Niels Bohr and the physics of the atomic nucleus

S. T. Belyaev and V. G. Zelevinskiĭ

I. V. Kurchatov Institute of Atomic Energy and Institute of Nuclear Physics, Siberian Branch of the USSR Academy of Sciences, Novosibirsk
Usp. Fiz. Nauk **147**, 210–251 (October 1985)

The development of three directions in nuclear physics, originating from three ideas of Niels Bohr, is analyzed: 1) the compound nucleus: is the compound state entirely chemical? what does the energy distribution of neutron resonances imply? "dynamic intensification" of weak effects; the role of fluctuations and the description of the kinetics; and, pre-equilibrium processes; 2) collective motions: how the collective and single-particle degrees of freedom coexist with one another; parallel formation of the shell model and the theory of collective oscillations; the generalized model; the problem of the moment of inertia; pair correlations and "superfluidity" in the nucleus; giant resonances; and theory of finite Fermi systems; and, 3) the fission process: fission in the liquid drop model and in the generalized model; shell corrections; double-hump fission barriers; fission isomers; nonconservation of parity in fission; and, "exotic" asymmetric fission. The emphasis is on the elucidation of the development of physical ideas, so that computational details are omitted.

CONTENTS

"An entirely new field of physics opens up here: the study of the internal structure of atomic nuclei."

N. Bohr[1]

 0038-5670/85/100854-24$01.80 © 1986 American Institute of Physics

INTRODUCTION

A future historian of science will probably note that during almost the entire first half of the 20th century the fundamental object of study, on which the leading physicists and the largest physical laboratories concentrated their scientific interests, was the atomic nucleus. From the source of the puzzling α and β rays located somewhere inside the atom, the nucleus materialized as the center, in which almost the entire atomic mass is concentrated, of the atomic "planetary system"; then, having given the physicists a glimpse of its internal constituents—the protons and neutrons, the nucleus transformed, once again changing its appearance, into a liquid drop, whose division releases an enormous amount of energy, carrying beneficial heat and at the same time endangering the existence of mankind.

During the second half of the century, physicists, arming themselves with experimental installations of unprecedented sophistication and cost and gigantic dimensions, were able to get a glimpse of the details of the internal structure of the innermost nuclear particles themselves, getting down to the deepest—the quark—level. The study of nuclear structure has receded from the frontier to the "rear guard" of research. At some institutes which have retained in their traditional names the word "nucleus" or its derivatives there is, for all practical purposes, no longer any nuclear physics as such.

Nuclear physics nevertheless developed even at this time probably more rapidly than previously. More spectroscopic information was accumulated and refined, new isotopes were synthesized, subtle polarization and correlation effects were measured, and the region of high excitation energies and high transferred momenta, high spins, short lifetimes, and nuclei far from the region of stability was entered. The theoretical apparatus adapted to the particular nature of the nucleus as a unique Fermi system with strong interactions and the powerful methods of the quantum field theory and statistical physics were perfected. Entirely new fundamental phenomena were discovered also. We shall recall (by no means exhaustively) only some of the achievements in nuclear research, beginning with the end of the 1950s.

We can speak conditionally about macroscopic and microscopic aspects of modern nuclear physics. To the former we refer phenomena which have direct analogs in the physics of condensed media: pair correlations of nucleons of the superconducting type leaving an imprint on all nuclear properties; double-hump fission barrier and fission shape isomers; family of new sound-like collective modes (giant multipole resonances and isobaric analog states); deep inelastic collisions of heavy ions, revealing the time-dependent unfolding of the processes involved in the establishment of equilibrium in a system with a finite number of degrees of freedom (nuclear kinetics); rapidly rotating nuclei with phase transitions to nonsuperconducting and superdeformed states. The most important microscopic aspects are those for which the nucleus is a natural laboratory for establishing the properties of fundamental interactions. Thus, in particular, variants of weak interactions were checked; the nonconservation of parity and longitudinal polarization of the neutrino were discovered; the most surprising was the discovery of nonconservation of parity in fission, enhanced by nuclear mechanisms up to the point of almost classical macroscopic motion. The question asked in school "What do nuclei consist of?" now takes on a new meaning. In addition to the principal (at low energies) nucleonic component of the wave function of the nucleus, experiments give information on other components: cluster, mesonic, isobaric, strange, charmed, and, finally, quark.[1)] At relativistic energies a new macroscopic nuclear physics, now linked to superdense nuclear matter, superstrong electric fields, and quark-gluon plasma arises.

This cursory list already demonstrates the richness of the nuclear phenomena which nature has buried in the atomic nucleus. Niels Bohr loved to quote the following Chinese proverb: (paraphrased from the Russian text) "We are all witnesses of and participants in the great pageant of life." In the course of half a century he witnessed the rise of the quantum physics of atoms, nuclei, and elementary particles and was its most active creator. Bohr not only laid the foundation of the new—quantum-physical—world view. He was the first to understand and to explain now trivial facts (the charge of the nucleus coincides with the number of the chemical element in the periodic table, and the nucleus fissions as a liquid drop), which it seems were always well-known to everyone. He also proposed the profoundest physical ideas, whose influence on the further development of physics in general and the physics of the atomic nucleus in particular has by no means yet been exhausted.

The discovery of the existence of atomic nuclei and the transformations of chemical elements accompanying the decay of nuclei by E. Rutherford was regarded by N. Bohr as an achievement heralding a "new epoch in physical and chemical science"[2] or, more expansively, "in the history of natural science."[1]

We shall selectively illustrate below (guided by our own scientific interest) the fate of some of Niels Bohr's ideas in nuclear physics. Making no effort to provide a complete bibliography and arranging it in chronological order, we shall in a number of instances refer to reviews. Each of the references to Bohr's work is also supplied (in the Russian text) with a reference to the volume of Russian translations of N. Bohr's works: N. Bohr, Selected Scientific Works, Vol. II. Articles from 1925 to 1961, Nauka, M. 1971 (Classics in Science Series).

1. FROM THE COMPOUND NUCLEUS TO NUCLEAR KINETICS

"He [N. Bohr] feared that the formal mathematical structure would obscure the physical core of the problem, and in any case, he was convinced that a complete physical explanation should absolutely precede the mathematical formulation."

W. Heisenberg (Ref. 179, p. 98)

1.1. Why are neutron resonances narrow?

In a short note entitled "Neutron capture and the structure of the nucleus,"[5] N. Bohr put forward a concept which had a strong or, as F. Friedman and V. Weisskopf write in a review article,[6] decisive influence on all further development of nuclear physics and many-body quantum systems in general. Starting from the discovery of narrow neutron resonances, where the reaction cross sections are large compared with nuclear sizes,[2)] Bohr proposed that there exist quasistationary excited states of nuclei with long lifetimes τ exceeding by orders of magnitude the times $\tau_{sp} \approx 10^{-21}$ sec corresponding to purely single-particle motion of a slow neutron through a complex nucleus.

The physical properties of this intermediate state (the compound nucleus) are determined, according to Bohr, by the strong interaction, which rapidly distributes the excitation energy between the particles. Discussing in greater detail the physics of the compound nucleus in the fundamental paper "On the transmutation of atomic nuclei by impact of material particles,"[8] N. Bohr and F. Kalckar wrote: "every nuclear transmutation will involve an intermediate stage in which the energy is temporarily stored in some closely coupled motion of all the particles of the compound system." In lectures and speeches Bohr repeatedly illustrated this hypothesis by the simple example of the entanglement of a billiard ball flying into a shallow plate filled with similar balls. In the presence of friction against the bottom of the plate it is entirely probable that the process will terminate without any balls being knocked out (analog of radiative deexcitation). Another possibility is that as a result of many-body collisions one of the balls will roll over the edge and escape.

Thus it is a question of the emission of particles or gamma quanta from the compound nucleus as a consequence of "fluctuations" with a sufficient concentration of energy on one particle. More accurately, one must talk about not individual particles, but rather degrees of freedom of the system, since the outgoing channel of the reaction could be, for example, emission of clusters, which have no analogs in the hard-sphere model. N. Bohr emphasized that an important feature of such reactions is the free competition between all possible (i.e., compatible with the strict laws of conservation) decay or emission processes.

In this picture, which obviously is not limited to neutron reactions, but also includes reactions induced by charged particles, gamma rays, or heavy ions, there arises a "division of nuclear reactions into well separate stages to an extent which has no simple parallel in the mechanical behavior of atoms."[5] The first stage corresponds to the creation of a compound nucleus c in some ingoing channel a, while the second stage—the decay of the compound nucleus into channel b—is independent of the first stage, being determined only by the relative probabilities for the concentration of energy in different degrees of freedom. The autonomy of the stages of the reaction is equivalent to the independence of the decay of the compound nucleus from its method of excitation. This concept can be expressed mathematically by the Breit-Wigner formula, obtained[9] independently from Bohr's theory for the cross section of the reaction $a \to c \to b$, proceeding through an isolated resonance of the intermediate system:

$$\sigma_{ba} = \frac{\pi}{k^2} \frac{\Gamma_a \Gamma_b}{(E-E_c)^2 + (\Gamma^2/4)}; \qquad (1)$$

where k is the momentum of relative motion in the ingoing channel, E is the total energy of the system, E_c is the energy of the resonance in the compound nucleus, Γ_a and Γ_b are the partial widths in the corresponding channels, and Γ is the total width, determining the lifetime of the compound nucleus. The resonance corresponding to the quasistationary state c is isolated if its width Γ is much smaller than the average energy separation $D = 1/\rho$ ($\rho(E_c)$ is the density of states) between the states of the compound nucleus with a given angular momentum J and parity P. The inequality $\Gamma \ll D$ holds in the region of neutron resonances and right up to excitation energies of ~ 10 MeV.

The formal description with the help of the dispersion formula (1) gives a convenient parameterization of the resonance component of the energy dependence of the cross sections, but says nothing about the nature of the resonant state or about the structure of its wave function in terms of nucleonic variables. According to Bohr, the wave function of the intermediate state is very complicated. At excitation energies of the nucleus exceeding several MeV the density of states becomes so high that a description in terms of some simple modes (for example, motion of independent particles) is no longer adequate.

"On account of the rapid increase of the possibilities of combination of the proper frequencies"[5] of simple motions as the excitation energy is increased the separation between the levels is much smaller than the characteristic energies of the interaction mixing these simple modes. As a result, the true wave functions of the stationary (or quasistationary, taking into account the possibility of emission of particles) states Ψ_c contains an enormous number of simple components φ_i:

$$\Psi_c = \sum_i A_i^{(c)} \varphi_i. \qquad (2)$$

Because of the normalization of the state Ψ_c, the typical magnitudes of the contributions $|A_i^{(c)}|$ of simple excitations are small,

$$|A_i^{(c)}| \sim \frac{1}{\sqrt{N_c}}, \qquad (3)$$

where $N_c \gg 1$ is the number of components φ_i which make an appreciable contribution to Ψ_c. And conversely, the representation of the simple states φ_i in the basis Ψ_c of the true nuclear states is just as complicated. If the expansion of φ_i encompasses the characteristic energy interval $\Gamma_\downarrow$ (the so-called spread width or fragmentation region), then $N_c \sim \frac{\Gamma_\downarrow}{D} \gg 1$. It may be expected that the phases of the coefficients in the superposition (2) or in the inverse expansion are random. Then coherent combinations of squares of moduli $|A_i^{(c)}|^2$, where the smallness of each term is compensated by the number of terms, will make the dominant contribution to the observed probabilities of processes.

1.2. The statistical model

Such arguments naturally lead from an exact formulation of the quantum many-body problem to a statistical formulation. In the region of isolated resonances, each resonance can be interpreted as the representative of an ensemble of states of the compound nucleus. The randomness of the phases causes the nondiagonal matrix elements of the statistical operator (density matrix) to vanish. The ingoing channel of the reaction leading to the formation of the compound nucleus plays the role of initial conditions; the compound nucleus itself, however, whose properties are independent of its method of excitation, corresponds to thermodynamic equilibrium. The average characteristics of the ensemble of resonances must be close to the results of time averaging of the properties of a typical state of the ensemble (erogodicity).

It is now possible to use the standard statistical language. The logarithm of the density of states of the nucleus gives the entropy $S(E)$, the derivative $\partial S/\partial E = \frac{1}{T}$ determines the temperature T, $\partial E/\partial T$ corresponds to the heat capacity, etc. Of course, the number of degrees of freedom of the nucleus is not as large as in macroscopic systems. Some quantities, which are negligibly small in the thermodynamic limit of large systems, can therefore give appreciable corrections in nuclei, so that in many cases statistical calculations for nuclei must be carried out with a higher degree of accuracy. Nevertheless, as long as the excitation energy of the compound nucleus is much lower than the total binding energy, it is entirely possible "to liken many properties of nuclear matter to the properties of ordinary solid or liquid substances."[8]

In this "thermodynamic" approximation the density of states is $\rho(E) \sim \exp S(E)$, but more detailed models are required in order to establish the function $S(E)$ and the preexponential factors. A quite good representation of the density of states of a nucleus can be obtained from the simplest model of a Fermi gas,[9] for which $S(E) = 2\sqrt{\alpha E}$ (where α is a constant proportional to the density of single-particle states at the Fermi surface).

An alternative version of the statistical approach can be obtained, as N. Bohr proposed, by combining multiquantum states of vibrational modes in whose excitation many particles of the nucleus participate, as in the case of the phonon states of an elastic medium. Bohr noted that in the mathematical sense the problem reduces to the determination of the number of ways $\rho(n)$ that a large number n can be represented as a sum of positive integers (the Hardy-Ramanujan formula[3)]):

$$\rho(n) \approx (4\sqrt{3}n)^{-1} \exp\left(\pi \sqrt{\frac{2n}{3}}\right).$$

The basic function $\rho(E)$ turns out to be the same in the model of vibrational modes and in the Fermi gas model. The dependence of E on the temperature T, however, may be substantially different, since the function $E(T)$ is determined by the dispersion laws of those elementary excitations whose gas models the system.

The concept of a compound nucleus, interpreted in statistical terms, enables one to make an enormous number of specific pedictions about the cross sections of diverse nuclear reactions.[4)] We should mention here first the theory of decay of the compound nucleus as an evaporation of particles.[9–12] The principle of detailed balance relates the probability of emission of a particle (or a cluster) by the excited nucleus to the cross section of the inverse capture process. The inverse process depends on the density of states of the nucleus which appears as the residual nucleus in the direct evaporation process. The energy distribution $w(E)$ of the emitted neutral particles is similar to the Maxwell distribution and is determined primarily by the competition between two factors: an exponential factor $\approx e^{-\frac{E}{T}}$, arising from the growth of the many-body density of states of the compound nucleus, because of a lower energy concentration in the emitted particle is favorable, and a power-law factor, arising from the increase in the single-particle phase volume accessible to the emitted particle as E increases. For charged particles the energy dependence of the penetrability of the Coulomb barrier is also significant. N. Bohr emphasized that in nuclei, in contrast to macroscopic systems, evaporation of particles can be accompanied by a substantial change in the thermal energy, so that the temperature determining the spectrum of the emitted particles is precisely the temperature of the residual nucleus.

1.3. The optical model

In N. Bohr's early works and speeches on the physics of the compound nucleus, this idea was formulated in a somewhat more categorical form than in other works where Bohr put forth fundamentally new physical ideas. We have already cited the paper by Bohr and Kalckar, where the necessity of the intermediate state for all nuclear reactions is described. This is easily explainable historically. Not long before this the efforts of a large number of independent experimenters showed that the simple picture of purely potential scattering of a slow neutron by a nucleus is inapplicable. New ideas were required in order to explain the existence of a sharp energy dependence with an average separation between resonances $D \approx 10^{-(5\div6)} D_{\rm s.p}$, where $D_{\rm s.p} \sim \frac{\hbar v}{R} \sim \frac{\hbar}{\tau_{\rm s.p}}$ is the energy separation (~ 1 MeV) between single-particle resonances in the scattering of a particle by an external potential well and v is the velocity inside the well. The picture of an intermediate nucleus explains in a natural manner both the existence of resonances and the large neutron-capture cross section (for low excitation energies the probability of radiative deexcitation of a long-lived compound nucleus is large, $\Gamma_\gamma \gtrsim \Gamma_n$).

The situation becomes much more complicated as the excitation energy increases. The separation between the levels decreases, and the total width of each level increases because of the rapid increase in the number of open channels for inelastic reactions. Thus there arises an inverse relation of the parameters $\Gamma \gg D$. This situation was discussed in a later paper by N. Bohr, R. Peierls, and G. Placzek.[13] Since separate resonances of the intermediate nucleus now overlap and the distinct resonant structure vanishes, the result of a specific experiment will depend on the detailed phase relations between the components of the particular superposition of overlapping states that is actually excited. Then the basic proposition of the statistical theory—the absence of a correlation between the formation and decay of the intermediate system—can be invalidated. The authors present a simple example of when the spatial structure of the state wholly linked to the ingoing channel and preferring a definite decay path arises: "If a fast particle collides with a system of comparatively large dimensions, then the excitation energy may turn out to be localized in a small region around the collision point and the emission of fast particles from this region could be more probable than in the case of statistical equilibrium." Experiments in many cases demonstrate the existence of "direct" processes, which do not proceed through a compound nucleus, as well as processes of intermediate types.

The necessary conditions for the applicability of the purely statistical approach can be formulated in the time-dependent language.[14] The average separation D between the levels corresponds to the best period $\tau_r \sim \hbar/D$, which can be interpreted as the quantum analog of the return time in quasiperiodic classical motion. It is precisely over this time that the ergodic equal probability of populating separate cells of phase space lying on a surface with fixed values of the exact integrals of motion is achieved. For $\Gamma \ll D$ the typical lifetime of the compound nucleus $\tau \sim \hbar/\Gamma$ is much longer than τ_r, i.e., real averaging over the phase space occurs over the lifetime, which corresponds to statistical equilibrium.

