Double beta decay and the neutrino mass

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Usp. Fiz. Nauk **143**, 513–551 (August 1984)

The present state of the problem of the double neutrinoless beta decay, $2\beta(0\nu)$, and related questions is reviewed. Various mechanisms for $\Delta L = 2$ transitions in models with spontaneous violation of lepton number (L) conservation are discussed. From the standpoint of unified gauge theories, a Majorana neutrino mass would be the most probable reason for processes with $\Delta L = 2$ (if they occur). The $2\beta(0\nu)$ decay is compared with other phenomena which are sensitive to the neutrino mass. It follows from this comparison that the $2\beta(0\nu)$ decay is essentially the only process which could be detected at the present level of experimental capabilities and which might tell us about the nature of the neutrino mass, if this mass is some tens of electron volts.

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INTRODUCTION

The double neutrinoless beta decay has been under discussion for several decades now.[1,2] The history of the formulation and development of the problem of 2β decay is described in detail in Refs. 3–5.

Attempts to observe the neutrinoless 2β decay experimentally are extremely complicated since the half-life of the $2\beta(0\nu)$ decay (if it occurs) would be no less than 10^{21}–10^{22} yr, and the energies of the decay products would typically be no more than a few MeV. Experiments on double β decay may be classified as direct and indirect. The indirect experiments are based on a geochemical analysis of ancient rocks containing the isotopes Te^{130}, Te^{128}, and Se^{82}, which convert in the course of double β decay into Xe^{130}, Xe^{128}, and Kr^{82}, respectively. Of particular interest here are data on the half-life ratio[7] $T_{1/2}(Te^{128})/T_{1/2}(Te^{130})$. The results of an analysis of these data are consistent with the existence of a neutrinoless decay, but the results of different experiments contradict each other.[8,9] It would thus clearly be premature to assert that the $2\beta(0\nu)$ decay does (or does not) occur on the sole basis of the indirect data available. In the category of direct experiments, all that we have at the moment are upper bounds on the probability for the $2\beta(0\nu)$ decay.

What is so interesting about the $2\beta(0\nu)$ decay? In the infancy of the theory of weak interactions this topic attracted interest for a variety of reasons. For example, before the V–A structure of weak charged currents was established it was asserted that a search for the $2\beta(0\nu)$ decay would be one way to test the identity of neutrinos and antineutrinos. At the time it was believed that the β-decay interaction was described by a combination of scalar and tensor versions of the interaction, and in this case the only obstacle to a neutrinoless 2β decay would be a difference between neutrinos and antineutrinos. The subsequent determination of the V–A structure of the Fermi interaction meant that there was another forbiddenness (the "chiral" forbiddenness) on neutrinoless 2β decay. It appeared that the chance that the $2\beta(0\nu)$ decay would occur had sharply diminished. At any rate it was clear that one could not trust estimates of the half-life found on the basis of the "old" theory, without the chiral forbiddenness.[3,10] Nevertheless, experimental searches for the $2\beta(0\nu)$ decay continued. The theoretical work on the problem which was carried out in parallel involved studying various modifications of the generally accepted models which allowed processes involving a change in lepton number.

The first calculation of the decay probability based on the assumption of a Majorana neutrino mass[2] appeared in 1960 in Ref. 11. Another possibility was examined simultaneously: an admixture of right-handed currents (V + A)

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with the additional assumption that neutrinos and antineutrinos are identical.[11-14]

In 1968 Pontecorvo[7] suggested the existence of an interaction with $\Delta L = 2$ on the basis of the analogy with the Wolfenstein interaction,[15] which changes strangeness by two units. It was shown that searches for the $2\beta(0\nu)$ decay would be a sensitive way to test this hypothesis if the constants of the $\Delta L = 2$ and $\Delta S = 2$ interactions were identical.[16,17]

Today, most of the interest in the double neutrinoless β decay stems from the possible existence of a Majorana neutrino mass. An experiment carried out at the Institute of Theoretical and Experimental Physics, Moscow,[18] to measure the β spectrum of tritium demonstrates that there is an effect which can be explained in terms of a nonzero mass of a neutrino emitted in β decay. The hidden mass of the universe is regarded as a serious although indirect argument in favor of a nonzero neutrino mass.[19] Both of these experimental facts, despite their markedly different nature, can be explained in a natural way by assuming that the mass of the electron neutrino is a few tens of electron volts. Here the Dirac and Majorana masses are completely equivalent. Consequently, a nonzero mass of a neutrino emitted in ordinary β decay, for example, would not be a sufficient condition for the existence of a neutrinoless β decay. At the same time, from the standpoint of the grand unified theories, which are enjoying much popularity today, a neutrino mass of some tens of electron volts would be interpreted more naturally as a Majorana mass. The Majorana mass, in contrast with the Dirac mass, could serve two functions. The first (which is common to both types of masses) is kinematic: The mass is treated as a quantity which determines the position of a pole in the neutrino propagator. The second function of the Majorana mass is that it characterizes the amplitude for a transition involving a change of ± 2 in lepton number. For this reason, the appearance of a Majorana mass would mean a breaking of the symmetry with which the conservation of lepton number is associated. The preferred mechanism, which does not affect the renormalizability of the theory, is of course the Higgs mechanism, for spontaneous symmetry breaking. The consequences of spontaneous violation of lepton-charge conservation would depend strongly on whether the corresponding symmetry is local (a gauge symmetry) or global. In turn, these consequences tell us how natural it would be to expect a violation of lepton-number conservation and the associated appearance of a Majorana mass. Let us assume that all the conservation laws, including lepton conservation, are associated with local symmetries. A massless gauge field then corresponds to each strictly conserved quantity. One such quantity is the electric charge, which interacts with the massless electromagnetic field. The existence of a massless gravitational field corresponds to the conservation of energy and momentum.[20] If lepton number is also a strictly conserved quantity, then there must exist a corresponding massless gauge field: a second "photon." The absence of such a "photon," having a significant coupling constant with leptons, raises the suspicion that the lepton number is not conserved and that the corresponding symmetry is broken. The "photon" mass which arises upon a spontaneous symmetry breaking can be quite large and could lie far outside the range of detectability. If the conservation of lepton number is associated with a gauge symmetry, it would then be more natural to expect that this symmetry is broken and that the neutrino has a Majorana mass.

Another possibility is that the conservation of lepton number is associated with a global symmetry, and there simply is no second "photon." This is the situation, for example, in the standard $SU(3)_c \times SU(2) \times U(1)$ model.[21-25] We know that the neutrino masses (Dirac and Majorana masses) are zero in the minimal version of this model. If we "add" a right-handed neutrino,then a Dirac mass can arise by the same mechanism which "gives" masses to the other particles. Here we would naturally expect that the neutrino mass would be of the order of the masses of the other fermions of the given generation. This situaiton, however, is in clear contradiction of experiment: The neutrino masses (if they exist) are far smaller than the masses of their charged partners for all known generations. The mechanism for the appearance of a Majorana mass is different from that which gives rise to Dirac masses. In the standard model, a "soft" incorporation of a Majorana mass implies a breaking of the global symmetry with which lepton-number conservation is associated. We know that a spontaneous breaking of a continuous global symmetry will be accompanied by the appearance of massless Goldstone particles.[26,27] The particle in this case has been labeled the "majoron," M^0 (Refs. 28–30). This is a real physical entity, whose existence should give rise to some new physical phenomena. One example is a double β decay accompanied by the emission of a majoron: $N \to N'e^-e^-M^0$. These possibilities for the appearance of neutrino masses in the unified gauge theories are examined in more detail using some concrete examples in Section 2.

It should also be noted that in models with a spontaneously broken global symmetry the appearance of a Majorana mass is not the only factor which would lead to a $2\beta(0\nu)$ decay. The expansion of the sector of Higgs particles with a nonzero vacuum expectation value would unavoidably lead to the appearance of mechanisms for a $2\beta(0\nu)$ decay[31-33] which would effectively reduce to Pontecorvo's $\Delta L = 2$ interaction.[7]

As we have already mentioned, a $2\beta(0\nu)$ decay can also occur if a charged lepton current contains an admixture of a right-handed component.

$$j_\mu = \bar{e}\gamma_\mu\,[1 + \gamma_5 + \eta'\,(1 - \gamma_5)]\,\nu,$$

and if the neutrino is a Majorana particle for which the condition $\nu = \bar{\nu}C$ holds. The mass of the neutrino can be arbitrarily small. Before the advent of the unified theories, it was this possibility which was discussed most frequently. In the gauge theories with a left-right symmetry there can be some very closely related possibilities, which lead to consequences which are essentially indistinguishable expeirmentally. Right-handed currents are embodied in these models from the outset, while the mixing of neutrinos and antineutrinos stems from the appearance of Dirac and Majorana masses. Just whose contribution to the amplitude of the $2\beta(0\nu)$ decay is the basic one—that of the left-handed Majorana mass or that of the right-handed currents—depends on the particular version of the model. The conditions which lead to $2\beta(0\nu)$

decay in the models with a left-right symmetry are discussed in more detail in Section 2.

We thus see that a rather large number of mechanisms might lead to $2\beta(0\nu)$ decay. Would it be possible to distinguish these possibilities experimentally—for example, to distinguish right-handed currents from a Majorana mass—if the $2\beta(0\nu)$ decay does in fact occur? As far as direct experiments are concerned, it would be necessary to measure the angular and energy distributions of the electrons. As we will see, for $0^+ \to 0^+$ nuclear transitions the Majorana mass and the right-handed currents lead to completely different one-electron spectra and completely different angular distributions. In principle, one could distinguish between these two possibilities simply by measuring the total probabilities for the decays of different elements (with different energy releases), but this approach would require an accurate knowledge of the amplitudes for nuclear transitions. Unfortunately, these amplitudes are not yet known very accurately; the estimates reported by different investigators differ by more than an order of magnitude. This is the fundamental reason for the discrepancy between the results calculated for upper bounds on lepton-nonconservation parameters on the basis of experimental restrictions on decay half-lives. Quite briefly, the results of these estimates appear as follows: The upper bound found, for example, for the Majorana mass of the electron neutrino from direct experiments on 2β decay lies between a few tens of electron volts and hundreds of electron volts. In practice, this is a very interesting region, as we see when we recall the tritium experiment,[18] whose result can be interpreted under the assumption that the mass of the neutrino emitted in β decay lies in the interval 14–46 eV. If this is a Majorana mass, then an improvement in the accuracy of the measurements of 2β decay by, say, an order of magnitude would very probably lead us to the desired result. Actually, however, the situation may turn out to be more complicated. Even if the Lagrangian contains only Majorana mass terms, we still do not have an unambiguous correspondence between the mass which determines the amplitude of the $2\beta(0\nu)$ decay and the mass which appears in the β spectrum of tritium. We will be discussing this point in more detail below.

Section 1 deals with certain questions of the phenomenology of the mass Lagrangian of neutrinos. Section 2 briefly review models in which left-handed Majorana masses arise. We then compare the 2β decay with other processes which are sensitive to neutrino masses. In the subsequent sections of this review we give expressions for the matrix elements, and we examine the spectra and angular distributions of the final electrons in various 2β transitions. These characteristics, in contrast with the absolute value of the decay amplitude, can be predicted essentially unambiguously. In the final sections we examine some numerical estimates and briefly summarize the results.

1. NEUTRINO MASS MATRIX; PHENOMENOLOGY

In this section we consider the most general case of the neutrino mass matrix and its diagonalization. We also look at some particular cases of interest for a study of double β decay. This analysis is required for discussing the various mechanisms for the appearance of neutrino masses in the unified gauge theories and also for a comparison of the $2\beta(0\nu)$ decay with other phenomena which are sensitive to neutrino masses.

If we focus on only a single generation of fermions, i.e., neutrinos of a common species ($\nu = \nu_e$, say), then the Lagrangian may in general contain mass terms of the type[34–40]

$$-L = m\bar{\psi}\psi + m_+\psi^T C\psi + m_-\psi^T C\gamma\gamma_5\psi + \text{H.a.}; \quad (1)$$

here m is the ordinary Dirac mass, while m_+ and m_- are Majorana masses. The Dirac mass term conserves any charge which the field ψ has, while the Majorana terms change this charge by two units. It is convenient to transform from the four-component spinors ψ to the chiral components ν_L and ν_R:

$$\nu_{L,R} = \frac{1 \pm \gamma_5}{2}\psi.$$

In the case of zero masses, there is a one-to-one correspondence between the indices L and R and the neutrino helicities. If the masses are nonzero, the ν_L and ν_R can have an arbitrary helicity.

In terms of the chiral fields ν_L and ν_R, Lagrangian (1) can be written as

$$-L = m_D\bar{\nu}_L\nu_R + m_L\nu_L^T C\nu_L + m_R\nu_R^T C\nu_R, \quad (2)$$

where $m_{L,R}$ are related to $m_\pm$ by

$$m_\pm = \frac{1}{2}(m_L \pm m_R), \qquad m = m_D. \quad (3)$$

If all three mass terms are nonzero, then ν_L and ν_R are not eigenstates of the Hamiltonian and do not have definite masses. Diagonal states (which we denote as φ_1 and φ_2) with definite masses μ_1 and μ_2 are orthogonal superpositions of the states ν_L and ν_R:

$$\begin{aligned} \varphi_{1L} &= \nu_L\cos\alpha + \nu_L^c\sin\alpha, \quad \varphi_{1R} = \nu_R^c\cos\alpha + \nu_R\sin\alpha, \\ \varphi_{2L} &= \nu_L\sin\alpha - \nu_L^c\cos\alpha, \quad \varphi_{2R} = \nu_R^c\sin\alpha - \nu_R\cos\alpha. \end{aligned} \quad (4)$$

The mixing angle α and the masses μ_1 and μ_2 can be expressed in terms of the parameters of the initial Lagrangian:

$$\mu_{1,2} = m_+ \pm \sqrt{m_-^2 + m_D^2}, \quad \text{tg}\, 2\alpha = \frac{m_D}{m_-}. \quad (5)$$

The states φ_1 and φ_2 describe Majorana states for which the following condition holds:

$$\varphi_{1,2}^T = \bar{\varphi}_{1,2}C,$$

where $C = i\gamma_2\gamma_0$ is the charge conjugation matrix. The inverse transformation, which expresses ν_L and ν_R in terms of the diagonal states $\varphi_{1,2}$, is

$$\begin{aligned} \nu_L &= \nu_{1L}\cos\alpha + \varphi_{2L}\sin\alpha, \\ \nu_R &= \varphi_{1R}\sin\alpha - \varphi_{2R}\cos\alpha. \end{aligned} \quad (6)$$

We turn now to two limiting cases of interest in a study of double β decay.

1. All the masses in Lagrangian (2) are comparable in magnitude: $m_D \sim m_L \sim m_R$ or $m_L \gg m_D, m_R$. The amplitude for the $2\beta(0\nu)$ decay is then proportional to m_L and independent of the two other masses.

2. $m_L = 0$ and $m_D \ll m_R$. In this case one of the diagonal states is light, with a mass $\mu_1 = m_D^2/m_R \ll m_D$, while the other is heavy, with $\mu_2 \sim m_R$. In this case the left-handed neu-

trino ν_L contains primarily an admixture of the light neutrino φ_1, while the right-handed neutrino primarily contains an admixture of the heavy neutrino, φ_2:

$$\nu_L \approx \varphi_{1L} + \frac{m_D}{m_R}\varphi_{2L}, \quad \nu_R \approx \frac{m_D}{m_R}\varphi_{1R} - \varphi_{2R}. \tag{7}$$

We further assume that m_R is significantly greater than the characteristic energies of the process under consideration (in this particular case, this would be the momentum of the virtual neutrino, which is equal in order of magnitude to the reciprocal of the distance between nucleons). In this case the amplitude for the 2β (0ν) decay will be dominated by the left-handed neutrino, and the contribution of the heavy neutrino would be exponentially small. This limiting case is therefore equivalent to the introduction of an effective left-handed Majorana mass $m_L = m_D^2/m_R$. The diagram in Fig. 1 illustrates the situation. The amplitude corresponding to this diagram gives us a Majorana mass term of the type $m_L \nu_L^T C \nu_L$, where m_L is proportional to the square of the Dirac mass m_D divided by m_R, which enters from the propagator of the right-handed neutrino. Here we have taken into account the fact that m_D and the neutrino momentum are small in comparison with m_R.

That is the situation in the case of a single generation of fermions. If we allow a mixing of neutrinos of different generations (ν_e, ν_μ, ν_τ, etc.), then in the most general case the Lagrangian will have the form in (2), where m_D, m_L, and m_R are to be understood as matrices which act in the generation space:

$$-L = m_{ik}^D \bar{\nu}_{iL}\nu_{kR} + m_{ik}^L \nu_{iL}^T C \nu_{kL} + m_{ik}^R \nu_{iR}^T C \nu_{kR}, \tag{8}$$

$$i, k = e, \mu, \tau, \ldots$$

The three $N \times N$ mass matrices, where N is the number of generations, which appear in (8) can be written as a single $2N \times 2N$ matrix

$$\begin{pmatrix} m_{ik}^{L*} & \frac{1}{2} m_{ik}^D \\ \frac{1}{2} m_{ik}^{DT} & m_{ik}^R \end{pmatrix}. \tag{9}$$

The diagonalization of the Lagrangian reduces to seeking a unitary $2N \times 2N$ matrix K which expresses the neutrinos of a particular species i($i = e, \mu, \tau, \ldots$) in terms of the diagonal states φ_A, of which there are $2N$. The details of this procedure can be found in Refs. 34, 35, 37, 39, and 40.

