Z. G. Berezhiani and Dzh. L. Chkareuli. *Horizontal symmetry: masses and mixing angles of quarks and leptons of different generations; neutrino mass and neutrino oscillation.*

1. Three generations of quarks and leptons

$$(\mathrm{u},\ \mathrm{d},\ \nu_e,\ \mathrm{e}),\ (\mathrm{c},\ \mathrm{s},\ \nu_\mu,\ \mu),\ (\mathrm{t},\ \mathrm{b},\ \nu_\tau,\ \tau) \qquad (1)$$

with identical strong and electroweak interactions with the standard, experimentally confirmed, symmetry $SU(3)_c \otimes SU(2) \otimes U(1)$[1] have now been discovered (although the t quark has not yet been discovered, its existence is not doubted).[1]

The identity of these generations strongly suggests that an additional symmetry transforming one generation into another, the so-called horizontal symmetry, could exist. It appears that an entire range of phenomena, which are essentially unexplainable within the framework of the standard scheme, such as the splitting of the masses between generations of quarks and leptons, mixing of quarks, CP violation, etc., could be associated with the spontaneous breaking of this symmetry. If it is assumed (in analogy to the standard $SU(3)_c \otimes SU(2) \otimes U(1)$ that the horizontal symmetry is local, then this indicates the existence of a new "horizontal" interaction, which changes the quark-leptonic flavors. It follows from the experimental limit on the rate of the processes caused by this interaction ($K^0 \to \overline{K}^0$, $K_L \to \overline{\mu}e$, etc.) that the distances characteristic for it must be very small: $R < 10^{-6}$ GeV^{-1}.

2. For the three generations of quarks and leptons (1), it is natural to adopt the group $SU(3)_H$ (22) as the horizontal symmetry group. The large splitting between the masses of the different generations (for example, for the bottom quarks $m_d : m_s : m_b \approx 1{:}20{:}600$) supports the chiral filling of this group, when the "right" (R) components of the quarks and leptons transform as triplets while their "left" (L) components transform as $SU(3)_H$ antitriplets:

$$q_L^\alpha = \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix}_L^\alpha, \quad l_L^\alpha = \begin{pmatrix} \nu \\ \mathrm{e} \end{pmatrix}_L^\alpha, \quad u_{R\alpha}, \quad d_{R\alpha}, \quad e_{R\alpha};$$

$$\alpha = 1,\ 2,\ 3\ (SU(3)_H) \qquad (2)$$

(or vice versa). In the case of vector filling, when the "left" and "right" quarks (and leptons) are both triplets, the large mass splitting observed experimentally is generally speaking impossible.

Spontaneous breaking of the horizontal symmetry $SU(3)_H$ is achieved by a simple set of scalars—two triplets ξ^α and η^α and the sextet $\chi_{\{\alpha\beta\}}$ $SU(3)_H$ ($\alpha, \beta = 1,2,3$)—generating vacuum averages (VA) of the following form (3,4):

$$\langle\xi^\alpha\rangle = (0,\ 0,\ p)^\alpha, \quad \langle\eta^\alpha\rangle = (q,\ 0,\ 0)^\alpha, \quad \langle\chi_{\{\alpha\beta\}}\rangle = \mathrm{diag}\,(r_1,\ r_2,\ r_3)_{\alpha\beta}, \qquad \hat{V}_H = \begin{pmatrix} r_1 & p & 0 \\ -p & r_2 & q \\ 0 & -q & r_3 \end{pmatrix}, \qquad (3)$$

where $\hat{V}_H$ is the full VA matrix of the horizontal symmetry whose matrix elements follow the hierarchy $r_3 \gg q \gg r_2 \gg p \gg r_1$ (on the average differing by an order of magnitude from one another).

