Bell's inequalities and experimental verification of quantum correlations at macroscopic distances

A. A. Grib

N. A. Voznesenskiĭ Finance and Economics Institute, Leningrad
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In recent years the fulfillment of the inequalities established by Bell has been tested in different laboratories in the world. These inequalities make it possible to establish which of the interpretations of quantum mechanics is correct—the interpretation due to Einstein, according to which the properties of a quantum system exist as elements of physical reality irrespective of their observation, or the Copenhagen interpretation of Bohr and Fock, according to which the properties of a microscopic system described by noncommuting operators do not exist independently of the means of measurement. The experiments can be divided into three classes: experiments with optical photons, experiments with γ rays, and experiments with nucleons. The experiments that have been performed convincingly indicate violation of Bell's inequalities. Thus, the Copenhagen interpretation of quantum mechanics and its associated principle of relativity with respect to the means of measuring the properties of a microscopic system give at the present the only description of quantum phenomena that does not contradict experiment.

In 1935, Einstein, Podolsky and Rosen[1] published a paper devoted to the foundations of quantum mechanics. In it, they formulated a thought experiment which, as they assumed, indicated an incompleteness of quantum theory. In their view, this experiment indicates the existence of a paradox, which has become known as the Einstein-Podolsky-Rosen paradox. Of course, now, after the passage of many years distinguished by successes of quantum mechanics in many fields—from the theory of superconductivity to the physics of elementary particles—doubts about the completeness of the quantum description seem hardly topical. However, the paradox formulated in 1935 led not only to discussions about the foundations of quantum theory but also to the carrying out of a number of important experiments to test quantum correlations over macroscopic distances. The experiments to be discussed in the present paper prove the impossibility of the existence of any reasonable hidden variables giving a complete description of an individual microscopic event and permitting the reduction of quantum mechanics to a classical model such as statistical mechanics, in which definite properties of objects exist before their measurement by instruments. Thus, we shall be considering experiments (known in the literature as Einstein-Podolsky-Rosen (EPR) experiments) in which one considers the concept of physical reality and the form of determinism for microscopic phenomena.

Before we turn to a description of the experiments, let us consider what are the problems that have been under discussion all these years in the dispute initiated by the supporters of the hypothesis of hidden variables.

The first problem is the absence in quantum theory of Laplace determinism. Thus, suppose we have five identical atomic nuclei. After a certain time, two of them have decayed, but three have not. The physics of the 19th century would say that there is a "reason" why the two given nuclei have decayed and not the others. From the point of view of quantum theory, there is no reason why precisely these nuclei decayed; they "simply" decayed, spontaneously. Quantum theory merely peredicts a probability for the decay of a nucleus, and this probability can be the same for each of the nuclei. People unsatisfied with this situation say that "in fact" there is some reason and that there is a "hidden variable," perhaps unknown at the present stage in the development of science but nevertheless existing in nature.

Another problem is associated with Bohr's complementarity, or, as Fock called it, the principle of relativity with respect to the means of observation.[2] This conception also has its origin in the theory of relativity. In view of the particular importance of this principle for all the following exposition, let us consider in more detail how we understand the concept of relativity with respect to the means of observation. According to the theory of relativity, the length of an object and the duration of a process described not so much the object or process itself but rather its relation to other objects that form the coordinate system. The length of a table is different for different observers, and it can be changed without applying any mechanical forces to the table simply by making one's frame of reference move sufficiently fast (which changes the relationship between the table and the coordinate system).

In quantum mechanics, an object has properties described by noncommuting operators A and B, so that $\hat{A}\hat{B} - \hat{B}\hat{A} \neq 0$. It is found that there does not exist a state in which the object can have simultaneously the properties A and B (if the operators do not commute, they do not have a common eigenfunction). This has the consequence that the properties A and B characterize the relationship of the object to different instruments, which play the part of frames of reference, so that if the operators $\hat{A}$ and $\hat{B}$ do not commute, one cannot speak of the existence of the corresponding properties independently of measurement. If after A one measures a quantity B that does not commute with it, and then A, the answer will, in general, be different. Since the first appearance of quantum theory people have frequently been

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tempted to think that this disturbance by the instrument can be "explained," by assuming that a very large macroscopic instrument "affects" the small microscopic particle so much that the instrument itself prevents the measurement of the complementary characteristics A and B. In fact, it was in this connection that Einstein, Podolsky, and Rosen suggested that one should measure the characteristics of quantum objects without subjecting them to any force, so that the appearance of a definite A (or B) in a measurement of a definite type will arise in a manner not involving forces. In the proposed method one first takes a system of two particles with a definite characteristic, for example, a total spin equal to zero. The particles then separate to a fairly large distance. After this, measuring the spin (the projection of the spin onto some axis) of one particle and knowing that the total spin of the system is equal to zero one can establish (and thus measure) the spin (the same projection) of the other. But the spin projections onto different axes are described by noncommuting operators; therefore, in such an experiment we can determine one or other relation to an instrument without using any force.

In their paper in 1935, Einstein, Podolsky, and Rosen put forward the following criterion for the reality of a physical quantity: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

The essence of the EPR paradox is that quantum mechanics contradicts this criterion. Of course, there is no paradox[3] if one accepts the principle of relativity with respect to the means of measurement, since physical reality then becomes relative and, independently of an instrument, one cannot speak of an element of physical reality (the Einsteinian element of reality becomes a relationship between the particle and the instrument). A new stimulus to the discussion and experimental verification of the EPR phenomenon was given by the proof of Bell's inequalities.[4] Here, we shall outline three ways in which these remarkable inequalities can be obtained.

First, however, let us say a few words about the fate of the theory of hidden variables (in this connection, see, for example, Ref. 5). These theories can be classified into theories of three types: zeroth, first, and second (this classification is similar but not completely identical to the one proposed in Ref. 6). In the theories of the zeroth type we have those whose inconsistency was proved already in the early stages in the development of quantum mechanics. First, there are theories that contradict von Neumann's theorem, which proves that hidden variables cannot exist if an assumption made in such theories is valid, namely, that the mathematical expectation of the sum of two quantities calculated using the rules of quantum mechanics is equal to the sum of the mathematical expectations of these quantities. Second, there are the so-called contextually independent theories of hidden variables. In these theories, it was assumed that the result of measurement of a property of a system can be predicted on the basis of knowledge of the values of the hidden variables of only the system itself. Gleason[7] and Kochen and Specker[8] showed that such theories are inconsistent and are of the zeroth type. Thus, the only possibilities are contextually dependent theories of hidden variables, in which the value of a particular characteristic of the system can be calculated on the basis of knowledge of the values of the hidden variables of not only the system itself but also the hidden variables of the instrument (so that for different devices there will be different hidden variables). Here, we already approach very close to relativity with respect to the means of observation.