Let us examine in more detail the case in which only the left-handed Majorana masses are nonzero. For three generations of leptons ($i = e, \mu, \tau$) the Lagrangian

$$-L = m_{ik}^L \nu_{iL}^T C \nu_{kL} + \text{H.a.} \tag{10}$$

can then be put in the following form after diagonalization:

$$-L = \sum_{A=1}^{3} \mu_A \bar{\varphi}_A \varphi_A$$

An analogous Lagrangian for $N = 2$ ($i = e, \mu$) was first examined by Gribov and Pontecorvo.[41] The diagonal states φ_A

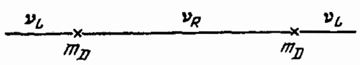

FIG. 1.

are Majorana states ($\varphi_A^T = \bar{\varphi}_A C$) and are related to ν_e, ν_μ, and ν_τ by the 3×3 unitary matrix U_{Ai}:

$$\varphi_{AL} = U_{Ai}\nu_{iL}, \quad \varphi_{AR} = U_{Ai}^* \nu_{iR}^c.$$

The inverse transformations are

$$\nu_{iL} = U_{iA}^+ \varphi_{AL}, \quad \nu_{iR}^c = U_{iA}^T \varphi_{AR}.$$

As before, the indices L and R have the following meaning:

$$\varphi_{L,R} = \frac{1 \pm \gamma_5}{2}\varphi.$$

We assume that all the masses in Lagrangian (8) are of the same order of magnitude. The amplitudes for transitions with $\Delta L_e = 2$ [including the 2β (0ν) decay] are then proportional to the mass-matrix element m_{ee}^L, which is expressed in terms of the masses μ_A of the diagonal states φ_A by means of the matrix U_{Ai}:

$$2m_{ee}^L = \sum_A \mu_A U_{eA}^2.$$

In the case in which the only nonzero term in Lagrangian (10) is the term

$$-L = m_{ee}^L \nu_{eL}^T C \nu_{eL} + \text{H.a.},$$

m_{ee}^L (or, for precisely, $2m_{ee}^L$) is interpreted as the mass of the Majorana electron neutrino.

We refer the reader to Refs. 34–43 for more details on the processes involving the study of the neutrino mass matrix, including the various limiting cases

2. NEUTRINO MASS IN GAUGE THEORIES

In this section we will examine the various mechanisms for the 2β (0ν) decay which arise in unified theories with a spontaneous breaking of the symmetry which corresponds to lepton-number conservation. For brevity we will refer to this symmetry as "L symmetry." Along with the Majorana mass, whose appearance we would naturally expect upon a spontaneous breaking of the L symmetry, there may be some other mechanisms which would lead to transitions with $\Delta L = \pm 2$.

We begin with the models based on the standard Weinberg-Salam group[21–25] $SU(3)_c \times SU(2) \times U(1)$, and for the moment we will consider only a single generation of fermions. We know that in the standard model with the minimal set of particle multiplets the masses of the neutrinos (both Majorana and Dirac) are zero. A Dirac neutrino mass can arise by analogy with the masses of other particles (quarks and charged leptons). Here we need to "add" a right-handed neutrino [a singlet according to the $SU(2) \times U(1)$ group] and to introduce an interaction of the neutrino with the doublet φ: $f \bar{\nu}_L \nu_R (\varphi)^+$. The spontaneous symmetry breaking caused by the nonzero vacuum expectation value $\langle \varphi^0 \rangle = v$ gives rise to a Dirac neutrino mass of fv. If there are no other contributions to the neutrino mass matrix we should assume that the interaction of the neutrino with the field φ should be many orders of magnitude weaker than the interaction of electrons with φ. This dramatic difference in the interaction constants of particles of a common generation for interactions with the same field φ does not seem natural.

The possibility of a "soft" incorporation of the Majorana mass was studied in Refs. 44 and 28–30. Let us consider two possibilities discussed in Refs. 28–30. For the appearance of a Majorana mass we must clearly assume that there exist some additional Higgs particles which carry a double lepton number.

We assume that there exists an interaction of left-hand and right-hand neutrinos with an SU(2) doublet φ of the form $f\bar{\nu}_L \nu_R(\varphi)^+$, where the constant f is of the order of the constant of the interaction of φ with other fermions of the same generation (in this case, these other fermions are the electron and light quarks). The spontaneous breaking of the SU(2)×U(1) symmetry thus gives rise in the Lagrangian to a Dirac mass of the electron neutrino; this mass is of the order of the masses of the other fermions. We further assume that the right-handed neutrino ν_R interacts in a Majorana-like fashion with the new scalar Higgs field χ, which is a singlet with respect to the standard group:

$$f_R \nu_R^T C \nu_R (\chi)^+. \tag{11}$$

Since ν_R is also an SU(2)×U(1) singlet, the introduction of this interaction does not alter the symmetry properties of the model. If we assign the field χ a double lepton number then interaction (11) will also conserve lepton number. Conservation of leptons is associated with the global group $U(1)_L$, which is external with respect to the gauge group SU(2)×U(1). The spontaneous breaking of $U(1)_L$ which can result from the appearance of a vacuum expectation value of the field χ should give rise to a right-handed Majorana mass $m_R = f_R, u$, where $u = \langle \chi^0 \rangle$. The breaking of the L symmetry should in turn lead to observable processes in which lepton number changes by two units [e.g., $2\beta(0\nu)$ decay]; the amplitudes for processes with $\Delta L = \pm 2$ will be determined by the magnitudes of the coupling constants of leptons with Higgs particles and the vacuum expectation values $\langle \varphi^0 \rangle$ and $\langle \chi^0 \rangle$. We assume that m_R is considerably larger than the Dirac mass and has a value of, say, ~ 100 GeV. The situation which we discussed in the preceding section then arises: the left-handed neutrino contains primarily an admixture of a light diagonal state with a mass $\sim m_D^2/m_R$, while the admixture of the heavy state (with mass $\sim m_R$) is small and proportional to m_D/m_R. As we have already stated, this model is effectively equivalent to the introduction of a left-handed Majorana mass $m_L = m_D^2/m_R$. This possibility was analyzed in Ref. 28. Since the L symmetry is global in this case, its spontaneous breaking should be accompanied by the appearance of a massless Goldstone boson.[26] In this case the continuous L-symmetry group is the $U(1)_L$ group, and the massless particle is described by an imaginary component ξ of the Higgs field χ:

$$\chi^0 = \langle \chi^0 \rangle + 2^{-1/2}(\rho + i\xi). \tag{12}$$

Because of the spontaneous symmetry breaking, the field ρ acquires a mass of the order of m_R and thus is not manifested in real physical processes. In contrast with that field, the massless pseudoscalar particle described by the field ξ should generate new physical phenomena even at low energies. The particle ξ is called a majoron since its Yukawa interaction with the neutrino has a Majorana-like form:

$$\frac{f_R}{\sqrt{2}}(\xi)^+ \nu_R^T C \nu_R. \tag{13}$$

The constant of the interaction of a majoron with left-handed neutrinos is determined within small corrections by the constant of the interaction of the majoron with light diagonal neutrinos; it is easy to see that this interaction constant is $f_L = 2^{-1/2} f_R \times (m_D/m_R)^2$. The interaction of a majoron with left-handed neutrinos also of course has a Majorana-like form,

$$f_L \nu_L^T C \nu_L (\xi)^+, \tag{14}$$

and means that there may be a (virtual) transition of the majoron into two left-handed neutrinos (the diagram in Fig. 2). One possible manifestation of an interaction of a majoron with left-handed neutrinos is double β decay accompanied by the emission of a majoron, $N \to N' e^- e^- M^0$. This decay may compete with the "ordinary" neutrinoless 2β decay if the constant f_L is large enough. These two processes will be compared in detail in the following sections of this paper. The double β decay is not the only process in which a massless majoron could be manifested. Processes involving a majoron and restrictions on the constant f_L are discussed in more detail in Refs. 28–30 and 45–47.

A left-handed Majorana mass may arise spontaneously, even without the participation of right-handed neutrinos. This process, however, requires the introduction of a large number of Higgs particles. Let us assume that a left-handed Majorana mass arises from the vacuum expectation value of a Higgs boson which is interacting with left-handed neutrinos:

$$g \nu_L^T C \nu_L (H^0)^+. \tag{15}$$

What would an Su(2)×U(1) interaction which leads to a Yukawa coupling of the type in (15) look like? Since ν_L is a component of an SU(2) doublet, $l_L = \binom{\nu_L}{e_L}$, the Majorana combination $\nu_L^T C \nu_L$ transforms as a component of an isovector. Accordingly, H^0 should also belong to a triplet. The charges of the two other particles are fixed by, for example, the requirement of invariance under U(1) transformations (by conservation of hypercharge, equal to twice the average electric charge of the multiplet):

$$\mathbf{H} = \begin{pmatrix} H^0 & H^-/\sqrt{2} \\ H^-/\sqrt{2} & H^{--} \end{pmatrix}.$$

The SU(2)×U(1) interaction of left-handed leptons with the **H** triplet is

$$g l_L^T \tau C l_L \mathbf{H}. \tag{16}$$

It is easy to see that expression (16) contains a term which describes the interaction of H^0 with ν_L in (15). If the vacuum expectation value $\langle H^0 \rangle$ is nonzero, a left-handed Majorana mass

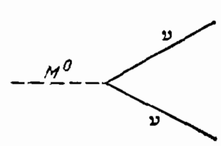

FIG. 2.

$$g\langle H^0\rangle = g\frac{v}{\sqrt{2}}$$

arises. The vacuum expectation value of the field H^0 also contributes to the masses of the gauge bosons $W^{\pm}$ and Z^0. Accordingly, the relation between the W and Z masses which holds in the standard $SU(2)\times U(1)$ model with a single doublet of Higgs particles φ,

$$M_W = \cos\theta_W \cdot M_Z,$$

where θ_W is the Weinberg angle, is broken, strictly speaking. To avoid spoiling the predictions of the standard model, which agree well with experiment, we must assume $\langle H^0\rangle \ll \langle\varphi^0\rangle$, where φ_0 is a component of a doublet. In the scheme which we considered earlier no corresponding restriction arises on the vacuum expectation value of the field χ interacting with right-handed neutrinos, since the singlet Higgs boson is coupled with gauge bosons only through a neutrino loop (Fig. 3).

A model with a triplet of Higgs particles **H** interacting with a Majorana-like combination of light leptons was studied in Refs. 29 and 30. In this model, as in that of Ref. 28, the spontaneous breaking of lepton number is accompanied by the appearance of a massless boson (a majoron). In this case the structure of the Higgs sector is more complicated than in the model of Ref. 28. The majoron M^0 is described by a combination of neutral components of the doublet φ_0 and the triplet H^0:

$$M^0 = [u(H^0 - H^{0*}) - 2v(\varphi^0 - \varphi^{0*})]\,(2\sqrt{2u^2 + 8v^2})^{-1},$$

where, we recall, $u = \langle\varphi_0\rangle$ and $v = \sqrt{2}\langle H^0\rangle$. As in the preceding model, the majoron is a pseudoscalar boson capable of converting into a pair of left-handed neutrinos. In discussing models with a spontaneous appearance of a Majorana mass in the standard model we considered only a single generation of fermions for simplicity. Both these models can obviously be generalized to an arbitrary number of generations, and a possible mixing of generations can be taken into account. The model of Refs. 29 and 30, for example, would then contain, instead of the single constant g for the interaction of the triplet H with leptons, a symmetric matrix g_{ik} (i, $k = 3, \mu, \tau, \ldots$), and the interaction would be

$$\sum_{i,k} g_{ik} l_{Li}^{T} C\tau l_{Lk} \mathbf{H}^{+} + \text{H.a.} \qquad (17)$$

The Majorana mass matrix is evidently

$$m_{ik}^{L} = g_{ik} v.$$

This is one of the particular cases, discussed in the preceding section, in which the Lagrangian contains only left-handed nonsterile states.

The appearance of a left-handed Majorana neutrino mass in the SU(5) model, which combines the strong, weak, and electromagnetic interactions,[48,49] was studied in Ref. 47. This is a direct generalization of the model proposed in Ref.

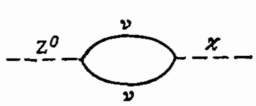

FIG. 3.

29. Transitions with $\Delta L = 2$ arise in the SU(5) model from a spontaneous breaking of the global symmetry associated with the conservation of B–L: the difference between the baryon and lepton numbers. It was shown[47] that all the low-energy predictions are the same as in the standard model with a spontaneous breaking of L symmetry.

We find a different situation in the SO(10) model,[50] in which the conservation of B–L is associated with a gauge symmetry. This symmetry cannot remain exact, since there is no second massless "photon" which interacts with the number B–L with a sufficiently large coupling constant (of the order of the electromagnetic coupling constant, say). The spontaneous breaking of the local B–L symmetry does not give rise to massless bosons: They are absorbed by the Higgs mechanism,[51,52,27] and the correspondong gauge field acquires a mass.

There is one more argument in favor of a spontaneous breaking of B–L in the SO(10) model. The fundamental representation of the SO(10) group is known to contain 16 parts,[50] including left-hand and right-hand neutrinos (of the same generation). Accordingly, the Dirac mass m_D should arise just as naturally as the masses of charged fermions do, and it should be of the order of the mass of the upper quark (the u quark in the first generation), i.e., of the order of a few MeV. In this form the model contradicts experiment, since the electron neutrino in this case is an eigenstate of the Hamiltonian with a mass m_D. As Gell-Mann, Ramond, and Slansky have pointed out,[53] this contradiction is eliminated if the B–L symmetry is spontaneously broken. Let us assume that the spontaneous breaking of the B–L symmetry has given rise to a large right-handed Majorana mass m_R. We would then have two diagonal states: a light one with a mass $\mu_1 \sim m_D^2/m_R$, and a heavy one with $\mu_2 \sim m_R$. The left-handed neutrino would contain primarily an admixture of the light state. A situation of this type has already been encountered: At low energies it is equivalent to the introduction of a left-handed Majorana mass $m_L = \mu_1 \ll m_D$.

Witten[54] studied two mechanisms for the generation of an m_R in the SO(10) model. In the first, m_R arises in the tree approximation because of the vacuum expectation value of a Higgs boson from a 126-plet which carries a double lepton number and which interacts with a Majorana-like combination of right-handed neutrinos, $\nu_R^T C \nu_R$. It is natural to assume that this vacuum expectation value is of the order of the energy at which the interactions become unified, i.e., no less than 10^{15} GeV. The "right-handed" mass is equal to the vacuum expectation value to within the constant of the Yukawa interaction. Such a large value of m_R leads to a vanishingly small effective mass $m_L = m_D^2/m_R$, which may be manifested in oscillations of solar neutrinos (if these oscillations occur).

If we do not introduce an interaction with a 126-plet of Higgs particles, then the right-handed neutrino automatically acquires masses by virtue of the two-loop diagram. The details of this mechanism are discussed in Ref. 54. The corresponding contribution to the mass m_R turns out to be considerably less than 10^{15} GeV. If this is the only mechanism which contributes to m_R, then we find the estimate[54]

$m_L \sim 10^{-7} m_q$, where m_q is the mass of the upper quark of the same generation. For the first generation, for example, we would have $m_L \sim 1$ eV. A Majorana mass of this order of magnitude for the electron neutrino leads to $2\beta(0\nu)$ decay times which, it may be hoped, will be amenable to experimental study not too far in the future.

There is yet another class of models for which a search for the $2\beta(0\nu)$ decay could in principle be a critical test. These are models with a left-right (L–R) symmetry. The basic idea underlying these models is that the parity breaking observed experimentally in weak interactions is a property of low-energy processes, while at energies above the masses of gauge bosons the symmetry between left and right is restored. In other words, the right-handed symmetry is broken to a greater extent than the left-handed symmetry, with the result that the mass of the left-handed W boson, m_{W_L}, is considerably smaller than that of the right-handed boson, m_{W_R}. This circumstance in turn explains the dominance of V–A currents in the effective weak interaction. If the breaking of L–R symmetry is accompanied by a violation of the conservation of lepton charge, then there may be mechanisms for the $2\beta(0\nu)$ decay other than those which arose in the models described above. This possibility is discussed most comprehensively in Refs. 55–59. Initially there is a symmetry under the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where B–L is the difference between the baryon and lepton numbers. The leptons (the electron and the electron neutrino, for example) form multiplets of the group:

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \left(\frac{1}{2},\ 0,\ -1\right), \qquad l_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \quad \left(0,\ \frac{1}{2},\ -1\right). \tag{18}$$

The numbers in parentheses here are values of $(T_i, T_R, B-L)$, which characterize the transformation properties of the left- and right-hand particles under the transformations of the group. The Higgs sector includes the following particles:

$$\left.\begin{aligned} \phi &= \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \quad \left(\frac{1}{2},\ \frac{1}{2},\ 0\right), \\ \Delta_L &= \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix} \quad (1,\ 0,\ 2), \\ \Delta_R &= \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix} \quad (0,\ 1,\ 2). \end{aligned}\right\} \tag{19}$$

The appearance of a vacuum expectation value $v_R = \langle \Delta_R \rangle$ implies a breaking of the initial symmetry to the standard symmetry:

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow[\langle \Delta_R \rangle \neq 0]{} SU(2)_L \times U(1).$$

At this stage the parity, like the local B–L symmetry, is broken. The breaking of B–L is accompanied by the appearance of a right-handed Majorana neutrino mass of the order of v_R. The subsequent breaking of the standard symmetry $SU(2)_L \times U(1)$ to $U(1)_{e.m}$ is due to the vacuum expectation value of the field ϕ:

$$\langle \phi \rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}.$$

In this stage, the Dirac masses of the fermions, including the neutrinos, arise; these masses are proportional to $k + k'$. Simultaneously, the vacuum expectation value[57,58]

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}$$

arises and contributes a Majorana mass to the left-handed neutrinos. This mass is of the order of $v_L \sim k^2/v_R$, if $v_R \gg k \gg k'$. The condition $k \gg k'$ guarantees a slight mixing[55,56] of W_L and W_R, while the requirement $v_R \gg k$ guarantees dominance of the left-handed interaction. Accordingly, the situation which arises in the neutrino sector is one with which we are already familiar, in which the diagonalization of the mass matrix gives rise to Majorana diagonal neutrinos with masses $\mu_1 \sim v_L \sim k^2/v_R$ and $\mu_2 \sim v_R \gg \mu_1$. The left-handed neutrino contains primarily an admixture of the light diagonal neutrino, and the right-handed neutrino contains primarily an admixture of the heavy neutrino:

$$\begin{aligned} \nu_L &\approx \frac{1+\gamma_5}{2} (\varphi_1 \cos\alpha + \varphi_2 \sin\alpha), \\ \nu_R &\approx \frac{1-\gamma_5}{2} (-\varphi_1 \sin\alpha + \varphi_2 \cos\alpha), \\ \sin\alpha &\sim \frac{m_D}{m_R} \ll 1, \quad \cos\alpha \sim 1, \quad m_D \sim m_e. \end{aligned}$$

Under the condition $v_R \gg k \gg k'$, the vacuum expectation value $v_R = \langle \Delta_R \rangle$ basically determines the mass of the right-handed gauge boson W_R, while $k = \langle \phi \rangle$ basically determines the mass of the left-handed boson W_L. The effective mass of the left-handed neutrino ($\approx \mu_1$) can therefore be written

$$m_{\nu_L} \sim \mu_2 \left(\frac{m_{W_L}}{m_{W_R}}\right)^2. \tag{20}$$

If the parameters of the Higgs potential are chosen in a special way,[57] we could have a situation with

$$m_{\nu_L} = \text{const} \cdot \frac{m_e^2}{m_{W_R}}. \tag{21}$$

This relation also holds if we assume that there exists an additional multiplet of Higgs fields ϕ_W which has the same transformation properties as ϕ, with the vacuum expectation value $\langle \phi_W \rangle \gg \langle \phi \rangle$, which contributes only to the masses of the gauge bosons. In this version, the model couples the small value of the neutrino mass with the dominance of the left-handed interaction. If we use the known bound[56] on the mass W_R ($m_{W_R} \gtrsim 3 m_{W_L}$) and set the constant in (21) equal to one, we find $m_{\nu_L} < \sim 1$ eV. All these estimates are of course order-of-magnitude estimates and depend on the constants

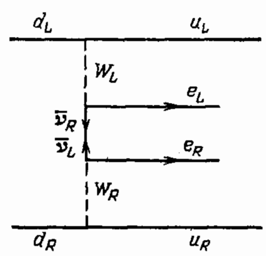

FIG. 4.

