The problem of scalar mesons

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A review is given of the dynamic structure of scalar resonances that exhibit a series of features that are unusual for two-quark systems. Recent experimental data on these resonances are analyzed by a very general method that is the most suitable for verifying different assumptions about the dynamic structure of the resonances. It is shown that the four-quark model is the most suitable for describing the known properties of scalar mesons. Attempts to identify any of the known scalar mesons with gluonium are unsuccessful. The phase ambiguity of elastic s-wave $\pi\pi$-scattering and the mixing of the S* and δ^0 resonances, which are model-independent for scalar resonances, are examined. A number of new experiments that may yield valuable information on the structure of scalar mesons is suggested. In particular, analysis of $\pi\pi\to K\bar{K}$ and $\pi\pi\to\pi\pi$ data predicts the appearance of well-defined effects due to the S* (980) and ε(1400) resonances in the $\pi^+\pi^-\eta\eta$ reaction.

CONTENTS

1. INTRODUCTION

Hadron spectroscopy[1)] has always played an important role in the evolution of our ideas on the dynamics of strong interactions. A clear example of this is the discovery of heavy quarkoniums—the J/ψ and Y meson families. There has also been recent increased interest in "pre-charm" hadron spectroscopy, following the hiatus during the general enthusiasm for small-distance physics. Once it became clear that quantum chromodynamics (QCD) was the only real pretender to the role of a theory of strong interactions, the central problem became the retention of color—the physics of large distances—which, in the final analysis, determines the spectrum of hadrons. One of the most surprising and so far unexplained dynamic features of light-hadron spectroscopy is the existence[1] of five already nearly filled nonets (two s-wave and three p-wave) in the quark-antiquark $q\bar{q}$ system predicted by the simplest nonrelativistic quark model. Moreover, "ideal" mixing of the SU(3) singlet and the isoscalar component of the octet occurs in the well-established $J^{PC}=1^{--}$ and 2^{++} nonets, which signifies the suppression of $(u\bar{u}+d\bar{d})/\sqrt{2}\leftrightarrow s\bar{s})$ transitions in these channels (this is the Okubo-Zweig-Iizuka rule). We note that the simplest nonrelativistic quark model predicts ideal nonets. The 1^{++} nonet is probably also ideal. The 1^{+-} nonet has not as yet been fully filled; eight out of the nine members have been established.

The pseudoscalar 0^{-+} nonet is not ideal. The reason for this is now understood, but is purely theoretical. It is connected with the solution of the U(1) problem in QCD, with instanton contributions (see, for example, Ref. 2).

Of course, within the framework of the simple quark model, one would expect nonet as radial excitations. In fact, eight out of the nine radial excitations of the pseudoscalar 0^+ nonet have not been discovered. There are also candi-

[1)]We shall confine our attention to mesons.

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dates for the 1^{--} radial excitations but, here, the interpretation is less obvious since some of the 1^{--} states can be orbital excitations with $L = 2$.

There are also nonets or orbital 3^{--} and 4^{++} excitations that are almost completely filled (eight out of the nine members of the nonets are known).

The simplicity of the light-hadron spectrum can probably be in many cases explained by the fact that, on the one hand, strong interactions due to gluon exchange remain small to distances of the order of $1/m_\rho$,[2)] as has been shown with the aid of the QCD sum rules,[3] and, on the other hand, for most channels (nonet quantum numbers), there are no nonperturbative contributions of direct instantons.[3]

At the same time, it is precisely the light-hadron spectrum that presents us with the first surprise. It turns out that the scalar-meson nonet[1] (δ(980), S*(980), ε(1300–1400) and $\varkappa$(1350–1500)) is difficult to understand as a set of two-quark states (p-wave nonet analogous to the 2^{++} nonet). The main problem is that the isoscalar S* and isovector δ have equal masses, whereas S* is more strongly bound to the $K\bar{K}$ than to the $\pi\pi$ channel. These two facts can be simultaneously explained within the framework of the simple two-quark model in which the mass of the resonances and the coupling constants to the hadrons are determined by simple "quark counting" rules. Similar considerations show that the masses of the ε and $\varkappa$ mesons are "incorrect" if these particles are placed in the same nonets with S* and δ mesons. Of course, one can hardly expect the 0^{++} nonets to be ideal since, in QCD, direct instanton contributions lead to the $(u\bar{u} + d\bar{d})/\sqrt{2} \leftrightarrow s\bar{s}$ mixing in both the scalar and the pseudoscalar channels (see, for example, Ref. 3). It is clear that this mixing would remove the degeneracy of the isovector and isoscalar and would not facilitate the solution of the principal scalar-meson problem. Moreover, it is shown in Ref. 4 with the aid of the QCD sum rules that, if scalar mesons are looked upon as $q\bar{q}$ systems, the "bare" masses of the S* and δ mesons (i.e., the masses without taking into account the finite width of the resonance and the interaction in the final states) should lie in the range 1.7–1.8 Gev.

At the same time, the degeneracy of the S* and δ meson masses and the suppression of the coupling of S* to the $\pi\pi$ channel as compared with its coupling to the $K\bar{K}$ channel is simply and naturally explained in terms of the four-quark nature of these mesons, as predicted by the MIT bag model of hadrons[5] (see Sec. 3 below). The question of multiquark hadrons arose together with the quark model. Multiquark mesons can be of two types: 1) clearly exotic, i.e., such that their quantum numbers are impossible in the $q\bar{q}$ system, and 2) latently exotic, i.e., such that their quantum numbers are possible in the $q\bar{q}$ system. In the former case, it is obviously possible to detect multiquark states, whereas, in the latter, one needs some well-defined effects connected with the manifestation of multiquark states, or some skillful theoretical analysis of experimental data on the coupling constants between resonances and the hadrons, etc., i.e, characteristics for which different models give different predictions.

In the twenty years since the advent of the quark model, not a single exotic state has been detected, and the great majority of cases leaves no doubt that the quark-antiquark interpretation was correct for the vast majority of detected mesons. The first difficulties arose in the interpretation of scalar mesons.

We have to say that, although multiquark states have long been under discussion, there have been no reliable dynamic calculations of their spectrum, coupling constants with hadrons, and so on. This became possible for the first time within the framework of the MIT bag model for hadrons,[5] which is a relativistic phenomenology of quark trapping. An abundant spectrum of low-lying (without orbital excitations) four-quark states ($qq\bar{q}\bar{q}$) was obtained in Ref. 5.

The four-quark mesons, in some sense, "consist" of pairs of pseudoscalar and vector "white" (and colored) $q\bar{q}$ mesons, and therefore have Zweig-superallowed coupling to the corresponding pairs. In most cases, Zweig-superallowed decay channels are not suppressed by the phase volume of the decay products and the four-quark states simply "break up" (without the creation of an additional $q\bar{q}$ pair from vacuum) into their "white components," $qq\bar{q}\bar{q} \to q\bar{q} + q\bar{q}$ (Fig. 1). The widths of these scalar mesons should be of the order of 1 GeV, so that these states do not appear as peaks in the particle collision cross sections, i.e., they are not like ordinary resonances.[5] In fact, these states appear as poles of the *P*-matrix.[6] At present, the only way information about them can be extracted from the background is by model-dependent procedures (the *P*-matrix method[6] is the only one known at present).

It is only in the rare cases where superallowed decay channels are suppressed by the phase volume that the "primitive" four-quark states should clearly show up. It is interesting to note that, according to the predictions reported in Ref. 5, it is expected that all the clearly exotic states have unsuppressed superallowed decay channels, and cannot appear as narrow resonances, with the exception, possibly, of the isotensor meson with a mass of 1650 MeV. Traces of this meson may have been recently detected[7] in the $\gamma\gamma \to \rho^0\rho^0$ process.

The most likely candidates for the four-quark states with latent exotic features are the scalar mesons S*(980) and δ(980). In addition, the scalar mesons ε(1400) and $\varkappa$(1500) may be four-quark states with latent exotic properties.

It is thus clear that the problem of scalar mesons is one of the most intriguing problems in pre-charm hadron spectroscopy. We have to add here that new excitations—gluoniums—have recently been predicted for the scalar sector. These "consist" of gluons rather than quarks, and their masses lie in the region of 600 and 1200–1400 MeV.

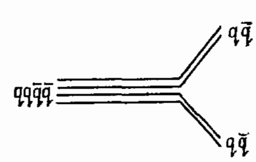

FIG. 1. Zweig superallowed decay of a $qq\bar{q}\bar{q}$ state into two $q\bar{q}$ mesons.

[2)] m_ρ is the mass of the ρ meson.

The aim of the present review is to examine in detail the situation that has arisen in the study of scalar mesons, and to identify theoretical assumptions about their possible dynamic structure that are not inconsistent with experimental data. We shall also try, as far as possible, to show the value of different conclusions about the dynamic structure of scalar mesons and, finally, draw attention to many new possibilities for the experimental investigation of scalar mesons which, in our view, could yield valuable information about the nature of these mesons.

The most important processes for the investigation of scalar mesons are the peripheral reactions with single-pion exchange in the t-channel, such as $\pi N\to\pi\pi N$, $\pi N\to K\bar{K}N$. Actually, it turns out that it is not at all a simple task to obtain clear experimental information about these states. The difficulty is that 0^+ mesons are either masked by resonances with higher spins, which have greater statistical weight $2J+1$ in the reaction cross section, or they are unusually broad and difficult to separate from the background and from one another, or, like the $S^*(980)$ and $\delta(980)$ resonances, they lie at the thresholds of inelastic channels and are greatly affected by them, which complicates the theoretical interpretation of such phenomena. It is even quite difficult to classify existing scalar mesons in accordance with the SU(3) multiplet scheme, since a given set of data can frequently be interpreted in different ways.