Theories of the first type include theories compatible with the special theory of relativity (there do not exist signals that propagate with a velocity greater than the velocity of light); these are local theories. Theories of the second type are nonlocal and allow invalidity of the special theory of relativity. As will be seen from what follows, only such theories are today possible as an alternative to quantum theory. Usually certain fields are assumed as the hidden variables. The most exotic variant is the suggestion by Burgers[9] (see also Shimony[10]) to introduce psychological hidden parameters. Thus, one can predict the behavior of a man in a case when no exterior criteria permit it by asking him "what he wants to do." From this point of view, quantum particles make a choice that cannot be predicted by means of external observations. A living system, in contrast to one that is not living, is like a ferromagnet, in which the directions of the magnetic moments are spontaneously aligned. A nonliving system is a collection of particles that make random choices, while a living system spontaneously observes a '"tradition." Such a point of view has the consequence that quantum mechanics cannot apply to living organisms (in this connection, see also Wigner's comments[11,60]). We note that psychological hidden variables must also be connected nonlocally. Interest in this variant was to a large degree stimulated by the well-known paradox of Schrödinger's "alive and dead cat."[58,59,61]

After this brief excursion into the problem of hidden variables, we turn to the proof and discussion of Bell's inequalities, which show that for whatever choice one makes of the local hidden variables the corresponding hypotheses can be experimentally refuted. In different papers these inequalities are expressed in different forms, but they are all called Bell's inequalities after their original discoverer.

1. Suppose there is an object characterized by three quantities A, B, C which take the values ± 1.

In quantum theory, these quantities can correspond to noncommuting operators $\hat{A}$, $\hat{B}$, $\hat{C}$.

Suppose however that a particle simultaneously possesses A, B, C. Then (see D'Espagnat[12]), considering an ensemble of identical particles and denoting by A^+ the case when A takes the value $+1$ (and similarly for B and C) and A^- the case when A takes the value -1, we obtain

$$N(A^+B^-) = N(A^+B^-C^+) + N(A^+B^-C^-),$$

where N is the number of particles with the corresponding properties. From the equalities

$$N(B^-C^+) = N(A^+B^-C^+) + N(A^-B^-C^+),$$
$$N(A^+C^-) = N(A^+B^+C^-) + N(A^+B^-C^-)$$

it obviously follows that

$$N(A^+B^-) \leqslant N(B^-C^+) + N(A^+C^-). \tag{A}$$

This is one of Bell's inequalities.

Suppose the operators $\hat{A}$ and $\hat{B}$ do not commute. Then, if, following Einstein, Podolsky, and Rosen, we assume that the properties A and B exist as elements of physical reality, so that only the interference of the instrument prevents their simultaneous measurement, to determine their values it is necessary to consider, not a system of identical particles, but pairs of particles. Each pair has the property that if for one particle A takes the value $+1$, then for the other $A = -1$ (for example, spin of one particle $S_z = 1/2$, and of the other $S_z = -1/2$ if the two form a state with zero spin). Then, denoting the number of pairs by n, we rewrite the inequality (A) as

$$n(A^+B^+) \leqslant n(B^-C^-) + n(A^+C^+). \tag{B}$$

In this inequality, n is the number of pairs of particles in which one has A^+, the other has B^+, etc. Thus, checking these inequalities experimentally, we test whether the particles simultaneously "have" A, B, C (a similar point of view is expressed, for example, in Ref. 62) or, in accordance with the principle of complementarity, they arise in the measurement as corresponding relations between particle and instrument. If A, B, C exist simultaneously (although $\hat{A}$, $\hat{B}$, $\hat{C}$ do not commute), this means that quantum mechanics is incomplete and one therefore needs some theory of hidden variables to describe the subquantum level.

The inequality (B) was tested experimentally at Saclay in France in 1976,[13] the properties A,B,C being the projections of the spin of a proton onto three different directions. For a system of two protons in the singlet state the experiment showed that Bell's inequalities are violated and thus quantum mechanics is valid: The characteristics A,B,C associated with noncommuting operators do not exist simultaneously. The arrangement of the experiment is as follows. Protons from an accelerator are aimed onto a hydrogen target and scattering occurs, the result being pairs of protons in the singlet state (total spin of the pair equal to zero). The particles of the pair separate to macroscopic distances. Then each particle passes through a carbon "analyzer," which transmits only a proton with a definite polarization (projection of the spin onto some axis) corresponding to the polarization of the carbon atoms (like the optic axis in an optical analyzer) (Fig. 1). Rotating the analyzer II with respect to analyzer I, we pass from measurement of B to measurement of C.

The most remarkable thing in this (and in other EPR experiments) is that rotating one analyzer that transmits a proton "influences" in some unusual manner the probability for transmission of the proton by the other analyzer, despite the fact that no material carrier (particle of field) of this influence exists. A dependence of the probability on the angle can also appear in a classical theory (see Appendix 3) due to the common past of the particles and the existence of a conservation law; it is the result of different sampling of pairs of events that take place independently of the rotation. However, the quantum correlations cannot be explained only in such a way. The point is that in classical physics the properties of the particles exist independently before their measurement. Therefore, if the particles do fly apart (for example, in an explosion of some device), in detecting particles with definite momentum at different angles to the right of the device, we appear to "change" the number of particles with opposite momenta to the left, but this is simply a different selection. But in the quantum case it is impossible to speak of the existence of a definite projection of the spin before its measurement that could be selected (or detected) differently by the observer. Here and in what follows, we use a term frequently encountered in the literature on EPR experiments: "influence," meaning by this the circumstance that if there is an observer capable of detecting only the passages of protons through his analyzer (for example, I) he can, rotating his analyzer through different angles, change the frequency of transmission of the protons coupled with his protons through the second analyzer II (the rotation changes the "particle-instrument" relationship). Note that the total probability of transmission of proton II remains the same for different angles and is 1/2.

