Spectra and kinetics of systems with magnetic-impurity states in a potential with finite range

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The results of theoretical and experimental investigations of the spectra of and the kinetic phenomena occurring in materials with magnetic-impurity states on impurities with a finite-range potential are reviewed. The spectra of magnetic-impurity states for different types of negative ions in strong magnetic fields are studied: weakly bound electron with an isotropic effective mass, negative ion with a markedly anisotropic electronic energy spectrum, and the H^- system. The current status of the theory of transverse galvanomagnetic phenomena, stemming from the interaction of carriers with neutral impurities having a potential with a finite range, is discussed. Current theoretical and experimental results concerning oscillation effects in quantizing magnetic fields on different types of negative ions in semimetals and semiconductors are compared.

CONTENTS

1. INTRODUCTION

The need to examine the interaction of charge carriers with impurities in strong magnetic fields arises in the study of various phenomena occurring in astrophysical objects, plasmas, and solids. The specific feature of this interaction is the possibility of the appearance of special quantum-mechanical states, whose structure is determined by both the magnetic field and the specific form of the impurity potential. Such magnetic-impurity (MI) states can, under certain conditions, determine all the kinetic properties of the material, leading to a series of new physical phenomena: nontrivial oscillatory dependences of the kinetic coefficients on the intensity of the magnetic field, both in static and time-dependent electromagnetic fields; very peculiar behavior of the longitudinal and transverse conductivities and coefficients of absorption at low temperatures, where resonance effects appear in scattering of carriers by the MI states; formation of additional channels for resonance absorption, etc.

For a long time, the study of impurity states of carriers and kinetic phenomena in substances with such states in quantizing magnetic fields was primarily restricted to problems involving a Coulomb impurity potential. At the same time, the nature of the MI states of charge particles in an impurity potential with a finite range remained little studied. This is explained to a large extent by the fact that such states, in contrast to Coulomb states, are realized under much more stringent conditions: at low temperatures and low impurity concentrations. For this reason, the available experimental data for a long time did not provide a strong enough stimulus for intensive theoretical research in this field.

The existence of MI states of an electron on neutral impurities was first pointed out by Bychkov in 1960.[1] He predicted that in the presence of a magnetic field, an arbitrarily shallow, attractive impurity potential, even one with zero range, binds a charged particle. The experimental confirmation of the possibility of the formation of such states,

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however, was obtained only in 1967 by Brandt and Lyubutina,[2] who observed beats in the de Haas–van Alfven effect in bismuth, due to the appearance of MI states of carriers on *neutral* impurities. A series of experiments, indicating the determining role of such states in the kinetics of low-temperature processes, then followed.[3–5] These experiments provided the stimulus for constructing a consistent theory of such phenomena, which also turned out to be much richer than in the case of the Coulomb impurity potential. This is explained, first of all, by the great variety of energy spectra of MI states on impurities with finite radius resulting from the large number of different ratios which are possible between the radius of the screening impurity, the Bohr radius α_B, and the magnetic radius, which in real materials vary over wide ranges. Second, the dynamics of the behavior of such MI energy levels is much more sensitive to the change in the magnetic field intensity and the depth of the impurity potential than in the analogous situation of the Coulomb problem. Third, aside from the magnetic field and the form of the impurity potential, the nature of the MI states is determined by the specific band structure of the material, which is most strongly manifested precisely in the MI states of a potential with finite range. Thus, for example, for a markedly anisotropic effective mass of charge carriers, the number of MI levels itself is controlled by the magnitude of the anisotropy parameter for the effective mass. The scattering of charged particles by neutral impurities in a quantizing magnetic field is also very curious, since the energy dependence of the scattering amplitude of an electron scattered by an isolated impurity reflects all the details of the structure of the spectrum of MI states.

The properties of the energy spectra indicated above are manifested directly in the kinetic coefficients, which are determined by the interaction of carriers with neutral impurities. Resonance effects of scattering of carriers, due to the presence of MI states, lead to the appearance of peaks or "troughs" in the absorption coefficient in certain frequency intervals. In static fields, when the carrier energies correspond to the energies of the MI levels, a change occurs in the temperature-field dependences of the kinetic coefficients—the longitudinal and transverse conductivities, in accordance with the different character of the electron-impurity interaction in different temperature ranges. This also concerns the dependence of the coefficient of absorption in many materials: in some regions of the magnetic field intensity H, the growth in the absorption can be replaced by a drop, related to the vanishing or appearance of MI levels with a change in H. In materials with a markedly anisotropic effective mass of carriers in a quantizing magnetic field, all kinetic coefficients oscillate with a "nonstandard" ($\propto \sqrt{H}$) dependence of the period of oscillations on the magnetic field intensity. The oscillations arise due to the displacement of MI levels into the continuous spectrum and by the transparency of the effective one-dimensional potential of impurities under conditions of an MI level with zero energy. Finally, we note that the temperature-field dependences of the conductivities are largely determined by the depth of the potential (U) of neutral impurities and, in addition, in certain situations they exhibit an oscillatory dependence on U, arising due to the presence of a magnetic field.

The development of a consistent theory of the phenomena listed above required going beyond the Born approximation with respect to the interaction of carriers with impurities, which has been the primary means for calculating the kinetic coefficients for more than 20 years. This in its turn lead to an understanding of many details of the electron-impurity interaction in quantizing magnetic fields.

In this review, we present the results of theoretical and experimental investigations of the energy spectra and of the kinetic phenomena occurring in materials with MI states on impurities with a finite-range potential. The analysis and the problems studied are examined in the following order. In Secs. 2–3, we discuss the spectral structure of MI states of different types of negative ions: 1) the spectrum of a particle with an isotropic effective mass in the potential of a neutral atom with small radius; 2) the spectrum of MI states of the H^- ion in a strong magnetic field; 3) the spectrum of an electron with a markedly anisotropic effective mass in the potential of a short-range impurity center. In Secs. 4 and 5, we analyze the oscillatory effects arising in systems of carriers with field-induced MI states. In Sec. 4, we examine photoabsorption on MI states of neutral impurities with a short radius, but arbitrary depth. We investigate the frequency-field dependences of the coefficient of absorption for all types of transitions of carriers between quasibound MI states and states in the continuous spectrum, as well as the dependence of the absorption coefficient on the depth of the impurity potential. In Sec. 5, we study the magnetic-impurity oscillations of kinetic coefficients in materials with a markedly anisotropic effective mass of charge carriers. We show that all kinetic coefficients, both in static and time-dependent fields, undergo specific oscillations as a function of H with a period proportional to the square root of the magnetic-field intensity.

When light is absorbed by MI states in such systems, the behavior of the absorption curve depends on the parities of the initial and final MI states, which are determined by the magnitude of the magnetic-field intensity. In Sec. 6, we review the current theory of transverse galvanomagnetic phenomena, arising due to the interaction of carriers with neutral impurities in quantizing magnetic fields.

We shall obtain a general formula for the transverse conductivity, expressed in terms of the matrix elements of the scattering operators for scattering of an electron by a center in an axisymmetric gauge. We shall then show that in the case of scattering of electrons by Coulomb centers with a large screening radius, the transverse conductivity has a very different temperature-dependence as a function of the characteristic energies of carriers, radius of the screening impurity and its Bohr radius, and the magnetic field intensity. This change is related to the change in the nature of the scattering of particles by impurities from weakly perturbed at very high temperatures to resonant at very low temperatures. In this section we also study the oscillations of the conductivity, arising with a change in the depth of the impurity potential with small radius.

Finally, in Sec. 7, we briefly report the results of the experiment described in Ref. 2 and of the experiments in Refs. 3–5, in which the photoabsorption by MI states of D^- impurities in n-CdS,[3] germanium,[4] and silicon[5] in quantizing magnetic fields was investigated. In the course of the presentation, we shall compare the presently available experimental data with the theory.

2. ENERGY SPECTRA OF MAGNETIC-IMPURITY STATES OF A WEAKLY BOUND ELECTRON (ISOTROPIC EFFECTIVE MASS)

a) Properties of the motion of a weakly-bound electron in an external magnetic field

At the beginning of this section we shall show, following Ref. 1, that in a magnetic field an arbitrarily shallow, attractive impurity potential binds a charged particle, and then we shall study qualitatively the character of the motion of a Landau electron in the potential of a center with small radius and arbitrary depth. Let an electron in a Landau band N with low energy $E \ll \hbar^2/m^* r_c^2 (E = \hbar\omega_H [N + (1/2)] + (\hbar^2 k^2/2m^*))$ be scattered by an impurity, described by the potential $U(r) < 0$ with a radius r_c that is short compared to the magnetic radius of the electron $L = \sqrt{c\hbar/eH}$. Since the wavelength of the particle is much larger than the radius of the impurity, in calculating the scattering amplitude we shall make the substitution

$$U(r) \to a_0 \frac{2\pi\hbar^2}{m^*} \delta(\mathbf{r}), \tag{2.1}$$

a_l is the scattering length for an electron with orbital angular momentum l scattered by the potential U with $H = 0$.

For the potential (2.1), the amplitude F_{mn} of the transition of an electron between states m and n in a magnetic field reduces to the sum of the terms of a geometric progression. Calculating this sum, we obtain

$$F_{mn} = a_0 \psi_m^*(0)\,\psi_n(0)\left\{1 + \frac{i\alpha}{kL}\left[1 - i\alpha 2^{-1/2}\sum_{M=0}^{M_{\max}}\left(\frac{E}{\hbar\omega_H} - \left(M + \frac{1}{2}\right)\right)^{-1/2}\right]\right\}^{-1}, \tag{2.2}$$

$\alpha = a_0/L$ is a dimensionless constant characterizing the interaction of the carrier with the center in a magnetic field. $M_{\max}$ is determined from the requirement that the smallest term in parentheses in (2.1) remain nonnegative.

The right side of (2.2) has poles in the complex k plane:

$$k \approx -i\alpha L^{-1}\left[1 - i\alpha \cdot 2^{-1/2}\sum_M (N - M)^{-1/2}\right], \tag{2.3}$$

corresponding, for $\alpha < 0$ and $|\alpha| \ll 1$, to a quasibound MI level in the N band. This level lies below the bottom of the Nth Landau band at the depth $\Delta E = -(\alpha^2/2)\hbar\omega_H$ and has a width $\Gamma_N = 2^{-1/2}|\alpha|^3 \Sigma (N - M)^{-1/2}\hbar\omega_H$. The appearance of the system of levels (2.3) is explained by the fact that the magnetic field, by restricting the transverse motion of the electron, reduces the effective potential of the center to a symmetrical one-dimensional potential, in which there always exists a shallow weakly bound state for the values of the parameter α indicated above. The appearance of an imaginary part in the energy for $N > 0$ is related to the possibility that the particle escapes into the continuous spectrum of the lower $M < N$ Landau bands. The "nonstandard" dependence of the width on the interaction constants $\Gamma_N \propto \alpha^3$ is a result of the actual one-dimensionality of the problem. The result (2.2) was obtained independently by Skobov.[6] The magnetic-impurity states, obtained in the approximation of Refs. 1 and 6, correspond to the model of a center with zero range and perturbation theory with respect to the scattering length.

