

# Niels Bohr's development of the quantum theory of the atom and the correspondence principle (his 1912–1923 work in atomic physics and its significance)

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*Usp. Fiz. Nauk* **147**, 253–301 (October 1985)

Niels Bohr's studies on the quantum theory of the atom over the years 1912–1923 are discussed. The sources of his work on these questions are analyzed. The beginning of his quantum studies in 1912 is described. His famous paper on the theory of the hydrogen atom and on the origin of spectra is analyzed in detail. The subsequent development of his ideas regarding atomic structure is discussed, with an emphasis on his postulates regarding stationary states and radiative transitions and on the development of the correspondence principle. How well he understood the difficulties of the model theory and how he strived for a profound understanding of quantum phenomena are shown.

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## 1. INTRODUCTION. TWO PARTS OF BOHR'S THEORY: HIS BASIC POSTULATES AND THE MODEL THEORY. ROLE OF THE CORRESPONDENCE PRINCIPLE

An extremely important step in the development of modern quantum theory was Niels Bohr's research on atomic physics over the years 1912–1923 (Refs. 1–4), primarily on the quantum theory of the atom, atomic spectra, and the development of the concept of correspondence between the quantum and classical theories. At the beginning of this period, Bohr worked four months, from March through July 1912, in Ernest Rutherford's laboratory at Manchester. This work was to determine the direction of his future research. L. Rosenfeld, Bohr's closest colleague, later wrote of this research that Bohr combined Rutherford's discovery of the atomic nucleus with the quantum of action and derived a theory for the structure of atoms and molecules on the basis of ideas foreign to classical electrodynamics, regarding stationary, nonradiative states of an atomic system and regarding radiative transitions between such states (Ref. 1, p. V). In 1913 Bohr published his famous paper on the structure of atoms and molecules in the *Philosophical Magazine*.<sup>5</sup> The first part of this paper contained the foundations of a quantum theory of the hydrogen atom and a discussion of its

spectrum. Bohr was brilliantly successful in theoretically deriving the value of the Rydberg constant and then applying the theory of one-electron atoms to the spectrum of the ionized helium atom.<sup>6</sup> Bohr gave a very clear description of his fundamental ideas in late 1913 in a report entitled "On the spectrum of hydrogen."<sup>1</sup> Especially noteworthy among his subsequent papers are several of a fundamental nature: in 1918, "On the quantum theory of line spectra" (Parts I and II)<sup>9</sup> (on the correspondence principle); in 1920, "On the serial spectra of the elements,"<sup>10</sup> with a further development of atomic theory; in 1921, "The structure of atoms in connection with the physical and chemical properties of the elements,"<sup>11</sup> where he gave a physical explanation for the periodic table worked out by Mendeleev; finally, in 1923, "On the application of the quantum theory to the structure of the atom. I. Fundamental postulates of quantum theory,"<sup>12</sup> where there was a long discussion of the fundamental difficulties which had appeared in this developing theory. These papers and several others were written by Bohr back in his own country, in Copenhagen. In 1920 he organized the Institute of Theoretical Physics there; the Institute was to become a leading center of the development of quantum theory. Bohr himself became the acknowledged leader of an international scientific school on theoretical physics (often

called the "Copenhagen school"). Bohr's ideas played a huge role in the establishment of quantum mechanics (see, for example, the collection in Ref. 13 and also the paper in Ref. 14).

It is extremely important to appraise correctly the exceptionally great contribution which Bohr made to the development of the quantum theory, as a theory of microscopic phenomena with their distinctive laws, different from the laws governing macroscopic phenomena and furthermore related to these macroscopic laws by the correspondence principle. Our purpose in the present review is to examine systematically the development of Bohr's ideas over the years 1912–1923 and thereby contribute to this appraisal. Another factor for making such a review very important is the fact that in many textbooks and monographs (and, not infrequently, in the popular literature) Bohr's quantum theory of the atom, or simply the "Bohr theory," as it is usually known, is described in a simplified and frequently totally incorrect way. This is particularly true of Bohr's postulates.

To achieve the goal we have set for this paper we need to analyze Bohr's original papers, his rough drafts, and his major revisions, drawing from the literature on the history of science regarding the development of quantum theory and directly regarding Bohr's scientific creativity. Our analysis is simplified considerably by the recent publication of Volumes 2–4 of his *Collected Works*,<sup>1–3</sup> whose publication was begun under the general editorship of Rosenfeld (Volume 1 appeared in 1972<sup>15</sup>). In these volumes, which cover precisely the period 1912–1923, there are photographic reproductions of Bohr's published papers along with extremely important archive materials: rough drafts of Bohr's papers, summaries of his lectures and reports, etc., selected correspondence. There are also useful introductory articles in Volume 2 by U. Hoyer (Hoyer had previously published a monograph<sup>16</sup> and a paper<sup>17</sup>; they were preceded by important papers by Rosenfeld<sup>18</sup> and by Heilbron and Kuhn<sup>19</sup>) and by J. R. Nielsen in Volumes 3 and 4. Russian translations of most of Bohr's published papers of these years (with brief commentaries) are contained in Volume I of *Izbrannye nauchnye trudy (Selected Scientific Works)*.<sup>4</sup> Questions regarding Bohr's development of the quantum of the atom and of the correspondence principle are discussed in well-known monographs by Jammer<sup>20</sup> and Hund.<sup>21</sup> Much factual material on these questions is contained in the monograph by Mehra and Rechenberg.<sup>22</sup> We might also note the collection in Ref. 13 and Bohr's biography in Ref. 23. The publications which we have cited here contain further references to materials regarding the scientific publications of Niels Bohr.\*

Before we go any further, we must emphasize that a distinction must be made between two parts or two aspects of Bohr's theory<sup>24–26</sup>. On the one hand, there are the basic assumptions regarding the existence of stationary states and of optical (radiative) quantum transitions between these states (the two basic postulates of Bohr); on the other hand, there is Bohr's specific model of the atom, Bohr's model theory, according to which the motion of electrons in stationary states is treated by classical mechanics with some auxiliary quantum conditions (in the simplest case of a single elec-

tron, there is the condition that the angular momentum of the electron is quantized in circular orbits).

Bohr originally formulated his two postulates as early as 1913, in the first part of Ref. 5. He refined this formulation over the course of several years (Bohr was always very careful about his formulations), particularly in Part I of Ref. 9 in 1918 and in Ref. 11 in 1921. In Ref. 12 in 1923, he carried out a special and extremely detailed examination of these postulates; in 1926, in his article on the atom in the *Encyclopedia Britannica*, he put these postulates in a final condensed form: 1. An atomic system is stable only for a certain set of states, "stationary states," which generally corresponds to a discrete sequence of values of the energy of the atom. Each change in this energy involves a complete "transition" of an atom from one stationary state to another. 2. The ability of an atom to absorb and emit radiation obeys a law according to which the radiation associated with a transition must be monochromatic and must have the frequency  $\nu_{\text{rad}}$  determined by the relation

$$E_1 - E_2 = h\nu_{\text{rad}}, \quad (1)$$

where  $h$  is Planck's constant, and  $E_1$  and  $E_2$  are the energies of the two corresponding stationary states<sup>21</sup> (Ref. 27, p. 263).

Bohr's postulates were exhaustively verified by experiments, in particular, the classic experiments by Franck and Hertz, and it was shown that they apply to both atoms and molecules (it was shown later that they also apply to nuclei). They have received a theoretical underpinning in quantum mechanics and in quantum electrodynamics; the word "postulates" survives only because of its historical meaning. It is on Bohr's postulates—characteristic quantum laws which are consequences of a systematic quantum theory—that modern spectroscopy is based.<sup>28</sup> This part of Bohr's theory, formulated in his postulates, has fully retained its importance and is today an inseparable part of the physics of microscopic phenomena.

A different role has been played in the history of quantum theory by the other part of Bohr's theory: his model of the atom. Bohr successfully applied this model to the hydrogen atom and to hydrogen-like ions. It was developed further by Bohr himself and, especially, Arnold Sommerfeld, who formulated some generalized quantum conditions (which are usually called the "Bohr-Sommerfeld rules"). Bohr's model theory, however, ran into some fundamental difficulties because the classical description of the motion is incorrect and because of the interaction of electrons in atoms. These difficulties were overcome only upon the advent of quantum mechanics. It was Bohr who clearly stated the unsatisfactory aspects of his model theory for the structure of atoms, despite its several successes (in particular, with regard to the periodic table and the spectra of the elements<sup>11,29,30</sup>). Bohr had a profound understanding of the need to develop a systematic and internally noncontradictory quantum theory. Bohr's model theory was only an intermediate step toward the development of such a quantum theory. Quantum mechanics has altered this part of Bohr's theory, as it applies to the structure of atoms and molecules.

Unfortunately, there are many examples of incorrect descriptions of Bohr's theory (this theory has been particularly unlucky in this regard); the two aspects of the theory have been mixed, and no distinction has been drawn between them. One often sees the condition of the quantization of circular orbits presented as one of Bohr's postulates, but Bohr himself did not include this condition, which refers to the model theory, among his basic postulates in his formulations. Starting in 1918, he always presented only two postulates. This error is found, in particular, in Volume 2 of *Fundamentals of Physics* by Yavorskiĭ and Pinskiĭ (Ref. 31, p. 237) and in *Introduction to Atomic Physics* by Gol'din and Novikova (Ref. 32, p. 142).<sup>3)</sup> Three postulates are given in those books. Evidence that Bohr's own formulations have fallen into oblivion also comes from the fact that later editions of *Encyclopedia Britannica* give four (!) Bohr's postulates in the article on the atom,<sup>34</sup> which replaced the corresponding article in the earlier editions, which had been written by Bohr<sup>27</sup> and which contained the formulation of two basic postulates as described above.

It should be emphasized, however, that in several well-known books Bohr's theory is presented absolutely correctly. One such book is Max Born's *Atomic Physics*,<sup>35</sup> and another is Shpol'skiĭ's *Atomic Physics* (Vol. 1).<sup>36</sup> The latter book sets forth Bohr's postulates and Bohr's model theory (the Bohr-Sommerfeld theory) separately, as would be methodologically correct. When Shpol'skiĭ speaks in terms of the "Bohr theory" he means the Bohr model theory. In the present paper we will use the term "Bohr theory" to mean the set of Bohr's postulates and his model theory of the atom. This terminology appears to be the most rational choice.

An exceedingly important role, often underestimated, was played in Bohr's development of the quantum theory of the atom by his development of the correspondence principle. The idea of a correspondence between the quantum and classical theories was used in its original form by Bohr as early as 1913 in a derivation of the value of the Rydberg constant (Ref. 5, p. 94), by the method which he regarded as the most convincing. He used this idea as the basis of Ref. 9 and later developed it and used it as a guiding idea in later papers. As we know, an approach to the development of a matrix formulation of quantum mechanics based on the correspondence principle was taken by Werner Heisenberg, on whom Bohr's ideas had a decisive influence (Ref. 37, p. 59). We might also note that, as was found after the development of quantum mechanics, the quantum conditions of the model theory (the Bohr-Sommerfeld rules) emerge as a limiting case in the semiclassical approximation, in agreement with the correspondence principle.

The major thrust of the present review is a detailed analysis of the pioneering first part of Bohr's 1913 work<sup>5</sup> and of his two postulates. We will deal in less detail with questions relating to the development of the model theory, which are covered quite extensively in the literature on the history of science. A considerable amount of attention is devoted to the development by Bohr of the correspondence principle, which is inseparably related to his development of the quantum theory of the atom.

## 2. SOURCES OF BOHR'S STUDIES OF THE QUANTUM THEORY OF THE ATOM

If we wish to reach an understanding of Bohr's studies on the quantum theory of the atom, which he began in mid-1912 in connection with the nuclear model of the atom which Rutherford had proposed in 1911 (Ref. 38)—studies in which Bohr achieved decisive successes in 1913 and which he later developed over a decade—it is extremely important to have a clear picture of the state of quantum theory and its basic concepts as they existed at the time. It is also important to know which paths Bohr took toward his first studies in the theory of the atom.

By 1911–1912, quantum theory had already been developed significantly, in studies first by Max Planck and Albert Einstein, and later by several other scientists.<sup>20–22,25,39,40</sup> The theme of the First Solvay Congress, which convened in the fall of 1911 and which attracted the most eminent physicists of the day, was "The theory of radiation and quanta."<sup>41,42</sup> Planck gave a talk on "The laws of thermal radiation and the hypothesis of an elementary quantum of action."<sup>43</sup> Einstein talked on "The present state of the theory of specific heat."<sup>44</sup> Questions of the theory of equilibrium thermal radiation, the interaction of radiation and matter, and the applications of quantum theory to atomic systems were widely discussed.

Characteristic of the development of quantum theory were the ideas regarding the discrete nature of energy and of energy transfer processes: First, there is the *idea of the quantization of the energy of matter*; second, there is the *idea of the quantization of the energy of radiation*; third, an idea related to the first two ideas is the *idea of the discrete nature of the elementary processes by which radiation and matter interact*—the discrete nature of the processes by which radiation is emitted and absorbed.<sup>4)</sup>

The idea of the discrete nature of the energy of matter—the quantization of the energy of matter was widespread. It was first applied by Planck to harmonic oscillators used as models of matter in his 1900 derivation of the law describing the spectral distribution of the energy of equilibrium thermal radiation (Planck's radiation law). As is well-known, in Planck's theory the total energy of such an oscillator, which is oscillating at a frequency  $\nu = \nu_{\text{vibr}}$  (the energy of a Planck "resonator" which emits and absorbs electromagnetic radiation at this frequency) is discrete, and is described by

$$E = nh\nu_{\text{vibr}} \quad (n = 0, 1, 2, \dots), \quad (2)$$

where  $h$  is Planck's constant (the "elementary quantum of action," as Planck called it in 1906; Ref. 45). This constant is characteristic of microscopic phenomena. Equation (1) represented a decisive break with the classical theory, but this circumstance was not immediately recognized. As Bohr was to point out later, Einstein was the first to point out the all-encompassing importance of Planck's theory for a discussion of the behavior of atomic systems<sup>1</sup> (Ref. 5, p. 88; in Ref. 1, Bohr cited Einstein's papers<sup>46–48</sup>). It was Einstein who, as early as 1906, clearly understood the incompatibility of the "special values of the energy" given by (1) with the classical theory (Ref. 47, p. 131; see also Ref. 49). In 1907 he applied this formula to the vibrations of atoms in a solid<sup>48</sup> and there-

by laid the foundation for the quantum theory of heat capacity, which he also discussed in a report<sup>44</sup> at the Solvay Congress.

Significantly, the quantization of the vibrational motion of atoms was beginning to be joined by a quantization of the rotational motion of molecules. In 1911 Walter Nernst pointed out the need to consider the quantization of the rotational energy of molecules in the theory of heat capacity, and in 1912 N. Bjerrum also used quantization to explain the structure of the bands in infrared absorption spectra (see the bibliographies in Refs. 14 and 39). Bjerrum equated the rotational energy of a molecule,  $J(2\pi\nu_{\text{rot}})^2/2$ , where  $J$  is the moment of inertia of the molecule, to the quantity  $nh\nu_{\text{rot}}$  ( $n = 1, 2, 3, \dots$ ) and obtained discrete rotation frequencies  $\nu = \nu_{\text{rot}} = nh/2\pi^2J$ , which he identified with the frequencies of observed spectral lines. Quantization of the energy of a rotator was also discussed in general terms by Hendrik Lorentz in a discussion of Einstein's report to the 1911 Solvay Congress (Ref. 44, p. 310). He set this energy to be  $q\nu_{\text{rot}}^2 = nh\nu_{\text{rot}}$ , where  $q$  is a constant, and  $n$  is an integer, finding rotation frequencies  $\nu_{\text{rot}} = nh/q$  and finding the energy itself to be  $q\nu_{\text{rot}}^2 = n^2h^2/q$ . The idea of quantization of rotational motion was also used to explain the discrete values of the magnetic moment (the so-called Weiss magneton<sup>50</sup>), on the basis of a representation regarding the relationship of the magnetic moment to the revolution of electrons in circular orbits, with allowance for the role played by the constant  $h$  (Ref. 19, p. 230). In particular, still in 1911, R. Gans (Ref. 50, p. 952) equated the kinetic energy of the revolution of an electron in a circular orbit of radius  $r$  to an integer multiple of  $h\omega/2\pi$ , where  $\omega$  is the angular velocity, finding

$$E_{\text{rot}} = \frac{1}{2} mr^2\omega^2 = \frac{nh\omega}{2\pi} \quad (n = 1, 2, 3, \dots) \quad (3)$$

( $m$  is the mass of the electron). As a result, this energy is related to the frequency of the revolution of the electron in its orbit (its revolution frequency),  $\nu_{\text{rot}} = \omega/2\pi$ , by the relation

$$E_{\text{rot}} = nh\nu_{\text{rot}} \quad (n = 1, 2, 3, \dots), \quad (4)$$

which is the same as that found by Lorentz and Bjerrum and is analogous to Planck relation (2) for the energy of the vibrations of a harmonic oscillator. It was the concept of a strictly determined relationship between the revolution energy of an electron in a circular orbit in an atom,  $E_{\text{rot}}$ —the expression for this energy incorporates the constant  $h$ —and the frequency of this revolution,  $\nu_{\text{rot}}$ , which Bohr used in 1912 (see Section 3 below).

It should be noted that expression (3) immediately leads to a quantization of the angular momentum of an electron in a circular orbit,  $M = mr^2\omega = 2E_{\text{rot}}/\omega$ , determined by the expression  $M = nh/\pi$ , which differs from the condition for the quantization of circular orbits in Bohr's model theory only by a factor of 2 (see Section 4 below). The same result is found for the magnitude of the magneton, which is proportional to  $M$  (in comparison with the Bohr magneton for  $n = 1$ ).

In addition to the concept of quantizing the energy of

matter, which came from Planck, there was the suggestion by Einstein in his pioneering 1905 paper<sup>46</sup> that the energy of radiation be quantized in the form of a *hypothesis of quanta of light*. According to this hypothesis, the energy of electromagnetic radiation ("the energy of light") "... consists of a finite number of quanta of energy which are localized at spatial points, which move as indivisible units and which may be absorbed or may appear only as a whole" (Ref. 46, p. 93).<sup>51</sup> The magnitude of these quanta, as Einstein demonstrated in this paper on the basis of statistical considerations, is  $h\nu_{\text{rad}}$ , where  $\nu_{\text{rad}}$  is the frequency of the radiation itself, in contrast with the situation in expressions (2) and (4). The hypothesis of quanta of light sharply contradicted classical electrodynamics, and most of the leading physicists of the day (in particular, those participating in the First Solvay Congress) did not accept this hypothesis in the period 1911–1913; in fact, they continued to reject it for the next decade. An important point is that the idea of quantization of the energy of radiation began to develop in another form, along with Einstein's concepts regarding the corpuscular structure of radiation. Initially Paul Ehrenfest<sup>51</sup> in 1906 and then independently Peter Debye<sup>52</sup> in 1910 applied this idea to the quantization of the natural oscillations of equilibrium radiation in a cavity. Debye was able to derive the Planck radiation law from the properties of the radiation itself, without appealing to a resonator model for matter.<sup>6)</sup> Consequently, by the time at which Bohr took up his study of the quantum theory of the atom the idea of the quantization of radiation energy was known both in the form of the hypothesis of quanta of light and in the form of the *suggestion of the quantization of the natural oscillations of a radiation field*.

The idea of a discrete nature of the elementary processes of the emission and absorption of light, an idea from which Bohr started in his postulates and in formulating condition (1) on the frequencies, was extremely important to Bohr's studies on the quantum theory of the atom. According to the quantum theory, the discrete nature of the emission and absorption is in fact a consequence of the discrete nature of the energy of a Planck resonator, in accordance with law (2), which expresses the quantization of the energy of a harmonic oscillator. The only allowed changes in the energy of such an oscillator are  $\epsilon = h\nu_{\text{vibr}}$  or some multiple of it.<sup>7)</sup> It should be emphasized, however, that the discrete nature of the transfer of energy, in portions  $h\nu_{\text{rad}}$ , in the interaction of radiation of matter followed in a more natural way from Einstein's hypothesis of quanta of light. It was in his 1905 paper<sup>46</sup> that Einstein used this hypothesis to explain the Stokes law, introduced his famous equation for the photoelectric effect, and discussed the ionization of molecules by individual quanta of light.

The concept of discrete emission and absorption of radiation "in quanta of energy"  $h\nu_{\text{rad}}$  was verified by many experimental results and came to be used widely by scientists in research on a variety of interactions with matter. Johannes Stark examined the emission of electromagnetic energy in portions  $hc/\lambda = h\nu_{\text{rad}}$  ( $c$  is the velocity of light, and  $\lambda$  the wavelength) in the capture of an electron by an atom into elliptical orbits, progressively closer to the center of the

atom [see Ref. 39, pp. 93 and 173, regarding these studies by Stark and their role in Bohr's 1913 formulation of frequency condition (1)]. The combination of the idea of the quantization of energy of matter and the idea of the discrete nature of the elementary processes by which radiation interacts with matter became a characteristic approach. On the basis of such a combination and of fundamental new ideas which Bohr introduced into quantum theory in his postulates and by means of the correspondence principle, a period of the development of this theory began in 1913 in which the major thrust was the development of the theory of the atom and of its spectra. Bohr himself, Sommerfeld, and many other scientists took part in this research.