Indeed, a simple calculation in accordance with the rules of quantum mechanics (see Appendix 1) shows that if analyzer I has transmitted a proton then the probability that II does so too is $w_{\mathrm{I\,II}} = (1/2)\sin^2(\theta/2)$, where θ is the angle between the spin projections that I and II measure. This "influence" (or nonlocality, in Bell's terminology[17,50]) is manifested as a correlation between the events of transmission of particles at I and II. To find this correlation it is necessary to know the events at both analyzers, and therefore transmision of a signal from I to II by means of the above influence does not occur. Indeed, the second observer will find that his analyzer transmits not only protons coupled with those that pass through I but also others, which are coupled with protons that do not pass through I. Since observer II does not know which protons pass through I, and which do not, the rules of the same quantum mechanics show that he will not discover any "influence" exterted by I. The probability of proton transmission through II in the case when a proton has not passed through I is $(1/2)\cos^2(\theta/2)$, so that the total probability of transmission through II is $(1/2)\sin^2(\theta/$

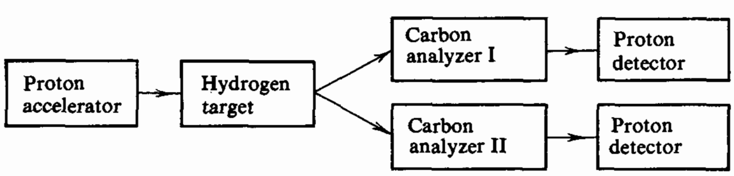

FIG. 1.

$2) + (1/2)\cos^2(\theta/2) = 1/2$. The same arguments hold for the "influence" of II on I. The situation here is again similar to the Lorentz contraction: Although different observers in different frames of reference contract the length of an object differently an observer in its rest frame does not note any contraction at all and thus cannot obtain information about the moving observer by virtue of the Lorentz contraction.

At the same time, just as the Lorentz contraction leads to definite physical consequences, the feature of quantum mechanics we are discussing (the relativity with respect to the means of measurement) leads to the appearance of very specific correlation functions, which violate Bell's inequalities.

Indeed, since the number of pairs in the Saclay experiment is obviously proportional to the corresponding probability, Bell's inequality (B) must mean

$$\frac{1}{2}\sin^2\frac{\theta_{AB}}{2} \leqslant \frac{1}{2}\sin^2\frac{\theta_{BC}}{2} + \frac{1}{2}\sin^2\frac{\theta_{AC}}{2},$$

which does not occur at all angles (for the expression of Bell's inequalities in this way and their violation for a system with half-integral spin, see also Ref. 11). We now turn to a different form of Bell's inequality.[14–16]

2. Suppose there are four quantities, each of them taking the values A, B, A', B' independently. Then it is readily seen that $AB + AB' + A'B - A'B' = \pm 2$.

For example, if each quantity is $+1$, this algebraic sum if 2; if $A = B = +1, A' = B' = -1$, then it is -2, etc.

Assuming that these quantities are random, we write down for the mean value

$$\left|\frac{1}{N}\sum_{n=1}^{N}(A_nB_n + A_nB'_n + A'_nB_n - A'_nB'_n)\right| \leqslant 2,$$

which leads to one further Bell's inequality for the mathematical expectations:

$$|P(A, B) + P(A, B') + P(A', B) - P(A', B')| \leqslant 2; \qquad \text{(C)}$$

where

$$P(A, B) = \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}A_nB_n.$$

As A, B, A', B' we can again take the spin projections onto different axes. Quantum mechanics and experiments, which confirm it, contradict the above inequalities in the case of two particles described by a common wave function. It is obvious that the presence of a correlation between A, B, A', B' due to the common past of the particles and some conservation law merely has the consequence that not all $+1$ pairs will be admissible but only a fraction, the final conclusion remaining the same (see the example in Appendix 3). What is wrong with our derivation of the inequalities? We have implicitly assumed the existence of an ensemble characterized by A, B, A', B' in accordance with the notion of hidden variables. Then the violation of the inequalities shows that we were wrong to assume that A, even when it is measured at a point at which no signal can have arrived from the point at which B is measured, does not depend on B, and this means nonlocality (contradiction with the special theory of relativity). For agreement with quantum mechanics, it is necessary to assume that A is a function of B and vice versa; therefore, if $A = 1$ when $B = 1$, then if B is rotated and made into B', then A, in general, cannot remain equal to $+1$. Therefore, if $AB = 1$, then AB' can become -1 when $B' = 1$, and instead of 2 we can obtain 0, 4, etc.

3. Finally, we give a proof of a Bell's inequality that directly uses the concept of local hidden variables and shows that quantum theory cannot be regarded as a theory of a relativistic probabilistic process.[17,18] Suppose that at some point I we measure the quantity A_a, and at II, which is separated by a spacelike interval from I, we measure some B_b. Both quantities can take the values ± 1, and the indices a and b indicate a dependence of these quantities on the direction. For example, if we measure the spin projection onto some axis, then a (or b) is the azimuthal angle. Suppose further that a definite result (± 1) of the measurement of A depends not only on the direction a but also on some hidden variable λ, and that the result of the measurement of B depends accordingly on the direction b and the same λ, localized in Ω, the intersection of the light cones of the past of the points I and II. Locality of the hidden variables means that A does not depend on b and B does not depend on a. Therefore, all correlations between A and B must be due solely to the common past, in which the λ values are given.

We write the mathematical expectation of the product of A_a and B_b as

$$P(A_aB_b) = \int d\lambda\rho(\lambda)\,\bar{A}(a, \lambda)\,\bar{B}(b, \lambda),$$

where $\rho(\lambda)$ is the probability distribution of the variables in Ω; $\bar{A}(a, \lambda)$ and $\bar{B}(b, \lambda)$ are the values of A_a and B_b averaged over the possible values of the hidden variables of the instruments (we consider contextually dependent theories of hidden variables!), so that $|\bar{A}| \leqslant 1$, $|\bar{B}| \leqslant 1$. Let a' and b' be positions of the instruments that measure A and B that are alternatives to a and b. Then, following Bell,[19]

$$\begin{aligned} P(A_aB_b) - P(A_aB_{b'}) &= \int d\lambda\rho(\lambda)\,[\bar{A}(a, \lambda)\,\bar{B}(b, \lambda) - \bar{A}(a, \lambda)\,\bar{B}(b', \lambda)] \\ &= \int d\lambda\rho(\lambda)\,\{\bar{A}(a, \lambda)\,\bar{B}(b, \lambda)\,[1 \pm \bar{A}(a', \lambda)\,\bar{B}(b', \lambda)]\} \\ &\quad - \int d\lambda\rho(\lambda)\,\{\bar{A}(a, \lambda)\,\bar{B}(b', \lambda)\,[1 \pm \bar{A}(a', \lambda)\,\bar{B}(b, \lambda)]\}. \end{aligned}$$