For a potential U with an arbitrarily small, but finite range $r_c \neq 0$, aside from the level (2.3), a system of shallow ($kr_c \ll 1$) quasibound MI levels,[7,8] classified according to the projection of the angular momentum of the electron $m \neq 0$ along the field, arises below the bottom of any Landau band. As the depth of the impurity potential increases, the spectrum of such states is restructured.[9–11] The qualitative picture of this phenomenon is as follows: let a shallow level with angular momentum l, so that in the corresponding wave scattering is anomalously large, be present in the potential $U(r)$. In the absence of an accidental degeneracy with respect to l, the energy interval between levels corresponding to different values of the angular momentum of the electron bound to the impurity is of the order of $\sim\hbar^2/m^* r_c^2 \gg \hbar\omega_H = \hbar^2/m^* L^2$, which indicates the smallness of their mixing by the field. For this reason, the orbital angular momentum, in spite of the absence of spherical symmetry ($H \neq 0$), is almost a "good" quantum number. We shall denote the effective radius of localization of the weakly-bound electron in the absence of a magnetic field by R_l. It is quite obvious that for depths of the impurity potential for which $R_l \ll L$, the effect of the magnetic field on the particle in the region of its localization is insignificant, and the spectrum is determined by the potential U. On the other hand, for large localization radii $R_l \gg L$, the motion of the electron in a plane perpendicular to H is determined by the magnetic field, while the motion along H is determined by the potential of the center averaged over the transverse motion. For this reason, the energy spectrum is restructured for values of U such that $R_l \gtrsim L$: a small change in the depth of the center has a large effect on the binding energy of the particle.

b) Structure of the energy spectrum

The MI states of an electron in a shallow ($|U| \ll \hbar^2/m^* r_c^2$) impurity with a short ($r_c \ll L$) radius were investigated in Refs. 7, 8, 12, and 13, and for an impurity potential with arbitrary depth, approximated by a rectangular well with small radius, in Refs. 9 and 10. A model-independent method for calculating the spectra of weakly bound states of a particle with an arbitrary angular momentum in external fields was developed in Ref. 11 (see Appendix). We shall describe, following Refs. 7–14,[1)] the general behavior of the spectrum of MI states of a short-range potential (see Figs. 1–3).

[1)]The states of the lowest Landau band with $l = 0$ were first studied in detail by Demkov and Drukarev in the approximation of a zero-range potential.[14]

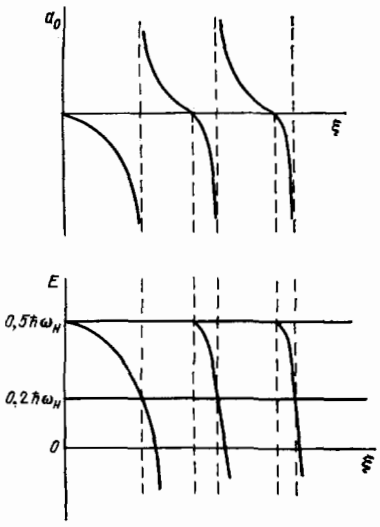

FIG. 1. Dependence of the energy of the MI level with $l = m = N = 0$ on the depth of the impurity potential[10] ($\xi = \sqrt{m^*|U|/\hbar^2}\cdot r_c$).

The spectrum is characterized by the number of the Landau band N, the orbital angular momentum l, and the projection of the angular momentum m; it does not depend on the specific form of the potential, and is determined by two parameters of the center: the scattering length a_l and the effective radius r_l.[11] The depth at which a level with fixed m lies below the bottom of the Landau band $N \ll L^2/r_c^2$ depends, for a small depth of the impurity potential, in a power-law fashion on the magnetic-field intensity, it has the following order of magnitude ($m \leqslant 0$)

$$\Delta E_N^{m\leqslant 0} = -\frac{\alpha^2}{2}\hbar\omega_H\left(\frac{r_c}{L}\right)^{4|m|}\left[\frac{N+|m|}{2^{|m|}N!\,(|m|!)^2}\right]^2$$

$$\propto H^{2|m|+2} \qquad (2.4)$$

and it increases with the band number N, on the whole, proportionally to $N^{2|m|}$. The width of these levels decreases with increasing $|m|$ according to a power law[8]

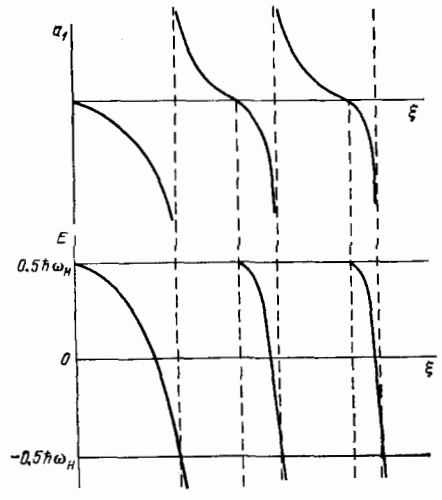

FIG. 2. Dependence of the energy of the MI level $N = 0$, $-m = l = l$ on the depth of the impurity potential.[10]

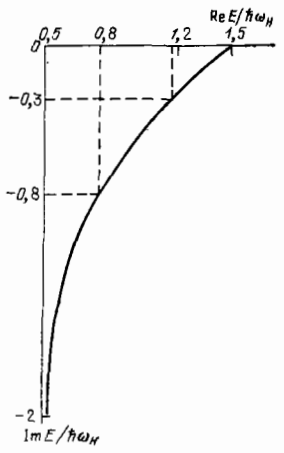

FIG. 3. Dependence of Re E on Im E for MI level $N = 1$, $m = l = 0$.[10]

$$\Gamma_N^{m\leqslant 0} = \frac{|\alpha|^3}{\sqrt{2}}\hbar\omega_H \times\left(\frac{r_c}{L}\right)^{6|m|}\frac{(N+|m|)!}{2^{3|m|}\cdot N!\,(|m|)^6}\sum_{k=0}^{N-1}\frac{(N+|m|)!\,(k+|m|)!}{N!k!\,(N-k)^{1/2}}$$

$$\propto H^{3|m|+(5/2)} \qquad (2.5)$$

and increases with N as $\propto N^{3|m|+(1/2)}$. The expressions for the shift of the level and the width for $m > 0$ are obtained from (2.4) and (2.5) by making the substitution $N + |m| \to N$. According to (2.4) and (2.5), there is only one state, corresponding to a given projection of the angular momentum, below the bottom of each Landau band. This property is specific to the spectrum of MI states of a particle with an isotropic effective mass in a short-range potential.

As the depth of the impurity potential increases, the energy of the MI levels decreases and their width increases. For states characterized by zero angular momentum and $N > 0$, the splitting of a level from the bottom of the N-th band and its width rapidly become equal (see Fig. 3), while for the uppermost $m \neq 0$ projections of the angular momentum the width of the level down to a narrow region near the bottom of the $(N-1)$-st Landau band is less than its depth,[15] due to the centrifugal barrier. At the same time, any Landau band N has one truly bound state with the projection of the angular momentum $m = N$. As the well approaches any of the resonances with given l ($a_l \to \infty$) the MI level with $|m| = l$ in the N band lies at a fixed depth, which depends on the value of the orbital angular momentum. As $|U|$ increases further, the MI levels of an arbitrary N band "fall through" the bottom of the zero Landau band, going over into the characteristic impurity states corresponding to $H = 0$.

Then, for depths of the potential for which the scattering length a_l vanishes, quasibound MI levels with angular momentum projections $|m| = l$ appear once again below the bottom of each band. For $a_l < 0$ and $|a_l| \ll L^{2l+1}$, their splitting away from the bottom of the band and their width are determined by the formulas obtained from perturbation theory with respect to the scattering length[11,27]:

$$\Delta E \propto \hbar\omega_H\left|\frac{a_l}{L^{2l+1}}\right|^2 \propto H^{2l+2},$$

$$\Gamma_l \propto \hbar\omega_H \left| \frac{a_l}{L^{2l+1}} \right|^s \propto H^{3l+5/2}, \tag{2.6}$$

which completely correspond to the results (2.4)–(2.5) for the case of a shallow impurity. At an exact resonance of the potential ($a_l = \infty$) the levels lie below the bottom of the Landau band at a fixed depth determined by the value of the quantum number l. (The position of the level near the resonance ($a_l = \infty$) is determined by the value of the effective radius r_l.[10]) In the post-resonance region ($a_l > 0$, $r_c^{2l+1} \ll a_l \ll L^{2l+1}$), when even in the absence of H there is a shallow bound state, the magnetic field only gives rise to its dia- and paramagnetic shift, while the MI levels with $|m| = l$ are absent in the higher bands. Finally, in the case $a_l \lesssim r_c^{2l+1}$, $a_l > 0$, the well does not have a shallow level and such a level does not appear when the magnetic field is switched on.

c) Magnetic impurity states on hydrogen atoms

As H increases and as the Larmor radius L becomes comparable to the dimensions of the neutral impurity, the magnetic field begins to determine the nature of the motion of the atomic electrons themselves. The spectrum of MI states of an electron on a hydrogen atom in the presence of a very weak magnetic field $\nu_0^{-1} \equiv (1/2)\ln(a_B/L) \gg 1$ was obtained analytically in Ref. 17. The lowest Landau band and considerably different values of the projections of the angular momentum of the atomic and of the additional electrons ($|m| \gg |m_{at}| \sim 1$), which permitted neglecting exchange effects, were examined. The region of localization of the atomic electron under conditions of such a strong magnetic field is strongly elongated along **H**—its cross-sectional area has the characteristic dimensions $\rho_{at} \sim L$, while the longitudinal dimensions is of the order of $z_{at} \sim \nu_0 a_B \gg L$. The MI levels have a different functional dependence on the intensity of the field and on the projection of the angular momentum of the electron, determined by the region of its transverse localization, $\rho_{eff} = \sqrt{|m|}\, L$. The spectrum of MI states of the H^- ion has primarily a power-law dependence on H and m for absolute magnitudes of the projection of the angular momentum which are much larger than the ratio z_{at}/ρ_{at}

$$\Delta E = -\frac{\hbar^2 k^2}{2m^*}, \quad k = \frac{11}{64}\frac{\nu_0}{a_B}\left(\frac{\nu_0 a_B}{\sqrt{|m|}\, L}\right)^3, \qquad \sqrt{|m|} \gg z_{at}\,|\rho_{at} \sim a_B\nu_0|L, \tag{2.7}$$

and primarily a logarithmic dependence for small values of the modulus of the projection of the angular momentum

$$\Delta E = -\frac{\hbar^2 k^2}{2m^*}, \quad k = \frac{2\nu_0}{a_B}\left(\ln\frac{\nu_0 a_B}{\sqrt{|m|}\, L}\right)^2, \qquad \ln\frac{\nu_0 a_B}{\sqrt{|m|}\, L} \gg 1 \quad \text{и} \quad 1 \ll \sqrt{|m|} \ll z_{at}\,|\rho_{at} \sim a_B\nu_0|\, L. \tag{2.8}$$

The binding energy of an electron bound by a hydrogen atom was calculated by numerical methods in Ref. 18. It was found that the bound state of the electron on a neutral atom, existing in the absence of a magnetic field, vanishes for large values of H. Heavy atoms were examined by the self-consistent field method in Ref. 19 and, according to Ref. 19, the bound state of an electron on an atom remains even in the limit of a strong magnetic field. The asymptotic extension of formula (2.9) to values $|m| \sim l$ agrees well with the results of calculations performed in Refs. 17 and 19.

3. SPECTRUM OF A WEAKLY-BOUND LANDAU ELECTRON WITH A PRONOUNCED ANISOTROPY OF THE EFFECTIVE MASS

In this section we shall show that when the effective mass of a charge carrier in an attractive potential with short radius has a pronounced anisotropy, a fundamentally new type of magnetic-impurity states appears.[20] These states disappear both when the magnetic field is switched off and when the anisotropy parameter of the masses decreases. For a given projection of the orbital angular momentum, not one, but a system of MI levels, which transform as H decreases into the continuous spectrum with period $\propto H^{(|m|+1)/2}$, appears below the bottom of each Landau band. The specific nature of such states is also manifested in the character of the scattering of carriers in the material, since the one-dimensional well becomes transparent when a zero-energy level appears in it.[21] This, in its turn, leads to oscillations of all kinetic coefficients, determined by scattering of carriers by these centers (see Sec. 5), primarily with a period $\propto \sqrt{H}$, which distinguishes them from the Gurevich-Firsov magnetophonon oscillations[22] and the Shubnikov–de Haas oscillations,[23] with a period $\propto H^{-1}$.

