I. O. Kulik. *Superconductivity of narrow-band metals and semiconductors and the model of superconducting glass.* In recent years, in connection with the problems of superconducting materials technology and the search for high-temperature superconductors,[1] considerable experimental data have been accumulated and the superconducting compounds exhibiting significant deviations from the Bardeen-Cooper-Schrieffer (BCS) model have been discovered. They include, in particular, the following: 1) superconducting oxides with the perovskite structure $BaPb_{1-x}Bi_xO_3$[2] and spinels $Li_{1+x}Ti_{2-x}O_4$,[3] Chevrel phases of $Eu_xMo_6S_8$,[4] tungsten M_xWO_3 and vanadium M_xVO_3 bronzes ($M = Rb$, K, Cs),[6] superconducting semiconductors $Pb_{1-x-y}Na_xTl_yTe$,[6] variable-valence compounds with "heavy fermions" $CeCu_2Si_2$, etc.[7] They have the following general properties: 1) quite high transition temperatures (~ 13 K in the case of $BaPb_{1-x}Bi_xO_3$); 2) nonmonotonic dependence of the superconducting transition temperature T_c on the composition and a transition into the dielectric phase at a definite value of x; 3) semiconducting behavior of the resistance as a function of temperature for $T > T_c$; and, 4) strong sensitivity to the structural state. In this report, the possibility of explaining such properties within the framework of the "superconducting glass" model[8–10] is examined. Analogous models were later studied by Aleksandrov and Ranninger,[1] Robashkevich *et al.*,[12] and Rice and Sneddon.[13]

The electron-phonon interaction (EPI) leads to the appearance of attraction between electrons $U = g^2/\langle\omega\rangle$, where g is the EPI constant and $\langle\omega\rangle$ is the average vibrational frequency. If the value of U is smaller than the Fermi energy (the width of the conduction band) t, i.e., the dimensionless constant $\lambda \sim U/t \lesssim 1$, then we are dealing with the BCS theory of superconductivity or its modifications to the case of "strong" coupling. A different situation appears for $\lambda > 1$ (Fig. 1) when pairs are localized at lattice sites and due to the polaron narrowing of the band ($t \sim t_0 e^{-g^2/\langle\omega^2\rangle}$) the transition temperature does not increase, but rather drops as the coupling constant λ increases. The coherence length (pair

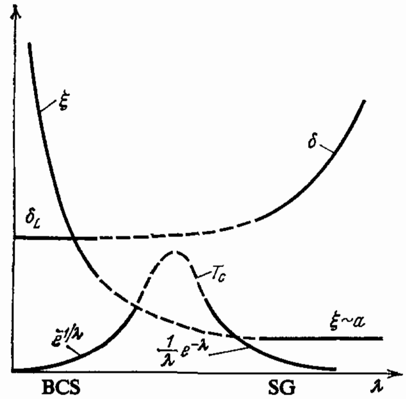

FIG. 1. Schematic behavior of the critical temperature (T_c), coherence length (ξ), and penetration depth of the transverse field (δ) as functions of the coupling constant λ.

radius) in this case is of the order of the lattice spacing, while the transition temperature is of the order t^2/U.

Since the bands are narrow, the disorder in the lattice—fluctuations of the position of the band bottom (diagonal disorder) and the site-to-site transition probability of an electron (off-diagonal disorder) strongly affect T_c. This situation is in some sense reminiscent of the concept of percolation and according to Refs. 14 and 15 the appearance of superconductivity is determined by the attainment of the mobility threshold.

It is precisely for this reason that it is natural to call superconductivity with $\lambda \gtrsim 1$ a "superconducting glass" (SG).

The Hamiltonian of the SG model has the form[10]

$$H = -\sum_i (U_i + \mu) N_i - W_1 \sum_{\langle ij \rangle} A_i^\dagger A_j + W_2 \sum_{\langle ij \rangle} N_i N_j, \qquad (1)$$

where $A_i^+ (A_i)$ is the creation (annihilation) operator of an electron pair at lattice site i, $N_i = A_i^+ A_i$, $W_1 = 2t^2/U$, $W_2 = 2t^2/U + V$, where V is the Coulomb repulsion at neighboring centers. The second term in (1) corresponds to the superconducting pairing and the third term corresponds to the formation of a nonuniform state of the Wigner crystal type—ordering of electron pairs in space.[16] The temperatures of the corresponding transitions in the mean-field approximation are[10]

$$T_c^0 = W_1 z \frac{1-2\nu}{\ln[(1-\nu)/\nu]}, \quad T_d^0 = 2W_2 z\nu(1-\nu) \qquad (2)$$

(z is the number of nearest neighbors in the lattice, ν is the average number of pairs per site, and $\nu = x/2$). The ratio of the quantities W_1 and W_2 determines the type of phase diagram of the SG (Fig. 2). Taking into account interstitial correlations (going outside the framework of the mean-field approximation)[17] does not qualitatively change the picture of the coexistence of the superconducting (SC), charge-ordered (CO), and mixed (M) phases. In the superconducting state, the gap is of the order of T_c (the ratio $2\Delta/T_c \gtrsim 4$), while in the charge-ordered states it is of the order of T_d. Diagonal disorder decreases both T_c and T_d. In this case, in the limit of strong disorder $\delta u \gg T_c^0$, we have (Fig. 2b)

$$T_c = c \frac{\operatorname{th} A}{A} W_1 \left(\frac{\delta U}{W_1}\right)^\alpha, \qquad (3)$$