It follows from $|\bar{A}| \leqslant 1$, $|\bar{B}| \leqslant 1$ that

$$\begin{aligned} |P(A_aB_b) - P(A_aB_{b'})| &\leqslant \int d\lambda\rho(\lambda)\,[1 \pm \bar{A}(a', \lambda)\,\bar{B}(b', \lambda)] \\ &\quad + \int d\lambda\rho(\lambda)\,[1 \pm \bar{A}(a', \lambda)\,\bar{B}(b, \lambda)], \end{aligned}$$

or

$$|P(A_aB_b) - P(A_aB_{b'})| \leqslant 2 + P(A_{a'}B_{b'}) + P(A_{a'}B_b),$$

$$|P(A_aB_b) - P(A_aB_{b'})| + |P(A_{a'}B_{b'}) + P(A_{a'}B_b)| \leqslant 2. \qquad \text{(D)}$$

It is easy to understand why and when quantum mechanics contradicts this inequality. The expectation value of a physical quantity for a known wave function ψ in quantum mechanics is calculated as

$$\langle\psi|\hat{A}\hat{B}|\psi\rangle = \int \psi^*(\mathbf{r}_{\mathrm{I}}, \mathbf{r}_{\mathrm{II}})\,\hat{A}_{\mathrm{I}}\hat{B}_{\mathrm{II}}\psi(\mathbf{r}_{\mathrm{I}}, \mathbf{r}_{\mathrm{II}})\,d\mathbf{r}_{\mathrm{I}}\,d\mathbf{r}_{\mathrm{II}},$$

but the wave function of a system of particles in the general case is not the product of the single-particle wave functions but some superposition of them, for example,

$$\psi(\mathbf{r}_{\mathrm{I}}, \mathbf{r}_{\mathrm{II}}) = \sum_i C_i \psi_{i\mathrm{I}}(\mathbf{r}_{\mathrm{I}}) \psi_{i\mathrm{II}}(\mathbf{r}_{\mathrm{II}}),$$

where C_i are certain coefficients. For example, the ordinary symmetrization of the product already leads to superposition.

In quantum theory there is a new connection between the whole and the part, which de Broglie calls complementarity between the whole and the part. If we have complete information about the parts of the system, this does not yet mean that we know the whole. Therefore, if the system is described by a wave function (is in a pure state), then a subsystem is characterized by a density matrix (mixed state). Because of this, the expectation value of the product AB calculated using the wave function (using knowledge of the whole) is not equal to the expectation value calculated by means of the density matrices for I and II. In the latter case, information about the correlation contained in the whole is lost. And it is only in the special case when the wave function is the product of single-particle functions that we obtain agreement with Bell's inequality.

We now turn to a description of the optical EPR experiments. They are arranged as follows (Fig. 2). There is a source of low-energy photons (atoms of calcium,[20,24] mercury-198,[22] mercury-202,[21] mercury-200[23]), which are produced in a cascade transition in the atom. In one transition $J=0 \to J=1 \to J=0$ a photon pair is produced in a state with wave function described by a superposition of states with total angular momentum $J=1$. The photons are emitted in opposite directions and pass through the analyzers 1 and 2. If a photon is polarized along an analyzer axis, the analyzer transmits it; if the polarization is perpendicular to the axis, it does not. The observable A_a has the value $+1$ if the photon passes through analyzer A_1, and -1 if it does not; B_b is associated similarly with the analyzer A_2. Then the quantum mechanical calculation gives (see Appendix 2 and Refs. 25 and 26) for the expectation value

$$P(A_a B_b) = \langle \psi | \hat{A}_a \hat{B}_b | \psi \rangle = \cos 2\alpha,$$

where $\alpha = a - b$ is the angle between the optical analyzers A_1 and A_2. After they have passed through the analyzers, the photons are caught by photon detectors. Experimentally, one measures the coincidence rate when both photons pass through the analyzers, both do not, one does and the other does not. Bell's inequality (D) can be written in the form

$$|P(A_a B_b) - P(A_a B_c)| + |P(A_d B_b) + P(A_d B_c)| \leqslant 2. \quad \text{(E)}$$

Choosing the angles $2a = 0°$, $2b = 135°$, $2c = 45°$, $2d = 90°$, for $\cos 2\alpha$ we obtain left-hand side equal to $2\sqrt{2}$, which obviously contradicts Bell's inequalities. So far, six optical experiments have been performed (University of California, 1972 (Ref. 20); University of Texas, 1976 (Ref. 23); Harvard University, 1973 (Ref. 22); Institute of Theoretical and Applied Optics, Orsay, 1981 and 1982 (Refs. 24 and 66)). Of these, five confirmed quantum mechancis, and one confirmed Bell's inequalities (Harvard). We note however that Bell's inequalities can be satisifed in some cases (when there is a product of single-particle wave functions) in quantum mechanics too (in this connection, we mention the attempt to explain the results of two experiments with optical photons and γ rays compatible with Bell's inequalities on the basis of quantum mechanics in Refs. 27 and 28), but their violation always rules out the possibility of local hidden variables (a classical picture).

Besides the optical experiments, experiments with γ rays have also been made. These experiments have much similarity to Wu and Shaknov's experiment[29] to determine the parity of positronium. A pair of photons is produced by the annihilation of an electron and positron in a state with zero total angular momentum when a positronium atom decays into γ rays. The γ rays then pass through analyzers (realized by means of Compton scattering) and are detected by counters. The basic arrangement is the same as in the case of the optical photons described above. In the optical experiments, the analyzers are better but the photon counters worse; in the experiments with the γ rays, the counters are better but the analyzers are worse. Six experiments have been performed with γ rays (University of Catania, 1974 (Ref. 30); University of Columbia, 1975 (Ref. 31); University of London, 1976 (Ref. 32); Institute of Physics at the University of Bologna, 1977 (Ref. 33), 1981 (Ref. 63); Freiburg University, 1979 (Ref. 34)). The first experiment did not contradict Bell's inequalities, but in all the others they were violated. The distance between the photons in the second experiment was 25 cm. Thus, correlations are indeed observed over macroscopic distances. At one time, Schrödinger[35] conjectured that the quantum mechanical correlations should disappear when the distance between the subsystems of a system desribed by a wave function becomes unquestionably macroscopic (see also Refs. 36 and 37). The experiment shows that this hypothesis must be rejected. Thus (together with the recent Frech experiment at Orsay), 11 experiments clearly contradict Bell's inequalities, and two are compatible with them.