We shall examine the spectrum of states of an electron whose mass is markedly anisotropic ($m_1 = m_2 \equiv m_\perp \ll m_\parallel = m_3$), in a magnetic field oriented along the long axis of the energy ellipsoid $\mathbf{H} \| m_\parallel$ and spherically symmetrical attractive potential $U(r) < 0$ with radius $r_c < L$. The motion of the electron in the plane perpendicular to **H** for sufficiently low transverse mass is always determined by the magnetic field, while the motion along **H** is determined by the potential of the center, averaged over the transverse motion,

$$U_N^m(z) = \int_0^\infty \rho R_{Nm}^2(\rho)\, U(\rho, z)\, d\rho \tag{3.1}$$

(R_{Nm} is a function which describes the radial motion in a magnetic field), which for the case of a center with a short radius $r_c < L$ can be represented as the product of a power-law function of H and a function which depends on the longitudinal coordinate $z \| \mathbf{H}$,

$$U_N^m(z) = \left(\frac{r_c^2}{L^2}\right)^{|m|+1} V_m(z) \propto H^{|m|+1}. \tag{3.2}$$

The potential $U_N^m(z)$ is an even function of z and drops off over characteristic distances of the order of the range of the potential of the center r_c. For a sufficiently large longitudinal mass

$$\frac{\hbar^2}{m_\parallel r_c^2} \ll |U_N^m(z)| \ll \frac{\hbar^2}{m_\perp r_c^2} \tag{3.3}$$

the one-dimensional well (3.2) contains many levels. This is the key difference between the spectrum of MI states of carriers with markedly anisotropic mass and the spectrum of an electron whose effective mass is isotropic. The number of

quasibound MI levels in the potential (3.2) below the bottom of any Landau band (for fixed H) increases with the anisotropy parameter of the masses $m_\parallel/m_\perp$.

The second characteristic of the spectrum of MI states of particles with markedly anisotropic effective mass is the dependence of the number of levels on the intensity of the magnetic field. The depth of the well (3.2) decreases as H decreases $\propto H^{|m|+1}$, while MI levels with period $T_H \propto (U_N^m)^{1/2} \propto H^{(|m|+1)/2}$ leave the well.

The spectrum can be calculated in an analytic form only for some models of impurity potentials, in particular, for a screened Coulomb potential with $m = 0$ and the lowest Landau band. The corresponding one-dimensional potential (3.2) has the form

$$U_0^0(z) = -U_0\left(\frac{r_c}{L}\right)^2 \exp\left(-\frac{|z|}{r_c}\right), \quad U_0 \equiv \frac{e^2}{\varkappa r_c}, \quad (3.4)$$

where $\varkappa$ is the dielectric permittivity. The spectrum of MI levels of the potential (3.4) is determined by the equation (J is a Bessel function)

$$J_{k_\parallel r_c}(\chi(H)) = 0, \quad k_\parallel r_c = \frac{2r_c}{\hbar}\sqrt{2m_\parallel \Delta E}, \qquad \chi(H) = \frac{2r_c}{\hbar}\sqrt{2m_\parallel U_0}\, r_c L^{-1}, \quad (3.5)$$

from which it follows in the asymptotic limit $\varkappa(H) \gg 1$ that for magnetic field intensities

$$H_n = \frac{\pi^2 c\hbar^3}{32 e m_\parallel r_c^4 U_0}\left(n + \frac{1}{2}\right)^2 \quad (3.6)$$

a level with zero energy appears in the one-dimensional potential (3.4). The periodicity of the appearance of states with zero energy is proportional to

$$\propto \sqrt{\frac{U_0}{\hbar^2/m^* r_c^2}}\,\frac{r_c}{L} \propto \sqrt{H}. \quad (a)$$

The range of values of H for which two neighboring MI levels cross the boundary of the continuous spectrum equals

$$\Delta H_n = \frac{dH_n}{dn} \approx \frac{\pi^2 c\hbar^3}{16 e m_\parallel r_c^4 U_0}\, n. \quad (3.7)$$

Substitution of the parameters of the electrons and shallow impurities n-Ge into (3.7) ($m_\parallel/m_\perp = 19.5$, $U_0 = 10^{-2}$ eV, $r_c = 4\cdot 10^{-7}$ cm) gives the interval $\Delta H_n = 4.6\cdot 10^4 \text{G}\cdot n$.

4. PHOTOABSORPTION BY MAGNETIC IMPURITY STATES OF AN ELECTRON WITH AN ISOTROPIC EFFECTIVE MASS

In this section we shall study the problem of absorption of light by MI states of an electron with an isotropic effective mass in a potential with a short range. We shall first investigate the frequency dependence of the coefficient of absorption of the electromagnetic field of a system of noninteracting carriers in the presence of a shallow ($|U| \ll \hbar^2/m^* r_c^2$) impurity potential and a quantizing magnetic field. Then, for centers with a small radius ($r_c \ll L$), we shall analyze the dependence of the absorption on the depth of the impurities, without making the assumption that they are shallow. As will be shown below, in intervals of depths of the impurity potential corresponding to the presence of shallow real or virtual MI levels in the ground and higher-order Landau bands the frequency dependence of the absorption is oscillatory, i.e., peaks situated at distances close to ω_H appear on the absorption curve. In this case, for centers of arbitrary depth the frequency dependence of the coefficient of absorption is the same as for shallow impurities, and the positions of the peaks on the absorption curve are determined by the frequencies at which, in the initial and final states, the charge carriers are resonantly scattered by the MI states. If, on the other hand, there are no such MI levels, then the peaks in the absorption vanish. All these questions are of interest in studying the properties of D^- and A^+ centers in semiconductors,[3-5] deep impurities whose optical properties can often be described by the model of a short-range potential, H^- ions, and a number of other objects. In studying them, we shall restrict our analysis for simplicity to the case of mutually perpendicular electric and magnetic fields.

In the absence of an impurity, absorption of a weak long-wavelength electromagnetic field by carriers can occur according to the selection rules for the dipole moment, only in their transitions between neighboring Landau bands ($N \rightleftarrows (N \pm 1)$). The center, mixing different bands, opens up additional absorption channels in any transitions $N_1 \to N_2$. If the impurity potential is shallow, then MI levels with arbitrary m occur below the bottom of any Landau band. As we already mentioned, the states of the upper bands with projection of the angular momentum $m \neq N$ are quasibound and, in addition, their width is by a factor of $|\alpha| \ll 1$ smaller than their depth above the bottom of the corresponding band. The ground MI level lies below the bottom of the zeroth Landau band and corresponds to zero projection of the angular momentum. In a quantizing magnetic field, primarily the states of the continuous spectrum and the truly bound MI states of the lowest band are populated. Four types of transitions of carriers between states of the continuous spectrum (C) and of the quasibound (QB) states contribute to absorption[8,24] (see Fig. 4):

a) continuous spectrum→continuous spectrum (C→C);
b) continuous spectrum→quasibound states (C→QB);
c) bound states→continuous spectrum (B→C);
d) bound states→quasibound states (B→QB).

Transitions of the electron out of the initial (B) state into the continuous spectrum of the first band, as well as (QB) and (C) states of the higher ($N \geqslant 2$) bands represents photodetachment of the electron, while absorption in transitions of the type (C→C, QB) between zero Landau bands and Landau bands with $N \geqslant 2$ are harmonics of the cyclotron resonance.

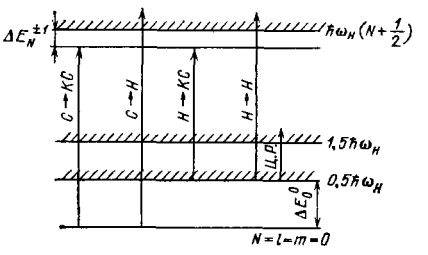

FIG. 4. Diagram of the electronic transitions responsible for the resonant absorption of an electromagnetic field.[24]

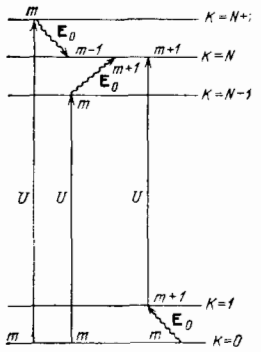

FIG. 5. Diagram of excitations of an electron into a Landau band N, responsible for resonant absorption in transitions of the type C→C.

Absorption of the field by transitions of the type (C→C) between 0 and 1 bands proceeds against the background of the usual cyclotron resonance and is of no special interest. The frequency dependence of the absorption coefficient can be easily understood using the example of any of the four types of transitions indicated above. Thus, for transitions of the type (C→C) between the zero Landau bands and the bands with $N \geqslant 2$ (harmonics of the cyclotron resonance), according to the selection rules for the dipole moment, resonance absorption occurs only along one of the channels (Fig. 5): 1) the center without changing m virtually excites the electron from the zeroth band to the $(N+1)$-st band, while the electric field (changing $m \to m-1$) excites the electron into the N-th resonance band; 2) The center excites the electron out of the zeroth band into the $(N-1)$-st band, while the field (changing m according to $m \to m+1$) transfers it into the N-th band. 3) The electric field transfers the electron from the band $k=0$ into the band $k=1$ $(m \to m+1)$, while the center transfers it into the N-th band. For simplicity, we shall assume, making use of the short-range nature of the potential, that the center affects only the motion of electrons with the smallest impact parameters $(m=0)$. In this case, the resonant component of the coefficient of absorption, related to the transition of the electron into the highest possible band, represents the sum of three terms, corresponding to the three possible channels for resonance transfer[24]:

$$\eta^{C\to C}(T, \Delta\omega) = A_N \frac{1}{\sqrt{T}} \int_0^\infty d\varepsilon \frac{\exp(-\varepsilon/T)}{\sqrt{\varepsilon(\varepsilon+\hbar\Delta\omega)}} \theta(\varepsilon+\hbar\Delta\omega) \times \left\{ [(N+1)^{-1} + N(N-1)^{-2}] \left(1 + \frac{\alpha^2}{2}\frac{\hbar\omega_H}{\varepsilon}\right)^{-1} + (N-1)^{-2}\left(1 + \frac{\alpha^2}{2}\frac{\hbar\omega_H}{(\varepsilon+\hbar\Delta\omega)}\right)^{-1} \right\}, \quad \Delta\omega = \omega - N\omega_H \tag{4.1}$$

(θ is the Heaviside function), and has a very simple physical interpretation. The integral over the energies arises due to the averaging over the initial states of the electron.[2)] The integrand includes the product of the initial $(\varepsilon^{-1/2})$ and final $(\varepsilon+\hbar\Delta\omega)^{-1/2}$ state densities by the coefficient of transmission of the electron

$$D_\varepsilon = \left(1 + \frac{\alpha^2}{2}\frac{\hbar\omega_H}{\varepsilon}\right)^{-1}$$

through a shallow one-dimensional potential, which contains an MI level with energy $|\Delta E| = (\alpha^2/2)\hbar\omega_H$. For the first and second transfer channels, D_ε, referring to the lowest Landau band, enters into (4.1) and for the third channel $D_{\varepsilon+\hbar\Delta\omega}$ for the final N band enters into (4.1) (see Fig. 5).