In the region of overlapping levels $\tau < \tau_r$, and there is not enough time for complete equilibrium to be established. Nevertheless the assumption that the decay of the compound nucleus is independent of its method of excitation could still be valid. A sufficient condition for this could be the separation of the processes of creation and decay of the compound nucleus in time, i.e., the condition that the lifetime τ must at least be longer than the time Δt of the interaction of the incident wave packet with the nucleus. If $(\tau, \tau_r) \gg \Delta t$, then the wave reflected during potential scattering does not interfere with the waves of particles in decay channels (the characteristic time for their appearance is τ) or with the wave elastically scattered through the compound nucleus (it appears after the return period τ_r).

It may therefore be expected that the ingoing and outgoing channels are independent if the uncertainty of the energy $\Delta E \sim \hbar/\Delta t$, associated with the duration Δt of the incident beam, extends the quantities $\Gamma \sim \hbar/\tau$ and $D \sim \hbar/\tau_r$. Then the processes which proceed via the compound nucleus will be noncoherent relative to the direct interaction processes. The quantity ΔE can be introduced artificially by averaging the experimental cross sections over a small (compared with the energy interval where the average value of the cross sections vary substantially) interval including many levels of the compound nucleus. Such an averaged description corresponds to the optical model of nuclear reactions,[15,16] where particle scattering is described with the help of an empirical complex potential whose imaginary part given a global description of the absorption of the incident wave, i.e., those processes which because of the formation of the intermediate nucleus are noncoherent relative to the ingoing channel (the aggregate of these processes is more extensive than the previously used concept of the compound nucleus, because here it is not assumed that total equilibrium is established).

The optical model is still almost the only tool for describing nuclear reactions in a wide range of energies. In the spirit of Bohr's principle of complementarity, the rejection of a detailed study of the rapidly varying energy behavior of the cross sections reveals another—essentially single-particle—aspect of the interaction of a particle with a nucleus. The average cross sections reflect not the statistical, but rather the purly dynamic (dynamics of averaged quantities) aspects of the scattering by a complex potential well. Here the characteristic energy dependence with typical energy intervals $\Delta E \gg D$, single-particle resonances associated with the dimensions of the well, transition to diffraction scattering ($\sigma_{tot} \to 2\pi(R + \lambda\!\!\!^-)^2$ with increasing energy, and so on, which are well known from elementary quantum mechanics, reappear.

1.4. Statistical spectroscopy. Are the levels of the compound nucleus randomly distributed?

Not only are the average "optical-model" quantities of great interest, but so are the fluctuation deviations from them. In the region of isolated resonances ($\Gamma \ll D$) it is a question of the statistics of nuclear levels and the corresponding wave functions,[17,18] manifested in terms of the squares of the modulus of some of their components (neutron and radiation widths). If, according to the concept of the compound nucleus, the wave functions of the true stationary or quasistationary states have a very complicated structure, consisting of a million incoherent components (2), in the simple (for example, shell) basis, then, as already mentioned, the statistical properties of this ensemble of levels can be studied. This rapidly growing area of theoretical study and computational modeling is called "statistical spectroscopy," encompassing, in addition to nuclei, the spectra of atoms, molecules, and model quantum systems.

Statistical spectroscopy arose precisely in the study of neutron resonances, where sequences of tens and hundreds of levels with fixed values of the integrals of motion J^P can be distinguished. The statistical properties of quite large pieces of such spectra turn out to be similar in different nuclei. With good accuracy the distribution $w(\varepsilon)$ of the energy separations ε between successive levels in the spectrum with average separation D is given by Wigner's function[19]

$$w(\varepsilon) = \frac{\pi}{2}\,\frac{\varepsilon}{D^2}\,e^{-\pi\varepsilon^2/4D^2}. \tag{4}$$

The vanishing of $w(0)$ expresses the "repulsion" of levels with the same symmetry, first discussed in Ref. 20 and observed in the analysis of real spectra in Ref. 21. The distribution (4) coincides with Rayleigh's distribution of the lengths of two-dimensional vectors whose components are independent Gaussian random variables. The vanishing of the length of a vector requires that both its components vanish simultaneously, which can happen only at one point in the plane (a set of measure zero).

In terms of the matrix elements of a Hamiltonian H for a two-level system, the square ε^2 of the separation between the eigenvalues is equal to

$$\varepsilon^2 = (H_{11} - H_{22})^2 + 4|H_{12}|^2. \quad (5)$$

In a system which is invariant under time reversal, the phases of the basis states can be chosen so that the nondiagonal matrix elements H_{12} and $H_{21} = H^*_{12}$ of a hermitian Hamiltonian H will be real and, therefore, equal to one another $H_{12} = H_{21}$. If the remaining variables $\xi = H_{12}$ and $\eta = (H_{11} - H_{22})/2$ are regarded as random normally distributed quantities, then we arrive at the Rayleigh-Wigner distribution (4) for the quantity ε. Thus the formula (4) actually expresses definite properties of a random Hamiltonian (*uncorrelated nature and Gaussian nature of the matrix elements, hermitian nature and* T *invariance*). This indicates, in some sense, a further step along the path to a limiting statistical description of the system—averaging not only over macroscopic states, but also over the unknown Hamiltonians themselves, which control the exact microscopic dynamics, but, in the presence of a very dense grid of levels, reveal only some of their general characteristics.

The prescription of global symmetry properties, invariant under transformations of the basis, determines the "canonical" ensembles of random matrices,[22] playing (in the sense of the minimum degree of input information), the same role as the Gibbs ensemble in ordinary statistical mechanics. Thus for T-invariant systems there arises a Gaussian orthogonal ensemble, which is the simplest (two-dimensional) case gives the distribution (4). For large segments of the spectrum the distribution function of the eigenvalues[22] has an exponential form $e^{-\mathrm{const}\cdot\mathrm{Tr}(H^2)}$, multiplied by a polynomial that vanishes when any pair of eigenvalues coincides; the distribution of the separations of neighboring levels in this case is very close to the result (4) for two-dimensional matrices, so that their difference falls outside the limits of error in the analysis of the experimental data.

If T invariance of the Hamiltonian breaks down, then the nondiagonal matrix elements $\xi = H_{12} = H^*_{21}$ are complex, so that the vanishing of ε in the equality (5) requires that three random variables (η, Re ξ and Im ξ) vanish. Such random matrices are described by a Gaussian unitary ensemble, for which the repulsion of levels is stronger, $w(\varepsilon) \sim \varepsilon^2$ in the limit $\varepsilon \to 0$. Here there arises an exceptional situation in which a particularly statistical effect, such as the ensemble-averaged correlation of the positions of close levels, can be used to search for an answer to the question of the existence of a component in nuclear dynamics that violates one of the fundamental symmetries of nature—time reversibility. Based on the available exerimental data on nuclear levels, no reliable indications of the absence of T invariance have been found.

Wigner's function (4) describes well the distribution of the smallest separations between levels for both neutron and proton resonances in cases when states of the compound nucleus with definite quantum numbers J^P are selected with an adequate degree of reliability. Typical atomic spectra (though here the quality of the comparison is not as good) and spectra found in the diagonalization of the shell Hamiltonian with residual forces exhibit analogous properties.

The Poisson distribution

$$w(\varepsilon) = \frac{1}{D} e^{-\varepsilon/D}, \quad (6)$$

where the probability density for observing a neighboring level with small separations is a constant $w(0) = 1/D$ and the maximum of the spectrum occurs precisely at small separations, is in some sense the opposite case.[5)] Here the levels on the energy scale create a random sequence of events, similar to the distribution of acts of radioactive decay as a function of time. It is possible to observe how the total distribution, when Wigner sequences of levels for different J^P are superposed on one another,[18] approaches *the Poisson distribution*, since the rotational invariance of the Hamiltonian and parity conservation strongly forbid matrix elements H_{12} between terms belonging to different sequences. Analogous results can also be obtained from an analysis of even the first excited states of nuclei, i.e., in the analysis of the "transverse" (to the energy scale) section of the ensemble of levels of different nuclei (another manifestation of ergodic properties). Here, however, distributions of the Wigner type for fixed values of J^P and of the Poisson type for levels with different values of the exact integrals of motion, arise only after the exclusion of systematic collective effects, which regularize the properties of the low-lying states.

Profound physical reasons for the resulting nature of the spectrum have apparently not yet been discovered. The interrelationship of these phenomena with the currently intensively studied dynamic chaos,[24,25] which can also arise in classical systems with a very small number of degrees of freedom manifesting itself as ergodic behavior of trajectories in definite regions of phase space and their instability relative to a small variation of the initial conditions, is only now beginning to be understood. Increasingly more examples of stochastic motion in simple quantum systems, where the quantum uncertainty relations and the concomitant spreading of wave packets can change the nature of the evolution at long times, have appeared in recent years.[26]

The initial stage of the theoretical study and numerical modeling provided arguments supporting the division (at least in the quasiclassical limit) of quantum systems into "regular" and "irregular" depending on whether their classical analog is characterized by quasiperiodic or ergodic behavior. Regular systems, in particular, include integrable systems with more than one degree of freedom. In the corresponding quantum systems, the separations between the levels are not correlated and the distribution must be the Poisson distribution (6). In contrast to this, systems which are chaotic in the classical limit manifest repulsion of levels and a nearly Wigner (4) distribution function of separations. A simple example is "Sinai's billiards" (see Ref. 27); the corresponding quantum problem was studied in Ref. 28 for the example of the solution of a free Schroedinger equation in a two-dimensional triangular region, from which a small section was cut out. At some energy, which decreases as the magnitude of the distortion of the region increases, the eigenvalue spectrum can be described by a Gaussian orthogonal ensemble with a characteristic "rigidity" (small and spectrally uniform fluctuations of the distribution of levels). The low-lying part of the spectrum, however, stores the memory about the starting triangular symmetry with its inherent degeneracy and can be calculated (for a small distortion of the form) by perturbation theory.

It is precisely this situation that appears to be a plausible model of nuclear spectra. The shell structure of the low-lying states arises[29] as the quantum analogy of quasiperiodic trajectories of wave packets in a field of definite symmetry. The configurations of nucleons, belonging to one shell, correspond[30] primarily to substantially different spatial-spin structures, while the selection rules for the principal parts of the residual interaction preferentially couple widely separated energy states. In this region of "regular" wave functions, quite large blocks of levels will be almost Poisson-like. In the region of "irregular" wave functions, on the other hand, the nondiagonal (in the shell basis) matrix elements of the interaction, creating, in agreement with Bohr's initial picture, stationary states with a close spatial-spin structure, will play the main role. Each of these states actually covers the entire classically accessible part of the phase space with a fixed energy, and an equal-probability population of phase cells, i.e., a microcanonical equilibrium ensemble, appears (after coarse-grained roughening) with the transition to the quasiclassical situation. In model quantum problems with a chaotic classical analog it is possible to follow directly[30] the transition of the quantum distribution function in phase space (which also is named after Wigner[31]) into the classical microcanonical distribution in the limit $\hbar \to 0$. In the quantum case, a distribution of levels close to the Wigner distribution (4) should be expected here.

The ensemble of random matrices determines the statistical properties not only of the eigenvalues, but also the eigenvectors (2). The observation of the realization of a separate channel i for the decay of a compound nucleus c implements the analysis of these complex superpositions, fixing the square of the modulus $|A_i^{(c)}|^2$ of the component linked with this channel. In a Gaussian orthogonal ensemble, the amplitudes $A_i^{(c)}$ in the limit $N_c \gg 1$ are described by the normal distribution with zero mean and with the variance (3)

$$\overline{|A_i^{(c)}|^2} = 1/N_c.$$

If only one channel i is open, then the partial decay width Γ_i coincides with the total width Γ of the compound nucleus and, since $\Gamma_i \sim |A_i^{(c)}|^2$, a Porter-Thomas distribution[32] is obtained for the distribution of widths:

$$w_1(\Gamma) = \frac{1}{\sqrt{2\pi\overline{\Gamma}\Gamma}} e^{-\Gamma/2\overline{\Gamma}}. \quad (7)$$

In a situation when several channels in which decays are uncorrelated are recorded, analogous arguments give for the total width $\Gamma = \Sigma_i \Gamma_i$ a χ-squared distribution with the number ν of degrees of freedom equal to the number of channels:

$$w_\nu(\Gamma) \sim \Gamma^{(\nu/2)-1} e^{-\nu\Gamma/2\overline{\Gamma}}. \quad (8)$$

The variance of this distribution (fluctuation of the widths) makes it possible to extract the number of open channels:

$$\nu = \frac{2\overline{\Gamma}^2}{\overline{\Gamma^2} - \overline{\Gamma}^2}. \quad (9)$$

For barrier reactions in which the widths are sharply energy dependent, the last formula determines some effective number of channels.[18] Important information has been obtained by this method in numerous experiments about fission channels (Sec. 3).

The search for statistical characteristics carrying information on the real nuclear Hamiltonian and differences from the limit picture of random matrices remains an extremely interesting and promising direction for further research.[18,25,33]

1.5. Nuclear kinetics

In many cases the high density of the compound-nuclear levels is a source of the so-called dynamic enhancement of effects which under ordinary conditions are small corrections.[34] This happens, in particular, in the mixing of nuclear levels with opposite parity when the weak interaction is taken into account.[35–37] If the weak interactions of nucleons induce the correction

$$U_w \sim \sigma P$$

to the potential of the mean nuclear field, then the single-particle orbitals will acquire an admixture of the order of

$$\alpha_{s.p} \sim \overline{U_w}/\omega_0,$$

where $\overline{U_w}$ is a typical matrix element of the weak interaction between nucleon states with different parity, belonging to neighboring shells separated in energy by ω_0.

In the compound nucleus the complex states which mix have an energy difference $\Delta E \sim D$, and the wave functions, neglecting the weak interaction, are represented by noncoherent superpositions (2), where φ_i are Slater's shell determinants. The single-particle operator U_w transfers in each determinant φ_i one particle to the nearest empty subshell of the other parity, removing in the final state the corresponding determinant φ_j. The amplitudes of these determinants in the functions of the compound-nuclear state are $1/\sqrt{N}$ and $(\Gamma_\downarrow/\omega_0\sqrt{N})$, because the fragmentation extends over an energy interval $\Gamma_\downarrow < \omega_0$ and the contributions of distant components to the superposition are suppressed in the ratio $\Gamma_\downarrow/\omega_0$. For low excitation energies the number of particles whose transitions are allowed is $\sim A^{2/3}$ so that the mean-square matrix element of the weak interaction is given by the nature of a noncoherent sum of $A^{2/3}N$ terms, the order of magnitude of each of which is $\overline{U_w}\frac{\Gamma_\downarrow}{\omega_0}\frac{1}{\sqrt{N}}\frac{1}{\sqrt{N}}$. Taking into account the fact that $D \sim \Gamma_\downarrow/N$ we find the mixing amplitude of close compound nuclear states:

$$\alpha_{cc'} \sim \frac{(H_w)_{cc'}}{D} \sim \frac{1}{D} A^{1/3}\sqrt{N}\,\overline{U}_w \frac{\Gamma_\downarrow}{\omega_0}\frac{1}{N} \sim \frac{\overline{U_w}}{w_0} A^{1/3}\sqrt{N}. \quad (10)$$

Thus the enhancement factor over mixing of single-particle states is given by

$$\frac{\alpha_{cc'}}{\alpha_{s.p}} \sim A^{1/3}\sqrt{N}. \qquad (11)$$

We now return to a discussion of the region of higher excitation energies, where the lifetime of the states is not high enough to ensure the establishment of total equilibrium ($\Gamma \sim D$ or $\Gamma > D$), so that the levels of the excited system overlap and the result of a process could depend on the detailed phase relations between the components of the initial state.[13] In reality this situation is more the rule than the exception,[38] since it is realized already at excitation energies of several MeV above the threshold of particle emission. If the contributions of a large number of overlapping levels to the amplitude of the reaction $f(E)$ at a fixed energy E are statistically independent, then the fluctuations of the cross sections $\sigma(E) \sim |f(E)|^2$ will be of the same order of magnitude as the reactions themselves. A well-known classical analog are the Rayleigh fluctuations of the intensity I of a field created by a large number N of identical sources with completely uncorrelated phases, $\overline{(\Delta I)^2}/\overline{I}^2 \sim 1$ when $N \gg 1$. The existence of such sharp fluctuation energy dependence was predicted theoretically ("Erikson fluctuations") and then observed in a large number of experimental studies. The correlations between the cross sections at different energies are determined by the diffuseness of the energies of separate intermediate states, i.e., the characteristic values of Γ or the times $\tau \sim \hbar/\Gamma$. Taking into account only the compound-nuclear processes that are linked to the most long-lived nuclear states is an extreme simplification. Potential scattering exists, distortion of the incident wave by absorption in the compound-nucleus state necessarily leads to elastic scattering of the shadow (diffraction) type, direct fast (quasielastic) nucleon or cluster knock-out processes occur, etc. These processes are not separated appreciably in time from the initial state and, as emphasized in Ref. 39, their identification is related to the problem of separating a signal from noise. The direct and compound-nucleus reactions are evidently only limiting cases of the actual situation, when the experimental picture of the cross sections and of the energy and angular distributions is constructed by superposing the results of many processes occurring with different time durations[40,41] and by mechanisms differing in complexity.