However, from these two experiments it cannot be concluded that in some cases a theory of hidden variables is possible. There are serious grounds for believing[38] that the reason for the discrepancy was a systematic error in the experiments. The point is that quantum mechanics predicts the presence of a strong correlation that may escape notice because of systematic error. But the detection of a strong correlation agreeing with high accuracy with quantum mechanics

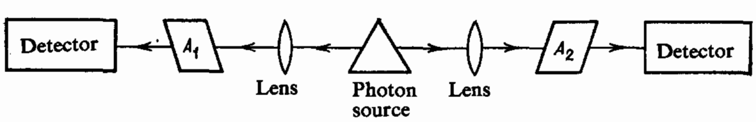

FIG. 2.

due to a systematic error is improbable.

At the present time, a much cleaner experiment has been designed[39,66] making it possible to confirm the correlations over macroscopic spacelike distances.

In this experiment, one analyzer is effectively rotated relative to the other. In contrast to other experiments in which the analyzers were rigidly fixed, in Ref. 66 the one analyzer is rotated relative to the other so rapdily that no signal could "warn" the second analyzer that the first has been rotated in the case of passage of photons through both analyzers. Therefore, this precludes any imitation of generation of correlations by a signal connecting the two analyzers.[4] The need to make such an experiment follows from Bell's remark[4] that in the case of rigidly fixed analyzers there can be a connection between them due to the exchange of signals at velocity less than or equal to the velocity of light, this resulting in a violation of Bell's inequalities that can be eliminated only if the analyzers are rotated very rapidly during the flight of the particles. In the experiment of Ref. 66, the effect of a rotation is achieved by deflection of a photon by optoacoustic interaction of the photon with a standing ultrasonic wave in water. For this (Fig. 3) the photons, before arriving at the analyzers, pass through a deflecting device C_1 on the left and C_2 on the right, where they interact with an ultrasonic wave. The light passes through C_1 and C_2 without refraction if the amplitude of the standing ultrasonic wave is zero, and it is completely refracted through angle $2\theta_B$ where θ_B is the Bragg angle, if the amplitude is maximal. If the light is not refracted, it arrives at the analyzers A_1 and A_2; if it is, it arrives at the analyzers B_1 and B_2. Choosing the frequency of the ultrasonic wave appropriately, one can ensure that the time of flight of the photons over the distance from the source to the deflecting device, $t = L/c$ (in Ref. 66, the time t is of the order of 40 nsec) is greater than the "deflection" time (in Ref. 66, of order 10 nsec). Thus, the events of the transmission of the photons through the left- and right-hand analyzers are separated by a space-like interval. The devices C_1 and C_2 to the left and right are connected to ultrasound generators working at different frequencies, so that it can be assumed that they are uncorrelated. The results of the experiment of Ref. 66 contradict Bell's inequalities and confirm quantum mechanics. The experiment of Ref. 66 is of significant interest as a demonstration of the fact that the wave packet reduction postulate introduced by von Neumann[42] in nonrelativistic quantum mechanics is also valid in the relativistic region (Fig. 3). What does the experimentalist directly measure in photon experiments that test Bell's inequalities?

If φ is the angle between the optic axis of the two polarizers, $R(\varphi)$ is the rate of coincidences in the readings of the photon counters in the presence of both analyzers, R_1 is the counting rate when the second polarizer is eliminated, and R_2 when the first is, but the second remains, and R is the rate when both are eliminated, then experimentally one measures

$$S_{\exp} = \frac{4\left[R(\beta)+R(\gamma)+R(\alpha+\beta)-R(\alpha+\gamma)-R_1-R_2\right]}{R}.$$

For the connection between $R(\varphi)$ and $P(A_a B_b)$, introduced above with allowance for the nonideality of the analyzers, see Appendix 2. If Bell's inequalities hold (see Refs. 25 and 26), then one must have $-1 \leqslant S_{\exp} \leqslant 0$; experimentally, for example, in the experiment of Ref. 20, $S_{\exp} = 0.05 \pm 0.008$; in the experiment of Ref. 24, $S_{\exp} = 0.126 \pm 0.014$; in the experiment of Ref. 66, $S_{\exp} = 0.101 \pm 0.020$. In the γ-ray experiments, one measures (see Ref. 31)

$$R(a, b) = \frac{\{N/N_{SS}\}}{\{n_1/N_{SS}\}\{n_2/N_{SS}\}},$$

where N_{SS} is the number of cases when the photons undergo Compton scattering, N is the number of cases when both photons undergo Compton scattering and both photons are detected. n_1 is the number of times when both photons undergo Compton scattering and only photon 1 is detected, n_2 is the same number for photon 2, and a and b are the azimuthal angles characterizing the Compton analyzers.

Quantum mechanics gives

$$R(a, b) = 1 - M_1 M_2 P(A_a B_b) = 1 - M_1 M_2 \cos 2(a - b),$$

where M_1, M_2 are certain instrumental factors of the Compton analyzers. In any local theory of hidden variables, $M_1 M_2$ must be replaced by some coefficient $B \leqslant M_1 M_2/\sqrt{2}$,[31] since for consistency with Bell's inequalities $\cos 2(a-b)$(must be replaced by $c \cos 2(a-b)$, where $c \leqslant +/\sqrt{2}$; in the Bohm-Aharonov[37] theory $B \leqslant M_1 M_2/2$, $c = \frac{1}{2}$. Finally, in connection with the various EPR experiments, we mention the inverse EPR experiment analyzed by Costa de Beauregard[40] in connection with the question of time reversal in quantum mechanics and macroscopic irreversibility (in this connection, see also Ref. 41).

The scheme of the inverse EPR experiment is as follows. Two lasers send beams toward each other; the photons pass through analyzers A_1 and A_2 and excite atoms due to an

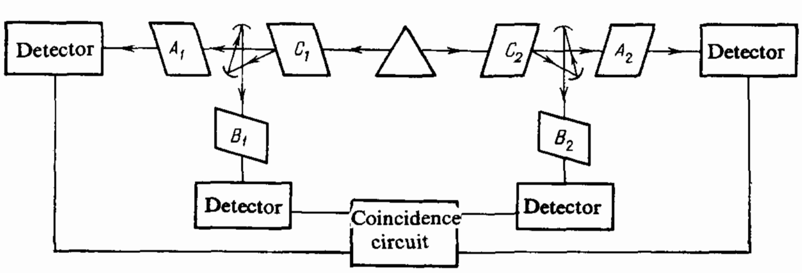

FIG. 3.

inverse cascade transition with absorption of both photons. The number of excited atoms is then measured by counters. In a direct EPR experiment the photons have a "common past"; in the inverse experiment, a "common future" (Fig. 4).