We shall now analyze the contour of the absorption line in different ranges of temperature and frequency detunings. In the range of high, compared to the energy of the ground MI level, temperatures and large detunings of the frequency, the coefficient of absorption reproduces the behavior of the density of final states:

$$\eta^{C\to C} \propto \frac{1}{\sqrt{\Delta\omega}}, \quad \frac{\alpha^2}{2}\hbar\omega_H \ll T \ll \hbar\Delta\omega. \tag{4.2}$$

With the temperature remaining constant, as the frequency decreases (the transmission coefficients, as before, are close to one), the square-root dependence is replaced by a logarithmic one

$$\eta^{C\to C} \propto \frac{1}{\sqrt{T}} \ln\frac{T}{\hbar\Delta\omega}, \quad \frac{\alpha^2}{2}\hbar\omega_H \ll \hbar\Delta\omega \ll T, \tag{4.3}$$

and the coefficient of absorption is determined by the density of both the initial and final states. Finally, for very small frequency detunings,

$$\eta^{C\to C} \propto \frac{1}{\sqrt{T}} \ln\frac{T}{(\alpha^2/2)\hbar\omega_H}, \quad 0 \leqslant \hbar\Delta\omega \ll \frac{\alpha^2}{2}\hbar\omega_H \ll T, \tag{4.4}$$

the coefficient of absorption no longer depends on the frequency detuning, and the logarithmic peak (4.3) becomes a logarithmic constant, whose value is determined by the energy of the MI level. The limiting cases of large frequency detunings (4.2) and (4.3) correspond to the Born approximation[25,26] with respect to the interaction of carriers with the impurities.

For temperatures at which the initial energy of the longitudinal motion of the electron is much lower than the energy of the ground MI state, for frequency detunings $\hbar\Delta\omega \gg T$, the contour of the absorption line is determined by the expression (see Fig. 6)

$$\eta^{C\to C} = \frac{A_N}{(N-1)^2}\sqrt{\frac{2\pi}{\hbar}}\frac{\sqrt{\Delta\omega}}{\Delta\omega + (\alpha^2/2)\omega_H}, \quad T \ll \frac{\alpha^2}{2}\hbar\omega_H, \ \hbar\Delta\omega, \tag{4.5}$$

while in the opposite limiting case of very small frequency detuning, the absorption coefficient approaches zero:

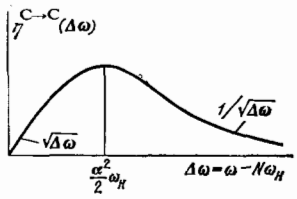

FIG. 6. Contour of the absorption line of harmonics of the cyclotron resonance at very low temperatures in transitions of the type C→C.[24]

[2)]Here and everywhere below, it is assumed that the carriers are described by Boltzmann statistics.

$$\eta^{C\to C} \sim \sqrt{T} \to 0, \quad \hbar\Delta\omega \ll T \ll \frac{\alpha^2}{2}\hbar\omega_H. \tag{4.6}$$

The absorption at low temperatures (4.5) has a peak at energies of the electron in the final Landau band equal to the energy of the ground MI level. The same quantity determines both the temperature behavior of the absorption coefficient and its frequency dependence (with fixed T), according to the analysis performed. The components of the coefficient of absorption for transitions of the type (C→QB) and (B→C) (photodetachment of the electron) are also determined by the product of the state density for the electrons, the transmission coefficients in the initial or final states through the one-dimensional well containing the MI level, and the distribution function. It is evident that both $\eta^{C\to QB}$ and $\eta^{B\to C}$ have maxima for frequency detunings corresponding to the energies of the MI states. For large detunings for transitions of the type (B→C), the coefficient of absorption reproduces the behavior of the density of final states $\eta^{B\to C} \propto (\Delta\omega)^{-1/2}$, while $\eta^{C\to QB}$ drops off exponentially due to the tail of the electron distribution function (Fig. 7). Finally, in transitions of the type (B→QB), the resonant component of the absorption coefficient near the peak, corresponding to the transition of carriers into the quasibound MI states of the band $N \geqslant 1$ with $m = \pm 1$, has a Breit-Wigner character, while when the bottom of the Landau band is approached from the peak from below, this dependence becomes more complicated[27] (see below).

We shall now examine the dependence of the absorption coefficient on the depth of the impurity potential U.[15] We shall start with transitions of the type (B→QB) from the ground state ($l = m = 0$) of the lowest Landau band into the quasibound MI state ($l = |m| = 1$) of the N-th band. If the depth of the center is such that the MI state with $|m| = 1$ has a small width (this will occur for $a_1 < 0$ and $|a_1| \ll L^3$), then independently of whether the ground state is induced by the magnetic field ($a_0 < 0$) or is a characteristic impurity state ($a_0 > 0$), the absorption coefficient has a sharp peak at the frequency of the resonance transition with the same dependence on ω. Only the factor characterizing the "strength" of the interaction of the electron with the center (see Table I) changes. Near the maximum the frequency dependence of the resonant component of the absorption coefficient[15,27]

$$\eta^{B\to QB} \sim \frac{(|\Delta E_0^0| - \hbar\Delta\omega)^{-1}}{\left(1 - \sqrt{\frac{|\mathrm{Re}\,\Delta E_N^1|}{|\Delta E_0^0| - \hbar\Delta\omega}}\right)^2 + \frac{(\Gamma_N^1)^2}{(\mathrm{Re}\,\Delta E_N^1)^2}} \tag{4.7}$$

is described by the Breit-Wigner formula with width $\Gamma_N^1 \propto \hbar\omega_H N^{7/2}|a_1/L|^3$, in the line wing it is replaced by a hyperbolic dependence. As the depth of the potential increases ($a_1 < 0, |a_1| \to \infty$) the width Γ_N^1 increases and, near the resonance of the well with the angular momentum equal to one it is of the order of $\Gamma_N^1 \propto \hbar\omega_H (N-1)^{2/3}(r_c/L)^{2/3}$, while the resonance absorption becomes indistinguishable from the background. The ratio of the maximum of the resonant component of the absorption coefficient to the background component is determined by the ratio of the shift in the energy of the final MI state to its width:

$$\eta^{B\to QB}: \eta_{\text{background}} \sim \left|\frac{\mathrm{Re}\,\Delta E_N^1}{\Gamma_N^1}\right|^2. \tag{4.8}$$

With the transition into the region of still greater depths of the potential U, the MI state of the N-th band with $l = 1$ falls through the bottom of the zeroth Landau band and the absorption oscillations vanish. Finally, if the impurity potential is such that the scattering length $a_1 > 0$, but $a_1 \ll L^3$, the resonant absorption peak (B→QB) is also absent.[27] (In this case, the sign in front of the square root in the denominator changes.)

In transitions of the type (B→C) from the ground MI bound state into the continuous spectrum of the band $N \geqslant 1$, for depths of the impurity potential such that the scattering length with $l = 1$ is very small ($|Na_1| \ll L^3, a_1 \leqslant 0$), the frequency dependence of the resonant component ($\eta^{B\to C}$) is given by the formulas of the shallow potential,[24] i.e., near the threshold the absorption coefficient grows in a square-root fashion ($\eta^{B\to C} \propto \sqrt{\Delta\omega}$); on the line wing it exhibits a square root drop-off, reproducing the behavior of the density of final states; and there is maximum in the region of ω corresponding to resonance scattering of the electron in the final state by a one-dimensional shallow potential. The square-root growth of $\eta^{B\to C}$ near the threshold is a result of the following circumstance: the resonant component of the absorption coefficient is proportional to the product of the density of final states of the electron $(\Delta\omega)^{-1/2}$ by the coefficient of its transmission through a shallow one-dimensional δ potential ($D_{\Delta\omega}$), which for low energies ($\Delta\omega \to 0$) gives the indicated frequency dependence. The dependence of $\eta^{B\to C}$ on the character of the ground MI level is the same as in the case of absorption in transitions of the type (B→QB) (see Table I).

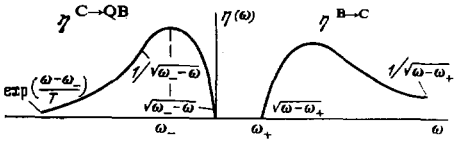

FIG. 7. Dependence of the resonance component of the absorption coefficient on the frequency of light in transitions of the type B→C and C→QB[24] ($\omega_\pm = \omega_H(N \pm \alpha^2/2)$.

TABLE I.

No. of region	1	2	3
Nature of the ground state	Field-induced MI level	Resonance MI states	Characteristic states of the impurity
Scattering length a_0	$a_0 < 0$, $\lvert a_0\rvert \ll L$	$a_0 \gtrsim 0$, $\lvert a_0\rvert \gg L$	$L \gg a_0 > 0$
$\eta^{B\to QB}$, $B\to C$	$\propto \lvert a_0 : L\rvert^3$	$\propto (0.13 + 0.2\, L : a_0)$	$\propto 2^{-3/2} L : a_0$

In transitions of the type (C→QB) from the lowest Landau band $N = 0$ into the resonant band N (with a change in the projection of the angular momentum $m = 0 \to |m| = 1$ or $m = -1 \to m = 0$), the frequency dependence of the resonant component of the absorption coefficient has an oscillatory character only for depths U corresponding to the presence of shallow MI states with $l = 0$ and $l = 1$ (i.e., with $a_0 < 0, |a_0| \ll L, a_1 < 0, |a_1| \ll L$ [3]). In the opposite case, either the penetration of the MI level below the bottom of the zeroth band or vanishing of the coefficient of transmission of the electron through the corresponding one-dimensional barrier suppresses the oscillations. The same also holds for absorption in transitions of the (C→C) type.

5. MAGNETIC-IMPURITY OSCILLATIONS IN THE CASE OF A MARKEDLY ANISOTROPIC ENERGY SPECTRUM OF CARRIERS

The theory of MI oscillations in materials with a markedly anisotropic effective mass of charge carriers was developed in Refs. 20, 28, and 29. The physical reason for such oscillations (see Sec. 3) lies in the fact that with a large longitudinal mass of the electron ($m_\parallel \| \mathbf{H}$), in a one-dimensional impurity potential $U^m(z)$ a system of MI levels, appears which is controlled by the field H and whose number increases with increasing anisotropy parameter of the masses. As the magnetic field intensity decreases, levels with period $\propto H^{(|m|+1)/2}$ go over into the continuous spectrum. If an electron (with the same m) is scattered by such a potential, then for it the well becomes transparent at the moment that a zero-energy level appears. In this case, all kinetic coefficients, determined by scattering, oscillate as a function of H. We emphasize that the oscillations under discussion are a result of two factors: the marked anisotropy of the effective mass and the finiteness of the range of the impurity potential, which leads to the existence of a restricted number of MI states in a one-dimensional well. In a purely Coulomb problem, even with a pronounced anisotropy of the effective mass, due to the infinite bunching of bound MI states toward the continuum, the oscillations are absent, although a radical restructuring of the spectrum occurs in the magnetic field.[30]

In the case of centers with a small radius ($r_c \ll L$), the kinetic coefficients are determined primarily by carriers with the smallest impact parameter ($m = 0$), while the oscillations have a distinct periodicity with a period $\propto H^{1/2}$. For impurities with arbitrary radius $r_c > L$, due to the competing contribution of states with different m, the oscillations are smoothed out. We shall first discuss the behavior of the longitudinal and transverse static conductivities. The longitudinal conductivity is determined by the flux of electrons reflected from the center[20]:

$$\sigma_{zz} \infty \int_0^\infty d\varepsilon\, \varepsilon \left(-\frac{\partial f}{\partial \varepsilon}\right) \frac{1}{\sum_{m=0}^{\infty} |T^{-m}(-k_\parallel)|^2}; \qquad (5.1)$$

f is the distribution function for electrons over the longitudinal energy ε, T^{-m} is the matrix element of the scattering operator for scattering by the potential $U^m(z)$, $\varepsilon = \hbar^2 k_\parallel^2 / 2m_\parallel$. The transverse conductivity is also expressed in terms of T operators[20,31,32] (see Sec. 6):

$$\sigma_{yy} \infty \int_0^\infty \frac{d\varepsilon}{\varepsilon} \left(-\frac{\partial f}{\partial \varepsilon}\right) \sum_{m=0}^{\infty} (m+1) |T^{-m} - T^{-m-1}|^2. \qquad (5.2)$$