Here we actually encounter a new area—nuclear kinetics, which is just beginning to be developed. The problem posed here is the analysis of the course of a nuclear reaction as a function of time and the establishment of the hierarchy of increasingly more complex states through which the process proceeds, and of the relationship between these states and the corresponding scales of the fluctuations in the energy spectra and the characteristic features in the angular distributions.[42,43] H. Feshbach introduced the concept of doorway states as the first stage of the initial process of relaxation of a strongly nonequilibrium, initially excited nucleus. These are most likely states of a comparatively simple nature (particle-hole states or their superposition of the giant-resonance type). These states are not steady states. Further relaxation leads to both mixing of the excitation among related states and to more complicated states. The entropy increases, and the evolution proceeds toward an equilibrium distribution of excitation energy among the degrees of freedom. At each stage particles carrying information on the duration of the corresponding stage can be emitted. Thus the reaction products could include direct, pre-equilibrium, and equilibrium (evaporative) products[44]; a properly set up experiment can separate them.

Several approaches to describing the picture of pre-equilibrium processes have now been developed.[45–47] Under definite assumptions about the random nature of the matrix elements describing the transitions, a phenomenological kinetic scheme of successive evolution of the system between classes of internal states is constructed. Existing theories have two basic drawbacks:

1) the absence of a direct relationship with the microscopic structure of the given nucleus and a specific nuclear Hamiltonian and

2) inadequate justification of plausible hypotheses about randomness.

Specific calculations can for the time being be performed only in cascade[48] or exciton[49] models. Cascade calculations model the process of pair nucleon collisions, but the applicability of this gas approximation remains doubtful, at least in the region of not too high excitation energies.

In exciton models the internal states are classified according to the number of particle-hole excitations. The validity of the hypothesis of fast relaxation within each such class as compared with transitions into more complicated configurations remains questionable. In excitations of the resonance type or deep hole states the reverse hypothesis appears to be more plausible. Of course, the difficulties mentioned above reflect the general problems involved in the development of a kinetic theory of quantum systems with strong interaction.

These difficulties are aggravated by the particular nature of the nucleus as a finite Fermi system, where the discreteness of the spectrum, the absence of additive integrals of motion of the momentum type, and coupling with the continuum must be taken into account. The development of a microscopic approach in nuclear kinetics remains a problem for the future.

We restricted ourselves here to the range of problems directly related to N. Bohr's hypothesis about the compound nucleus. The entire problem of statistical regularities in a small quantum system such as the atomic nucleus appears most vividly in the physics of heavy ions, whose achievements in recent years are impressive. We mention only the discovery of deep inelastic collisions of heavy ions,[50] in which a comparatively long-lived double nuclear system, which relaxes to equilibrium but decays before reaching it, is formed. The angular distribution of the products actually shows the unfolding of the relaxation process, providing thereby a unique possibility for following the behavior of different nuclear characteristics as a function of time.

One of the most interesting results here is the indication of the important role of coherent collective phenomena, which are usually ignored in the statistical analysis.

Finally, collisions of relativistic heavy ions, to which the considerations regarding the compound nucleus are fully applicable, open up a new region in the phase diagram of nuclear matter, making possible a search for anomalous phases—superdense matter, the mesonic condensate, and the quark-gluon plasma.

2. EXCITATION IN NUCLEI: INDEPENDENT PARTICLES OR COLLECTIVE MOTIONS

> *"While most people tend to notice the difference between similar things, it was natural for him (Bohr) to see what was common to apparently different ones."*
>
> *O. Klein (Ref. 179, p. 75)*

2.1. Macroscopic collective modes

In an article entitled "The Unity of Knowledge"[51] N. Bohr wrote: "...no experimental fact can be formulated apart from some system of concepts," and further: "...every apparent disharmony between experimental facts can be eliminated only by approxpriately expanding this system of concepts." Together with the concept of the compound nucleus N. Bohr introduced into nuclear physics one more key concept—collective motion. It is precisely the picture of the collective excitation of a system of many bodies, customarily used in the analysis of macroscopic systems, that he juxtaposed with the concept of a nucleus as a collection of almost independent nucleons.

N. Bohr did not yet have at his disposal adequate spectroscopic information to speak with certainty about specific types of collective motion in nuclei. However, he immediately called attention[4] to the dominant role of the electric quadrupole component in nuclear γ emission. Dipolar emission of long-wavelength quanta by a nucleus is necessarily linked to the motion of protons relative to neutrons, i.e., it cannot be completely coherent. There is no such restriction for quadrupole radiation. N. Bohr also indicated[8] classical situations where quadrupole radiation or even emission of γ rays in general can be suppressed (radial pulsations of a uniform body or its rotation around an axis of symmetry).

Starting from the comparison with condensed media which forms the foundation of the idea of the intermediate nucleus, N. Bohr and F. Kalckar gave[8] the first estimates of "macroscopic" collective modes which are possible in a nucleus. Two considerations make these estimates almost unique. First, according to the uncertainty relation the dimensions of a nucleus fix the average kinetic energy $\bar{\varepsilon}$ of the particles. Second, the intensive properties of most nuclei, except the lightest ones, are roughly universal. Then their elastic characteristics must also be approximately constants. Using macroscopic analogs, N. Bohr and F. Kalckar showed that elastic oscillations with an energy quantum varying smoothly as a function of the mass number $\hbar\omega \sim \bar{\varepsilon}A^{-1/3}$, if the oscillations are of a volume character, or $\hbar\omega \sim \bar{\varepsilon}A^{-1/2}$ for surface oscillations should be expected in a nucleus. In these two cases the volume elastic force and the surface tension act as the return force.

Such oscillations of the liquid-drop type correspond to energy quanta > 2 MeV, even in the heaviest nuclei. For this reason, N. Bohr and F. Kalckar further note that the lowest excited states of nuclei must have a different nature. They note the evidence for the existence of other types of excitations in nuclei. They include here, first of all, the periodic changes in the binding energy of nuclei along the periodic chart of the elements and the much higher energies of the first excited states in even-even nuclei than in nuclei which are odd with respect to one or both types of nucleons. The question of the coupling of the orbitals and spin moments of separate particles, which could lead to an analog of the fine structure in nuclear spectra, are discussed separately. Finally, they hypothesize that collective superposition of orbital moments, giving a picture similar to that of a rotating rigid body, is possible. The corresponding energy intervals ΔE_r between the lower rotational states must be inversely proportional to the moment of inertia $\mathcal{Y}$, i.e., according to the rigid-body estimate they are small, $\Delta E_r \sim \mathcal{Y}^{-1} \sim A^{-5/3}$. To be sure, the authors do point out the contradictions appearing between such "quasirigid" rotation and the liquid-drop properties of the nucleus.

Today, almost half of a century after N. Bohr and F. Kalckar published their study, we clearly see that it incorporates, just like an embryo, the program of future experimental and theoretical studies of nuclear excitations; in addition, they anticipated precisely those pivotal points out of which grew the most important results which now comprise the generally accepted ideas about nuclear structure.[17,52]

2.2. Can single-particle degrees of freedom be manifested?

An enormous amount of experimental information, including the periodic change in the nuclear properties noted by N. Bohr and F. Kalckar, points to the fact that in spite of the strong internucleon interaction, single-particle motion remains in nuclei as an independent type of excitation. Not being an exact stationary state, this motion decays with time, which process is described in the optical model in a gross manner with the help of the complex potential. The sharp change in the density of states with decreasing excitation energy should increase in a natural manner the lifetime of single-particle states. In the limit there appears a picture of a simple motion of particles in a real potential well, where the lowest single-particle levels are filled in accordance with the Pauli principle, thereby determining the configuration of the ground state of the nucleus. The simplest excited states are then described by some number of particle-hole pairs.

As mentioned in Sec. 1, there exist[29] simple quasiclassical reasons for the clustering of energy eigenvalues in shells, as a result of which the single-particle density of states is not uniform, but rather is modulated with the frequency ω_0 (the typical separation between shells, which is determined by the conditions of reflection of stable wave packets at the boundary of the nuceus, i.e., by the geometry of the potential, and therefore coincides with Bohr's estimate $\bar{\varepsilon}A^{-1/3}$ for volume elastic waves). In the hierarchy of increasingly more complicated states, the single-particle motion corresponds to maximum correlation widths $\Gamma_{s.p} \sim \hbar\omega_0 \sim \hbar/\tau_{s.p}$, where

$\tau_{s.p} \sim R/v \sim \hbar A^{1/3}\bar{\varepsilon}$ is the transit time of a particle through the nucleus.

The problem that reasonable shell scheme must solve is not only to provide a qualitative explanation of the periodicity of nuclear properties but also, primarily, to predict the sequence of energies and quantum numbers of the single-particle orbitals and thereby of the magic numbers also. The first variants of the shell model gave the correct magic numbers only for the lowest shells. This circumstance, together with the unknown degree of substantiation of the naive picture of independent strongly interacting particles, gave rise to distrust of all models of this type. This distrust was also clearly stated by N. Bohr, who suggested[5] as a counterbalance the idea of entangled collective motion in the compound nucleus. It was nevertheless possible to find[53] a simple modification of the shell model, in which with the help of the introduction of spin-orbital splitting of nucleonic orbitals with respect to the total angular momentum $j = l + s$, added to the central potential, all magic numbers could be correctly determined and the spins of the ground states of almost all nuclei could be predicted correctly. Further developments led to an internal synthesis of the ideas of single-particle and collective motions in nuclei.

The recipes of the simple model of spherical shells can be completely unique only for magic and neighboring nuclei, having one extra particle or hole. Filled shells necessarily have the quantum numbers $J^P = 0^+$ while the spin and parity of nuclei adjacent to magic nuclei must coincide with the angular momentum j and parity of the closest orbital to which the last nucleon (or hole) belongs. For other nuclei the shell model determines only the basic configuration of the valence nuleons, but requires additional hypotheses similar to Hund's rules for atoms in order to provide an answer to the question mentioned by N. Bohr regarding the scheme for adding angular momenta in partially filled orbitals. The empirical fact that all ground states of even-even nuclei are characterized by the quantum numbers 0^+ suggests the hypothesis that nucleons in the degenerate orbitals (j,m) and $(j, -m)$, which transform into one another under time reversal, are paired. The energy advantageousness of pairing clearly follows from the mass difference of even and odd nuclei, which is added to shell variations, and from the difference in the densities of low-lying states (as mentioned above, this was also pointed out by N. Bohr and F. Kalckar[8]). This uniquely identifies the spin J_0 of the ground state of an odd nucleus with the angular momentum j of the orbit of the unpaired nucleon.

The shell model with spin-orbital coupling and a pairing rule is, in one or another modification, the basis of practically all calculations of the structure of complex nuclei. On this level, however, we can talk about only the single-particle characteristics: spins and parities of the ground states and excitations of particle-hole type, static multipole moments of the states with one unpaired nucleon, probability of β decay, electromagnetic single-particle transitions and reactions of the direct knock-out type, and stripping or capture of nucleons. A big qualitative success of the shell model was the explanation of islands of isomery—groups of nuclei, in which the level with high angular momentum $j = l + (1/2)$, dropping into a lower shell because of spin-orbital splitting, turns out to be located in an environment of levels with different parity and appreciably different values of j, which strongly suppresses γ transitions between them and gives rise to the appearance of isomeric states.

On the whole there are no doubts about the reality of nucleon shells. Numerical predictions, however, are in agreement with experiment in the best case in order of magnitude and with respect to some qualitative trends. The magnetic moments of odd nuclei systematically deviate inwards from simple shell predictions (Shmidt lines[17]) by an amount of the order of one nuclear moment, though it would appear that they are determined completely by one unpaired nucleon. The situation is even worse in the case of the electric quadrupole moments, which as a shell is filled begin to exceed appreciably the shell model predictions and, moreover, are of the same order of magnitude both for proton-odd and neutron-odd nuclei. There exist regions of nuclei (for example, rare-earth and subsequent nuclei with $A = 150$–180 and heavy nuclei with $A > 220$) where the shell model cannot explain even the spins of the ground states.

2.3. On the path to a generalized model

The accumulation of experimental information on a large collection of nuclei made possible a new step to be taken in the early 1950s in the understanding of nuclear structure, associated with the idea of nuclear deformation proposed by J. Rainwater, A. Bohr, and B. Mottelson.[54–56] It is clear that the independent-particle model, where the interaction of nucleons is entirely reduced to the mean potential field, can seve only as a first approximation to a more accurate description. "Residual" forces, giving rise to correlations between nucleons, must obviously exist. One type of such correlations—pairing correlations—has already been mentioned. When residual interactions are seriously taken into account, mixing of shell configurations arises. Since the residual forces are not known beforehand, physical considerations which distinguish the basic components of these forces are important.

Assume that the forces induce a maximum overlapping of the nucleonic wave functions. The addition of a nucleon above the magic core into a definite level with angular momentum j and projection $j_z = m$ fixes in the quasiclassical approximation the "orbital plane." Then, from the viewpoint of correlations of nucleons, it is advantageous to distort somewhat the spherical core, deforming it in accordance with the orbit of the outer nucleon. In terms of quantum perturbation theory, this "coupling of nucleons with the surface" indicates a polarization of the core—an admixture to the closed configuration of virtual excitations of the particle-hole type with the "correct" spatial orientation. Of course, the distortion by one outer nucleon of the mean field created by all particles is small; but this smallness can to a large extent be compensated by coherent contributions to the polarization of the core from a large number of inner nucleons. As a result, the multipole moments of an odd nucleus (primarily the electric quadrupole moment) acquire a correction determined by the residual nuclear forces, whose exact magnitude depends on the polariability of the core by a field of given symmetry. We note that the polarizability of a nucleus was apparently also first mentioned by N. Bohr in Ref. 57 in 1938.

In magic nuclei the quadrupole polarizability of a nucleus is small, since because of the conservation of parity quadrupole transitions with the creation of a particle-hole pair require a large energy of $\sim 2\hbar\omega_0$. As the number of valence nucleons increases, the situation changes: their efforts to polarize the core add coherently and additional low-energy virtual transitions of the valence nucleons increases, the situation changes: their efforts to polarize the core add coherently and additional low-energy virtual transitions of the valence nucleons themselves are added; it may be expected that the system becomes increasingly "softer." At the same time the fundamental difference between odd and even-even nuclei is lost: in the second case the virtual breaking of a spherical pair of nucleons can play the role of a bare field. Such an excitation induces a polarization of the core, and if the potential energy of deformation increases at the same time, then self-maintaining oscillations around the spherical shape arise.

Indeed, in almost all even-even nuclei the first excited state has the quantum numbers 2^+. The energy $E(2^+)$, which is naturally compared with the frequency of the vibrational quantum, drops rapidly as the shell is filled. In magic and near-magic nuclei the value of $E((2^+)$ approximately agrees with the frequency of quadrupole surface oscillations of a charged liquid drop. Away from a magic nucleus, however, hydrodynamic estimates overestimate the value of $E(2^+)$ by a factor of 4–5. The probabilities of $E2$-transitions between the ground and single-quantum states grow at the same time that the quadrupole oscillatory mode softens. In typical spherical nuclei these probabilities are several tens of times greater than the characteristic probabilities of single-particle transitions, which clearly demonstrates the collective nature of the motion.

2.4. Phonons in nuclei

The vibrational picture of quadrupole excitations is confirmed by the presence of a series of states, similar to vibrational bands in molecules, in the low-lying region of the spectrum. In many spherical nuclei triplets close to the states $4^+, 2^+, 0^+$, which can be interpreted as two-quantum quadrupole levels, have been found near the energy $2E(2^+)$. In the ideal case of noninteracting quanta (the harmonic approximation[58]) the quadrupole moment operator, inducing $E2$ transitions, is proportional to the coordinate of the quadrupole oscillator. For this reason, only transitions with a change of the quantum numbers n by $\Delta n = \pm 1$ are allowed; in addition, for all levels of the two-quantum triplet, the reduced probability of a transition into the one-phonon state must be two times greater than the probability of a transition from the one-phonon state into the ground state.

There also exist states to which a multiquantum structure can be assigned. In particular, a great deal of information has been collected on the so-called yrast bands, unifying the lowest energy levels of a nucleus for different values of the total angular momentum I. Yrast bands with even $I = 2n$ and positive parity, having, probably, an n-quantum nature (these are the terms of the n-quantum multiplet where the angular momenta of the quanta are aligned in the maximum possible value $2n$), can be followed in spherical nuclei. In the harmonic situation intense $E2$ transitions with the typical "laser" stimulation factor $\sqrt{n}$ in the amplitude of the transition $n \to n-1$, should occur along the yrast band.[52]

The phenomenological theory of harmonic multiple oscillations was first constructed[58] on the basis of the quantization of small deviations from equilibrium in a drop of ideal fluid. In reality, however, the basic features of the phenomenological description depend only on the assumed symmetry of the excitations and remain in force irrespective of the liquid-drop model. The experimental picture of the low-lying collective states, on the whole, fits into the scheme of quadrupole oscillations.[52] A strong argument in favor of this scheme is the intense collectivization of quadrupole transitions with $|\Delta n| = 1$ and the weakness of those transitions which are absent in the harmonic approximation ($|\Delta n| = 2$ and $\Delta n = 0$). The harmonic approximation is still clearly inadequate. The energy intervals and the probabilities of the electromagnetic transitions deviate substantially from the simple predictions of the harmonic model, the multiplets are strongly split with respect to the total angular momentum, and nonzero average values of the quadrupole moment in the first excited state 2^+, strictly forbidden in the harmonic approximation (the analog of vanishing of the average coordinate of an oscillator in the state with a definite number of quanta), have been observed. The quadrupole vibrational motion is strongly anharmonic.[59]

This is understandable, since because of the smallness of the frequencies ω in soft spherical nuclei the amplitude of the quadrupole deviations from the equilibrium shape is too large for the theory of small oscillations to remain valid. We are dealing with a slow collective motion on large scales, to which the single-particle degrees of freedom adjust adiabatically. Thus for an appreciable fraction of the time nucleons move in a deformed mean field, and the concept of the shape of the nucleus becomes somewhat arbitrary.