As in the direct EPR experiment, the probability of photon transmission through both analyzers with subsequent two-cascade excitation of an atom depends sinusoidally on the difference between the angles between the optic axes of the analyzers. Technically, the inverse EPR experiments are simpler, and the experiments that have been done do indeed demonstrate the existence of sinusoids. But what does this sinusoid mean from the point of view of theory? The probability of excitation of an atom is described by the same expression as the probability of emission; this last means that the initial wave function of the photons absorbed by the atom is not the product of single-particle wave functions of photons emitted independently by lasers but some symmetrized function. Hence, a wave function representing the product of single-particle photon functions has been transformed at some time—reduced in accordance with von Neumann,[49]—to a symmetrized function because (or in order that?) at a future time the atom should be excited. Since the situation in this experiment is the time reverse of the direct EPR experiment, one can say in accordance with the theory of the direct experiment that the wave function for the chosen cases of excitation of the atoms representing the product of the single-particle functions of the photons emitted by the lasers is reduced to a symmetrized function on the passage through the analyzers, so that the expectation value for the product of the spins is the same cosine of the angle as in the direct experiment. However, can one then transmit any information from the future (create an "antitelegraph")? The answer is no, in the sense that one cannot control the transmission of such information.

Suppose that in some way we can recognize the transmission of the photons through the analyzers (for example, using Compton scattering in the case of γ rays). Then the inverse EPR experiment indicates the presence of a correlation between the events of transmission of photons through the analyzers for a definite angle between their optic axes and excitation of an atom at a subsequent time. As in the case of the direct EPR experiment, to establish the fact of correlation it is necessary to know both events. Now suppose we do not know the future and follow only how the photons are transmitted by the analyzers, the atoms being excited in some cases and in the others not. Then, generally speaking, we shall not see sinusoids. And it is only in the case of an improbable piece of 'luck" that we can "predict" the excitation of an atom. The fundamental absence of Laplace determinism in quantum mechanics mentioned at the beginning of the paper has the consequence that the fact of excitation of an atom is entirely random and beyond the control of man, and therefore, if atoms are excited successively several times, then later they also cease to be excited randomly, and a "prediction" obtained once is completely unreproducible. The inverse EPR experiments also confirm quantum mechanics (see Refs. 64 and 65).

However, the example we have given is interesting as an indication of the possibility of extending the principle of relativity with respect to the means of measurement for quantum systems not only in the present but also in the past.[43,44,45]

In conclusion, we say a few words about the significance of Bell's inequalities and their verification in EPR experiments.

First, the establishment of these inequalities and the experimental verification of their violation makes it possible to rule out entirely a local theory of hidden variables, and thus we obtain an experimental confirmation of the correctness of the Copenhagen interpretation of quantum mechanics. The simplest variant of a nonlocal theory of hidden variables[46] also currently contradicts experiment.[47] In principle, a nonlocal theory of hidden variables is not ruled out (see, for example, Ref. 48). However, quantum mechanics is much simpler than such a nonlocal theory, and therefore a nonlocal theory of hidden variables can hardly be regarded as a serious alternative to quantum mechanics.

Second, the general nature of Bell's inequalities makes it possible to perform experiments to test them in other fields of science where we encounter random processes not directly related to quantum phenomena (for example, in biology). Then the experimental detection of violation of these inequalities would make it possible to establish a correlation between phenomena assumed entirely unrelated to one another and not due to a common origin.

Third, it is of interest to investigate correlations over macroscopic distances, not for two-particle states as in the discussed EPR experiments, but for more complicated many-particle states (see Ref. 49).

In connection with the important part played by particle production in cosmology, it has been suggested that correlations of EPR type may be responsible for the observed isotropy of the background radiation received from regions that are causally unconnected. In fact, from the vacuum, which plays the part of the common wave function of the Universe in the early stage of evolution, pairs of particles could be produced at spacelike distances,[51] so that the observation of one particle (with definite charge) necessarily entails the appearance at another point of a particle with the opposite charge (a similar situation for decays of K mesons is analyzed in Refs. 52 and 53). Finally, the quantum correlations make it possible to give one further proof of the quantum nature of light.[54,55] In the literature on the interaction of light with matter, one frequently encounters the opinion that

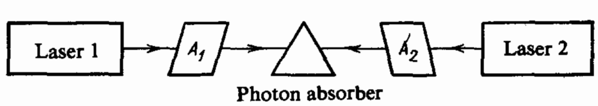

FIG. 4.

a classical theory can be used to describe many phenomena involving light. Thus, in a semiclassical theory of radiation light energy is emitted and absorbed in quanta not because the electromagnetic field is quantized but because there are discrete energy levels of atoms. Therefore, one can speak of a portion of electromagnetic energy described classically and characterized by a definite polarization. But then in the optical EPR experiments using cascade transitions it follows from quantum theory that the ratio of the coincidence rate when both analyzers are present, $R(\varphi)$, to the analogous quantity R_0 (Ref. 55) in the absence of analyzers in $R(\varphi)/R_0=(\frac{1}{2})\cos^2\varphi$, i.e., for $\varphi=\pi/2$ (optic axes perpendicular) this quantity is zero.

In the classical theory, zero is obtained only in the exceptional case when the axes of the analyzers exactly agree with the direction of the polarization vector. If there is any rotation of the two analyzers that maintains the perpendicularity of the axes, a different result is obtained (both analyzers begin to transmit photons). But in quantum theory a null result is obtained in all cases. The experimental proof of this fact prompted Jaynes[56] who is actually a well-known supporter of the neoclassical theory of radiation, to write: "I wish John von Neumann were here to see it."