Another direction in the development of the quantum theory, to a large extent an independent direction, involved the idea of quantizing radiation energy in the form of the hypothesis of quanta of light. Einstein continued to work in this direction. In 1909 he examined the corpuscle-wave dualism for radiation and then carried out several other studies related to the quantum theory of radiation (see Ref. 49 regarding the importance of these studies). Although Bohr continued to disagree with the hypothesis of light quanta for a long time (until 1925), he held Einstein's work in high regard. In 1918, in a paper on the correspondence principle,<sup>9</sup> Bohr used the results of Einstein's investigations of the probabilities for quantum transitions (see Section 6 below).

Let us examine Bohr's attitude toward the quantum theory in 1911–1912, i.e., in the period preceding the beginning of his research on the quantum theory of the atom.<sup>19</sup>

In May 1911, Bohr successfully defended his dissertation, *Studies on the Electron Theory of Metals*, in Copenhagen [at the time, his dissertation was published only in Danish (see Ref. 15, p. 167; see also the English translation, p. 291)]. In this dissertation Bohr drew from work by Lorentz in 1905 on the statistical theory of free electrons in metals<sup>53</sup> and posed the problem of "... attempting to carry out calculations on various phenomena which can be explained in terms of the presence of free electrons and metals in the most general possible form, in accordance with the principles of the Lorentz theory" (Ref. 15, p. 299). An extremely interesting point was that Bohr reached the conclusion that classical mechanics was inadequate for a study of the interaction of bound electrons. He wrote, "Although, on the one hand, the kinetic theory of gases has yielded some unusual results, under the assumption that the forces between individual molecules are mechanical, there are, on the other hand, many properties of solids which cannot be explained by assuming that the forces acting within individual molecules (which, according to the conventional point of view, consist of systems containing a large number of "bound" electrons) are also mechanical" (Ref. 15, p. 175). With regard to the theory of thermal radiation, Bohr assumed that the Rayleigh-Jeans radiation law (for conduction electrons in a metal, found from the classical theory of Lorentz<sup>54</sup>) does not apply in the short-wave part of the spectrum. Bohr reached the following conclusion: "The reason for the failure is very probably the fact that the electromagnetic theory does not agree with the actual conditions in matter and can predict

correct results only in application to a large number of electrons (as in ordinary objects) or for determining the average velocity of an individual electron over a comparatively long time (as in a calculation of the motion of cathode rays); it cannot be used to study the motion of an individual electron over a short time" (Ref. 15, p. 378). These were the calculations that Lorentz reported in Ref. 54. In his dissertation, Bohr also discussed the Langevin theory of magnetism. He drew the conclusion that classical mechanics is inapplicable for describing magnetism and compared it with the "ultra-violet catastrophe" (i.e., the inapplicability of the classical Rayleigh-Jeans radiation law in the short-wave part of the spectrum). The explanation of magnetism in the electron theory was linked with concepts regarding the orbital motion of bound electrons in atoms; taking this approach, Bohr was led to questions in the quantum theory of the atom (Ref. 19, p. 222).

After Bohr defended his dissertation, he spent a year in England for further development, choosing as the site for his post-doctorate (as we refer to it today) research, the famous Cavendish Laboratory at Cambridge, under the direction of J. J. Thomson. Thomson was very interested in electron theory. In 1904 he had proposed his own model of the atom,<sup>55</sup> which had become extremely popular and was widely used. Bohr arrived at Cambridge in September 1911 and pursued his interests in electron theory.

In particular, he was interested in the "doublet model" which Thomson had proposed in 1910 to explain the photoelectric effect.<sup>56</sup> According to this model, an electron moves in a circular orbit under the influence of an electric dipole of moment  $P$ , near the axis of this dipole. For the allowed orbits, the kinetic energy of the electron,  $mv^2/2$ , turns out to be  $A\nu_{\text{rot}}$ , where  $\nu_{\text{rot}}$  is the frequency of the revolution of the electron in its orbit, and the constant  $A$  is proportional to  $P$ . Thomson estimated that this constant was close in order of magnitude to the Planck constant  $h$ . The relation

$$E_{\text{kin}} = E_{\text{rot}} = \frac{1}{2} mv^2 = h\nu_{\text{rot}}, \quad (5)$$

may be thought of as the condition which determines the state of the electron in the atom. It was a condition analogous to this one which Bohr subsequently used in his first outline of the quantum theory of the atom in the summer of 1912 (see Section 3 below).

Bohr intended to discuss questions of electron theory with Thomson, but the latter showed no interest in Bohr's ideas (as is evidenced by Bohr's letter to his brother Harald of 23 October 1911; Ref. 15, p. 527). In Thomson's laboratory, Bohr took up experimental studies of electric discharges and also continued his theoretical work on electron theory. He discussed these studies at length in a letter to his friend, C. W. Oseen in 1911, concluding this letter in the following way<sup>81</sup>: "I am at the moment very enthusiastic about the quantum theory (I mean its experimental side), but I am still not sure this is not due to my ignorance. I can say the same in a far higher degree, about my relation to the theory of magnetons. I very much look forward to trying to get all these things straight next term." Just before this letter, P.



Weiss published his paper on magnetons<sup>50</sup> (with a brief discussion of it, in which Gans took part); Bohr apparently had that paper in mind.

Bohr worked at Cambridge, pursuing the electron theory, until March 1912, when he transferred to Manchester to work in Rutherford's laboratory, where he began his fundamental studies of the quantum theory of the atom.

Bohr had first seen Rutherford, heard him speak, and made his acquaintance in late 1911. Rutherford's personality made a major impression on Bohr and he decided to go to Manchester for the rest of his training in England. After first reaching an agreement about his stay with Rutherford in a personal meeting (Ref. 57, p. 546), Bohr wrote him from Cambridge on 18 January 1912 (Ref. 1, p. 576). He had now set his plans for his stay here in England, and he had spoken to professor Thomson about them. He (Bohr) wished to remain here at Cambridge for the semester and to carry out some theoretical work. Then, if Rutherford would arrange it, Bohr would be very happy to come to Manchester in late March and to work in Rutherford's laboratory. On 27 January 1912, Rutherford answered Bohr; he would be glad to allow Bohr to work in his laboratory if Bohr decided to leave Cambridge. Rutherford thought that it would be best for Bohr to take leave in late March, since he (Rutherford) would probably be away on vacation for most of April. However, Rutherford would probably be able to organize some preliminary work for Bohr in April so that Bohr could acquire some experience in radioactive methods. Bohr did not postpone his move to Manchester; he in fact arrived there in the middle of March.

### 3. THE BEGINNING OF THE RESEARCH. THE RUTHERFORD MEMORANDUM: THE DIMENSIONS OF THE ATOM AND THE QUANTUM OF ACTION (1912)

In Rutherford's laboratory at Manchester, Bohr became acquainted with experimental studies of radioactivity. Later, in May, he began his experimental work on these questions, which was related to radium, and he began to meet regularly with Rutherford. Bohr continued to be primarily interested in questions of the electron theory of metals, as can be seen from his letters to his brother on 27 and 28 May 1912 (Ref. 15, p. 549). He hoped to explain "various difficulties of a general nature" and planned "to write a little about this."

Beginning in June 1912, however, Bohr began to work extremely actively on the theory of the passage of charged particles through matter and, later, on questions of the structure of atoms and molecules. He was now working on the basis of Rutherford's model of the atom, which was at center stage at the Manchester laboratory and with which Bohr naturally became familiar. He had previously become quite familiar with Thomson's model of the atom,<sup>55</sup> which he had also used in his own earlier theoretical work.

We can see the decisive turn in the direction of Bohr's studies, which proved to be exceedingly important to the development of quantum theory, in his letter to his brother on 12 June 1912, where he wrote (Ref. 15, p. 555), "A cou-

ple of days ago a modest idea came to me regarding understanding of the absorption of  $\alpha$  rays. [It occurred in the following way: A young mathematician, C. G. Darwin, had just published a theory on this problem, and I believed that this theory not only was not completely correct mathematically (although the error was minor) but was also extremely unsatisfactory in terms of its basic concepts.] I developed a modest theory in this regard, which, even if it does not prove to be very important, may be able to cast a little light on certain questions associated with the structure of atoms. I propose to publish a short paper on this matter in the near future." Bohr further mentioned that Rutherford recently "... developed a theory of atomic structure which apparently had a far firmer base than anything we had had before. Although my own theory was not of the same kind or of the same value, my results agreed fairly well with it (you will understand that all I intend to say is that the basis of my own simple calculations can be reconciled with his ideas)."

Darwin, who was working with Rutherford, in the paper<sup>58</sup> which Bohr cited, made two suggestions which Bohr criticized: that in the brief interaction of an  $\alpha$  particle passing through an atom it was possible to ignore the forces acting within the atom on a bound electron and that the velocity of the  $\alpha$  particle does not change if the particle does not pass directly through the atom but simply passes by it. Bohr developed a theory in which he took into account the energy transfer from a charged particle passing through matter to an electron bound in an atom. In June through August 1912 he wrote his paper, "On the theory of the decrease of velocity of moving electrified particles on passing through matter," which he discussed with Rutherford. This paper was submitted for publication in August 1912 (when Bohr gave it to Rutherford), and it was published<sup>59</sup> in early 1913. This paper was the beginning of Bohr's research on atomic physics.<sup>9)</sup>

In Ref. 59, Bohr considered the collisions of  $\alpha$  and  $\beta$  particles with atoms and worked using Rutherford's model of the atom. He wrote that it is assumed that the atoms of matter consist of a cloud of electrons which are confined to a nucleus by attractive forces. It is furthermore assumed that this nucleus, which has a positive charge equal to the sum of the charges of the electrons, has most of the mass of the atom, while its dimensions are exceedingly small in comparison with those of the atom. According to this theory, an  $\alpha$  particle is simply the nucleus of a helium atom (Ref. 59, p. 63). After criticizing Darwin's paper,<sup>58</sup> Bohr set forth his own theory of the transfer of energy to a bound electron from a fast charged particle. He made assumptions regarding the characteristics of the orbits of the electrons and regarding the motion of electrons in these orbits, in accordance with Rutherford's model of the atom<sup>10)</sup> (Ref. 59, p. 73).

In applying his theory to experimental data on the absorption of  $\alpha$  rays, Bohr reached agreement with the assumption that the number of electrons is 2 for the diatomic hydrogen molecule and for the helium atom, while it is 16 for the diatomic oxygen molecule (these figures correspond to nuclear charges  $Z = 1$  for the  $H$  atom,  $Z = 2$  for the  $He$  atom, and  $Z = 8$  for the  $O$  atom). For aluminum, tin, gold, and lead, Bohr found 14, 38, 61, and 65, respectively, as the

numbers of electrons in the atom (and thus as the values of  $Z$ ).

Bohr pointed out that according to Rutherford's model of the atom we would expect the oxygen molecule to have 16 electrons. Bohr judged the agreement between this value and the value obtained by himself to be completely satisfactory (Ref. 59, p. 78; Bohr calculated 18 instead of 16 from the experiment). There is every reason to believe that Bohr had already associated the value of 8 as the charge of the nucleus with the position of oxygen as the eighth element in the periodic table; at the time, he was very interested in the theory of this table, working from Rutherford's model of the atom (more on this below, in the present section of the paper).

In his conclusion,<sup>59</sup> Bohr wrote that in adopting Rutherford's theory of the structure of the atom one can very confidently assert, on the basis of data on the absorption of  $\alpha$  rays, that the hydrogen atom contains one electron, while the helium atom contains two electrons outside a positively charged nucleus. Bohr stated that the latter circumstance followed directly from Rutherford's theory. The absence of the number of electrons (and thus of the nuclear charge) of other atoms from the list of conclusions was apparently the result of a discussion of this paper with Rutherford, who was always very cautious with regard to drawing conclusions from a theory.

Bohr's studies on the theory of the stopping of charged particles led him into close contact with the problem of the structure of atoms and molecules, in which he became actively involved. Bohr rapidly began to acquire results in this field.

As early as 19 June 1912, Bohr wrote his brother (Ref. 15, p. 559), "It is possible that I have found out a little about the structure of atoms . . . This all stemmed from a small bit of information which I had obtained from the absorption of  $\alpha$  particles (the modest theory about which I wrote you in my last letter<sup>11</sup>). You will understand that I may still be wrong, because this matter has not yet been worked out finally (however, I believe that I am not making any errors). I do not believe that Rutherford regards these ideas as completely insane either, but he is a correct man and will never say that he believes in something which has not been completely worked out."

Bohr prepared material for a discussion of his results with Rutherford. He wrote Rutherford in July 1912, apologizing for writing so late, but stating that he had seen some difficulties in explaining the experimental results on dispersion. He had not seen these difficulties earlier, and he had not yet been able to put everything back in order. He stated that he was sending along some ideas regarding the structure and stability of molecules which Rutherford had been gracious enough to request (Ref. 1, p. 577).

This material, which has survived in the Bohr archives under the label applied by Bohr himself, as the first draft of the ideas contained in the paper, "On the constitution of atoms and molecules" (written to show the ideas to Professor Rutherford), is of much interest. In their paper,<sup>19</sup> Heilbron and Kuhn called this material the "Rutherford memorandum," while they called Bohr's 1913 paper,<sup>5</sup> "On the

constitution of atoms and molecules," consisting of three parts, a "trilogy." The same terminology was later used in Volume 2 of Bohr's Collected Works,<sup>1</sup> and we will retain it in the discussion below. A facsimile of the Rutherford memorandum (written in English) is reproduced in Ref. 1, and a revised version is also given. It contains text and several calculations of the energy of electrons in atoms and simple molecules. There are also some schematic figures which illustrate the corresponding electron structure of these models.

At the beginning of the memorandum, Bohr wrote that according to the model of the atom proposed by Rutherford, with the goal of explaining the "large scattering" of  $\alpha$  particles, atoms consist of a positive charge which is concentrated at a point (in a region with a dimension very small in comparison with the dimensions of atoms) and surrounded by a system of electrons whose total charge is equal to the charge of the positive "Kern" (Bohr was using the German word for "nucleus"). The Kern was also assumed to be the place where the mass of the atom was concentrated (Ref. 1, p. 136). Bohr immediately emphasized an extremely important property of this model: In such an atom there could be no equilibrium configuration, without a motion of electrons (nothing was given for determining the dimensions or "length"). Significantly, in his later introduction to Part I of the trilogy (Ref. 5, p. 84), Bohr made special note of the absence of a characteristic length for Rutherford's model of the atom, in contrast with Thomson's model, for which there is such a length: the radius of a positively charged sphere (Section 4 below).

As Thomson had done in his model, Bohr examined the distribution of electrons in an atom on coaxial rings (at equal angular intervals), but on the basis of Rutherford's model of the atom. He raised the question of the conditions for the stability of a ring of  $n$  electrons which are revolving around a positive point charge of magnitude  $n \times e$ . He drew the conclusion that a ring like the one under consideration here would not be stable in the usual mechanical sense and that the question of stability must therefore be approached from a completely different direction.

Bohr emphasized the distinctions between Thomson's and Rutherford's models of the atom and stated that Rutherford's model apparently made it possible to explain the periodic law of the chemical properties of the elements (the chemical properties were assumed to depend on the stability of the outermost ring of "valence electrons"), while Thomson's model cannot offer a satisfactory explanation for the periodic law.

Bohr carried out calculations on the energy of the electrons of the ring, finding their kinetic, potential, and total energies as functions of the number of electrons (under the assumption of a Coulomb interaction). He found as a result that a ring would be mechanically stable at  $n \leq 7$  and unstable at  $n > 7$ . He then made the case for the formation in complex atoms of several rings, inner and outer. This result was erroneous (Ref. 19, p. 246) and contradicted the known relation between the kinetic energy  $E_{\text{kin}}$  and the potential energy  $E_{\text{pot}}$ , specifically,  $E_{\text{pot}} = -2E_{\text{kin}}$ , which holds for the Coulomb interaction of charged particles and which leads to the

following expression for the total energy  $E$ :

$$E = E_{\text{pot}} + E_{\text{kin}} = -E_{\text{kin}} = E_{\text{pot}}/2. \quad (6)$$

Bohr himself, in the last part of his memorandum, derived the relation  $E_{\text{pot}} = -2E_{\text{kin}}$  for a system of electrically charged particles having a symmetry axis (a common revolution axis; Ref. 1, p. 142). In other words, he derived a particular case of the virial theorem. However, Bohr, apparently in haste in the preparation of the memorandum, did not notice the contradiction.

Despite the incorrect derivation, the idea of the separation of the electrons in an atom into groups turned out to be valid, and Bohr was to make successful use of it in future in his derivation of a physical theory of the periodic table. Bohr laid the foundation for this derivation right in the memorandum.

The most important feature of the memorandum was Bohr's use of quantum theory in the form of the "special hypothesis  $E = K\nu$ "; (Ref. 1, p. 141; the emphasis is Bohr's), where  $E$  is the kinetic energy of an electron moving in a circular orbit at a revolution frequency  $\nu = \nu_{\text{rot}}$ , and  $K$  is a *quantum constant of the dimensionality of action*, which is of the order of Planck's constant  $h$ . We rewrite this relation as

$$E_{\text{kin}} = E_{\text{rot}} = K\nu_{\text{rot}} = \alpha h\nu_{\text{rot}}, \quad (7)$$

where  $\alpha$  is a numerical factor. Bohr himself in his memorandum did not give a value of  $K$ , but it follows from an analysis of the equations which he did use and from the numerical values of the other constants that he was using a value of approximately  $0.6h$  for  $K$  (i.e.,  $\alpha = 0.6$ ; see Ref. 18, p. XXX, and Ref. 19, p. 250).

Relation (7) is analogous to relations of the type in (4) (with  $n = 1$ ) and (5), which were known to Bohr, but Bohr did not simply set  $K = h$ , apparently preferring to determine  $K$  from experimental data. With the value  $\alpha = 1/2$ , we find from the relation  $E_{\text{rot}} = m r^2 \omega^2 / 2 = h \nu_{\text{rot}} / 2 = h \omega / 4\pi$  [cf. (3)] an angular momentum  $M_0 = h / 2\pi$ : the condition for the quantization of the circular orbit for the ground state of an electron in a hydrogen atom. In Bohr's writings, this condition appears later, in Part I of his trilogy (Ref. 5, p. 97), but it appears as a consequence of general relations based on the concept of the correspondence principle.

Extremely interesting was Bohr's basis for his hypothe-

sis, which he offered after beginning the memorandum with a discussion of the stability of Rutherford's model of the atom.

He wrote that in studying the configuration of electrons in atoms we immediately encounter the difficulty (associated with the instability mentioned earlier) that a ring, provided that the magnitude of the central charge and the number of electrons are given, can rotate with an infinitely large number of different rotation times, corresponding to the different ring radii which are adopted. It appears (by virtue of the instability) that there is nothing which would make it possible to choose, on the basis of mechanical considerations, among different radii and different oscillation times.<sup>12)</sup> Bohr stated that in the study to follow he would therefore introduce and make use of a hypothesis which would make it possible to determine these quantities. This hypothesis was as follows: For any stable ring (for any ring which is met in a natural atom) there is a definite relation<sup>13)</sup> between the kinetic energy of the electrons in the ring and the rotation time. This hypothesis, for which Bohr would not attempt to offer a mechanical explanation (since the attempt appeared to him to be hopeless<sup>14)</sup>), was adopted simply as the only hypothesis which apparently opened up a way to explain the entire set of experimental results, which are thereby brought together and which Bohr perceived as confirming the ideas proposed by Planck and Einstein regarding the mechanism for emission (Ref. 1, p. 137).

Bohr later stated that his hypothesis could explain (1) the periodic law of the atomic volumes of the elements, (2) Whiddington's law relating the velocity required for the excitation of the characteristic x rays of an element to the atomic weight of this element, (3) (approximately) Bragg's law for the absorption of  $\alpha$  rays by various elements, and (4) (apparently) the stability and heat of formation of certain simple compounds.

In the memorandum Bohr emphasized the electron structure and properties of simple molecules (cf. his letter to Rutherford, discussed earlier in this section). He offered a model for the structure of such molecules (Fig. 1), according to which the electrons (shown by the crosses) are moving along rings perpendicular to the axis of the molecule (in the case of methane,  $\text{CH}_4$ , perpendicular to lines connecting the carbon nucleus to the hydrogen nuclei).