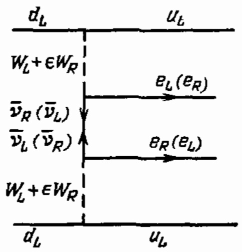

FIG. 5.

of the Yukawa interaction and the parameters of the Higgs potential.

Since the diagonal states φ_1 and φ_2 are Majorana states with substantially different masses, the left- and right-handed neutrinos may annihilate with an amplitude proportional to the mixing angle α ($\approx \sin\alpha$). In this manner, the mechanism in Fig. 4 for the $2\beta\,(0\nu)$ decay arises. Right-handed currents "work" at the lower vertices, and left-handed currents at the upper vertices. The amplitude corresponding to the diagram in Fig. 4 contains the product of constants

$$G_L G_R \alpha = G_L^2 \left(\frac{m_{W_L}}{m_{W_R}}\right)^2 \alpha.$$

By definition, G_L is equal to the Fermi constant G_F. The parameter

$$\eta = \alpha\left(\frac{m_{W_L}}{m_{W_R}}\right)^2 \sim \frac{m_e}{m_R}\left(\frac{m_{W_L}}{m_{W_R}}\right)^2 \tag{22}$$

is thus a measure of the breaking of lepton number. The numerical value of η depends on the particular parameters of the model. With $m_R \gtrsim 100$ GeV, for example, we find from the restriction on the mass of the right-handed boson the condition

$$\eta < 10^{-6}.$$

If there is a mixing of the left- and right-handed gauge bosons, characterized by a parameter ϵ, then we have yet another analogous mechanism for the $2\beta\,(0\nu)$ decay (Fig. 5). Here the lepton current is

$$\bar{e}\,\gamma_\mu\,[1+\gamma_5+\epsilon\alpha\,(1-\gamma_5)]\nu, \tag{23}$$

so that the amplitude corresponding to the diagram in Fig. 5 is proportional to $G_L^2\epsilon\alpha$. We know from β and μ decay[56] that $\epsilon < 10^{-2}$. The parameter of the breaking of lepton number, $\eta = \epsilon\alpha$, is thus an order of magnitude smaller in this case than in the case corresponding to the diagram in Fig. 4. We might note that the diagrams in Figs. 4 and 5 predict essentially identical characteristics (spectra and angular distributions) for the $2\beta\,(0\nu)$ decay, so that it would not be possible to distinguish between these diagrams experimentally.

To conclude this section we consider yet another mechanism for $2\beta\,(0\nu)$ decay in models with doubly charged Higgs bosons. In these models there can be transitions of a doubly charged boson χ^{--} into a pair of left- or right-handed (depending on the model) electrons: $\chi^{--}\to e^-e^-$. In addition, there is the interaction of χ^{--} with singly charged scalars and the interaction with gauge fields ($\chi^{--}\to\chi^-\chi^-, \chi^-W^-, W^-W^-$). As a result, the diagram in Fig. 6 contributes to the amplitude for $2\beta\,(0\nu)$ decay. Here

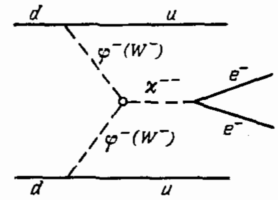

FIG. 6.

the heavy lines are quarks belonging either to different nucleons or to, for example, a π meson. The contribution of this mechanism was studied in Refs. 31–33. Because of the large masses of the intermediate particles (large at least in comparison with the characteristic energies of the process), the diagram in Fig. 6 effectively reduces to a Pontecorvo interaction with $\Delta L = 2$ (Ref. 7). To see this, we assume that the initial and final quarks in Fig. 6 belong to π^- and π^+ mesons, respectively. The diagram then reduces to an effective $\Delta L = 2$ interaction of the type shown in Fig. 7.

Modifications of the standard model which allow a breaking of lepton number have also been examined in Refs. 60–63. Among these modifications there are some mechanisms analogous to those which we have discussed here which allow $2\beta\,(0\nu)$ decay.

3. DOUBLE β DECAY AND OTHER PROCESSES SENSITIVE TO NEUTRINO MASS

As we have already mentioned, there are many phenomena whose study might yield information on neutrino masses and the breaking of lepton number. Among these possibilities, searches for $2\beta\,(0\nu)$ decay would be the most sensitive method for determining the nature of the neutrino mass at the present level of experimental capabilities. This conclusion is based on a comparison of the $2\beta\,(0\nu)$ decay with such phenomena as neutrino oscillations and the β decay of tritium.

1. Let us begin with neutrino oscillations. This is perhaps the most elegant phenomenon which should occur if the current states (in the weak Lagrangian) of the neutrinos are not the same as eigenstates of the mass matrix.[64–66,42,43] The neutrino oscillations may be initiated by either Dirac or Majorana masses. We restrict the discussion to the three known generations of leptons: e, μ, and τ. In the general case of the Dirac mass matrix there are three diagonal Dirac neutrinos with masses μ_1, μ_2, and μ_3; three mixing angles, which express ν_e, ν_μ, and ν_τ in terms of the diagonal states φ_1, φ_2, and φ_3; and one *CP*-odd phase.[37,39,40] We note that the mixing matrix is completely identical to the Kobayashi-Maskawa matrix[67] for quarks. Measurements of the oscillations $\nu_i \rightleftarrows \nu_k$ $(i,k = e,\mu,\tau)$ present the possibility of

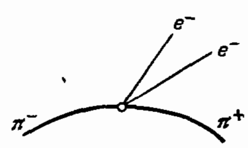

FIG. 7.

determining the difference between the squared masses $\mu_A^2-\mu_B^2$, the mixing angles, and the CP-odd phase.

In the case of a Majorana mass matrix, the number of physical CP-odd phases is three,[35,37,39,40,68] but two of them would be essentially impossible to see experimentally.[37,69] It would thus be necessary to seek transitions $\nu\to\bar{\nu}$, whose amplitudes contain a small factor of the order of m_ν/E_ν. It follows that it would be impossible to distinguish Dirac masses from Majorana masses (if any of these masses exist) on the basis of oscillations.

There is the possibility, however, that the Lagrangian contains both Dirac and Majorana mass terms. This is the most general case of the mass matrix.[34,40] In this case there should be transitions to sterile states, e.g., $\nu_{eL}\to\bar{\nu}_{eL}$. However, in studying reactions caused by charged currents we still cannot tell whether there is an active component in the neutrino beam, so we cannot draw conclusions about the occurrence of transitions $\nu\to\bar{\nu}$. It may be suggested, for example, that there exist other generations of neutrinos with rather heavy charged partners. Such a neutrino, having been formed as a result of oscillations, would appear as a sterile neutrino simply because there is not enough energy for the creation of "its" charged lepton. Neutral currents are more favorable in this regard: there is no threshold for them. By studying the dependence of the cross section for neutrino scattering by hadrons due to neutral currents on the distance from the neutrino source, one could draw conclusions about transitions to sterile states.

2. We turn now to the β spectrum of tritium. As we will see, precise measurements of the edge of the β spectrum of tritium would make it possible to determine a rather large number of parameters of the mass matrix.[37] Obviously, however, again in this case we would not be able to distinguish Dirac masses from Majorana masses, since the manifestation of the neutrino mass is a purely kinematic manifestation, and the β spectrum is sensitive to both. For example, let us assume that there are three types of neutrinos, $\nu_i (i = e,\mu,\tau)$. If the mass matrix is either a Dirac matrix or a Majorana matrix, with all possible mixings of the different ν_i, then ν_e would be a superposition of three diagonal states with masses μ_1, μ_2, and μ_3:

$$\nu_e=U_{e1}\varphi_1+U_{e2}\varphi_2+U_{e3}\varphi_3,\quad \sum_{A=1}^{3}|U_{eA}|^2=1. \qquad (24)$$

The spectrum of electrons in β decay should then look like that in Fig. 8. The quantity plotted along the ordinate axis here is the function $F(E_e)=\sqrt{W_\beta/P_eE_e}$, and E_0 is the maximum electron energy at a zero neutrino mass. With decreasing electron energy, thresholds for the production of more massive neutrinos are crossed. As can be seen from this figure, it is possible in principle to find from the spectrum all three diagonal masses $\mu_A (A = 1,2,3)$ and two mixing angles (of the three in the mixing matrix). At $E_e < E_0-\max\{\mu_A\}$, the spectrum is sensitive to some effective mass of a neutrino emitted in β decay. This effective mass is given approximately by

$$\mu_{eff}\approx\left(\sum_{A=1}^{3}\mu_A^2|U_{eA}|^2\right)^{1/2}. \qquad (25)$$

Let us compare this mass with the mass which appears in the amplitude for the $2\beta(0\nu)$ decay:

$$\mu_{2\beta}=\sum_{A=1}^{3}\mu_AU_{eA}^2. \qquad (26)$$

We assume that the neutrino mass matrix is a Majorana matrix. Using the properties of the mixing matrix U_{Ai}, we can easily show that

$$\mu_{2\beta}\leqslant\mu_{eff}. \qquad (27)$$

In particular, we could have a situation with $\mu_{eff}\neq0$ but $\mu_{2\beta}=0$. This possibility arises, for example, in the model of Zee,[63] in which the different diagonal neutrinos have opposite CP parities.[62] This situation corresponds effectively to the vanishing of the matrix element m_{ee}^L of the left-handed Majorana mass matrix m_{ik}^L $(i, k = e, \mu, \ldots)$.

Diagonal neutrinos cannot always have definite C and CP parities. If the initial mass Lagrangian, written in terms of current neutrinos, is invariant under CP conjugation, then the diagonal neutrinos φ_A generally do not transform into themselves under C and CP transformations. It follows from the explicit expression for φ_A that

$$C\bar{\varphi}_A^T=\varphi_A,\qquad C=i\gamma_2\gamma_0;$$

and the diagonalization procedure can be chosen such that the sign on the right side of this relation is positive for all φ_A. Here $\mu_{2\beta}$ is related to the masses of the diagonal neutrinos by expression (25). Questions related to the charge parity of the states which arise upon a diagonalization of the neutrino mass matrix are discussed in more detail in Refs. 62, 63, and 37.

Returning to the comparison of $\mu_{2\beta}$ and μ_{eff}, we note that inequality (27) may also hold in general. In the particular case in which the masses are Dirac masses, we would obviously have $\mu_{2\beta}=0$.

A nonzero effective mass of a neutrino emitted in β decay would thus by no means imply that a $2\beta(0\nu)$ decay must exist. As for the oscillations, we note that they may also be absent if, for example, we assume that all the diagonal masses are identical. The result of the comparison of the three processes may be formulated as follows: A nonzero effective mass of a neutrino emitted in β decay is not a sufficient condition for the existence of oscillations in a $2\beta(0\nu)$ decay; neutrino oscillations and the $2\beta(0\nu)$ decay may also occur independently of each other. There is, nevertheless, a coupling between these effects: If the $2\beta(0\nu)$ decay or the oscillations (involving electron neutrinos) do occur, then the effective mass of the neutrino emitted in β decay should be nonzero. The situation is shown schematically in Fig. 9.

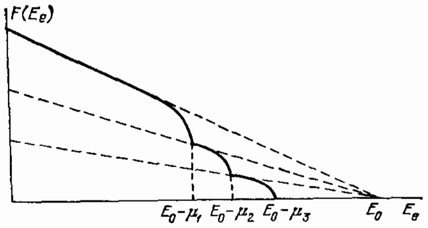

FIG. 8.

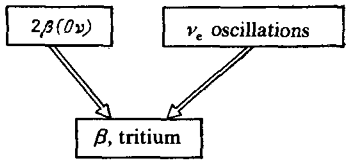

FIG. 9.

3. The possibility of determining the neutrino mass from measurements of the spectrum of bremsstrahlung γ rays produced in the capture of an orbital electron,

$$Z \rightarrow (Z-1) + \gamma + \nu_e, \qquad (28)$$

was recently studied in detail.[70,71] This method is obviously analogous to determining the neutrino mass from measurements of the electron spectrum in β decay. Therefore, everything we said in the preceding subsection on the β spectrum of tritium also applies to reaction (28). As before, a determination of the neutrino mass from the γ spectrum cannot distinguish between Dirac and Majorana masses, but it would allow us to set upper limits on masses of both types.

4. Both Dirac and Majorana neutrino masses can initiate the processes $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ (Refs. 72–74), muon-antimuon oscillations[75,76] $[\mu^+ e^-] \rightleftarrows [\mu^- e^+]$ and the decay $K^0 \rightarrow \mu e$. Experiments on these processes thus cannot distinguish between Dirac and Majorana masses. Furthermore, the amplitudes of these processes are quadratic in the neutrino masses, so that experimental searches for them could hardly shed light on the neutrino mass matrix. This assertion does not, of course, mean that attempts to observe these processes would be pointless. In the models of Refs. 29 and 30, for example, diagrams with an exchange of heavy Higgs mesons arise, and these mesons contribute substantially to the amplitudes of the decays[77] $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$.

The processes listed in Subsections 1–4 look the same in the Dirac and Majorana cases. A simple rule can be stated: If the total lepton number $L_e + L_\mu + L_\tau$ is conserved in some process, then measurements of this process cannot distinguish Dirac from Majorana masses. Those processes in which the total lepton number is not conserved can occur only if the masses are Majorana masses (or if some other mechanism is operating). Among these processes are those which we will list in the following subsections.

5. The decays[76] $K^+ \rightarrow \pi^- e^+ e^+$, $K^+ \rightarrow \pi^- \mu^+ e^+$, $K^+ \rightarrow \pi^- \mu^+ \mu^+$. For these decays we have $\Delta L_e + \Delta L_\mu = 2$. The decay $K^+ \rightarrow \pi^- e^+ e^+$ is obviously a double neutrinoless β decay of the K meson. Its amplitude is determined by the same neutrino mass as that which determines the $2\beta(0\nu)$ decay of a nucleus.[76] The restriction on the mass m^L_{ee} which can be extracted from the experimental data available is weak, of the order of 2 MeV.

6. The reaction $\bar{\nu}_e + Z \rightarrow (Z+1) + e^-$. The amplitude of this reaction is also proportional to m^L_{ee}. The restriction found on the ratio m^L_{ee}/E_ν from the experiment by Davis[78] is again quite weak, about 0.2.

7. μ capture accompanied by the emission of a positron: $\mu^- + Z \rightarrow (Z-2) + e^+$. This reaction is analogous to K capture (of an electron) accompanied by the emission of a positron, which is in turn a crossing process with respect to the $2\beta(0\nu)$ decay.[76,79] The μ capture with $\Delta L_e + \Delta L_\mu = 2$ is interesting because its amplitude is determined by a parameter of the Majorana mass matrix, $m^L_{e\mu}$, and is independent of the other masses. Present experiments are, however, insensitive to small masses $m^L_{e\mu}$ and can only place a lower limit on the large "transitional" mass $M_{e\mu}$, 10 GeV (Ref. 79).

In summary, from this comparison of the various processes we conclude that at the present level of experimental capabilities searches for the $2\beta(0\nu)$ decay represent the most sensitive method for measuring the Majorana mass of the electron neutrino at a few tens of electron volts.

4. PROBABILITY OF DOUBLE BETA DECAY

If a neutrinoless 2β decay were to be discovered, we would have to ask what caused it. A large number of mechanisms for 2β decay arise in the models which we discussed earlier which have a spontaneous violation of lepton-number conservation. As a rule, the spontaneous breaking of the L (B–L) symmetry would give rise to Majorana masses which would contribute to the amplitude of the $2\beta(0\nu)$ decay. An exceptional case is Zee's model,[63] in which the Majorana-like coupling of the leptons with Higgs bosons is anti-symmetric with respect to generations, so that diagonal elements of the left-handed Majorana neutrino mass matrix (including m^L_{ee}) would be zero.

In this section we give expressions for the probabilities for all possible 2β transitions to ground and excited levels of the daughter nucleus. We should point out the various estimates of integral probabilities found by different investigators diverge substantially, primarily because of the calculations of the nuclear matrix elements. The existing estimates of these matrix elements differ by more than an order of magnitude. The results found by the different investigators will be summarized at the end of this section.