APPENDICES

1. Spin $\frac{1}{2}$

Suppose there is singlet state of two particles with spin $\frac{1}{2}$:

$$\psi=\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right]. \tag{1}$$

We denote the plane formed by the optic axes of the polarizers by z, x. If the axis of one of the polarizers is taken as z, then to the axis of the other, rotated through the angle θ about the y axis, there corresponds the operator

$$\sigma_z'=\sigma_z\cos\theta+\sigma_x\sin\theta, \tag{2}$$

where σ_z and σ_x are Pauli matrices, and

$$\sigma_z'=\begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{pmatrix}. \tag{3}$$

The eigenvectors of σ_z' are

$$\psi_+=\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{pmatrix}=\cos\frac{\theta}{2}\begin{pmatrix}1\\0\end{pmatrix}+\sin\frac{\theta}{2}\begin{pmatrix}0\\1\end{pmatrix} \tag{4}$$

$$\psi_-=\begin{pmatrix}-\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\end{pmatrix}=-\sin\frac{\theta}{2}\begin{pmatrix}1\\0\end{pmatrix}+\cos\frac{\theta}{2}\begin{pmatrix}0\\1\end{pmatrix}. \tag{5}$$

Conversely,

$$\begin{pmatrix}1\\0\end{pmatrix}=\cos\frac{\theta}{2}\psi_+-\sin\frac{\theta}{2}\psi_-, \tag{6}$$

$$\begin{pmatrix}0\\1\end{pmatrix}=\sin\frac{\theta}{2}\psi_++\cos\frac{\theta}{2}\psi_-. \tag{7}$$

Then it is obvious that if the probability of transmission of a particle by analyzer 1 (the state ψ_+) is $\frac{1}{2}$, then the probability of both analyzers transmitting particles is $(\frac{1}{2})\sin^2(\theta/2)$ (since the state ψ_- for the second particle is reduced to $\begin{pmatrix}1\\0\end{pmatrix}$ with probability $\sin^2\left(\frac{\theta}{2}\right)$). The probability is the same that neither analyzer transmits a particle. The probability of the first analyzer not transmitting a particle (the state ψ_-) and the second transmitting one is $(1/2)\cos^2(\theta/2)$. The total probability that the second analyzer transmits a particle is obviously

$$\frac{1}{2}\sin^2\frac{\theta}{2}+\frac{1}{2}\cos^2\frac{\theta}{2}=\frac{1}{2}.$$

The correlation function is

$$\langle\psi|\sigma_z^{(1)}\cdot\sigma_z'^{(2)}|\psi\rangle=-\cos\theta. \tag{8}$$

2. Spin 1

Suppose the two-photon spin function of photons emitted in an atomic cascade is[57]

$$\psi=\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\right] \tag{9}$$

The analyzer 1 measures the projection of the photon spin onto the optic axis of the analyzer in the x, y plane at angle a to the x axis; a similar angle b is associated with analyzer 2. Then (see, for example, Ref. 26)

$$A_a=\begin{pmatrix}\cos 2a & \sin 2a & 0\\ \sin 2a & -\cos 2a & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{10}$$

The eigenvectors of A_a are

$$\chi_+=\cos a\chi_x+\sin a\chi_y,\quad \chi_-=-\sin a\chi_x+\cos a\chi_y, \tag{11}$$

$$\chi_x=\begin{pmatrix}1\\0\\0\end{pmatrix},\quad \chi_y=\begin{pmatrix}0\\1\\0\end{pmatrix}. \tag{12}$$

The correlation function is

$$P(A_aB_b)=\langle\psi\mid A_aB_b\mid\psi\rangle=\cos 2(a-b). \tag{13}$$

If we bear in mind that the real photons are not emitted strictly along the axes $\mathbf{n}_1=z$ and $\mathbf{n}_2=-z$ but in cones $\mathbf{n}_1\in\Omega_1$, $\mathbf{n}_2\in\Omega_2$, then we must introduce a characteristic such as the angle θ, which is equal to the half-sum of the angles of the cones Ω_1 and Ω_2. Finally, we take into account the imperfection of the polarizers and introduce (Refs. 25 and 26) ε_M^i ($i=$ I, II), the probability of a photon polarized along the polarization axis of the polarizer i passing through it, and ε_m^i, the probability of a photon polarized perpendicular to this axis passing through the polarizer. The effect of the lenses that direct the photon onto the polarizer has the consequence that the idealized photon wave function used above is transformed into $\psi'=D_{+z}D_{-z}\psi$, where

$$D_{\pm z}=\begin{pmatrix}\pm\cos^2\varphi\cos\theta+\sin^2\varphi, & \pm\sin\varphi\cos\varphi\cos\theta-\cos\varphi\sin\varphi, & \mp\cos\varphi\sin\theta\\ \pm\cos\varphi\sin\varphi\cos\theta-\sin\varphi\cos\varphi, & \pm\sin^2\varphi\cos\theta+\cos^2\varphi, & \mp\sin\varphi\sin\theta\\ \pm\cos\varphi\sin\theta, & \pm\sin\varphi\sin\theta, & \pm\cos\theta\end{pmatrix}. \tag{14}$$

The operators $D_{\pm z}$ rotate the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, respectively, transforming them into z and $-z$. With allowance for these corrections, we obtain (see Refs. 25 and 26) instead of the idealized case

$$\hat{A}_a=\begin{pmatrix} 2\varepsilon_{\mathrm{M}}^{\mathrm{I}}\cos^2 a+2\varepsilon_{\mathrm{m}}^{\mathrm{I}}\sin^2 a-1, & 2(\varepsilon_{\mathrm{M}}^{\mathrm{I}}-\varepsilon_{\mathrm{m}}^{\mathrm{I}})\cos a\sin a, & 0\\ 2(\varepsilon_{\mathrm{M}}^{\mathrm{I}}-\varepsilon_{\mathrm{m}}^{\mathrm{I}})\cos a\sin a, & 2\varepsilon_{\mathrm{M}}^{\mathrm{I}}\sin^2 a+2\varepsilon_{\mathrm{m}}^{\mathrm{I}}\cos^2 a-1, & 0\\ 0, & 0, & 1\end{pmatrix}. \tag{15}$$

Then the calculation of Ref. 26 gives

$$\begin{aligned} P(A_aB_b)=P(\alpha)&=\langle\psi'\mid\hat{A}_a\hat{B}_b\mid\psi'\rangle\\ &=[1-(\varepsilon_M^{\mathrm{I}}+\varepsilon_m^{\mathrm{I}})]\,[1-(\varepsilon_M^{\mathrm{II}}+\varepsilon_m^{\mathrm{II}})]\\ &+(\varepsilon_{\mathrm{M}}^{\mathrm{I}}-\varepsilon_{\mathrm{m}}^{\mathrm{I}})(\varepsilon_{\mathrm{M}}^{\mathrm{II}}-\varepsilon_{\mathrm{m}}^{\mathrm{II}})F_1(\theta)\cos 2(a-b), \end{aligned} \tag{16}$$