2.5. Deformation of nuclei

The softness of a nucleus in which the zero-point oscillations have a high amplitude indicates that the nucleus is close to the point of instability. Vanishing of ω would indicate that the ground state must be restructured (the analog of a second-order phase transition) in the direction indicated by the nature of the soft mode. Rapid changes[52] of nuclear properties from one nucleus to an adjacent one (for example, when a pair of neutrons is added to the isotopes with $N = 88$, the soft spherical nucleus is replaced by a statically deformed nucleus) are well known. The deformation apparently appears somewhat earlier than the point where $\omega \to 0$, and, following the terminology of statistical physics, the transition could be called a first-order transition in the absence of a soft mode. Her the deformed mean field produces a deeper energy minimum than a spherical field. After the appearance of the static deformation, the quantum numbers of the single-particle orbitals change—instead of the central symmetry of the field only axial symmetry remains (nuclei which do not have a symmetry axis in the ground state have not yet been reliably identified, though this question is being intensively discussed[60]), and the energy distribution of the orbits becomes more uniform. In the case of quadrupole deformation (it is possible that nuclei in the region of $A = 220$–230 have a stable octapole deformation[61]) degeneracy with respect to the sign of the projection of the angular momentum of the particles on the symmetry axis and together with it the pairing of particles in degenerate orbits coupled by time reversal are preserved.

Deformed nuclei are usually more rigid then typical spherical nuclei. Of five quadrupole oscillation modes in a spherical nucleus, which are degenerate with respect to the projection of the angular momentum, in the case of axial deformation two remain (oscillations which are longitudinal and transverse with respect to the axis of the spheroid). These are the so-called β and γ modes, whose frequencies vary little from nucleus to nucleus, having a value of the order of 1 MeV. The corresponding amplitude of the oscillations is much smaller than the static deformation, and anharmonic effects are not as fundamental as in soft, spherical nuclei. The remaining three quadrupolar degrees of freedom determine the spatial orientation of the deformed nucleus. In this case there arises the possibility of rotational motion as predicted by N. Bohr and F. Kalckar.

2.6. Rotation of nuclei. Rigid body or liquid drop?

Rotational bands are the most vivid distinguishing feature of the spectra of nonspherical nuclei.[62] As follows from Bohr's estimates (Sec. 1), the rotational motion is the slowest nuclear excitation. It is adiabatic relative to other degrees of freedom, and a separate rotational band can be constructed for every single-particle configuration (or vibrational state). Collective rotation can occur only around axes which are perpendicular to the symmetry axis. Each band, in this case, is approximately characterized by the projection K of the internal angular momentum of the configuration on the axis of symmetry. Because of the noninertial nature of the reference system fixed to the rotating nucleus, centrifugal and Coriolis effects, which mix bands with different values of K, arise.

For small angular momenta J the Coriolis forces are small and the spectrum of the rotational band is described by the simple expression for a rotator with a fixed moment of inertia $\mathscr{Y}$,

$$E_J = \text{const} + \frac{\hbar^2 J(J+1)}{2\mathscr{Y}}. \tag{12}$$

The probabilities of transitions between bands under the assumption that their internal structure remains unchanged satisfy the geometric intensity rules,[63] which hold approximately experimentally.

The moment of inertia is associated with profound physical problems.[64] This quantity is easily calculated in molecular systems, where there exits a natural framework with which the axes of a moving coordinate system can be associated. It is then easy to express the angles determining the orientation of the framework in terms of the coordinates of the nuclei and to distinguish the rotational energy (12) as a constituent part of the kinetic energy operator of the system. In a nucleus consisting of identical particles there does not exist an unequivocal recipe for distinguishing collective angles. The standard methods of analysis consist of forced external rotation of the system with some angular velocity Ω and the search for a coefficient of proportionality between the average value of the angular momentum arising $\langle J \rangle$ and the quantity Ω: $\hbar\langle J \rangle = \mathscr{Y}\Omega$. The microscopic justification of this procedure (the cranking model[65]) for nuclei with a stable axial deformation was demonstrated much later.[66]

The forced-rotation model gives substantially different results for an ideal liquid and for a gas of independent fermions. We have already mentioned Bohr's juxtaposition of the liquid-drop and rigid-body pictures. A Fermi gas rotates in a nonspherical mean field[65,67] (to within quasiclassical accuracy) as a rigid body with the same spatial density distribution. This result is closely related to a famous theorem of N. Bohr and Van Leuwen[68,69] about the absence of magnetism in a classical equilibrium system of charges. The reason is that the isotropy of the velocity distribution is preserved at each point even when a uniform magnetic field is applied. Analogously to this, in a rotating Fermi gas the Coriolis term $-\mathbf{\Omega J}$ in the quasiclassical approximation does not change the isotropy of the velocity distribution relative to the rotating system, so that the velocity field is purely translational, $\mathbf{v} = [\mathbf{\Omega r}]$, and the rotation has a rigid-body character, $\mathscr{Y} = \mathscr{Y}_{\text{rb}}$.

On the other hand, a potential flow of an ideal liquid in a nonspherical rotating vessel requires much higher energy in order to create the same total angular momentum, which corersponds[62] to a small hydrodynamic moment of inertia $\mathscr{Y}_{\text{h.d.}} \approx \mathscr{Y}_{\text{rb}}\beta^2$, where $\beta = \Delta R / R$ is the nonsphericity parameter (in typical deformed nuclei $\beta \approx 0.2$–0.3). The rotation picture here is closer to the propagation of a surface wave with rolling over of the nonspherical "crest." The experimental moments of inertia lie between the hydrodynamic and rigid-body boundaries, $\mathscr{Y} \approx (1/2–1/3)\mathscr{Y}_{\text{rb}}$.

2.7. Pair correlations in the nucleus. Superfluidity

An important step in increasing the depth of understanding of nuclear structure further was the development of a microscopic theory of pair correlations.[70–72] The phenomenological introduction of pairing into the independent-particle model, required in order to make its predictions unique, was replaced by the realization of the fact that the system of nucleons in complicated nuclei is a superfluid (superconducting) system. Of course, such a relationship could be established only after the microscopic theory of superconductivity of metals was constructed. The theory of pair correlations in nuclei was then formulated following this example. It became clear that in addition to surface phenomena, which in the past were associated with the influence of pairing, in reality pair correlations affect almost all properties of both single-particle and collective excitations.

Because of the presence of a condensate of Cooper pairs, the unpaired particles must be regarded as new elementary excitations—quasiparticles, which are superpositions of old particles and holes. The spectrum of shell excitations ε_λ acquires a gap $\varepsilon_\lambda \to E_\lambda = \sqrt{\varepsilon_\lambda^2 + \Delta^2}$, the density of low-lying states decreases substantially, and all single-particle matrix elements are renormalized. The magnitude of the gap $\Delta \sim 1$ MeV agrees with the mass differences of even-odd nuclei. We note that on the average the value of Δ decreases as $\sim A^{-1/2}$ in the direction of heavy nuclei. Is this not an indication of the predominant role of the surface region in pair correlations? Such questions have been raised repeatedly, but an unequivocal answer still does not exist.

Pairing of the superconducting type radically alters the characteristics of the low-lying collective excitations.[71] It is well known that in a normal Fermi gas at low temperatures waves with velocity c less than the velocity v_F of particles at the Fermi surface cannot propagate: the continuum of particle-hole excitations contains an excitation into which a wave quantum can transform without violating the laws of conservation of energy and momentum, which will give rise to rapid absorption of waves (their transformation into the motion of uncorrelated particles and holes). This also concerns ordinary sound, which requires a quite high frequency of collisions between particles $\omega_{\rm col} > \omega_{\rm s}$, in order for there to be enough time for an equilibrium momentum distribution to be established in the presence of compressions and rarefactions of the medium; in a degenerate Fermi gas, however, collisions die out. L. D. Landau[73] pointed out the possibility of propagation of waves of so-called zero sound with $c > v_F$ and $\omega_{\rm s} > \omega_{\rm col}$, whose dispersion law does not fall into the particle-hole continuum. These collisionless waves can be represented as the propagation of the deformation of the Fermi surface, where the elasticity is created in a self-consistent manner by the interaction between the particles. The low-lying collective waves are restored[74] in superconducting systems, where the lower boundary of the pair continuum is not equal to zero, but rather 2Δ.

The qualitative features of this situation remain in nuclei also. Soft quadrupole oscillations of nuclei have a frequency $\omega < 2\Delta$ (and in typical spherical nuclei $\zeta = \omega/2\Delta \ll 1$), and they can be interpreted as waves of breaking of Cooper pairs and creation of correlated two-quasi-particle states–quadrupolar phonons, whose energy is lower than 2Δ because of the quadrupolar component of the residual interaction of quasiparticles, amplified by the polarization of the core. In near magic nuclei the oscillations of the "natural" multipolarities ($2^+, 3^-, \ldots$) are similar to Bohr's liquid-drop modes.[75] Because of the small gap ω_0 in the single-particle density of states, the vibrational quanta here are rather analogous to excitons in semiconductors. In soft spherical nuclei with $\zeta \ll 1$, however, the "metallic" behavior of the superconducting nucleons in the outer shells, giving rise to the observed low-energy modes, begins to dominate. Finally, the high-lying giant resonances are less sensitive to shell effects and the closest to Landau's zero sound.

In the region of low-lying modes the structure of the spectra of soft spherical nuclei remains the main problem. A systematic microscopic description of the anharmonicity, initiated by Ref. 59, entails very complicated numerical calculations. It would therefore be useful to separate—from theoretical considerations and with the help of an analysis of the data—the main anharmonic effects. On the basis of the classical phenomenological theory, the leading corrections to the harmonic approximation[58] are given, as a result of the low frequency ω and high amplitude of the oscillations $\sim 1/\sqrt{\omega}$, by the nonlinear terms in the potential energy. A collective phonon is a superposition of the type (2) of a large number of particle-hole excitations with a given symmetry. Unlike (2), however, the phonon superposition is coherent, which is what increases the probabilities (collectivization) of the transitions. The existence of small adiabaticity $\zeta = \omega/2\Delta$ and collectivity parameters $1/\sqrt{N}$ makes possible the discrimination and selection from mass of possible correction terms[76] the main ones. The main terms in typical cases are[77] the strong four-phonon anharmonicity and the corrections of the rotational type (24) to the energies of multiphonon states, associated with the virtual deformation of a slowly oscillating nucleus and the degree of freedom of the collective rotation arising as a result of this. This analysis reveals new nontrivial regularities in the vibrational spectra. It is remarkable that in spite of the strong nonconservation of the number of quanta by the anharmonic corrections, the general structure of the spectra of phonon multiplets is preserved in accurate nonlinear solutions.[78] A detailed adjustment of the spectra for many nuclei, however, requires the introduction of additional parameters and has not yet been made.

A different approach—the so-called model of interacting bosons (MIB),[79–81] where only terms of the anharmonic Hamiltonian which conserve the total number of quanta remain has been developed further. In the first formulation of MIB the phonons were interpreted as images of pairs of fermions and only the s and d quanta, modeling the condensate ($l = 0$) and quadrupole excited ($l = 2$) nucleonic pairs, were taken into account. This literal interpretation has not been adequately substantiated microscopically[77] and leads to a number of much too rigid predictions, which have not been confirmed experimentally. For example, in the $2N$-fermion system of a supermagic nucleus, the total number of bosons of the s and d types must be set equal to $n = N$ and all rotational bands must be cut off at a maximum angular momentum $J = 2N$, when the transition s→d has been completed. This cutoff of the bands is not observed. The striving to get rid of these artificial effects makes it necessary to introduce many new parameters.

Thus the description of soft spherical and translational (to deformed) nuclei turns out to be a more complicated theoretical problem than in the case of nuclei with a stable deformation. This is associated with the high amplitude of collective motion, because of which the basis of the starting spherical shells is inadequate for describing fluctuations of the nuclear field.

2.8. Giant resonances

A separate and very interesting area of study of the collective nuclear motion is the physics of giant resonances.[82]

The specific peculiarity of nuclear zero sound lies in the fact that the characteristic frequencies of coherent superpositions of particle-hole pairs fall into the continuous spectrum. Excitation of giant resonances with different orbital-spin and isospin structure is observed in numerous nuclear processes; they serve, as discussed in Sec. 1, as the input states for the subsequent relaxation process. The observed widths of these resonances are large ($\gtrsim 1$ MeV) and are linked[83,84] to the degradation of the coherent motion into less ordered complex configurations.

One of the first discussions of the physics of giant resonances was given in the same paper by N. Bohr and F. Kalckar that we have repeatedly cited.[8] Experiment (the Bothe-Gentner experiment[85] on the nuclear photoeffect) gave at that time only an indication of the concentration of the strength of γ transitions in the region of frequencies substantially exceeding the frequencies which would characterize the equilibrium thermal emission of a compound nucleus. The authors wrote: "We have in transitions from these highly excited nuclear states to the normal state to do with some peculiar features of the radiative mechanism connected perhaps with the appearance of dipole moments." Indeed, it is precisely the dipole giant resonance (as we have already pointed out, associated with the relative oscillations of neutrons and protons, i.e., of an isovector character), which was destined to be for a long time the only representative of a large class of collective motions, that was observed here.

Two notes by N. Bohr in 1938 are devoted to a special discussion of the nuclear photoeffect.[57,86] He points out the apparent inconsistency between the high density of nuclear levels in the region including the frequency of the resonance and the resonance character of the effect itself. This indicates a phased collective motion, which turns out to be comparatively long-lived even being immersed in a sea of compound-nuclear states (the corresponding oscillator has a high Q-factor). It is therefore possible to observe cases of emission of γ rays, carrying away all of the excitation energy and returning the nucleus into the starting state, prior to the relaxation to equilibrium: "The photoeffect is primarily conditioned by certain special vibratory motions with singular radiation properties." The same physics could be even more vividly manifested in isobaric analog resonances discovered approximately 30 years later[87]—raised by the Coulomb energy and therefore falling into the continuous spectrum of states belonging to the same isospin multiplets to which the low-lying states of the neighboring nucleus belong.)

N. Bohr later introduced the language borrowed from optics and associated with the frequency-dependent susceptibility or polarizability of the system—"the degree of excitability" of forced oscillations in it, which is more suitable for the description of resonance collective phenomena. Using this approach A. B. Migdal obtained[88] the first microscopic estimates of the characteristics of the giant dipole resonance. With the help of relations analogous to the classical dipole sum rule and arguments based on requirements of self-consistency, it is possible to arrive at almost model-free estimates of the frequencies of giant resonance of higher multipolarities.[52]

Giant resonances, including spin and charge-exchange excitation modes, have been intensively studied in recent years.[89–91] The question of the possible role of internal excitations of the nucleon[92–93] (creation of a pion or transition of a nucleon into the isobar), transferred in a wavelike fashion from particle to particle and forming the unique modes of giant resonances, remains intriguing.

2.9. High-spin rotational states

Superconducting pair correlations lead to a unique nuclear rotation. An increase in the energy of the virtual breaking of pairs and pairing renormalization of single-particle matrix elements weaken the reaction of the system to a slow external rotation (analog of the Meissner effect). This explains[71] the fact that the moments of inertia of nuclei are lower than the rigid-body value. We note that in nuclei a standard estimate of the coherence length of the superconducting state $\xi \sim \hbar v_F/\Delta$ gives a value exceeding the size of the nucleus. The nucleus is therefore closer to a type-I superconductor with nonlocal response to perturbations. The response becomes local only in the formal limit $\Delta > \hbar\omega_0 \sim \varepsilon_F A^{-1/3}$, and the moment of inertia of the nucleus approaches[94] the value for vortex-free flow of an ideal liquid. Taking into account specifically the corrections to the forced-rotation model, arising in a more rigorous microscopic analysis,[66–95] makes it possible to reproduce the experimental values of the moments of inertia of well-deformed nuclei.[96]

Progress in the area of high angular momenta, attained primarily with the formation of a compound nucleus in reactions involving the coalescence of heavy ions followed by the emission of neutrons and γ rays, lowering the nucleus into the vicinity of yrast lines,[6),97] has opened up an entirely new area of research—rapidly rotating nuclei.[98,99] The phenomenon of "Coriolis antipairing"—the transition into the normal state with increasing angular momentum, analogous to the effect of the destruction of superconductivity by a magnetic field—was predicted immediately after the explanation of the effect of pair correlations on the observed values of the moments of inertia.[100,101] The sharp S-like growth of effective moments of inertia $\mathscr{Y} = \hbar J/\Omega$ with some angular rotational velocity $\hbar\Omega = dE_J/dJ$ was observed experimentally.[102] Similar moment-of-inertia anomalies are characteristic for a very large number of nuclei with $J \approx 14$–20.[103,104] However, the physics of phenomenon turned out to be somewhat different from what was expected.

