V. A. Khoze, *Heavy quarks and perturbation theory in quantum chromodynamics*. Since the physics of heavy quarks Q is associated with short distances, where asymptotic freedom is in effect, many problems concerning them look much simpler than is the case for light quarks. Here there arise a number of predictions, which reflect the most fundamental properties of quantum chromodynamics (QCD) and which follow from its first principles. The large magnitude of the quark mass $M \gg R^{-1}$ (R is the characteristic value of the decolorization radius $R^{-1} = \mu \approx 250$–300 MeV) enables one to use standard perturbation-theory (PT) methods in the analysis, for example, summation of leading logarithms (SLL) (V. N. Gribov and L. N. Lipatov (1972) and Yu. L. Dokshitser (1977).

In the report, which is based on work$^{1-5}$ performed together with Ya. I. Azimov and Yu. L. Dokshitser, the properties of the final hadronic states in strong processes with heavy quarks and in decays of $Q\bar{Q}$ quarkonia are discussed within the framework of perturbative QCD. An expression is presented for the fragmentation function D_H of a heavy Q quark into a hadron H_Q ($H_Q = Q\bar{q}$, Qqq). A check of the consequences of the picture of cascade multiplication of gluons in direct decays of quarkonia is analyzed.

a) Radiation physics of heavy quarks. Because of the massiveness of the Q quark the time for the formation of bremmstrahlung with energy $\omega = z_g E_Q$ and transverse momentum $K_\perp$

$$t_{\text{rad}} \approx \frac{E_Q z_g (1 - z_g)}{k_\perp^2 + M^2 z_g^2}$$

with $z_g \sim 1$ is always less than the gluon hadronization time $\tau_{\text{had}} \approx \omega R^2$, when the interaction becomes strong. For energetic gluons $(k_\perp)_{\text{char}} \gtrsim M$ and only parametricaly soft gluons with $z_g \lesssim \mu/M \ll 1$ are emitted. From here follows the important result that the energy loss by emission is small and is controlled by perturbation theory. The formation of the hadron H_Q occurs not as a result of perturbative effects associated with the capture of light q quarks. At this stage, for $\mu \ll M$, parton considerations show that the heavy quark is only slightly slowed down and the dominant fraction z of its energy E_Q is carried away by the heavy hadron. The leading role of heavy particles is explained along these lines (Ya. I. Azimov, L. L. Frankfurt, V. A. Khoze (1976); M. Suzuki (1977), J. D.Bjorken (1978)). The emitted gluons form an accompanying jet of light hadrons. Due to the leading role and the presence of noticeable clearly identifiable modes of decay of heavy hadrons, a production process for "tagged" heavy particles is essentially realized.

The spectrum of hadrons H_Q, generated by the quark Q, formed at small distances with energy E, can be represented in a simplified manner in the form ($x = E_H/E$)

$$D_H(x) = \int_x^1 \frac{dx_Q}{x_Q} D_Q(x_Q)\, w_H\left(\frac{x}{x_Q}\right), \tag{1}$$

where $D_Q(x_Q)$ is the spectrum of Q quarks ($x_Q = E_Q/E$) and $w_H(z)$ is a function which describes the fragmentation of the Q quark into the hadron H_Q ($\int_0^1 w_H(z)dz = 1$). Perturbative QCD permits obtaining a rigorous model-free expression for $D_Q(x_Q)$, which is infrared-stable in the region $1 - x_Q \gg \mu/M$. For real energies E, when the creation of additonal $Q\bar{Q}$ pairs can be neglected, $D_Q(x_Q)$ is determined only by the fragmentation of the heavy quark $Q \to Q + \ldots$ and is written with the help of the Mellin transformation of the contour integral. In this case, the spectrum of H_Q can be found, generally speaking, only by numerical integration. The average fraction of the energy carried away by the quark, however, is given by the simple expression

$$\langle x_Q \rangle = \left(\frac{\alpha_s(M^2)}{\alpha_s(E^2)}\right)^{-32/81} \tag{2}$$

(the nonlogarithmic corrections weaken the PT emission and increase $\langle x_Q \rangle$). At $E \approx 20$ GeV it follows from (2) that $\langle x_c \rangle \approx 0.75$ and $\langle x_b \rangle \approx 0.85$. For real energies, a simple interpolation formula for $D_Q(x_Q)$ holds to within $\sim 5\%$:

$$D_Q(x_Q) = A\,\frac{1 + x_Q^2}{2}(1 - x_Q)^{-1 + (16/3)\Delta\xi}, \tag{3}$$

$$\int_0^1 D_Q(x_Q)\, dx_Q = 1, \quad \Delta\xi = \frac{1}{9} \ln \frac{\alpha_s(M^2)}{\alpha_s(E^2)}.$$

The presence of a dependence on $\Delta\xi$ determines the energy behavior of $D_H(x)$.