We shall try to examine experimental data by using simple and readily interpreted formulas to process these data. The amplitudes for s-wave processes, for example, $\pi\pi\to\pi\pi$, $\pi\pi\to K\bar{K}$, $\pi\pi\to\eta\eta$, and so on, will be written in the form of the sum of inelastic, i.e., coupled to several channels, resonances that can transform into each other and mix as a result of common decay channels. Whenever necessary, we shall take into account nonresonant background contributions. Our formulas are explicitly unitary and are direct generalizations to the relativistic case of the well-known quantum-mechanical formulas used to describe systems of resonances with common decay channels. Essentially, the only difference will be in the relativistic kinematics and in the inclusion of finite-width corrections that are important for the scalar mesons that we shall consider. It seems to us that this approach is the most suited to attempts to obtain information on the masses and coupling constants of resonances to different channels. Moreover, it is very convenient as a way of verifying relationships between the coupling constants that are among the most important characteristics of any quark-gluon model of resonances. The particular feature of experimental data of scalar mesons is that they contain resonances that are strongly distorted by interactions between them, a large continuous background, and very narrow features that appear at the thresholds of inelastic channels. All this is a manifestation of the strong coupling between the scalar mesons and one or more two-particle decay channels. Naturally, in the case of strong coupling, finite-width corrections (FWC) modify the usual Breit-Wigner resonance formula in an essential way. Inclusion of these corrections lead, for example, to fundamentally new possibilities in the interpretation of data on the $S^*(980)$ and $\delta(980)$ resonances (cusps), and highlights a new theoretical ambiguity in the behavior of the elastic s-wave phase shift in $\pi\pi$ scattering above the threshold in the $K\bar{K}$ channel. The coupling constants between the resonances and pseudoscalar and vector mesons, and their masses, serve as parameters in our formulas. Using reasonable values and relationships for them, we arrive at the very interesting conclusion that experimental data on scalar mesons can be interpreted as supporting the four-quark structure. It is considered that this fact must be looked upon as a new physical basis for further experimental and theoretical studies of these interesting objects. Of course, we shall also examine other possibilities and some difficulties in the description of the data, for example, those on the $\pi\pi\to\pi\pi$ process (or discrepancies between the data?). We emphasize that the evidence we have found in favor of the four-quark model is indirect, but even this is impossible without detailed analysis of the experimental situation which, in this case, is very complex and sometimes contradictory and varying with time.

We now summarize the content of the successive sections of this review. Section 2 enumerates briefly the basic experimental data on scalar mesons (general information). Some of them were obtained quite recently. Specific examination of data and their interpretation actually runs through the entire review. Section 3 discusses the popular four-quark ($qq\bar{q}\bar{q}$) model of scalar mesons put forward by R. L. Jaffe.[5] We also discuss the two-quark ($q\bar{q}$) model for the 0^+ resonances, and the possibility that the 0^+ gluonium exists. Section 4 reproduces formulas that will be used in the analysis of the experimental data. The analysis of data on scalar mesons is carried out in Section 5, which deals with a number of points. First, we examine the complex of resonances $S^*(980)$ and $\varepsilon(1300\text{–}1400)$ with isospin $I=0$ in the $\pi\pi\to\pi\pi$ and $\pi\pi\to K\bar{K}$ processes. The data clearly support strong "superallowed" coupling of S^* to the $K\bar{K}$ and $\eta\eta$ channels. At the same time, they indicate strong suppression of coupling between S^* and $\pi\pi$. All this constitutes the characteristic features of a four-quark $S^*(980)$ meson—a member of the first ("lightest") $(\underline{9}, 0^+)$ nonet of scalar $qq\bar{q}\bar{q}$ mesons predicted by the MIT bag model.[5] Moreover, we shall show that the $\varepsilon(1400)$ resonance can be interpreted as a member of the second ("heavier") four-quark $(\underline{9}^*, 0^+)$ nonet. At the same time, the explanation of existing experimental data on the $\pi\pi\to K\bar{K}$ process does not require the introduction of a further new 0^+ "$g_S(1240)$" resonance in addition to $S^*(980)$ and $\varepsilon(1470)$, as in Ref. 8. It is sufficient to include $S^*(980)$ and $\varepsilon(1400)$ resonances and their mixing. We note that particles from the $(\underline{9}^*, 0^+)$ $qq\bar{q}\bar{q}$ nonet should have relatively small widths, since the "fraction" of pairs of pseudoscalar mesons in their wave functions is smaller by an order of magnitude as compared with the members of the $(\underline{9}, 0^+)$ nonet, and the channels for the decay into pairs of vector mesons, whose "fraction" in $(\underline{9}^*, 0^+)$ is large, are suppressed by the phase volume ($\Gamma_{\varepsilon(1400)\pi\pi}\approx 100\text{–}150$ MeV). The question is what can be said about other models of the $\varepsilon(1400)$ resonance? Analysis of the data shows that $\varepsilon(1400)$ can hardly be gluonium [pure SU(3) singlet]. At the same time, the two-quark $q\bar{q}$ model of it is not inconsistent with experimental data. As for $S^*(980)$, we do not think it can be described as a $q\bar{q}$ resonance [in the same nonet as $\varepsilon(1400)$ and $\delta(980)$] without abandoning generally accepted simple ideas according to which the masses and coupling constants to different channels for members of a given multiplet are determined simply by their quark composition. In addition, in this section, we use an analysis of the $\pi\pi\to K\bar{K}$, $\pi\pi$, processes to predict well-defined effects due to the scalars S*(980) and $\varepsilon(1400)$ resonances in the cross section for the process $\pi^+\pi^-\to\eta\eta$ in the invariant-mass region $2m_\eta\sim1.5$ GeV, for which contributions of the tensor f and f′ mesons are highly suppressed by the phase volume. A theoretical prediction for the cross section $\sigma(\pi^+\pi^-\to\eta\eta)$ is made for the first time. The $\pi^+\pi^-\to\eta\eta$ process has not yet been studied experimentally, but it can be effectively investigated, for example, by using the neutral-particle detectors on the Serpukhov accelerator in the $\pi^-p\to\eta\eta\, n\to4\gamma n$ process with single-ion exchange in the t-channel.

We next consider data on the δ (980) resonance, whose effects are seen in channels with isospin $I=1(\pi\eta$ and $K\bar{K})$, and the data on the $\pi K\to\pi K$ process, which contain information about the $\varkappa(1350–1550)$ resonance with $I=1/2$. Existing data are not inconsistent with the four-quark scheme for the δ (980) resonance, in which, like S*(980), the δ (980) belongs to the first "light" ($\underset{\sim}{9}$, 0^+) nonet and, like $\varepsilon(1400)$, $\varkappa(1500)$ belongs to the second "heavier" ($\underset{\sim}{9}^*$, 0^+) nonet. A more specific prediction for the $\varkappa(1500)$ cannot be made at present. We emphasize that the δ and S* data can be described together within the framework of the $qq\bar{q}\bar{q}$ model with the aid of large ("superallowed") coupling constants between δ and $\pi\eta$, and between $K\bar{K}$, S* and the $K\bar{K}$ and $\eta\eta$ channels.

Sections 6 and 7 report two model-independent results. It is shown in Section 6, that, for strong coupling between S* and the $K\bar{K}$ channel in the $\pi\pi\to\pi\pi$ process, there is a theoretical ambiguity in the interpretation of data on the behavior of the s-wave phase shift in $\pi\pi$ scattering above the threshold of the $K\bar{K}$ channel. This can only be settled experimentally by improving the resolution in the invariant mass of the $\pi\pi$ system. Section 7 examines the mixing of S*(980) and δ^0(980) mesons, which violates the conservation of isotopic spin. This differs radically from, for example, the well-known $\rho^0-\omega$ mixing. The mixing of S* and δ^0 mesons is determined by the strong coupling constants between S* and δ and the $K\bar{K}$ channel, and by the quantity $\sqrt{(m_{K^0}-m_{K^+})/m_{K^0}}$, rather than $m_{K^0}-m_{K^+})/m_{K^0}$. The latter might have been expected from general considerations. Section 7 lists processes in which this phenomenon can be investigated.

In Section 8, we discuss Törnqvist's analysis of scalar mesons.[9] According to the Particle Data Group,[1] the problem of the scalar meson spectrum is resolved in Ref. 9, i.e., it is considered that they can be described as ordinary $q\bar{q}$ mesons with a large admixture of the $qq\bar{q}\bar{q}$ component in the form of virtual two-meson states, without introducing a special 0^+ $qq\bar{q}\bar{q}$ nonet. We shall show, using arguments that are, in our view, quite convincing, that the conclusion reported in Ref. 1 is premature. We shall also show that the objections put forward in Ref. 10 against the interpretation of δ (980) as a broad $qq\bar{q}\bar{q}$ state, which were based on the

TABLE I. Coefficients of the expansion of the $qq\bar{q}\bar{q}$-meson wave functions into pairs of "$q\bar{q}$ mesons." The P, V singlet and $\underset{\sim}{P}$, $\underset{\sim}{V}$ color octets of pseudoscalar and vector $q\bar{q}$ mesons, respectively.

Multiplets of $qq\bar{q}\bar{q}$ mesons, J^P	PP	VV	$\underset{\sim}{PP}$	$\underset{\sim}{VV}$
($\underset{\sim}{9}$, 0^+)	0.743	−0.041	−0.169	0.646
($\underset{\sim}{9}^*$, 0^+)	−0.177	0.644	0.623	0.407

$\eta'\to\eta\pi\pi$ and D, $E\to\eta\pi\pi$ decays, are simply incorrect within the framework of the $qq\bar{q}\bar{q}$ model.