Instead of a phase transition with the destruction of the Copper condensate, something like the Paschen-Back effect occurs: a pair on the specific orbit which is most strongly subjected to the action of the Coriolis forces breaks and this pair is now oriented not along the axis of deformation, but rather along the axis of rotation perpendicular to it.[105] From the viewpoint of single-particle motion in a rotating potential, there occurs an intersection of bands, and it is not the band constructed on the paired configuration of the ground state, but rather the two-quasi-particle band with aligned quasiparticle angular momenta that becomes the lowest (yrast) band. Since the depleted condensate still exists, there arises the nuclear analog of gapless superconductivity. This scheme describes well the observed phenomena.[106] As the angular momentum further increases, the picture becomes more complicated: new pair breakings, i.e., band intersections, occur, and the effective moment of inertia now refers to the envelope of those bands which successively fall into the yrast line; it may be expected that the structure and shape of the nucleus change at the same time, for example, centrifugal stretching and nonaxial deformation appear.[107]

Finally, in the presence of even higher angular momenta, the particles are not paired and the oriented along the axis of rotation. It is here that the situation in which the orbital angular momenta of the particles are superposed into the rotational moment of the nucleus, mentioned by N. Bohr and F. Kalckar,[8] should occur. This rotation[108] does not resemble the collective rotation observed with low angular momenta; here the axis of rotation coincides with the axis of symmetry and a change in the total angular momentum requires a redistribution of the particles along the orbits, i.e., a transition to a band corresponding to a different configuration. Averaging over a small interval of angular momenta, i.e., over several intersections, however, will again yield the rigid-body moment of inertia.[52] Because of the single-particle nature of the rotation here it may be expected that amplified collective transitions along the yrast line and the appearance of isomeric states ("traps") on the yrast line[109] linked to the single-particle irregularities will be absent. The maximum angular momenta ($J \sim 80$) that a heavy nucleus can acquire are limited[110] by centrifugal ejection of nucleons or fission, which can be estimated by appealing again to the macroscopic effects of surface tension and Coulomb repulsion. On the whole, we can see that the inconsistency between the drop and rigid-body features of nuclear rotation, pointed out by N. Bohr and F. Kalckar, is resolved in a far from trivial manner, revealing a profound and in many ways yet unstudied physics.

2.10. Why does the "shell model" survive in a strongly-interacting collective system?

In discussing the shell model and the residual interactions and collective oscillations and rotations associated with them, we had set aside the most fundamental question: why can the independent-particle model be used at all, even as a zeroth-order approximation, in a system with strong interaction? This question was answered by L. D. Landau in his theory of the Fermi liquid[73] and rigorously substantiated by V. M. Galitskiĭ and A. B. Migdal[111] with the use of the methods of the quantum theory of fields. Qualitatively the answer lies in the fact that the interaction between particles "dresses them," transforming them into new objects—quasiparticles, obeying as before the Fermi statistics. Models of independent "particles" in reality operate precisely with these objects. The dressing process in normal Fermi systems can be imagined with the help of adiabatic inclusion of interactions between the starting particles. Up to nonadiabatic corrections, the classification of levels of an ideal Fermi gas, associated with the Fermi surface and the concepts of particles and holes, is preserved in this process. However, the interaction cannot be turned on too slowly: the duration of this process must be shorter than the lifetime of the quasiparticles. We can therefore talk only about some region of excitations near the Fermi surface, where the quasiparticles are long-lived objects and indeed form the response of the system to an external perturbation.

Near the Fermi surface the lifetime τ_q of quasiparticles increases ($\tau_q \sim 1/(\varepsilon - \varepsilon_F)^2$, where ε is the energy of the quasiparticle), because the Pauli principle forbids possible interaction processes between quasiparticles and the Fermi background. For this reason, here, the uncertainty $\Delta\varepsilon \sim \hbar/\tau_q$ of the energy of a quasiparticle is much smaller than its excitation energy itself $|\varepsilon - \varepsilon_F|$, which grows linearly away from the Fermi surface. As a result, the system can be modeled as a gas of Fermi quasiparticles, whose properties (effective mass, gyromagnetic ratio, dispersion laws, etc.) do not coincide with the properties of the "bare" particles.

The real dispersion law of quasiparticles, the reaction of the system to external fields, the properties of the collective motions which are possible here are controlled by the effective interaction of the quasiparticles. This interaction which implicitly takes into account the repulsive core, multiple scattering, the role of the surrounding medium, and so on, must be parametrized phenomenologically, if there are no additional simplifications that would permit calculating it from first principles. Comparison of the computational results with experiment serves as a check. Such a program was carried out for nuclei by A. B. Migdal and his coworkers, who formulated the theory of finite Fermi systems.[112] In the modern variants of this theory,[113] the effective interaction is close to the widely used Skyrme forces—short-range forces, including spin and spin-orbital components, as well as the dependence on velocities and on the local density, which ensures saturation and the correct transition to vacuum interactions of nucleons. Pair correlations are also easily incorporated into the general scheme. The effective interaction, in principle, determines in a self-consistent manner all observed low-energy nuclear properties, including the density distribution, the form of the mean field, and the quasiparticle spectrum, i.e., the parameters of the shell model, as well as global characteristics such as the parameters in the mass formula.[114]

The theory of finite Fermi systems has achieved substantial successes in the description of magic and nearly magic nuclei. The idea of local similarity between the shape oscillations and a displacement of the nucleus as a whole makes it possible to develop a theory of the low-lying oscillations and giant resonances in these nuclei. At the same time, the classical Bohr liquid-drop approximation appears as a natural limiting case of surface oscillations of a Fermi liquid. The quantum structure of the excitations induces volume components of the oscillations. The agreement between the calculations, especially for the collective modes of the type 3^- in the region of nuclei near doubly magic ^{208}Pb, and the results of modern precision measurements (in electron scattering) of the transient form factors (their classical analogs are the Fourier components of the density, corresponding to the excitation frequency) is impressive.

The theory is nevertheless confronted with more serious problems. When the theory is generalized to nuclei with a large number of valence nucleons, difficulties of both a technical character (it has not yet been possible, for example, to develop acceptable computational algorithms for deformed nuclei) and of a fundamental character arise. Going into the depth of a shell, collective excitations, as discussed above, become increasingly softer, and fluctuations of the mean field grow with them. The interaction of quasiparticles with collective excitations[115] become increasingly more important, determining, for example, the spectra of odd nuclei.[56,116] It is therefore necessary to introduce retardation, i.e., an energy dependence, into the effective interaction of quasiparticles. This leads to a more complicated Lagrangian formulation[114] of the theory of finite Fermi systems. Moreover, since the energies of soft collective modes and of unpaired quasiparticles are of the same order of magnitude, there arise unique resonance effects[117] and quasiparticles become surrounded with coherent phonon clouds. The energy dependences associated with this are not contained in the usual formulation of the theory of a Fermi liquid, which presumes smooth variations of all quantities near the Fermi surface, Thus, to describe soft modes and phase transitions it would be desirable to construct the theory at the outset in a consistent manner in terms of elementary excitations of both types—quasiparticles and phonons.

An even more difficult problem was formulated[118] by N. Bohr and is still far from being solved: "...The problem of nuclear structure cannot be separated from the problem of expressing the laws of nuclear forces." The relationship between the properties of complex nuclei and the fundamental strong interactions remains unclear, as indicated by the much too large number of variants of effective phenomenological forces which lead to more or less equivalent predictions. Effects linked to exchange mesonic currents and the internal structure of nucleons have been studied to some degree only in light nuclei. The power of Landau's approach to the theory of the Fermi liquid lies in the fact that in problems of low-energy nuclear physics the questions associated with first-principles calculations of the parameters of the theory, i.e., the effective interaction of the quasiparticles, are successfully separated from the problems involved in finding the observed nuclear properties with the help of this effective interaction. At higher excitation energies and higher transferred momenta, however, nuclear physics must unavoidably be joined with the laws of the fundamental interactions.

In spite of the decades which have passed, N. Bohr's words remain valid: "...In nuclear physics proper we stand only at the threshold of development. The deep ties between experimental and theoretical studies, distinguishing research in this field, is what gives us the foundation for the greatest hope for further success."[4]

3. NUCLEAR FISSION

"It was typical of Bohr's approach to the problems that he willingly yielded—like Goethe—to the "Forderung des Tages," the exigency of the day. No worthwhile challenge left him unresponsive."

L. Rosenfeld (Ref. 179, p. 116)

In the last section of this article we shall discuss the phenomenon which in the eyes of millions of people is primarily associated with the role of nuclear physics in the modern world—nuclear fission. Neutron-induced fission was discovered experimentally by O. Hahn and F. Strassman[119] and was immediately correctly interpreted by L. Meitner and O. Frish[120] as, in the words of N. Bohr, "a new hope of disintegration of heavy nuclei, consisting in a fission of the nucleus into two parts of approximately equal masses and charges with release of enormous energy."[121]

The basic physical notions regarding the process of fission are expounded very clearly by N. Bohr in two brief notes[121,122] and in a fundamental paper by N. Bohr and J. Wheeler.[123] Although in the 45 years which have passed since then an enormous number of "pure" and "applied" studies of different aspects of nuclear fission have been carried out, the picture of this process and the vocabulary used to describe it have remained essentially the same as those introduced by N. Bohr. This was pointed out in the book by Halpern[124] and remains in force today.[182]

3.1. From the compound nucleus to fission

Bohr's theory can be summarized by several assertions: a) the rough features of the phemonena are described by the classical model of a charged liquid drop, where the electrostatic repulsion forces overcome the short-range nuclear attraction, responsible for the surface energy and preventing large deformation of the nucleus; b) the process exhibits a barrier character, which explains the "remarkable stability of heavy nuclei in their normal state or in states of low excitation, in spite of the large amount of energy which would be liberated by an imaginable division of such nuclei"[121]; c) an activation energy is required to overcome the barrier, since the deformational motion is quasiclassical and the tunneling quantum transitions predicted in Ref. 123 are still unlikely (spontaneous subbarrier fission was discovered by G. N. Flerov and K. A. Petrzhak later[125]); d) the height of the fission barrier in heavy nuclei "is of the same order of magnitude as the energy necessary for the escape of a single nuclear particle,"[121] so that ^{235}U is fissioned by slow neutrons (see also Refs. 126 and 127); e) after the excitation energy is injected into the nucleus, a compound nucleus, which we discussed in detail in Sec. 2, forms; its lifetime is long (fission widths $\Gamma_f \approx 0.1$ eV correspond to $\tau_f \approx 10^{-14}$ sec) and activation of fission proceeds through those fluctuations in which "the quasi-thermal distribution of energy is largely converted into some special mode of vibration of the compound nucleus involving a considerable deformation of the nuclear surface."[121] Starting from such ideas, N. Bohr not only explained the main features of the fission mechanism, but he also predicted[122] the marked difference between odd an even-even isotopes with respect to neutron-induced fission, associated with the fact that in the first case the excitation energy and the density of states of the compound nucleus will be appreciably higher because of pairing effects. N. Bohr also pointed out that because of the fluctuation character of the process "a wide range of mass and charge numbers of the fragments may occur," so that "a closer study of the statistical distribution of the fragments"[122] is required.

The quantitative analysis of the mechanism of nuclear fission performed by N. Bohr and J. Wheeler[123] and Ya. I. Frenkel'[128] is based on the concept of critical deformation, corresponding to the point of unstable equilibrium of the nucleus (saddle point), beyond which further deformation into two separated fragments now proceeds with a lowering of the potential energy. The magnitude of the critical deformation in the liquid-drop description depends only on the ratio of the Coulomb energy ($\sim Z^2/R \sim Z^2 A^{-1/3}$) to the surface energy ($\sim R^2 \sim A^{2/3}$), i.e., on the fissionability parameter Z^2/A, and when some limiting value $(Z^2/A)_c$, equal according to modern estimates to 45.5, is reached the critical deformation vanishes, i.e., the drop becomes unstable relative to fission already for arbitrarily small deformations.[7)]

As N. Bohr and J. Wheeler pointed out,[123] "Exactly how the excitation energy originally given to the nucleus is gradually exchanged among the various degrees of freedom and leads eventually to critical deformation proves to be a question which needs not be discussed in order to determine the fission probability." In the picture of the compound nucleus, described by an equilibrium thermal ensemble with a fixed excitation energy E, fission is determined by the density of "transitional states" $\tilde{\rho}\,(E - E_f)$, where E_f, the fraction of the excitation energy which is concentrated in the fissioning degree of freedom ($E_f = U_f + K_f$, is the sum of the potential and kinetic energies of deformation, corresponding to ordered motion). If thermal equilibrium is present, then near the saddle point the fissioning system with an excitation energy E moves slowly near the fission threshold $(U_f)_{max}$ and is almost "cold" relative to the internal (nonfissioning) degrees of freedom. These arguments later formed the foundation of the concept of fission channels.[129]

Reference 123 gives a detailed discussion of the experimental data available up to that time on the fissioning induced by thermal and fast neutrons, deuterons, protons, and photons; it is shown that the "delayed emission of neutrons indeed arises as a result of nuclear excitation, following the beta-decay of the nuclear fragments"; two possible mechanisms for the emission of prompt neutrons are examined: emission from the "neck" at the moment of fission or evaporation from the excited fragments[8)] (later experiments confirmed that neutrons are mainly evaporated isotropically from the moving heated fragments); and, qualitative effects of the dependence of the mass distributions on the excitation energy of the fissioning nucleus are predicted. On the basis of Bohr's model different observed facts "fit together in a reasonable way to give, a satisfactory picture of the mechanism of nuclear fission."[123]

The subsequent 15 years were years of intensive study of fission and even more intensive practical applications of fission both for peaceful and destructive purposes. As pointed out in the review by Grant,[130] the questions associated with fission were to a significant degree "isolated from the rest of nuclear physics." Only gradually, as general progress in the understanding of nuclear structure was made, has it become apparent that fission is not a detached event: it is only the most vivid example of collective motion in a quantum system with strong interaction. Experimental discoveries and the development of the theory have "brought fission back into the mainstream" of science.[130]

3.2. From phenomenology to microscopic analysis

The beginning of this process can be conditionally related to the generalized model,[56] unifying on a microscopic foundation the description of single-particle and collective aspects of nuclear structure (Sec. 2). In the study by D. Hill and J. Wheeler[131] the dynamics of fission was studied from precisely this viewpoint. The authors point out that they are indebted to numerous discussions with N. Bohr for their understanding of how the liquid-drop model can be reconciled with the independent-particle picture and that the initial variant of their paper was prepared with N. Bohr as a coauthor.

The successes of the shell model for the ground and weakly excited nuclear states enable one to extend the single-particle analysis to slow collective motion with large amplitude, which fission is. Now the question concerns the time-dependent self-consistent field (though the requirements of self-consistency in real calculations are sometimes only implied or are not only very roughly) in which the nucleons move. The smallness of the characteristic single-particle times τ_{sp} compared with the deformation times ensures that the nucleons adjust adiabatically to the slowly varying form of the general field, though such a sharp separation of fast and slow degrees of freedom as in molecules (owing to the low ratio of the electron mass to the nuclear masses) does not occur here.

The specific dynamics of large amplitude collective motion depends on the choice of collective variables —the coordinates Q_i and the conjugate momenta P_i. An *a priori* recipe, determining the transformation from the starting nucleonic variables to the collective variables, does not exist. Aside from the macroscopic analogies, the weakness of the coupling between the collective degrees of freedom and the remaining ("internal") degrees of freedom can serve as a practical guiding principle—only in this case does it make sense to separate the collective motion. It would be preferable to talk about, as is now customary, the separation of a collective subspace within which the dynamics are expressed entirely in terms of the collective operators Q and P while the matrix elements of the true Hamiltonian H and of the collective operators themselves are small for transitions to states of a different nature from the complete space of states of the nucleus.[132] Neglecting the coupling with noncollective degrees of freedom, we can talk about a conservative collective Hamiltonian $H_c\,(P,Q)$, which is a mapping of the multiparticle dynamics controlled by the starting operator H onto the collective subspace.

The form of the collective Hamiltonian $H_c\,(P,Q)$ is restricted by the requirements of the strict conservation laws

(T invariance), rotational symmetry, conservation of spatial parity). In the case when the collective motion is adiabatic, only terms with the lowest powers of the collective momenta are significant, i.e.,

$$H_c(P, Q) = U(Q) + \frac{1}{2} P_i B_{ij}^{-1}(Q) P_j, \qquad (13)$$

the collective Hamiltonian consists of the potential energy $U(Q)$ and the kinetic energy, quadratic in the momenta P_i and containing the mass tensor $B_{ij}(Q)$, which depends on the collective coordinates.

The basic principles of the calculation of the terms in the adiabatic Hamiltonian (13) were formulated in Ref. 131. As in the case of molecules, the potential energy $U(Q)$ is identified with the total energy of the frozen configuration of nucleons with fixed values of Q (nucleonic or, more accurately, quasiparticle terms). The coordinates Q in the simplest approach can be fixed by introducing the corresponding Lagrangian factor analogously to the way this is done in the forced-rotation model[65] for fixing the average angular momentum (see Sec. 2); this procedure can be improved by seeking a self-consistent set of numerical values of Q at all points of the collective space.[133–135] With regard to the collective kinetic energy it arises[71] from the nonadiabatic mixing of the Q-dependent internal wave functions via a change $\dot{Q}$ of the collective coordinates. The main effect of mixing is the addition of a phase factor to the internal function whose gradient with respect to the nucleonic variables is proportional to the collective velocity $\dot{Q}$, whence arises the correction to the total energy $\sim \dot{Q}^2$, identified with the kinetic energy of the collective motion. This description, used also in the modern approaches[133–134] to the derivation of the nuclear collective Hamiltonian, is close in spirit to the theory of the macroscopic quantum coherence,[136] where the hydrodynamic velocity is also the gradient of the phase of a macroscopic wave function.

The physical origin of the phase lies in the necessity for bringing the internal function, whose quantum numbers are adiabatic invariants, into correspondence with the variable conditions on the surface, where the collective motion is concentrated (because of the saturation of the nuclear forces and the weak compressibility of nuclear matter). This consideration, in turn, enables one to assert that the internal wave function depends substantially on the form and symmetry properties of the mean field, so that it is precisely the shape parameters that it is natural to choose, following N. Bohr and J. Wheeler,[123] as the collective variables Q_i in the fission problem. The simple relationship between these parameters and the density $\rho(\mathbf{r})$ makes it possible to formulate easily the self-consistency conditions, while the coherence of the contributions of many nucleons guarantees the collective nature of these variables, the on-the-average weak sensitivity to changes of the states of separate particles, and the relative slowness of the collective motion. One of the possible parameterization is given by the standard expansion

$$R(\mathbf{n}) = R_0 \left[1 + \sum_{l,m}' Q_{lm}^* Y_{lm}(\mathbf{n})\right], \qquad (14)$$

where for a classical "leptodermal" (thin-skinned) drop $R(\mathbf{n})$ is the radius of the sharp boundary in the $\mathbf{n}$ direction, while in the real problem with a diffuse edge $R(\mathbf{n})$ describes the form of the equipotential surfaces.