Some of the electrons are shown on rings of smaller

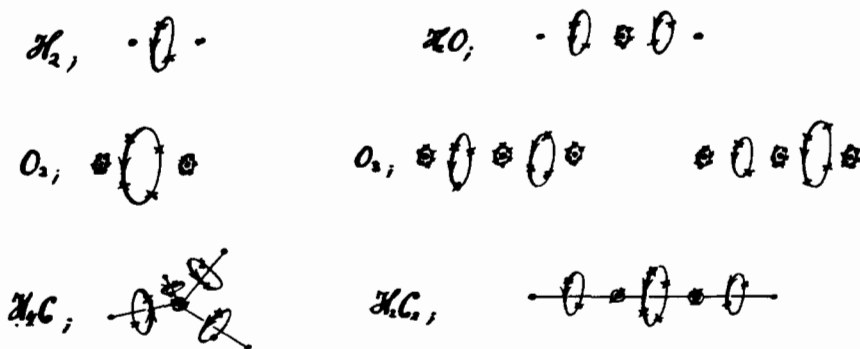


FIG. 1. Electronic structure of the simplest molecules ( $\text{H}_2$ ,  $\text{H}_2\text{O}$ ,  $\text{O}_2$ ,  $\text{O}_3$ ,  $\text{CH}_4$ , and  $\text{C}_2\text{H}_2$ ).



dimensions near the oxygen and carbon nuclei. The total number of electrons corresponds to nuclear charges of 1 for hydrogen, 6 for carbon, and 8 for oxygen (only for the acetylene molecule,  $C_2H_2$ , is Bohr two electrons shy on his central ring of the six required for a triple bond).

For hydrogen and helium, Bohr calculated the energies for the H and He atoms and for the corresponding  $H_2$  and  $He_2$  molecules. He showed that the  $H_2$  molecule is stable (its energy turned out to be lower than the energy of two isolated H atoms), while the  $He_2$  molecule was unstable (its energy is greater than the energy of two isolated He atoms).

In these calculations Bohr considered the Coulomb interactions for electrons moving along rings in accordance with the models which he adopted, and he made use of the condition for mechanical equilibrium. To find numerical values for the energy he used hypothesis (7), with a value for the constant  $K$  corresponding to the resonance frequency of molecular hydrogen, found from experiments on refraction and dispersion (see Ref. 19, p. 251, and also Bohr's papers in Ref. 59, p. 75, and Ref. 5, p. 138, where he gives the value of the resonance frequency).

Bohr's application of relation (7), based on the concept of discrete values of the energy of matter, to Rutherford's model of the atom in the Rutherford memorandum was the beginning of Bohr's studies in the quantum theory of the atom. By the fall of 1912, after returning to Copenhagen, where he was working at Copenhagen University, Bohr began to write a paper on the structure of atoms and molecules. He overcame the difficulties which he had previously encountered. On 4 November 1912 he wrote Rutherford, apologizing that he was not yet able to complete his paper on atoms and to send it to Rutherford, explaining that he had been busy with lectures and laboratory work (he was assisting Professor Knudsen), so that he was left with very little time. Bohr hoped that he would be able to finish the paper in a few weeks (Ref. 1, p. 577). Bohr was delayed until March 1913 in finishing the paper, but by that time he had managed to make great progress and had achieved a decisive success in the creation of a quantum theory of the atom.

#### 4. BOHR'S STRIKING SUCCESSES: A THEORY OF THE HYDROGEN ATOM AND AN EXPLANATION OF THE SPECTRAL LAWS; FIRST USE OF THE CONCEPT OF CORRESPONDENCE (1913)

During the preparation of his paper on atoms (about which Bohr wrote Rutherford on 4 November 1912—his future trilogy<sup>5</sup>), through to the beginning of February 1913, Bohr characteristically concerned himself with the states of atoms and molecules with the lowest energy: ground states, as we would call them today. To find definite values of the energy for such states, Bohr applied a quantum condition like (7) to atomic systems formed from nuclei and electrons interacting by Coulomb's law. He was thus combining Rutherford's model of the atom with the concept of a quantization of the energy of matter [for which the starting point was the Planck formula for the quantization of a harmonic oscillator, (2); see Section 2 above], but Bohr restricted his calculations to finding the energy of a specific atomic system in

its ground state, i.e., the lowest possible energy.

For the motion of an electron in a circular orbit in a simple H or He atom or  $H_2$  or  $He_2$  molecule, Bohr calculated the energy in the Rutherford memorandum. The use of condition (7) corresponded to the use of an expression like (4) with  $n = 1$  (when the kinetic energy of the revolution of the electron is at its minimum possible value). Bohr used condition (7) as a universal condition in his calculations. A corollary of this universality is the universality of the value of the angular momentum,  $M_0 = 2E_{\text{rot}}/\omega = 2\alpha h\nu_{\text{rot}}/2\pi\nu_{\text{rot}} = \alpha h/\pi$  (i.e.,  $h/2\pi$  with  $\alpha = 1/2$ ; see Section 2 above) for individual electrons in the ground state of an atomic system which are moving in the circular orbits of the smallest radius (see Ref. 17, p. 193, where there is a discussion of the universal constancy of the angular momentum and of Bohr's first studies of the quantum theory of the atom). The suggestion that the angular momentum of each electron in the ground state of any atomic system consisting of nuclei and electrons is  $h/2\pi$  was subsequently made by Bohr in his trilogy (Ref. 5, pp. 106, 148), and he was to use it again in the future. In the trilogy, however, this suggestion arose as a consequence of more general assumptions and was not simply postulated (see the discussion below in this section).

We know of the state of Bohr's work on his paper in late January and early February 1913 from his letters to Rutherford, Oseen, and Hevesy. At this time he was, as before, concerned with the ground states of atoms and molecules, but he had also been attracted to the emission of radiation by atoms—to the field of atomic spectra, i.e., to excited states of atoms (in the modern terminology). By the end of 1912 Bohr had become acquainted with Nicholson's papers on astrophysics<sup>64</sup>; Nicholson used quantum ideas to explain unidentified spectral lines in the spectra of nebulae and the solar corona (regarding the studies by Nicholson see Ref. 65 and also Ref. 19, p. 258, and Ref. 20, p. 72).

Nicholson worked from Nagaoka's Saturn-like model of the atom (which, of course, appeared before Rutherford's model of the atom), which consisted of a heavy central positive charge around which an electron ring was rotating. The ratio of the energy of the ring to its rotation frequency was set equal to an integer multiple of Planck's constant  $h$  by Nicholson, who suggested that the discrete frequencies  $\nu_{\text{rad}}$  of the observed spectral lines are equal to the frequencies  $\nu_{\text{vibr}}$  of the mechanical vibrations (perpendicular to the plane of the orbit) and corresponded to the emission of quanta of energy  $h\nu_{\text{rad}}$ . Nicholson attributed the lines themselves to hypothetical elements, "nebulium" and "proto-fluorine," with positive nuclear charges of  $4e$  and  $5e$ . Nicholson was thus using the concept of quantization of the energy of matter and the concept of a discrete nature of the processes by which radiation is emitted.

Bohr first wrote about Nicholson's works on 23 December 1912 in a Christmas greeting to his brother (Ref. 15, p. 563): "P. S. Although this may not be suitable for a Christmas card, one of us<sup>15)</sup> would like to point out that he does not regard Nicholson's theory as incompatible with his own. Indeed, his calculations will be valid for the final, classical state of atoms, while Nicholson is dealing with atoms which are

emitting radiation, when electrons are in the process of losing energy, prior to taking up their final positions. The radiation will then occur in pulses (there is much evidence for this assertion), and Nicholson is dealing with atoms while their energy is still so large that they are emitting light in the visible part of the spectrum. Later on, light is emitted in the ultraviolet region, until, finally, all the energy which can be radiated has been lost . . . ."

This picture corresponds to that which Nicholson drew for the loss of energy by an atom. He stated that if the atom loses its energy in definite amounts, rather than continuously, it should give rise to a series of spectral lines corresponding to each of the steps. Furthermore, the inability of the atom to radiate energy continuously would make the lines quite sharp (Ref. 64, p. 730). We see that Bohr was already forming his ideas regarding the abrupt loss of energy by an atom during the capture of electrons to the ground state. This interpretation acquired a firm basis when, in Part I of his trilogy, Bohr postulated the existence of stationary states with discrete values of the energy, ground and excited states (see the discussion below in the present section).

In a very interesting letter to Rutherford on 31 January 1913, Bohr wrote (Ref. 1, p. 576) that he hoped to be able very soon to send Rutherford his paper on atoms. He had spent far more time on this paper than he had expected. Bohr believed, however, that he had recently made some progress. Bohr wrote that the foundations of his interpretation were now far clearer, and he believed that he now also had a better understanding of the relationship and the differences between his own calculations and, for example, calculations similar to those which had been published in recent papers by Nicholson regarding the spectra of stellar nebulae and the solar corona.

Bohr went on to compare Nicholson's and his own approaches. He wrote that Nicholson in his calculations, like Bohr himself, was dealing with systems having the same structure as Rutherford's model of the atom. He wrote that when Nicholson was determining the dimensions and energy of the system he, like Bohr, sought a basis in the relation between the energy and the frequency which had been proposed in the Planck radiation theory.<sup>16)</sup> However, Bohr wrote, the state of the systems which he had considered in his calculations—from among the number of states which agree with this relation—was characterized as that state in which the systems have the lowest possible amount of energy, i.e., that state whose formation is accompanied by the emission of the greatest possible amount of energy.

It thus seemed to Bohr to be a plausible hypothesis that the state of the systems with which he was dealing in his calculations should be identified with a constant (natural) state of the atoms. (He felt that this hypothesis was justified by the agreement between the theory and experiments on the atomic volumes and on x rays which he had found from the very beginning and which he intended to pursue in the future.)

According to this hypothesis, continued Bohr, the states of the system considered by Nicholson have, on the contrary, a lower stability. Bohr perceived these states as the

states which are traversed during the formation of the atoms and as states in which there is an emission of the energy corresponding to the spectral lines characteristic of the given element. From this point of view systems in a state of the sort considered by Nicholson would be present in significant number, according to Bohr, only in places where the atoms continuously decay and form anew, i.e., in places similar to excited vacuum tubes or stellar nebulae. Bohr felt obliged to note, however, that the relations mentioned above do not play any important role in the study in his own paper. He stated that he was totally not involved in calculating the frequencies corresponding to lines in the visible spectrum. All that Bohr was trying to do was to use a simple hypothesis, which he had used from the very beginning, to discuss the structure of atoms and molecules in a "constant" state. By this, Bohr meant that he had attempted to extract certain general properties of the systems which he was considering without—in accordance with Rutherford's advice—going through detailed calculations for any specific system other than the simplest.

We see that Bohr was not yet taking up the questions of the characteristics of the excited states or of an explanation of the spectra—questions which became central in Part I of his trilogy.<sup>5</sup>

In a letter of 5 February 1913, Bohr wrote his friend Oseen, whom he had previously met in Sweden, that on returning to Copenhagen he had quickly resumed his work and would attempt to publish it very rapidly. Bohr feared that he would have to hurry so that his work would be new when it appeared; he perceived the question as being of burning interest at the time (Ref. 1, p. 551).

Bohr later wrote that when he returned home he found two articles in *Physikalische Zeitschrift*. One had been written by Hevesy and dealt with the chemical properties of the radioactive elements; Hevesy's results agreed completely with Bohr's own ideas. The other paper had been written by "V. d. Broek,"<sup>17)</sup> who had empirically arrived at representations regarding the periodic table similar to those which Bohr had reached on the basis of his own "theoretical speculations." Here we are dealing with his ideas associated with the explanation of the law of radioactive displacement and with the equality of the order number of an element in the periodic table to the nuclear charge  $Z$ . Bohr later discussed questions of radioactivity with D. Hevesy; the latter—on 15–29 January 1913—wrote Bohr about his (Hevesy's) paper<sup>66)</sup> on the valence of radioactive elements, stating that there was a close relationship between the transfer of charge and the change in valence (Ref. 1, p. 528). In Ref. 67, van den Broek first considered the equality of the order number to the nuclear charge  $Z$  (see the monograph in Ref. 68, Chapters 8 and 9, and the paper in Ref. 69). In his memoirs regarding Rutherford, Bohr writes (Ref. 57, p. 549) that these ideas came to him when he was working in Rutherford's laboratory in Manchester in 1912. Bohr's memoirs apparently give a basically correct reflection of the appearance of these ideas at the time he was preparing the Rutherford memorandum (in contradiction of the opinion expressed in the monograph in Ref. 68, on p. 131) (see Section

3 above).

On 7 February 1913, Bohr wrote Hevesy a long letter (Ref. 1, p. 529). He stated that he had become extremely interested in Hevesy's papers. The elegant results were exactly what Bohr had expected from his point of view regarding the structure of atoms. Bohr wrote that since he had left Manchester he had been continuously pursuing his work on the theory of the structure of atoms and molecules on the basis of Rutherford's model of the atom. Bohr hoped that he would be able to publish a paper on the subject in the very near future.

Bohr went on to characterize the ideas which he had used as the basis of his calculations. He mentioned, in particular, the assumption that the energy emitted in the form of radiation during the binding of electrons is equal to the Planck constant (Bohr put an asterisk on "Planck's constant"), multiplied by the revolution frequency of the electron, considered in its final orbit. In the footnote referred to by the asterisk he wrote that the constant which enters the calculations is not exactly equal to Planck's constant, differing from it by a numerical factor, in accordance with theoretical predictions. Here Bohr was talking about the constant  $K$  in relation to (7) above (see also Ref. 19, p. 251).

Bohr felt that in addition to the very stimulating indications of an understanding of the periodic table of elements, these ideas lead to a theory of chemical compounds, and he cited the result that two hydrogen atoms combine into a molecule, while two helium atoms do not.

Emphasizing that radioactivity depends only on the internal structure of nuclei, Bohr wrote that in radioactivity phenomena we are observing an explosion of nuclei; the chemical and physical properties of the new elements which form depend only on the charge of the new nuclei, which in turn depend on the charge of the ejected rays. He pointed out that the latter relation was just what Hevesy had found in his own experiments, so that Hevesy's results were what Bohr had expected and had hoped for.

Bohr concluded his letter by expressing hope for and faith in a future (possibly very early) tremendous and "unexpected?"<sup>18)</sup> development of our understanding.

Bohr's hope was fulfilled very soon. In February 1913 he became acquainted with the Balmer formula and the combination principle (see, for example, Ref. 18, p. XXXIX, and Ref. 39, p. 173). Bohr was later to say that as soon as he saw the Balmer formula the entire question immediately became clear to him. By the beginning of March Bohr had already written a paper expressing fundamentally new ideas about stationary states and the frequencies of radiative quantum transitions.

On 6 March 1913 Bohr sent Rutherford this paper—the first version of Part I of the trilogy<sup>5</sup>—with a letter in which he wrote (Ref. 1, p. 581) that he was sending Rutherford part of his (Bohr's) paper on the structure of atoms. Bohr hoped that the subsequent chapters would follow in the next few weeks. His work had been coming along well recently, and Bohr hoped that he would be successful in extending the arguments which he was using to a variety of different phenomena, such as the emission of line spectra and magnetism

(Bohr also mentioned the possibility of an indication of a theory of the formation of crystal structures). Bohr asked Rutherford to submit the first chapter of his paper to the *Philosophical Magazine*. Bohr further wrote that this calculation made it possible to achieve close quantitative agreement with experiments. Bohr reproduced, in parentheses, the numerical value which he had derived for the theoretical Rydberg constant, multiplied by the velocity of light,  $c$ :  $2\pi^2 me^4/h^3 = 3.290 \cdot 10^{15}$ ; (Ref. 5, pp. 91–92). Regarding the content of the subsequent chapters, Bohr stated that the second chapter would be devoted to atoms, the third to molecules, and the rest to magnetism and certain general considerations. In fact, Bohr subsequently published only Parts II and III after Part I, and these three parts constituted his trilogy.<sup>5</sup>

In this letter, Bohr asked Rutherford about the "delicate problem" of simultaneously using the old mechanics and the new assumptions introduced by the theory of Planck radiation. Rutherford responded to this question in a letter to Bohr on 20 March 1913 (Ref. 1, p. 583; this letter is completely reproduced in Bohr's memoirs of Rutherford, on p. 556 in Ref. 57). In particular, Rutherford wrote that it seemed to him that Bohr's hypothesis had a serious difficulty, which Rutherford thought that Bohr himself undoubtedly was fully aware of: How does an electron decide which frequency it is to oscillate at when it goes from one stationary state to another? It seemed to Rutherford that Bohr would be forced to assume that an electron knows beforehand where it intends to end up. The question which Rutherford raised was to be resolved only later, on the basis of concepts regarding the probabilities for quantum transitions.

Bohr continued his work. Not waiting for a response from Rutherford, he sent him a revised version of the first part of his article on 21 March 1913, accompanied by a letter in which he wrote (Ref. 1, p. 584) that since he had last written Rutherford he had continued to work on these questions and as a result felt it necessary to introduce some slight changes and additions. The changes and additions which he had made in the accompanying copy, however, were only formal in nature. In the first place, Bohr pointed out that they referred to Nicholson's theory. Another addition was the introduction (pp. 25–34) of some comments about the absorption of radiation. Bohr said that he was trying to put the theory in a general form and thought that the result might be of some interest. He said that his arguments did not amount to much more than a reexpression of the experimental results in new words. On the other hand, he felt that his arguments indicated a way in which it might be possible to explain several facts in a very simple way. Furthermore, the arguments point to a really elegant analogy between the old electrodynamics and the considerations in his own paper. Here Bohr was talking about his use of the concept of a correspondence between the quantum and classical theories in its original form—an idea which he was to go on to develop and which he felt was very important.

After Bohr's trip to Manchester in early April and a discussion of his paper with Rutherford (see, for example,

Ref. 23, p. 57), his paper—Part I of the trilogy—was finally, on 5 April 1913, submitted to the *Philosophical Magazine*, where it was published in July 1913.

In the introduction to the trilogy Bohr stresses that the Rutherford model of the atom has no characteristic length of the order of the linear dimension of the atom, in contrast with Thomson's model of the atom, where there is such a length: the radius of a positively charged sphere (see Section 3 above). Bohr says that if we introduce in the laws of motion a quantity foreign to classical electrodynamics, namely Planck's constant, or the elementary quantum of action, as it is frequently called, then the question of stable configurations of electrons and atoms changes substantially, since the dimensionality and magnitude of this constant are such that it makes it possible to determine, together with the mass and charge of the particles, a distance of the necessary order of magnitude (Ref. 5, p. 85). Bohr characterizes the contents of Part I of this trilogy by saying that in the first part of this study he uses Planck's theory to analyze the mechanism for the binding of electrons with a nucleus. He said that he shows that this point of view leads to a simple explanation for the behavior seen in the spectrum of hydrogen. He says that he will go on to state the initial prerequisites for the fundamental hypothesis on which all the arguments in the subsequent parts of the paper will be based.

The first part of the trilogy is devoted primarily to a study of the simplest system, consisting of a positively charged nucleus of very small size and an electron moving in a closed orbit around the nucleus (Ref. 5, p. 86), a one-electron atom, primarily, the hydrogen atom ( $Z = 1$ ).<sup>19)</sup>

Bohr writes that for simplicity he is assuming that the mass of the electron is negligibly small in comparison with that of the nucleus and that the velocity of the electron is small in comparison with the velocity of light. Bohr took the finite mass of the nucleus into account in a paper<sup>6</sup> in October 1913, and he introduced a relativistic correction later, in February 1915 (see Section 5 below; see also Part II of the trilogy,<sup>5</sup> p. 110). He considered the electron orbits to be circular, stating that this assumption would cause no changes for systems containing a single electron (Ref. 5, p. 87). Bohr subsequently extended the idea of circular orbits to multi-electron atoms (for the case of a circular orbit, the constancy of the kinetic energy of the revolution of the electron implies constancy of the angular momentum, as discussed below in the present section of the paper).

Using the example of a one-electron atom, Bohr solved two related problems on the basis of quantum concepts. One problem was that of determining the properties of atomic systems in stable states, primarily that of finding the possible values of the energy; the other was the problem of explaining the behavior in the line spectra of such systems. While Bohr had previously (in the Rutherford memorandum and later, until February 1913) been concerned with using the idea of quantization of the energy of matter to study the first problem, and then for the ground states of the atomic systems of interest, he now applied the idea of a discrete nature of the elementary processes by which radiation is emitted and absorbed to the solution of a second problem: the spectral prob-

lem. In the first problem, he extended his study to excited states.

New ideas which were of fundamental importance and which marked the most decisive break with classical concepts were the idea of stationary states of an atomic system and the idea of frequencies of radiative transitions between stationary states differing from the frequencies of the motion in these states of atomic systems. The combination of the first of these ideas with the concept of quantization of the energy of matter, on the one hand, and the combination of the second idea with the concept of the discrete nature of the processes by which radiation is emitted and absorbed, on the other, were expressed in two postulates by Bohr (Section 1 above). Specifically, in Part I of the trilogy Bohr stated these postulates, offered their original formulations, and on this basis explained the basic physics of the regular behavior observed in the line spectra of atoms. This work founded theoretical spectroscopy (Ref. 28, p. 32). In addition, Bohr suggested the ideas which were the basis of his model theory of the atom, and—an especially important point—first made use of the idea of a correspondence between the quantum and classical theories (which he described as a very beautiful analogy in a letter to Rutherford on 21 March 1913; see the discussion just above). The most important concrete result which Bohr derived, and which made a particularly great impression on the scientists who were his contemporaries, was a theoretical determination of the Rydberg constant.