It is no less important to know the differential characteristics of the decay, primarily the distribution in the total energy of the electrons. Clearly, without measurements of these distributions we would not be able to distinguish, for example, neutrinoless decay from two-neutrino decay, in the calculated probabilities for which there are again large discrepancies. Furthermore, one could in principle work from the spectra and angular distributions to distinguish among the various mechanisms for lepton-number breaking, e.g., to distinguish right-handed currents from a Majorana mass. In contrast with the total probabilities, it is possible to predict essentially unambiguously the shape of the electron spectra and angular distributions, at least for transitions to the ground level of the daughter nucleus. In addition to $2\beta(0\nu)$ decay, studies are being made of the crossing process—the capture of an orbital electron accompanied by the emission of a positron—and also the 2β decay accompanied by the emission of a scalar particle (a majoron): $N \rightarrow N' e^- e^- M^0$.

a) Light Majorana neutrino

The decay $2\beta(0\nu)$ with the Majorana neutrino mass has been studied in several places.[11,80–95] As we will see, the Majorana mass can cause $2\beta(0\nu)$ transitions between initial and final nuclei having identical spins and parities. All known

2β-active nuclei have $J^P = 0^+$, so that our calculations actually pertain to $0^+ \to 0^+$ nuclear transitions.

The amplitude for the $2\beta\,(0\nu)$ decay is

$$S = -\frac{i}{2!}\int d^4x\, d^4y\, \langle N'e^-e^- |\, TL(x)\, L(y) \,|\, N\rangle. \tag{29}$$

(We will not retain the common phase factor in the calculations below.) Here N and N′ characterize the states of the initial and final nuclei, and $L(x)$ is the Lagrangian of the interaction of weak charged currents, given by

$$L = \frac{G_F}{\sqrt{2}} J_\mu(x)\, \bar{e}(x)\, \gamma_\mu (1+\gamma_5)\, \nu(x),$$
$$G_F = \frac{10^{-5}}{m_p^2},$$

where J_μ is the hadron current. Let us calculate the contribution of the two-nucleon mechanism in Fig. 10. (The so-called Δ mechanism, $\Delta \to pe^-e^-$, does not contribute because Δ and p have different spins.) The effective weak Lagrangian $L(x)$ can be rewritten as

$$L(x) = \frac{G_F}{\sqrt{2}} \cos\theta_C \bar{p}(x)\, O_\mu n(x)\, \bar{e}(x)\, \gamma_\mu (1+\gamma_5)\, \nu(x),$$
$$O_\mu = \gamma_\mu (1 + g_A\gamma_5), \quad g_A = 1{,}25 \tag{30}$$

(the terms with neutral currents of course do not contribute). The symbols for particles denote the corresponding second-quantization operators. The final electrons are described by plane waves

$$\frac{1}{\sqrt{2\varepsilon}}\, e^{ikx}\bar{e}(k),$$

where $k = k_1$ or k_2 are the 4-momenta of the electrons, $\varepsilon_{1,2} = k_{0,1,2}$, and $\bar{e}(k)$ is a 4-spinor. After the standard transformations (Appendix I), we can write the matrix element of the S matrix in the form

$$S = \frac{2\pi\delta(\varepsilon_1+\varepsilon_2-\Delta)}{\sqrt{2\varepsilon_1}\sqrt{2\varepsilon_2}} \frac{G_F^2 m_\nu}{2\pi} \cos^2\theta_C \cdot l(k_1, k_2) \left\langle \frac{1}{r} \right\rangle. \tag{31}$$

The last factor in this expression is the nuclear matrix element,

$$\left\langle \frac{1}{r} \right\rangle = \left\langle N' \left| \sum_{a\neq b} H (1 - g_A^2 \sigma_a\sigma_b) \right| N \right\rangle, \qquad H = \frac{e^{iQ_0 r}}{r}, \tag{32}$$

and $\mathbf{r} = \mathbf{x} - \mathbf{y} = \mathbf{r}_a - \mathbf{r}_b$. In (32) it is assumed that there are isospin operators $\tau_a^+\tau_b^+$ which transform two neutrons into two protons. The expression $\langle N'| \ldots |N\rangle$ denotes the integral

$$\left\langle N' \left| \sum_{a\neq b} A_{ab} \right| N \right\rangle = \int dx\, dy\, \Phi_{N'}^+ \sum_{a\neq b} A_{ab}\Phi_N, \tag{33}$$

where Φ_N and $\Phi_{N'}$ are the wave functions of the initial and final nuclei, and the indices a and b specify the individual nucleons of the nucleus. In the integrand we have ignored the dependence on the energy of the electrons, since the condition $|\varepsilon_1 - \varepsilon_2| r \ll 1$ holds in all cases of practical interest. The presence of the exponential function $e^{iQ_0 r}$ means that the energy region of the virtual neutrino, $Q_0 < 1/r$, makes the basic contribution. It is easy to see that nuclear matrix element (32) is nonzero only for transitions which do not involve changes in the spin or parity of the nucleus. The $0^+ \to 0^+$ transitions are of practical interest.

The dependence on the 4-momenta and spins of the electrons is embodied in the lepton brackets,

$$l(k_1, k_2) = \bar{e}(k_1)(1-\gamma_5)\, C\bar{e}^T(k_2), \tag{34}$$

which are antisymmetric with respect to lepton variable:

$$l(k_1 k_2) = -l(k_2, k_1).$$

The dependence on the lepton and nuclear variables in the S matrix can thus be factorized. The part which depends on the lepton variables can be calculated exactly in a Lorentz-covariant manner. The doubly differential probability for the $2\beta\,(0\nu)$ decay is

$$\frac{dw}{d\varepsilon_1\, d\cos\vartheta} = \frac{G_F^4 m_\nu^2 \cos^4\theta_C}{2(2\pi)^5} \left| \left\langle \frac{1}{r} \right\rangle \right|^2 \times \varepsilon_1^2 (\Delta - \varepsilon_1)^2 (1 - v_1 v_2 \cos\vartheta)\, v_1 v_2 F_c(v_1)\, F_c(v_2), \tag{35}$$

where Δ is the mass difference between the initial and final nuclei, v_1 and v_2 are the velocities of the final electrons, the Fermi factor

$$F_c(v) = \frac{2\pi\alpha z/v}{1 - \exp(-2\pi\alpha z/v)}$$

reflects the effect of the nuclear Coulomb field on an individual electron, ε_1 is the energy of one of the electrons, and ϑ is the electron divergence angle. We rewrite (35) as

$$\frac{dw}{d\varepsilon_1\, d\cos\vartheta} = f(\varepsilon_1)\,[1 - a(\varepsilon_1)\cos\vartheta].$$

Figure 11 shows the functions $f(\varepsilon_1)$ and $a(\varepsilon_1)$. We set $v_{1,2} = 1$

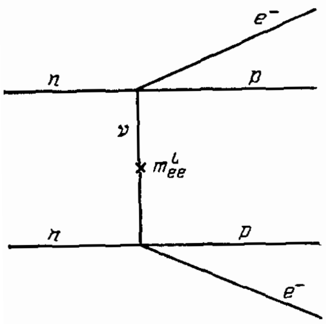

FIG. 10.

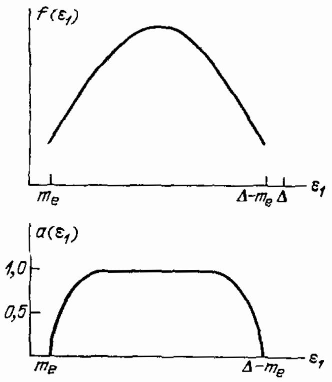

FIG. 11.

in the exponential functions which arise from the Coulomb corrections; it then becomes a simple matter to calculate the total probability:

$$w = \frac{G_F^4 m_\nu^2 \cos^4\theta_C}{30\,(2\pi)^5}\left|\left\langle\frac{1}{r}\right\rangle\right|^2$$

$$\times [F_C(1)]^2 \Delta^5 (1-20x^3+30x^4-12x^5), \qquad (36)$$

$$x = \frac{m_e}{\Delta}.$$

b) Heavy Majorana neutrino

The calculation of the probability for the $2\beta(0\nu)$ decay caused by the exchange of a Majorana neutrino with a mass M_ν considerably higher than the characteristic energies of the process is completely analogous to the preceding calculation. There is a difference that in the above equations the effective neutrino propagator was proportional to m_ν/q^2, in accordance with a Coulomb-like interaction between nucleons, which exchange a light virtual neutrino. In the exchange of a heavy Majorana particle the forces acting between the decaying nucleons are described by the Yukawa potential $e^{-M_\nu r}/r$ with a constant which is, as before, proportional to the mass of the corresponding neutrino. Accordingly, the nuclear matrix element, denoted by $\langle 1/r\rangle$ in (31), should be replaced in the present case by

$$\left\langle\frac{e^{-M_\nu r}}{r}\right\rangle = \int dx\,dy\,\Phi_{N'}^{+}\sum_{a\neq b}\frac{e^{-M_\nu r}}{r}e^{iQ_0 r}(1-g_A^2\sigma_a\sigma_b)\,\Phi_N. \qquad (37)$$

In the exchange of a light neutrino the nuclear matrix element $\langle 1/r\rangle$ is obviously determined by the nuclear radius. In this case, in evaluating the nuclear matrix element in (37), we need to consider short-range nucleon-nucleon correlations. Corresponding calculations were carried out in Ref. 96, where the following expression was derived for the nuclear matrix element:

$$\frac{3e^{-M_\nu r_c}(1+M_\nu r_c)-e^{-2M_\nu R}(1+2M_\nu R)}{M_\nu^2[(2R)^3-r_c^3]}\sum_{a\neq b}(1-g_A^2\sigma_a\sigma_b),$$

where $r_c \sim 0.4$–0.5 fm is the core radius, and R is the nuclear radius. No changes occur in the dependence on the spins and momenta of the electrons.

Estimates of the contribution of the Δ mechanism ($\Delta^-\to pe^-e^-, n\to\Delta^{++}e^-e^-$) to the probability for the $2\beta(0\nu)$ decay initiated by a heavy Majorana neutrino were also found in Ref. 96. However, these estimates are not pertinent, since the operator $(1-g_A^2\sigma_a\sigma_b)$ where a and b specify nucleons or quarks, have nonzero matrix elements between hadron states with identical quantum numbers J^P.

c) Right-handed currents

A $2\beta(0\nu)$ decay can also occur if there are right-handed currents, and the neutrino is a Majorana particle. In the most general case the weak-interaction Lagrangian incorporating left- and right-handed charged currents is

$$L = \frac{G_L}{\sqrt{2}}J_\mu^L J_\mu^{L+} + \frac{G_R}{\sqrt{2}}J_\mu^R J_\mu^{R+} + \frac{G_{LR}}{\sqrt{2}}(J_\mu^L J_\mu^{R+} + J_\mu^R J_\mu^{L+}). \qquad (38)$$

The last term in this expression arises if there is a mixing of left- and right-handed W bosons. From the standpoint of models with a left-right symmetry, the amplitude for the $2\beta(0\nu)$ decay is dominated by terms proportional to $G_L G_R = G_F^2(m_{WL}/m_{WR})^2$. The contribution proportional to $G_L G_{LR}$ is determined by the angle of the mixing of W_L and W_R. The contribution to the S matrix proportional to $G_R G_{LR}$ contains an additional power of the ratio $(m_{W_L}/m_{W_R})^2$ in comparison with the preceding contribution. Furthermore, the contributions of three terms ($G_L G_R$, $G_L G_{LR}$ and $G_{LR} G_R$) are multiplied by the mixing angle of the (nominally) left- and right-handed neutrinos, α (Section 2).

The matrix element of the S matrix of second order in L, proportional to $\eta G_F^2 = \alpha G_L G_R$, where $\eta = \alpha(m_{W_L}/m_{W_R})^2$, is

$$S = -\frac{i}{2!}\eta\frac{G_F^2}{2}\int d^4x\,d^4y\,\langle N'e^-e^- \mid T\{J_\mu^L(x)$$

$$\times J_\lambda^R(y)\,j_\mu^{L+}(x)\,j_\lambda^{R+}(y) + [L\leftrightarrow R]\}\mid N\rangle, \qquad (39)$$

where J_μ and j_μ are respectively the hadron and lepton currents, and L and R correspond to the Lorentz structure of the Dirac operators. The transformations of the S matrix are analogous to those above, but, in contrast with the case of the $2\beta(0\nu)$ decay with a Majorana mass, we must add a matrix element for Lorentz-noncovariant structures, and we must separately consider the spatial and temporal terms. The basic steps in the transformation of the S matrix are given in Appendix II. The expression for the matrix element can be written as the sum of three terms:

$$S = \eta\frac{G_F^2\cos^2\theta_C}{2\pi}\frac{2\pi\delta(\varepsilon_1+\varepsilon_2-\Delta)}{\sqrt{2\varepsilon_1}\sqrt{2\varepsilon_2}}(l_0 M_0 + l_{ik}M_{ik} + l_i M_i), \qquad (40)$$

where

$$\left.\begin{aligned} l_0 &= \bar{e}(k_1)\gamma_0 C\bar{e}^T(k_2),\\ l_{ik} &= (\mathbf{k}_1-\mathbf{k}_2)_i\,\bar{e}(k_1)\gamma_k C\bar{e}^T(k_2),\\ l_i &= (\mathbf{k}_1+\mathbf{k}_2)_i\,\bar{e}(k_1)\gamma_0\gamma_5 C\bar{e}^T(k_2); \end{aligned}\right\} \qquad (41)$$

and M_0, M_{ik}, and M_i are the nuclear matrix elements

$$\left.\begin{aligned} M_0 &= \langle N' \mid \sum_{a\neq b} H(1-g_A^2\sigma_a\sigma_b)\mid N\rangle,\\ M_{ik} &= \langle N' \mid \sum_{a\neq b} rH'n_i\{n_k(1+g_A^2\sigma_a\sigma_b)\\ &\quad + 2ig_A[\sigma_a \mathbf{n}]_k - 2g_A^2\sigma_{ak}(\mathbf{n}\sigma_b)\}\mid N\rangle,\\ M_i &= \langle N' \mid \sum_{a\neq b} RH'n_{+i}\{2g_A(\mathbf{n}\sigma_a) - ig_A^2\mathbf{n}[\sigma_a\sigma_b]\}\mid N\rangle. \end{aligned}\right\} \qquad (42)$$

The notation is the same as in (32) and (33); $\mathbf{n}_+$, H, and H' have the following meaning:

$$\mathbf{n}_+ = \frac{\mathbf{R}}{R}, \quad H = \frac{e^{iQ_0 r}}{r}, \quad H' = \frac{\partial H}{\partial r}.$$

The term with M_0 contributes to the amplitude for a transition without a change in the spin or parity of the nucleus.

The contribution M_i is nonzero for $0^+ \to 1^+$ transitions. Finally, the tensor matrix element M_{ik} contributes to the amplitudes of the transitions $0^+ \to 0^+$, 1^+, 2^+.

1) $0^+ \to 0^+$ transitions

The contribution to the amplitude is determined by the first two terms in (40):

$$S = \frac{2\pi\delta(\varepsilon_1+\varepsilon_2-\Delta)}{\sqrt{2\varepsilon_1}\sqrt{2\varepsilon_2}}\,\eta\,\frac{G_F^2}{2\pi}\cos^2\theta_C \times \{A(\mathbf{k}_1-\mathbf{k}_2)\,\bar{e}(k_1)\,\gamma C\bar{e}^{\mathrm{T}}(k_2) + B(\varepsilon_1-\varepsilon_2)\,\bar{e}(k_1)\,\gamma_0 C\bar{e}^{\mathrm{T}}(k_2)\}, \tag{43}$$

where

$$A = \frac{1}{3}\langle \mathrm{N}' \mid \sum_{a\neq b} rH'\,[1+g_A^2\sigma_a\sigma_b - 2g_A^2(\mathbf{n}\sigma_a)(\mathbf{n}\sigma_b)] \mid \mathrm{N}\rangle,$$

$$B = \langle \mathrm{N}' \mid \sum_{a\neq b} H\,[1-g_A^2\sigma_a\sigma_b] \mid \mathrm{N}\rangle.$$

The distribution in the energy of a single electron and in the angle between the electron momenta, ϑ, is

$$\frac{\mathrm{d}w}{\mathrm{d}\varepsilon_1\,\mathrm{d}\cos\vartheta} = \eta^2\frac{G_F^4\cos^4\theta_C}{4(2\pi)^5}F_c(v_1)\,F_c(v_2)\,v_1v_2\varepsilon_1^2\varepsilon_2^2 \times\Big\{(\varepsilon_1-\varepsilon_2)^2\Big(1+v_1v_2\cos\vartheta-\frac{m_e^2}{\varepsilon_1\varepsilon_2}\Big) \times|A-B|^2-4|A|^2\frac{m_e^4}{\varepsilon_1\varepsilon_2} + 4\frac{m_e^2}{\varepsilon_1\varepsilon_2}[\mathrm{Re}\,A^*(A-B)(\varepsilon_1-\varepsilon_2)^2 + |A|^2\varepsilon_1\varepsilon_2(1-v_1v_2\cos\vartheta)]\Big\}, \tag{44}$$

where $\varepsilon_1+\varepsilon_2=\Delta$.

We write this distribution as

$$\frac{\mathrm{d}w}{\mathrm{d}\varepsilon_1\,\mathrm{d}\cos\vartheta} = f(\varepsilon_1)\,[1-a(\varepsilon_1)\cos\vartheta].$$

Figure 12 shows typical functions $f(\varepsilon_1)$ and $a(\varepsilon_1)$. These distributions should be compared with the analogous distributions for the $2\beta(0\nu)$ decay caused by a Majorana neutrino mass. We see that a Majorana mass and right-handed currents lead to completely different angular dependences and completely different single-electron spectra. If we ignore the electron mass in comparison with the energy release (the effect is to reduce the accuracy of the predictions of the spectra near the minimum), we can factorize the dependence of $\mathrm{d}w/\mathrm{d}\varepsilon_1\mathrm{d}\cos\vartheta$ on the lepton and nuclear variables:

$$\frac{\mathrm{d}w}{\mathrm{d}\varepsilon_1\,\mathrm{d}\cos\vartheta} = \eta^2\frac{G_F^4\cos^4\theta_C}{4(2\pi)^5}F_c(v_1) \times F_c(v_2)\,v_1v_2\,|A-B|^2\varepsilon_1^2\varepsilon_2^2(\varepsilon_1-\varepsilon_2)^2(1+\cos\vartheta). \tag{45}$$

Consequently, inaccuracies in the estimates of the nuclear matrix elements have essentially no effect on the theoretical predictions of the angular distribution or the shape of the spectra of single electrons.