The function $D_Q(x_Q)$ is smooth over the entire range of x_Q, and for a quite sharp form of $w_H(z)$ the entire behavior of $D_H(x)$ in the region

$$(1 - x) \gg \frac{\mu}{H} \tag{4}$$

is determined by the *quark spectrum* D_Q. This behavior is analogous to the well-known physics of "radiation tails" of resonances: in the region (4) emission of gluons with $z_g = 1 - x_Q \approx 1 - x$ turns out to be most favorable in order that $z = E_H/E_Q \to 1$ and $w_H(z)$ act like a delta function, practically independent of its specific form. The entire behavior of $D_H(x)$ in this region actually represents the "radiation tail" and is a universal expression for the spectra of H_Q. For making comparisons with experiment in real cases of c and b quarks ($\mu/M_c \approx 0.2$, $\mu/M_b \approx 0.06$), it is convenient to use an approximate expression of the form

$$D_{\mathrm{H}}(x) \approx \frac{1}{\langle z\rangle} D_{\mathrm{Q}}\left(\frac{x}{\langle z\rangle}\right), \qquad \langle x\rangle = \langle x_{\mathrm{Q}}\rangle \langle z\rangle. \tag{5}$$

To obtain an idea of the behavior of $D_{\mathrm{H}}(x)$ for all x, attention can be restricted to the minimal variant, when a quantum-mechanically well-founded expression is chosen for $w_{\mathrm{H}}(z)$ $(w_{\mathrm{H}} \sim 1/(\Delta E)^2$, $\Delta E = E_{\mathrm{Q}} - E_{\mathrm{H}} - E_{\mathrm{q}})$ (C. Petersson *et al.*, 1983)

$$w_{\mathrm{H}}(z) \approx \frac{N}{z}\left(-1 + \frac{1}{z} + \frac{\varepsilon_{\mathrm{Q}}}{1-z}\right)^{-2}, \qquad \varepsilon_{\mathrm{Q}} = \left(\frac{\mu}{M}\right)^2.$$

In the case of leptonic spectra from decays of heavy hadrons the hadronization of Q quarks is less important, and the theoretical description in the entire region $x_l = E_l/E$ is clearly more reliable.

Existing experimental data on the spectra of D, D*, and F mesons and leptons from the decay of b and c quarks agree with the theory. In particular, experimentally $\langle x\rangle_{\mathrm{c}} \simeq 0.6 \pm 0.1$ and $\langle x\rangle_{\mathrm{b}} \simeq 0.8 \pm 0.1$. To make a detailed check of perturbative QCD, the experimentally obtained statistical sample must be substantially increased.

b) Multihadronic direct decays of quarkonia $\mathrm{Q\bar{Q}}$. Present data on the multiple formation of hadrons in e^+e^- annihilation agree well with the specific predictions of the picture of cascade multiplication of gluons and their subsequent hadronization made by perturbative QCD.

According to this picture, with the use of known distributions of gluons in the decays of quarkonia relations can be obtained which link the experimental characteristics of the direct decays of $\mathrm{Q\bar{Q}}$ directly to the corresponding quantities in e^+e^- annihiltion. In particular, for more or less symmetrical configurations of gluons in the decays[3] of $^3\mathrm{S}_1$ quarkonium $V_{\mathrm{Q}} \to 3g$ simple relations are obtained for the yield of different types of hadrons:

$$\langle n_h(V_{\mathrm{Q}})\rangle = \frac{3}{2}\cdot\frac{9}{4}\,\Delta\left\langle n_{\mathrm{h}}\left(e^+e^-,\ W = \frac{2}{3}M_{V_{\mathrm{Q}}}\right)\right\rangle + \langle n_{\mathrm{h}}(\mathrm{J}/\psi)\rangle, \tag{6}$$

where $\Delta\langle n_{\mathrm{h}}\rangle$ is the increment to the multiplicity in e^+e^- annihilation from gluonic cascades, the factor 3/2 accounts for the transition from two jets (e^+e^-) to three jets (V_{Q}), and the factor 9/4 accounts for the higher probability of emission of soft gluons by a gluon than by a quark (at present energies corrections $\sim\sqrt{\alpha_s}$ modify the combinatorial factor 9/4 by not more than — 10%). The use of the relation (6) for the case of Υ mesons leads, in particular, to values of $\langle n_{\mathrm{ch}}\rangle$, $\langle n_{\bar{\mathrm{p}},\mathrm{p}}\rangle$, and $\langle n_{\bar{\Lambda},\Lambda}\rangle$ which agree well with the data obtained by the CLEO and ARGUS groups. For toponium $^3\mathrm{S}_1(\mathrm{t\bar{t}})$ with $M_{\mathrm{T}} \gtrsim 50$ GeV the value of $\langle n_{\mathrm{ch}}\rangle$ in the direct decays of the resonance and in the background must differ by almost a factor of two, whereas the ratios of the baryon to meson yields at resonance should be close to their values away from resonance.