2. EXPERIMENTAL DATA ON SCALAR MESONS. GENERAL IDEAS

We begin with the generally accepted resonances listed in published tabulations.[1,11,12] A few narrow structures are known: S*(980, $I^G(J^P)=0^+(0^+))\to\pi\pi$, $K\bar{K}$, δ (980, $1^-(0^+))\to\pi\eta$, $K\bar{K}$; $\Gamma_{S^*}\approx\Gamma_\delta\approx50$ MeV, and the more or less ordinary resonances $\varepsilon(1300–1400?$, $0^+(0^+))\to\pi\pi$, $K\bar{K}$, $\Gamma_\varepsilon\approx200–400$ MeV; $\varkappa(1350–1500?$ $1/2(0^+))\to\pi K$, $\Gamma_\varkappa\approx250$ MeV. The narrow feature S* in the $\pi\pi\to\pi\pi$ process, which appears at the $K\bar{K}$ channel threshold, the ε resonance noted above in the same process, and the $\varkappa$ meson in the $K\pi\to K\pi$ process, are observed against a background of smoother (nonresonant?) contributions (Fig. 2a; see also Fig. 8 in Section 5). The S* and ε resonances appear in a "purer" form in the inelastic process $\pi\pi\to K\bar{K}$ (see Figs. 3 and 4 in Section 5). S* is most clearly seen as a peak in the two-pion spectrum in the $K\text{-}P\to\pi^+\pi^-(\Lambda,\Sigma^0)$ processes[13] and, recently, in the $e^+e^-\to J/\Psi\to S^*+X\to\pi^+\pi^-X$ process.[14] The dynamic structure of precisely this nonet of scalar mesons is now under extensive discussion (see, for example, Refs. 1, 5, 6, 9, and 15–25). We shall also deal mainly with the above S*, ε, δ, and $\varkappa$ resonances. In addition, we shall frequently consider the status of $\varepsilon(700–900)$—a very broad and considerable enhancement of the $\pi\pi\to\pi\pi$ amplitude, which is difficult to interpret as a separate resonance structure.[16,26,27] Several experiments have recently been performed on the $\pi^+\pi^-\to K^0_SK^0_S$ processes,[8,28,29] which reveals the further 0^+ resonances S*′ with a mass of about 1770 MeV and width equal to $200\pm^{156}_{9}$ MeV.[29] We shall not consider this feature because our analysis is restricted to lower energies (actually,

TABLE II. Flavor structure of the wave functions of four-quark 0^+ mesons in the ($\underset{\sim}{9}$, 0^+) nonet [and similarly for the ($\underset{\sim}{9}^*$, 0^+) nonet], $\pi^0(\rho^0)=(u\bar{u}-d\bar{d})/\sqrt{2}$, $\pi^+(\rho^+)=u\bar{d}$, $\pi^-(\rho^-)=\bar{u}d$, $\eta_0(\omega)=(u\bar{u}+d\bar{d})/\sqrt{2}$, $\eta_s(\varphi)=s\bar{s}$, $K^+(K^{*+})=u\bar{s}$, $K^0(K^{*0})=d\bar{s}$, $\bar{K}^0(\bar{K}^{*0})=\bar{d}s$, $K^-(K^{*-})=\bar{u}s$.

$$|S^*\rangle=\left|\frac{1}{2}K^+K^-+\frac{1}{2}K^0\bar{K}^0-\sqrt{\frac{1}{2}}\,\eta_0\eta_s\right\rangle$$
$$|\varepsilon\rangle=\left|\sqrt{\frac{1}{2}}\,\pi^+\pi^-+\frac{1}{2}\pi^0\pi^0-\frac{1}{2}\eta_0\eta_0\right\rangle$$
$$|\delta^0\rangle=\left|-\frac{1}{2}K^+K^-+\frac{1}{2}K^0\bar{K}^0+\sqrt{\frac{1}{2}}\,\eta_s\pi\right\rangle$$
$$|\varkappa^0\rangle=\left|\sqrt{\frac{1}{2}}\,K^+\pi^--\frac{1}{2}K^0\pi^0-\frac{1}{2}K^0\eta_0\right\rangle$$

the region of the $\rho\rho$ threshold) and its extension to higher energies would involve major technical complications. Moreover, the S*′ data require further refinement. The basic experimental data on scalar mesons (that are known to us), published up to the beginning of 1983, were obtained as a result of s-wave studies in the following two-particle reactions with pseudoscalar mesons, and also studies of the mass spectra of the pair of pseudoscalar mesons created in different reactions of decays:

process	references
$\pi^+\pi^- \to \pi^+\pi^-$	27, 30–40
$\pi^+\pi^- \to \pi^0\pi^0$	26, 27, 40–45
$\pi^+\pi^- \to K^+K^-$	35, 36, 46–51
$\pi^+\pi^- \to K_s^0K_s^0$	8, 28, 29, 52–54
$\pi K \to \pi K$	55–58
$K^-P \to [(\eta\pi^-), (K^0K^-)]\,\Sigma^-_{1385}$	59
$D(0^+(1^+)),\ E(0^+(1^+)) \to \eta\pi\pi,\ K\bar{K}\pi$	60–64
$J/\Psi \to X + \pi\pi,\ \varphi + \pi\pi$	14
$p\bar{p} \to K_s^0K_s^0(n\pi)$	65
$K^-p \to \pi^+\pi^-(\Lambda, \Sigma^0)$	13

The first five processes are separated from the more complicated $\pi^\pm N \to [(\pi\pi), (K\bar{K})], (N,\Delta)$ $KN \to K\pi N$ by the single-pion exchange in the t-channel. We note that there have been several reviews[15,27,66–69] of hadron resonance spectroscopy in which the problem of scalar mesons was considered to a greater or lesser extent.

3. qqq̄q̄ AND qq̄ MODELS OF SCALAR MESONS

The theoretical quark structures of scalar mesons considered below will be used later to analyze experimental data (see Section 5).

A. qqq̄q̄ model[5]

We begin by considering this popular model in which there is a large number of nontrivial relationships for the coupling constants between the scalar resonances and different decay channels. The MIT-bag model predicts[5] many scalar four-quark states with masses in the range 0.6–2.4 GeV: these are the (9̲, 0⁺), (9̲*, 0⁺), 36̲(, 0⁺) and (36̲*, 0⁺) U(3) multiplets. As metioned in the Introduction, the "live" resonance manifestations of qqq̄q̄ states in $\pi\pi$, $\pi\eta$, and $K\bar{K}$ channels could be the S*(980) and δ (980) mesons from the first ("lighter") four-quark nonet (9̲, 0⁺), for which Zweig-superallowed $K\bar{K}$ channel is suppressed by the phase volume.[21,23] Moreover, it is expected that there are narrow scalar mesons belonging to the second ("heavier") four-quark nonet (9̲*, 0⁺). This expectation is based on two facts. Firstly, it is clear from Table I, which lists the expansion coefficients for the wave functions of the qqq̄q̄ states in the (9̲, 0⁺) and (9̲*, 0⁺) multiplets in terms of qq̄ meson pairs,[5,7,70] that the "fraction" of pseudoscalar mesons (PP) in the (9̲*, 0⁺) is smaller by a factor of 18 than in the (9̲, 0⁺). Secondly, channels involving decays into vector mesons, whose "fraction" (VV̄) is large in (9̲*, 0⁺), are suppressed by the phase volume, since their thresholds lie above the expected masses of the scalar resonances in the (9̲*, 0⁺) multiplet.[5] In the simplest variant, the coupling of the four-quark states in the (9̲, 0⁺) and (9̲*, 0⁺) multiplets to the pseudoscalar (and vector) mesons, of which they "consist" in some way, is determined by the single constant g_0. Tables I and II can now be used in an obvious way to calculate the coupling constants of the S*(980), ε(1300–1400), and other resonances to specific channels. For example, $g_{S^*K^+K^-} = 1/2 \times 0.743 \times g_0$, and so on. We shall try to identify S*(980) and ε(1300–1400) with the heaviest isoscalar representative of the (9̲, 0⁺) multiplet and the lightest member of the (9̲*, 0⁺) multiplet. It may be that suitable candidates for other (9̲*, 0⁺) states are[1,11] $\varkappa$(1350–1500, 1/2(0⁺)) and the recently discovered[29] S*′(1770, 0⁺(0⁺)). The prediction made in Ref. 5 is $m_{S^*} = m_\delta = 1100$ MeV, instead of the observed $m_{S^*} \simeq m_\delta \simeq 980$ MeV. The predicted

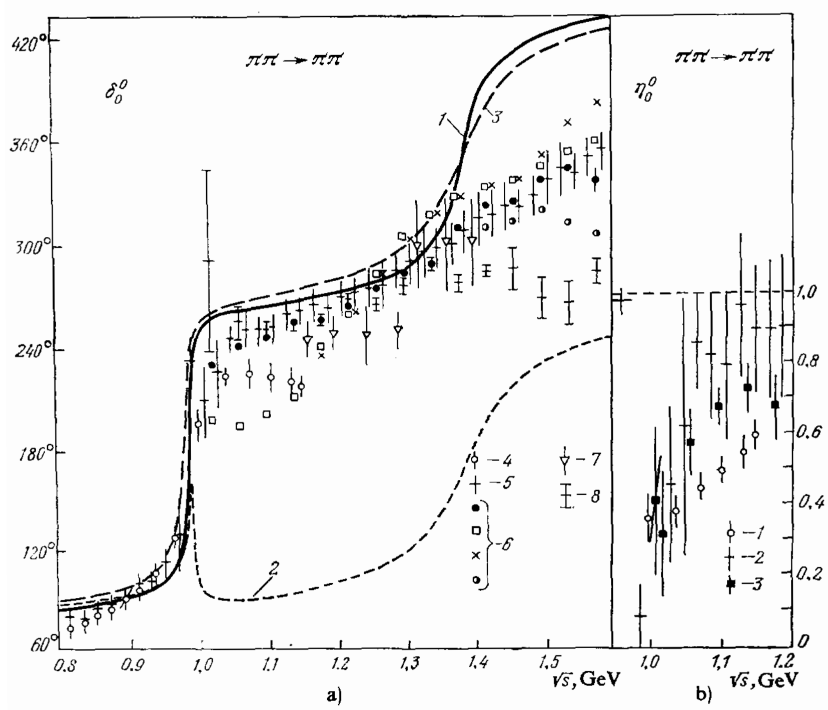

FIG. 2. (a) The phase δ_0^0 in elastic s-wave $\pi\pi$ scattering with $I = 0$ (curves 1–3 correspond to the sets listed in Table III; details are given in the text; 4—Ref. 31; 5—Refs. 32 and 33; 6—four solutions for the phase taken from Ref. 33; 7—Refs. 49 and 38; 6—Ref. 37); (b) elasticity parameter for the η_0^0 amplitude for the same process (1—Ref. 31; 2—Ref. 33; 3—Ref. 19).