For weak mixing of Landau levels by a separate scatterer ($U_{\text{eff}} \ll \hbar\omega_\perp = \hbar eH/m_\perp c$) the calculation of T^{-m} reduces to the solution of the one-dimensional Schroedinger equation with the potential $U^m(z)$. For an impurity potential in the form of a screened Coulomb potential (see (3.4)–(3.5)), this gives simple analytic expressions for the longitudinal and transverse conductivities in the asymptotic limit of a very large longitudinal mass, $\chi(H) \equiv (2r_c/L)\sqrt{2m_\parallel U_0} r_c/\hbar \gg 1$ and low temperatures $T \ll \hbar^2/m_\parallel r_c^2$ [20,29]

$$\sigma_{zz}(H) \infty \int_0^\infty \varepsilon\, d\varepsilon \frac{\nu^2 (-\partial f/\partial \varepsilon)}{\frac{\cos^2 2\chi(H)}{\cos^2 2\chi(H) + \pi^2\nu^2} + \sum_{m=1}^{\infty} |g_m(-k_\parallel)|^2},$$

$$\nu = 2k_\parallel r_c,$$

$$g_m \infty T^{-m}. \qquad (5.3)$$

$$\sigma_{yy}(H) \infty \int_0^\infty d\varepsilon \left(-\frac{\partial f}{\partial \varepsilon}\right) \frac{(1+\sin 2\chi(H))(1-\sin 2\chi(H) + \pi^2\nu^2)}{\cos^2 2\chi(H) + \nu^2\pi^2}. \qquad (5.4)$$

It follows from (5.3) that for values of the magnetic field intensity satisfying relation $2\chi(H) = \pi[n + (1/2)]$, the coefficient of reflection of electrons with $m = 0$ from the well vanishes and the longitudinal conductivity increases. This occurs each time when a level with zero energy appears in the one-dimensional potential $U_0^0(z)$ (compare with (3.6)). The amplitude of the oscillations is bounded due to the fact that states with the highest values of the projection of the angular momentum contribute to the scattering. The transverse conductivity (5.4) likewise oscillates; in addition, σ_{yy} vanishes for values of H corresponding to odd $n = 2k + 1$, and reaches a maximum for even values $n = 2k$. This behavior is explained by the different parity of the wave functions of the bound states in a one-dimensional well for successive values of n. The period of the oscillations of the conductivities (5.3) and (5.4) is proportional to the square-root of the magnetic-field intensity. We note that for values of H corresponding to the existence of an MI level with zero energy in a one-dimensional potential $U_0^0(z)$, the longitudinal magnetoresistance has a minimum. This leads to the appearance of a negative longitudinal magnetoresistance for values of the magnetic field intensity close to (3.6):

$$\frac{\rho_\parallel(H) - \rho_\parallel(0)}{\rho_\parallel(0)} < 0, \quad \rho_\parallel = \sigma_{zz}^{-1}.$$

The absorption by MI states of carriers with markedly anisotropic effective mass also exhibits a number of peculiarities.[28,29] Thus, with intraband absorption of light in parallel fields in transitions of the (B→C) type from the ground MI state into the continuous spectrum of the lowest Landau band, the absorption coefficient either approaches zero according to a square-root law $\eta \propto \sqrt{\Delta\omega}$ or becomes infinite $\eta \propto (\Delta\omega)^{-1/2}$ (Fig. 8), depending on whether the even or the

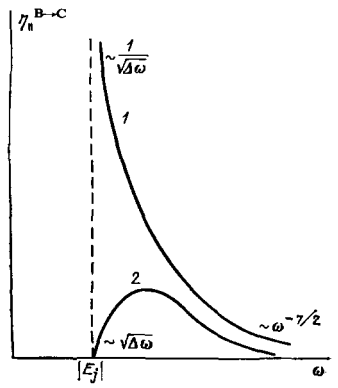

FIG. 8. Dependence of the coefficient of intraband absorption in parallel fields on the frequency of light in the case of a markedly anisotropic effective mass of the electron. The depth of the well corresponds to the presence of an MI level with energy close to zero. 1) odd-even transition; 2) even-even transition.

odd level goes over into the continuous spectrum. This difference in the behavior of η is explained by the fact that in the presence of an odd level with zero energy the wave function within the well of a slow electron undergoing scattering is almost odd, the transition from the even ground state under the action of the electric field $\mathbf{E}_0\|\mathbf{H}\|z$ is allowed, the one-dimensional well is transparent, i.e., the transmission coefficient equals unity, and due to the density of final states η diverges in a square-root fashion. In the presence of an even MI level of zero energy, on the other hand, the wave function of the slow electron in the well is primarily even, and its odd component approaches zero with decreasing energy of the electron. For this reason, due to the oddness with respect to z of the operator characterizing the interaction of the electron with the field E_0, the absorption coefficient vanishes as $\Delta\omega\to 0$.

The coefficient of interband ($N=0\to N=1$, $m=0\to m=+1$) absorption in transitions of the (C→QB) type in transverse fields behaves in a similar manner. The upper MI state, characterized by a nonzero projection of the angular momentum, is a shallow state by virtue of the condition $r_c \ll L$ and, correspondingly, it is even. For this reason, if an odd MI level approaches the boundary of the continuous spectrum of the lowest Landau band, then the wave functions of the initial and final states have a different parity with respect to z, and $\eta^{C\to B}$ approaches zero in a square-root fashion. If, on the other hand, the level with zero energy in the zeroth band is even, then the wave functions have the same parity with respect to z, their overlap integral is small, and due to the density of initial states $\eta^{C\to B}$ diverges in a square-root fashion (see Fig. 8).

For interband transitions ($N=0\to N=1$, $m=0\to m=+1$) of the (B→C) type in transverse fields the behavior of the absorption coefficient likewise differs considerably depending on the parity of the initial bound MI state. If the initial state is odd, then near the threshold of absorption we have a square-root dependence $\eta^{B\to C}\propto\sqrt{\hbar\Delta\omega-|\Delta E_{2n+1}|}$, ($\Delta\omega=\omega-\omega_H$), while in the absorption line wing $\eta\propto(\Delta\omega)^{-3/2}$. In the opposite case of an even MI state, the previous dependence of η near the threshold should be observed, but in the tail the dependence of the absorption coefficient turns out to be different: $\eta\propto(\Delta\omega)^{-5/2}$.

The possibilities of observing the oscillations under study are most favorable in group-IV semiconductors Ge and Si, in which the effective mass of carriers is markedly anisotropic. In the case of shallow hydrogen-like impurities in Ge, the corresponding one-dimensional potential $U_0^0(z)$ has four MI levels[20] for $H=4\cdot10^5$ G. As the magnetic field intensity drops to $H=4\cdot10^4$ G, three of the MI levels leave the well. This indicates that as H varies up to the value indicated, three oscillations should be observed experimentally. Here, it is useful to make the following remark. Since real semiconductors have multivalley structures, the contribution to absorption (or conductivity) of "foreign" valleys determines the background, since in them due to the absence of axial symmetry states with different projections of the orbital angular momentum m are mixed. To eliminate the background, it is useful to observe oscillations in samples subjected to strain, which permits "pumping" electrons over into the main ($m_\parallel\|\mathbf{H}$) valley.

In concluding this section we note that Eaves, *et al.*,[3] in the case of scattering of free carriers by electron drops in a magnetic field in Ge, and Nicholas *et al.*,[34] in the case of scattering of carriers by impenetrable cylindrical impurities in InSb, observed periodic oscillations of the conductivity with period $\sqrt{H}$ arising due to the size effect, i.e., due to the resonant behavior of the scattering amplitude for scattering by an impenetrable impurity with magnetic field intensities such that $\sqrt{2}r_c/L=\pi(n+\gamma)$, $n=0,1,2,\ldots$, and $r_c>L$.

6. TRANSVERSE CONDUCTIVITY ON DEFECTS WITH FINITE RADIUS

a) General formula for transverse conductivity

It is well known that in free space an electron placed in crossed $\mathbf{E}_0\perp\mathbf{H}$ electric and magnetic fields drifts in a direction perpendicular to the plane of the fields. Only its collision with some defects in the material will lead to the appearance of a dissipative—along the electric field—component of the current. The transverse conductivity (or the coefficient of diffusion related to it by Einstein's relations) characterizing this component is usually calculated in a quantizing magnetic field using Titeica's formula[35] in the representation of Landau numbers:

$$\sigma_{yy}=\frac{2\pi e^2}{\hbar}\sum_{\alpha,\alpha'}\left(-\frac{\partial f(E_\alpha)}{\partial E_\alpha}\right)\delta(E_\alpha-E_{\alpha'})\times|\langle\varphi_\alpha^*|U|\psi_\alpha\rangle|^2(y_\alpha-y_{\alpha'})^2 \tag{6.1}$$

($\alpha\equiv\{N,y_0,p_z\}$ is the set of Landau quantum numbers, φ_α and ψ_α are the wave functions of the electron for free motion and in the scattering problem, respectively; $U(\mathbf{r})$ is an arbitrary scattering potential), which represents the mean-square displacement of the electron along the field as a result of collisions.

The Landau gauge is very convenient and, as a rule, is used to obtain the general formulas for the conductivity, since the Schroedinger equation for the electron in crossed

fields can be solved exactly in this gauge. However, the analysis of the scattering problem in this gauge is usually restricted to perturbation theory. A calculation of the transverse conductivity (6.1) with the scattering probability found in the Born approximation leads in a quantized magnetic field to the appearance of a logarithmic divergence at low energies, due to the large density of states of the electron $g(\varepsilon) \propto \varepsilon^{-1/2}$ for low $\varepsilon \to 0$ energies. Skobov[36] was the first to show[3)] that going beyond perturbation theory in the case of scatterers with a zero radius automatically eliminates the divergence. We note that according to Ref. 36 the cutoff of the divergence in σ occurs at energies of the order of the energy of the MI state of the electron in the field of an isolated center. At the same time, if in the high-temperature case there arises a natural lower limit in the logarithmically diverging expression for the conductivity, then in the low-temperature limit the nature of the dependence $\sigma(H,T)$ itself changes. It is evident, however, that Landau's gauge does not take into account the natural axial symmetry of the problem in all cases of scattering of carriers by spherically symmetrical defects.

