B. A. Volkov, A. A. Gorbatsevich, and Yu. V. Kopaev. *Anomalous diamagnetic properties of systems with spontaneous current.* The possible existence of materials that differ from semiconductors and have high diamagnetic susceptibilities is of considerable interest. The concept of a diamagnetic reaction of a system presupposes that it contains a large-scale current structure and that we accept the classical interpretation that relies on the Larmor diamagnetic precessions of a current-carrying circuit in a magnetic field. The current structure must be stiff enough to suppress the paramagnetic associated with the change in the area of the current circuits projected onto the direction of the magnetic field. Actually, the latter effect detemines the paramagnetic sign of magnetic susceptibility of the usual ferro- and antiferromagnets. From the point of view of point symmetry, ordinary magnets are described by the axial vector **M** that changes sign under time reversal. The physical meaning of **M** in a crystal is that it is the local magnetization density. However, this does not exhaust the possible symmetry classification of materials with a current structure. The state in which current order is characterized by a polar vector T that is antisymmetric under time reversal has recently attracted some attention.[1–4] This state represents the ordering of elementary toroidal moments.[5,6] The ordering of respectively magnetic and electric moments occurs in a similar way in magnetic and ferroelectric materials, and the point symmetry of the crystal is set by the symmetry of the elementary moment (the polar vector **P**, which is symmetric under time reversal, in the case of ferroelectrics).

The electric, magnetic, and toroidal moments are generated by three independent families of electromagnetic multipoles.[6] The description of a system with given electron density $\rho(\mathbf{r})$ and current density $\mathbf{j}(\mathbf{r})$ in external electric and magnetic fields is complete only when all three types of multipole are taken into account. The simplest object that has only a toroidal moment is a solenoid in the form of a antibifilar coil bent into a torus. The magnetic field produced by this system is confined to the interior of the torus.

In the expression for the free energy density, the interaction between the toroidal moment density and the magnetic field is described by the term[3,6]

$$\delta F_{\mathrm{int}\,1} = -\gamma\,\mathrm{curl}\,\mathbf{B}\cdot\mathbf{T} = -\gamma\mathbf{J}\cdot\mathbf{T};$$

where γ is a coefficient determined by the parameters of the micromodel, **J** is the total current, and **B** the magnetic induction. Above the temperature of transition to the toroidal state (TS), the external current $\mathbf{J}_{\mathrm{ext}} = \mathrm{curl}\mathbf{H}$ (**H** is the magnetic field) plays the role of the field associated with the order parameter. In crossed electric and magnetic fields, the free energy density acquires the additional contribution

$$\delta F_{\mathrm{int}\,2} = \lambda\,[\mathbf{E}\times\mathbf{B}]\cdot\mathbf{T},$$

which means that the magnetoelectric effect can now appear. The vector **T** is then the dual of the antisymmetric *component* of the magnetoelectric tensor. It follows that materials in which the vector **T** exists belong to the subset of a set of classes in which the magnetoelectic effect can be present.

There are 31 classes of magnetic symmetry in which the vector **T** can exist.[5,7]

The realization of the toroidal state has now been investigated in detail, but only within the framework of one microscopic model, namely, the excitonic dielectric.[8] Studies of the possible realization of the toroidal state[9] can be briefly summarized as follows:

1. Systems with strong interelectron correlations are suitable objects for searches for the toroidal state, for example, narrow-gap semiconductors and semimetal-type compounds with coincident electron and hole band extrema in momentum space.[10]
2. The toroidal state formed against the background of ferroelectric or antiferromagnetic states is the most likely situation.
3. The conditions for the realization of the toroidal state are easier to satisfy in the neighborhood of macroscopic inhomogeneities (domain walls, intergrain boundaries, dislocations, impurity clusters, and so on).
4. The appearance of the toroidal state is facilitated by the presence of impurities (charged in the case of ferroelectrics and charged and magnetic in the case of antiferromagnetic materials).
5. Strong spin-orbit interaction will also facilitate the appearance of effects associated with toroidal ordering.

The magnetic properties of toroidal states are unusual. The magnetic susceptibility in the neighborhood of a transition to the toroidal state above the transition temperature is paramagnetic and nonzero only in the case of a nonuniform magnetic field.[11] The most interesting property of the toroidal system is that it may have high diamagnetic susceptibility (superdiamagnetism). The structure of a toroidal system in a magnetic field below the transition temperature can be determined from the equilibirum equation.

$$\nabla U = -\mathbf{f}_{\mathrm{ext}}; \tag{1}$$

where U is related to the free energy density by the Legendre transformation. Under given boundary conditions, U can be interpreted as the Gibbs energy density. The right-hand side of (1) contains the Lorentz force $\mathbf{J}\times\mathbf{B}$. The expression for the macroscopic current density in the toroidal state is

$$\mathbf{J}(\mathbf{r}) = \gamma\,\mathrm{curl}\,\mathrm{curl}\,\mathbf{T}(\mathbf{r}), \tag{2}$$

i.e., the spontaneous current density is nonzero in the case of a transversely-inhomogeneous vector $\mathbf{T}(\mathbf{r})$. Equations (1) and (2) are used in Ref. 4 to deduce the following expression for the diamagnetic response of a toroidal system to a uniform magnetic field:

$$\chi = \chi_L \left(\frac{\lambda_J}{r_a}\right)^2,$$

where χ_L is the Landau diamagnetic susceptibility of a noninteracting electron gas, λ_J is a new characteristic scale of the theory (effective radius of current correlations), and r_a is of the order of the interatomic separation. Since λ_J can also be of the order of r_a, the identification of the toroidal state must involve the entire range of electromagnetic properties of the toroidal state, for example, magnetoelectric and photogalvanic effects, as well as anomalies in the electrical conductivity tensor near the *TS* transition temperature.[12]

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