masses for the ε, $\varkappa$ and S*' states in the ($\underline{9}$*, 0^+) multiplet are 1450, 1600, and 1800 MeV, respectively. The fact that the observed masses are slightly lower than the theoretical predictions cannot be a serious objection because, on the one hand, the model is based on simplifying assumptions and, on the other, the observed peaks may be shifted by interactions in the final state and by the overlap of the resonances. It follows from Table I that, in the "ideal" variant of the $qq\bar{q}\bar{q}$ model, the S* is coupled to $K\bar{K}$, $\eta\eta$, $\eta\eta'$, and $\eta'\eta'$, and the ε to the $\pi\pi$, $\eta\eta$, $\eta\eta'$, and $\eta'\eta'$ channels. Experimental data clearly show that the coupling of the S* to the $\pi\pi$ is actually much weaker than to the $K\bar{K}$ (Refs. 18, 19, 21, 22, 30, 31, 35, 36, 41, 48, 49, and 71), and that the absolute magnitude of the coupling between S* and $K\bar{K}$ is large (Refs. 21, 22, 71) (this is the superallowed coupling). There is also evidence for strong coupling between S* and $\eta\eta$ (Refs. 21, 22, and 71). According to Refs. 1 and 11, the ε(1300–1400) resonance is coupled mainly to the $\pi\pi$-channel.[1,11,38,50] We shall use the following quark structure (in flavor) for the physical S* state in order to introduce the coupling of S* to $\pi\pi$:

$$S^* = s\bar{s}\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\cos\alpha + u\bar{u}\,d\bar{d}\sin\alpha. \quad (1)$$

The second term in (1) is responsible for the coupling between S* and $\pi\pi$. We emphasize that this representation is sensible when there is little mixing, i.e. $|\mathrm{tg}\alpha| \ll 1$:

$$\frac{g_{S^*\pi^+\pi^-}}{g_{S^*K^+K^-}} = \sqrt{2}\,\mathrm{tg}\,\alpha, \quad \frac{g_{S^*\eta\eta}}{g_{S^*K^+K^-}} = 1 - \frac{\mathrm{tg}\,\alpha}{2\sqrt{2}}, \quad (2)$$
$$g_{S^*K^+K^-} = \frac{1}{2}\cdot 0.743 g_0 \cos\alpha \approx \frac{1}{2}\cdot 0{,}743 g_0;$$

where we have taken into account η–η' mixing, the mixing angle is $\theta_{\eta\eta'} \approx -10°$, and we have assumed that $\eta = [(u\bar{u}+d\bar{d})/\sqrt{2} - s\bar{s}]/\sqrt{2} \equiv \eta_8\cos\theta_{\eta\eta'} - \eta_1\sin\theta_{\eta\eta'}$. For the ε meson, we can write, by analogy with (1),

$$\varepsilon = u\bar{u}\,d\bar{d}\cos\beta + s\bar{s}\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\sin\beta, \quad (3)$$

and thus take into account the (small) coupling between ε and $K\bar{K}$. In this expression, α and β are independent parameters [S*(980) and ε(1300–1400) belong to different multiplets], and

$$\frac{g_{\varepsilon K^+K^-}}{g_{\varepsilon\pi^+\pi^-}} = \frac{\mathrm{tg}\,\beta}{\sqrt{2}}, \quad \frac{g_{\varepsilon\eta\eta}}{g_{\varepsilon\pi^+\pi^-}} = -\frac{1}{2\sqrt{2}}(1 - 2\,\mathrm{tg}\beta), \quad (4)$$

$$\cos\alpha\, g_{\varepsilon\pi^+\pi^-} = -\frac{0{,}177}{0{,}743}\sqrt{2} g_{S^*K^+K^-}\cos\beta, \quad (5)$$

where the last relation follows from (1) and (3) and from Tables I and II. We note at once that, although the parameter $|\mathrm{tg}\alpha|$ is much less than unity, we cannot set it equal to zero. On the other hand, the zero value of $\mathrm{tg}\,\beta$ is entirely allowed by experimental data ($\cos\alpha$ and $\cos\beta$ are both equal to unity with good precision). At the same time, the coupling of ε to the $K\bar{K}$ channel arises only because of S* $-$ ε mixing, which is due to the presence of common $\pi\pi$, $\eta\eta$, etc., decay channels (see next Section).

In the $qq\bar{q}\bar{q}$ model, the δ^0 (980, $1^-(0^+)$) resonance has the structure $(u\bar{u} - d\bar{d})s\bar{s}/\sqrt{2}$ and, together with S*, belongs to the ($\underline{9}$, 0^+) multiplet. Since $S^* \approx (u\bar{u} + d\bar{d})s\bar{s}/\sqrt{2}$ [see (1)], the degeneracy of the masses of δ and S*, and the strong coupling of S* to the $K\bar{K}$ channel, can both be explained by the four-quark model in a very natural way.

The question is: what can be said about the lighter $qq\bar{q}\bar{q}$ states ε(650) $-$ $u\bar{u}d\bar{d}$ and $\varkappa^+$(900) $-$ $\bar{s}ud\bar{d}$ in the ($\underline{9}$, 0^+) multiplet for which the predicted masses are[5] 650 and 900 MeV, respectively? First of all, they must be exceedingly broad, since their masses lie well above the thresholds of the superallowed decay channels ($\varepsilon(650)\to\pi\pi$, $\varkappa(900)\to K\pi$). If their widths are of the order of 1 GeV, the resonance interpretation, the problem of separation from the background, and even the question of the "mass" of states must be considered in the light of the decay dynamics, i.e., within the framework of some particular model of strong interactions (large-distance quantum chromodynamics). It is striking that, in the elastic s-wave region of the $\pi\pi\to\pi\pi$, $K\pi\to K\pi$ processes (for the first of these, the elastic region is bounded by the $K\bar{K}$ threshold[30–34] and, for the second, it reaches up to 1300–1400 MeV[55–58] and, possibly, up to the $K^*\rho$ threshold, to within experimental error), there are actually very broad structures producing large amplitudes. The phase of the elastic amplitude passes smoothly through 90° at about 850 MeV for $\pi\pi\to\pi\pi$ and at about 1300 MeV for $K\pi\to K\pi$, with the amplitude reaching the unitary limit[27,32–34,55,58] (see Figs. 2a and 8). In the phenomenological analysis of the data, these contributions are usually taken into account by interpreting them as a large continuous elastic background. We note that the P-matrix method[6] can be applied to the phase data to "expose" the "primitive" four-quark states ε(650) and $\varkappa$(900) in the ($\underline{9}$, 0^+) nonet in a more or less graphic way (see Ref. 6 for details).

In "grid" calculations, the mass of the scalar gluonium sometimes turns out to be approximately 600 MeV. The width is not predicted. A narrow state of this mass has not been seen, at least in the $\pi\pi$ channel. If, on the other hand, this state is very wide, the interpretation of this kind of resonance must involve the complex dynamics of the process, just as it does in the wide four-quark 0^+ state.

Apart from the ($\underline{9}$, 0^+) and ($\underline{9}$*, 0^+) nonets of $qq\bar{q}\bar{q}$ mesons mentioned above, the four-quark model also predicts the ($\underline{36}$, 0^+) multiplet[5] in the region between 1.1 and 2 GeV. The masses of all its representatives lie appreciably above the threshold for superallowed decay channels. As far as these states are concerned, the situation is therefore analogous to that considered for the ε(650) and $\varkappa$(900) mesons in the ($\underline{9}$, 0^+) multiplet. At present, it is very difficult to say anything specific about them. All the foregoing refers to the heaviest four-quark multiplet ($\underline{36}$*, 0^+) that lies[5] in the mass range between 1.8 and 2.4 GeV and is coupled mainly to decay channels to vector mesons.[70] We shall actually confine our application of the $qq\bar{q}\bar{q}$ model to clearly resonant structures that can be assigned to ($\underline{9}$, 0^+) and (9*, 0^+) nonets (see, however, Section 8B).

B. $q\bar{q}$ Model for the ε(1300–1400) resonance

We shall consider the $q\bar{q}$ model only for the ε(1300–1400) resonance, since it does not seem to us possible to describe S* as a $q\bar{q}$ resonance (with the $(u\bar{u}+d\bar{d})/\sqrt{2}$ or $s\bar{s}$

structure) without abandoning the generally accepted simple considerations,[3)] according to which the masses and coupling constants to different channels for members of a particular multiplet [in the case of the $q\bar{q}$ model, it is natural to consider $S^*(980)$, $\varepsilon(1300–1400)$, $\delta(980)$, and $\varkappa(1350–1500)$ as members of a particular nonet] are determined simply by their quark composition. This is a well-known difficulty in the two-quark model of S^*. From the phenomenological point of view, the four-quark structure seems the most appropriate for S^*, if not in the sense of the MIT-bag model, then in the sense of a system consisting largely of virtual $K\bar{K}$ pairs.[9,72] Thus, let us suppose that, for example, $\varepsilon = (u\bar{u} + d\bar{d})/\sqrt{2}$. Then, if we take $\eta - \eta'$ mixing into account, we obtain the following ratios of coupling constants for the ε resonance:

$$g_{\varepsilon\pi^+\pi^-} : g_{\varepsilon K^+K^-} : g_{\varepsilon\eta\eta} = 1 : \frac{1}{2} : \frac{1}{2\sqrt{2}}. \qquad (6)$$

It will mix with the S^* resonance because of the presence of common channels. We have verified that the ratios of the coupling constants $g_{\varepsilon\pi^+\pi^-}$ and $g_{\varepsilon K^+K^-}$ in this model are not inconsistent with experimental data on the $\pi\pi\to K\bar{K}$, $\pi\pi$ processes. We note that the ratio $g^2_{\varepsilon\eta\eta}/g^2_{\varepsilon\pi^+\pi^-} \approx 1/8$ does not depend on whether ε has a two-quark $(u\bar{u} + d\bar{d})/\sqrt{2}$ or four-quark $u\bar{u}d\bar{d}$ structure [see (4) and (6)].