By setting $v_{1,2}\to 1$ in the exponential functions in the Coulomb functions $F_c(v_{1,2})$, we can easily integrate expression (45); we find an expression for the probability for the $2\beta(0\nu)$ decay:

$$w = \eta^2\frac{G_F^4\cos^4\theta_C}{210\,(2\pi)^5}|A-B|^2\Delta^7[F_c(1)]^2. \tag{46}$$

Some more accurate expressions, incorporating the electron mass, are given in Refs. 84, 87, and 89, for example.

Comparing this expression with expression (36) for the probability for the $2\beta(0\nu)$ decay initiated by a left-handed Majorana neutrino mass, we see that in the case of right-handed currents the probability is more sensitive to the energy release Δ. Another difference between the two mechanisms (the Majorana mass and the right-handed currents) is that in the former case both of the final electrons are (primarily) left-handed, while in the latter case the electrons have opposite helicities.

2) $0^+ \to 1^+$ transitions

The angular distribution and the spectrum of single electrons are determined by the expression

$$\frac{\mathrm{d}w}{\mathrm{d}\varepsilon_1\,\mathrm{d}\cos\vartheta} = \eta^2\frac{G_F^4\cos^4\theta_C}{3(2\pi)^5}\{(\mathbf{k}_1+\mathbf{k}_2)^2(\varepsilon_1\varepsilon_2+\mathbf{k}_1\mathbf{k}_2+m_e^2)(N_k^{(1)}N_k^{(1)}) + 2[(\mathbf{k}_1-\mathbf{k}_2)^2(\varepsilon_1\varepsilon_2+m_e^2)+(\mathbf{k}_1\mathbf{k}_2-\mathbf{k}_1^2)(\mathbf{k}_1\mathbf{k}_2-\mathbf{k}_2^2)](N_k^{(2)}N_k^{(2)}) + 4[\mathbf{k}_1\mathbf{k}_2]^2\,\mathrm{Im}\,(N_k^{(1)}N_k^{(2)})\}\,\varepsilon_1\varepsilon_2v_1v_2F_c(v_1)\,F_c(v_2), \tag{47}$$

where $(L_K^{(\alpha)}N_K^{(\beta)}) = \Sigma_{J_z}N_K^{(\alpha)*}N_K^{(\beta)}$ $(\alpha,\beta=1,2)$ are products of the nuclear elements

$$N_k^{(1)} = \langle \mathrm{N}' \mid \sum_{a\neq b} RH'n_{+k}\{2g_A(\mathbf{n}\sigma_a) - ig_A^2\mathbf{n}\,[\sigma_a\sigma_b]\} \mid \mathrm{N}\rangle,$$

$$N_k^{(2)} = \langle \mathrm{N}' \mid \sum_{a\neq b} rH'\{ig_A\,[\mathbf{n}\,[\mathbf{n}\sigma_a]]_k - g_A^2(\mathbf{n}\sigma_a)\,[\mathbf{n}\sigma_b]_k\} \mid \mathrm{N}\rangle$$

summed over the spins of the daughter nucleus. If we ignore the electron mass in comparison with Δ, the probability can be calculated quite easily; it is

$$w(0^+\to 1^+) = \eta^2\frac{G_F^4\cos^4\theta_C}{(2\pi)^5}\frac{2^4}{3\cdot 7!}\Delta^7[F_c(1)]^2 \times\{2(N_k^{(1)}N_k^{(1)}) + 5(N_k^{(2)}N_k^{(2)}) + 2\,\mathrm{Im}\,(N_k^{(1)}N_k^{(2)})\}. \tag{48}$$

In this case, in contrast with the $0^+\to 0^+$ transitions, both the integral probability and the spectral shape of the single electrons depend on the nuclear matrix elements $N^{(1)}$ and $N^{(2)}$.

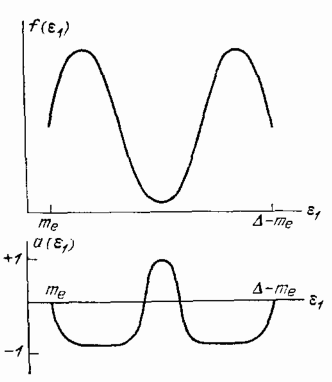

FIG. 12.

3) $0^+ \to 2^+$ transitions

The decay probability is

$$\frac{dw}{d\varepsilon_1 \, d\cos\vartheta} = \eta^2 \frac{G_F^4 \cos^4\theta_C}{15\cdot(2\pi)^5} v_1 v_2 F_c(v_1) F_c(v_2) (N_{ik}^T N_{ik}^T) \varepsilon_1\varepsilon_2$$
$$\times \{3(\mathbf{k}_1\mathbf{k}_2)^2 - (\mathbf{k}_1\mathbf{k}_2)[10(\varepsilon_1\varepsilon_2 + m_e^2) + \mathbf{k}_1^2 + \mathbf{k}_2^2]$$
$$+ 5(\varepsilon_1\varepsilon_2 + m_e^2)(\mathbf{k}_1^2 + \mathbf{k}_2^2) - \mathbf{k}_1^2\mathbf{k}_2^2\}, \quad (49)$$

where $(N_{ik}^T N_{ik}^T) = \Sigma_{J_z} N_{ik}^T N_{ik}^{T*}$ is the square of the nuclear matrix element

$$N_{ik}^T = \langle N' \mid \sum_{a\neq b} rH' n_i \{n_k (1 + g_A^2 \sigma_a\sigma_b) + 2ig_A [\mathbf{n}\sigma_a]_k$$
$$- 2\sigma_{bk}(\mathbf{n}\sigma_a)\} \mid N\rangle$$

summed over the projections of the spin of the final nucleus. Integrating over the electron energies and angles, we find

$$w(0^+ \to 2^+) = \eta^2 \frac{G_F^4 \cos^4\theta_C}{(2\pi)^5} \frac{2^5}{7!} \Delta^7 [F_c(1)]^2 (N_{ik}^T N_{ik}^T). \quad (50)$$

Some more accurate expressions, which incorporate the electron mass, can be found in Ref. 84, among other places.

In all cases of practical interest the ground level of the final nucleus has the quantum numbers $J^P = 0^+$. Levels with quantum numbers 1^+ and 2^+ are excited levels of the daughter nucleus. The lifting of degeneracy is accompanied by the emission of γ rays, which is another characteristic feature of $0^+\to 1^+$ and $0^+ \to 2^+$ transitions. The total energy of the electrons is lower than in the case of the $0^+ \to 0^+$ transition.

Up to this point we have been talking about the two-nucleon mechanism for the $2\beta(0\nu)$ decay. We know that nuclei contain an admixture of the Δ isobar at the level of a few percent. Consequently, there can be an additional mechanism for the $2\beta(0\nu)$ decay of the nucleus: due to the transition $\Delta^- p + e^- + e^-$ or $n \to \Delta^{++} + e^- + e^-$. Since the spin of the Δ isobar is 3/2, this mechanism could contribute only when the $2\beta(0\nu)$ decay results from the admixture of right-handed currents. The decay matrix element is proportional to[84]

$$(\mathbf{k}_1 - \mathbf{k}_2)_i \bar{e}(k_1) \gamma_k C \bar{e}(k_2) (i\varepsilon_{ikj} T_j + T_{ik}) \quad (i, k = 1,2,3),$$

where

$$T_j = \langle n \mid \sum_{a\neq b} rH' \sigma_{aj} \mathbf{1}_b \mid \Delta^{++}\rangle,$$
$$T_{ik} = \langle n \mid \sum_{a\neq b} rH' \sigma_{ai}\sigma_{bk} \mid \Delta^{++}\rangle.$$

The distributions in the electron energies and angles are

$$\frac{dw}{d\varepsilon_1 \, d\cos\vartheta} = \eta^2 \frac{G_F^4 \cos^4\theta_C \cdot 2^7}{3^3\cdot(2\pi)^5}$$
$$\times \mid M_\Delta \mid^2 P_\Delta v_1 v_2 F_c(v_1) F_c(v_2) \varepsilon_1\varepsilon_2 \{2(\mathbf{k}_1\mathbf{k}_2)^2$$
$$- (\mathbf{k}_1\mathbf{k}_2)[(\varepsilon_1 + \varepsilon_2)^2 + 4(\varepsilon_1\varepsilon_2 + m_e^2)]$$
$$+ 3(\varepsilon_1\varepsilon_2 + m_e^2)(\mathbf{k}_1^2 + \mathbf{k}_2^2)\},$$
$$M_\Delta = \langle rH'\rangle \langle \Phi_f \mid \Phi_i\rangle, \quad (51)$$

where P_Δ is the probability for the presence of the Δ isobar in the nucleus, and the factor $\langle \Phi_i | \Phi_f\rangle$ reflects the overlap of the wave function of the initial and final nuclei. The decay probability is

$$w_\Delta \approx \frac{G_F^4 \cos^4\theta_C}{3^3\cdot 7!\,(2\pi)^5} \eta^2 \mid M_\Delta \mid^2 P_\Delta [F_c(1)]^2 \Delta^7. \quad (52)$$

It has been shown in several studies[13,84,97] that the contribution of the Δ mechanism to the amplitude of a 2β transition without a change in the quantum numbers of the nucleus is about an order of magnitude larger than the contribution of the two-nucleon mechanism. These calculations, however, ignore the fact that the electrons carry off a nonzero total angular momentum, so that the contribution of the Δ mechanism to the amplitude for a $0^+ \to 0^+$ transition may be nonzero only if we take into account the recoil of the nucleon formed in the decay $\Delta^- \to pe^-e^-$. Let us assume that the Δ isobar is in the S wave with respect to the "fragment" of the nucleus consisting of N–1 nucleons. Since the initial nucleus has the quantum numbers 0^+, this fragment must have the same quantum numbers as the Δ. A transition to the ground level of a daughter nucleus with the quantum numbers 0^+ is possible if the proton formed in the decay of the Δ isobar is in the D wave with respect to the fragment. This conclusion means that the contribution of the Δ mechanism to the amplitude for a $0^+ \to 0^+$ transition contains an additional small factor on the order of $(pR)^2$, where p is the recoil momentum, and R the nuclear radius. The same factor arises when we take into account the contribution of the transition $n \to \Delta^{++} e^- e^-$ to 2β decay to the ground level of the daughter nucleus. In all cases of practical interest this factor is less than 1/100.

Doi *et al.*[89] assert on the basis of an analysis of the nuclear matrix elements that the contribution of the Δ mechanism to the amplitude for a $0^+ \to 0^+$ transition is strictly zero.

d) Higgs mechanisms for $2\beta(0\nu)$ decay

As we mentioned in the Introduction, additional mechanisms for $2\beta(0\nu)$ decay arise in models with doubly charged Higgs bosons. In the model of Refs. 29 and 30, for example, there is the doubly charged Higgs boson H^{--}, which is capable of decaying into e^-e^-. On the other hand, the interaction potential of Higgs bosons and the kinetic terms in the Lagrangian (with spontaneous breaking of lepton-number conservation) generate an interaction of H^{--} with, respectively, singly charged Higgs bosons and W bosons, which are in turn coupled with quarks. As a result, the diagrams for the $2\beta(0\nu)$ decay shown in Fig. 6 arise. The dependence on the electron momenta and spins of the amplitudes corresponding to these mechanisms is described by

$$\bar{e}(k_1)(1 \pm \gamma_5) C \bar{e}^T(k_2) \quad (53)$$

(the sign of γ_5 depends on the particular version of the model). This is the only Lorentz-invariant scalar which can be constructed from the wave functions of the final electrons. It is easy to show that expression (53) is antisymmetric under an interchange of spins and momenta of the electrons. We recall that the same functional dependence is found in the case of a Majorana neutrino mass [see (31) and (34)].

The contributions of the mechanisms in Fig. 6 were compared with that of a Majorana mass in Ref. 100. It was shown that in all the specific models based on the standard $SU(2)_L \times U(1)$ group the contribution of the Majorana mass turns out to be dominant.

Analogous mechanisms for $2\beta(0\nu)$ decay arise in models with a left-right symmetry.[55-59] In this case, the diagrams with doubly charged bosons may prove important. Of the three diagrams here (shown as one diagram in Fig. 6), that with gauge bosons is dominant (Fig. 13).[32] The matrix element corresponding to this diagram is

$$M = \frac{g^2 \varkappa f}{192 m_{W_R}^4 m_{\Delta_R}} \bar{e}(k_1)(1+\gamma_5) C \bar{e}(k_2) \times \int dx\, dy \langle N' | J_\mu^R(x) J_\mu^R(y) F(r) | N \rangle,$$

$$F(r) = (3 + 3M_A r + M_A^2 r^2) e^{-M_A r}, \quad \mathbf{r} = \mathbf{x} - \mathbf{y}; \qquad (54)$$

here g is the gauge constant, and f is the constant of the interaction of the gauge bosons with Δ_R^{--}. The constant f can be expressed in terms of the gauge constant (which is related to G_F) and the masses of the gauge bosons[101]:

$$f = \sqrt{\frac{4\sqrt{2} G_F \eta_R}{(1+\eta_R)^2}}, \quad \eta_R = \frac{m_{W_L}^2}{m_{W_R}^2}.$$

Finally, $\varkappa$ is the constant of the transition of the Δ_R^{--} into a pair of electrons. We recall that the same constant, $\varkappa$, is a measure of the Majorana mass of the right-handed neutrino. At $\sim m_{W_R} \sim m_{\Delta_R^{--}}$, the constant $\varkappa$ must be of the order of 0.1–0.2.

The function $F(r)$ arises because of the incorporation of the vector form factor, for which the dipole approximation is used:

$$F(k^2) = \left(1 - \frac{k^2}{M_A^2}\right)^{-2}, \quad M_A = 0{,}85 \text{ GeV}.$$

From this point on, the manipulations of the matrix element are similar to the calculations of the amplitude for $2\beta(0\nu)$ decay with a heavy Majorana neutrino. Formally, the form factor plays the same role as that played by the propagator of the heavy Majorana neutrino.

The probability for the $2\beta(0\nu)$ decay is

$$w \approx \frac{G_F^4 \varkappa^2}{72(2\pi)^5} \frac{\eta_R^6}{(1+\eta_R)^2} \frac{M_A^6}{m_{W_L}^2} [F_c(1)]^2 |M_N|^2 |\langle F \rangle|^2 \cdot \frac{2}{15} \Delta^5. \qquad (55)$$

In deriving this expression we used the approximations

$$\int dx\, dy \langle N' | J_\mu^R(x) J_\mu^R(y) F(r) | N \rangle \approx \langle F \rangle M_N,$$

$$M_N = \int dx\, dy \langle N' | J_\mu^R(x) J_\mu^R(y) | N \rangle,$$

$$\langle F \rangle = \int F(r) P(r)\, dr,$$

where $P(r)$ takes into account nuclear core. The function $\langle F \rangle$ is given explicitly in Ref. 32.

Numerical estimates of the decay probability from (55) will be found in the following sections. We wish to point out, however, that there is some arbitrariness in these estimates because of the adjustable parameters of the model.

e) The $2\beta^+(0\nu)$ decay and $e^- \to e^+$ transitions at a nucleus

If a $2\beta^-(0\nu)$ decay does occur, then a $2\beta^+(0\nu)$ decay should also occur.[102] The expression for the probability for the $2\beta(0\nu)$ decay differs from (36) by a factor which takes into account the nuclear Coulomb field, and which is given in this case by

$$F_{c+}(v) = \frac{2\pi\alpha z/v}{\exp(2\pi\alpha z/v) - 1}.$$

Along with the $2\beta^+(0\nu)$ decay there should be a capture of an orbital electron, accompanied by the emission of an e^+. This process has been examined in detail.[103] If both of these processes are caused by a Majorana neutrino mass, then the ratio of their probabilities would be

$$\frac{w(e^- \to e^+)}{w(e^+e^+)} = \frac{\frac{2}{\pi}\Delta^2 |\psi(0)|^2 F_{c+}(v)\, v}{\frac{1}{2} \cdot \frac{1}{2\pi^3} \int_{m_e}^{\Delta - m_e} \varepsilon_1^2 (\Delta - \varepsilon_1)^2 v_1 v_2 F_{c+}(v_1) F_{c+}(v_2)\, d\varepsilon_1}, \qquad (56)$$

where, as before, Δ is the mass difference between the initial and final nuclei, v is the positron velocity in the reaction $e^- \to e^+$, v_1 and v_2 are the velocities of the positrons in the $2\beta^+(0\nu)$ decay, and $\psi(0)$ is the wave function of the k electron at $r = 0$, $|\psi(0)|^2 \approx (\alpha z m_e)^3/\pi$. Since the dependence on the lepton and nuclear variables is factorized, ratio (56) does not depend on the nuclear matrix elements. Table I lists numerical values of ratio (56) for all the $2\beta^+$-active nuclei.

In this case, if the $2\beta(0\nu)$ decay results from an admixture of right-handed currents, the ratio (56) contains an additional factor $(\varepsilon_1 - \varepsilon_2)^2/\Delta^2$ in the integrand in the denominator. This factor increases the ratio $w(e^- \to e^+)/w(e^+e^+)$ by an order of magnitude, on the average. In principle, this fact could help us distinguish between different mechanisms for lepton-number nonconservation.

Unfortunately, the energies released in all the $2\beta^+$ de-

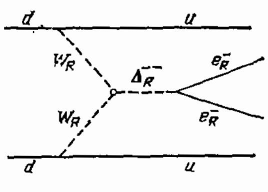

FIG. 13.

TABLE I.