3. The appearance of a quark-leptonic mass spectrum depends on the form of the Yukawa couplings in the theory. The simplest variant of the studied model contains only the standard Higgs doublet $\varphi = \binom{\varphi^+}{\varphi^0}$ and the horizontal scalars (3). In the case of $SU(3)_H$ the symmetry admits only unrenormalizable quadrilinear (with respect to the fields) couplings (5). For example, for the bottom quarks these couplings have the form:

$$\frac{1}{M_P}\,(\bar{q}_L)_\alpha\, d_{R\beta}\varphi\,(f_1\varepsilon^{\alpha\beta\gamma}\bar{\xi}_\gamma + f_2\varepsilon^{\alpha\beta\gamma}\eta_\gamma + f_3\bar{\chi}^{\{\alpha\beta\}}), \qquad (4)$$

where M_p is the Planck mass, which enters as a natural regulator. It is generally believed that gravitation must induce such couplings at supershort distances. They arise naturally in schemes with supergravitation.

After the horizontal scalars develop their VA and the scalar $\varphi(\langle\varphi^0\rangle = v)$ condenses, the mass matrices of quarks and leptons $\hat{m}^u$, $\hat{m}^d$, and $\hat{m}^e$, practically repeating the structure of the matrices of the horizontal VA(3), $\hat{m} \sim (v/M_P)\hat{V}_H$, arise. To have quark and lepton masses of the correct order of magnitude, it is evidently necessary to adopt for the largest of the VA, breaking $SU(3)_H$, the value $r_3 = O(10^{-1})M_P$.

The obtained mass matrices $\hat{m}^u$, $\hat{m}^d$, and $\hat{m}^e$, in principle correctly reflect the regularity in the growth of the quark-leptonic masses from one generation to another and give (in order of magnitude) the experimentally observed small quark mixing angles. The number of independent parameters in these matrices, however, is still too large to obtain accurate values of the masses and mixing angles.

4. Additional restrictions on the obtained mass matrices appear in grand unification models, in particular, in the most popular of these models SU(5).[6] If it is assumed that the quarks and leptons of each of the generations (1) fill the reducible multiplet $\bar{5} + 10$ of the SU(5) group, then all three generations (1) must form the multiplet $(\bar{5} + 10, \bar{3})$ of the group $SU(5) \otimes SU(3)_H$ in accordance with the chiral nature of $SU(3)_H$.

We now discuss the scalar composition of the theory. The introduction of a single Higgs 5-plet into the Yukawa couplings of the type (4) (as in the standard model) gives mass matrices of bottom quarks and leptons $\hat{m}^d$ and $\hat{m}^e$ which are trivially proportional to one another, which does not agree with experiment. On the other hand, in the general case, when together with the 5-plet a scalar 45-plet is also introduced, the matrices $\hat{m}^d$ and $\hat{m}^e$ are completely independent, as also in the case of the standard $SU(3)_c \otimes SU(2) \otimes U(1)$ scheme discussed above.

Empirically acceptable mass matrices $\hat{m}^d$ and $\hat{m}^e$ arise if in the Yukawa couplings of the type (4) the scalar 5-plet "acts" together with the horizontal scalars ξ and χ, while the 45-plet acts only together with the scalar η. This can be achieved by introducing together with $SU(3)_H$ an additional local symmetry $U(1)_H$ (5), corresponding to the horizontal hypercharge Y_H. It is remarkable that the values of Y_H for all fields of fermions and scalars which fix the required structure of $\hat{m}^d$ and $\hat{m}^e$ automatically lead to a diagonal form of the matrices of the top quarks $\hat{m}^u$ [3,4]:

$$\hat{m}^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad \hat{m}^d = \rho \begin{pmatrix} m_u & a & 0 \\ -a & m_c & b \\ 0 & -b & m_t \end{pmatrix},$$

$$\hat{m}^e = \rho \begin{pmatrix} m_u & a & 0 \\ -a & m_c & -3b \\ 0 & 3b & m_t \end{pmatrix}, \tag{5}$$

where, aside from the masses of the top quarks u, c, and t, the matrices include the complex parameters a and b and the dimensionless parameter ρ. Diagonalization of these matrices leads[4] to the mass relations

$$m_d m_s \approx m_e m_\mu, \quad m_b \approx (m_\tau - m_\mu) \pm m_s,$$

$$m_t \approx 8 m_c \left(9 \frac{m_s}{m_b} \pm \frac{m_\mu}{m_\tau} \right)^{-1} \tag{6}$$

and the Kobayashi-Maskawa quark mixing angles ($s_i = \sin \vartheta_i$, $i = 1,2,3$):