C. (1300–1400) resonance as gluonium

The possibility of a gluonium state with quantum numbers $J^{PC} = 0^{++}$ in the region of 1300 MeV is now being actively pursued. The extent to which theoretical predictions can be regarded as reliable is not clear. There is, as yet, no clear evidence for the existence of this state.[68] However, general considerations suggest that the structure of the wave function of gluonium, which is a "flavor" singlet, should produce the same coupling of this state to all pseudoscalar mesons. We identify it with the $\varepsilon(1300–1400)$ meson. We then have

$$g_{\varepsilon\pi^+\pi^-} = g_{\varepsilon K^+K^-} = \sqrt{2}\, g_{\varepsilon\eta\eta}. \qquad (7)$$

As in the previous formulas, the identity of particles is taken into account in this expression directly in the definition of the constants. We note that (7) has enabled us to achieve a satisfactory fit to all the data on the $\pi\pi\to K\bar{K}$ process.

4. RESONANCE PARAMETRIZATION OF THE s-WAVE AMPLITUDES FOR THE $\pi\pi\to(\pi\pi, K\bar{K}, \eta\eta)$, $K\pi\to(K\pi, K\eta)$ PROCESSES

Suppose that the s-wave amplitude for the process $ab\to cd$ (a, b, c, d are pseudoscalar mesons) includes contributions due to several scalar resonances coupled to several channels. The resonance amplitude ($T^{\rm res}_{ab\to cd}$) can then be written in the following form (which satisfies the unitarity condition):

$$T^{\rm res}_{ab\to cd} = \sum_{R,\, R'} \frac{g_{Rab}\, g_{R'cd}}{16\pi}\, G^{-1}_{RR'}(s), \qquad (8)$$

where the sum is evaluated over the resonances R, R' (R (R') $= S^*, \varepsilon, \ldots$), g_{Rab} is the coupling constant between R and the channel ab, and $G_{RR'}(s)$ is the inverse propagator matrix:

$$G_{RR'}(s) = \begin{pmatrix} D_{S^*}(s) & -\Pi_{S^*\varepsilon}(s) & \ldots \\ -\Pi_{S^*\varepsilon}(s) & D_\varepsilon(s) & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix}, \qquad (9)$$

where

$$D_R(s) = m_R^2 - s + \sum_{ab} [\mathrm{Re}\,\Pi_R^{ab}(m_R^2) - \Pi_R^{ab}(s)],$$

$$\mathrm{Im}\,\Pi_R^{ab}(s) = \sqrt{s}\,\Gamma_{Rab}(s) = \frac{g^2_{Rab}}{16\pi}\,\rho_{ab}(s), \qquad (10)$$

$$\rho_{ab}(s) = \sqrt{s-m_+^2}\sqrt{s-m_-^2}\,\frac{1}{s}, \quad m_\pm = m_a \pm m_b,$$

$$\Gamma_R(s) = \sum_{ab} \Gamma_{Rab}(s), \quad \Pi_R(s) = \sum_{ab} \Pi_R^{ab}(s), \qquad (11)$$

and M_R, Γ_R, and Γ_{Rab} are, respectively, the mass, total width, and partial widths of the resonance R. The finite-widths corrections to the decay of the resonance appear in the expressions $\Pi_R^{ab}(s)–\mathrm{Re}\Pi_R^{ab}(m_R^2)$, which are single-loop contributions to the proper energy of the resonance R due to two-particle intermediate states ab, subtracted at the point $s = m_R^2$. A detailed discussion of their role in the case of scalar resonances can be found, for example, in Refs. 21–23 and 71. When a and b are 0^- mesons and $m_a \geqslant m_b$, $s > m_+^2$, we have

$$\Pi_R^{ab}(s) = \frac{g^2_{Rab}}{16\pi}\Big[\frac{m_+ m_-}{\pi s}\ln\frac{m_b}{m_a} + \rho_{ab}\Big(i + \frac{1}{\pi}\ln\frac{\sqrt{s-m_-^2}-\sqrt{s-m_+^2}}{\sqrt{s-m_-^2}+\sqrt{s-m_+^2}}\Big)\Big]. \qquad (12)$$

When $s < m_+^2$, $\Pi_R^{ab}(s)$ can be obtained by analytic continuation.[21] Near the ab threshold (either above or below it), the function $\Pi_R^{ab}(s)$ varies much more rapidly than $m_R^2 - s$ (even for $m_R \approx m_a + m_b$), so that there is a substantial modification (distortion) of the 0^+–meson propagator for large coupling constants.

The off-diagonal elements of the matrix $G_{RR'}(s)$ are the amplitudes for the $R\leftrightarrow R'$ transitions due to the mixing of resonances that results from the interaction in the final state, which occurs when there are common decay channels, $R\to(ab)\to R'$ ("unitary mixing"). We shall write them in the form

$$\Pi_{RR'}(s) = \Big[\sum_{ab} \frac{g_{R'ab}}{g_{Rab}}\,\Pi_R^{ab}(s)\Big] + C_{RR'}, \qquad R \neq R'. \qquad (13)$$

All experimental data on the scalars $S^*, \varepsilon, \delta, \varkappa, \ldots$ show that, at energies of about 1–1.5 GeV, the most important for them are the two-particle decay modes to pseudoscalar mesons. We shall take them into account in the first instance in the sums over the intermediate states of (ab) in (10) and (13). The suppression of four-particle, etc. decays of the form $0^+\to 4 0^-, \ldots$ in this energy range can be understood by assuming, for example, that they occur in a quasi-two-particle manner through the vector mesons $0^+\to VV\to 4 0^-$ and, con-

[3)]In Sec. 8, we shall discuss the attempt[9] to describe the observed spectrum of 0^+ resonances within the framework of the $q\bar{q}$ model with allowance for finite-width corrections. This attempt is based, in our view, on the unjustified assumption of a specific violation of the quark model [U(3) symmetry] for the coupling constants between the 0^+ resonances and the (0^-0^-) decay channels.

sequently, are suppressed under the VV threshold (for $\sqrt{s} < m_{V_1} + m_{V_2}$). One could try to estimate the contribution of vector mesons top $\mathrm{Im}\Pi_R(s)$ for particular models, for example, in the way it is done below for the ε resonance (see Section 5). The question then is: what is the nature of constants C_{RR} in (13), which are free parameters in our case? The answer is that, firstly, they include subtraction constants for diagrams corresponding to the $R\to(0^-0^-)\to R'$ transitions, the finite parts of which are written our explicitly in the square brackets in (13). Secondly, C_{RR}, efffectively represent contributions of all other intermediate states, for example, VV, 40^- and so on, which we cannot calculate, but which are naturally assumed to be smooth functions of the energy in the range 1–1.5 GeV, and can be approximated by a constant. At this point, we note one favorable fact. In the four-quark model of the S* and ε resonances (in its "ideal" variant;[5] see Section 3), the sum of diagrams corresponding to the $S^*\leftrightarrow\varepsilon$ transition due to common superallowed decay channels into pseudoscalar mesons is found to converge because of the orthogonality of the wave functions of the four-quark S* and ε resonances. In that case, the portion of the constant that is connected with the contribution due to pseudoscalar mesons turns out to be known (it is equal to $(g_{S^*\eta\eta}g_{\varepsilon\eta\eta}/16\pi^2 \ln(m_{\eta'}^2/m_\eta^2)\approx 0.045\ \mathrm{GeV}^2)$, but the constant $C_{S^*\varepsilon}$ due to the other intermediate states is at our disposal.

We emphasize that the parameter $C_{S^*\varepsilon}$ is very imporant when a fit is made to the data (it cannot be set equal to zero), although its order of magnitude (and sign) is the same as the Re-part of the explicitly calculated contributions to $\Pi_{S^*\varepsilon}(s)$. The cross section corresponding to the amplitude given by (1) is

$$\sigma^{\mathrm{res}}_{ab\to cd}(s)=\frac{16\pi}{s}\,\frac{\rho_{cd}}{\rho_{ab}}\,|\,T^{\mathrm{res}}_{ab\to cd}(s)\,|^2. \qquad (14)$$

Thus, the adjustable parameters now available to us are the masses m_R of the resonances, the coupling constant g_{Rab}, and the mixing constants $C_{RR'}$. Naturally, to reduce the number of independent parameters, we must use considerations involving SU(3) or U(3) symmetries, or some particular quark model for the scalar resonances. Whatever scheme is actually used, there are certain relationships, for example, between the constants $\pi\pi\to K\bar{K}$, $\pi\pi$, so that, if we use the data on the $\pi\pi\to K\bar{K}$, $\pi\pi$ processes, we can make some predictions for the $\pi\pi\to\eta\eta$ process. Our formulas readily enable us to investigate a large number of channels. We note that, so far, the analysis of the data (for example, by the K-matrix method) was performed with allowance for only the two channels, $\pi\pi$ and $K\bar{K}$ (Refs. 15, 19, 24, and 48–50). This assumes, right from the start, that there is no coupling between the $\eta\eta$-channel and the resonances. However, as we have already mentioned, this coupling is present and may be considerable in any composite model. We shall see this later in the case of the S* resonance. Moreover, additional data on scalar mesons in the $\pi\pi\to\eta\eta$ channel can, in principle, be helpful even in connection with their classification in accordance with the SU (3) multiplets on which, as we have already mentioned, there is at present, a lack of unanimity.[8,9,15–25,48–50,67–69]

5. ANALYSIS OF DATA ON SCALAR MESONS

A. Scalar mesons with $I=0$ in the $\pi\pi\to K\bar{K}$, $\pi\pi\to\pi\pi$ processes; S*(980)–ε(1300–1400) resonance complex. Predictions for the $\pi\pi\to\eta\eta$ process

