron. **21**, 2433 (1976).
[50]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Nauka, M., 1982, Sec. 117 (Engl. Transl. of earlier edn., Pergamon, Oxford, 1960).
[51]M. B. Weissman, J. Appl. Phys. **51**, 5872 (1980).
[52]M. B. Weissman, R. D. Black, and W. M. Snow, *ibid.* **53**, 6276 (1982).
[53]W. M. C. Van Bokhoven, Arch. Elektron. Übertragungstechn. **32**, 349 (1978).
[54]R. D. Black, P. J. Restle, and M. B. Weissman, Phys. Rev. Lett. **51**, 1476 (1983).
[55]Sh. M. Kogan and K. E. Nagaev, Fiz. Tverd. Tela (Leningrad) **24**, 3381 (1982) [Sov. Phys. Solid State **24**, 1921 (1982)].
[56]J. Kilmer, E. R. Chenette, C. M. van Vliet, and P. H. Handel, Phys. Status Solidi A **70**, 287 (1982).
[57]C. J. Adkins and R. H. Koch, J. Phys. C **15**, 1829 (1982).
[58]L. J. De Felice, J. Appl. Phys. **47**, 350 (1976).
[59]D. M. Fleetwood, J. T. Masden, and N. Giordano, Phys. Rev. Lett. **50**, 450 (1983).
[60]F. N. Hooge, Phys. Lett. A **29**, 139 (1969).
[61]H. C. Montgomery, Bell Syst. Tech. J. **31**, 950 (1952).
[62]J. J. Brophy, J. Appl. Phys. **27**, 1383 (1956).
[63]V. E. Noble and J. E. Thomas, *ibid.* **32**, 1709 (1961).
[64]M. Epstein, *ibid.* **36**, 2590 (1965).
[65]P. Van de Voorde, C. K. Iddings, and W. Love, Phys. Rev. B **19**, 4121 (1979).
[66]L. K. J. Vandamme, Phys. Lett. A **49**, 233 (1974).
[67]F. N. Hooge and A. M. F. Hoppenbrouwers, Physica (Utrecht) **45**, 386 (1969).
[68]D. M. Fleetwood and N. Giordano, Phys. Rev. B **25**, 1427 (1982).
[69]J. Clarke, in: SQUID-80: Proc. of Intern. Conference, West Berlin, 1980, Berlin, New York, 1980, p. 187.
[70]A. Ya. Shul'man, Zh. Eksp. Teor. Fiz. **81**, 784 (1981) [Sov. Phys. JETP **54**, 420 (1981)].
[71]M. Surdin, J. Phys. Radium **10**, 188 (1939); **12**, 777 (1951).
[72]F.K. Du Pré, Phys. Rev. **78**, 615 (1950).
[73]A. Van der Ziel, Physica (Utrecht) **16**, 359 (1950).
[74]A. L. McWhorter, in: Semiconductor Surface Physics, ed. R. H. Kingston, Univ. of Pennsylvania Press, Philadelphia, 1957, p. 207.
[75]P. Dutta, P. Dimon, and P. M. Horn, Phys. Rev. Lett. **43**, 646 (1979).
[76]B. I. Shklovskii, Solid State Commun. **33**, 273 (1980).
[77]Sh. M. Kogan and B. I. Shklovskiĭ, Fiz. Tekh. Poluprovodn. **15**, 1049 (1981) [Sov. Phys. Semicond. **15**, 605 (1981)].
[78]B. I. Shklovskii and A. L. Éfros, Elektronnye svoĭstva legirovannykh

poluprovodnikov (Electronic Properties of Doped Semiconductors), Nauka, M., 1979 (Engl. Transl., Springer-Verlag, Berlin, 1984).
[79]B. P. Smolyakov and E. P. Khaĭmovich, Usp. Fiz. Nauk **136**, 317 (1982) [Sov. Phys. Usp. **25**, 102 (1982)].
[80]J. L. Black, in: Glassy Metals. I., eds. H.-J. Güntherodt and H. Beck, Springer-Verlag, Berlin, 1981, p. 167.
[81]P. W. Anderson, B. I. Halperin, and C. Varma, Philos. Mag. **25**, 1 (1972); W. A. Phillips, J. Low Temp. Phys. **7**, 351 (1972).
[82]C. N. Guy, J. Phys. F **8**, 1309 (1978); J. Appl. Phys. **50**, 7308 (1979).
[83]V. V. Daniel, Dielectric Relaxation, Academic Press, London, 1967.
[84]L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika, Ch. 1 (Statistical Physics, Part 1), Nauka, M., 1976 (Engl. Transl. of 3rd edn., Pergamon, Oxford, 1979).