Let us examine in more detail the contents of the first part of the trilogy as Bohr's basic study in his development of the quantum theory of the atom—a study in which he was strikingly successful and which determined the subsequent development of this theory, deservedly known as the "Bohr theory." In the first three sections of this paper (general considerations; emission of line spectra; and a continuation of general considerations, respectively), Bohr discussed the foundations of a theory of one-electron atoms. Section 4 dealt with the absorption of radiation, and Section 5 (on the ground state of an atomic system) contained ideas dealing with complex atoms and also molecules. These ideas were discussed in more detail in the second and third parts of the trilogy.

In Section 1 Bohr analyzes, for one-electron atoms, on the basis of a Rutherford model, the motion of an electron around a nucleus along different stationary circular orbits, characterized by definite discrete values of the electron binding energy  $W$  (which is equal to the electron's kinetic energy  $E_{\text{kin}}$ , which in turn is equal to the revolution energy  $E_{\text{rot}}$  in the case of a circular orbit), of the revolution frequency  $\nu_{\text{rot}}$ , and of the orbital radius  $a$ . Bohr obtains these values from the quantization condition

$$W = E_{\text{kin}} = \frac{1}{2} n h \nu_{\text{rot}} \quad (n = 1, 2, 3, \dots), \quad (8)$$

which is analogous to the Planck quantization law for a harmonic oscillator, (2), and which gives the ground state in the case  $n = 1$  and excited states<sup>20)</sup> in the case  $n > 1$ .

Using condition (8) and the relations between  $E_{\text{rot}}$  and  $a$  ( $E_{\text{rot}} = E_{\text{kin}} = -E_{\text{pot}}/2 = Ze^2/2a$  and  $E_{\text{rot}} = ma^2\omega^2/$

$2 = 2\pi^2 m a^2 \nu_{\text{rot}}^2$ ), we find the values of  $W = E_{\text{rot}}$ ,  $\nu_{\text{rot}}$ , and  $a$  in terms of  $n$  and the constants  $m$ ,  $e$ , and  $h$  for a one-electron atom with a given  $Z$  value<sup>21)</sup>:

$$W = \frac{2\pi^2 m e^4 Z^2}{n^3 h^2}, \quad \nu_{\text{rot}} = \frac{4\pi^2 m e^4 Z^2}{n^3 h^2}, \quad 2a = \frac{n^2 h^2}{2\pi^2 m e^2 Z}. \quad (9)$$

Applying (9) to the hydrogen atom ( $Z = 1$ ,  $E = eZ = e$ ) in the ground state ( $\tau = n = 1$ ), Bohr finds results which he describes as quantities of the same order of magnitude as the linear dimensions of the atom, optical frequencies, and ionization potentials (Ref. 5, p. 88):  $2a = 1.1 \cdot 10^{-8}$  cm,  $\nu_{\text{rot}} = 6.2 \cdot 10^{15}$  s<sup>-1</sup>, and  $W/e = 13$  V.

Circular orbits with different values of  $n$  and with  $n$ -dependent discrete values of  $W$ ,  $\nu_{\text{rot}}$ , and  $a$  were perceived by Bohr as being a series of configurations of a system which correspond to states of the system in which there is no emission, so that these are stationary states as long as the system is not perturbed from without (Ref. 5, p. 88). Here Bohr gives the first definition of a stationary state. The combination of the idea of the quantization of the energy of an atomic system with the idea of the existence of stationary states which conserve their energy (in contradiction of classical electrodynamics) is a characteristic feature of Bohr's first postulate, which Bohr has not yet formulated as a separate position but which he has already made the basis of the entire discussion which follows.

For Bohr the most important point is the question of the discrete nature of the processes by which radiation is emitted. He writes that an important aspect of the Planck theory of radiation is the assertion that the emission of energy by an atomic system does not occur continuously, as is assumed in classical electrodynamics, but, on the contrary, in definite, distinct emission events (Ref. 5, p. 87). In considering discrete events of the emission of quanta of energy  $h\nu_{\text{rad}}$ , where  $\nu_{\text{rad}}$  is the frequency of the radiation, Bohr is taking a decisive step forward, in particularly sharp contradiction of classical electrodynamics, consisting of a rejection of the identification of the radiation frequency with the frequency of a motion in the radiating atomic system. This identification had been regarded as obvious and had been generally accepted in accordance with the concepts of classical electrodynamics. Its adherents included Planck, Einstein, and other scientists after them.

Planck assumed that the frequency of the radiation which was emitted and absorbed was equal to the frequency of oscillations of a corresponding "resonator"  $\nu_{\text{rad}} = \nu_{\text{vibr}}$ . In the quantum theory of heat capacity,<sup>48</sup> Einstein assumed that the frequency of infrared radiation which was absorbed is equal to the frequency of an ion vibrating in a solid. The same assumption was made for the rotation of molecules; for example, Bjerrum set  $\nu_{\text{rad}} = \nu_{\text{rot}}$  (see Section 2 above). A. Haas treated the hydrogen atom as an "optical resonator"; Haas was the first to have used the quantum theory, in 1910, to determine the characteristics of the ground state of this atom (Bohr cited the paper<sup>70</sup> by Haas; Ref 5, p. 88).

Examining Nicholson's papers<sup>64</sup> (see the discussion above in the present section of this paper), and criticizing them, Bohr emphasizes that in Nicholson's calculations the

frequency of the lines in a spectrum is identified with the frequency of oscillations of a mechanical system in an exactly given equilibrium position. Bohr says that since Planck's theory is used we can expect that the radiation will be emitted in quanta. However, Bohr continues, systems like those considered here, for which the frequency is a function of the energy, are not capable of emitting a finite amount of monochromatic radiation, for the energy of the system and thus the frequency change in proportion to the radiation (Ref. 5, p. 89). Bohr goes on to say of Nicholson's theory that in this form the theory is incapable of explaining the Balmer and Ritz laws describing the frequencies of lines in the spectra of ordinary elements. Bohr explains these laws on the basis of an assumption regarding the emission of monochromatic radiation in a transition between stationary states. He formulates this assumption at the end of Section 1 and develops it in Section 2. Bohr writes that this assumption is in explicit contradiction of the commonly accepted interpretation of electrodynamics, but it is necessary in order to explain experimentally established facts (Ref. 5, p. 90). For the stationary states themselves, on the other hand, Bohr makes the assumption that he can apply classical mechanics to them on the basis that it is known that ordinary mechanics loses its absolute applicability in calculations on the motion of electrons and is valid only for average values. On the other hand, in calculations of a dynamic equilibrium in a stationary state, where there are no relative displacements of particles, there is no need to distinguish between real motions and average motions (Ref. 5, p. 90).

Bohr formulates his basic assumptions as follows: 1. The dynamic equilibrium of a system in stationary states can be treated by ordinary mechanics, while a transition of the system from one stationary state to another cannot be treated on this basis. 2. This type of transition is accompanied by the emission of *monochromatic* radiation, for which the relation between the frequency and the amount of energy released is precisely the relation given by the Planck theory (Ref. 5, p. 90).

The second of these assumptions is the original formulation of Bohr's second postulate [of frequency condition (1), according to which the change in the energy of the system is  $h\nu_{\text{rad}}$ ]. Here Bohr is combining the idea of the discrete nature of the emission of radiation with the idea of a transition frequency  $\nu_{\text{rad}}$  differing from the frequency of mechanical motion in combining stationary states.

The first assumption lies at the basis of the model theory of the atom which was developed in the second and third parts of the trilogy, in subsequent papers by Bohr himself, and, especially, in studies by Sommerfeld and later by many other scientists. We should point out immediately that the question of the applicability of the model theory which Bohr had been developing, beginning with the Rutherford memorandum, was a question with which Bohr was always concerned. In late 1913, in a report on the hydrogen spectrum, he stated that if we wish in general to form a clear concept of stationary states we have no means other than ordinary mechanics, *at least at the present time* (the italics are introduced by the author of this article M. E.; Ref. 7, p. 161). In later



years, Bohr's doubts about the applicability of classical mechanics to stationary states grew progressively stronger, as can be seen in his many statements, some of which will be reproduced below.

Here it is pertinent to stress that the two assumptions which Bohr made referred to two different aspects of his theory (Section 1 above) and that it was only later that he clearly stated the basic postulates regarding stationary states and the frequencies of radiative quantum transitions, as the most general assumption, independent of specific models of atomic systems.

We thus cannot label these assumptions as "Bohr's two famous postulates," as has been done in an interesting monograph on the methodology of quantum theory (Ref. 71, p. 119). The first of these assumptions was only an intermediate step in the development of the quantum theory, while the second survived in the systematic quantum theory which was subsequently developed, and where that assumption is derived.

An exceptionally important passage was Bohr's discussion, in Section 2 of his paper, of the regularities observed in the line spectra of atoms. For the spectral lines of the hydrogen atom, this behavior is described, as we know, by the generalized Balmer formula, which we write as

$$\sigma = \frac{1}{\lambda} = R \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) = T(n_2) - T(n_1) \quad (10)$$

as the difference between two *spectral terms* of the type  $T(n) = R/n^2$  [ $\sigma$  and  $\lambda$  are the wave number and wavelength of the radiation;  $R$  is the Rydberg constant; and  $n_1, n_2$ , and  $n$  are integers (with  $n_2 = 2$  and  $n_1 = 3, 4, 5, \dots$ , we find the Balmer spectral series)]. In the case of multielectron atoms, for which spectral series are observed, the Rydberg-Ritz combination principle holds, according to which we have

$$\sigma = \frac{1}{\lambda} = T_r(n_2) - T_s(n_1), \quad (11)$$

where  $T_1(n), T_2(n), T_3(n), \dots$ , are functions of the integer  $n$  which are of the approximate form  $T_i = R/(n + \alpha_i)^2$ , where the  $\alpha_i$  are constants ( $r$  and  $s$  are possible values of  $i$ ).

It was by working from expressions of the types in (10) and (11) that Bohr interpreted the regular behavior observed in the spectra. He cited a 1908 paper<sup>72</sup> by Ritz in which a combination principle had been formulated. In general form, this principle can be written for atoms as  $\sigma = T_j - T_k$ , where  $T_j$  and  $T_k$  are two terms of the complete set of spectral terms characterizing the given atom (see, for example, Ref. 73, p. 13).

Multiplying (10) and (11) by the velocity of light  $c$ , we find the frequencies of the spectral lines,  $\nu_{\text{rad}} = c\sigma = c/\lambda$ , as the difference between two terms of the type  $cT(n)$ ; a further multiplication by the Planck constant  $h$  yields the discrete value of the energy emitted (or absorbed) by the atom,  $h\nu_{\text{rad}}$ , as the difference between two terms of the type  $hcT(n)$ , with the dimensionality of energy. For the hydrogen atom, Bohr identified these terms with the electron binding energies  $W$  in two stationary states with  $n = n_1$  and  $n = n_2$ . According to the first expression in (9), with  $Z = 1$  we have  $W_n = 2\pi^2 me^4/n^2 h^2$ ; Bohr found that the amount of

energy emitted in a transition of the system from the state corresponding to  $n = n_1$  to another state, with  $n = n_2$  is

$$W_{n_2} - W_{n_1} = \frac{2\pi^2 me^4}{h^2} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right). \quad (12)$$

Bohr said that if we now assume that this radiation is monochromatic and that the amount of energy emitted is  $h\nu_{\text{rad}}$ , where  $\nu_{\text{rad}}$  is the frequency of the radiation, we find

$$W_{n_2} - W_{n_1} = h\nu_{\text{rad}}, \quad (13)$$

and thus

$$\nu_{\text{rad}} = \frac{2\pi^2 me^4}{h^3} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right). \quad (14)$$

Bohr recognized that this relation explains the regularity which links the lines in the hydrogen spectrum (Ref. 5, p. 91).

Relation (13) is an explicit statement of Bohr's famous frequency condition (1) (for the binding energies we have  $W_{n_1} = -E_1$  and  $W_{n_2} = -E_2$ , where  $E_1$  and  $E_2$  are the energies of the combining stationary states, measured from the ionization boundary of the hydrogen atom). In this condition, the frequency  $\nu_{\text{rad}}$  is *not the same as* the frequencies of motion,  $\nu_{\text{rot}} = 4\pi^2 me^4/n^3 h^3$  [see the second expression in (9) with  $Z = 1$ ] in stationary states.

For the constant  $2\pi^2 me^4/h^3$  (equal to<sup>22</sup>)  $cR$  in (14) Bohr finds a value of  $3.1 \cdot 10^{15} \text{ s}^{-1}$  from the values available for  $e, e/m$ , and  $h$  at the time. He compares this value with the corresponding empirical value of  $3.290 \cdot 10^{15} \text{ s}^{-1}$  and writes that the correspondence between the theoretical and observed values lies within the errors in the measurements of the constants which appear in the theoretical formula (Ref. 5, p. 92).

Bohr was thus able to find theoretically the value of the Rydberg constant, and he wrote Rutherford about the matter as early as March 1913 (see the discussion in the present section of this paper). This derivation was a major achievement of Bohr.

Bohr explains the different numbers of lines in the Balmer series observed in emission spectra on the basis that the diameter of the orbit of an electron is proportional to  $n^2$  [in accordance with the last expression in (9)], and only in a sufficiently low-density gas can there be transitions from stationary states with large values of  $n$ , with large orbital diameters.

Bohr then uses his theory to explain the origin of the observed spectral series with semi-integer values of  $n$ . He makes the case that they belong not to hydrogen, as was previously assumed, but to helium. For the helium atom, with  $Z = 2$ , we have  $W_n = 8\pi^2 me^4/n^2 h^2$  [see (9)], and the expression analogous to (14) is

$$\nu_{\text{rad}} = \frac{8\pi^2 me^4}{n^2 h^3} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) = \frac{2\pi^2 me^4}{n^2 h^3} \left[ \frac{1}{(n_2/2)^2} - \frac{1}{(n_1/2)^2} \right]. \quad (15)$$

Bohr writes that the theory described above can explain these series in a natural way if they are attributed to helium (Ref. 5, p. 92). This suggestion by Bohr was confirmed brilliantly, and very soon, still in 1913, by experiments by E. Evans and A. Fowler. Bohr convincingly explained the

slight discrepancies with expression (15) in Ref. 6 (see also the report in Ref. 7), on the basis of a difference between the reduced mass of the hydrogen atom and that of the helium atom [in the refined versions of expressions (14) and (15), the electron mass  $m$  is replaced by the reduced mass  $mM/(M+m)$ , where  $M$  is the mass of the nucleus, which is different for the hydrogen and helium atoms]. This explanation became yet another major success of Bohr's theory.

Working from the combination principle in the form in (11), Bohr also briefly examines the question of the regular behavior in the spectra of multielectron atoms. He writes that the circumstance that the frequency can be represented as the difference between two functions of integers leads to the conclusion that the origin of lines in these spectra is similar to that which we have adopted for hydrogen. Bohr concluded that this similarity means that the lines correspond to that radiation which occurs when a system undergoes a transition from one stationary state to another (Ref. 5, p. 94).

We might note that Bohr successfully explains the identical nature (for all elements) of the constant  $R$  in the Rydberg expression  $T_i = R/(n + \alpha_i)^2$  [see (11)], on the basis that the force acting on an electron far from a nucleus, i.e., in the case of large values of  $n$ , in a multielectron atom is roughly the same force as in the preceding case, when an electron is bound by a hydrogen nucleus (Ref. 5, p. 94). The reader is of course aware that so-called "Rydberg states" of an outer electron are presently the subject of much research interest.

Despite the successes of his theory, Bohr himself was not satisfied by its underpinnings, based on energy quantization law (8), analogous to Planck law<sup>23)</sup> (2). At the end of the first section of his paper, Bohr already writes that we are, however, able (here he refers the reader to Section 3 of his paper) to derive relations (9) for stationary states by using assumptions of a slightly different type (Ref. 5, p. 90). After examining (in Section 2 of his paper) the emission of line spectra and discussing an explanation of the regular behavior observed in the spectra, Bohr offers a second derivation of the basic relations, (9) (and thus of the Rydberg constant), in which he makes the first use of the idea of a correspondence between the quantum and classical theories.

To find the electron binding energy  $W$ , Bohr first replaces quantization condition (8) by the more general condition  $W = E_{\text{rot}} = f(n)h\nu_{\text{rot}}$ , where  $f(n)$  is some function of the integer  $n$ . Instead of the first expression in (9), he finds  $W = W_n = \pi^2 me^4 Z^2 / 2h^2 f^2(n)$ , and a comparison with (12) leads him to conclude that in order to derive an expression analogous to the formula for the Balmer series we must set  $f(n) = \alpha n$  (Ref. 5, p. 95), where  $\alpha$  is a coefficient to be determined [this assertion corresponds to  $W_n = E_{\text{rot}} = \alpha h\nu_{\text{rot}}$ ; see (7) in the Rutherford memorandum<sup>24)</sup>]. To find  $\alpha$ , Bohr makes use of the idea of a correspondence. He requires that, in accordance with classical electrodynamics, in the limiting case of large values of  $n$  the radiation frequency  $\nu_{\text{rad}}$  in a transition between adjacent stationary states with energies  $W_n$  and  $W_{n-1}$  must be equal to the revolution frequency of the electron,  $\nu_{\text{rot}}$  (the limiting case of low radiation frequencies, i.e., of large wavelengths).

In an expression of the type in (14) we need to replace

$n_1/2$  and  $n_2/2$  by  $f(n_1)$  and  $f(n_2)$ , so that with  $f(n) = \alpha n$ ,  $n_1 = N$ ,  $n_2 = N - 1$  we find, at large values of  $N$ ,

$$\nu_{\text{rad}} = \frac{W_{n_2} - W_{n_1}}{h} = \frac{\pi^2 me^4 Z^2}{2\alpha^2 h^3} \frac{(2N-1)}{N^2(N-1)^2} \approx \frac{\pi^2 me^4 Z^2}{\alpha^2 h^3 N^3}. \quad (15')$$

For  $\nu_{\text{rot}}$ , replacing  $n/2$  by  $f(n)$  in the second expression in (9), we find, with  $n = N$ ,

$$\nu_{\text{rot}} = \frac{\pi^2 me^4 Z^2}{2h^3 f^2(N)} = \frac{\pi^2 me^4 Z^2}{2\alpha^2 h^3 N^3}. \quad (16)$$

The frequencies  $\nu_{\text{rad}}$  and  $\nu_{\text{rot}}$  are approximately the same, while at large values of  $N$  this is true only if  $\alpha = 1/2$ . We thus find condition (8) and expressions (9).

Bohr also considers a transition between stationary states with  $n = N$  and with  $n = N - s$ , where  $s \ll N$ , and under the condition  $f(n) = \alpha n = n/2$  he finds  $\nu_{\text{rad}} = s\nu_{\text{rot}}$ . Regarding this point he writes that the possibility of the emission of radiation with such a frequency can also be explained by analogy with ordinary electrodynamics, since an electron moving in an elliptical orbit around a nucleus emits radiation which can, in accordance with the Fourier theorem, be expanded in components with frequencies  $s\nu_{\text{rot}}$ , where  $\nu_{\text{rot}}$  is the electron revolution frequency (Ref. 5, p. 96). Bohr subsequently used relations of this sort in deriving selection rules from the correspondence principle (see Section 6 below).

In offering his second derivation of the basic relations, (9), Bohr returns to the question of the meaning of the correspondence between the observed and calculated values of the constants in (14) for the Balmer series in the hydrogen spectrum (Ref. 5, p. 96). He writes that from the expression given above we arrive at the same expression for the constant as in (14). Bohr states that we need to assume only (first) that the radiation is emitted in the form of quanta  $h\nu_{\text{rad}}$  and (second) that the frequency of the radiation emitted in a transition between successive stationary states is equal to the electron revolution frequency in the region of large wavelengths (Ref. 5, p. 96).

We immediately note that in the second derivation Bohr, while using the idea of a correspondence between the quantum and classical theories in its original form, continues to make use of the condition of quantization of the electron energy, as in the first derivation. This is a condition of the Planck type, although of more general form, with  $n/2$  replaced by the function  $f(n)$  in (8). Bohr offered his third and most systematic derivation in a report<sup>7</sup> in December 1913. In this very important and clear derivation, Bohr does not make any use at all of an equality analogous to Planck law (2). He stressed that since this equality is used in very different ways here and in the Planck theory, however, he thought it was a mistake to adopt this formal analogy as a basis. He said that he would attempt to get away from this analogy to the extent possible in his work (Ref. 7, p. 163).