As already noted, the s-wave amplitude for the $\pi\pi\to\pi\pi$ processes contains a large continuous background in addition to the S* and ε resonance (see, for example, Fig. 2a, which shows the phase δ_0^0 of the amplitude). The phase "take-off" in the narrow region near the $K\bar{K}$ threshold is due to the contribution of the S* resonance. It is interesting to note that, in the cross section for the $\pi\pi\to\pi\pi$ process, the resonance S* corresponds not to a peak, but to a narrow valley, since the background phase δ_B in this region is of the order ot 90° and the elasticity is $\eta_0^0=1$ with high precision up to the $K\bar{K}$ threshold.[15,27,30–34] We shall consider the s-wave amplitude for the $\pi\pi\to\pi\pi$ process with $I=0$ in the form of the sum of the inelastic resonant amplitude $\Gamma^{\mathrm{res}}_{\pi\pi\to\pi\pi}$ [cf. (8)], in which we shall include contributions due to the S* (980) and ε(1300–1400) resonances, and the amplitude for the elastic background

$$T(\pi\pi\to\pi\pi)=\frac{\eta_0^0 e^{2i\delta_0^0}-1}{2i\rho_{\pi\pi}}=\frac{e^{2i\delta_B}-1}{2i\rho_{\pi\pi}}+e^{2i\delta_B}T^{\mathrm{res}}_{\pi\pi\to\pi\pi}, \qquad (15)$$

where $\delta_0^0=\delta_B+\delta_{\mathrm{res}}$. Formulas such as (15), which include the elastic background, are frequently used in data-fitting procedures in a very broad range of energies.[15,18–22,30,31,48,49] This is, of course, the simplest approximation, but it works well in the region of the S* resonance, and in the $\pi\pi$ and $K\bar{K}$ channels. The amplitudes for the inelastic processes $\pi\pi\to K\bar{K}(\eta\eta)$ can be taken in the following form:

$$T(\pi\pi\to K\bar{K}(\eta\eta))=e^{i\delta_B}T^{\mathrm{res}}_{\pi\pi\to K\bar{K}(\eta\eta)}e^{i\tilde{\delta}_B} \qquad (16)$$

(in accordance with the unitarity condition). Here, $\tilde{\delta}_B$ is the smooth phase of the nonresonant elastic background in the $K\bar{K}\to K\bar{K}$ process (or the $\eta\eta\to\eta\eta$ process), which we shall neglect, since there are no clear indications that it plays an appreciable role. This can definitely be done, at least near the $K\bar{K}(\eta\eta)$ threshold, where $\tilde{\delta}_B \ll \delta_B$.

When the moduli and phase of the amplitudes for the s-wave process $\pi^+\pi^-\to\pi^+\pi^-$, K^+K^- are determined from experimental angular distributions of pseudoscalar mesons in the $\pi N\to(\pi\pi, K\bar{K})+(N,\Delta)$ processes, there are usually several possible solutions[26,27,32–34,37,40,44–50,66] The number of solutions can usually be reduced with the aid of data on a process with identical particles in the final state: $\pi^+\pi^-\to\pi^0\pi^0$, $\pi^+\pi^-\to K_s^0K_s^0$ (Refs. 26–29, 40–45, 52, and 66). We note that the process $\pi^+\pi^-\to K_s^0K_s^0$ has the advantage that it does not involve $I=2$ contributions which complicate the analysis of the data in the $\pi\pi\to\pi\pi$ case. It is precisely this situation that occurred, for example, for the ε(1300–1400) resonance, which has been under intensive study since 1976 in the $\pi^\pm N\to K\bar{K}N$ processes.[8,28,29,46–54] Some of the solutions for the $\pi^+\pi^-\to K^+K^-$ process contained this s-wave resonance, whereas, in other solutions, the resonance enhancement "migrated" into the p-wave.[46–50] There is no p-wave in the $\pi^+\pi^-\to K_s^0K_s^0$ process, and the D-wave (the interference with which by the s-wave is very important in the determination of the latter) is well known and is dominated by the f and f' resonances. Recent data on this

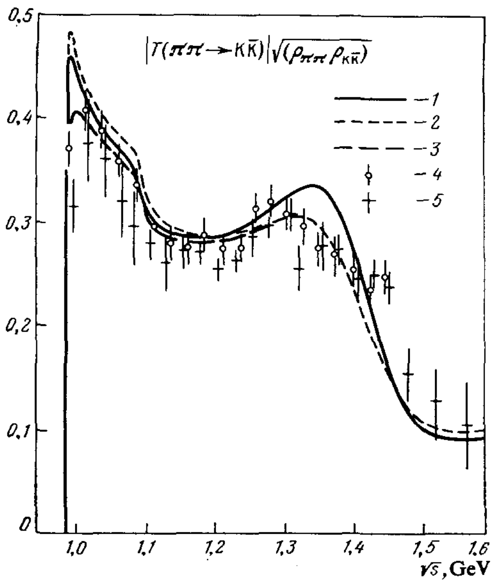

FIG. 3. Modulus of the amplitude for the $\pi\pi\eta\to K\bar{K}$ process. s-wave $I = 0$; the value of 0.5 is the unitary limit for this process. The curves represent the sets listed in Table III. 4—Ref. 20, 5—Refs. 48 and 49.

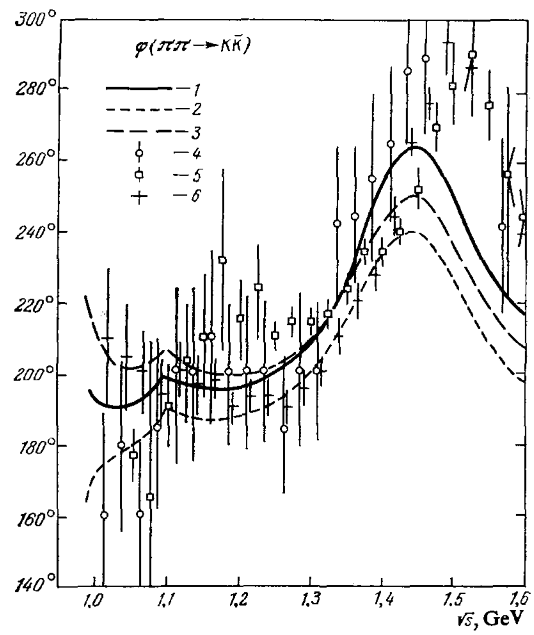

FIG. 5. Phase of the amplitude for the process $\pi\pi\to K\bar{K}$. s-wave, $I = 0$. The curves correspond to the sets listed in Table III. The data are taken as follows: 4—Ref. 15, 5—Ref. 29, 6—Refs. 48 and 49.

process confirm the resonance enhancement in the region of 1300–1400 MeV (Refs. 8, 29, 48–50).

Let us begin with this particular process $\pi\pi\to K\bar{K}$; Figs. 3–5). Here, we can clearly see a very rapid and large rise in the amplitude (the height of the maximum is 80% of the unitary limit; see Fig. 3) near the threshold, which is due to the S* resonance (strong coupling between S* and $K\bar{K}$ channel), the ε resonance in the form of the rise in the region of 1250–1450 MeV, and also traces of an interaction between them, represented by the abrupt fall in amplitude above 1450 MeV and the clearly constructive interference in the region between the resonances. Naturally, this mutual interaction between the S* and ε resonances complicates the investigation of their individual properties.

Figures 3–5 show the results of various fits based on (8)–(16) and restrictions (1)–(5) on the coupling constants of the S* and ε mesons, which follow from the $qq\bar{q}\bar{q}$ model (cf. Sections 3 and 4). Table III lists three sets of values of the adjustable parameters for the S* and ε resonances that correspond to the curves of Figs. 3–5. Other (close) parameter sets are also possible and yield satisfactory agreement. Case 2 (Table III) with $m_{S^*} = 1.4$ GeV should be no surprise. It corresponds to the S* cusp.[21,22,71] All such non-standard cases that arise because of the finite-width corrections in $D_{S^*}(s)$ have been described in sufficient detail in the literature.[21–23,71] Here, we merely note that $m_{S^*} \approx 1.4$ GeV corresponds to a zero of Re $D_{S^*}(s)$, which is not reflected in the amplitude for the process against the background of the large Im $D_{S^*}(s) \approx \sqrt{s}\,\Gamma_{S^*K\bar{K}}(s)$, and that Re $D_{S^*}(s)$ is close to zero for $\sqrt{s} \approx 2m_K$. On the other hand, the fact that the real part of the S* propagator is close to zero for $\sqrt{s} \approx 2m_K$ pro-

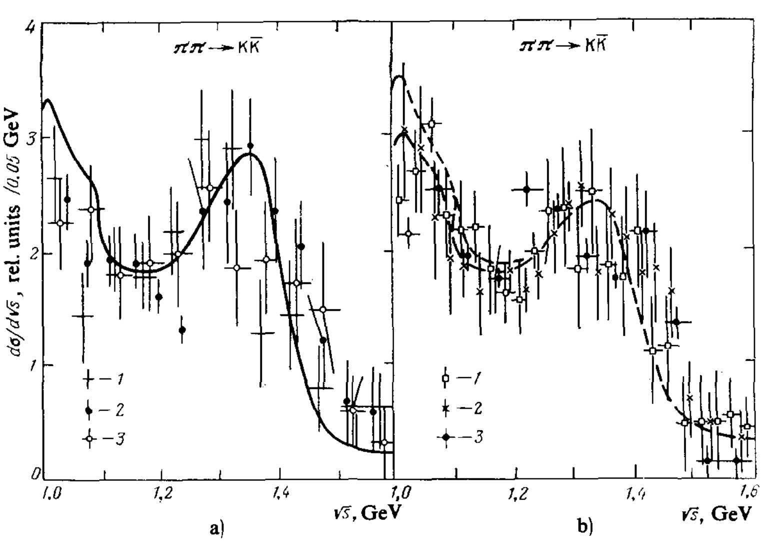

FIG. 4. The $K\bar{K}$ mass spectrum in the $\pi N\to K\bar{K}N$ process with one-pion exchange in the t-channel. $\sqrt{s}$ is the invariant mass of the $K\bar{K}$ system. $d\sigma/d\sqrt{s} = \text{const}\cdot\sqrt{s}\,\rho_{K\bar{K}} \times |T(\pi\pi\to K\bar{K})|^2$. The curves are marked as in Fig. 3. The data are as follows: (a) 1—Ref. 28, 2—Ref. 30, 3—Ref. 32; (b) 1—Ref. 29, 2—Refs. 48 and 49, 3—Ref. 51.

