Group-theoretical methods in physics

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Symmetries play an enormous role in physical processes and by Noether's theorem they are closely linked to the laws of conservation of physical quantities in mechanics, field theory, and quantum physics. The concept of the symmetry of an object, which made use of graphic representations of the spatial form of perfect crystals for the first time in physics, has undergone a substantial evolution: the discovery of the symmetry of Maxwell's equations in Minkowski space with respect to the Lorentz transformations, followed by the higher symmetry of the equations of gravitation, the equations of the general theory of relativity under arbitrary transformations of coordinates in space and time, and today the idea of gauge symmetries, which form the foundation of the present understanding of the nature of interactions between elementary particles. The concept of isotopic symmetry of strong interactions plays a large role in nuclear physics and in the physics of elementary particles. The mathematical apparatus, on the basis of which it is possible to describe rigorously the symmetry of physical objects and the laws of nature, is the apparatus of the theory of groups and the theory of representations of groups, and in recent years also the theory of supergroups and the theory of representations of supergroups, which make use not only of transformations of the ordinary coordinates, but also supertransformations, and this permits unifying into a single multiplet bosons or fermions not only separately, but also together, i.e., bosons and fermions are placed into one supermultiplet.

Thus, the methods of the theory of representations of groups (theory of symmetries and supersymmetries) have been extensively developed during the last ten years and are used in different areas of physics, primarily in the quantum theory of fields, the physics of elementary particles, and nuclear physics, as well as in quantum optics, traditionally in solid-state physics, mathematical physics and statistics, and the theory of nonlinear equations (solitons). In this connection, since the group-theoretical approach is used in so many areas of theoretical physics, a separate branch "Group-Theoretical Methods in Physics" has appeared. In 1972, an international colloquium on "Group-Theoretical Methods in Physics," was organized and has become traditional and meets annually in the countries of America, Europe, and Asia,. The first colloquium was held in Marseilles (France, 1972); the second in Nihmegen (Holland, 1973); the third in Marseilles (France, 1974); the fourth in Nijmegen (Holland, 1975); the fifth in Montreal (Canada, 1976); the 6th in Tübigen (FRG, 1977); the seventh in Austin (USA, 1978); the eighth in Qiryat Anaville (Israel, 1979); and ninth in Newbury (England, 1980); the tenth in Coquojaca (Mexico, 1981); the eleventh in Istanbul (Turkey, 1982); the twelfth at the International Center for Theoretical Physics in Trieste (Italy, 1983); the thirteenth at the University of Maryland (USA, 1984). The 1985 colloquium (the fourteenth one) will be held in Seoul (South Korea) and the fifteenth colloquium will be held in 1986 in Philadelphia (USA). The colloquia on Group-Theoretical Methods in Physics are organized by a permanent international committee which includes theoreticians from different countries (E. Wigner, L. Bidenharn, L. Michel, M. Moshinsky, M. Hamermesh, Y. Neeman, A. Bohm, H. Bakri, B. Judd, and others), who are actively applying the methods of the theory of symmetries in their research, as well as a representative from the USSR. The proceedings of all colloquia are published by different publishers. The purpose of these colloquia is to bring together for discussion physicists and mathematicians who are working in different areas but use the same method, and this permits rapid transfer of new results obtained in one area into all other areas where group-theoretical methods are used. For example, the theory of representations of simplectic groups, used in classical and quantum mechanics to describe small oscillations (of a harmonic oscillator), have turned out to be important in the solution of the problem of joining two light guides without radiation losses at the joint (antireflection optics of light guides), which can be important for collecting light in underwater experiments on the detection of the neutrino within the framework of the DUMAND project. Such possibilities were discussed at the colloquium at the University of Maryland.

To give an idea of the scope of the physical problems addressed at the colloquia on "Group-Theoretical Methods in Physics" we shall mention the parts of the program at the 13th colloquium in Maryland: plasma, chaos and nonlinear dynamics, mathematical physics and general questions, atomic physics, nuclear physics, chemical physics, solid-state physics, elementary particles, and the theory of relativity. Most of the programs at the last colloquia involved a discussion of results on the solution of nonlinear equations, particularly soliton solutions. As part of the activity of the colloquia on "Group-Theoretical Methods in Physics" a prize–the Wigner Medal–is awarded every two years to physicists and mathematicians actively working on the application of group methods. Its laureates, in addition to E. Wigner, Include Professor V. Bargman, I. M. Gel'fand (Corresponding Member of the USSR Academy of Sciences), Professor Y. Neeman, and Professor L. Michel.

In connection with the large number of papers on the use of group-theoretical methods in different areas of physics, the Division of Nuclear Physics together with the P. N. Lebedev Physics Institute of the USSR Academy of Sciences organized in 1979 and 1982 international seminars in Zvenigorod on "Group-Theoretical Methods in Physics," which are held in conjunction with the work of the permanent in-

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ternational committee of the colloquium with the same name. The proceedings of each seminar were published in the form of two-volume sets by Nauka Press. The proceedings of the second seminar will also be published in the English language by Gordon and Breach. The third seminar will be held in 1985. The subject matter of "Group-Theoretical Methods in Physics," encompassing a wide range of problems in different areas of physics, has become one of the interesting areas of research in modern theoretical physics.

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