c) Check of the spin and parity of a gluon. To assess the criticality of any particular prediction of QCD, the prediction is compared with the predictions of other models, in particular, with the modified quantum numbers. Of course, such models are only auxiliary models, because they are not variants of a *consistent theory*. At the present time there exists a number of methods based on perturbation theory for determining the spin and parity of a gluon, which have already been compared with experiment. They all confirm the hypothesis that $J_g^{PC} = 1^{--}$.

The most reliable is the comparison of the widths of direct decays of heavy quarkonia based on the minimal assumption of completeness of parton states. The predicted value of the ratio of the widths

$$r^{\mathrm{S}} = \frac{\Gamma^{\mathrm{dir}}(^1\mathrm{S}_0)}{\Gamma^{\mathrm{dir}}(^3\mathrm{S}_1)}$$

is $r^{\mathrm{S}}_{\mathrm{QCD}} \approx 6(\alpha_s/\pi)^{-1}(1 + O(\alpha_s))$. For the J/ψ, η_{c} case $r^S_{\mathrm{QCD}} \sim 10^{-2}$. For $J_g^P = 0^-(^1\mathrm{S}_0 \not\to 2\mathrm{g},\ ^1\mathrm{S}_0,\ ^3\mathrm{S}_1 \to 3\mathrm{g})$ the difference in the widths is much smaller ($r^S_P \approx 10$ for J/ψ, η_{c}).

For $J_g^P = 0^+(^1\mathrm{S}_0 \to \geqslant 4\mathrm{g},\ ^3\mathrm{S}_1 \to 3\mathrm{g})$ it may be expected that $\Gamma^{\mathrm{tot}}_{\eta_{\mathrm{c}}} < \Gamma^{\mathrm{tot}}_{\mathrm{J}/\psi}$. For $J_g^P = 1^+$, $\Gamma^{\mathrm{dir}}(^1\mathrm{S}_0)$ is expected to be much smaller than in the standard case and could be smaller than $\Gamma^{\mathrm{dir}}(^3\mathrm{S}_1)$. Thus the ratio $r^{\mathrm{S}}_{\mathrm{exp}} \approx (3 \pm 1)\cdot 10^{-2}$ measured for J/ψ and η_{c} is in excellent agreement with QCD and refutes other possibilities.

The observed properties of the distribution of events in direct hadronic decays of Υ and Υ' also agree well with QCD and completely refute the hypotheses $J_g^P = 0^-$ and 0^+. Another example is the comparison of the widths of cascade transitions of ψ' and Υ'.

d) Outlook ("tagged" quarks as a working tool).

1. Further detailed study of the fragmentation functions of b and c quarks over a wide range of energies for different types of hadrons. Check of the specific predictions of perturbative QCD. Study of a gluonic jet in its "pure" form in the reaction $e^+e^- \to \mathrm{Q\bar{Q}g}$ (B. L. Ioffe, 1978).

2. study of the characteristic properties of the distributions of light hadrons in events with heavy quarks. Clarification of the effect of such events on the general inclusive characteristics of strong processes.

3. Detailed study of the structure of weak neutral currents of heavy quarks, including in the e^+e^- annihilation reaction with polarized initial particles.

4. Search for t quarks and analysis of the background to their production processes, associated with the creation of b and c. Searches for the decays of $W^\pm$ and Z^0 bosons into heavy quarks.

5. Search for Higgs H bosons, whose essential property (for $m_{\mathrm{H}} < 2m_{\mathrm{W}}$) is the dominance of decays into heavy quarks: $\mathrm{H} \to \mathrm{Q\bar{Q}}$.

6. Study of the yield of different types of particles in direct hadronic decays of toponium. The structure of spectra and distribution over multiplicities of hadrons in these decays. Comparison with data on e^+e^- annihilation into hadrons.

7. Check of QCD by comparing the widths of S-wave states of $\mathrm{b\bar{b}}$ and $^3\mathrm{P}_0$ and $^3\mathrm{P}_2$ states of $\mathrm{c\bar{c}}$ and $\mathrm{b\bar{b}}$.

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