It turns out that it is possible to go beyond the Born approximation and to obtain temperature-field dependences of the transverse conductivity over a wide temperature range and wide range of quantizing magnetic fields for the case of scatterers with finite, but arbitrary radius by transforming to an axisymmetrical gauge.[31,32,38] The idea of this transformation is this: the diagonal component of the transverse conductivity tensor has the form[39] (Kubo)

$$\sigma_{yy} = \frac{e^2}{2\pi VT} \left\langle \mathrm{Sp}\, \hat{\rho}\hat{v}_y \int_{-\infty}^{+\infty} dE \frac{1}{E-\hat{H}-i\delta} \hat{v}_y \frac{1}{E-\hat{H}+i\delta} \right\rangle,$$

$$\delta = +0, \tag{6.2}$$

$\hat{H} = H_0 + \Sigma_a U(|r - \mathbf{R}_a|)$ is the total Hamiltonian of the system in the absence of an electric field, v is the volume, $\hat{\rho}$ is the density matrix of the electron, and $\hat{\mathbf{v}}$ is the electron velocity operator. The notation $\langle \ldots \rangle$ indicates averaging over the position of the impurities. In the approximation $\hbar\tau^{-1} \ll T$, where τ is the momentum relaxation time of the carrier, the current in the system is the product of the volume-averaged current from an isolated scatterer by the number of scatterers in the volume. In finding the current from an isolated impurity the Hamiltonian $\hat{H}$ must be replaced by the Hamiltonian of the single-center problem $\hat{H}_1$. In the expression obtained in this case from (6.2), we shall take the trace with respect to the complete system of functions of the single-center problem. As a result, we arrive at a formula for the transverse conductivity, expressed in terms of the functions of the axial gauge[4),31,32]

$$\sigma_{yy} = 2\pi e^2 n_0 \int_{-\infty}^{+\infty} dk\, dk'\, \delta(\varepsilon - \varepsilon') \left(-\frac{\partial f}{\partial \varepsilon} \right)$$

$$\times \sum_{m=0}^{\infty} |T_{kk'}^{-m} - T_{kk'}^{-m-1}|^2 (m+1), \tag{6.3}$$

T^{-m} are the matrix elements of the scattering operator for an electron scattered by an isolated impurity in an axisymmetrical gauge, n_0 is the density of scatterers, and $\varepsilon = \hbar^2 k^2 / 2m^*$ is the energy of longitudinal motion of the electron.

b) The case of a screened Coulomb impurity potential

The calculation of the transverse conductivity according to formula (6.3) permits obtaining the dependence $\sigma_{yy}(T,H)$ for arbitrarily low temperature $0 \leqslant T \ll \hbar\omega_H$ in quantizing magnetic fields, if we know the solution of the single-center scattering problem in the axisymmetrical gauge (see Refs. 17, 20, 27, 38, 40, and 41, which are concerned with the analysis of scattering in different potentials). Attractive screened Coulomb centers are of greatest interest, because the effect of the nature of scattering of carriers on the conductivity is most clearly manifested here.[32] (The coefficient of diffusion is calculated for a number of other impurity potentials in Ref. 38.) We shall restrict our analysis to the case of impurities with a large screening radius $r_c \gg a_B$ and very strong magnetic field $a_B \gg L$. The last condition permits neglecting the excitation of electrons into the higher-order Landau bands. Neglecting such excitation leads to the one-dimensionality of scattering of an electron by an isolated center. In this case, the carrier with impact parameters $\rho = \sqrt{2|m|}\, L$ moves in the effective potential

$$U_0^m(z) = -\frac{e^2}{\varkappa} \frac{\exp(-\sqrt{2|m|L^2+z^2})}{\sqrt{2|m|L^2+z^2}}, \tag{6.4}$$

and finding the matrix elements of the scattering operators T reduces to solving the one-dimensional Schroedinger equation with the potential (6.4). Comparing (6.4) with the energy $\varepsilon_c = \hbar^2/m^* r_c^2$ it is easy to see that the one-dimensional wells $U^m(z)$ are deep for particles with impact parameters $\rho \ll \rho_c = r_c \ln(r_c/a_B)$ and they are shallow for $\rho > \rho_c$. It is also evident that as the impact parameter increases the strength of the potential (6.4) decreases, which leads to a transfer of bound states into the continuous spectrum.[5)] In this case, in the low-energy range $\varepsilon \ll \varepsilon_c$ the scattering of electrons with impact parameters corresponding to the presence of a shallow level in $U^m(z)$ has a resonant character.[32]

We will not present the details of the calculation of the transverse conductivity. We shall instead give a general description of its behavior.[6),32,38] Excluding the case of very low temperatures, we can identify the following regions of electron energies, where the dependences $\sigma(T,H)$ differ considerably:

I. Region of resonant scattering of carriers:

$$\left(\frac{L}{r_c}\right)^4 \left(\frac{r_c}{a_B}\right) \varepsilon_c \ll T \ll \varepsilon_c, \quad \sigma_{yy} \sim H^{-2}T^{-2}. \tag{6.5}$$

At such temperatures, the transverse conductivity is deter-

[3)] Using the Landau gauge.

[4)] Formula (6.3) is essentially identical to formula (18.19) obtained in Ref. 37 and expressed in terms of the phase shifts of the scattered waves (in a one-dimensional problem) of different parity, but it is much more convenient in performing actual calculations.[20,32,38]

[5)] The situation here is analogous to that examined in Sec. 3 for MI states of particles with markedly anisotropic effective mass.

[6)] The regions of applicability of the results in Ref. 32 are clarified in Ref. 35 and the appearance of the logarithmic dependence $\propto \ln T \ln H$ (6.10) at high temperatures is pointed out.

mined by the carriers being scattered resonantly by deep wells.

II. Region of quasiclassical scattering:

$$\text{a)}\ \varepsilon_c \ll T \ll \varepsilon_B \frac{a_B}{r_c}, \quad \sigma_{yy} \propto r_c H^{-2} T^{-1/2}, \tag{6.6}$$

$$\text{b)}\ \varepsilon_B \frac{a_B}{r_c} \ll T \ll \varepsilon_B, \quad \sigma_{yy} \propto H^{-2} T^{-3/2} \ln^2 \left(\frac{T}{2\varepsilon_B} \frac{r_c}{a_B} \right); \tag{6.7}$$

where ε_B is the Bohr energy of the electron. The transverse conductivity is determined by carriers with impact parameters $\rho \gg a_B$, for which scattering in the entire temperature interval $\varepsilon_C \ll T \ll \varepsilon_B$ is quasiclassical, and forward scattering makes the main contribution to σ. In this case, in the energy range a), in calculating the quasiclassical phase of the scattering we can neglect the energy of the electron, while in the region b) it must be included.

III. Region of perturbation theory:

$$\varepsilon_B \ll T \ll \hbar\omega_H,$$

$$\sigma_{yy} \propto H^{-2} T^{-3/2} \left(\ln \frac{r_c}{L} \ln \frac{T}{\varepsilon_B} + \frac{1}{2} \ln^2 \frac{r_c}{a_B} \right). \tag{6.8}$$

The first term in the transverse conductivity arises due to the contribution to σ from electrons whose energies greatly exceed the Bohr energy, and the second term is due to carriers with energy $\varepsilon \ll \varepsilon_B$. The temperature-field dependence of the transverse conductivity (6.8) is determined by a number of parameters. If the screening radius of the impurity is large enough, so that $\ln^2(r_c/a_B) \gg \ln(r_c/L)\ln(T/\varepsilon_B)$, then the dependence

$$\sigma_{yy} \propto H^{-2} T^{-3/2} \ln^2 \frac{r_c}{a_B} \propto H^{-2} T^{-3/2} \tag{6.9}$$

holds. If, on the other hand, the temperature is high enough or the magnetic field is strong enough, i.e., $\ln(r_c/L)\ln(T/\varepsilon_B) \gg \ln^2(r_c/a_B)$, then the dependence of the transverse conductivity on H and T is proportional to $\ln T$ and $\ln H$

$$\sigma_{yy} \propto H^{-2} T^{-3/2} \ln T \ln H. \tag{6.10}$$

The formula for the transverse conductivity, first obtained by Adams and Goldstein[42] in the Born approximation with respect to the interaction of carriers with the Coulomb centers, had the form

$$\sigma_{yy} \propto H^{-2} T^{-3/2} \ln \frac{T}{\varepsilon_{\min}} \ln \frac{r_c}{L}, \tag{6.11}$$

where $\varepsilon_{\min}$ is some (unknown) energy. From the analysis performed it follows that the basic temperature-field dependence of the conductivity $\sigma \propto H^{-2} T^{-3/2}$, found in Ref. 42, is valid beginning with much lower temperature $\varepsilon_B a_B / r_c \ll T \ll \varepsilon_B$, and only the functional dependence of the logarithmic factors changes. In addition, as the screening radius of the impurity increases, the emergence of σ into the logarithmic regime (6.10) is increasingly delayed.

c) Impurity oscillation effects

In the case of impurities with small radius ($r_c \ll L$), the matrix elements of the scattering operator can be found for an arbitrary depth of the potential.[10,11] The main contribution to the transverse conductivity on such centers comes from S electrons, which are incident with minimum impact parameter. The neglect of the contribution of carriers with the highest projections of the angular momentum reduces (6.3) to the expression

$$\sigma_{yy} = \sqrt{2\pi} \frac{n_e n_0 e^2 \hbar^2 L^2}{(m^* T)^{3/2}} \alpha^2 \int_0^\infty \frac{d\varepsilon}{\varepsilon} \frac{\exp(-\varepsilon/T)}{1 + (\alpha^2/2)\hbar\omega_H/\varepsilon}, \quad \alpha \equiv \frac{a_0}{L}, \tag{6.12}$$

first obtained by Skobov.[36] The physical interpretation of the last formula is very simple. The integrand in (6.12) consists of the product of the densities of initial ($\varepsilon^{-1/2}$) and final ($\varepsilon^{-1/2}$) states of the electron by the coefficient of transmission of the electron through the effective one-dimensional δ potential. For $a_0 < 0$ and $|\alpha| \ll 1$, i.e., in the presence of a shallow MI level on the impurity the transverse conductivity carriers direct information on its energy. Indeed, at high temperatures we have a logarithmic dependence of σ on the magnetic field intensity and on the whole a power-law dependence on the temperature:

$$\sigma_{yy} \propto T^{-3/2} \ln \frac{T}{H^2}, \quad T \gg \frac{\alpha^2}{2} \hbar\omega_H, \tag{6.13}$$

while at low temperatures,

$$\sigma_{yy} \propto T^{-1/2} H^{-2}, \quad T \ll \frac{\alpha^2}{2} \hbar\omega_H \tag{6.14}$$

the dependence of the conductivity on T and H is purely a power-law dependence. Since the energy $\Delta E = -(\alpha^2/2)\hbar\omega_H$, for $|\alpha| \ll 1$, is the energy of the bound MI state in the field of a short-range center, it follows from here that the temperature-field dependence $\sigma(T,H)$ reflects the structure of the spectrum of the MI states.

The behavior of the transverse and longitudinal conductivities as a function of the depth of an isolated carrier with small radius $r_c \ll L$ was investigated in Ref. 10. Oscillations have been observed in $\sigma(U)$ near the resonance depths of the center $|a_0| \gtrsim L\sqrt{T/\hbar\omega_H} \gg r_c$, which intensify with increasing temperature—the amplitude of the peaks increases, but the width decreases (see Fig. 9). According to (6.12), in

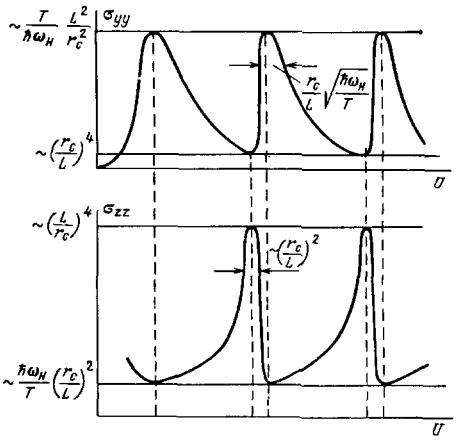

FIG. 9. Dependence of the longitudinal and transverse conductivities on the depth of the impurity potential of small radius $r_c \ll L$ at high temperatures.[10]

the case of centers with a small radius the conductivity

$$\sigma_{yy} \propto \frac{|a_0|^2}{L^2}\left(1+\frac{1}{2}\frac{|a_0|^2}{L^2}\frac{\hbar\omega_H}{T}\right)^{-1} \qquad (6.15)$$

represents the product of two factors, the first of which, increasing as $\propto |a_0|^2$, is the square of the electron-impurity interaction constant, and the second, which decreases with increasing $|a_0|^2$, is the coefficient of transmission of the electron with characteristic kinetic energy $\sim T$ through an effective one-dimensional δ potential, arising due to the combined action of the center and of the magnetic field. As $|a_0|^2$ increases from zero to infinity (6.15) increases, approaching a constant value

$$\propto \frac{T}{\hbar\omega_H} \propto \frac{T}{\hbar\omega_H}\left(\frac{L}{r_c}\right)^2, \qquad (6.16)$$

which corresponds to the relative amplitude of the oscillations shown in Fig. 9. The width of the peak of the oscillations is easily obtained from (6.15). Indeed, near the resonance $a_0 \propto r_c/\Delta U$, and from here, estimating the effective value of ΔU from the condition that the first and second terms in the denominator (6.15) are equal, we find that the width of the peak is proportional to $(r_c/L)\sqrt{\hbar\omega_H/T} \ll 1$. The maximum longitudinal conductivity occurs for values of U where $a_0 = 0$. In this case, electrons with $m = 0$ do not "feel" the center ($T^0 = 0$), and the state with $m = -1$ contributes to the conductivity. The matrix element T^{-1}, in this case, is of the order of $\sim r_c/L^2$, which gives a relative amplitude of oscillations $\propto |T^{-1}|^{-2} \propto L^4/r_c^4$.