Transition	$w(e^- \to e^+)/w(e^+e^+)$	Δ/m_e
$Kr^{78} \to Se$	2.6	3.64
$Ru^{96} \to Mo$	13	3.33
$Cd^{106} \to Pd$	16	3.44
$Xe^{124} \to Te$	11	4.00
$Ba^{130} \to Xe$	115	3.045
$Ce^{136} \to Ba$	650	2.71

cays are significantly smaller than those in $2\beta^-$ decays. Furthermore, the Coulomb corrections reduce the rate of the $2\beta^+$ decay. Consequently, the $2\beta^+(0\nu)$ decay is less probable than the $2\beta^-(0\nu)$ decay. Comparing the $2\beta^+$-active and $2\beta^-$-active nuclei with the maximum energy release (Ca^{48} and Xe^{124}), and assuming that their nuclear matrix elements are comparable in magnitude, we find

$$\frac{w\,(Xe^{124}\to Te+e^+ +e^+)}{w\,(Ca^{48}\to Ti+e^- +e^-)}\sim\frac{1}{400}\,.$$

f) Double β decay involving a majoron

If a majoron M^0 capable of undergoing a virtual conversion into a pair of neutrinos exists, then the decay $N\to N'e^-e^-M^0$ should occur. Figure 14 shows a Feynman diagram which describes this decay. The matrix element of this decay is calculated by analogy with the matrix element for the $2\beta(0\nu)$ decay initiated by a Majorana neutrino mass; the result is

$$M\,(e^-e^-M^0)=\frac{G_F^2\cos^2\theta_c f\sqrt{2}}{\pi}\,\bar{e}(k_1)\,(1-\gamma_5)\,C\bar{e}(k_2)\,\varphi^+_{M^0}\left\langle\frac{1}{r}\right\rangle, \tag{57}$$

where φ_{M^0} is the wave function of the majoron M^0, and $f/\sqrt{2}$ is the constant of the coupling of the majoron with the neutrino. The nuclear matrix element, $\langle 1/r\rangle$, is the same as that in the $2\beta(0\nu)$ decay with a Majorana neutrino mass [see (32)]. The reason is that the Fourier transform of the expression $[\hat{q}(\hat{q}-\hat{k})]^{-1}$ gives us the same result as $1/q^2$. The amplitude for the 2β decay accompanied by the emission of a majoron is thus nonzero only for transitions without changes in the spin or parity of the nucleus.

The differential probability for the decay $N\to N'e^-e^-M^0$ is

$$\frac{dw}{d\varepsilon_1\,d\varepsilon_2\,d\cos\vartheta}=\frac{2G_F^4 f^2\cos^4\theta_c}{(2\pi)^7}\left|\left\langle\frac{1}{r}\right\rangle\right|^2 \times v_1v_2F_c(v_1)\,F_c(v_2)\,\varepsilon_1^2\varepsilon_2^2\,(\Delta-\varepsilon_1-\varepsilon_2)\,(1-v_1v_2\cos\vartheta), \tag{58}$$

where ε_1 and ε_2 are the electron energies. The spectrum of single electrons is given by

$$\frac{dw}{d\varepsilon_1}\approx\frac{G_F^4 f^2\cos^4\theta_c}{3\cdot(2\pi)^7}\left|\left\langle\frac{1}{r}\right\rangle\right|^2 \times[F_c(1)]^2\,\varepsilon_1^2\,[(\Delta-\varepsilon_1)^4-4\,(\Delta-\varepsilon_1)+3]\,m_e^7 \tag{59}$$

(Δ and ε_1 are expressed in units of the electron mass). In integrating (57) we set $v_{1,2}=1$ in the arguments of the exponential functions in the functions $F_c(v_1)$ and $F_c(v_2)$. Figure 15 shows a typical spectrum. If we ignore the electron mass in comparison with the energy release Δ, we find that the spectrum reaches a maximum at $\varepsilon_1=\Delta/3$.

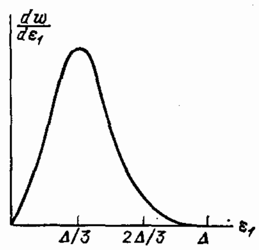

FIG. 15.

A basic distinguishing feature of the neutrinoless 2β decay $N\to N'e^-e^-$ is that the total energy of the electrons is constant: $E=\varepsilon_1+\varepsilon_2=\Delta$. In the decay $N\to N'e^-e^-M^0$, part of the energy is carried off by the majoron. The distribution in the total energy of the electrons is given by

$$\frac{dw}{dE}=\frac{G_F^4 f^2\cos^4\theta_c}{15\,(2\pi)^7}\left|\left\langle\frac{1}{r}\right\rangle\right|^2 \times[F_c(1)]^2\,m_e^7\,(\Delta-E)\,(E^5-20E^2+30E-12). \tag{60}$$

Figure 16 shows a typical distribution in $E=\varepsilon_1+\varepsilon_2$. At $\Delta\gg m_e$ the maximum of the distribution dw/dE lies at $E=(5/6)\Delta$. Incorporating the electron mass shifts the peaks in Figs. 15 and 16 in the soft direction. The probability for the decay $N\to N'e^-e^-M^0$ is

$$w\,(e^-e^-M^0)\approx\frac{G_F^4 f^2\cos^4\theta_c}{315\,(2\pi)^7}\,\Delta^7\left|\left\langle\frac{1}{r}\right\rangle\right|^2[F_c(1)]^2. \tag{61}$$

The ratio of the probability for the decay $N\to N'e^-e^-M^0$ to that for the $2\beta(0\nu)$ decay initiated by a Majorana neutrino mass does not depend on the nuclear matrix element and has the value

$$\frac{w\,(e^-e^-M^0)}{w\,(e^-e^-)}=\frac{f^2}{84\pi^2}\left(\frac{\Delta}{2m_{ee}^L}\right)^2\Phi\left(\frac{\Delta}{m_e}\right). \tag{62}$$

In the limit $\Delta\gg m_e$ we have $\Phi(\Delta/m_e)=1$.

In the model of Refs. 29 and 30, the matrix element m_{ee}^L of the left-handed Majorana mass matrix of the neutrino is related to the interaction constant f and the vacuum expectation value of the neutral component of the triplet, $\langle H^0\rangle$, by

$$m_{ee}^L=\frac{f}{\sqrt{2}}\langle H^0\rangle.$$

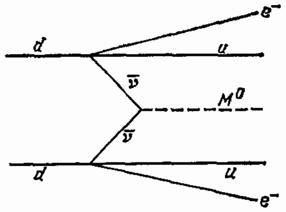

FIG. 14.

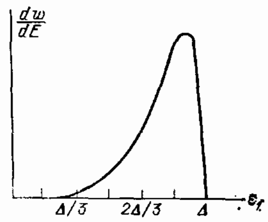

FIG. 16.

In this model, the ratio (62) does not contain the coupling constant f and is given by

$$\frac{w\,(\mathrm{e^-e^-}M^0)}{w\,(\mathrm{e^-e^-})}=\frac{1}{84\pi^2}\left(\frac{\Delta}{v}\right)^2,\qquad v=\frac{\langle \mathrm{H}^0\rangle}{\sqrt{2}}. \tag{63}$$

Some other physical consequences of the existence of a majoron and limitations on the parameters of the model were analyzed in Refs. 28–30 and 45–47. An astrophysical limitation was found on the vacuum expectation value $v=\langle H^0\rangle/\sqrt{2}$ in Ref. 30: $v<100$ keV. Substituting this value into (63), we find that for most of the 2β-active nuclei the decay accompanied by the emission of a majoron (if it exists) is more probable than the $2\beta\,(0\nu)$ decay initiated by a Majorana neutrino mass.

g) Double β decay accompanied by the emission of two neutrinos

The decay $\mathrm{N}\to\mathrm{N'e^-e^-}\bar{\nu}\bar{\nu}$ is allowed by lepton-number conservation, and there is no doubt that it occurs. The probability for the $2\beta\,(2\nu)$ decay is small because of the small phase space. In most cases of practical interest, the possible lepton-number-nonconservation effects would have no significant influence on the decay probability and can thus be ignored. In numerical estimates there are uncertainties which stem, as before, from the calculations of the nuclear matrix element. The probability for a transition to the ground level of the daughter nucleus, integrated over the neutrino momenta, is[84]

$$\frac{dw}{d\varepsilon_1\,d\varepsilon_2\,d\cos\vartheta}=\frac{G_F^4\cos^4\theta_C}{480\pi^7}\left|\left\langle \mathrm{N}'\left|\frac{1}{\mu_0}\sum_{a\neq b}(1-g_A^2\sigma_a\sigma_b)\right|\mathrm{N}\right\rangle\right|^2\times v_1v_2\varepsilon_1^2\varepsilon_2^2(\Delta-\varepsilon_1-\varepsilon_2)^5(1-v_1v_2\cos\vartheta)\,F_c(v_1)\,F_c(v_2),$$

where $\mu_0=\langle E_{\mathrm{int}}\rangle-(M+M')/2$, $\langle E_{\mathrm{int}}\rangle$ is the expectation value of the energy of the intermediate nucleus, and M and M' are the masses of the initial and final nuclei, respectively. The spectrum of single electrons is

$$\frac{dw}{d\varepsilon_1}\sim\varepsilon_1^2(T-t_1)^6\,[(T-t_1)^2+8m_e(T-t_1)+28m_e^2],$$
$$t_{1,2}=\varepsilon_{1,2}-m_e$$

and is shown in Fig. 17. The probability for the $2\beta\,(2\nu)$ decay is

$$w\,(2\beta,\ 2\nu)=\frac{2G_F^4\cos^4\theta_C}{11!\,\pi^7}\left|\left\langle \mathrm{N}'\left|\frac{1}{\mu_0}\sum_{a\neq b}(1-g_A^2\sigma_a\sigma_b)\right|\mathrm{N}\right\rangle\right|^2\times[F_c(1)]^2T^7(T^4+22T^3m_e+220T^2m_e^2+990Tm_e^3+1980m_e^4),$$

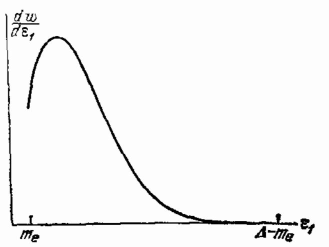

FIG. 17.

where $T=\Delta-2m_e$ is the maximum kinetic energy of a single electron.

The amplitudes for transitions to excited levels of the daughter nucleus ($0^+\to1^+$, 2^+) contain powers of the momenta of the final leptons higher than those in the amplitude for the $0^+\to0^+$ transition. These transitions to excited levels are thus forbidden. The probability for a $2\beta\,(2\nu)$ decay accompanied by a transition to the 2^+ level, integrated over the neutrino momenta, is

$$\frac{dw}{d\varepsilon_1\,d\varepsilon_2\,d\cos\vartheta}=\frac{G_F^4\cos^4\theta_C}{336\pi^7}\left[\sum_{J_z}\left|\left\langle \mathrm{N}'\left|\frac{g_A^2}{\mu_0^3}\sum_{a\neq b}\sigma_{ai}\sigma_{bk}\right|\mathrm{N}\right\rangle\right|^2\right]\times F_c(v_1)\,F_c(v_2)\,v_1v_2\varepsilon_1^2\varepsilon_2^2(\varepsilon_1-\varepsilon_2)^2\times(\Delta-\varepsilon_1-\varepsilon_2)^7\left(1+\frac{1}{3}v_1v_2\cos\vartheta\right).$$

The expression in square brackets here is the square of the nuclear matrix element summed over the projections of the spin of the final nucleus. The spectrum of single electrons is

$$\frac{dw}{d\varepsilon_1}\sim\varepsilon_1^2(T-t_1)^8\,[(T-t_1)^4-6t_1(T-t_1)^3+459m_e^2t_1^2+11(t_1^2-4m_et_1+m_e^2)(T-t_1)^2+110m_et_1(t_1-m_e)(T-t_1)].$$

The integral probability for the $0^+\to2^+$ transition is

$$w\,(2\beta,\ 2\nu)_{0^+\to2^+}=\frac{2^33^2G_F^4\cos^4\theta_C}{15!\,\pi^7}\times\left(\sum_{J_z}\left|\left\langle \mathrm{N}'\left|\frac{g_A^2}{\mu_0^3}\sum_{a\neq b}\sigma_{a_i}\sigma_{b_k}\right|\mathrm{N}\right\rangle\right|^2\right)\times[F_c(1)]^2T^{11}(T^4+30m_eT^3+420m_e^2T^2+1820m_e^3T+2730m_e^4).$$

The probability for the $0^+\to1^+$ transition is twice that for $0^+\to2^+$ transition. The nuclear matrix element for the $0^+\to1^+$ transition is

$$\left\langle \mathrm{N}'\left|\frac{g_A}{\mu_0^3}\sum_{a\neq b}\sigma_{a_i}1_b\right|\mathrm{N}\right\rangle.$$

h) Nuclear Coulomb field

Incorporation of the effect of the Coulomb field on an individual electron in the expressions for the probabilities for 2β decay given above reduces to a multiplication of the probabilities by the Fermi factor

$$F_c(v)=\frac{2\pi\alpha z/v}{1-\exp(-2\pi\alpha z/v)}.$$

(In evaluating the integral probability, we used the approximations $v_{1,2}=1$.) The accuracy is quite good here if the S-wave part is important in the electron wave functions, as it is, for example, in the $2\beta\,(0\nu)$ decay with the Majorana neutrino mass. We recall that the terms of zeroth order in the electron momenta in the expansion of the wave functions make the predominant contribution. A more accurate expression for $F_c(v)$, which incorporates the finite size of the nucleus, is[104]

$$F_c(v) = 4\left|\frac{\Gamma(\gamma+iy)}{\Gamma(1+2\gamma)}\right|^2 e^{\pi y}(2kR)^{2(\gamma-1)},$$
$$\gamma = \sqrt{1-(\alpha z)^2}, \quad y = \frac{\alpha z}{v}. \tag{64}$$

In those cases in which the P wave is significant, we should use, instead of plane waves, the exact solution for the electron wave functions in a Coulomb field.[89,104] This comment applies to the $2\beta(0\nu)$ transitions caused by an admixture of a right-handed interaction. The use of the exact Coulomb wave functions complicates all the calculations considerably and leads to different numerical results for the decay half-life. On the otherhand, the selection rules, the spectra, and the angular distributions of the electrons remain the same as when the electron wave functions are replaced by plane waves.[104]

In the numerical estimates below we will use the tabulated values of the functions $F_c(v)$ given in Ref. 105.

i) Numerical estimates; limitations on the lepton non-conservation parameters.

The 2β transitions are classified in Table II. A larger circle means an allowed transition, a smaller circle a suppressed transition, and a cross a forbidden transition.

To find restrictions on the Majorana neutrino mass and other parameters characterizing the mechanisms for $\Delta L = 2$ transitions, we need to know the nuclear matrix elements. The results of calculations of the nuclear matrix elements by different investigators differ substantially. This is the primary (but not the only) reason for the discrepancies between the estimated upper limits on the parameters of processes with $\Delta L = 2$. The results obtained by the various investigators are summarized in Table III. The estimates shown here were found from expressions for the probability for the two-nucleon (2n) mechanism for $2\beta(0\nu)$ decay.

From the experimental data on $2\beta(0\nu)$ decay we can also find a lower limit on the mass of the heavy Majorana neutrino. According to Ref. 96, this restriction is 3 GeV. A similar restriction was found in Ref. 83 from data on Ca^{48}. An increase in the experimental upper limit on the decay half-life could not substantially change this result, since the matrix element contains an exponential functional dependence on the mass of the heavy neutrino, M_ν [see (37)].

In models with an L–R symmetry, diagrams with doubly charged Higgs bosons (Fig. 13) may contribute substantially to the amplitude for $2\beta(0\nu)$ decay. The parameters of the model enter the probability as the combination

$$\varkappa^2\frac{\eta_R^5}{(1+\eta_R)^2} \approx \varkappa^2\eta_R^5 \quad \text{for} \quad \eta_R \ll 1.$$

We set the nuclear matrix element M_N equal to 1 in (55). We then find the following limitation from the data of Ref. 107 on Ca^{48}:

$$\eta_R^5\varkappa^2 < 6\cdot10^{-9}.$$

With $\chi \sim 0.1$ we find the following restriction on η_R:

$$\eta_R = \frac{m^2_{W_L}}{m^2_{W_R}} < \frac{1}{20}.$$

In the particular version of the model of Ref. 57 which relates the small value of the neutrino mass to the dominance of the left-handed interaction we would have

$$m_{\nu_L} = \text{const}\cdot\frac{m_e^2}{m_{W_R}} < \sim 1 \text{ eV}.$$

Approximately the same result was found in Ref. 32. This limitation on the Majorana mass of the left-handed neutrino should not be taken too literally, since it involves a special choice of parameters and the Higgs sector of the model. This example demonstrates the possible dominance of the Higgs mechanism in the amplitude for the $2\beta(0\nu)$ decay

5. WHAT WOULD THE DISCOVERY OF THE $2\beta(0\nu)$ DECAY MEAN?

In order to distinguish experimentally among the various mechanisms for $2\beta(0\nu)$ decay it is necessary to measure the angular or energy distributions of the decay products. For example, a Majorana neutrino mass and right-handed currents lead to completely different spectra for single electrons and to completely different distributions in the angle between the electron momenta. Measurements of the differential characteristics, however, constitute a more complicated problem than identifying events with a fixed total energy of the final electrons. Furthermore, there may be mechanisms which lead to the same differential distributions as would result from a neutrino mass. Examples are the mechanisms which reduce to the effective $\Delta L = 2$ Pontecorvo in-

TABLE II.

Type of 2β transition	2β, 2ν		2β, 0ν						2β, M^0	
			Majorana neutrino mass		Right-handed currents		Higgs mechanism			
Mechanism	$2n$	Δ	$2n$	Δ	$2n$	Δ	$2n$	Δ	$2n$	Δ
Transition:										
$0^+ \to 0^+$	◯	×	◯	×	◯	○	◯	×	◯	×
$0^+ \to 1^+$	○	○	×	×	◯	◯	×	×	×	×
$0^+ \to 2^+$	○	○	×	×	◯	◯	×	×	×	×

TABLE III.