$$s_1 \approx \sqrt{\frac{m_d}{m_s}}, \quad s_2 \approx \frac{1}{2\sqrt{2}} \sqrt{\frac{m_\mu}{m_\tau} \pm \frac{m_s}{m_b}}, \quad s_3 \approx \frac{m_s}{m_b} s_2, \tag{7}$$

where the small uncertainties in the values of the masses and angles correspond to the arbitrariness in the CP-violating phase (not computed in the model), associated with the phases of the off-diagonal masses a and b in the matrices $\hat{m}^d$ and $\hat{m}^e$. The values of the physical masses of the d, s, b, and t quarks in the transition from the SU(5) limit to the laboratory energies

$$m_d^0 \approx 7\ \text{MeV}, \quad m_s^0 \approx 130\ \text{MeV},$$

$$m_b^0 \approx 4.8\ \text{MeV}, \quad m_t^0 \approx (35-70)\ \text{GeV} \tag{8}$$

as also the values of the angles

$$s_1 \approx 0.22, \quad s_2 \approx 0.08 - 0.11, \quad s_3 \approx (2-3)\cdot 10^{-3}, \tag{9}$$

are in good agreement with experiment.

5. The chiral character of the horizontal symmetry $SU(3)_H$ requires the introduction of additional fermions, which compensate the triangular anomalies (with respect to $SU(3)_H$) of quarks and leptons (2). The simplest choice consists of the introduction of 15 horizontal triplets $N_\alpha^{(n)}$ ($\alpha = 1,2,3$ and $n = 1,\ldots,15$).[4] These fields acquire large Majorana masses as a result of the spontaneous breaking of $SU(3)_H$, which within each of the triplets $N_\alpha^{(n)}$ ($\alpha = 1,2,3$) follow the "horizontal" hierarchy

$$m_{N1} : m_{N2} : m_{N3} = r_1 : r_2 : r_3 = m_u : m_c : m_t. \tag{10}$$

On the other hand, these fields are mixed (independently for each generation) with the neutrino fields ν_L^α ($\alpha = 1,2,3$) and thereby generate their low Majorana masses ($m_{\nu_\alpha} \sim v^2/r_\alpha$), which now satisfy not the direct, but rather the inverse "horizontal" hierarchy (7):

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = m_u^{-1} : m_c^{-1} : m_t^{-1}. \tag{11}$$

Thus the heaviest is the neutrino in the first generation (electron neutrino) with a mass of the order of O (1 eV), which is of experimental interest.[8] In the standard SU(5) model it is only of the order of $O(10^{-5}$ eV).[6]

The appearance of neutrino masses must lead to oscillation of the neutrino (9). For leptonic mixing angles obtained in the model ($s_i' = \sin \vartheta_i'$, $i = 1,2,3$) (7):

$$s_1' \approx \sqrt{\frac{m_e}{m_\mu}}, \quad s_2' \approx \frac{3}{2\sqrt{2}} \sqrt{\frac{m_\mu}{m_\tau} \pm \frac{m_s}{m_b}}, \quad s_3' \approx \frac{m_\mu}{m_\tau} s_2', \tag{12}$$

the average probabilities of the transitions $\nu_e \to \nu_\mu$, $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\tau$ are equal to 0.01, 0.2, and 0.001, respectively. The smallness of the $s_1' \approx 7 \cdot 10^{-2}$ naturally explains the difficulty of observing experimentally the neutrino oscillations $\nu_e \to \nu_\mu$.

6. The studied model with $SU(5) \otimes SU(3)_H \otimes U(1)$ symmetry gives a reasonable description of the quark-leptonic mass spectrum and predicts the small quark mixing angles s_2 and s_3 (which is already confirmed experimentally), the mass of the t quark in the range 35–70 GeV, and oscillation of the neutrino in reactor experiments $\nu_e \to \nu_\mu$ at a 1% level.

The unification of the horizontal symmetry $SU(3)_H \otimes U(1)_H$ with SU(5) leads to the universal SU(8) unification of all quark-leptonic flavors.[2] The chiral symmetry $SU(3)_H$, which we examined here, first arose precisely in this model.[2] All basic applications of the horizontal symmetry are contained in subsequent papers.[3,4,5,7,10,11]

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