In the third derivation Bohr uses the relation<sup>25)</sup>  $\nu_{\text{rot}}^2 = 2W^3/\pi^2 me^4$  for a circular orbit of an electron in a hydrogen atom, and for a stationary state with a binding energy  $W = W_n = hcR/n^2$  he finds

$$\nu_{\text{rot}}^2 = \frac{2R^3 h^3 c^3}{\pi^2 me^4 n^6}. \quad (17)$$

On the other hand, for a transition between stationary states with  $n_1 = n + 1$  and  $n_2 = n$  we find from the generalized Balmer formula (10)

$$\nu_{\text{rad}} = c\sigma = cR \left[ \frac{1}{n^2} - \frac{1}{(n+1)^2} \right]. \quad (18)$$

From this expression we find, at large values of  $n$ , the approximation  $\nu_{\text{rad}} = 2cR/n^3$  (i.e.,  $\nu_{\text{rad}}^2 = 4c^2R^2/n^6$ ). Bohr equates the radiation frequency  $\nu_{\text{rad}}$  to the number of electron revolutions  $\nu_{\text{rot}}$ , finding  $\nu_{\text{rot}} = 2cR/n^3$ , (i.e.,  $\nu_{\text{rot}}^2 = 4c^2R^2/n^6$ ), and he writes (Ref. 7, p. 162) that a comparison of this expression with (17) reveals that  $n$  cancels out in this equation, yielding the following expression for  $R$ :

$$R = \frac{2\pi^2me^4}{ch^3}. \quad (19)$$

In his report,<sup>7</sup> Bohr gives only this, the third derivation, which is based entirely on the concept of a correspondence between the quantum and classical theories.<sup>26)</sup>

Having used the idea of correspondence already in the second derivation of (9) and thereby justifying the coefficient  $\alpha = 1/2$  in quantization condition (8), Bohr finds, at the end of his Section 3, quantization of the angular momentum of an electron in a circular orbit *as a consequence*. He writes (Ref. 5, p. 96) that although he could of course not say that the calculations in that paper had a mechanical basis it was nevertheless possible to offer a very simple interpretation of the calculations on<sup>27)</sup> (p. 87) in terms of concepts from ordinary mechanics. Denoting by  $M$  the angular momentum of an electron revolving around a nucleus, he states that we immediately find  $\pi M = E_{\text{kin}}/\nu_{\text{rot}}$  for a circular orbit, where  $\nu_{\text{rot}}$  is the revolution frequency, and  $E_{\text{kin}}$  is the kinetic energy of the electron. For a circular orbit we have  $E_{\text{kin}} = W$  (see<sup>28)</sup> p. 86), so we find from (8)

$$M = nM_0, \text{ where } M_0 = \frac{h}{2\pi} = 1,04 \cdot 10^{-27}. \quad (20)$$

Bohr later made extensive use of specifically condition (20), the quantization of the angular momentum for circular orbits, in his model theory of the atom. He formulated this quantization condition as follows: In a stationary state of a system, the angular momentum of an electron revolving around a nucleus is equal to an integer multiple of some universal quantity, regardless of the charge of the nucleus (Ref. 5, p. 97).

At the end of his Section 3, Bohr writes that a large number of different stationary states will be observed only in a study of the absorption and emission of radiation (Ref. 5, p. 97). He then discusses absorption processes in Section 4 (in the earlier sections, Sections 1–3, he had been discussing emission processes exclusively). He states that in most other physical phenomena the atoms of matter are in only one definite state, specifically, in a state at a low temperature. Bohr discusses this case of a ground state, that stationary state whose formation is accompanied by the emission of the greatest amount of energy, in Section 5.

In Section 4, Bohr discusses the absorption of radiation as the process which is the inverse of the emission of radiation, i.e., as a transition from a stationary state with a lower

energy to a stationary state with a higher energy; for the hydrogen atom, this would be a transition  $n_2 \rightarrow n_1$ , where  $n_1 > n_2$ . Bohr explains experiments on the absorption of sodium vapor in terms of transitions of a system between two states, one of which is a ground state, while the occurrence of absorption in hydrogen only in an emitting state he explains in terms of an absorption by hydrogen atoms in stationary states with  $n \geq 2$  (he is talking about the Balmer and Paschen series, for which  $n_2 = 2$  and  $n_1 = 3$ ). He again stresses the difference between the transition frequency  $\nu_{\text{rad}}$  and the frequency of the motion of the electrons, noting that just how greatly his explanation differs from the explanation based on ordinary electrodynamics can be seen most clearly from the fact that we are forced to assume absorption by a system of electrons of radiation whose frequency differs from the electron oscillation frequency calculated in the usual way (Ref. 5, p. 98).

Extremely important is Bohr's study of the absorption of radiation which leads to ionization (photoionization, as we would call it today), and which is associated with a transition from stationary states with discrete values of the energy to a state of the system in which an electron is free, i.e., in which an electron has enough energy to move an infinite distance away from the nucleus. Bohr continues that if we assume that the motion of an electron is described by ordinary mechanics and that there is no (significant) radiation of energy then the total energy of the system is constant (as in the stationary states considered previously). Bohr argues that there must be a complete continuity between the states of the two types, because the difference between the frequencies and dimensions of the system in the successive stationary states decreases with increasing  $n$  (Ref. 5, p. 98). Consequently, Bohr is considering an infinite motion, along with a finite motion, for an electron: the complete energy spectrum. He reaches the conclusion that absorption of radiation is possible not only between two different stationary states<sup>29)</sup> but also between a stationary state and a state in which an electron is free (Ref. 5, p. 98). Bohr states that the frequency of this radiation must be determined by the equality  $E = h\nu_{\text{rad}}$ , where  $E$  is the difference between the total energies of the system in the two states, and that this absorption is exactly the same as that which is observed in experiments on ionization by ultraviolet light and x rays (Ref. 5, p. 99). At this point Bohr, citing Einstein's paper,<sup>46)</sup> writes that in this manner we find the same expression for the kinetic energy of an electron which is torn from an atom, and he writes the equation  $E_{\text{kin}} = h\nu_{\text{rad}} - W$ , where he identifies  $W$  as the total energy released upon the original attachment of an electron.

Bohr also explains the results of Wood's famous experiments<sup>74)</sup> on the absorption of light in sodium vapor, emphasizing that we must assume that this absorption is followed by an emission of energy which returns the system to its original state. If no collisions occur between systems, this energy is emitted in the form of radiation, whose frequency is equal to the frequency of the radiation absorbed. We are thus essentially dealing not with an absorption but simply a scattering of the original radiation. An actual absorption occurs

only if, as a result of collisions, this energy is converted into the kinetic energy of free particles (Ref. 5, p. 99).

Finally, Bohr suggests that monochromatic x radiation is emitted upon the recombination of a system after a preliminary removal of a strongly bound electron, e.g., in a collision with cathode particles (Ref. 5, p. 100). Bohr later took up this question in the second part of his trilogy (Ref. 5, p. 127); in 1914, Kossel<sup>75</sup> offered a more detailed explanation of the characteristic x radiation by working from Bohr's theory.

Bohr's ideas related to experiments with x rays are of major interest. He writes that these experiments suggest that ordinary electrodynamics fails in studies of not only the emission and absorption of radiation but even collisions of two electrons, one of which is bound in an atom (Ref. 5, p. 127). He states that an examination of a collision between a free electron and a bound electron leads to the conclusion that a bound electron cannot acquire an energy less than the difference between the energies of two successive stationary states. Consequently, a free electron colliding with it cannot lose a lesser amount of energy (Ref. 5, p. 101). Consequently, in addition to *radiative* quantum transitions, Bohr considered *radiationless* quantum transitions of a bound electron in an atom. Similar transitions in collisions of electrons of a given velocity (i.e., of a definite kinetic energy) with mercury atoms were, of course, observed experimentally in the famous experiments by James Franck and Heinrich Hertz,<sup>76</sup> in agreement with Bohr's prediction (of which Franck and Hertz were unaware).

Especially noteworthy is the fact that all the conclusions which Bohr drew from his theory in his Section 4, including those concerning multielectron atoms, were correct. These conclusions were based on that part of his theory which was associated with his two basic postulates regarding stationary states and radiative transitions, not with the particular results of his model theory (Section 1 above). In contrast, Bohr's discussion in his Section 5 of the ground state of a multielectron atomic system is based on model representations regarding the motion of electrons in accordance with the laws of classical mechanics, with auxiliary quantum conditions; furthermore, this model deals with the motion in circular orbits (only for circular orbits is the kinetic energy of an electron constant and unambiguously related to the angular momentum of the electron).

We will not discuss the contents of his Section 5 in detail here. We simply note that Bohr uses what he refers to as an easily proved theorem: *In any system consisting of a nucleus at rest and electrons which are moving in circular orbits at velocities small in comparison with the velocity of light the kinetic energy is numerically equal to half of the potential energy* (Ref. 5, p. 105). He thus concludes that, as for a one-electron atom, the relation  $W = E_{\text{kin}}$  holds, where  $W$  is the formation energy of the system for formation from particles which are separated by an infinite distance and which have no velocity relative to each other, while  $E_{\text{kin}}$  is the kinetic energy of the electrons in their final arrangement in the system.

At the end of his Section 5, Bohr offers the following

hypothesis: *In any molecular system consisting of positively charged nuclei and electrons, where the nuclei are at rest with respect to each other, while the electrons are moving in circular orbits, the angular momentum with respect to the center of the orbit for each electron in the ground state is equal to  $h/2\pi$ , where  $h$  is Planck's constant* (Ref. 5, p. 106). Bohr used this hypothesis as a fundamental hypothesis in the second and third parts of his trilogy, which will be discussed briefly in Section 5 below). However, this hypothesis has proved to be unsatisfactory, even within the framework of the model theory.

To conclude this section of the review we must stress that it was the first part of the trilogy, Sections 1–4, which contained Bohr's fundamental new ideas on quantum theory, their successful application to one-electron atoms, and several important conclusions of a general nature regarding multielectron atoms and their spectra. For this reason, the first part of the trilogy occupies an exceedingly important place in the development of quantum concepts. Bohr gave a very clear exposition of his ideas and results in Ref. 7, a report which we have cited several times. Bohr concludes that report by stating that before he ends he wishes to express the hope that he has expressed himself sufficiently clearly and that his audience has understood the sharp contradiction between the ideas presented here and the remarkably harmonious set of concepts which are known as classical electrodynamics. Bohr said that at the same time he had been attempting to arouse the hope in his audience that the very fact of stressing this contradiction might eventually lead to a certain relationship in terms of the new concepts also (Ref. 7, p. 167).

## 5. DEVELOPMENT OF A THEORY OF ONE-ELECTRON ATOMS AND ATTEMPTS TO DERIVE A THEORY OF MULTIELECTRON SYSTEMS (1913–1917)

In Bohr's studies over the decade (1913–1923) after his striking successes in the spring of 1913, in the first part of his trilogy,<sup>5</sup> we can distinguish three periods. In the first period, from the summer of 1913 to the end of 1917, Bohr, while continuing to develop his theory of one electron atoms, devoted much effort to multielectron systems, developed a model theory of the periodic motion of electrons in circular orbits, and—running into serious difficulties—attempted to overcome them. In the second period, from late 1917 to 1920, Bohr achieved some successes in the development of the correspondence principle and in the theory of the serial spectra of atoms; on this new basis, he took up a study of the physical explanation of the periodic properties of the elements. In the third period (1921–1923), he developed a theory of the periodic table in close connection with the theory of the optical and x-ray spectra of atoms. He sought ways to overcome the fundamental difficulties of the model theory which did not satisfy him; these fundamental difficulties were to be resolved only in quantum mechanics. We will discuss Bohr's investigations during these three periods in the present section and the two following sections of this review, respectively.

In the first period, Bohr continued to teach at Copenha-

gen University during the academic year 1913–1914; after the beginning of the First World War, in the fall of 1914, and through to the summer of 1916, he worked two years in England at Manchester University, at the invitation of Rutherford. On 20 May 1914 Rutherford wrote Bohr (Ref. 1, p. 593) that a lectureship had opened up and that he wished to find a young man with some originality. On 19 June 1914, Bohr responded (Ref. 1, 594) that he did not know how to express his joy that Rutherford had offered him the vacant lectureship for the following year. Bohr was very pleased and agreed to take the position. Bohr spent two years in Manchester in close contact with Rutherford. In the summer of 1916 he returned to Denmark, and in September 1916 he became a professor at Copenhagen University.

Bohr published his work in this period in English in *Philosophical Magazine* and *Nature*. He wrote several long articles: Parts II and III of his trilogy,<sup>5</sup> in September and November 1913, Refs. 77 and 78 in March 1914 and September 1915, and Ref. 60 in November 1915 (this was a continuation of his paper in Ref. 59; see Section 3 above). Bohr also published several letters: Ref. 6 in October 1913 (see Section 4 above), Ref. 79 in January 1914, Ref. 80 in February 1915 (in that letter he called attention to the need to take into account the relativistic dependence of the mass of the electron on its velocity), and Ref. 81 in July 1915. In January 1916 Bohr submitted to *Philosophical Magazine* a paper, "On the application of the quantum theory to periodic systems,"<sup>82</sup> whose publication he subsequently stopped (see the discussion further on in this section of the review). In 1916 and 1917 he did not publish research results. Some materials characterizing this research are published in Ref. 1.

Already by the beginning of this period, the results which Bohr had obtained in the first part of his trilogy had attracted considerable interest among scientists (Ref. 1, p. 122, and Ref. 13, p. 107). In particular, his results were discussed in September 1913 at the annual meeting of the British Association for the Advancement of Science at Birmingham<sup>83</sup> with Bohr participating. Several leading scientists, in particular, James Jeans, who presented a paper at Birmingham, spoke highly favorably of Bohr's results. Particularly noteworthy is the fact that Sommerfeld was very interested in Bohr's work. On 4 September 1913 Sommerfeld had sent Bohr a card on which he wrote that he had been concerned for a long time with the problem of expressing the Rydberg-Ritz constant in terms of Planck's constant  $h$ . Although Sommerfeld was at that time still somewhat skeptical of atomic models in general, nevertheless he regarded the calculation of this constant as being undoubtedly a major accomplishment (Ref. 13, p. 168). As we know, Sommerfeld himself was subsequently to make a very large contribution to the development of a model theory of the atom.

Here we should emphasize that many scientists did not agree with Bohr's ideas and criticized his theory. It took several years before these ideas gained widespread acceptance.

The second and third parts of the trilogy,<sup>5</sup> published in

the fall of 1913, were devoted primarily to the theory of the ground state of multielectron systems: systems containing only a single nucleus, i.e., atoms (Part II), and systems with several nuclei, i.e., molecules (Part III). In this regard, these papers were a direct continuation and a development of the Rutherford memorandum (Section 3 above). In them he also discussed the electron configurations and the stability of atoms and molecules. There was the distinction that Bohr was working not from quantum condition (7) but from the hypothesis that the angular momentum of each electron with respect to the center of its own circular orbit is a universal quantity  $h/2\pi$  in the ground state (Ref. 5, pp. 108 and 132). This hypothesis corresponds to quantum condition (8) with  $n + 1$ . This hypothesis had been suggested at the end of Part I of the trilogy.

For atoms, Bohr again examined (in §§2–4 of Part II) the distribution of electrons on coaxial rings. He determined the stability of a ring of  $n$  electrons which are revolving around a nucleus with a given  $Z$  (which he denoted by  $N$ ) with respect to oscillations perpendicular to the plane of the ring. He found that a single ring is stable at  $n \leq 8$  if  $Z = 10$ , at  $n \leq 10$  if  $Z = 20$ , and at  $n \leq 13$  if  $Z = 40$ . Bohr concluded that the electrons in complex atoms must be distributed in a series of rings. For the lithium and beryllium atoms ( $Z = 3, 4$ ) he assumed that there were two electrons in an inner ring and one and two electrons, respectively, in an outer ring. For the following light atoms with  $n \leq 24$  he found a distribution in rings, taking into account the periodicity in the properties of the elements, in particular, the existence of a characteristic period of 8. According to his model, there can be eight, four, or two electrons in inner rings, while the outer ring can have from one to four electrons, in accordance with the valence of the element.

For molecules, Bohr examined (in §§2–5 of Part III) the stability of systems consisting of two nuclei and a single ring of electrons, perpendicular to the axis of the molecule. In particular, he studied the stability of the hydrogen molecule and the process by which a diatomic molecule is formed from two atoms.

Bohr was not able, on the basis of his analysis of the motion of electrons in the ground state of multielectron systems along circular orbits with  $n = 1$ , to derive important and convincing results of a quantitative nature in his subsequent studies of this period. That Bohr himself was not satisfied with the results of his research and felt the need to reject several ideas can be seen from his letter to Oseen on 28 September 1914. Bohr wrote (Ref. 1, p. 562) that for systems consisting of more than two particles there is no simple relationship between the energy and the number of revolutions, and for this reason arguments like those which Bohr had used previously cannot be used to determine the stationary states of a system. Bohr tended to believe that some very substantial difficulties were hidden in this problem and that these difficulties could be overcome only by rejecting the ordinary concepts to an ever greater extent than had already been necessary. Bohr felt that the only reason for the successes which had been achieved was the simplicity of the systems which had been considered (i.e., one-electron



atoms).

In the same letter, Bohr wrote about his trip to Germany. At the beginning of the summer vacation he had made a brief trip to Germany with his brother and had met several physicists: Debye, Wien, and Sommerfeld. Bohr had not previously met any of the German physicists, and he was quite pleased to talk with them. In the future, Bohr was to establish very close contact with several German physicists.

In the second part of his trilogy, Bohr also discussed questions related to the characteristic  $x$  radiation (in §5) and radioactive phenomena, including the law of radioactive displacement; in some concluding comments in the third part of the trilogy, Bohr enumerated the basic assumptions which had been made in the trilogy. He formulated them as follows (Ref. 5, p. 147): 1. The emission (or absorption) of energy does not occur continuously, as is assumed in ordinary electrodynamics, but only upon a transition of the system from one stationary state to another. 2. The dynamic equilibrium of a system in stationary states is determined by the ordinary laws of mechanics, while for a transition of a system between different stationary states these laws do not apply. 3. The radiation which is emitted upon a transition of a system from one stationary state to another is monochromatic, and the frequency of this radiation,  $\nu_{\text{rad}}$ , is related to the total amount of energy radiated,  $E$ , by  $E = h\nu_{\text{rad}}$ , where  $h$  is Planck's constant. 4. The different stationary states of a simple system consisting of an electron revolving around a positive nucleus are determined from the condition that the ratio of the total energy emitted upon the formation of the given configuration to the number of revolutions of electrons is an integer multiple of  $h/2$ . The assumption that the orbit of an electron is circular is equivalent to the requirement that the angular momentum of an electron revolving around a nucleus must be an integer multiple of  $h/2\pi$ . 5. The ground state of any atomic system, i.e., the state in which the emitted energy is at a maximum, is found from the condition that the angular momentum of each electron is equal to  $h/2\pi$ .

We see that Bohr has not yet singled out the most general assumptions, including among the basic assumptions also several postulates of the model theory, which furthermore differ in degree of generality. It was only later that Bohr did this.

Bohr did not include in Ref. 5 his original discussion of magnetism (about which he had written Rutherford; see Section 4 above). However, he kept his rough drafts (Ref. 1, pp. 254–265) on magnetism, which demonstrated the difficulties that Bohr ran into in his attempts to explain the Zeeman effect on the basis of quantum theory. Another question which arose was that of finding an explanation for the splitting of spectral lines which Johannes Stark discovered<sup>84</sup> in November 1913 and which is now known as the "Stark effect."

Rutherford wrote Bohr on 11 December 1913 (Ref. 1, p. 589) about Stark's recent discovery that, in Rutherford's words, an electric field causes a splitting of the lines of hydrogen and helium very similar to the Zeeman effect. Rutherford advised Bohr to write something about the Zeeman effect and this electric effect if it was possible to reconcile

them with Bohr's theory. Bohr responded to Rutherford on 21 December 1913 (Ref. 1, p. 590), saying that he was grateful for Rutherford's kindness in calling Bohr's attention to Stark's discovery. Bohr said that no one in Copenhagen had received a reprint of this paper, so that Bohr had written Stark, requesting a reprint. Bohr said he had received the reprint a few days before but that he had not been able to form an exact opinion regarding an interpretation of the results. All that Bohr could say was that one of the most characteristic facts—the rapid increase in the effect of an electric field with increasing order number of the line in the corresponding spectral series—was precisely what would be expected on the basis of the assumption of stationary states. Bohr said that as soon as he could work out a more detailed formulation of a possible interpretation of Stark's results he would write Rutherford. Bohr said that, following Rutherford's advice, he would publish something on this matter and on the Zeeman effect as soon as he could.