In the representation (14) collective motion unfolds in the multidimensional space of the variables $Q_{lm} = (-1)^m Q_{l,-m}^*$. For an incompressible liquid drop the dynamics contains only the contributions of Q_{lm} with $l \geqslant 2$, while under the assumption of axially symmetrical deformations only the terms with $m = 0$ remain (m is the projection of the angular momentum, associated with the given mode, on the axis of symmetry). Different variants of the drop model, including also those with parameterizations differing from (14), paint approximately the same picture of the potential energy $U_{\rm drop}(Q)$, including Coulomb and surface terms, and determine the path passing through this landscape from the ground state to fission for nuclei with different Z^2/A.[137] It turns out that the liquid drop model is stable with respect to deformations of odd order (for example, Q_{3m}), which are capable of explaining the mass asymmetry of fission products observed with not very high excitation energies.[138]

3.3. Shell corrections: double-hump barriers, shape isomers, and all that...

Of course, even a macroscopic description must contain corrections to the simplest drop model. The compressibility of nuclear matter, the difference between the neutron and proton densities, the effect of the diffuseness and local curvature of the surface, and so on must be taken into account.[139] The total contribution of all these effects to the potential energy $U_{\rm drop}(Q)$, varying smoothly from nucleus to nucleus, however, is apparently small. At the same time, in order to ensure a consistent description of internal (quasiparticle) variables in the presence of a variation of the form of the field, it is absolutely necessary to take into account shell effects: quasiparticles adiabatically adjust their orbits to the form of the field, and the shell structure unavoidably arising in the process (see Secs. 1 and 2) modulates the energy of the "stopped" nucleus. The deformed shells are, generally speaking, not at all filled for those values of the magic numbers which correspond to nuclear states near the unperturbed configurations of the mean field. Thus a new distinct picture of shells, associated with the resonance degeneracy of quasiperiodic wave packets, arises[140] in the region of large deformations with a 2:1 ratio of the axes of the spheroid.

In the first attempts to include the influence of shell effects in a consistent manner, the uncertainties were so large that satisfactory accuracy could not be obtained. Progress in this direction was achieved after V. M. Strutinskiĭ was able to reformulate in the spirit of the theory of the Fermi liquid the procedure for calculating shell corrections,[141] so that it was explicitly determined only by the quasiparticle levels near the Fermi surface. It is precisely these levels that make rapidly varying contributions to the energy of the nucleus accompanying a change in shape. According to Strutinskiĭ, a shell correction can be identified with the difference between the total quasiparticle energies for two distribution functions: the real distribution over the levels n_λ with a sharp Fermi boundary at zero temperature and a smoothed function n_λ. The latter can be compared, for example, with the smeared Fermi distribution that corresponds to a temperature T when the shell effects vanish. The smoothing procedure is insensitive to the precise algorithm used to find n_λ.

Beginning with Refs. 141 and 142, many efforts were made to calculate the shell corrections for different, including hypothetical, nuclei. Such calculations are necessary in order to extend systematically nuclide charts both toward superheavy nuclei and toward the boundaries of the region of stability. Although the accuracy of the calculations and their sensitivity to a change in the computational details are not completely understood from the theoretical point of view,[143] the general opinion now apparently is that there is hope that this method can be used to calculate the shell corrections with an accuracy of $\sim$0.5–1.0 MeV. The nonuniqueness of the procedure could be lowered by using modern variants of the microscopic theory of finite Fermi systems (Sec. 2), which permit finding theoretically both the parameters of the mass formula and the quasiparticle contributions. As already mentioned, however, the calculations here for the time being are limited to spherical forms of the mean field.

A remarkable fact, discovered, in the study of shell corrections, is the double-humped form of the potential energy as a function of the longitudinal deformation parameter. Two minima in the potential energy in heavy nuclei correspond to the correctly predicted deformation of the ground state and a large deformation (ratio of the axes equal to about 2), whose value makes the number of neutrons in the nucleus close to the magic number ($N \approx 146$); the neutron shell correction is then negative and large. With this number of neutrons in the region of the internal barrier the density of quasiparticle states is high, which gives a positive shell correction.

Among many experimental phenomena[144] that fit into the framework of the notions of the complex structure of the potential surfaces of nuclei, the existence of fission isomers stands out.[145–146] Spontaneously fissioning isomeric states, now found in many nuclei (the longest lived isomer is ^{242m}Am, with a lifetime of 14 msec, are interpreted as quasistationary states, corresponding to a value of the collective deformational variable in the second (external) well. Fission from here proceeds by a tunneling transition through the outer barrier to the outside. Experimental estimates of the heights and widths of potential barriers on the whole are in agreement with the calculations of the shell corrections. The ratio of the outer and inner barriers can differ in different nuclei. Thus in thorium isotopes the inner barrier is apparently lower than the outer barrier, which has moreover, a fine structure, and there is enough time for the populated state in the second well to tunnel into the inner stable minimum (or into one of the low-lying collective states above it) accompanied by the emission of a gamma ray before the fissioning tunneling transition occurs through the outer barrier. In thorium isotopes, therefore, fissioning isomers are not observed. The parameters of the inner barriers, however, vary more slowly as a function of the nuclear charge, with the exception of the drop in even-even nuclei compared with odd nuclei accompanying pairing.

It is well known[124,138] that in the case of fission induced by thermal neutrons all nuclei from thorium to einsteinium give a mass asymmetry of the primary fragments with a heavy peak near $A \approx 142$ and a light peak gradually approaching the heavy peak as A increases. There exists quite convincing evidence supporting the fact that this is also attributable to shell effects. According to many calculations,[147,148] in actinides the states at the outer barrier have nonzero odd deformation parameters (Q_{3m} in (14)), i.e., already at this stage of the process the equilibrium form corresponds to mass asymmetry. Its source is the favorable shell structure of the deformed heavy fragment, which determines the approximate constancy of its mass. In the heaviest nuclei with a large neutron excess, another peak, associated with the appearance of a doubly magic spherical nucleus $^{132}_{50}Sn_{82}$, begins to appear. The complicated picture of the mass distributions (the shell effects also give a fine structure) accompanying a transition to symmetrical fission with an increase in the excitation energy, when the shell and pairing corrections become weaker, indicates that the probabilities of different paths to fission in the space of collective variables (different "types of fission") are comparable. If the shell explanations of the form of the potential surfaces are correct, then we arrive at the conclusion that in the region of the saddle point the fission mechanism is already predetermined. There exists, however, the complicated and unanswered question of whether or not a dynamic change can occur in the symmetry along the further path from the saddle point to the act of separation. This idea was stated, in particular, by B. T. Geĭlikman.[124]

The dynamics of the slow collective motion from the ground configuration to the saddle point is controlled by, aside from the potential energy, the kinetic energy also, including the generalized inertial tensor $B_{ij}(Q)$. Indeed the classical motion must occur along the trajectory satisfying the canonical equations generated by the Hamiltonian (13), and in the classically inaccessible region along the path with minimum action $\int |P| dQ$. For the time being the only practical method for calculating[71,133] the inertial tensor is the adiabatic perturbation theroy, constructed on the basis of the frozen internal functions Ψ_{nQ} with fixed collective coordinates Q. Here the change in Q induces an admixture of higher states to the ground configuration Ψ_{0Q}

$$B_{ij}(Q) = 2\hbar^2 \sum_{n \neq 0} \frac{\langle \Psi_{0Q} \mid \partial/\partial Q_i \mid \Psi_{nQ} \rangle \langle \Psi_{nQ} \mid \partial/\partial Q_j \mid \Psi_{0Q} \rangle}{E_n(Q) - E_0(Q)}; \tag{15}$$

where $E_n(Q)$ are the running energies of the terms. In the quasiparticle approximation for the internal functions, the excited configurations making the main contribution to (15) are associated in even-even nuclei with pair breaking or, for filled shells, with the excitation energy of the order of the energy separation between the shells. From here follows the strong periodic dependence of the mass tensor (15) on the collective coordinates and its correlation with the magnitude of the shell corrections (the effective masses $B(Q)$ are small near the shell minima of the potential energy). At the same time the true trajectory can differ from the trajectory found with constant inertial parameters. If the assumptions regarding surface pairing (Sec. 2) are correct, then it will appreciably change in the process of fission, modifying the dependence (15) on the collective coordinates. This question remains virtually unstudied, and the experimental data are contradictory.

The ideas concerning slow collective motion, weakly coupled to the remaining degrees of freedom of the nucleus, and adiabatic nucleonic terms $E_n(Q)$ lead to the conclusion that when the excitation energy E is close to the fission barrier E_f, almost all of this energy is concentrated in the fissioning degree of freedom, if the coordinates Q of the collective motion have approached the saddle point. Very little energy remains in the internal degrees of freedom, and they become frozen. The density $\tilde{\rho}(E-E_f)$ of states $E_n(Q)$ in this region, as also near the normal ground state, is low. The nucleus can fission from each state Ψ_{nQ}, and N. Bohr[129] introduced accordingly, the concept of fission channels.

3.4. Fission channels

The quantum numbers of separate fission channels and the characteristics of the fission products associated with them (primarily, the angular distribution of the fragments[149]) are determined by the trajectory in the collective space, leading to the outer saddle point. If, as apparently occurs in actinides, the nucleus here is axially symmetrical, then the state Ψ_{nQ} is characterized by the projection K of the angular momentum onto the symmetry axis. If, further, K does not change with the descent from the barrier and separation then the angular distribution of the fragments is determined by the probability density for observing the symmetry axis along with the separation directed in a given direction **n** occurs, i.e., for an initially excited nucleus having an angular momentum J and a projection M on an axis distinguished in the experiment (for example, the direction of the incident beam of particles) in terms of the squared modulus $|D^J_{MK}(\mathbf{n})|^2$; where D is Wigner's function. The function D^J_{MK} describes a rotator with fixed values of J and M and with a fixed projection K of the angular momentum on an internal axis of symmetry rigidly fixed to the rotator.

Thus the quantum numbers of the adiabatic states near the saddle point form the angular distribution of the fission fragments. In particular, if for an even-even nucleus pair breaking does not occur in the process of deformation, then $K=0$ is preserved and the distribution of fragments is proportional to $|D^J_{M0}(\mathbf{n})|^2 \sim |Y_{JM}(\mathbf{n})|^2$. Only the dipole $(J=1)$ and quadruple $J=2)$ photoabsorption are important in photofission with not too high gamma-ray energies. In the presence of mirror symmetry of the nuclei at the saddle point the main rotational band, constructed on this deformation, has only even angular momenta $0^+, 2^+, \ldots$. In such nuclei the quadrupole photoabsorption dominates at the fission threshold.[146] In the absence of mirror symmetry the rotational band contains[152] all moments $0^+, 1^-, 2^+, \ldots$ and the main component is the dipole component. When the contributions the dipole and quadrupole components are comparable, because of the different parity of their angular distribution there arises an interference pattern which is asymmetrical with respect to 90°. It is precisely in this asymmetry that one can try to search experimentally for the contribution of the so-called direct fission[150,151] which does not proceed through the compound-nucleus stage. On averaging over the photon energy in the interval $\Delta E \approx 100$ keV, the contribution of the compound states to the asymmetry is lowered (compare the analogous estimates in Sec. 1) by a factor of $\sqrt{\Delta E/\Gamma} \approx 10^3$, where Γ is the typical width of compound resonances.

Experiments confirm the interpretation of angular distributions linked with the fission channels.[130,146] In odd nuclei the quantum number K in the presence of deformation coincides near the ground state with the projection of the angular momentum Ω of the unpaired quasiparticle. As the deformation increases, the quasiparticle levels with different Ω intersect, but because of the conservation of K the quasiparticle remains in its orbit, which is no longer the lowest orbit. The "specific" energy[152] of odd fissioning nuclei is related to this. It is possible to extract from experiment information on the single-particle and oscillatory structure of the levels near the saddle point and inside the second well, where in many cases it is possible to develop a "secondary" spectroscopy.[153] In particular, rotational bands constructed on the second minimum are observed, and the magnitude of the moment of inertia coincides with the value expected for a superfluid nucleus with large deformation. If the fission threshold is determined by a higher internal barrier, where the equilibrium shape is probably not axially symmetric, then K is no longer an integral of the motion. Coriolis forces also break down the conservation of K, but for comparatively low angular momenta J their influence is small.

The nuclear spectroscopy of quasistationary states is coupled potential wells gives diverse physical phenomena, corresponding to different ratios of the parameters of the wells and, therefore, different lifetimes relative to tunneling between wells, radiative transitions, and fission. Cases when the density of states at a fixed energy is substantially different in the two wells, so that the collective motion is characterized by a different degree of coupling with the noncoherent background giving different widths $\Gamma_\downarrow$ (see Sec. 1) of collective levels, are of special interest. where it is possible to resolve a sufficient number of levels in the second well, statistical analysis of the levels shows[154] a Wigner structure (4).

The explanation of the energy dependences of the fission cross sections on a larger energy scale (tens of MeV) unavoidably requires turning to Bohr's arguments about the compound nucleus. A statistical analysis with the help of the principle of detailed balance, in many ways analogous to the problem of the evaporation of particles (Sec. 1), was formulated by N. Bohr and J. Wheeler.[123] An interesting recent result is the demonstration that single-particle states are not sufficient to describe the density of states $\tilde{\rho}(E-E_f)$ at the saddle point (the fission width $\Gamma_f(E)$ is proportional precisely to this quantity), even when pair breaking and thermal smearing of the shells are taken into account. In accordance with the idea proposed by N. Bohr and F. Kalckar,[8] the contribution of collective states of the vibrational and rotational type to the total density of states must apparently be taken into account.[156] Of course, these states can be represented by linear combinations (2) of excitations of particles and holes, so that they "are contained" in the system of levels of independent particles. But collective effects substantially lower the energy of coherent superpositions, transferring them into an entirely different energy range, whre as a result the density of states increases appreciably. The question of how to avoid double counting and obtain the correct density of states for high excitation remains open.

3.5. Dissipation of the energy of collective motion

Thus far we have primarily discussed fission as a purely collective motion along a potential surface, corresponding to an adiabatically varying internal structure. The dynamics of the interaction of global collective variables with the internal degrees of freedom, playing the role of a thermostat or of a surrounding medium, is thus far the least understood aspect of the process. This thermostat is comparatively "poor"—unlike large thermodynamic systems its intensive parameters (for example, the temperature) cannot be assumed to be fixed, which was always emphasized by N. Bohr.[8] Conversely, we are interested precisely in the large fluctuation, freezing out the thermostat and transferring almost all of the energy into global motion. Such a fluctuation tends to decay, returning the system into the equilibrium compound-nucleus state. In the deep-inelastic collisions of heavy ions[50] with energy of several MeV/nucleon, mentioned in Sec. 1, a significant fraction of the kinetic energy of relative motion rapidly (over a time of $\sim 10^{-22}$ s, less than the lifetime of a double nuclear system) dissipates, exciting other degrees of freedom, including also collective ones—deformation of the constituent components and different vibrational modes.

The mechanism of dissipation accompanying slow fission motion in the classically accessible region (the excitation energy lies above the barrier) was discussed in detail by D. Hill and J. Wheeler[131] and is actually well-known from molecular physics. The position of the adiabatic terms $E_n(Q)$ varies, and when the levels converge the probabilities of real transitions between them induced by nonadiabatic perturbations become appreciable. If when $Q = Q^\circ$ the terms $E_1(Q)$ and $E_2(Q)$ with the same values of the exact integrals of motion (angular momentum and parity) intersect, $E_1(Q^\circ) = E_2(Q^\circ)$, then, taking into account the nondiagonal matrix elements of the Hamiltonian H_{12} in the vicinity of the intersection, we obtain repulsion of levels $E_\pm(Q)$ (see (5)), changing their character in the region of intersection, for example, $E_+(Q<Q^\circ) \to E_2(Q)$ and $E_+(Q>Q^\circ) \to E_1(Q)$. Going over to the nonstationary problem with one-dimensional collective motion $Q(t)$, in the approximation of a constant velocity $\dot{Q}$ in the vicinity of the point of intersection, we obtain the Landau-Zener formula[157,158]

$$w = \exp\left(-\frac{2\pi}{\hbar}\frac{|H_{12}|^2}{|\dot{Q}\mathrm{d}(E_1-E_2)/\mathrm{d}Q|}\right) \tag{16}$$

for the probability of transition from the lower term to the upper term on passing through the intersection of the levels. In the absence of the interaction H_{12} or with very rapid passage through the region where this interaction is significant, the system moves along the term $E_1(Q)$, coinciding with E_- far to the left and E_+ far to the right of the intersection, $w \to 1$. Conversely, in the limiting case of adiabaticity, even when the interaction H_{12} is weak the interaction time is long, and there is enough time for the state of the system to become restructured while the system moves all the time along the lower term ($w \to 0$) i.e., it makes the transition from $E_1(Q)$ and $E_2(Q)$.