[85]Sh. M. Kogan, Solid State Commun. **38**, 1015 (1981).
[86]J. F. Fernandez and R. Medina, Phys. Rev. B **19**, 3561 (1979).
[87]D. Kumar and J. Stein, J. Phys. C **13**, 3011 (1980).
[88]T. Li, Phys. Rev. B **24**, 6579 (1981).
[89]I. Morgenstern and H. Horner, *ibid.* **25**, 504, 6067 (1982).
[90]H.-H. Chen and S. Ma, J. Stat. Phys. **29**, 717 (1982).
[91]G. P. Zhigal'skiĭ and A. V. Karev, Radiotekh. Elektron. **22**, 2610 (1977).
[92]E. M. Lifshitz and L. P. Pitaevskiĭ, Statisticheskaya fizika, Ch. 2 (Statistical Physics, Part 2), Nauka, M., 1978, Sec. 88.
[93]K. M. Van Vliet, A. van der Ziel, and R. R. Schmidt, J. Appl. Phys. **51**, 2947 (1980).
[94]M. B. Weissman, Phys. Rev. Lett. **41**, 1 (1978).
[95]L. A. Vaĭnshteĭn, Zh. Eksp. Teor. Fiz. **83**, 1841 (1982) [Sov. Phys. JETP **56**, 1064 (1982)].
[96]Yu. L. Klimontovich, Zh. Eksp. Teor. Fiz. **80**, 2243 (1981) [Sov. Phys. JETP **53**, 1171 (1981)]; see also: Statisticheskaya fizika (Statistical Physics), Nauka, M., 1982, p. 271.
[97]Sh. M. Kogan and K. E. Nagaev, Solid State Commun. **49**, 387 (1984).
[98]A. Ludviksson, R. Kree, and A. Schmid, Phys. Rev. Lett. **52**, 950 (1984).
[99]A. U. McRae and H. Levinstein, Phys. Rev. **119**, 62 (1960).
[100]T. G. Maple, L. Bess, and H. A. Gebbie, J. Appl. Phys. **26**, 490 (1955).
[101]G. L. Pearson, H. C. Montgomery, and W. L. Feldman, *ibid.*, **27**, 91 (1956).
[102]C. G. B. Garrett and W. H. Brattain, Phys. Rev. **99**, 376 (1955).
[103]C. T. Sah and F. H. Hielscher, Phys. Rev. Lett. **17**, 956 (1966).
[104]G. Abowitz, E. Arnold, and E. A. Leventhal, IEEE Trans. Electron Devices **ED-14**, 775 (1967).
[105]E. H. Nicollian and H. Melchior, Bell Syst. Tech. J. **46**, 1935 (1967).
[106]S. T. Hsu, D. J. Fitzgerald, and A. S. Grove, Appl. Phys. Lett. **12**, 287 (1968).
[107]R. J. Hawkins, I. R. Mansour, and G. G. Bloodworth, Br. J. Appl. Phys. Ser. 2, **2**, 1063 (1969).
[108]A. Mircea, A. Roussel, and A. Mittonneau, Phys. Lett. A **41**, 345 (1972).
[109]O. V. Snitko, Problemy poverkhnosti poluprovodnikov (Problems of the Surface of Semiconductors), ed. O. V. Snitko, Naukova Dumka, Kiev, 1981, p. 7.
[110]W. E. Spicer, P.W. Chye, C. M. Garner, L. Lindau, and P. Pianetta, Surface Sci. **86**, 763 (1979).
[111]H. Suhl, Bell Syst. Tech. J. **32**, 647 (1953).
[112]H. M. K. Vaes and T. G. M. Kleinpenning, J. Appl. Phys. **48**, 5131 (1977).
[113]T. G. M. Kleinpenning and D. A. Bell, Physica (Utrecht), B **81**, 301 (1976).
[114]T. G. M. Kleinpenning, *ibid.* B **77**, 78 (1974).
[115]T. G. M. Kleinpenning, *ibid.* B **103**, 340 (1981).
[116]T. G. M. Kleinpenning, *ibid.* B **113**, 189 (1982).
[117]A. K. Naryshkin and A. C. Vrachev, Teoriya nizkochastotnykh shumov (Theory of Low-Frequency Noise), Énergiya, M., 1972.
[118]V. S. Pryanikov, Prognozirovanie otkazov poluprovodnikovykh priborov (Prognosis of Failure of Semiconductor Devices), Énergiya, M., 1978.
[119]J. W. Eberhard and P. M. Horn, Phys. Rev. Lett. **39**, 643 (1977).
[120]P. Dutta, J. W. Eberhard, and P. M. Horn, Solid State Commun. **27**, 1389 (1978).