TABLE III. Parameters of the S* and ε resonances in the qq$\bar{q}\bar{q}$ model, obtained by fitting the $\pi\pi\to K\bar{K}$ data (with allowance for the data on the $\pi\pi\to\pi\pi$ process).

Parameter set	m_{S^*}, GeV	m_ε, GeV	$\frac{g^2_{S^*K^+K^-}}{4\pi}$, GeV^2	tg α	tg β	$C_{S^*\varepsilon}$, GeV^2
1	0,991	1,4	3	0,163	−0,12	−0,336
2	1,4	1,4	4,5	0,163	−0,098	−0,384
3	0,985	1,4	4,5	0,163	−0,098	−0,384

duces exactly the observed abrupt increase in the cross section for the $\pi\pi\to K\bar{K}$ process near the threshold [since $\Gamma_{S^*K\bar{K}}$ $(s\approx 4m_K^2)$ is small]. Sets 1 and 3 and set 2 also lead to qualitatively different behavior of the phase of the amplitude for $\pi\pi\to K\bar{K}(T(\pi\pi\to K\bar{K}))=e^{i\varphi}|T(\pi\pi\to K\bar{K})|$ near the threshold itself (see Fig. 5). Because of the large uncertainties in the phase φ near the threshold, it is difficult to decide between these parameter sets. It is important to note, however, that different experiments reveal generally different behavior of φ near the threshold (Refs. 15, 20, 29, 48, and 49). This is emphasized, for example, in Ref. 29.

It is clear the simple parametrization of the $\pi\pi\to K\bar{K}$ amplitude (16) with an elastic background in the $\pi\pi$ channel (the phase was taken in the simplest form $\delta_B = ap_{\pi\pi}$ in all the sets, where $a\approx 80°$ was determined from data on the phase $\delta_0^0(\pi\pi\to\pi\pi)$ near the $K\bar{K}$ threshold) produced a successful fit to all the main features of the behavior of $|T(\pi\pi\to K\bar{K})|$ and the phase φ. The various fits to the data showed that the coupling between S* and $K\bar{K}$ was large (superallowed) and that $g^2_{S^*K^+K^-}/g^2_{S^*\pi^+\pi^-}\approx 19 \gg 1$, as should be the case in the qq$\bar{q}\bar{q}$ model. At the same time, for example, the width in set 1 is $\Gamma_{S^*\pi\pi}(\sqrt{s}\approx 1\ \text{GeV})\approx 60$ MeV. Next, $(\text{tg}\,\beta/\sqrt{2})^2 = g^2_{\varepsilon K^+K^-}/g^2_{\varepsilon\pi^+\pi^-} \ll 1$, where (as already noted), there is actually no change in the fit if tg β is set equal to zero. It is interesting that the most stringent restriction of the qq$\bar{q}\bar{q}$ model (5), $g^2_{\varepsilon\pi^+\pi^-}\approx 0.11 g^2_{S^*K^+K^-}$, turns out to be completely admissible. Values of the order 90–140 MeV are obtained for $\Gamma_{\varepsilon\pi\pi}$ $(\sqrt{s}=1.4\ \text{GeV})$. According to the qq$\bar{q}\bar{q}$ model, the ε resonance [which belongs to the ($\underline{9}$*, 0$^+$) multiplet] has a considerable coupling to the decay channel to vector mesons, $\varepsilon\to VV$ (see Tables I and II). By using this coupling we take into account the decays $\varepsilon\to\rho\rho\to 4\pi$, $\varepsilon\to\omega\omega\to 6\pi$. The contributions of these decays to the total width $\Gamma_\varepsilon^{\text{tot}}(s)$ for $\sqrt{s}<2m_V$ are suppressed by the phase volume for the creation of unstable vector mesons, so that for $\sqrt{s}\approx m_\varepsilon\approx 1.4$ GeV they amount to about 25% of $\Gamma_\varepsilon^{\text{tot}}$ but, as $\sqrt{s}$ increases, their role also increases. For example, for $\sqrt{s}\approx 1.6$ GeV $\Gamma_{\varepsilon 4\pi}\approx 2\Gamma_{\varepsilon\pi\pi}$. Such fits to the data yield reasonable values for the parameter $C_{S^*\varepsilon}$ [cf. (13)] that characterizes the $S^*\leftrightarrow\varepsilon$ transition due to the presence of common decay channels. It is found that $C_{S^*\varepsilon}$ (see Table II) is of the same order as the contribution to the amplitude for the $S^*\leftrightarrow\varepsilon$ transition (13), which can be calculated explicitly [see (12) and (13)].

Thus, existing data on the $\pi\pi\to K\bar{K}$ process can be satisfactorily interpreted in the spirit of the qq$\bar{q}\bar{q}$ model of the S* and ε resonances.

The fit to the experimental data that we have achieved is not at all inferior to that reported in Ref. 8 where, in addition to S* (980) and ε(1470), a further new scalar resonance, the "g_S(1240)", was introduced. One may well ask why the authors of Ref. 8 have had to introduce the new resonance. The point is that the data analysis performed in Ref. 8 did not take into account the mixing of the S* and ε resonances, which cannot be justified when the resonances have common decay channels. It is now interesting to consider how the presence of the complex consisting of the S*(980) and ε(1400) resonances is reflected in the process $\pi^+\pi^-\to\eta\eta$. We have verified that the coupling of the S* to the $\eta\eta$ channel can be considerable [the same as with the $K\bar{K}$, $g^2_{S^*\eta\eta}\approx g^2_{S^*K^+K^-}$; cf. (2) and Table III]. Strong coupling of S* to $\eta\eta$ is evidently indicated by the abrupt change in the variation of the amplitude for the $\pi\pi\to K\bar{K}$ process at the $\eta\eta$ threshold[21,22,71] (see Fig. 3).

We already know that the strong coupling to the $\eta\eta$ channel is characteristic for the four-quark S* resonance. The ε resonance is also coupled to $\eta\eta$, $g^2_{\varepsilon\eta\eta}\approx g^2_{\varepsilon\pi^+\pi^-}/8$ [cf. (4) and Sec. 3B]. Our predictions for $\sigma(\pi^+\pi^-\to\eta\eta)$ are shown in Fig. 6. The nature of the $S^*-\varepsilon$ interference is here uniquely determined by the magnitude and relative signs of the resonance parameters, deduced from data on the

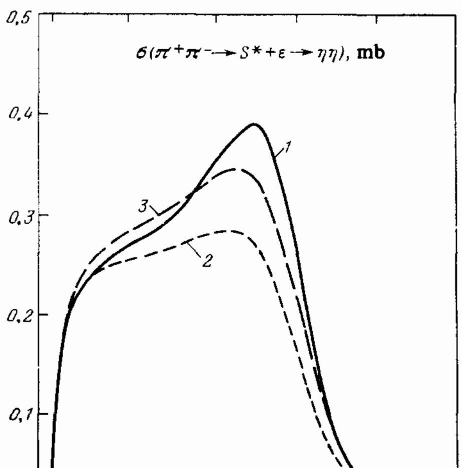

FIG. 6. Predictions for the s-wave cross section for the $\pi^+\pi^-\to\eta\eta$ process. The theoretical curves correspond to the parameter sets for the S* and ε resonances in Table III.

$\pi\pi \to K\bar{K}$ process [the dominant feature is the relative sign of the constants $g_{S^*\pi^+\pi^-}$ and $C_{S^*\varepsilon}$; see (2), (4), and (13) and Table III]. The authors Ref. 73 had already noted the possibility of studying the ε resonance in the $\pi^- p \to \eta\eta n \to 4\gamma n$ process (the cross section for which for small $|t|$ is proportional to $\sigma(\pi^+\pi^- \to \eta\eta)$ because of the single-pion exhange in the t-channel). We recall that the contributions of the f and f′ mesons with $J = 2$ to the $\pi^+\pi^- \to \eta\eta$ process are highly suppressed relative to the s-wave contribution, since the corresponding partial cross section is proportional to $\rho^5_{\eta\eta}$. It follows that the s-wave $S^* - \varepsilon$ complex may be the dominant entity. In this sense, the process $\pi^+\pi^1 - \eta\eta$ is unique. The s-wave cross sections for the processes $\pi^+\pi^- \to \eta\eta$ and $\pi^+\pi^- \to K^+K^-$ are predicted to be very similar in magnitude and character for $\sqrt{s} \lesssim 1.5$ GeV. The cross section for the process $\pi^- p \to (S^* + \varepsilon)n \to \eta\eta n \to 4\gamma n$ can be estimated readily with the aid of the formula for single-pion exchange. For $|t|_{\min} \leqslant |t| \leqslant 0.1$ $(\text{GeV}/c)^2$ and $2m_\eta \leqslant m_{\eta\eta} \leqslant 1.6$ GeV, the result for $q_{\text{lab}} = 6$, 20, and 40 GeV/c is, respectively,

$$\sigma\,(\pi^- p \to (S^* + \varepsilon)n \to \eta\eta n \to 4\gamma n) \approx 1{,}8;\ 0.16;\ 0.04\ \mu\text{b}. \tag{17}$$

These values are measurable, for example, with systems designed for the detection of the neutral decay modes of resonances. The h-meson ($J^P = 4^+$) was discovered with this system at Serpukhov in the $\pi^- p \to \pi^0\pi^0 n \to 4\gamma n$ process. The system has also been used to measure the fraction of η-meson decays into $\pi^0\gamma\gamma$ (Refs. 74 and 75). We note, for comparison, that the cross section for the process $\pi^- p \to hn \to \pi^0\pi^0 n \to 4\gamma n$ is roughly the same as for the process considered here.