In an actual nuclear situation the applicability of the simple result (16) is more an exception than the rule. First of all, the nature of the transition between multidimensional potential surfaces can differ[131] from the one-dimensional case (16). Second, the formula (16) presupposes an isolated intersection of two terms—a process for which there is enough time to pass to completion (the wave functions emerge into the asymptotic region) before the interaction with some third term becomes appreciable. This assumption is often violated. The analogy between the picture of Landau-Zener intersections and collisions of particles in a rarefied gas is obvious: at low density rare pair encounters occur and each interaction is completed before the next one begins. The kinetic equation operating only with the probabilities of elementary acts is then applicable. If the collisions occur often, then the phase relations inherited from preceding collisions are also important. Interference phenomena can occur in the presence of several converging paths. Thus in the presence of collective motion of large amplitude there arises a concomitant motion in the space of the inner terms of the complicated system. Averaging over the intervals in which the collective coordinates vary and which include several collisions of terms leads[159] to a stochastic process of the Brownian type.

3.6. Nuclear kinetics revisited

In recent years, progress in computational physics has made possible complex calculations describing nuclear processes on a large scale by the method of the time-dependent mean field.[160] In particular, collisions of heavy ions are calculated in this manner.[161] Digressing from the difficulties of this method in the interpretation of specific reaction channels, we point out that here the evolution of the mean field, created in a self-consistent manner by particles populating orbits in this field, is studied. At the same time, in standard calculations using the nonstationary Hartree-Fock method only those orbits come into play which are genetically related to the initially filled states. Admixtures of Slater determinants, where the particles are located in orbits originating from initially empty states, are thereby dropped. The only thing missing here is the random collisional element that is responsible for creating the dissipative coupling between the global motion and the noncoherent background of internal excitations, leading to relaxation (thermalization) of the global degrees of freedom.

In this sense, the problems of fission, heavy-ion collisions, the widths of giant resonances (Sec. 1), or the quadrupole low-energy motion of large amplitude (Sec. 1) are interrelated: in all cases it would be desirable to replace the Schrödinger equation for collective variables, generated by a Hamiltonian of the type (13), by a Langevin equation with a random force, responsible for fluctuations and dissipation of the collective motion. The characteristics of the random force must be calculated microscopically. If the collective motion is subjected only to Brownian noise, associated with diffusion along energy surfaces, which correspond to different particle-hole configurations, then it may be expected that the process will exhibit a Markov character and the phase-memory time will not appreciably exceed the typical hopping time. Because the change in the collective motion is comparatively small for each hop, this Markov process can be reduced to a Fokker-Planck equation (drift toward the equilibrium energy distribution over degrees of freedom and diffusion, where the coefficients must be interrelated by the fluctuation dissipation relations[162,163]). The phenomenological introduction of the friction coefficients is only the first step in this direction and cannot completely describe, for example, the widths of the angular and energy distributions in deep-inelastic heavy-ion collisions, though in order of magnitude the same coefficient of viscosity of the nuclear liquid is required in order to describe dissipative effects in different nuclear phenomena.

It is interesting that the Brownian motion itself is not, strictly speaking, of a Markov nature: a particle excites weakly damped elastic waves in the medium, which act on its subsequent motion (viscous after-effect). In large-amplitude nuclear collective motion the analog of this is[163] the excitation of long-lived vibrational modes of the giant resonance type, constructed on slowly varying values of the global variables $Q(t)$. The frequencies of such modes depend strongly on the forms of the intermediate nuclear states. Because of the large Coulomb perturbations in fission, just as in heavy-ion collisions, the excitation of the lowest isovector modes, which can serve as the input states (Sec. 2) for the process of relaxation of global motion, most likely occurs. In addition to the decay of the global energy, such vibrations play a very important role in the rapid establishment of equilibrium with respect to the ratio Z/A which occurs.[164,165] These processes are included in the general scheme of the introduction[166] of new collective variables, describing mass and charge distributions.

3.7. Quantum effects in macroscopic phenomena. Nonconservation of parity accompanying fission. Unusual rare decays

We can see that the idea of quantum collective motion in nuclei, originated by N. Bohr, lies at the foundation of our understanding of fission and unifies this process with the phenomena (Sec. 2) which do not required large departures from equilibrium. In recent years the fundamental problems of the existence of "real quantum" macroscopic effects have been intensively discussed.[167,168] Phenomena in which quantum coherence is observed on macroscopic scales are well known (quantum vortices in a superfluid liquid and in superconductors, Josephson tunneling effects in weak superconducting contacts, etc.). Here, however, we are talking about something different: roughly speaking, is, for example, the motion of the center of mass of an oscillating macroscopic pendulum, consisting of an enormous number of atoms, a quantum motion? In a certain sense nuclear fission is precisely an example of such a real quantum process (tunneling along a collective coordinate, i.e., coordinated passage of 200 nucleons through the classically forbidden region). As follows from the results of Ref. 167, the coupling with noncollective degrees of freedom must lower the probability of such purely quantum collective events. These questions have been very poorly investigated thus far.

A unique manifestation of fundamental quantum laws in fission—nonconservation of spatial parity—was discovered not too long ago.[169] The direction of predominant emission of light fragments accompanying fission by slow polarized neutrons turns out to be correlated with the direction of the spin of the neutrons. According to the theory expounded in Ref. 150 the mixing of states with opposite parity by the weak interaction is most effective at the "hot" stage of the compound nucleus, when the density of state is high and the dynamic amplification factor (11), proportional to $\sqrt{\Gamma_\downarrow/D}\sim 10^3$ and giving rise to effects of the order of 10^{-4} instead of 10^{-7}, which one would expect on the basis of the weak interaction constant, operates. Here it is a question of the excitation of the nucleus after capture of monochromatic neutrons (the energy spread ΔE is much smaller than the widths of the resonances Γ, while $\Gamma < D$). We are therefore dealing with a definite, though very complex, wave function of the quasistationary state. Expanding this function with respect to the fission channels, i.e., the basis of transitional states around the saddle point, we obtain mixing of the same order of magnitude $\sim 10^{-4}$ for the lowest (rotational) states of opposite parity, characteristic for a mirror asymmetrical (pear-shaped) rotator, which a fissioning nucleus with a saddle deformation is. Fission channels, as we have already discussed, form angular and mass distributions of fragments which carry the observed information on nonconservation of parity. Thus the mechanics of this process incorporates the basic ideas of Bohr's theory of fission: a compound nucleus with a high density of states of a complicted nature, motion along a collective variable, and concentration of energy in the fission degree of freedom. The result is a "real quantum" macroscopic motion, seemingly contradicting the second law of thermodynamics which intensifies the weak details of elementary interactions.

One of the interesting unsolved problems in the physics of fission is the relationship between Bohr's liquid-drop mechanism and cluster processes of the α decay type. The discovery of radioactive decays of the radium isotopes $^{222,223,224}Ra$ with the emission of a ^{14}C nucleus[170,171,172] shows that even here the observed reaction is controlled by shell mechanisms (the decay product is the magic lead nucleus). It has not been excluded that the probability of emission of ^{14}C is increased by the octopole deformation of the ground state, which apparently exists in the isotopes of Ra[61] (in other words, in the context of an answer to the question of the constituent parts of nuclei, the pear-shaped deformation is created by the component of the wave function with an excess ^{14}C cluster above a doubly magic core), so that the process is delayed only by the lowered (approximately by ten orders of magnitude is compared with α decay) penetrability of the barrier. Attempts have been made[172] to give a complete explanation of fission by the mechanism of diffusion growth of the initially appearing quasimolecular cluster state. Study of the transition from deep-inelastic transfers in heavy-ion collisions to complete coalescence in the compound nucleus indicates that here the diffusion path of successive transfer of nucleons can be very significant.[50] In the quasimolecular hypothesis α decay or ^{14}C radioactivity would only be limiting cases of superasymmetrical fission. Since in principle both fission methods are possible, the question is actually one of comparing the probabilities of "liquid-drop" and "cluster" fluctuations. The collection of data on ordinary fission (dependence on the drop parameter Z^2/A, the mass and angular distributions, consistency of data on neutron- and photon-induced fission, experiments on nonconservation of parity, etc.) fits in its entirety into the Bohr interpretation. With regard to the radioactivity with emission of heavy clusters, where a different mechanism could be operating, this question can be solved by collecting experimental data on the charge and energy dependences of the probabilities of these "exotic" decays.[173]

We mention in conclusion that a new range of problems opens up in the region of high excitation energies: is the drop picture with the dominant role of the paramter Z^2/A preserved? How do the fission barriers and the nuclear densities of states change? Is there enough time for a compound nucleus to form, independent of the excitation method? What new information is obtained by opening channels with the creation of mesons nad nucleonic resonances, etc.? It is especially interesting to study the fission of medium nuclei, where existing data are contradictory.[174]

CONCLUSIONS

"In all of world science in our day there has never existed a man who influenced natural science as did Bohr. Amongst all theoretical paths, Bohr's path was the most significant."
P. L. Kapitsa[183]

We have arrived at the end of our protracted review. We have ignored many aspects of N. Bohr's scientific legacy which are of direct relevance to nuclear physics. The series of works devoted to the passage of nuclear particles through matter was completely ignored. The fundamental studies of the philosophy of scientific knowledge and the profound foundations of quantum theory, which could have been illustrated by instructive examples from modern nuclear physics, were also ignored. We discussed only the development of N. Bohr's ideas about nuclear structure proper which are relevant today: the concept of the compound nucleus and the statistical description, collective motion and nuclear fission as phenomena whose understanding is based on the synthesis of ideas presented in the foregoing sections. We have seen that N. Bohr's vivid ideas enriched nuclear physics and gave it a powerful impetus, whose influence has not waned to this day.

It is undoubtedly useful to study the works of the founders of modern physics and to try to understand their style of thinking and their approach to problems. It is not in vain that P. Ehrenfest wrote, having in mind N. Bohr and A. Einstein: "... For them new things are a necessity, because they understand well the old things and clearly see the impossibility of the old, classical explanation." Of course, some specific areas of N. Bohr's work appear now to be obsolete. But, as N. Bohr himself emphasized, speaking about the accomplishments of J. Maxwell, "the basic concepts of physics, which we owe to the great masters are of greatest value.[175] We can completely agree with N. Bohr's wording, appearing in the same work: "... The utmost any theory can do" is that, aside from the interpretation of observed phenomena, it "should be instrumental in suggesting and guiding new developments beyond its original scope."

Now, when it seems that the editors of many scientific journals are concerned mostly with economizing pages, N. Bohr's articles may appear to be overextended and "too convincing." Acknowledging the great influence of the Danish philospher S. Kierkegaard on the formation of his world view, N. Bohr by no means followed his principle that "geniuses have no need for proofs." Again and again N. Bohr, always having in mind the presumed reader and opponent, examines the subject from different sides and revamps and refines the arguments, striving to foresee the objections and provide nuances of his deeply dialectical thought, tirelessly seeking the truth. N. Bohr's well-known utterance "on the two kinds of truth"[176] illuminates his attitude toward science and to the inner world of a true scientist: "To the one kind belong statements so simple and clear that the opposite assertion obviously could not be defended. The other kind, the so-called "deep truths," are statements in which the opposite also contains deep truth. Now the development in a new field will usually pass through stages, in which chaos becomes gradually replaced by order: but it is not least in the intermediate stage, where deep truth prevails that the work is really exciting and inspires the imagination to search for a firmer hold."

This is precisely how the ideas in the field of nuclear structure developed. The single-particle and collective motion—two interacting aspects of nuclear structure—embody two deep truths with regard to nature, originating in the depths of the nucleus. The motion of nucleons and clusters, in its turn, corresponds to the collective aspects of quark physics, underscoring in a specifically nuclear manner the effects of fundamental interactions. This field is the main arena for the experimental and theoretical research of the next generation.

Recalling his teacher E. Rutherford as "the founder of nuclear science,"[177] N. Bohr wrote: "The generations who in coming years pursue the exploration of the world of atoms will continue to draw inspiration from the life and work of the great pioneer." We are fully justified in applying these words to Niels Bohr himself.

[1)]The modern formulation of this problem essentially goes back to N. Bohr. After determining that the proton-electron model of the nucleus is unrealistic, he anticipated the Fermi theory of β decay, arriving at the conclusion that "the expulsion of a β ray from a nucleus may be regarded as the creation of an electron as a mechanical entity."[3] Later on he repeatedly emphasized the probabilistic character of this problem.[4] It is interesting that Bohr nevertheless was able to state: "α particles can be considered to a large extent to enter as separate entities into the constitution of these nuclei."[3] Modern data indeed indicate the appreciable weight of strong α-particle correlations of nucleons in the wave function of the nucleus. These four-particle correlations, possibly, could be important in the manifestation of quark effects in nuclei.[178]

[2)]The entire history of this stage of nuclear physics is expounded in detail in Ref. 7.

[3)]It is curious that this problem was already solved by L. Euler in 1753 (see Ref. 155).

[4)]The theory of multiple processes at high energies was formulated under the influence of Bohr's idea of the intermediate state, rapidly relaxing to equilibrium and subsequent thermodynamic decay.

[5)]The Poisson distribution of the energy separations between levels of heavy nuclei was first used by I. I. Gurevich.[23]

[6)]Yrast—from the ancient Scandinavian yr—"rapidly turning": the collection of states of highest energy with successively increasing angular momenta.

[7)]According to a theorem already established by Rayleigh, a charged drop becomes unstable when $E_{coul}/E_{surf} = 2$.

[8)]Ya. Zel'dovich and Yu. Zysin calculated the excitation energy of fragments and proved the inevitability of "evaporative" neutrons.[181]

[9)]Articles dedicated to the memory of N. Bohr were also published in the June 1963 issue of Usp. Fiz. Nauk (Vol. 80, No. 2 [Sov. Phys. Usp. Vol. 6]).