[121]J. W. Eberhard, P. Dutta, and P. M. Horn, Phys. Rev. B **19**, 1307 (1979).
[122]G. P. Zhigalskiĭ, É. A. Zhil'kov, N. D. Terekhov, and A. S. Fedorov, in: Sb. Nauchnykh trudov po problemam mikroélektroniki, Ser. "Poluprovodnikovye pribory i integral'nye skhemy" (Collection of Scientific Studies of Problems of Microelectronics, Series "Semiconductor Devices and Integrated Circuits"), MIÉT, M., 1976. No. 27, p. 134.
[123]G. P. Zhigal'skiĭ and A. S. Fedorov, *ibid.*, p. 147.
[124]G. P. Zhigal'skiĭ, Yu. E. Sokov, and N. G. Tomson, Radiotekh. Elektron. **24**, 410 (1979).
[125]G. P. Zhigal'skiĭ and I. S. Bakshi, *ibid.* **25**, 410 (1980).
[126]A. S. Nowick and B. S. Berry, Anelastic Relaxation in Crystalline Solids, Academic Press, New York, 1972 [Russ. Transl., Atomizdat, M., 1975].
[127]M. Weller and P. Moser, J. Phys. (Paris) **42**, Colloque C5, Suppl. No. 10, 741 (1981).
[128]J. Kedzia and L. K. J. Vandamme, Phys. Lett. A **66**, 313 (1978).
[129]F. N. Hooge, *ibid.* **33**, 169 (1970).
[130]A. A. Verveen and H. E. Derksen, Proc. IEEE **56**, 906 (1968).
[131]V. V. Kolachevskaya, N. N. Kolachevskiĭ, V. V. Rozhdestvenskiĭ, and L. V. Strygin, Radiotekh. Elektron. **16**, 1211 (1971).
[132]M. V. Bukharov, N. N. Kolachevskiĭ, and V. V. Rozhdestvenskiĭ, in: Fizika magnitnykh materialov: Mezhvuz. tem. sb. (Physics of Magnetic Materials: Intercollege Topical Collected Volume), KGU, Kalinin, 1978, p. 3.
[133]M. V. Bukharov and N. N. Kolachevskiĭ, Radiotekh. Elektron. **26**, 999 (1981).
[134]B. I. Lebed', I. I. Marchik, V. A. Noskin, and V. G. Shirko, Fiz. Tverd. Tela (Leningrad) **24**, 1997 (1982) [Sov. Phys. Solid State **24**, 1140 (1982)].
[135]R. F. C. Vessot, in: Experimental Gravitation: Proc. of Intern. School of Physics "Enrico Fermi," Course 56, Academic Press, New York, 1974, p. 111.
[136]J. J. Gagnepain and J. Uebersfeld, in: Proc. of Symposium on 1/*f* Fluctuations, ed. T. Musha, Tokyo, 1977, p. 173.
[137]T. Musha and H. Higuchi, Jpn. J. Appl. Phys. **15**, 1271 (1976); **17**, 811 (1978).
[138]M. J. Campbell and B. W. Jones, Science **177**, 889 (1972).
[139]P. Manneville, J. Phys. (Paris) **41**, 1235 (1980).
[140]I. Procaccia and H. Schuster, Phys. Rev. A **28**, 1210 (1983).
[141]K. S. Ralls, W. J. Skocpol, L. D. Jackel, R. E. Howard, L. A. Fetter, R. W. Epworth, and D. M. Tennant, Phys. Rev. Lett. **52**, 228 (1984).
[142]V. G. Karpov, M. I. Klinger, and F. N. Ignat'ev, Zh. Eksp. Teor. Fiz. **84**, 760 (1983) [Sov. Phys. **57**, 439 (1983)].
[143]T. S. Kuan, S. I. Raider, and R. E. Drake, J. Appl. Phys. **53**, 7464 (1982).
[144]Sh. M. Kogan and K. É. Nagaev, Pis'ma Zh. Tekh. Fiz. **10**, 313 (1984).
[145]C. T. Rogers and R. A. Buhrman, IEEE Trans. Magn. **MAG-19**, 453 (1983).
[146]J. Clarke and G. Hawkins, Phys. Rev. B **14**, 2826 (1976).
[147]M. E. Levinshteĭn and S. A. Rumyantsev, Fiz. Tekh. Poluprovodn. **17**, 1830 (1983) [Sov. Phys. Semicond. **17**, 1167 (1983)].
[148]R. G. Palmer, D. L. Stein, E. Abrahams, and P. W. Anderson, Phys. Rev. Lett. **53**, 958 (1984).

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