7. EXPERIMENTS

a) Magnetic impurity oscillations in the de Haas–van Alfvén effect

Brandt and Lyubutina investigated the de Haas–van Alfvén oscillations in Bi samples doped with small concentrations of donor-type impurities—selenium and tellurium. The measurements were performed in a uniform magnetic field, varying in the range 1.5–13 kG and at temperatures of 1.6–4.2 K and 14–20.4 K, on suspended samples with the trigonal axis perpendicular to the axis of suspension. The dependence of the torque M on the magnetic field intensity was measured for different orientations of the field ψ relative to the trigonal axis. In a number of cases, a distinct periodic variation of the amplitude of the oscillations, reminiscent of a beat pattern (Fig. 10), was observed on curves of the anisotropy of the magnetic susceptibility $\Delta\chi$ as a function of $1/H$. The beats were most distinct in Bi–Te alloys with low impurity concentrations (Fig. 10, 1, 2). As a rule, the frequency of the beats was proportional to the frequency of the observed oscillations and constituted 15–20% of the main frequency. As a result of this, in going over to small sections of the electronic surface, the period of the beats increases[7)] and for orientations of the magnetic field close to the bisector axis, encompasses practically the entire range of fields from 1.5 to 13 kG.

[7)]The frequency of oscillations in the de Haas–van Alfvén effect is related to the extremal section S of the Fermi surface, perpendicular to the direction of the magnetic field, by the relation $\Omega = (c/eH)S$.

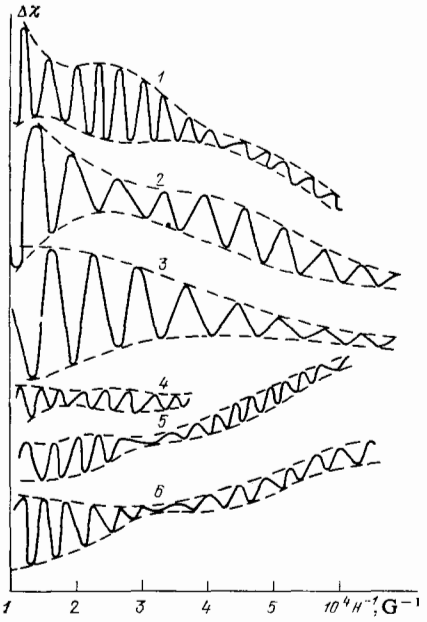

FIG. 10. Dependence $\Delta\chi(1/H)$ for Bi–Te and Bi–Se alloys, according to Ref. 2. ψ is the angle between the field **H** and the trigonal axis of the sample. The computed concentrations of impurities in at. % and the angles are: 1—$8\cdot10^{-4}$ Te, $\psi = 165°$; 2—$8\cdot10^{-4}$ Te, $\psi = 140°$; 3—$8\cdot10^{-4}$ Te, $\psi = 78°$; 4—$5.3\cdot10^{-3}$ Se, $\psi = 160°$; 5—$2.4\cdot10^{-3}$ Te, $\psi = 160°$; 6—$1.6\cdot10^{-4}$ Te, $\psi = 155°$.

An explanation of the beats was proposed in Ref. 43 (Ermolaev, Kaganov) and their theory for the model of shallow impurities is developed in Refs. 12 and 13. According to Ref. 43, beating[2] is a result of a superposition of two oscillatory dependences. One is due to the system of Landau levels and the other to the system of quasibound MI levels, induced by the magnetic field on neutral donor impurities. Beats appear when the system of MI levels crosses the electronic Fermi surface. The presence of such MI levels leads to a change in the electronic density of states, as a result of which, in their turn, oscillatory corrections to the thermodynamic quantities appear.[12,13] In the approximation which is linear with respect to the impurity concentration (zero radius) and temperatures which are much smaller than the Fermi energy $T \ll \varepsilon_F$, this correction to the free energy has the form[12]

$$\Delta F = 2n_0 T \sum_{p=1}^{\infty} \frac{(-1)^p}{p} \exp\left(-\frac{2\pi p}{\omega_H \tau_F}\right) \times \frac{\cos\left[2\pi p\left(\varepsilon_F + |\Delta E_N^0|/\hbar\omega_H\right)\right]}{\operatorname{sh}(2\pi^2 pT/\hbar\omega_H)}. \qquad (7.1)$$

($\tau_F = \hbar^2\omega_H/4|\Delta E_N^0|^{3/2}\varepsilon_F^{1/2}$ is the lifetime of an electron). As follows from (7.1), the frequency of the oscillations, arising due to quasibound MI levels is greater than the frequency of the fundamental oscillations by the amount $\delta\Omega = \Omega\left[|\Delta E_N^0|/\varepsilon_F\right]$. The angular dependences of the beats were analyzed in Ref. 13 and the effect of the anisotropy of the effective mass of the carriers in the model of a shallow impurity potential with small but finite radius was taken into account. The theory[12,13] describes well the qualitative characteristics of the phenomena observed in Ref. 2 and also confirmed by two experimental facts: the vanishing

of beats when donor-type impurities are replaced by acceptor-type impurities (the quasibound MI levels also disappear) and the stability of the beats relative to the technology used to prepare the samples.[8)] To obtain a quantitative agreement between theory and experiment, however, it is apparently necessary to perform calculations outside the framework of the Born impurity potential[12,13] and to use a general approach to find the energies of the MI levels.[11]

b) Photodetachment of an electron from magnetic impurity states of D^- impurities

Aside from bismuth, MI states of negative ions have been observed in samples of n-CdS[3] and the group-IV semiconductors Ge[4] and Si[5] by measuring the photoabsorption in D^- centers in a quantizing magnetic field. As we have already mentioned above, in Refs. 3 and 5 a series of peaks situated at distances close to ω_H and related to the participation of successive Landau bands in the absorption was observed in the curve of the cross section for photodecay of D^- ions $\sigma^H(\omega) \propto \eta(\omega)$. (Analogous oscillations were observed in Ref. 45 for negative sulfur ions S^-.) In all the experiments enumerated above attempts were made to determine the binding energy $E_i(H)$ of D^- ions from the dependence $\sigma^H(\omega)$ obtained. In the experiments performed by Cohn and coworkers on n-CdS, doped with shallow donor impurities, the carriers had a temperature of 1.4 K, while the magnetic field varied over the range 30–126 kG. The cross section for photodetachment[3] was virtually independent of the orientation of H relative to the crystallographic axes of the sample and represented the sum of two components, one of which described the cyclotron resonance absorption line with the effective electron mass $m^* = (0.194 \pm 0.005)\, m_e$, while the second corresponded to some variable effective mass, varying from $(0.130 \pm 0.005)\, m_e$ at $H = 30$ kG to $(0.170 \pm 0.005)\, m_e$ for $H = 126$ kG. Larsen[16] proposed that the second component is due to absorption accompanying photodetachment of the electron from the ground MI level of a neutral impurity. To calculate the dependence of the energy of this level on the magnetic field intensity, Larsen[16] used a trial wave function with three adjustable parameters. For $H \neq 0$, the calculation[16] gives good agreement between theory and experiment, but in the absence of a magnetic field there is a somewhat larger disagreement between the computed and observed values of the energy.

Since the effective mass of the electron in n-CdS is a scalar, in order to describe the ground MI level of the D^- impurity, the theory in Refs. 14, 10, and 11 was applied directly in Ref. 3. The dependence of the energy ε_i of this level, measured from the bottom of the first Landau band (and measured in rydbergs R) on the magnetic field intensity $\gamma = \hbar\omega_H/2R$, in accordance with Refs. 11 and 14, is given by the equation (see Appendix)

$$\frac{a_B}{a_0} = \varepsilon_i^{1/2} - \frac{\gamma}{(\varepsilon_1 - 2\gamma)^{1/2}} - \frac{\gamma}{2\sqrt{\varepsilon_1}} - \frac{\gamma^2}{12\varepsilon_i^{3/2}} + \frac{r_0}{2a_B}(3\gamma - \varepsilon_1), \tag{7.2}$$

[8)] Analogous[2] oscillatory dependences, stemming from the appearance of quasibound MI levels, must also appear in the thermogalvanomagnetic coefficients in metals.[44]

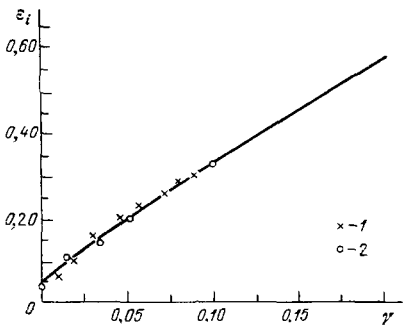

FIG. 11. Dependence of the binding energy of the D^- ion on H in n-CdS. The continuous curve shows the measurements[3]; 1) Larsen's calculation,[16] 2) calculation in Refs. 10 and 14.

where a_B is the Bohr radius of the neutral impurity. The ratios a_B/a_0 and r_0/a_B can be determined from independent experiments. They can be found from any two points.[3] In so doing, to within several percent, the terms proportional to r_0 can be dropped. The ratio a_B/a_0 obtained from one experimental point[3] ($\gamma = 0.1$, $\varepsilon_i = 0.32$) equals 0.193. Substituting the ratio found into the left side of (7.2), we obtain an equation for determining the dependence $\varepsilon_i(\gamma)$. Figure 11 shows a comparison of the results of the calculation using formula (7.2) with the experimental data from Ref. 3 and Larsen's results.[16] In the absence of a magnetic field ($\gamma = 0$), the experimental value of the binding energy $\varepsilon_i^{exp} = 0.0555$, the calculation using formula (7.2) gives the quantity $\varepsilon_i^{theor} = 0.372$, while in Larsen's model $\varepsilon_i^{Lar} = 0.051$. An analogous approach to the analysis of the data on the dependence of the binding energy of D^- ions on H in Ge[4] and Si[5] likewise gives quite good agreement between theory and experiment. In this case, it is necessary to take into account carefully the anisotropy of the effective mass of the carriers in the semiconductors as well as the contribution of all valleys.

With regard to measurement of the dependence $E_i(H)$, we must make the following remark.[9)] Its determination by a direct method is, in principle, a determination of the threshold absorption frequency ω_{th} in the dependence $\sigma^H(\omega)$.[3–5] It was proposed in Ref. 3 that ω_{th} lies below the first maximum, and in Refs. 4, 5 it was proposed that it coincides with the maximum. If, however, the depth of the impurity is such that $a_1 < 0$, then a quasibound MI state with $|m| = l = 1$ is present below the bottom of any Landau band. In this case, the absorption curve near the bottom of any band has two peaks: one at a frequency corresponding to the transition of the electron into this state and less than ω_H, and the other at a frequency greater than ω_H and corresponding to resonant scattering by this MI level (see Fig. 4). If, on the other hand, $a_1 > 0$, then there is only one peak occurring at a frequency greater than ω_H. In other words, ω_{th} can lie both above and below the first maximum, and its determination requires more detailed experiments. We note also that in order to analyze the oscillations it is desirable to perform measurements with circular polarization, since in linear polarization[3–5] peaks with $m = \pm 1$ are superposed.