Bohr quickly wrote a paper<sup>77</sup> on the Stark and Zeeman effects. In this paper he explained the effect of an electric field on the spectrum of hydrogen in terms of a deformation of the circular orbit of an electron. He showed that the problem allowed only two stationary orbits of an electron (Ref. 77, p. 176). He found a splitting of spectral lines into two components, and he found that the magnitude of the splitting was proportional to the electric field and to the difference  $n_1^2 - n_2^2$ , between the squares of the quantum numbers ( $n_1$  and  $n_2$ ) of the combining stationary states.

The splittings which he calculated for the lines of the Balmer series,  $H_\beta$  ( $n_1 = 4, n_2 = 2$ ) and  $H_\gamma$  ( $n_1 = 5, n_2 = 2$ ), turned out to agree in order of magnitude with experimental data on the distances between the two components polarized parallel to the electric field (the calculated values exceeded the experimental values by about 1/3).

This was the first theory, although not yet perfected, of the Stark effect.

To explain the Zeeman effect, and to establish a relationship with ordinary mechanics while maintaining agreement with experiment (Ref. 77, p. 182), Bohr suggested that the energy of a hydrogen atom in stationary states does not change in a magnetic field (which causes a rotation around the field direction at a frequency  $\tau$ ), and the frequency condition  $E_1 - E_2 = h\nu_{\text{rad}}$  is replaced for oscillations perpendicular to the field by the relation  $E_1 - E_2 = h(\nu_{\text{rad}} \mp \tau)$ . Although this suggestion turned out to be wrong, the concept of a degeneracy of energy levels (as we would say today) and the concept of a splitting of these levels were unknown at the time. As a result, Bohr was also unable to explain the splitting of spectral lines in the Stark effect into more than two components. The imperfections of Bohr's model theory during this period, which was limited to a study of circular orbits, characterized by only a single quantum number  $n$  (which determined both the energy of the electron and its angular momentum), could be seen particularly clearly in a study of the effect of external fields on spectral lines. For the same reason, Bohr suffered setbacks in his investigations of the electronic configurations of complex atoms in the second part of his trilogy and in attempts to

explain the periodic table. In the model theory, all these problems could be solved only on the basis of the generalized Bohr-Sommerfeld quantum conditions.

Bohr's study of questions relating to atomic structure in a 1915 paper<sup>78</sup> was more successful. At the beginning of this paper he said that since his theory had been criticized and, furthermore, since important experimental results on these questions had recently been obtained, he would attempt to analyze in more detail several questions in that paper. Bohr later characterized his general assumptions as six points (A, B, C, D, E, F), the first two of which he formulated as follows (Ref. 78, p. 195): A. An atomic system has states in which there is no radiation involving a loss of energy, even if particles are moving with respect to each other, so that radiation should occur according to ordinary electrodynamics. Such states are called "stationary states" of the system. B. Any emission or absorption of energy will correspond to a transition between two stationary states. The accompanying radiation will have a definite frequency, given by  $h\nu_{\text{rad}} = E_1 - E_2$ , where  $h$  is Planck's constant, and  $E_1$  and  $E_2$  are the values of the energy of the system in the two stationary states.

Point C is the same as assumption 2 in Part II of his trilogy (see Section 4 above) regarding the applicability of classical mechanics to a dynamic equilibrium of a system in stationary states. According to point D, the energy and the frequency are related by (8) for a one-electron atom. Bohr went on to write that he had previously been discussing systems containing only a single electron, but assumptions A and B apparently apply in the general case, since they yield a simple explanation for the general combination principle for spectral lines (Ref. 78, p. 196). As direct confirmation of assumption A, Bohr cited the experiments by Einstein and de Haas,<sup>85</sup> which Bohr perceived as indicating that electrons can revolve in atoms without radiating energy (Ref. 78, p. 197).

For multielectron systems Bohr again presented as point E the hypothesis that the angular momentum of each electron is equal to  $h/2\pi$  for the ground state (this hypothesis was formulated at the end of the first part of his trilogy—see Section 4 above). As point F he offered the suggestion that a configuration which satisfies point E is stable if the total energy corresponding to it is lower than for any other configuration which satisfies the same condition for the angular momenta of the electrons (Ref. 78, p. 197). It was this condition that Bohr used in the trilogy.

An important point is that Bohr was singling out the postulate of stationary states as a separate—and first—assumption (point A) here. As the second assumption (B) he offered the frequency condition, stressing the generality of these assumptions. Here we are seeing Bohr's two basic postulates in the two leading positions (see Section 1 above); only after these postulates are stated does he list the assumptions of the model theory.

In Ref. 78 Bohr first discussed (at the end of §3, on the spectra of systems containing more than one electron) the results of the experiments by Franck and Hertz.<sup>78</sup> He correctly interpreted the value of 4.9 V, corresponding to the

appearance of an ultraviolet line of mercury with a wavelength of 2536 Å (Ref. 78, p. 210), as an excitation potential, not the ionization potential, of the mercury atom (as had been suggested by Franck and Hertz). Bohr wrote that if these arguments are correct, the implication is that the measurements by Franck and Hertz confirm the theory discussed in this paper (Ref. 78, p. 210).

In the last section of the paper<sup>78</sup> (§4, on high-frequency spectra), Bohr discussed the results of Moseley's studies of characteristic x-ray spectra. He stated that the entire problem can be clarified to a significant extent by the recent interesting studies by Kossel.<sup>75</sup> Bohr emphasizes that Kossel's suggestions lead to simple relations between the frequencies  $\nu_{\text{rad}}$  of different lines and that these relations correspond to the usual combination principle (Ref. 78, p. 212).

Bohr wrote his next, long paper<sup>82</sup> continuing to develop model concepts regarding the motion of electrons in circular orbits, by the beginning of 1916. This paper dealt with the application of quantum theory to motions characterized, as was stressed above, by a single quantum number for each electron.

This paper was submitted by Rutherford to *Philosophical Magazine* in January 1916, and it was to have been published in the April issue. However, after learning of Sommerfeld's studies (see the discussion below in the present section of this paper), Bohr stopped the publication of his paper.<sup>30)</sup>

In the introduction to the paper Bohr writes that the quantum theory was established as an attempt to overcome certain characteristic difficulties which arise in the application of ordinary mechanics or electrodynamics to atomic systems. Bohr wrote that the basic assumptions of the theory must therefore be regarded as postulates, having no basis of any sort in ordinary mechanics and electrodynamics. On the other hand, there was the natural question of whether these postulates could be put in a mutually consistent form to cover the various, extremely different applications. Bohr thought that this question could not be answered in general at the time since the theory at that time had been worked out in a definite form only for periodic systems. Bohr felt that a generalization of the theory to other systems would run into serious difficulties. In the particular case of periodic systems, however, Bohr thought that it was apparently possible to answer the question in the affirmative, and he stated that he would attempt to prove that assertion in that paper (Ref. 1, p. 433).

We will not analyze in detail that paper, which Bohr did not publish; we will simply summarize it. In it we see clearly Bohr's efforts to achieve a general formulation of questions and clear formulation of basic positions.

Right in the introduction, Bohr presents his postulate on stationary states, including the concept of quantum transitions. He states that a fundamental assumption on which his arguments are based is that an atomic system can be constant only in a certain series of states which correspond to discrete values of the energy of the system. Any change in the energy of the system, including the absorption and emission of electromagnetic radiation, must occur through a transition between two such states. These states would be called

stationary states of the system by Bohr (Ref. 1, p. 434).

It was this formulation which was the foundation for Bohr's subsequent formulations of his first postulate (see Section 6 below).

In §1 of the paper, Bohr uses the model theory to examine the conditions which must hold in stationary states of a periodic system. He writes that for any periodic system which contains only a single moving particle he will replace condition (8) for circular orbits, in which the kinetic energy  $E_{\text{kin}} = E_{\text{rot}} = nh\nu_{\text{rot}}/2$ , is constant, by the more general condition  $\bar{E}_{\text{kin}} = nh\nu_{\text{per}}/2$ , where  $\nu_{\text{per}}$  is the frequency of the periodic motion, and  $\bar{E}_{\text{kin}}$  is the average value of  $E_{\text{kin}}$  over a complete period. For periodic systems containing several particles Bohr suggests that this condition applies to each particle separately. Bohr emphasizes the importance for the quantum theory of the invariance of the quantity  $E_{\text{kin}}/\nu_{\text{per}}$ , and he cites Ehrenfest's well-known paper<sup>87</sup> on adiabatic invariants. This approach, however, yields no new results for an electron in an atom, revolving around a nucleus under the influence of the Coulomb force, when the revolution frequency  $\nu_{\text{per}} = \nu_{\text{rot}}$  is the same for each state corresponding to the same value of  $\bar{E}_{\text{kin}}/\nu_{\text{rot}}$  (i.e., for different elliptical orbits for a given total energy  $E$ ). Bohr argues that he can consider only a circular orbit of an electron (with a maximum angular momentum  $nh/2\pi$ ). Bohr could not derive new results as long as he used only a single quantum number ( $n$ ) to characterize the orbit of an electron, and as long as the concept of degenerate stationary states differing in the values of other quantum numbers had not yet emerged.

In §2 Bohr uses the frequency condition to discuss the radiation emitted and absorbed upon a transition between two stationary states. Working from the idea of correspondence, he emphasizes that the frequency condition as applied to the hydrogen atom,  $E_{n_2} - E_{n_1} = h\nu_{\text{rad}}$ , may be thought of as a generalization of the relation  $E_{n+1} - E_n = h\nu_{\text{rad}}$  for a Planck oscillator. Bohr also states that these relations are formally in agreement with the Debye theory of temperature radiation (he is talking about Debye's paper of Ref. 52; see the discussion in Section 2 above).

Bohr also applies the frequency condition to the rotational spectra of diatomic molecules, citing the studies by Bjerrum (see Section 2 above). Working from the quantization of the rotation energy of a molecule,  $E_{\text{rot}} = n^2 h^2 / 8\pi^2 J$  (in accordance with the condition  $E_{\text{kin}} = nh\nu_{\text{rot}}/2$ ), Bohr derives a transition frequency  $\nu_{\text{rad}} = h(n_2^2 - n_1^2) / 8\pi^2 J$ . Interestingly, he offers the suggestion (by analogy with the manifestation of only the fundamental harmonic oscillation, not overtones, in infrared absorption spectra) regarding the presence of transitions only between adjacent stationary states, with  $n_1 = n$  and  $n_2 = n + 1$ . For such transitions he found  $\nu_{\text{rad}} = (h/4\pi^2 J)(2n + 1)/2$ . The idea of selection rules is embodied here. Bohr subsequently formulated several selection rules on the basis of the correspondence principle (as discussed in Section 6 below).

Finally, in §3, the last section of the paper, Bohr first discussed the question (citing Planck) of the probabilities for stationary states in the case of a statistical equilibrium of periodic systems. He used the concept of phase space and of

phase integrals, which he writes for a single degree of freedom as  $Q = \oint p dq = 2\bar{E}_{\text{kin}}/\nu_{\text{per}} = nh$ , while for a system with  $r$  degrees of freedom he writes them in the form  $Q = C(nh)^r$ , where  $C$  is described by Bohr as a numerical constant which depends on the nature of the system. Bohr works from the assumption that the probability for a stationary state of a system for which each orbit is periodic can be found by replacing the energy differential  $dE$  in the expression derived from ordinary statistical mechanics by the quantity  $h\nu_{\text{per}}$ . It is this assumption which he presents at the end of the paper as a fourth (and last) assumption. As the first and third assumptions Bohr offers the first and second basic postulates; the second assumption is the relation  $\bar{E}_{\text{kin}} = nh\nu_{\text{per}}/2$  of the model theory, where  $\nu_{\text{per}}$  is the frequency of the periodic motion (i.e.,  $\nu_{\text{vibr}}$  and  $\nu_{\text{rot}}$  in particular cases). Bohr makes no other assumptions here.

Soon after the paper<sup>82</sup> had been submitted to *Philosophical Magazine*, Bohr became acquainted with Sommerfeld's papers.<sup>88,89</sup> Sommerfeld had formulated generalized quantum conditions for systems with many degrees of freedom and had applied these conditions with great success to the solution of the three-dimensional problem of the motion of an electron in a one-electron atom. Sommerfeld introduced three quantum numbers, and to explain the fine structure of spectral lines as a relativistic effect (Bohr had mentioned this possibility previously<sup>80</sup>) he introduced a fine-structure constant  $\alpha$  (Ref. 73, p. 22). This was a major success in the development of a model theory of the atom (which may with justification be called the "Bohr-Sommerfeld theory"). This major result was followed by a rapid development of the theory, culminating in the foundation of quantum mechanics in 1925.

Bohr immediately concluded that Sommerfeld's accomplishments were very important; he stopped publication of his paper<sup>82</sup> and took up a major revision of it, now making use of Sommerfeld's results.

Bohr immediately wrote Sommerfeld (on 19 March 1916; Ref. 1, p. 603) about Sommerfeld's papers which Bohr described as elegant and exceptionally interesting, and Sommerfeld's results, which Bohr described as brilliant and extremely important. In a letter to Ossen (17 March 1916; Ref. 1, p. 571), Bohr wrote that this work by Sommerfeld has significantly altered the present state of quantum theory. Bohr commented that Sommerfeld's excellent results fitted in exceptionally well with Bohr's own ideas.

We might note that generalized quantum conditions had been formulated before Sommerfeld by W. Wilson and also by D. Ishiwara (see, for example, Ref. 20, p. 92); Bohr cited their work in Ref. 82 (Ref. 1, p. 275). However, it was only Sommerfeld who applied these conditions to atomic spectra and immediately achieved striking successes, causing Bohr to decide to revise Ref. 82.

Bohr's revision of Ref. 82 took more than a year and a half, to the end of 1917. During this time, Einstein published some famous papers<sup>90,91</sup> in which Bohr's two basic postulates regarding stationary states and radiative quantum transitions were used to study the equilibrium of radiation with matter. In these papers, Einstein introduced the coefficients

(which bear his name) which determine the probabilities for radiative quantum transitions, spontaneous and stimulated. In Bohr's studies, Einstein's ideas found important applications (along with Sommerfeld's results), when Bohr composed his fundamental paper in Ref. 9 on the basis of a complete revision of Ref. 82. This new paper became the starting point of the next period in Bohr's scientific activity.

## 6. THE CORRESPONDENCE PRINCIPLE AND THE THEORY OF SERIAL ATOMIC SPECTRA (1917-1920)

From the end of 1917 to the end of 1920 Bohr's research was devoted primarily to the development of the idea of a correspondence between the quantum and classical theories and to the use of this idea in the theory of atomic spectra.

During this period Bohr was a professor at Copenhagen University. In the fall of 1916 he was joined by H. Kramers, who had come from Holland and who worked as an assistant to Bohr, becoming his closest aide. After the First World War, in 1919, Bohr organized an Institute of Theoretical Physics at Copenhagen University. By this time, Bohr's work had become widely known, and he was maintaining scientific communications with many foreign scientists, in particular, Sommerfeld in Germany. He carried on a voluminous correspondence and met with these scientists.

After the 1918 publication of Ref. 9 (the first part appeared in March, and the second part in December), Bohr continued to develop his new ideas about intensities in spectra and selection rules; these ideas were expressed in that paper on the basis of the correspondence principle. In 1919-1920 he made several reports of his research: in Leyden on the problems of the atom and the molecule (Ref. 2, p. 201), in December 1919 and February 1920 back in Copenhagen on the problem of the newest atomic physics, in the Chemical Society (Ref. 2, p. 221), and on the interaction between radiation and matter at the Royal Danish Academy (Ref. 9, p. 227), and in April 1920 on the serial spectra of the elements at Berlin, at the German Physical Society. He developed his Berlin report into a paper which he published in September 1920, in the recently founded German journal *Zeitschrift für Physik*. This important paper<sup>10</sup> (which subsequently became the second paper in the collection in Ref. 8) was the next major study after his 1918 paper.<sup>9</sup> Finally, in December 1920, Bohr gave a report on some considerations regarding the structure of atoms (Ref. 3, p. 43) at the Physical Society in Copenhagen; in the following year, 1921, he developed the ideas in that report into a physical theory of the periodic table of elements (Section 7 below).

In his basic study of this period, "On the quantum theory of line spectra,"<sup>9</sup> in a very interesting introduction, Bohr characterized the progress which had been achieved in the development of a theory of these spectra by Sommerfeld, followed by other scientists. Part of this progress was the explanation of the Stark and Zeeman effects. Bohr went on to write that despite the major progress which has been achieved in this research we still have many unresolved difficulties of a fundamental nature, concerning not only the limited applicability of the method used in calculating the frequencies in the spectrum of a given system but also, and

especially, concerning the polarization and intensity of the spectra lines which are emitted. Bohr stated that these difficulties were intimately related to a radical departure from the ordinary ideas of mechanics and electrodynamics, manifested in the basic principles of the quantum theory, and the circumstance that it had not yet been possible to replace these ideas by others, equally consistent and developed (Ref. 2, p. 70). Bohr said that again in this direction there had recently been major progress, citing papers by Einstein<sup>90,91</sup> and Ehrenfest<sup>87</sup> (and also some subsequent work by Ehrenfest). Bohr stated his purpose in Ref. 9 by saying that with the theory as it exists today it may be of interest to attempt to discuss, from a common standpoint, various applications of this theory, in particular, to discuss the assumptions on which it is based as they relate to ordinary mechanics and electrodynamics. Bohr said that he was attempting to do this in that paper, and he would show that it appeared possible to cast some light on some as yet unresolved difficulties by attempting to pursue the analogy between quantum theory and the ordinary theory as far as possible (Ref. 2, p. 70).

His introduction was dated November 1917. In it he said that the work was divided into four parts: Part I contained a brief discussion of the general principles of the theory and a discussion of the applications of the general theory to periodic systems with a single degree of freedom and to the class of aperiodic systems which he had mentioned.<sup>31)</sup> Part II would contain a detailed discussion of the theory of the hydrogen spectrum, for the purpose of illustrating the general ideas. Part III would contain a discussion of questions which arise in connection with the explanation of the spectra of other elements. Part IV would have a general discussion of the theory of the structure of atoms and molecules on the basis of the application of quantum theory to the nuclear model of the atom.

Bohr published only Parts I and II in 1918; Part III was not published until November 1922,<sup>92</sup> in the form in which it had been written in the spring of 1918 but with a supplement dated September 1922. This supplement characterized the development of the theory of atomic spectra over the years 1919-1922. At the end of Part III, Bohr thanked his colleagues for their valuable assistance, especially Kramers, who, as Bohr stated, not only made an important contribution to the subject but also was of gracious assistance to Bohr in editing the manuscripts of all parts of the study (Ref. 2, p. 184). Part IV was never published at all; only the rough drafts survive (Ref. 2, p. 186).

Bohr begins Part I of his study,<sup>9</sup> devoted to the general theory of line spectra, by setting forth the general principles and, primarily, stating two fundamental assumptions on which this theory was based: the postulate of stationary states and the postulate of radiative quantum transitions. Beginning with this paper, Bohr asserted only these two basic postulates and attempted to formulate them as precisely as possible.

Bohr formulates his first postulate as in Ref. 82 (see the discussion above) and uses the expression "complete transition"; he would go on to use this expression in the future. He formulates his second postulate as follows: Radiation ab-

sorbed or emitted in a transition between stationary states is monochromatic and has a frequency  $\nu_{\text{rad}}$  given by the relation  $E' - E'' = h\nu_{\text{rad}}$ , where  $h$  is Planck's constant, and  $E'$  and  $E''$  are the energies of the two states under consideration (Ref. 9, p. 71).

Bohr emphasizes the discrete nature of the radiative transitions between stationary states and the fact that ordinary electrodynamics cannot be applied to such transitions. He discusses the Einstein coefficients  $A_{n''n'}$ ,  $B_{n''n'}$ , and  $B_{n'n''}$  for spontaneous and stimulated emission and for absorption as quantities characterizing *probabilities for transitions* between stationary states with energies  $E'$  and  $E''$ . He writes that these coefficients are constants which depend on only the stationary states under consideration (Ref. 2, p. 73). In discussing Einstein's derivation of the Planck radiation law and with the help of frequency condition (1), Bohr states that Einstein's theory may be regarded as a very direct confirmation of this condition (Ref. 2, p. 73). Bohr further writes that in the subsequent discussion the use of quantum theory to determine the line spectrum of a given system is not mandatory, in precisely the same way as it is not necessary to introduce detailed arguments regarding the mechanism for a transition between two stationary states in the theory of thermal radiation. Bohr said that he would show, however, that the conditions which will be used to determine the energies in the stationary states are of such a nature that the frequencies calculated from (1) in the limit in which the motions in successive stationary states differ only slightly will tend toward the frequencies which would be expected on the basis of the ordinary theory of radiation for the motion of a system in stationary states. To find the necessary relationship with the ordinary theory of radiation in the limit of slow oscillations, Bohr said that he would therefore directly reach definite conclusions regarding the probability for a transition between two stationary states. This path would again lead to certain general ideas regarding the relationship between the probability for a transition between any two stationary states and the motion of the system in such states. Bohr would show that this discussion casts light on the question of the polarization and intensity of various lines in the spectra of a given system (Ref. 2, p. 74).