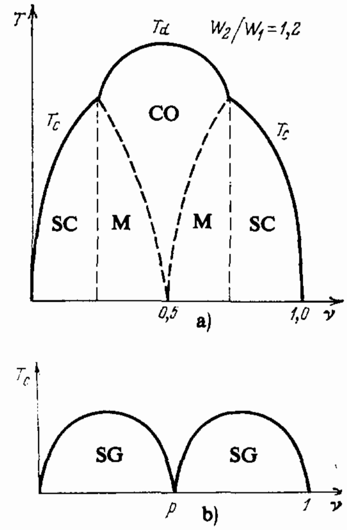

FIG. 2. Phase diagram of a "superconducting glass." a) Ignoring disorder; b) superconductivity in the presence of strong diagonal disorder.

where $0 < \alpha < 1$, whie the value of A is determined from the relation $\tanh A = (2\nu - p)/p$ for $\nu < p$ and $\tanh A = (2\nu - 1 - p)/(1-p)$ for $\nu > p$; p is the relative number of sites with $U_i = \langle U \rangle + \delta U/2$ and $1-p$ is the number of sites with $\langle U \rangle - (\delta U/2)$.

Above the transition temperature, in the presence of a narrow polaron band, the conductivity is primarily determined by single electrons, so that it will increase with the temperature. The SG model actually describes the semiconductor-superconductor transition in exactly the same manner (due to the presence of CO) as the transition into the dielectric state accompanying a change in composition. On the whole, in spite of the apparent large discrepancy in the starting assumptions forming the foundation of the BCS and SG theories, they differ quantitatively rather than qualitatively and because superconductivity in reality is impossible either for very low ($T_c \sim e^{-1/\lambda}$) or for very high ($T_c \sim (1/\lambda) e^{-\lambda t/\langle\omega\rangle}$) values of λ, it can be revealed by a detailed study of quantities such as $2\Delta T_c$, $\Delta c/c$ (Δc is the jump in the heat capacity), H_{c2}/H_{c1} and so on in the cases $\lambda \lesssim 1$ and $\lambda \gtrsim 1$. It is interesting that the value of the critical field H_{c2} in the SG model turns out to be very high[18] and greatly exceeds the so-called paramagnetic limit.

[1] V. L. Ginzburg, Usp. Fiz. Nauk **118**, 315 (1976) [Sov. Phys. USP **19**, 174 (1976)].
[2] B. Batlogg, J. P. Remeika *et al.*, Superconductivity in d- and f-band Metals, edited by W. Buckel and W. Weber, Karlsruhe, 1982, p. 401.
[3] D. C. Johnson, J. Low Temp. Phys. **25**, 145 (1976).
[4] R. Lacoe, S. A. Wolf *et al.*, Phys. Rev. Let. **48**, 1212 (1982).
[5] M. Sato, B. H. Grier *et al.*, J. Phys. Ser. C **16**, 5217 (1983).
[6] I. A. Chernik, S. N. Lykov, and N. I. Grechko, Fiz. Tverd. Tela **24**, 2931 (1982) [Sov. Phys. Solid State **24**, 1661 (1982)].
[7] F. G. Aliev and N. B. Brandt *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 435 (1982) [JETP Lett. **35**, 539 (1982)].
[8] E. Abrahams and I. O. Kulik, Normal-state conductivity, screening and possible transition to the pair-correlated state in superconducting glasses, Preprint FTINT (1977).
[9] I. O. Kulik and A. G. Pedan in: Trudy Vsesoyuznogo soveshchaniya po fizike nizkikh temperatur (Proceedings of the All-Union Conference on Low-Temperature Physics), Moscow, 1978, Vol. 3, p. 87.

[10] I. O. Kulik and A. G. Pedan, Zh. Eksp. Teor. Fiz. **79**, 1469 (1980) [Sov. Phys. JETP **52**, 742 (1980)].
[11] A. Alexandrov and J. Ranninger, Phys. Rev. B **24**, 1164 (1981).
[12] S. Robaszkiewicz, R. Micnas, and K. A. Chao, Phys. Rev. B **24**, 1579 (1981).
[13] T. M. Rice and L. Sneddon, Phys. Rev. Lett. **47**, 689 (1981).
[14] A. G. Pedan and I. O. Kulik, Fiz. Nizk. Temp. **8**, 236 (1982) [Sov. J. Low Temp. Fiz. **8**, 118 (1982)].
[15] R. Micnas, S. Robaszkiewicz, and K. A. Chao, Charge Ordering and Superconductivity in Bipolaronic Systems with Diagonal Disorder, Phys. Rev. (to be published).
[16] S. P. Ionov, Fiz. Khim. **56**, 2385 (1982).
[17] I. O. Kulik and A. G. Pedan, Fiz. Nizk. Temp. **9**, 256 (1983) [Sov. J. Low Temp. Fiz. **9**, 127 (1983)].
[18] L. N. Bulaevskiĭ, A. A. Sobyanin, and D. I. Khomskiĭ, Zh. Eksp. Teor. Fiz. **87**, 1490 (1984) [Sov. Phys. JETP **60**(4), (1984)].

Translated by M. E. Alferieff