[9)] The following arguments were formulated by Yu. A. Gurvich and A. S. Zil'bermints.[27]

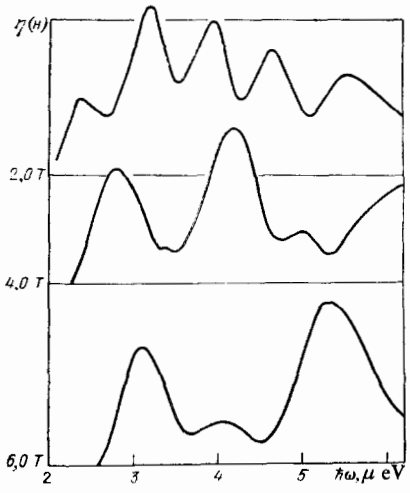

FIG. 12. Dependence of the coefficient of photodetachment of an electron from the ground state of As^- in Si on the intensity of the magnetic field.[5] The concentration of neutral As impurities is $6\cdot10^{14}$ cm^{-3}; $T = 1.46$ K.

The distance $\Delta\omega_H$ between neighboring peaks in the absorption curve[5] (Figs. 12, 13) was calculated in Ref. 27. According to Ref. 27, almost all $\Delta\omega_H$ do not coincide with the cyclotron frequency ω_H and, as a rule, $\Delta\omega_H < \omega_H$. This corresponds to the situation when the scattering length a_1 is negative, i.e., to the absence of a purely impurity bound state with angular momentum $l = 1$ in a zero magnetic field (and the presence of MI levels with $|m| = l = 1$ for $H \neq 0$). In this case, the deviation of $\Delta\omega_H$ from ω_H reaches tens of percent, i.e., errors are clearly present in the measurements. For this reason,[27] the maxima in the absorption in Ref. 5 are due to the lower peaks, while the threshold frequency must lie above the first maximum. The relation between the maxima of the peaks in the photoconductivity qualitatively coincides with the results of the calculation in Ref. 27. Thus, both in the case of As (see Fig. 12) and Li (see Fig. 13) impurities in a field of $H = 2T$, the peak corresponding to the transition of the electron in a D^- impurity into the Landau band $N = 1$ must be the highest peak, and as H increases the maxima of

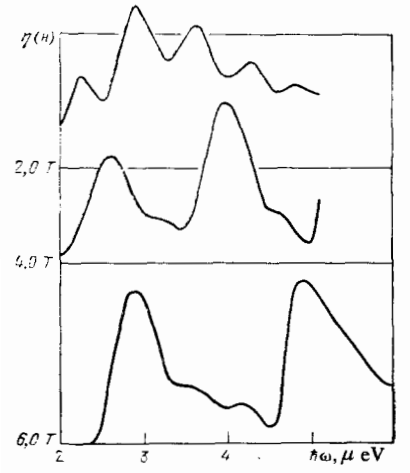

FIG. 13. Dependence of the coefficient of photodetachment of an electron from the ground state of Li^- in Si on the intensity of the magnetic field.[6] The concentration of neutral Li impurities is $6\cdot10^{14}$ cm^{-3}; $T = 1.46$ K.

peaks with $N = 0$ and $N = 1$ must tend to become equal, which does in fact occur in experiments.[5]

The pattern of peaks in the photoionization cross section of isolated D^- impurities in a quantizing magnetic field was also observed in Ge doped with Sb and As. The temperature of the samples was 0.35 K, the neutral impurity concentrations was $5\cdot10^{13}$ cm^{-3} (Sb) and $6\cdot10^{13}$ cm^{-3} (As), and the magnetic field varied over the range 0–20 kG. Oscillations (whose maximum number reaches four), arising in fields of the order of 5 kG, are clearly evident in the absorption curves.[4] However, for a number of reasons it is difficult to make comparisons between the experiments described in Ref. 4 and the theory.[16,27]

We have presented the current ideas concerning the spectra of MI states of impurity potentials with finite range and the kinetic phenomena occurring in systems which contain such states. It appears to us that from the point of view of a qualitative description of physical processes which are due to the appearance of such states, the general features of the theory are complete. However, its quantitative agreement with experiment is only in an initial stage and will require an analysis of the effect of different collective mechanisms of broadening in the interaction of carriers with isolated impurities. Some efforts to study the effects of multiple scattering of carriers by impurities and phonons have been made,[46–48] but these questions have not been investigated in detail.

It appears to us that the possibility of the realization of magnetic-field controllable MI states of charged particles on neutral impurities opens up great prospects in three important directions: first, in connection with the study of deep impurities in semiconductors and especially in dielectrics, where the mechanisms responsible for the formation of the impurity potential are entirely unclear, the presence of such states permits performing direct experiments designed to clarify these mechanisms; second, in connection with the extraordinary interest shown recently in the development of active systems, where the use of materials with magnetic-field-induced MI states permits creating relatively simply a population inversion of the levels; and, third, in investigations of the theory of metal-insulator phase transitions in disordered systems containing such MI states.

The author takes this opportunity to thank L. P. Pitaevskiĭ for encouragement in writing this review and for a number of useful remarks, and V. F. Elesin, Yu. M. Kagan, I. B. Levinson, V. N. Sobakin, and I. S. Shapiro for discussions.

APPENDIX[11]

We examine the problem of determining the energy spectrum of weakly-bound states (shallow real or quasidiscrete levels) of a particle described by the Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2m^*}\Delta + U(r) + V_f(\mathbf{r}) \equiv \hat{H}_f + U(r), \qquad (A.1)$$

where $U(r)$ is a short-range central potential with radius r_c, and V_f can describe both the action of the external field and the interaction with other centers, which can themselves bind the particle. Weakly bound states are states with energy

$E = \hbar^2 k^2/2m^* \ll \hbar^2/m^* r_c^2$. The interaction V_f in the region $r \lesssim r_c$ is assumed to be weak ($V_f \ll \hbar^2/m^* r_c^2$), varying considerably only over distances $L_f \gg r_c$ and admitting an exact solution of the Schroedinger equation with the Hamiltonian $\hat{H}_f$.

In this problem, the Schroedinger equation has an exact solution both at large ($r \gg r_c$) distances, where $U = 0$, and at small distances ($r \ll L_f$), where V_f can be neglected, so that the problem becomes spherically symmetrical and we can use the effective-radius approximation.[21] The energy spectrum is determined from the condition of joining the solutions in the region of overlapping

$$r_c \ll r \ll \min\{L_f, k^{-1}\}. \tag{A.2}$$

To solve the problem of joining, which involves the characteristic feature of the problem—the presence of a shallow level in the field U, we first introduce the complete system $G^f_{lm}(\mathbf{r},E)$ of solutions of the Schroedinger equation with the Hamiltonian $\hat{H}_f$ as follows. We require that, together with satisfying the boundary condition as $r \to \infty$ (damped or outgoing wave), these solutions in the limit $r \to 0$ contain singular terms of the form $r^{-l'-1}Y_{l'm'}(\mathbf{n})$ only with $l' = l$, $m' = m$. Also, in the region (A.2)

$$G^f_{lm} = (r^{-l-1}Y_{lm}(\mathbf{n}) + \ldots) + \sum_{l'm'} A^{l'm'}_{lm}(E, f)(r^{l'}Y_{l'm'}(\mathbf{n}) + \ldots). \tag{A.3}$$

The indicated boundary conditions, generally speaking, uniquely determine both G^f_{lm} and $A^{l'm'}_{lm}$.

For $r \gg r_c$, the wave function of the state sought is represented in the form $\Psi = \Sigma C_{l'm'} G^f_{l'm'}$; in addition, the asymptotic form (A.3) can be used in the region (A.2). At the same time, in this region, if we proceed from the side of small r, we have[21]

$$\Psi = \sum \tilde{C}_{l'm'} Y_{l'm'} [(r^{-l'-1} + \ldots) + B_{l'}(E)(r^{l'} + \ldots)],$$
$$[(2l-1)!!]\,[(2l+1)!!]\,B_l = k^{2l+1} \operatorname{ctg} \delta_l \approx -\frac{1}{a_l} + r_l \frac{m^* E}{\hbar^2}, \tag{A.4}$$

where δ_l, a_l, and r_l are the phase shift, scattering length, and effective radius for the potential U. If there is a shallow level with angular momentum l in $U(r)$, then in this partial wave the scattering is anomalously large, the phsaes $\delta_{l' \neq l}$ are small and in (A.4) the singular terms $\propto r^{-l'-1}$ with $l' \neq l$ can be dropped. Joining the wave function in the region (A.2), we find that $C_{l'm'} \neq 0$ only for $l' = l$ and in this case $C_{lm} = \tilde{C}_{lm}$ and

$$B_l C_{lm} - \sum_{m'} A^{lm'}_{lm} C_{lm'} = 0,$$
$$B_{l'} \tilde{C}_{l'm} = \sum_{m'} C_{lm'} A^{l'm'}_{lm}, \quad l' \neq l. \tag{A.5}$$

The condition for the existence of a nontrivial solution of this system is

$$\det\left\{\left[-\frac{1}{a_l} + r_l \frac{m^*}{\hbar^2} E\right]\delta_{mm'} - \tilde{A}^{lm'}_{lm}(E, f)\right\} = 0 \tag{A.6}$$

and represents the relation determining the energy spectrum.[10)11] (Here $\tilde{A}^{lm'}_{lm} = (2l-1)!!(2l+1)!!\, A^{lm'}_{lm}$).

[10)]The method is described in detail in Ref. 49.

For a uniform magnetic field, in view of the conservation of l_z, (A.6) decomposes into $(2l+1)$ independent relations. For $m = \pm l$ we have

$$G^f_{l,\, m=\mp l}(r, E) = \left[\left(\frac{\partial}{\partial x} + L^{-2}x\right) \mp i\left(\frac{\partial}{\partial y} + L^{-2}y\right)\right]^l G_0 \quad (\mathbf{r}, \mathbf{r}' = 0, \tilde{E}); \tag{A.7}$$

$\tilde{E} = E - \hbar\omega_H(|m| + m)$, and G_0 is the Green's function of a particle in a uniform magnetic field. By comparing (A.3) and (A.7) we find $A^{l'm'}_{lm}$, after which (A.6) gives the equation for determining the energies of MI states with $m = \pm l$:

$$-\frac{1}{a_l} + r_l \frac{m^*}{\hbar^2}(E - m\hbar\omega_H) = \frac{2^{2l+3/2}}{\sqrt{\pi}\, L^{2l+1}} \int_0^\infty \frac{dt}{\sqrt{t}} \frac{d^{l+1}}{dt^{l+1}} \left\{\left(\frac{t}{1-e^{-2t}}\right)^{l+1} \exp\left[-\left(1 + |m| + m - \frac{E}{\hbar\omega_H}\right)t\right]\right\}. \tag{A.8}$$

Setting $l = 0$ in (A.8), we arrive at an equation determining the behavior of the MI level $N = l = m = 0$

$$-\frac{1}{a_0} + r_0 \frac{m^*}{\hbar^2} E = \frac{1}{\sqrt{2}L} \zeta\left[\frac{1}{2}, \left(\frac{1}{2} - \frac{E}{\hbar\omega_H}\right)\right]; \tag{A.9}$$

where ζ is the Riemann zeta function. This equation differs from the equation in Ref. 14 by the presence of a term that takes into account the finiteness of the radius of the center. Using the approximate expression for the ζ function[14]

$$\zeta\left(\frac{1}{2}, \xi\right) = -2(\xi+1)^{1/2} + \xi^{-1/2} + 2^{-1}(\xi+1)^{-1/2} + \frac{1}{24}(\xi+1)^{-3/2}, \tag{A.10}$$

and introducing the variables $\gamma = \hbar\omega_H/2R$ and $\varepsilon_i = [(3/2)\hbar\omega_H - E]/R$, we obtain Eq. (7.2) from (A.9).

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