I	II	III	IV	V
Transition	$2\beta\,(2\nu)$, $T_{1/2}$, yr (theory)	$T_{1/2}$, yr (experiment)	m_ν, eV	η
$\frac{Te^{130}\to Xe}{Te^{128}\to Xe}$	$\frac{T_{1/2}(128)}{T_{1/2}(130)}=5.1\cdot10^{3}$	$T_{1/2}(128)/T_{1/2}(130) = (1.59\pm0.06)\cdot10^{3}$ [8]	$=34$ [84] >0.8 [92] $=10$ [86] $=10$ [89]	$=1{,}5\cdot10^{-4}$ [81] $=4.4\cdot10^{-5}$ [89]
		$T_{1/2}(128)/T_{1/2}(130) = (0.9\pm0.95)\cdot10^{4}$ [9]	<4.7 [89]	
$Te^{130}\to Xe$	$T_{1/2}=2.55\cdot10^{21}$	$T_{1/2}=(2.2\pm0{,}7)\cdot10^{21}$ [106] $T_{1/2}=(2.55\pm0.20)\cdot10^{21}$ (c)		$<10^{-3}$ [13]
$Ca^{48}\to Ti$	$T_{1/2}=3.6\cdot10^{19}$	$T_{1/2}(0\nu)>2\cdot10^{21}$ [107] $T_{1/2}(2\nu)>3.6\cdot10^{19}$ [107]	$<50\ (\times\sqrt{2})$ [81] <700 [84] <45 [89] <80 [83] <73 (*)	$<6.2\cdot10^{-4}$ [84] $<4\cdot10^{-5}$ [89] $<4\cdot10^{-4}$ [98] $<3\cdot10^{-6}$ [99]
$Se^{82}\to Kr$	$T_{1/2}=1.45\cdot10^{20}$	$T_{1/2}(0\nu)>3{,}1\cdot10^{21}$ [108] $T_{1/2}=(1.45\pm0.15)\cdot10^{20}$ (c) $T_{1/2}(2\nu)=(1.0\pm0{,}4)\cdot10^{19}$ [109]	<280 [84] <12 [85] <33 [89] <89 (*)	$<3.4\cdot10^{-4}$ [84] $<4{,}4\cdot10^{-5}$ [84]
$Ge^{76}\to Se$	$T_{1/2}=2.3\cdot10^{21}$	$T_{1/2}(0\nu)>5\cdot10^{21}$ [110]	<550 [84] <15 [85] <45 [89] <146 (*)	$<10^{-3}$ [84] $<8\cdot10^{-5}$ [89]
		$T_{1/2}(0\nu)>2\cdot10^{22}$ [111]	<22 [112] <8.3 [88, 111] <73 (*)	$<4\cdot10^{-5}$ [111] $<1.5\cdot10^{-5}$ [111, 88]
$Mo^{100}\to Ru$		$T_{1/2}(0\nu)>2.1\cdot10^{21}$ [112]	$<25-80$ [112] <88 (*)	

Notes

1. Column III shows experimental data on $0^+\to0^+$ transitions for Ca^{48}, Ge^{76}, Se^{82}, and Mo^{100}.
2. The geochemical data[8,9] on the ratio $T_{1/2}(128)/T_{1/2}(130)$ contradict each other. The data of Ref. 8 cannot be explained on the basis that only the $2\beta\,(2\nu)$ decay occurs. It is thus concluded that the Majorana neutrino mass is nonzero (IV) or the parameter η is nonzero (V). The new data of Ref. 9 are consistent with the nonoccurrence of the $2\beta\,(0\nu)$ decay and can be used to determine upper limits on m_ν and η.
3. Column II gives theoretical predictions of $T_{1/2}$ for $2\beta\,(2\nu)$ decay taken from Ref. 84. There may be some uncertainties because of the estimate of the nuclear matrix element.
4. The geochemical data on the absolute value of $T_{1/2}(Te^{130})$ can be explained by the occurrence of $2\beta\,(2\nu)$ decay, and they can also be used as a lower limit on $T_{1/2}$ for the $2\beta\,(0\nu)$ decay. On this basis, a restriction was found on the parameter η' in Ref. 13. In this case (see Ref. 13), this parameter is a measure of the admixture of the right-handed component in the lepton current:

$$\bar{e}\gamma_\mu\,[1+\gamma_5+\eta'\,(1-\gamma_5)]\,\nu.$$

   For the same value of the nuclear matrix element, the same restriction applies to the parameter η.
5. A restriction on m_ν was found in Ref. 81 from expressions for the probability for the $2\beta\,(0\nu)$ decay found in Ref. 11. Actually, the expression for $w(2\beta,0\nu)$ is only half as large as that given in Ref. 11. The corresponding restrictions on m_ν are larger by a factor of $\sqrt{2}$.
6. Exact solutions of the Dirac equation in a Coulomb field were used as the electron wave functions in Ref. 89. As Doi *et al.*[89] assert, this circumstance is responsible for the difference between the estimates of the upper limits found there and some earlier results[84] found by the same investigators, where the Coulomb field was taken into account by simply multiplying $w(2\beta,0\nu)$ by the Fermi factor $F_c(\nu_1)F_c(\nu_2)$. We note, however, that using different ways to take the Coulomb field of the nuclei into account could not substantially change estimates of the upper limits on the Majorana neutrino mass. The difference between the results of Refs. 84 and 89 is apparently also due to the calculations of the nuclear matrix elements.
7. The (*) shows results calculated for the upper limits on the neutrino mass from expression (36), in which the nuclear matrix element is taken to be $1/R$, where R is the nuclear radius. The values given in Ref. 105 are used for F_c.
8. The expression found for the probability $w(2\beta,0\nu)$ in Ref. 99 is four times as large as (45). The corresponding upper limit on η should be doubled.
9. Table III ignores data on transitions to excited levels, $0^+\to2^+$:

$$Ge^{76}\to Se^{76}:\ T_{1/2}>5\cdot10^{21}\ {}^{110},\quad T_{1/2}>0{,}8\cdot10^{22}\ {}^{111},$$
$$Nd^{150}\to Sm^{150}:\ T_{1/2}>10^{18}\ {}^{118}.$$

   The restrictions on η which can be extracted from these results are weaker than the restrictions listed in this table which follow from data on $0^+\to0^+$ transitions.
10. The (c) indicates expectation values given in Ref. 89 for the decay half-lives of Te^{130} and Se^{82} (Refs. 9, 106, and 113–117), found by taking an average over all the geochemical experiments.

teraction. Experimental measurements of the differential characteristics of the $2\beta\,(0\nu)$ decay can thus substantially reduce the list of tenable hypotheses, but they cannot unambiguously identify the mechanism which generates the $\Delta L = 2$ transitions.

However, it can be shown[100] on the basis of certain, shall we say, generally accepted assumptions regarding the structure of weak interactions that the discovery of a $2\beta\,(0\nu)$ decay would be evidence of a nonzero Majorana neutrino mass. Among these assumptions are the following.

1. Weak interactions are described by a gauge theory.
2. The gauge fields (W bosons) are coupled with both a lepton $(\bar{l}\nu_l)$ current and a quark current, $(\bar{u}d)$ $(\bar{s}c)$, etc.
3. There is a crossing symmetry.

For a proof we consider the reaction $e^-uu \to dde^+$, which is a crossing reaction with respect to $2\beta\,(0\nu)$ decay. The discovery of a $2\beta\,(0\nu)$ decay will mean that the six-fermion diagram in Fig. 18 is nonzero. The hatched block in this diagram corresponds to some mechanism for $2\beta\,(0\nu)$ decay. We "close" the quark lines and connect them with the lepton W bosons. We find the diagram in Fig. 19. This diagram means that there is a Majorana mass term of the form

$$m^{\mathrm{L}}_{ee}\nu^{\mathrm{T}}_{e\mathrm{L}}C\nu_{e\mathrm{L}}.$$

We note that this theorem does not prove that a Majorana neutrino mass (and it alone) is the reason for the $2\beta\,(0\nu)$ decay. Furthermore, the contribution of the Majorana mass is not necessarily the dominant one.

As for concrete models with a spontaneous violation of lepton-number conservation, we note that in most of the examples which we have considered here the contribution of the Majorana neutrino mass is the basic one. Exceptional cases may be models with a left-right symmetry, which allow some latitude in the choice of adjustable parameters.

CONCLUSION

It has been nearly half a century since the appearance of the first study of double neutrinoless β decay. The purpose of the search for this phenomenon is clearer today than even before: $2\beta\,(0\nu)$ is the only process whose study can allow the detection and measurement of a Majorana neutrino mass. There can be no denying that independent measurements of the parameters of the neutrino mass matrix from the β spectrum of tritium and from neutrino oscillations are equally important. In terms of sensitivity these experiments are no poorer than, and are perhaps more promising than, the present capabilities for detecting a neutrino mass in a $2\beta\,(0\nu)$ decay. However, among these phenomena in which a neutrino mass might be manifested the $2\beta\,(0\nu)$ decay is the only one which would tell us about the nature of this mass. In fact, the

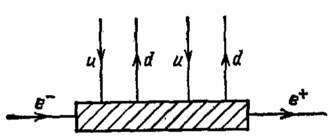

FIG. 18.

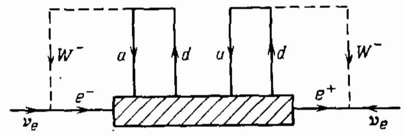

FIG. 19.

qualitative question of the nature of the neutrino mass matrix (Dirac or Majorana) is no less important than the quantitative question. One argument in favor of the Majorana neutrino mass is that it is small in comparison with the masses of charged fermions. This argument is based on the unified gauge theories in which the particle masses arise as a consequence of spontaneous symmetry breaking. The existence of a Majorana neutrino mass would mean that in addition to the standard $SU(2)\times U(1)$ symmetry the L $(B-L)$ symmetry is also broken. An additional argument that $B-L$ is not a strictly conserved quantity (and that a neutrino acquires a Majorana mass) comes from the theories in which this symmetry is a spontaneously broken gauge symmetry. In the opposite case we have the problem of the existence of a corresponding "photon."

A Majorana neutrino mass is not the only reason for a $2\beta\,(0\nu)$ decay, although it is the most likely reason. A detailed study of the $2\beta\,(0\nu)$ decay (if it occurs) would make it possible to shorten considerably the list of tenable hypotheses about the interaction which breaks lepton number. In this sense a search for the $2\beta\,(0\nu)$ decay can serve as a method for testing the models of the unified gauge theories. Furthermore, the discovery of a neutrinoless 2β decay would essentially erase all doubt that the Majorana neutrino mass is nonzero even if its contribution to the amplitude for the $2\beta\,(0\nu)$ decay is not the predominant one.

I wish to thank B. A. Dolgoshein, Yu. G. Zdesenko, O. Ya. Zel'dovich, A. Kalinovskiĭ, I. Yu. Kobzarev, V. A. Lyubimov, B. V. Martem'yanov, L. B. Okun', A. A. Pomanskiĭ, A. Smol'nikov, S. Fayans, and V. A. Khodel' for useful discussions.

APPENDIX I

We wish to calculate the matrix element for the $2\beta\,(0\nu)$ decay initiated by a Majorana neutrino mass m_ν. The S matrix of second order in the weak interaction is

$$S = -\frac{i}{2!}\int d^4x\, d^4y\, \langle N'e^-e^- \,|\, TL(x)\,L(y)\,|\, N\rangle, \qquad (\mathrm{I}.1)$$

where L is the Lagrangian of the weak interaction. For the moment we ignore the nuclear Coulomb field, assuming that the electrons are described by plane waves

$$\frac{1}{\sqrt{2\varepsilon}}\, e^{ikx}\bar{e}\,(k),$$

where $k = k_1, k_2$ are the 4-momenta of the electrons, $\varepsilon_{1,2} = k_{01,2}$ are the energies of the electrons, and $e(k)$ is a four-component spinor. The product of lepton currents in the T product is

$$j_\mu(x)\, j_\lambda(y) = \bar{e}(x)\,\gamma_\mu(1+\gamma_5)\,\nu(x)\cdot\bar{e}(y)\,\gamma_\lambda(1+\gamma_5)\,\nu(y).$$

For the calculations below it is convenient to transpose one of them, writing it as

$$j_\lambda(y) = \nu^{\mathrm{T}}_{\lambda}(y)(1+\gamma_5)\gamma^{\mathrm{T}}_{\lambda}\bar{e}^{\mathrm{T}}(y).$$

A convolution of the T product of the electron operators yields

$$\bar{e}(k_1)\,\gamma_\mu\,(1+\gamma_5)\langle 0\,|T\nu(x)\,\nu^{\mathrm{T}}(y)|\,0\rangle\,(1+\gamma_5)\,\gamma^{\mathrm{T}}_{\lambda}\bar{e}^{\mathrm{T}}(y)\,e^{i(k_1x+k_2y)} - (k_1 \leftrightarrow k_2). \tag{I.2}$$

The vacuum expectation value of the T product of the neutrino operator, $\nu(x)\nu^{\mathrm{T}}(y)$ is nonzero if the electron neutrino is a Majorana particle (or if it is a superposition of a Majorana diagonal states $\nu = \nu_e = \Sigma_{A=1}^{N} U_{eA\varphi_A}$). We denote by m_ν the Majorana mass of the electron neutrino (we recall that in the general case m_ν would be understood as

$$\sum_{A=1}^{N} \mu_A U^2_{eA},$$

where μ_A are the masses of the Majorana diagonal neutrinos). For a Majorana particle we know that the equality $\nu^{\mathrm{T}} = \bar{\nu}C$ holds, where $C = i\gamma_2\gamma_0$ is the charge conjugation matrix. The vacuum expectation value $\langle 0|T\nu(x)\nu^{\mathrm{T}}(y)|0\rangle$, which may be called the propagator of the Majorana particle, can then be expressed in terms of the propagator of an "ordinary" Dirac particle as follows:

$$G_{\mathrm{M}} = \langle 0\,|\,T\nu(x)\,\nu^{\mathrm{T}}(y)\,|\,0\rangle = G_{\mathrm{D}}C. \tag{I.3}$$

We know that

$$G_{\mathrm{D}}(x-y) = \int \frac{\mathrm{d}^4q}{(2\pi)^4}\,\frac{e^{iq(x-y)}}{\hat{q}-m_\nu}, \qquad \hat{q} = q_\alpha\gamma_\alpha.$$

We substitute $(\hat{q} - m_\nu)^{-1}$ into (I.2) and proceed to a transformation of the product of γ matrices (up to the integration over d^4q):

$$\gamma_\mu(1+\gamma_5)\,\frac{q_\alpha\gamma_\alpha + m_\nu}{q^2-m_\nu^2}\,C\,(1+\gamma_5)\,\gamma^{\mathrm{T}}_{\lambda} = -\frac{2m_\nu}{q^2-m_\nu^2}\,\gamma_\mu\gamma_\lambda\,(1-\gamma_5)\,C.$$

At this point we can see the conditions under which the amplitude for the $2\beta(0\nu)$ decay will be nonzero if the lepton current is left-handed:

1) ν_e is a Majorana particle;

2) $m_\nu \neq 0$.

Ignoring m_ν^2 in comparison with q^2, we find

$$\int \frac{\mathrm{d}^4q}{(2\pi)^4}\,\frac{e^{iq(x-y)}}{q^2} = -\frac{1}{4\pi}\,\delta_+(t^2-r^2), \quad t = x_0 - y_0, \quad \mathbf{r} = \mathbf{x} - \mathbf{y}. \tag{I.4}$$

The T product of the lepton operators thus becomes

$$\frac{m_\nu}{2\pi}\,\delta_+(t^2-r^2)\,[l_{\mu\lambda}(k_1,\,k_2)\exp(i\mathbf{k}_1\mathbf{x}+i\mathbf{k}_2\mathbf{y}) - (k_1 \rightleftarrows k_2)], \tag{I.5}$$

where $l_{\lambda\mu}(k_1,k_2) = \bar{e}(k_1)\gamma_\mu\gamma_\lambda(1-\gamma_5)C\bar{e}^{\mathrm{T}}(k_2)$.