[1]N. Bohr, Naturwissenschaften **18**, 73 (1930).
[2]N. Bohr, Nature. Suppl. **118**, 51 (1926).
[3]N. Bohr, J. Chem. Soc. 349 (1932).
[4]N. Bohr. Ann. Phys. (Leipzig) **32**, 5 (1938).
[5]N. Bohr, Nature **137**, 344 (1936).
[6]F. Friedman, and V. Weisskopf in: Niels Bohr and the Development of Physics, Pergamon Press, London (1955).
[7]E. Amaldi, Phys. Rept. **111**, 1 (1984).
[8]N. Bohr and F. Kalckar, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **14**, 10 (1937).
[9]G. Breit and E. Wigner, Phys. Rev. **49**, 519, 642 (1936); H. Bethe, Phys. Rev. **50**, 332 (1936); Rev. Mod. Phys. **9**, 69 (1937).
[10]Ya. I. Frenkel', Phys. Z. Sowjetunion **9**, 533 (1936).
[11]L. D. Landau, Phys. Z. Sowjetunion **11**, 556 (1937).
[12]V. F. Weisskopf, Phys. Rev. **52**, 295 (1937).
[13]N. Bohr, R. Peierls, and G. Placzek, Nature **144**, 200 (1939).
[14]V. F. Weisskopf, Helv. Phys. Acta **23**, 187 (1950).
[15]H. Feshbach, C. E. Porter, and V. Weisskopf, Phys. Rev. **90**, 166 (1953).
[16]P. E. Nemirovskiĭ, Sovremennye modeli atomnogo yadra (Contemporary Problems in Nuclear Physics), Atomizdat, Moscow (1960).
[17]A. Bohr and B. Mottelson, *Nuclear Structure*, W. A. Benjamin, Reading, MA (1969), Vol. 1.
[18]T. A. Brody *et al.*, Rev. Mod. Phys. **53**, 385 (1981).
[19]E. P. Wigner, Ann. Math. **53**, 36 (1951); **62**, 248.
[20]J. Von Neumann and E. P. Wigner, Phys. Z. **30**, 467 (1929).
[21]I. I. Gurevich and M. I. Pevsner, Nucl. Phys. **2**, 575 (1957).
[22]F. J. Dyson, J. Math. Phys. **3**, 140, 157, 166 (1962); M. L. Mehta, *Random Matrices and the Statistical Theory of Energy Levels*, Academic Press, N.Y. (1967).
[23]I. I. Gurevich, Zh. Eksp. Teor. Fiz. **9**, 1283 (1939).
[24]B. V. Chiricov, Phys. Rept. **52**, 263 (1979).
[25]G. M. Zaslavsky, *ibid.* **80**, 157 (1981).
[26]B. V. Chiricov, F. M. Izrailev, and D. L. Shepelyansky, Sov. Sci. Rev. Sect. C **2**, 209 (1981).
[27]M. V. Berry, Ann. Phys. (N.Y.) **131**, 163 (1981).
[28]O. Bohigas *et al.*, Phys. Rev. Lett. **52**, 1 (1984).
[29]V. M. Strutinskiĭ and A. G. Magner in: Proceedings of the 10th Winter School at the Leningrad Institute of Nuclear Physics, Leningrad Institute of Nuclear Physics of the USSR Academy of Sciences, Leningrad (1975), Ch. 1, p. 251.
[30]P. Pechukas, Phys. Rev. Lett. **51**, 943 (1983).
[31]V. I. Tatarskiĭ, Usp. Fiz. Nauk **139**, 587 (1983) [Sov. Phys. Usp. **26**, 311 (1983)].
[32]C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).
[33]J. P. Draayer, J. B. French, and S. S. M. Wong, Ann. Phys. (N.Y.) **106**, 472, 503 (1977).
[34]V. V. Flambaum and O. P. Sushkov, Nucl. Phys. A **412**, 13 (1984).
[35]O. P. Sushkov and V. V. Flambaum, Yad. Fiz. **33**, 59 (1981) [Sov. J. Nucl. Phys. **33**, 31 (1981)].
[36]R. J. Blin-Stoyle, Phys. Rev. **120**, 181 (1960).
[37]I. S. Shapiro, Usp. Fiz. Nauk **95**, 647 (1868) [Sov. Phys. Usp. **11**, 582 (1968/1969)].
[38]T. Ericson, Ann. Phys. (N.Y.) **23**, 390 (1963).
[39]E. L. Feĭnberg in: Proceedings of Symposium on Problems in Nuclear Physics, Tbilisi (1967), Institute of Theoretical and Experimental Physics, Moscow (1967), Vol. 2, p. 389.
[40]A. I. Baz' in: Proceedings of the 13th Winter School at the Leningrad Institute of Nuclear Physics, Leningrad Institute of Nuclear Physics of the USSR Academy of Sciences, Leningrad (1978), p. 5.
[41]V. S. Ol'khovskiĭ, Fiz. Elem. Chastits At. Yadra **15**, 289 (1984) [Sov. J. Part. Nucl. **15**, 130 (1984)].
[42]D. Agassi, H. A. Weidenmuller, and G. Mantzouranis, Phys. Rept. C. **22**, 145 (1975).
[43]H. Feshbach, A. K. Kerman, and S. Koonin, Ann. Phys. (N.Y.) **125**, 429 (1980).
[44]F. Zhivopistsev and V. G. Sukharevskiĭ, Fiz. Elem. Chastits At. Yadra **15**, 1208 (1984) [Sov. J. Part. Nucl. **15**, 539 (1984)].
[45]B. A. Rumyantsev and S. A. Kheifets, Yad. Fiz. **21**, 510 (1975) [Sov. J. Nucl. Phys. **21**, 267 (1975)].
[46]W. A.Friedman *et al.*, Phys. Rept. **77**, 47 (1981).
[47]K. W. McVoy and X. T. Tang, *ibid.* **94**, 139 (1983).
[48]K. Zaidel' *et al.*, Fiz. Elem. Chastits. At. Yadra **7**, 499 (1976) [Sov. J. Part. Nucl. **7**, 192 (1976)].
[49]J. J. Griffin, Phys. Rev. Lett. **17**, 478 (1966); Phys. Lett. B **24**, 5 (1967).
[50]V. V. Volkov, Yadernye reaktsii glubokoneuprugikh peredach (Nuclear Reactions with Deep Inelastic Transfer), Ėnergoizdat, Moscow (1982).
[51]N. Bohr, *The Unity of Knowledge*, Doubleday and Co., New York (1955), p. 17.
[52]A. Bohr and B. Mottelson, *Nuclear Structure*, Benjamin, New York (1974), Vol. II, Vol. 2.
[53]M. G. Mayer, Phys. Rev. **78**, 16, 22 (1950).
[54]J. Rainwater, *ibid.* **79**, p. 432.
[55]A. Bohr, *ibid.* **81**, 134 (1950).
[56]A. Bohr and B. Mottelson, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **27**, No. 16 (1953).
[57]N. Bohr, Nature **141**, 1096 (1938).
[58]A. Bohr, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **26**, 14 (1952).
[59]S. T. Belyaev and V. G. Zelevinsky, Nucl. Phys. **49**, 582 (1962).
[60]I. Hamamoto and B. R. Mottelson, Phys. Lett. B **132**, 7 (1983).
[61]G. A. Leander *et al.*, Nucl. Phys. A **388**, 425 (1983).
[62]A. Bohr, *Rotational States of Atomic Nuclei*, Munksgaard, Copenhagen (1954).
[63]G. Alaga *et al.*, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **29**, No. 9 (1955).
[64]A. Bohr and B. Mottelson, *ibid.* **30**, No. 1.
[65]D. R. Inglis, Phys. Rev. **103**, 786 (1956).
[66]S. T. Belyaev and V. G. Zelevinskiĭ, Yad. Fiz. **11**, 741 (1970) [Sov. J. Nucl. Phys. **11**, 416 (1970)].
[67]R. M. Rockmore, Phys. Rev. **118**, 1645 (1960).
[68]N. Bohr, Studies over metallernes elektrontheori, Thaning og Appel, Copenhagen (1911).
[69]H. J. Van Leuwen, J. Phys. Radium **2**, 361 (1921).
[70]A. Bohr, B. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).
[71]S. T. Belyaev, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. **31**, No. 11 (1959).
[72]V. G. Solov'ev, Vliyanie parnykh korrelyatsiĭ sverkhprovodyashchego tipa na svoistva atomnykh yader (Influence of Pair Correlations of the Superconducting Type on the Properties of Atomic Nuclei), Atomizdat, Moscow (1963).
[73]L. D. Landau, Zh. Eksp. Teor. Fiz. **30**, 1058 (1956); **32**, 59 (1957) [Sov. Phys. JETP **3**, 920 (1956); **5**, 101 (1957)].
[74]V. G. Vaks, V. M. Galitskiĭ, and A. I. Larkin, Zh. Eksp. Teor. Fiz. **41**, 1655 (1961) [Sov. Phys. JETP **14**, 1177 (1962)].
[75]E. E. Sapershtein, S. A. Fayans, and V. A. Khodel', Fiz. Elem. Chastits At. Yadra **9**, 221 (1978) [Sov. J. Part. Nucl. **9**, 91 (1978)].
[76]G. Gneuss and W. Greiner, Nucl. Phys. A **171**, 449 (1971).
[77]V. G. Zelevinskiĭ, Izv. Akad. Nauk SSSR, Ser. Fiz. **48**, 79 (1984); **49**, 65 (1985) [Bull. Acad. Sci. USSR Phys. Ser. **49**, 68 (1985)].
[78]O. K. Vorov and V. G. Zelevinskiĭ, Yad. Fiz. **37**, 1392 (1983) [Sov. J. Nucl. Phys. **37**, 830 (1983)].

[79]D. Janssen, R. V. Jolos, and F. Donau, Nucl. Phys. A 224, 93 (1974).
[80]A. Arima and F. Iachello, Ann. Phys. (N.Y.) 99, 253 (1976); 111, 201 (1978); 123, 468 (1979).
[81]I. Kh. Lemberg and V. M. Mikhaĭlov, Izv. Akad. Nauk SSSR, Ser. Fiz. 46, 2170 (1982) [Bull. Acad. Sci. USSR Phys. Ser. 46, No. 11, 106 (1982)].
[82]J. Speth and A. vander Woude, Rep. Prog. Phys. 44, 719 (1981).
[83]M. G. Urin, Fiz. Elem. Chastits At. Yadra 15, 245 (1984) [Sov. J. Part. Nucl. 15, 109 (1984)].
[84]R. Bergere in: Lecture Notes in Physics, Springer-Verlag, Berlin (1977), Vol. 61, p. 1.
[85]W. Bothe and W. Gentner, Z. Phys. 112, 45 (1939).
[86]N. Bohr, Nature 141, 326 (1938).
[87]N. Auerbach *et al.*, Rev. Mod. Phys. 44, 48 (1972).
[88]A. B. Migdal, J. Phys. USSR 8, 331 (1944).
[89]P. G. Hansen, Adv. Nucl. Phys. 7, 159 (1973).
[90]Yu. V. Gaponov and Yu. S. Lyutostanskiĭ, Fiz. Elem. Chastits At. Yadra 12, 1324 (1981) [Sov. J. Part. Nucl. 12, 528 (1981)].
[91]N. I. Pyatov and S. A. Fayans, *ibid.* 14, 953 (1983) [Sov. J. Part. Nucl. 14, 401 (1983)].
[92]A. B. Migdal, Rev. Mod. Phys. 50, 107 (1978).
[93]E. Moniz, Nucl. Phys. A 374, 557 (1982).
[94]B. A. Rumyantsev, Yad. Fiz. 15, 46 (1972) [Sov. J. Nucl. Phys. 15, 28 (1972)].
[95]S. T. Belyaev and V. G. Zelevinskiĭ, Yad. Fiz. 17, 525 (1973) [Sov. J. Nucl. Phys. 17, 269 (1973)].
[96]V. G. Zelevinskiĭ and M. I. Shtokman, Izv. Akad. Nauk SSSR, Ser. Fiz. 36, 2577 (1972) [Bull. Acad. Sci. USSR Phys. Ser. 36, 2236 (1972)].
[97]J. O. Newton *et al.*, Nucl. Phys. A 141, 631 (1970).
[98]M. J. A. de Voigt *et al.*, Rev. Mod. Phys. 55, 949 (1983).
[99]I. M. Pavlichenkov, Usp. Fiz. Nauk 133, 193 (1981) [Sov. Phys. Usp. 24, 79 (1981)].
[100]B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. 5, 511 (1960).
[101]Yu. T. Grin' and A. I. Larkin, Yad. Fiz. 2, 40 (1965) [Sov. J. Nucl. Phys. 2, 27 (1966)].
[102]A. Johnson, H. Ryde, and J. Sztarkier, Phys. Lett. B 34, 605 (1971).
[103]R. A.Sorensen, Rev. Mod. Phys. 45, 353 (1973).
[104]J. D. Garrett, Nucl. Phys. A 409, 259 (1983).
[105]P. Stephens, Rev. Mod. Phys. 47, 43 (1975).
[106]R. Bengtsson and S. Frauendorf, Nucl. Phys. A 314, 27 (1979); 327, 139.
[107]R. Bengtsson *et al.*, *ibid.* 405, 221 (1983).
[108]A. Bohr and B. R. Mottelson, J. Phys. Soc. Jpn. Suppl. 44, 157 (1978).
[109]J. Pederson, *et al.*, Phys. Rev. Lett. 39, 990 (1977).
[110]S. Cohen, F. Plasil and W. J. Swiatecki, Ann. Phys. (N.Y.) 82, 557 (1974).
[111]V. M. Galitskiĭ and A. B. Migdal, Zh. Eksp. Teor. Fiz. 34, 139 (1958) [Sov. Phys. JETP 7, 96 (1958)].
[112]A. B. Migdal, Teoriya konechnykh fermi-sistem i svoĭstva atomnykh yader (Theory of Finite Fermi Systems and the Properties of Atomic Nuclei), Nauka, Moscow (1965); 2nd Edition (1982).
[113]V. A. Khodel and E. E. Saperstein, Phys. Rep. 92, 183 (1982).
[114]E. E. Sapershteĭn and V. A. Khodel', Zh. Eksp. Teor. Fiz. 81, 22 (1981) [Sov. Phys. JETP 54, 12 (1981)].
[115]S. T. Belyaev, Yad. Fiz. 1, 3 (1965) [Sov. J. Nucl. Phys. 1, 3 (1965)].
[116]S. T. Belyaev and V. G. Zelevinskiĭ, Yad. Fiz. 1, 13 (1965); 2, 51 (1965) [Sov. J. Nucl. Phys. 1, 10 (1965); 2, 35 (1966)].
[117]S. T. Belyaev and V. G. Zelevinskiĭ, *ibid.* 2, 615 (1965) [Sov. J. Nucl. Phys. 2, 442 (1966)].
[118]N. Bohr, Congres du Palais de la decouverte, Paris (1938).
[119]O. Hahn and F. Strassman, Naturwissenschaften 27, 11 (1939).
[120]L. Meitner and O. Frisch, Nature 143, 239 (1939).
[121]N. Bohr, *ibid.* 330.
[122]N. Bohr, Phys. Rev. 55, 418 (1939).
[123]N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).
[124]I. Halpern, Delenie yader (Nuclear Fission), IL, M., (1962).
[125]K. A. Petrzhak and G. N. Flerov, Zh. Eksp. Teor. Fiz. 10, 1013 (1940); Usp. Fiz. Nauk 73, 655 (1961) [Sov. Phys. Usp. 4, 305 (1961)].
[126]N. Bohr and J. A. Wheeler, Phys. Rev. 56, 1065 (1939).
[127]N. Bohr, *ibid.* 59, 1042 (1941).
[128]Ya. I. Frenkel', Zh. Eksp. Teor. Fiz. 9, 641 (1939).
[129]A. Bohr in: Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955.
[130]I. S. Grant. Rep. Prog. Phys. 39, 955 (1976).
[131]D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953).
[132]S. T. Belyaev and V. G. Zelevinskii, Yad. Fiz. 16, 1195 (1972) [Sov. J. Nucl. Phys. 16, 657 (1972)].
[133]S. T. Belyaev, Nucl. Phys. 64, 17 (1965).
[134]V. G. Zelevinskiĭ, Nucl. Phys. A 337, 40 (1980).
[135]M. Baranger and M. Veneroni, Ann. Phys. (N.Y.) 114, 123 (1978).
[136]P. W. Anderson, Rev. Mod. Phys. 38, 298 (1966).
[137]S. Cohen and W. J. Swiatecki, Ann. Phys. (N.Y.) 22, 406 (1963).
[138]Plutonium Project, Rev. Mod. Phys. 18, 539 (1946).
[139]W. D. Myers and W. J. Swiatecki, Ann. Phys. (N.Y.) 55, 395 (1970).
[140]J. R. Nix, Ann. Rev. Nucl. Sci. 22, 65 (1972).
[141]V. M. Strutinsky, Nucl. Phys. A 95, 420 (1967); 122, 1 (1968).
[142]M. Brack *et al.*, Rev. Mod. Phys. 44, 320 (1972).
[143]D. P. Grechukhin and G. A. Pik-Pichak, Preprint IAE-2115, Moscow (1971).
[144]S. Bjornholm and J. E. Lynn, Rev. Mod. Phys. 52, 75 (1980).
[145]S. M. Polikanov *et al.*, Zh. Eksp. Teor. Fiz. 15, 1016 (1962).
[146]Yu. M. Tsipenyuk *et al.*, Usp. Fiz. Nauk 144, 3 (1984) [Sov. Phys. Usp. 27, 649 (1984)].
[147]V. V. Pashkevich, Nucl. Phys. A 169, 275 (1971).
[148]P. Moller and R. Nix, Nucl. Phys. A 229, 269 (1974).
[149]V. M. Strutinskiĭ, Zh. Eksp. Teor. Fiz. 30, 603 (1956); 39, 781 (1960) [Sov. Phys. JETP 3, 636 (1956); 12, 546 (1961)].
[150]O. P. Sushkov and V. V. Flambaum, Usp. Fiz. Nauk 136, 3 (1982) [Sov. Phys. Usp. 25, 1 (1982)].
[151]V. V. Flambaum, Preprint No. 84–119, Institute of Nuclear Physics, Siberian Branch of the USSR Academy of Sciences, Novosibirsk (1984).
[152]J. Wheeler in: Ref. 6.
[153]H. J. Specht, Rev. Mod. Phys. 46, 773 (1974).
[154]P. Glassel, H. Rosler, and H. J. Specht, Nucl. Phys. A 256, 220 (1975).
[155]T. Ericson, Adv. Phys. 9, 425 (1960).
[156]A. I. Vdovin *et al.*, Fiz. Elem. Chastits At. Yadra 7, 952 (1976) [Sov. J. Part. Nucl. 7, 380 (1976)].
[157]L. Landau, Phys. Z. Sowjetunion 1, 88 (1932); 2, 46.
[158]C. Zener, Proc. R. Soc. London Ser. A 137, 696 (1932).
[159]L. Wilets, Phys. Rev. 116, 372 (1959).
[160]J. W. Negele, Rev. Mod. Phys. A 54, 913 (1982).
[161]H. Flocard *et al.*, Nucl. Phys. A 339, 336 (1980).
[162]H. Hofmann and P. J. Siemens, Nucl. Phys. A 257, 165 (1976); 275, 164 (1977).
[163]V. G. Zelevinskiĭ in: Proceedings of the 12th Winter School at the Leningrad Institute of Nuclear Physics, Leningrad Institute of Nuclear Physics of the USSR Academy of Sciences, Leningrad (1977), p. 53.
[164]H. Hofmann *et al.*, Z. Phys. A. 293, 229 (1979).
[165]P. N. Isaev, Yad. Fiz. 34, 1207 (1981) [Sov. J. Nucl. Phys. 34, 671 (1981)].
[166]S. Yamaji *et al.*, Z. Phys. A 278, 69 (1976).
[167]A. O. Caldeira and A. J. Legett, Ann. Phys. (N.Y.) 149, 374 (1983).
[168]K. K. Likharev, Usp. Fiz. Nauk 139, (1983) [Sov. Phys. Usp. 26, 87 (1983)].
[169]G. V. Danilyan *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. 26, 197 (1977) [JETP Lett. 26, 186 (1977)].
[170]H. J. Rose and G. A. Jones, Nature 307, 245 (1984).
[171]P. B. Price *et al.*, Phys. Rev. Lett. 54, 297 (1985).
[172]V. A. Shigin, Yad. Fiz. 27, 67 (1978) [Sov. J. Nucl. Phys. 27, 35 (1978)].
[173]D. N. Poenaru *et al.*, J. Phys. G 10, L183 (1984).
[174]L. A. Vaishnene *et al.*, Z. Phys. 302, 143 (1981).
[175]N. Bohr, Nature 128, 691 (1931).
[176]N. Bohr in: Albert Einstein—Philosopher-Scientist (Ed. P. A. Schilpp), Tudor, N.Y., 1951, p. 240.
[177]N. Bohr, Proc. Phys. Soc. London 78, 1083 (1961).
[178]L. A. Sliv, M. I. Strikman, and L. L. Frankfurt, Usp. Fiz. Nauk 145, 553 (1985) [Sov. Phys. Usp. 28, 281 (1985)].
[179]Niels Bohr, His Life and Work. Collection of Articles, (Ed. S. Rozental) North-Holland, Amsterdam, 1967 [Russ. Transl. Nauka, M., 1967.]
[180]D. V. Aleksandrov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. 40, 152 (1984) [JETP Lett. 40, 909 (1984)].
[181]Ya. Zel'dovich and Yu. Zysin, Zh. Eksp. Teor. Fiz. 10, 831 (1940).
[182]Ya. B. Zel'dovish and Yu. B. Khariton, Usp. Fiz. Nauk 139, 501 (1983) [Sov. Phys. Usp. 26, 279 (1983)].
[183]P. L. Kapitsa, Priroda, No. 1, 67 (1963).

Translated by M. E. Alferieff