Bohr also mentions a relationship between his ideas and Ehrenfest's adiabatic hypothesis (which Bohr called the principle of the mechanical convertibility of stationary states).

For the case of a system with a single degree of freedom, Bohr expands a periodic motion with a frequency  $\nu_{\text{per}}$  in the harmonic frequencies  $\tau\nu_{\text{per}}$  ( $\tau = 1, 2, 3, \dots$ ) with expansion coefficients  $C_\tau$  which directly determine, in accordance with ordinary electrodynamics, the intensities of the radiations corresponding to different values of  $\tau$  (Ref. 2, p. 81). He writes that we must expect for large values of the quantum number  $n$  that these coefficients will, in accordance with the quantum theory, determine the *probability for a spontaneous transition* from a given stationary state, for which we have  $n = n'$ , to a neighboring state for which we have  $n = n'' = n - \tau$ . He goes on to say that we can expect that for small values of  $n$  also the amplitude of the harmonic oscilla-

tions corresponding the given value of  $\tau$  will somehow give us a measure of the probability for a transition between two states for which we have  $n' = n'' = \tau$ . Consequently, there would in general be a definite probability that the atomic system in a stationary state would spontaneously go into any other state of lower energy, but if the coefficients  $C$  in expression<sup>32)</sup> (14) are zero for all motions for certain values of  $\tau$  then we should expect that transitions for which  $n' - n''$  is equal to one of these values will not be possible (Ref. 2, p. 82). We see that the use of the idea of correspondence leads to selection rules, in particular, the selection rule  $\Delta n = n' - n'' = 1$  for a harmonic oscillator ( $C_\tau = 0$  for  $\tau > 1$ ).

For systems with many degrees of freedom Bohr examines a conditionally periodic motion characterized by  $s$  frequencies of motion and  $s$  quantum numbers, and he arrives at analogous results (the coefficients  $C_\tau$  are replaced by coefficients of the type  $C_{\tau_1, \tau_2, \dots, \tau_s}$ , and the vanishing of these coefficients determines selection rules for the given system).

In Part II of his study,<sup>9</sup> Bohr successfully applies the results which he derived in Part I to the hydrogen atom and discusses, from a common standpoint, the fine structure of spectral lines and their splitting in external electric and magnetic fields, treating these effects as small perturbations (following Sommerfeld). In contrast with Ref. 77, back in 1914, when Bohr had not been able to interpret the Stark and Zeeman effects in detail (see Section 5 above), he was now able to do this, taking into account the lifting of a degeneracy by an external field and the selection rules for the quantum number (which has been introduced by Sommerfeld) determining the projection of the angular momentum, onto the field direction. In taking this approach, Bohr also found an explanation for the polarization of the components of the Stark and Zeeman splittings.

The successful results achieved in his 1918 study<sup>9</sup> were discussed by Bohr in late 1919 in a report on the program of the newest atomic physics (Ref. 2, p. 223). Bohr said there that ideas regarding an analogy between the ordinary theory of radiation and the quantum theory lead to a consideration of the conditions determining stationary states in a slightly different light. While the starting point had previously been an appropriate generalization of a condition which gave possible values of a Planck oscillator, it was also possible to approach the problem by a path along which one attempts to determine stationary states, while taking this analogy into account. Bohr is still talking here, as in Ref. 9, about an analogy and has not yet used the word "correspondence."

Among Bohr's general ideas at this time we should also mention his appraisal of the present state of the theory of the interaction of radiation with matter which he offered in a report on this topic in February 1920 (Ref. 2, p. 235). He said that we must assume that at present we completely lack anything in the way of an actual understanding of the interaction between light and matter. Actually, he continued, many physicists believe that it would hardly be possible to suggest any sort of picture which would simultaneously explain interference and the photoelectric effect without introducing radical changes in the point of view from which we



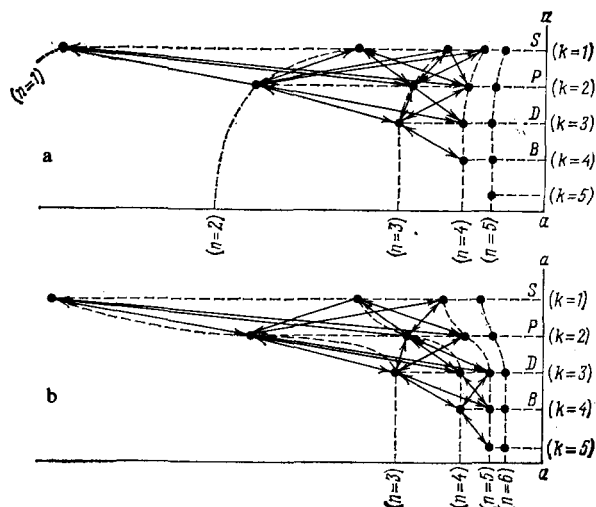


FIG. 2. Scheme of the serial spectrum of the sodium atom according to Ref. 10(a) and Ref. 11(b).

have previously attempted to explain natural phenomena. It is necessary to emphasize that also in the following years up to the development of quantum mechanics and quantum electrodynamics, Bohr was acutely aware of the absence of an adequate theory of the interaction of radiation with matter.

Bohr summarized his research over the years 1918–1920 in a paper on the serial spectra of elements.<sup>10</sup> Here we will consider only the general characteristic of this widely known paper. Bohr offers a clear formulation of the correspondence between the quantum and classical series, now using the terms “correspondence” and “correspondence principle,” which he would go on to use in subsequent papers. Bohr wrote that the process of radiation, involving a transition from one stationary state to another, cannot be pursued in detail with the help of the usual electromagnetic concepts. The properties of the radiation of an atom from the point of view of these concepts are determined directly by the motion of the systems and by an expansion of these motions in harmonic components. Nevertheless, he continued, there is a far-reaching *correspondence* between different types of possible transitions from one stationary state to another, on the one hand, and the different harmonic components of the expansion, on the other. Consequently, the theory of spectra under consideration here may be regarded as, to some extent, a generalization of the concepts of the usual theory of radiation (Ref. 10, p. 250). Bohr later speaks of “the correspondence principle” mentioned above (Ref. 10, p. 252).

In addition to the hydrogen spectrum, Bohr examined the serial spectra of multielectron atoms and offered (Ref. 10, p. 256) what he called a theoretical scheme for the formation of the serial spectrum of sodium (Fig. 2a), which he would use to show the transitions allowed by the selection rules between stationary states characterized by two quantum numbers  $n$  and  $k$ . The first of these numbers determines the successive states of one type ( $S, P, D, \dots$ ), while the sec-

ond determines the angular momentum  $M = kh/2\pi$  for an elliptical orbit with a ratio of minor and major semiaxes equal to  $k/n$  ( $k = 1, 2, 3, \dots$  for states of the  $S, P, D, \dots$  type;  $n \geq k$ ). This scheme is the original version of the energy-level scheme with which we are quite familiar, and the characteristics of the stationary states with the help of the quantum numbers  $n$  and  $k$  for each electron in a complex atom were widely used by Bohr in his subsequent studies.

Bohr also pays considerable attention to the theory of the Stark and Zeeman effects, emphasizing the importance of using the correspondence principle to find selection rules and to find the intensity distribution of the components in the splitting pattern.

At the end of the paper Bohr briefly discusses the structure of atoms and molecules and moves away from his earlier positions regarding the electron rings which he had used in the second and third parts of his trilogy<sup>5</sup> (see Section 3 above). He writes that already it is not possible to justify the assumption, introduced for a preliminary orientation, that in normal states the electrons move along orbits which have a particularly simple geometry, similar to “electron rings” (Ref. 10, p. 283). He further states that we are forced to seek some more complex types of motion (Ref. 10, p. 284). This search led Bohr quite soon, in 1921, to his theory of the periodic table of elements (see the following section of this review).

## 7. PHYSICAL EXPLANATION OF THE PERIODIC TABLE, DIFFICULTIES OF THE MODEL THEORY AND THE SEARCH FOR WAYS TO OVERCOME THEM (1921–1923)

The years 1921–1923 were years of exceptional intensity in the multifaceted work of Bohr. He achieved some major specific results in the development of a physical theory of the periodic table of elements (which he had taken up back in 1912; see Section 3 above) and in explaining their optical and x-ray spectra. Bohr also devoted considerable effort to general questions of quantum theory and an understanding of its foundations; he was profoundly aware of the difficulties of the model theory of the atom and was striving to overcome them. In contrast with the preceding period, when he had written only two long papers,<sup>9,10</sup> Bohr published several papers, including some of fundamental importance, in 1921–1923. He repeatedly gave reports at Copenhagen and in other countries; he gave a series of lectures in England, Germany, and the USA; he met with many foreign scientists; and he expanded his scientific links.

Bohr was doing his work in the Institute of Theoretical Physics which he had organized at Copenhagen University in 1920. The ceremony marking the opening of the Institute was held in March 1921, and Bohr gave a speech. He had much to say about the international collaboration of scientists, on which he placed much importance (Ref. 2, p. 283). Working in addition to Bohr at the Institute were H. Kramers, O. Klein, S. Rosseland, and many other young colleagues. Many physicists from various countries came to work with Bohr. A Copenhagen scientific school in theoretical physics began to take shape. Bohr's own studies became widely known and generally acknowledged. In 1922 he was

awarded the Nobel Prize for attainments in the study of the structure of the atom and its radiation.

Regarding questions of atomic structure, Bohr published in 1921 two important letters<sup>93,94</sup> (in March and October) and a general paper<sup>95</sup> (in April). In a collection<sup>86</sup> of translations of his papers from 1913–1916 he provided a foreword.<sup>96</sup> His rough drafts on atomic structure from these years have survived (Ref. 3, p. 99). In October 1921 Bohr made a major report on the structure of the atom in connection with the physical and chemical properties of elements at the Physical Society in Copenhagen. He later developed this report into an extremely important and comprehensive paper,<sup>11</sup> which he published first in Danish and later, in March 1922, in German translation in *Zeitschrift für Physik* (this paper appears in augmented form as the third paper in the collection in Ref. 8). In January and May of 1923 Bohr published two long papers on spectra, Ref. 29 (in German) (in collaboration D. Koster) and Ref. 30.

Several of his papers dealt with questions related to the application of the correspondence principle and general problems of quantum theory and its difficulties. In 1921 he published a paper on the polarization of radiation<sup>97</sup> (this paper appeared in August) and prepared a report for the Third Solvay Congress on the application of quantum theory to the problems of the atom.<sup>98</sup> This Congress convened in April 1921, but Bohr was not able to attend because of severe overwork, and his report was read by Ehrenfest, who supplemented the report with his own ideas regarding the correspondence principle.<sup>99</sup> Bohr continued to work on the application of quantum theory to general problems of the atom (his rough drafts survive; Ref. 2, p. 397). In March 1922 Bohr delivered a report to the Physical Society of London on the effect of electric and magnetic fields on spectral lines. This report contained an application of the correspondence principle (the report was later published,<sup>100</sup> in July 1923). In June he wrote a letter regarding selection rules.<sup>101</sup> In January 1923 he published in *Zeitschrift für Physik* a long and fundamentally important paper<sup>12</sup> on the basic postulates of quantum theory (Bohr did not publish a continuation of this paper, dealing with the theory of serial spectra, but a rough draft survives; Ref. 2, p. 501).

Bohr summarized the results of the development of the quantum theory of the atom over the decade in December 1922 in his Nobel Lecture on the structure of the atom,<sup>102</sup> at the beginning of which he said that he believed he would be conforming to the traditions of the Nobel Foundation if he structured his lecture as a review of the developments which had occurred in recent years in the field of physics to which his paper belongs (Ref. 3, p. 429).

Particularly successful among his series of lectures were seven lectures on the quantum theory of the structure of the atom which he read in Germany, at Göttingen University in June 1922; these lectures are now known as the "Bohr festival" (Ref. 22, p. 259). Detailed transcripts of these lectures, of major interest, have survived; they are published in Ref. 3, on p. 341. Summaries of a later series of six lectures which Bohr gave at Yale University in New Haven in October 1923 (the "Silliman lectures") have also been published (Ref. 2,

p. 581). In his lectures Bohr always paid close attention to fundamental questions of quantum theory.

In this review we do not have room for a detailed analysis of even Bohr's most important papers of the years 1921–1923, including papers<sup>11</sup> on the physical and chemical properties of the elements,<sup>33</sup> a paper<sup>12</sup> on the basic postulates of quantum theory, and a summary Nobel Lecture on the structure of the atom.<sup>102</sup> We will restrict the discussion here to questions related to Bohr's general approach to the formulation and solution of problems of the quantum theory of the atom, which can be seen particularly clearly in these papers, and his appraisal of the difficulties of this theory.

We must stress at the outset that Bohr devoted a great deal of effort to the general positions of the theory: its two postulates and the correspondence principle. In the paper in Ref. 11 and the lecture in Ref. 102 he clearly characterized these postulates, and in a paper<sup>12</sup> devoted to them the major thrust is a detailed examination of these general positions. Chapter I of this paper, on stationary states, begins with a section on the first basic postulate (§1), while Chapter II, on radiation processes, begins with a section on the second basic postulate (§1). Of the first postulate Bohr writes that the requirement contained in this postulate contradicts the classical theory (Ref. 12, p. 483); of the second postulate, he writes that it furthers the break with classical electrodynamics which was marked by the first postulate (Ref. 12, p. 502). Bohr devoted a large fraction of these papers to the correspondence principle, as the most important common principle. In Ref. 11 he writes of results to which we are led by the correspondence principle with regard to the question of the feasibility of transitions between various pairs of stationary states. These results were regarded by Bohr as being of decisive importance for the subsequent materials [Ref. 11, p. 335; he is talking about the selection rules for quantum numbers, in particular, in serial spectra of the type in Fig. 2a, which Bohr presented in Ref. 10 (see the discussion above in Section 6); this figure reappears in Ref. 11, on p. 333]. In his Nobel Lecture Bohr devotes a separate section to the correspondence principle (Ref. 102, p. 437) and notes its importance for the interpretation of the Zeeman and Stark effects (Bohr discussed this matter in more detail in Ref. 100). Bohr was very concerned with the correspondence principle in a paper<sup>12</sup> on the basic quantum postulates, in §§2–4 of Chapter II (on the correspondence principle, on the correspondence principle and a determination of stationary states, and on the correspondence principle and the structure of radiation). At the beginning of the first of these sections, Bohr wrote that despite the fundamental discrepancies between the postulates of quantum theory and classical electrodynamics it is still possible to use relations<sup>34</sup> (A) and (B) to establish some relationship between the radiation process and motion in an atom. The method of comparison explains why the laws of the classical theory are valid for describing phenomena in a certain boundary region (Ref. 12, p. 505).

Bohr's success in creating a physical theory of the periodic table resulted from a successful combination of general postulates with graphic concepts of the model theory of el-

litical orbits of electrons in complex atoms, characterized by two quantum numbers: a main quantum number  $n$  (Bohr introduces this term in Ref. 11, on p. 337), which is analogous to the quantum number which determines the energy of an electron in a hydrogen atom, and a quantum number  $k$  ( $k = 1, 2, \dots, n$ ), which determines the magnitude of the electron's angular momentum  $M = kh/2\pi$  (Bohr designated these orbits as  $n_k$ ).

Bohr discussed the successive binding of electrons by a nucleus with an atomic number  $Z$ , referring to this process as *one which decomposes into  $Z$  steps*, and he writes that for each such *attachment process* we should expect to find a corresponding spectrum (Ref. 11, p. 330).

Bohr was able to find the correct values of the main quantum number  $n$  for the electrons which became attached to the nucleus by working from spectral data. In particular, on p. 351 in Ref. 11 he gives a model for the serial spectrum of sodium (Fig. 2b), with the correct values for the stationary states of the outer electron of the sodium atom. In particular, he gives  $n = 3$  for the ground state (in Fig. 2a, only the relative number  $n$  is given; he sets  $n = 1$  for the ground state).

An extremely important point is that Bohr takes into account, along with the concept of the model theory and data on the serial spectra of atoms, the periodicity of the chemical and physical properties of the elements in determining the sequence in which electrons are attached to orbits of various types with increasing  $n$ . He also makes use of symmetry considerations regarding the spatial positions of orbits in determining the numbers of electrons in orbits with given values of  $n$  and different values of  $k$ . In Ref. 11 he offers no calculations at all and writes at a qualitative level. In particular, he discusses penetrating elliptical orbits, for which the orbit of a valence electron partially enters the region of the internal configurations (Ref. 11, p. 352). The Nobel Lecture contains an instructive diagram showing the penetration of electrons with small values of  $k$  ( $k = 1, 2$ ), moving in highly prolate elliptical orbits, into the inner regions of an atom (Ref. 102, p. 441).

Bohr was able to find the correct occupation numbers of the electron orbits with various values of  $n$  (the successive electron shells  $K, L, M, N, \dots$ ); in addition, and particularly importantly, he was able to show convincingly how the groups of transition elements form in the fourth, fifth, and sixth rows of the periodic table and how the group of rare earth elements forms in the sixth row, through the filling of  $3_d, 4_d, 5_d$ , and  $4_f$  inner orbits (the  $3d, 4d, 5d$ , and  $4f$  electron shells in today's notation<sup>28</sup>).

Bohr presented a table (Ref. 11, p. 363) of the distribution of electrons among orbits for the inert gas atoms, assuming, however, on the basis of symmetry considerations, that for a given value of  $n$  the numbers of electrons with different values of  $k$  are identical (for Ar, for example, he finds that there are four  $3_1$  electrons and four  $3_2$  electrons, instead of two and six, while for Xe he finds six  $3_1$  electrons, six  $3_2$  electrons, and six  $3_3$  electrons instead of two, six, and ten). We know that the correct occupation numbers could be found even in the model theory of the atom. This was done in

1924 by E. Stoner and, independently, by D. Main Smith; in 1925, Wolfgang Pauli derived these occupation numbers on the basis of his famous exclusion principle<sup>104</sup> (see, for example, Ref. 22, p. 667, and Ref. 73, p. 36).

In some supplements which Bohr added to the text of Ref. 11 upon its publication in the collection in Ref. 8 and also in the Nobel Lecture<sup>102</sup> Bohr presented the periodic table in a form which graphically demonstrated the filling of inner layers (Ref. 4, pp. 364 and 420).

The exceptional intuition of Niels Bohr is obvious in his physical theory of the periodic table. He achieved further successes in explaining the spectra of complex atoms in Refs. 29 and 30.

The results of the development of the model theory of the atom and of its spectra by both Bohr and Sommerfeld, on the one hand, and many other scientists, on the other, are reflected in Sommerfeld's well-known monograph, *Structure of the Atom and Spectral Lines* (in German), which appeared in four editions<sup>105</sup> in 1919–1924.

However, in parallel to the major achievements of the Bohr-Sommerfeld model theory of the atom, this theory ran into some major and fundamental difficulties when classical mechanics with artificially imposed quantum conditions was applied to stationary states. These difficulties were particularly obvious in 1922–1923. In particular, Bohr and Kramers were not able to solve correctly even the problem of the simplest two-electron system—the helium atom—which they studied from 1916 to 1923 (Ref. 3, p. 36). In explaining the structure of the electron shells of atoms and spectral lines, they were forced to eliminate states of an electron with  $k = 0$ , which corresponded to zero angular momentum (in the model theory, this is the case in which the elliptical orbit degenerates into a straight line passing through the nucleus). We now know that only in quantum mechanics has it been shown that the orbital angular momentum of an electron is  $M = (h/2\pi)\sqrt{l(l+1)}$ , where  $l = 0, 1, 2, \dots, n-1$ , and the  $S$  states ( $k = 1$ ),  $l = n-1 = 0$ ) are now correctly interpreted as corresponding to zero angular momentum.

Bohr clearly understood that the model theory of the atom was unsatisfactory and had some fundamental shortcomings, as can be seen from his words in the spring of 1922—at a time when his achievements in the development of a physical theory of the periodic table had attracted general attention.

In a letter to A. Haas on 11 April 1922, Bohr wrote about the quantum theory of the atom (Ref. 2, p. 647). In his opinion, an exposition of this theory (the newest theories of the spectra and structure of the atom) which is found in most special textbooks, e.g., the well-known book by Sommerfeld, is not particularly suitable either with regard to the preceding course of the development of the theory or with regard to his current points of view, for giving the reader a picture of the principles of the quantum theory which would correspond to the actual content of the theory. In a well-known letter to Sommerfeld on 20 April 1922, Bohr was even more definite (Ref. 2, p. 691). Here he wrote that in recent years he had frequently felt himself very alone in a scientific sense, under the impression that his efforts to de-

rive systematically, to the extent his abilities allowed, the principles of a quantum theory, were meeting with very little understanding. What Bohr was concerned with was not didactic trifles but a serious attempt to find an internal relationship which would raise the hope that it would be possible to develop a solid basis for a future derivation. Bohr said that he understood quite well that little was yet known about the questions and that he was helpless in attempting to explain his thoughts in an easily accessible form. Bohr further wrote that he would pursue questions of this type further in a paper which would soon appear in *Zeitschrift für Physik*<sup>35)</sup> and which would contain a more detailed discussion of the ideas in his report on the structure of the atom<sup>36)</sup> (Ref. 2, p. 691).