The convolution of the T product of the nucleon operators can be written as a sum over all possible pairs of neutrons of the nucleus:

$$\sum_{a\neq b} J^a_\mu(\mathbf{x})\,J^b_\lambda(\mathbf{y})\,\exp(i\Delta E_a x_0 + i\Delta E_b y_0). \tag{I.6}$$

Here $J^a_\mu(\mathbf{x})$ should now be understood as the amplitude for the conversion of neutron a, at the point $\mathbf{x}$, into a proton. In this expression we have singled out the exponential time dependence of the nuclear wave function, and ΔE_a is the change in the energy of the nucleus as a result of the decay of neutron a. The sum $\Delta E_a + \Delta E_b = \Delta$ is evidently equal to the mass difference between the initial and final nuclei, while the difference $\Delta E_a - \Delta E_b = Q_0$ is a measure of the energy of the virtual neutrino. In the nonrelativistic limit the spin structure of the matrix elements of the currents J_μ is

$$J_0 = 1, \quad \mathbf{J} = g_A\boldsymbol{\sigma}, \quad g_A \approx 1{,}25.$$

It is now convenient to integrate over the time variables $T = \frac{1}{2}(x_0 + y_0)$ and $T = x_0 - y_0$; the S matrix can then be written as

$$S = \frac{2\pi\delta(\varepsilon_1+\varepsilon_2-\Delta)}{\sqrt{2\varepsilon_1}\,\sqrt{2\varepsilon_2}}\,m_\nu\,\frac{G_{\mathrm{F}}^2\cos^2\theta_{\mathrm{C}}}{2\pi}\,l(k_1,\,k_2)$$
$$\times\int \mathrm{d}x\,dy\,\Phi^{+}_{\mathrm{N'}}\sum_{a\neq b}\frac{e^{iQ_0r}}{r}\,(1-g_A^2\sigma_a\sigma_b)\,\Phi_{\mathrm{N}}. \tag{I.7}$$

Here Φ_{N} and $\Phi_{\mathrm{N'}}$ are the wave functions of the initial and final nuclei. We are ignoring in the integral the exponential function which contains the electron energies, since the condition $|\varepsilon_1 - \varepsilon_2| r \ll 1$ holds in all cases of practical interest. The quantity $l(k_1,k_2)$ is a measure of the dependence on the electron momenta and spins:

$$l(k_1,\,k_2) = \bar{e}(k_1)\,(1-\gamma_5)\,C\bar{e}^{\mathrm{T}}(k_2).$$

APPENDIX II

Let us outline the transformation of the S matrix of the $2\beta(0\nu)$ decay [see (39)] which results from an admixture of right-handed currents. A convolution of the neutrino operators gives us

$$(\mathrm{N'}\,|J^{\mathrm{L}}_{\mu}(x)\,J^{\mathrm{R}}_{\lambda}(y)|\,\mathrm{N}\rangle\,[\bar{e}(x)\,\gamma_\mu\gamma_\alpha\gamma_\lambda\,(1+\gamma_5)\,C\bar{e}^{\mathrm{T}}(y)]\,\frac{q_\alpha}{q^2}$$
$$+\langle\mathrm{N'}\,|J^{\mathrm{R}}_{\mu}(x)\,J^{\mathrm{L}}_{\lambda}(y)|\,\mathrm{N}\rangle\,[\bar{e}(x)\,\gamma_\mu\gamma_\alpha\gamma_\lambda\,(1-\gamma_5)\,C\bar{e}^{\mathrm{T}}(y)]\,\frac{q_\alpha}{q^2}.$$

As before, we assume that the electrons are described by plane waves, while the time dependence of the nuclear wave functions is determined by an exponential function. A convolution of the electron and nucleon operators then gives us

$$\sum_{a\neq b}\{\langle J^{\mathrm{L}(a)}_{\mu}J^{\mathrm{R}(b)}_{\lambda}\rangle\,[\bar{e}(k_1)\,\gamma_\mu\gamma_\alpha\gamma_\lambda\hat{0}_{+}C\bar{e}^{\mathrm{T}}(k_2)$$
$$\times\exp(i\varepsilon_1x_0+i\varepsilon_2y_0)\exp(-i\mathbf{k}_1\mathbf{x}-i\mathbf{k}_2\mathbf{y})$$
$$-\bar{e}(k_2)\,\gamma_\mu\gamma_\alpha\gamma_\lambda\hat{0}_{+}C\bar{e}^{\mathrm{T}}(k_1)\exp(i\varepsilon_2x_0+i\varepsilon_1y_0)$$
$$\times\exp(-i\mathbf{k}_1\mathbf{y}-i\mathbf{k}_2\mathbf{x})]\,e^{i\Delta E_ax_0+i\Delta E_by_0} \tag{II.1}$$
$$+\langle J^{\mathrm{R}(b)}_{\mu}J^{\mathrm{L}(a)}_{\lambda}\rangle\,[\hat{0}_{+} \rightleftarrows \hat{0}_{-}]\,e^{i\Delta E_ay_0+i\Delta E_bx_0}\},$$
$$\hat{0}_{\pm} = 1 \pm \gamma_5.$$

This expression is antisymmetric under the interchange of fermion variables. We introduce the notation $t = x_0 - y_0$, $T = \frac{1}{2}(x_0 + y_0)$, $\mathbf{R} = \frac{1}{2}(\mathbf{x} + \mathbf{y})$, $\mathbf{r} = \mathbf{x} - \mathbf{y}$, $E = \varepsilon_1 + \varepsilon_2$, $\mathbf{p} = \mathbf{k}_1 + k_2$, $\varepsilon = \varepsilon_1 - \varepsilon_2$, $\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$. The S matrix is

$$S = \eta G_{\mathrm{F}}^2\cos^2\theta_{\mathrm{C}}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\frac{d^4q}{(2\pi)^4}\,e^{iET}e^{-i\mathbf{pR}}\,\frac{q_\alpha}{q^2}$$
$$\times\sum_{a\neq b}\left\{e^{i(\Delta E_a-\Delta F_b)\frac{t}{2}}\,\langle J^{\mathrm{L}(a)}_{\mu}J^{\mathrm{R}(b)}_{\lambda}\rangle\right. \tag{II.2}$$

$$\times\left[\bar e(k_1)\,T^{(+)}_{\mu\alpha\lambda}\bar e^{\rm T}(k_2)\exp\left(i\frac{\varepsilon t}{2}-i\frac{\mathbf{kr}}{2}\right)\right.$$

$$\left.-\bar e(k_2)\,T^{(+)}_{\mu\alpha\lambda}\bar e^{\rm T}(k_1)\exp\left(-i\frac{\varepsilon t}{2}+i\frac{kr}{2}\right)\right]+e^{i(\Delta E_b-\Delta E_a)\frac{t}{2}}$$

$$\times\langle J^{\rm R\,(a)}_{\mu}J^{\rm L\,(b)}_{\lambda}\rangle\,[T^{(+)}\rightleftarrows T^{(-)}]\Big\},$$

where $T^{\pm}_{\mu\alpha\lambda}=\gamma_\mu\gamma_\alpha\gamma_\lambda(1\pm\gamma_5)\,C$. The matrix element of the product of hadron currents, denoted by $\langle J^{\rm L(a)}_{\mu}J^{\rm R(b)}_{\lambda}\rangle$ is

$$\langle J^{\rm L\,(a)}_{\mu}J^{\rm R\,(b)}_{\lambda}\rangle=(\delta_{0\mu}1^a+g_A\delta_{i\mu}\sigma^a_i)\,(\delta_{0\lambda}1^b-g_A\delta_{k\lambda}\sigma^b_k),\qquad\text{(II.3)}$$

where a and b specify the nucleons (or quarks, in which case we would have $g_A=1$). The integration over q_0 reduces to an integration of the δ-functions which arise in the evaluation of the integral over the variable t. We find

$$S=\eta G_{\rm F}^2\cos^2\theta_{\rm C}\int d^3x\,d^3y\,\frac{d^3q}{(2\pi)^3}\,e^{i\mathbf{pR}}$$

$$\times\left\{\langle J^{\rm L\,(a)}_{\mu}J^{\rm R\,(b)}_{\lambda}\rangle\left[\bar e(k_1)\,T^{(+)}_{\mu\alpha\lambda}\bar e^{\rm T}(k_2)\,e^{-i\,\mathbf{kr}/2}\,\frac{q_\alpha}{q^2}\,e^{-i\mathbf{qr}}\Big|_{q_0=q_0^{(1)}}\right.\right.$$

$$\left.-\bar e(k_2)\,T^{(+)}_{\mu\alpha\lambda}\bar e^{\rm T}(k_1)\,e^{i\mathbf{kr}/2}\,\frac{q_\alpha}{q^2}\,e^{-i\mathbf{qr}}\Big|_{q_0=q_0^{(2)}}\right]$$

$$\left.+\langle J^{\rm R\,(a)}_{\mu}J^{\rm L\,(b)}_{\lambda}\rangle\,[T^{(+)}\rightleftarrows T^{(-)}]\right\}\cdot 2\pi\delta(\varepsilon_1+\varepsilon_2-\Delta),\qquad\text{(II.4)}$$

where $q_0^{(1,2)}$ are the energies of the virtual neutrino for diagrams differing by an interchange of the final electrons; here $q_0^{(1)}-q_0^{(2)}=\varepsilon_1-\varepsilon_2$. The integration over $\mathbf{q}$ gives us

$$\int\frac{d^3q}{(2\pi)^3}\,e^{-i\mathbf{qr}}\,\frac{q_\alpha}{q^2}=\frac{1}{4\pi r}\,Q_\alpha e^{-iq_0r},$$
$$Q_0=q_0,\quad \mathbf{Q}=\left(q_0+\frac{i}{r}\right)\mathbf{n},\quad \mathbf{n}=\frac{\mathbf{r}}{r}\,.\qquad\text{(II.5)}$$

We also use $\bar e(k_1)T^{+}_{\mu\alpha\lambda}\bar e^{\rm T}(k_2)=\bar e(k_2)T^{(-1)}_{\lambda\alpha\mu}\bar e^{\rm T}(k_1)$. We write the S matrix as

$$S=\eta\,\frac{G_{\rm F}^2}{4\pi}\cos^2\theta_{\rm C}\sum_{a\neq b}\int dx\,dy\,\frac{e^{iQ_0r}}{r}\,e^{-i\mathbf{pR}}\,\{\langle J^{\rm L\,(a)}_{\mu}J^{\rm R\,(b)}_{\lambda}\rangle$$

$$\times[\bar e(k_1)\,T^{(+)}_{\mu\alpha\lambda}\bar e^{\rm T}(k_2)\,e^{-i\mathbf{kr}/2}\,Q^{(1)}_\alpha-\bar e(k_1)\,T^{(-)}_{\lambda\alpha\mu}\bar e^{\rm T}(k_2)\,e^{i\mathbf{kr}/2}Q^{(2)}_\alpha]$$

$$+\langle J^{\rm R\,(a)}_{\mu}J^{\rm L\,(b)}_{\lambda}\rangle\,[T^{(+)}\rightleftarrows T^{(-)}]\}\,2\pi\delta(\varepsilon_1+\varepsilon_2-\Delta).\qquad\text{(II.6)}$$

In the exponential function e^{iQ_0r} we have set $Q_0^{(1)}\approx Q_0^{(2)}=Q_0$, since the characteristic energies of the virtual neutrinos are of the order of the reciprocal of the distance between the nucleons, and the difference $Q_0^{(1)}-Q_0^{(2)}$, which is equal to the difference between electron energies, is considerably smaller than Q_0 in all cases of practical interest.

The tensors constructed from the lepton variables can be broken up into irreducible representations which transform as a scalar, a vector, and a second-rank tensor under three-dimensional rotations. These tensors are contracted with the nucleon tensors, which can be broken up into the same irreducible representations. The scalar, the vector, and the tensor describe the nuclear transitions $0^+\to0^+$, $0^+\to1^+$, and $0^+\to2^+$, respectively.

For the subsequent transformations of the S matrix it is convenient to break up the tensors $T^{(\pm)}_{\mu\alpha\lambda}$ into parts which are symmetric and antisymmetric with respect to the indices μ and λ, $T^{(\pm)}_{\mu\alpha\lambda}=S^{(\pm)}_{\mu\alpha\lambda}+A^{(\pm)}_{\mu\alpha\lambda}$, through the use of the equality

$$\gamma_\mu\gamma_\alpha\gamma_\lambda=\gamma_\mu\delta_{\alpha\lambda}+\gamma_\lambda\delta_{\alpha\mu}-\gamma_\alpha\delta_{\lambda\mu}+i\epsilon_{\mu\alpha\lambda\beta}\gamma_5\gamma_\beta.$$

We note that $A^{(\pm)}_{\mu\alpha\lambda}$ is antisymmetric with respect to all three indices.

The product of hadron currents is also conveniently broken up into parts which have a definite symmetry under a simultaneous permutation $\mu\leftrightarrow\lambda$ and permutation of the indices specifying the nucleons, $a\leftrightarrow b$:

$$\left.\begin{matrix}\langle J^{\rm L\,(a)}_{\mu}J^{\rm R\,(b)}_{\lambda}\rangle\\ \langle J^{\rm R\,(a)}_{\mu}J^{\rm L\,(b)}_{\lambda}\rangle\end{matrix}\right\}=[\delta_{0\mu}\delta_{0\lambda}1^a1^b-\delta_{i\mu}\delta_{k\lambda}\sigma^a_i\sigma^b_kg_A^2]$$

$$\pm g_A\,[\delta_{0\lambda}\delta_{i\mu}\sigma^a_i1^b-\delta_{0\mu}\delta_{i\lambda}\sigma^b_i1^a].\qquad\text{(II.7)}$$

Since the condition $R\Delta\ll1$ holds in all cases of practical interest, as we have already mentioned, we expand the exponential functions containing $(k_1+k_2)R$ and $(k_1-k_2)r$ and retain the first nonvanishing terms. We recall that in the calculation of the probability for $2\beta\,(0\nu)$ decay with a Majorana mass it was the zeroth-order terms in the expansions of these exponential functions which made the basic contribution. Here we must deal separately with the contributions of the spatial and temporal components of the vectors $\mathbf{Q}^{(1,2)}_\alpha$.

The term with $Q_0^{(1,2)}$. It is easy to see that the terms with $S^{(\pm)}_{\mu\alpha\lambda}$ make a nonvanishing contribution when the exponential functions are replaced by unity. This contribution is proportional to

$$\bar e(k_1)\,(S^{(+)}_{\mu0\lambda}+S^{(-)}_{\mu0\lambda})\,\bar e^{\rm T}(k_2)\cdot(\varepsilon_1-\varepsilon_2).\qquad\text{(II.8)}$$

The contribution of the antisymmetric part vanishes after a sum is taken over the nucleons of the nucleus. The contribution of the other terms of the expansion of exponential functions contains a small factor of the order of $(R\Delta)^2$.

The term proportional to $Q^{(1,2)}$. Since the vector $\mathbf{Q}$ is directed along the vector $\mathbf{n}=\mathbf{r}/r$, the zeroth-order term of the expansion of the exponential functions vanishes after an integration over angles in (48). We consider the first terms in the expansion of $\exp(\pm i\mathbf{kr}/2)$ and $\exp(i\mathbf{pR})$ where $\mathbf{p}=\mathbf{k}_1+\mathbf{k}_2$. In carrying out this expansion we need to use $\mathbf{r}=\mathbf{r}_a-\mathbf{r}_b$ and $2\,\mathbf{R}=\mathbf{r}_a+\mathbf{r}_b$, so that under the interchange $a\leftrightarrow b$ we have $\mathbf{r}\to-\mathbf{r}$ and $\mathbf{R}\to\mathbf{R}$. We introduce the notation $\mathbf{Q}=a\mathbf{n}/r$ and $a=q_0r+i$; here $|a|\sim1$ and $q_0=q_0^{(1)}$ or $q_0^{(2)}$.

1. $\exp(\pm i\mathbf{kr}/2)\to1\pm i\mathbf{kr}/2$; $\exp(i\mathbf{pR})\to1$. The contribution of the symmetric part is proportional to

$$i\,\frac{kr}{2}\,\bar e(k_1)\,(S^{(+)}_{\mu i\lambda}+S^{(-)}_{\mu i\lambda})\,\bar e^{\rm T}(k_2)\,(Q_i^{(1)}+Q_i^{(2)})$$
$$\approx Q_i\bar e(k_1)\,(S^{(+)}_{\mu i\lambda}+S^{(-)}_{\mu i\lambda})\,\bar e^{\rm T}(k_2)\,i\,(\mathbf{kr}).\qquad\text{(II.9)}$$

This tensor is contracted with the part of tensor (II.7) which is symmetric under the interchanges $\mu\leftrightarrow\lambda$, $a\leftrightarrow b$. The corresponding contribution of the antisymmetric part is proportional to

$$i\,(\mathbf{kr})\,Q_i\bar e(k_1)\,(A^{(-)}_{\mu i\lambda}-A^{(+)}_{\mu i\lambda})\,\bar e^{\rm T}(k_2).\qquad\text{(II.10)}$$

The tensor (II.10) is contracted with the part of tensor (II.7) which is antisymmetric under the simultaneous interchanges $\mu\leftrightarrow\lambda$ and $a\leftrightarrow b$.

2. We now consider the expansion $\exp(i\mathbf{pR})\approx1+i\mathbf{pR}$, replacing $\exp(\pm i\mathbf{kr}/2)$ by unity. The symmetric part, pro-

portional to $S^{(\pm)}_{\mu a\lambda}$, gives us

$$i\,(\mathbf{pR})\,\bar{e}\,(k_1)\,(S^{(+)}_{\mu i\lambda}+S^{(-)}_{\mu i\lambda})\,\bar{e}^{\mathrm{T}}\,(k_2)\,Q_i. \tag{II.11}$$

This tensor is contracted with the part of tensor (II.7) which is antisymmetric under the interchanges $\mu\leftrightarrow\lambda$ and $a\leftrightarrow b$.

The contribution of the antisymmetric part can be calculated in an analogous way:

$$i\,(\mathbf{pR})\,\bar{e}\,(k_1)\,(A^{(+)}_{\mu i\lambda}+A^{(-)}_{\mu i\lambda})\,\bar{e}^{\mathrm{T}}\,(k_2)\,Q_i. \tag{II.12}$$

The tensor (II.12) is contracted with the part of the tensor (II.7) which is symmetric under the interchanges $\mu\leftrightarrow\lambda$ and $a\leftrightarrow b$. Summing the contributions of the various terms, we can write the S matrix as

$$S=\eta\,\frac{G_F^2}{2\pi}\cos^2\theta_C\,\frac{2\pi\delta\,(\varepsilon_1+\varepsilon_2-\Delta)}{\sqrt{2\varepsilon_1}\,\sqrt{2\varepsilon_2}}\,[l_0M_0+l_{ik}M_{ik}+l_iM_i]$$

$$(i,\ k=1,\ 2,\ 3);$$

$$l_0=\bar{e}\,(k_1)\,\gamma_0C\bar{e}^{\mathrm{T}}\,(k_2),$$

$$l_{ik}=(\mathbf{k}_1-\mathbf{k}_2)_i\,\bar{e}\,(k_1)\,\gamma_k C\bar{e}^{\mathrm{T}}\,(k_2),$$

$$l_i=(\mathbf{k}_1+\mathbf{k}_2)_i\,\bar{e}\,(k_1)\,\gamma_0\gamma_5 C\bar{e}^{\mathrm{T}}\,(k_2),$$

$$M_0=\langle \mathrm{N}'\,|\sum_{a\neq b}H\,(1-g_A^2\sigma_a\sigma_b)\,|\,\mathrm{N}\rangle,$$

$$M_{ik}=\langle \mathrm{N}'\,|\sum_{a\neq b}rH'n_i\,\{n_k\,(1+g_A^2\sigma_a\sigma_b)+2ig_A\,[\boldsymbol{\sigma}_a\mathbf{n}]_k-2g_A^2\sigma_{ak}\,(\mathbf{n}\boldsymbol{\sigma}_b)\}\,|\,\mathrm{N}\rangle,$$

$$M_i=\langle \mathrm{N}'\,|\sum_{a\neq b}RH'n_{+i}\,\{2g_A\,(\mathbf{n}\boldsymbol{\sigma}_a)-ig_A^2\mathbf{n}\,[\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b]\}\,\mathrm{N}\rangle;$$

$$H=\frac{e^{iQ_0r}}{r},\quad H'=\frac{\partial H}{\partial r}.$$

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Translated by Dave Parsons