$F_1(\theta)\cos 2\alpha,\ \alpha=a-b,$

$$\alpha=a-b \quad F_1(\theta)=\frac{(7-3\cos\theta-3\cos^2\theta-\cos^3\theta)^2}{12(8-16\cos\theta+9\cos^2\theta-2\cos^4\theta+\cos^6\theta)}. \tag{17}$$

If a theory of hidden variables were true, then Bell's inequalities would require

$$S=P(\beta)+P(\gamma)-P(\alpha+\beta)-P(\alpha+\gamma)-2\leqslant 0. \tag{18}$$

For $\varepsilon_{\mathrm{m}}^{\mathrm{I}}=\varepsilon_{\mathrm{m}}^{\mathrm{II}}=0$, $\varepsilon_{\mathrm{M}}^{\mathrm{I}}=\varepsilon_{\mathrm{M}}^{\mathrm{II}}=0.9$, $\theta=30°$, quantum mechanics already gives $S=0.306$ for $\alpha=45°$, $\beta=-22\frac{1}{2}°$, $\gamma=22\frac{1}{2}°$.

The quantity $P(A_aB_b)=P(\alpha)$ is related to the rate of coincidence of photon pairs by

$$P(\alpha)=\frac{R_{pp}(\alpha)+R_{bb}(\alpha)-R_{pb}(\alpha)-R_{bp}(\alpha)}{R_{pp}(\alpha)+R_{bb}(\alpha)+R_{pb}(\alpha)+R_{bp}(\alpha)}, \tag{19}$$

where $R_{pp}(\alpha)$ is the coincidence rate when both photons pass through the polarizers, and $R_{bb}(\alpha)$ is the same quantity when both do not; $R_{pb}(\alpha)$ and $R_{bp}(\alpha)$ are the corresponding quantities when one of the photons does not pass through a polarizer. The quantities $R(\alpha)$, R_1, R_2 introduced in the main text are related to R_{pp}, R_{bb}, R_{pb}, R_{bp} by

$$\left.\begin{aligned} &R_{pp}(\alpha)=R(\alpha),\quad R_{pb}(\alpha)=R_1-R(\alpha),\quad R_{bp}(\alpha)=R_2-R(\alpha),\\ &R_{bb}(\alpha)=R-R_{pp}(\alpha)-R_{pb}(\alpha)-R_{bp}(\alpha)=R+R_1(\alpha)-R_1-R_2,\\ &\qquad R=R_{pp}(\alpha)+R_{bb}(\alpha)+R_{bp}(\alpha)+R_{pb}(\alpha). \end{aligned}\right\} \tag{20}$$

3. Example of a classical and a quantum case of the occurrence of a correlation for two particles[15]

Suppose there is some body initially at rest that breaks up into two parts with angular momenta $\mathbf{J}_1$ and $\mathbf{J}_2=-\mathbf{J}_1$.

Two observers measure the projections of $\mathbf{J}_1$ and $\mathbf{J}_2$ onto certain directions fixed by unit vectors α and β, and determine

$$r_\alpha=\operatorname{sign}(\boldsymbol{\alpha}\mathbf{J}_1),\quad r_\beta=\operatorname{sign}(\boldsymbol{\beta}\mathbf{J}_2).$$

Suppose the experiment is repeated N times and the directions $\mathbf{J}_1$ and $\mathbf{J}_2$ are distributed randomly. Then the mean values

$$\langle r_\alpha\rangle=\frac{1}{N}\sum_j r_{j\alpha},\quad \langle r_\beta\rangle=\frac{1}{N}\sum_j r_{j\beta}$$

are zero but $\langle r_\alpha r_\beta\rangle=(1/N)\Sigma_j r_{j\alpha}\,r_{j\beta}$ must be nonzero. If $\boldsymbol{\alpha}=\boldsymbol{\beta}$, then $r_{j\alpha}=-r_{j\beta}$ and $\langle r_\alpha r_\beta\rangle=-1$ always.

It is obvious that conservation of the total angular momentum leads to a correlation between the events observed by the first and second observers. Rotating the axis $\boldsymbol{\alpha}$ and following, for example, only the cases of motion of the fragments along $\boldsymbol{\alpha}$, we could thus make different samplings of cases of motion of the fragments in opposite directions. This would change r_β, but it is obvious that this change is not any kind of "influence" but merely a consequence of different sampling of events.

If we consider the unit sphere divided by the equatorial plane perpendicular to $\boldsymbol{\alpha}$, so that in the upper half $r_\alpha=1$ and in the lower $r_\alpha=-1$, and we then do the same for r_β, we obtain four regions in which $r_\alpha r_\beta=\pm 1$. Their areas stand in the ratio $\frac{\theta}{\pi-\theta}$, where θ is the angle between $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. Then

$$\langle r_\alpha r_\beta\rangle=\frac{\theta-(\pi-\theta)}{\pi}=-1+\frac{2\theta}{\pi}.$$

Quantum mechanics for the analogous case of the singlet state of two particles with spin $\frac{1}{2}$, for which one measures $r_\alpha=2\boldsymbol{\alpha}\mathbf{s}_1$, $r_\beta=2\boldsymbol{\beta}\mathbf{s}_2$, gives $\langle r_\alpha r_\beta\rangle=\langle\psi|2\boldsymbol{\alpha}\mathbf{s}_1 2\boldsymbol{\beta}\mathbf{s}_2|\psi\rangle=-\cos\theta$. The classical mean value $\langle r_\alpha r_\beta\rangle$ obviously satisfies Bell's inequality $|\langle r_\alpha r_\beta\rangle+\langle r_{\alpha'}, r_\beta\rangle+\langle r_\alpha r_{\beta'}\rangle-\langle r_{\alpha'} r_{\beta'}\rangle|\leqslant 2$. For example, if $\boldsymbol{\alpha}=\boldsymbol{\beta}$, and $\boldsymbol{\alpha}'$ and $\boldsymbol{\beta}'$ are such that $\boldsymbol{\alpha}\cdot\boldsymbol{\beta}'=\boldsymbol{\alpha}'\cdot\boldsymbol{\beta}=\cos\theta$, $\boldsymbol{\alpha}'\cdot\boldsymbol{\beta}'=\cos 2\theta$, then

$$-1-2\left(1-\frac{2\theta}{\pi}\right)+\left(1-\frac{4\theta}{\pi}\right)=-2.$$

But in the quantum case

$$|1+2\cos\theta-\cos 2\theta|=|2+2\cos\theta(1-\cos\theta)|,$$

which is greater than 2 for any $\theta<90°$.

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