Bohr's extremely profound approach, which he subsequently developed during the period of the establishment of quantum mechanics, after advancing the principle of complementarity, can be seen in his discussion with Heisenberg in June of 1922. Heisenberg, who was a student of Sommerfeld at Munich University at the time, heard Bohr's lectures at Göttingen (see the discussion above in this section of the review), where Heisenberg had become acquainted with Bohr. In his very interesting book of memoirs,<sup>37)</sup> Heisenberg wrote about his first discussions with Bohr during a stroll. Heisenberg said that that stroll had an exceedingly great effect on his subsequent scientific development, or he thought it might be more accurate to say that his genuine scientific development did not begin until that stroll (Ref. 37, p. 59). Heisenberg reports several comments which Bohr made regarding the history of the quantum theory at the beginning of their discussion. He said that the starting point was by no means the thought that the atom is a planetary system of small dimensions and that we could apply the laws of astronomy. He said that he never took any of that stuff literally. He said that for him the starting point was the stability of matter, which is an absolute wonder from the standpoint of the earlier physics (Ref. 37, p. 60). Regarding the stability of matter Bohr later stated that because of the stability of matter Newtonian physics cannot be valid inside an atom; at best, it may episodically give us a starting point. For that reason, Newtonian physics was also incapable of giving us anything in the way of a clear description of the structure of the atom, since such a description—precisely because it would have to be clear—would have to make use of the concepts of classical physics, but these concepts no longer apply to what has emerged. Bohr said that it is understandable that in such a theory attempts will be made essentially to achieve something which is totally impossible. He said that we must say something about the structure of the atom but we do not have any sort of language in which we could make ourselves understood. In that situation, the theory would be totally incapable of furnishing an explanation in the usual scientific sense. Bohr was talking about determining relationships and cautiously feeling one's way ahead (Ref. 37, p. 62). Bohr responded to Heisenberg's question regarding the meaning of the pictures of atoms which Bohr had shown and which he had discussed in his lectures. Bohr said that these pictures were constructed from experiments, or if you wish, were the results of guesses, not the products of theoretical calculations

of any sort. Bohr hoped that these pictures would give a good description of the structure of the atom, but this description could be only as good as was possible in the graphic language of classical physics (Ref. 37, p. 63). Regarding the future of physics Bohr said that we must expect that the paradoxes of the quantum theory and the puzzling aspects associated with the stability of matter will come further and further out of the shadows with each new experiment. When this happens, it can be hoped that new concepts will eventually be developed which we can successfully apply even to these puzzling processes in the atom, but we are still extremely far from that day (Ref. 37, p. 63).

Heisenberg's memoirs undoubtedly convey accurately the general nature of Bohr's views in this period and his hopes for the future resolution of the difficulties of the quantum theory.

Bohr devoted a particularly large effort to the difficulties of the quantum theory in the last year of this period: 1923. At that time, the question of the relationship between the wave and corpuscular concepts of electromagnetic radiation loomed extremely large, after Arthur Compton's discovery in late 1922 of the change in the wavelength of x rays when they are scattered by free (or weakly bound electrons)<sup>106)</sup> (an effect now known as the "Compton effect"). This effect was explained by Compton himself<sup>107)</sup> and, independently, by Debye,<sup>108)</sup> as the result of collisions of quanta of radiation with electrons, in accordance with Einstein's hypothesis of light quanta. Bohr continued to disagree with that hypothesis<sup>37)</sup> and sought other ways to resolve the difficulties of the quantum theory, still striving somehow to reconcile quantum concepts with classical electrodynamics.<sup>38)</sup>

Bohr's interest in the general questions of the physical theory at the end of 1923 is clearly reflected by a summary of a lecture he gave at a US university on 29 October of that year, on some philosophical aspects of the modern theory of the atom (Ref. 2, p. 46).

The topics which he covered were the gradual development of the concepts on which science is based; the development of mechanics; Galileo, Newton, and Einstein; the independence of the description from subjective conditions; the electromagnetic theory; Faraday and Maxwell; the electromagnetic theory of light; Einstein's works; and the form of the laws of classical physics, possibly the final form. Other topics were the laws of the atomic world; the discovery of the atomistic structure of electricity and the structure of the atom; the change in the nature of physical ideas; the inverse problem; the quantum theory of radiation; a paradox; the impossibility of a compromise; a description of an atomic process by means of probability considerations; analysis of the concept of probability; the problematical nature of the benefit of constructing a picture based on classical ideas; analysis of the meaning of explanation in science.

In his rough drafts of 1923–1924 on the problems of the theory of the atom, Bohr wrote (Ref. 2, p. 569) that the present state of the development of the atomic theory is characterized by the problem of explaining the typical physical and chemical properties of the elements on the basis of a general knowledge of the constituent parts of atoms extract-

ed from a study of phenomena which are not directly related to these properties. Bohr thought that the most characteristic feature of attempts to solve this problem was the conviction that it would not be possible to achieve progress of any sort without making a substantial break with the points of view with which we have attempted in the past, with striking success, to explain phenomena which are directly accessible to observation with the help of our senses and which depend on the collective action of a huge number of atoms. However, he continued, the effort to discover the laws controlling the behavior of individual atoms will necessarily be of an exploratory nature, and there can hardly be a better way to emphasize this exploratory nature than to recognize that not only a description of the properties of the elements based on an interpretation of observations in the language of the concepts of classical physics but even our knowledge of the constituent parts of atoms depends, by the nature of things, on the use of the laws of classical physics.

We see that here Bohr's ideas regarding the role of classical concepts in a description of microscopic phenomena are being formed. Bohr embarked on a profound analysis of the theory of such phenomena from very general standpoints; this approach was particularly characteristic of the later periods of his scientific activity.

## 8. CONCLUSION

In appraising the importance of the pioneering studies by Bohr in the quantum theory of the atom and the correspondence principle over the years 1912–1923, we should first stress the difficulty of the decisive step which Bohr took in the spring of 1913 in the first part of his trilogy,<sup>5</sup> making a sharp break with the ideas of classical physics. This point is demonstrated clearly by Rutherford's and Einstein's comments regarding Bohr's theory. Later, in 1936, Rutherford wrote (Ref. 110, p. 490) that he considered the original quantum theory of spectra proposed by Bohr to be one of the most revolutionary of all theories which had ever been developed in science. Rutherford knew of no other theory which would have been more successful. Then there are Einstein's well-known comments in 1949 in his *Autobiographical Notes* (Ref. 11, p. 275). In discussing the consequences of the Planck law, Einstein said that all his own attempts to make the theoretical foundations of physics accommodate these results were complete failures. It was a matter of the earth slipping away from under my feet; there was no firm ground anywhere in sight on which to build. It was always a wonder to me that this foundation, swaying and full of contradictions, was good enough that Bohr—a man of brilliant intuition and keen perception—could find the basic laws of the spectral lines and the electron shells of atoms, including their importance for chemistry. Einstein said that he was still filled with wonder. He said that Bohr exemplified the highest level of musical talent in the field of thought.

Analysis of Bohr's studies in this period convincingly shows that the most important part of his research concerned the fundamental new ideas of stationary states and of quantum transitions between these states. These ideas are embodied in the first postulate and, in application to radia-

tive transitions, in the second postulate [frequency condition (1)].

The idea of stationary states was based on Bohr's profound understanding of the problems of the stability of atomic systems, which was unexplainable on the basis of either classical electrodynamics or classical mechanics. We must stress that Bohr was examining questions of the mechanical stability of atoms and molecules as early as 1912 in this "Rutherford memorandum," and then again in 1913 in Parts II and III of his trilogy.<sup>5</sup> Bohr always emphasized questions of the stability of atomic systems with respect to radiation, beginning in 1913 (in Part I of the trilogy), as a characteristic property of these systems.

The idea of quantum transitions—abrupt (or complete, as Bohr said) changes in the energy of atomic systems, or "quantum jumps"—was also developed by Bohr starting in 1913, when he basically explained the spectral laws and offered an explanation for the Rydberg-Ritz combination principle. An important point is that as early as Part I of the trilogy Bohr was studying radiationless as well as radiative quantum transitions.

In close connection with the ideas of stationary states and radiative quantum transitions, Bohr developed the idea of correspondence between the quantum and classical theories, beginning in 1913, with respect to frequencies, and then, beginning in 1917, with respect to the intensities and polarizations of these transitions. An extremely important factor was Bohr's understanding, based on the idea of correspondence, of the probabilistic nature of radiative quantum transitions, as can be seen clearly in his discussion of the Einstein coefficients in Part I of Ref. 9. The correspondence principle was used to much advantage in Bohr's derivation, in 1921–1923, of a physical theory of the periodic table of elements and his further development of the theory of atomic spectra on the basis of a model theory. Particularly, important, however, was the role played by the correspondence principle in 1925, when Heisenberg worked directly from this principle to develop a matrix formulation of quantum mechanics and thus start the foundation of quantum mechanics.<sup>14</sup>

A point which deserves special emphasis is that the development of modern physics has shown that the correspondence principle, in its general form as a principle relating new and old physical theories, must be regarded as one of the most important methodological principles of physics.<sup>112</sup> Bohr began the development of the correspondence principle.

Today in marking the 100th anniversary of the birth of Niels Bohr, we are acutely aware of the lasting importance of his research in 1912–1923 on the quantum theory of the atom and the correspondence principle.

<sup>11</sup>This was published in Danish<sup>7</sup> in early 1914 and in the first paper in the collection in Ref. 8 in 1922.

\*Transl. Editor's Note: Some of the quotations of items retranslated from Russian translations of the original sources could not be checked and probably deviate from the precise wording of the original while retaining the general meaning.

<sup>2</sup>For clarity we will be using notation which differs in part from that of the original papers. In the present instance we are designating the radiation frequency as  $\nu_{\text{rad}}$  (rather than simply  $\nu$ ).



- <sup>3)</sup>In which the authors erroneously assumed that Bohr was talking only about allowed orbits, rather than about stationary states (the concept of which Bohr had introduced already in the first part of his paper<sup>3</sup>). Savelev also failed to avoid inaccuracies in his presentation of Bohr's theory in Volume 3 of his *Course in General Physics* in formulating Bohr's first postulate not for stationary states but for "discrete orbits which satisfy certain quantum conditions" (Ref. 33, p. 55).
- <sup>4)</sup>The development of the concepts of "energy quanta" and "radiation quanta" was the topic of the section "Formation of quantum concepts" in Hund's monograph.<sup>21</sup> Two lines in the development of quantum theory, associated with "quanta of matter" and "quanta of the electromagnetic field," respectively, were clearly distinguished in the monograph by Mehra and Rechenberg.<sup>22</sup> The three ideas listed here were emphasized in my previous papers,<sup>14,25</sup> and their importance to the development of quantum concepts was traced.
- <sup>5)</sup>The Russian translation of this quotation, and of certain others, has been rendered more accurately.
- <sup>6)</sup>Debye used a probabilistic method analogous to that which had been used in 1900 by Planck. He applied it directly to the problem of the distribution of radiation energy over the quantized eigen-oscillations of this radiation.
- <sup>7)</sup>Only later was it to be learned that for a harmonic oscillator there is a selection rule  $\Delta n = \pm 1$  (for dipole radiation), which leads to  $\nu_{\text{rad}} = |\Delta n| \nu_{\text{vibr}} = \nu_{\text{vibr}}$ .
- <sup>8)</sup>A letter of 1 December 1911, cited on p. 230 in Ref. 19.
- <sup>9)</sup>Although Bohr was at that time moving on to the development of a quantum theory of the atom, he remained interested in the passage of charged particles through matter. In 1915 he published the paper of Ref. 60, and in 1925 he took up these questions in Ref. 61. After the development of quantum mechanics and the appearance of several studies on the quantum-mechanical theory of collisions, in 1948, Bohr analyzed many types of collisions accompanying the passage of atomic particles through matter, stressing the quantum and classical aspects of the phenomenon.<sup>62</sup> In 1954, in collaboration with J. Lindhard, he published the paper of Ref. 63.
- <sup>10)</sup>That Bohr was not working from the concept of harmonic oscillations of electrons in accordance with Thomson's model of the atom [as was assumed erroneously by Heilbron and Kuhn (Ref. 19, p. 249)], but was working from Rutherford's model of the atom, is shown convincingly by an analysis by Hoyer (Ref. 17, p. 180) of the paper in Ref. 59, with an appeal to archive data.
- <sup>11)</sup>The letter in question was written on 12 June; see the discussion above.
- <sup>12)</sup>He was talking about revolution times.
- <sup>13)</sup>Bohr erroneously wrote "ratio" instead of "relation." The ratio determined by (7) is the ratio of the kinetic energy of the electron to the frequency, i.e., the quantity which is the reciprocal of the revolution time.
- <sup>14)</sup>Here Bohr commented that this was to be expected since, in his words, it appears that it had been rigorously proved that mechanics was incapable of explaining the experimental facts in problems involving individual atoms. He went on to write that by analogy with what is known about other problems, however, it appears to be legitimate to use mechanics to study the behavior of a system, provided that we do not take up questions of stability (or of a final statistical equilibrium).
- <sup>15)</sup>That is, Bohr himself.
- <sup>16)</sup>That is, in relation (2).
- <sup>17)</sup>A. van den Broek.
- <sup>18)</sup>Bohr's question marks.
- <sup>19)</sup>Bohr introduces the nuclear charge  $E$ , and for the hydrogen atom he sets  $E = e$ , where  $e$  is the charge of the electron. We thus have  $E = Ze$ , where  $Z$  is the atomic number.
- <sup>20)</sup>Condition (8) is a generalization of condition (7) in the Rutherford memorandum (see Section 3 above). Bohr derives it in Section 1 of his paper, assuming that as an electron is captured by a nucleus to a circular orbit with a given value of  $n$  there is an emission of  $n$  quanta of energy  $h\nu_{\text{rad}}$  with a total energy  $nh\nu_{\text{rad}}$ , and under the assumption  $\nu_{\text{rad}} = \nu_{\text{rot}}/2$ . Working from the quantization condition (2),  $E = nh\nu_{\text{vibr}}$ , and noting that the average kinetic energy of a harmonic oscillator is equal to its average potential energy, we find  $\bar{E}_{\text{kin}} = nh\nu_{\text{vibr}}/2$ ; condition (8) is found by analogy. Bohr later (Ref. 7, p. 162) mentioned this approach both the derivation of this condition. The reader is directed to Ref. 19, p. 267, for a more detailed discussion of Bohr's original approach (which he subsequently abandoned) and the contradictory nature of that approach.
- <sup>21)</sup>We are presenting expressions (8) and (9) in the modern notation. In Ref. 5 (p. 87), Bohr uses  $\tau$  for  $n$ ,  $\omega$  for  $\nu_{\text{rot}}$ , and  $E$  for the charge of the nucleus ( $E = Ze$ ).<sup>19)</sup> His expressions for  $\bar{W}$  and  $\omega = \nu_{\text{rot}}$  corresponding to (9) have  $e^2 E^2 = e^4 Z^2$ , and his expression for  $2a$  has  $eE = e^2 Z$ .
- <sup>22)</sup>In his trilogy<sup>5</sup> and in several later papers, Bohr denotes this constant as  $K$ . The quantity  $R$  is expressed in reciprocal centimeters, while  $K$  is in reciprocal seconds (in Ref. 6,  $K$  is a constant which is the inverse of the constant  $Z^2 R$ ; in this paper,  $K_H = 1/R_H$  and  $K_{He} = 1/4R_{He}$ ).
- <sup>23)</sup>In Section 3 (Ref. 5, p. 94), Bohr rejected the assumption that during the capture of an electron by a nucleus to a circular orbit with a given  $n$  there is an emission of  $n$  quanta of energy  $h\nu_{\text{rad}} = h\nu_{\text{rot}}/2$  (see footnote<sup>20)</sup>).
- <sup>24)</sup>Bohr uses the notation  $\alpha = c$  and, as before,  $n = \tau$ .
- <sup>25)</sup>Which follows from the relations  $\bar{W} = -E_{\text{pot}}/2 = e^2/2a$  and  $\bar{W} = E_{\text{pot}} = 1/2ma^2\omega^2 = 2\pi^2ma^2\nu_{\text{rot}}^2$  (see the discussion above in the present section of this paper).
- <sup>26)</sup>It is this derivation, which is the most characteristic of Bohr's approach, which I believe should be presented in an exposition of the Bohr theory in courses in atomic physics (as, for example, Born does in Ref. 38).
- <sup>27)</sup>He is talking about Eqs. (8) and (9).
- <sup>28)</sup>Bohr cites the beginning of this Section 1.
- <sup>29)</sup>He is talking about stationary states with discrete values of the energy, i.e., a case of a finite motion.
- <sup>30)</sup>The proof of this paper has survived; it is reproduced in Ref. 1. Its German translation was published in collection<sup>86</sup> along with some translations of some earlier papers by Bohr.
- <sup>31)</sup>In the Introduction, Bohr speaks in terms of aperiodic motions of a simple type and of a broader class of aperiodic systems in connection with the theory of the Stark effect.
- <sup>32)</sup>Bohr refers to the coordinate expansion formula  $\xi = \sum C_\tau \cos 2\pi(\tau\nu_{\text{per}}t + c_\tau)$  where  $C_\tau$  and  $c_\tau$  are constant coefficients.
- <sup>33)</sup>This paper, which contains an exposition of a physical theory of Mendeleev's periodic table of the elements, is discussed in detail in Ref. 73 (see also Ref. 103).
- <sup>34)</sup>The Bohr-Sommerfeld quantization rules (A) and the frequency condition (B).
- <sup>35)</sup>He is talking about Ref. 12.
- <sup>36)</sup>In he report of Ref. 11.
- <sup>37)</sup>See Section 2 above. Before Compton's discovery, in Ref. 12 (in §1 of Chapter III, on the hypothesis of light quanta), Bohr wrote that the hypothesis of light quanta is not suitable for giving a general picture of the processes which could include the entire set of phenomena considered in applications of the quantum theory (Ref. 12, p. 518). We note, however, that Bohr had a positive attitude toward the concept of the quantization of the natural oscillations of a radiation field (see Section 2 above). As early as late 1913, in a report,<sup>7</sup> he mentions Debye's derivation (in Ref. 52) of the Planck radiation law (Ref. 7, p. 160). In a very interesting paper,<sup>97</sup> he makes a special comparison of the derivations of this law by Einstein<sup>91</sup> and Debye,<sup>52</sup> saying that both derivations should be regarded as very important, although all they have in common are their initial assumptions (Ref. 97, p. 294).
- <sup>38)</sup>As we know, in 1924 Bohr attempted (in Ref. 109, in collaboration with Kramers and J. Slater) to preserve the concepts of the electromagnetic theory of light by abandoning energy and momentum conservation in elementary processes. Only in May of 1925, in a postscript to Ref. 61, did Bohr reach the conclusion that the hoped-for generalization of electrodynamics would require a decisive break with the concepts on which our description of nature has previously rested (Ref. 61, p. 560). However, an examination of these questions goes beyond the scope of the present discussion, which is restricted to the preceding period.

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<sup>2)</sup>N. Bohr, Collected Works, Vol. 3. The Correspondence Principle (1918–1923) (ed. J. R. Nielsen), North-Holland, Amsterdam, 1976.

<sup>3)</sup>N. Bohr, Collected Works, Vol. 4. The Periodic System (1920–1923) (ed. J. R. Nielsen), North-Holland, Amsterdam, 1977.

<sup>4)</sup>N. Bohr, Selected Scientific Works. Vol. I. Papers 1909–1925 (Russ. Transl., Nauka, Moscow, 1970).

<sup>5)</sup>N. Bohr, Philos. Mag. 26, 1 (Part I), 476 (Part II), 857 (Part III); see also Ref. 1, p. 159 (Russ. Transl., Ref. 4, p. 84).

<sup>6)</sup>N. Bohr, Nature 92, 231 (1913); see also Ref. 1, p. 273 (Russ. Transl., Ref. 4, p. 149).

<sup>7)</sup>N. Bohr, Phys. Tidsskr. 2, 97 (1914); see also Ref. 1, p. 281 (Russ. Transl., Ref. 4, p. 152).

<sup>8)</sup>N. Bohr, Drei Aufsätze über Spektren und Atombau, Vieweg, Braunschweig, 1922 (Russ. Transl., Gostekhizdat, Moscow–Leningrad, 1923).

- <sup>9</sup>N. Bohr, K. Danske Vidensk. Selsk. Skr. 4, No. 1, p. 1 (Part I), 37 (Part II) (1918); see also Ref. 2, p. 65.
- <sup>10</sup>N. Bohr, Z. Phys. 2, 423 (1920); see also Ref. 2, p. 241 (Russ. Transl., Ref. 4, p. 247).
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