We now turn to the s-wave process $\pi\pi \to \pi\pi$. The S^* resonance is indicated in this case by the abrupt change in smooth variation of the phase η^0_0 in the region of the $K\bar{K}$ threshold (see Fig. 2a). In addition, it is known independently of the process $\pi\pi \to K\bar{K}$ that the elasticity parameter η^0_0 in the amplitude $T(\pi\pi \to \pi\pi)$ also undergoes an abrupt reduction for $\sqrt{s}$ just above the $K\bar{K}$ threshold (Refs. 19, 27, 31, and 32; see Fig. 2b). It is generally accepted that the $\pi\pi \to K\bar{K}$ channel is the first main inelastic s-wave $\pi\pi$-scattering channel, i.e., that

$$\eta^0_0 = \sqrt{1 - 4\rho_{\pi\pi}\rho_{K\bar{K}}\,|T\,(\pi\pi \to K\bar{K})\,|^2} \tag{18}$$

at least up to the $\eta\eta$ threshold. The η^0_0 data near the $K\bar{K}$ threshold (see Fig. 2b) are in agreement with experimental data on $|T(\pi\pi \to K\bar{K})|$, and there is no problem in describing them together with the δ^0_0 data for $\sqrt{s} < 1.2$ in terms of the S^* resonance (which is strongly coupled to the $K\bar{K}$ and weakly to the $\pi\pi$ channels) and the elastic nonresonance background in accordance with (15) (Refs. 19, 21, 22, 30, and 71). For $\sqrt{s} > 1.2$ GeV, data on the process $\pi\pi \to K\bar{K}$ show that it is important to take into account the ε resonance. Unfortunately, in the $\pi\pi$ channel in the region $\sqrt{s} > 1.2$ GeV, not a single solution has so far been obtained for the phase and modulus of the $\pi\pi \to \pi\pi$.[1,15,32–34,37,38,40,44,45,66] The solution with the almost elastic ε resonance[11,24,38,40,44,45,66] is frequently considered to be the most acceptable. Because of the considerable coupling to the $\pi\pi$ channel, this resonance appears as a further "burst" in the phase δ^0_0 between about 270° (which it has reached up to $\sqrt{s} \approx 1.2$ GeV) and 360° (at $\sqrt{s} \approx m_\varepsilon$). Moreover, if the parameter η^0_0 is not small in the resonance region (this is indicated by the data for $\sqrt{s} \approx 1.2$–1.5 GeV; Refs. 11, 33, 40, and 50), the ε resonance should appear in $|T(\pi\pi \to \pi\pi)|$ as a minimum at $\sqrt{s} \approx m_\varepsilon$ (analogously to the S^* resonance). This picture is found in many of the solutions[1,11,15,32–34,38,40,44,66] for the processes $\pi^+\pi^- \to \pi^+\pi^-$, $\pi^0\pi^0$: $\delta^0_0 \to 360°$ for $\sqrt{s} \approx 1.5$–1.6 GeV, with a corresponding deep minimum on $|T(\pi\pi \to \pi\pi)|$ at these energies. However, it is clear that the region in which the events connected with the ε resonance develop is much broader in the $\pi\pi \to \pi\pi$ process than in the $\pi\pi \to K\bar{K}$ process, and the mass m_ε of the resonance, if it is identified with the position of the minimum on $|T(\pi\pi \to \pi\pi)|$, does not correspond with the value found from the $\pi\pi \to K\bar{K}$ process, i.e., $m_\varepsilon \simeq 1.4$ GeV. When theoretical curves for δ^0_0 (see Fig. 2a) are constructed by using the above fits to the $\pi\pi \to K\bar{K}$ data, good agreement with experiment is not achieved for $\sqrt{s} > 1.35$ GeV, δ^0_0 (theor.) passes through 360° at $\sqrt{s} \approx m_\varepsilon \approx 1.4$ GeV, and its variation is more rapid than indicated by the experimental data (here we have in mind curves 1 and 3 in Fig. 2a; curve 2 for δ^0_0 will be separately discussed in the next section). What is the conclusion from all this? We recall that the behavior of δ^0_0 and $|T(\pi\pi \to \pi\pi)|$ is now more or less established only up to $\sqrt{s} \approx 1400$ MeV, as indeed has been emphasized in many publications.[33,37,38,40,44,66] For larger $\sqrt{s}$, the situation is actually as follows: there are as many different sets of data on $|T(\pi\pi \to \pi\pi)|$ and δ^0_0 as there are different experiments, and this obtains under the condition that a unique solution can be obtained.[15,32–34,37–45,66] Actually, the measured modulus of the amplitude is known to within a substantial uncertainty.[33,37,38,42,44,45] When the s-wave with $I = 0$ is separated from the data on $\pi^+\pi^- \to \pi^+\pi^-$, $\pi^0\pi^0$, there is also the problem of correct extrapolation of the $I = 2$ wave to the region $\sqrt{s} \gtrsim 1.5$ GeV. This is particularly important for solutions for which $|T(\pi\pi \to \pi\pi)|$ has a minimum at $\sqrt{s} \gtrsim 1.5$ GeV, since there are serious doubts about the correct value of the $I = 0$ amplitude when the amplitude is small. For example, there are solutions for δ^0_0 and $|T(\pi\pi \to \pi\pi)|$ without a hint of a resonance structure for $\sqrt{s} > 1.2$ GeV and $\delta^0_0 \approx$ const (Ref. 37). Some of the solutions violate unitarity.[33,34,45,66] A very important experimental problem in this situation is the reduction of the data from $\sqrt{s} > 1.4$ to a "common denominator." On the other hand, our predictions in this region for the $\pi\pi \to \pi\pi$ process cannot, of course, be regarded as indispensable (although they were based on a good fit to the $\pi\pi \to K\bar{K}$ data) mainly because our assumption of an elastic and roughly constant background ($\delta_B = a\rho_{\pi\pi}$) for large $\sqrt{s}$ does not seem convincing. Other authors have faced a similar problem.[15,24,48,49] This conclusion is not, of course, very constructive, but one has to face reality. The problem of constructing a nonformal, i.e., not purely adjustable, model of the background in $\pi\pi$ scattering for S^*, ε, and other resonances is, in

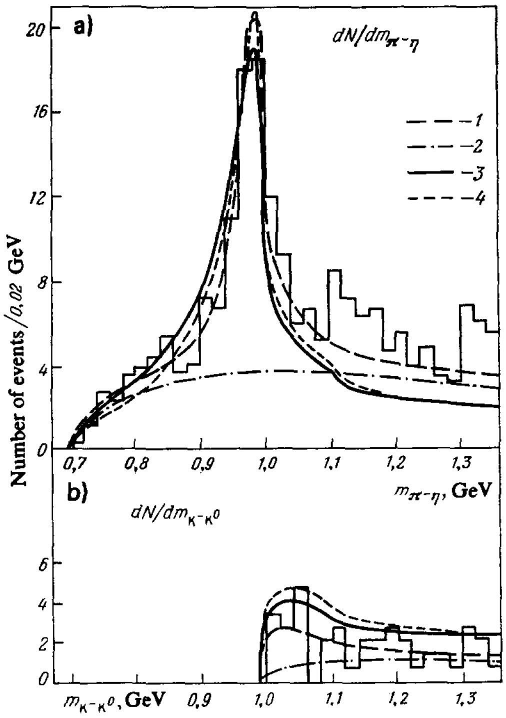

FIG. 7. $\pi^-\eta$ (a) and K^-K^0 (b) mass spectra in the $K^-p\to\delta^-\Sigma^+_{1385}\to[(\pi^-\eta), (K^-K^0)]\Sigma^+_{1385}$ p process (Ref. 59). 1—the fit produced in Ref. 59 with the aid of a narrow δ resonance ($\Gamma_{\delta\pi\eta}\approx 72$ MeV) and a high noncoherent background (cf. curve 2). Our curves 3 and 4 correspond to sets 1 and 2 in Table IV.

fact, a serious physical problem that will have to be tackled in future investigations.[4)]

We note that these difficulties are not confined to the $qq\bar{q}\bar{q}$ model of ε resonance, but are also found in the $q\bar{q}$ model and any other model in which the ε resonance is mainly coupled to the $\pi\pi$ channel.

B. Can the δ(980) resonance be a four-quark object?

Although, by now, there is no doubt about the existence of the δ resonance, there are still very few data available concerning it. The δ resonance has been clearly seen in the mass spectrum of the $\pi\eta$ and $K\bar{K}$ mesons in one experiment[59] on the process $K^-p\to\delta^-\Sigma^+_{1385}\to[(\pi^-\eta), (K^-K^0)]\Sigma^+_{1385}$ (Fig. 7) and in a few experiments on the creation of the D(1285) and E(1420) axial mesons with the decay mode[60–64] $\delta\pi\to\eta\pi\pi$, $K\bar{K}\pi$).

The mass spectra of $\pi^-\eta$ and K^-K^0 in the first of these processes have been analyzed by many authors,[18,19,21–23,59,76] using two-channel parametrization. Analyses[19,59,76] that take into account the relation $g_{\delta^-K^-K^0}=\sqrt{3/2}\,g_{\delta^-\pi^-\eta}$ [predicted by SU (3) symmetry at $\theta_{\eta\eta'}=0$] yield $\Gamma_{\delta\pi\eta}\approx 80$ MeV and a much smaller value (because of the influence of the $K\bar{K}$ channel) for the total effective width of the observed peak in the $\pi\eta$ channel: $\Gamma^{\rm eff}_\delta\approx 50$ MeV. One of these fits[59] is shown in Fig. 7. As can be seen, the fit is very good. It is clear that the four-quark model faces the most critical situation in the case of the δ resonance. Actually, the $\delta\to\eta\pi$ decay, which is suppressed by the phase volume, cannot be regarded as superallowed in this case, since $\Gamma_{\delta\pi\eta}\lesssim 100$ MeV, which is not consistent with the spirit of the $qq\bar{q}\bar{q}$ model that predicts a Zweig-superallowed coupling of scalar mesons to pseudoscalar mesons. However, let us examine the situation more carefully from different points of view, following largely the work reported in Refs. 22 and 23.

The first point is: how is information usually extracted from the data?[59] Essentially, it is assumed right from the outset that the $\pi\eta$ mass spectrum (Fig. 7a) is a narrow resonance above a high noncoherent background that is proportional to the $\pi\eta$ phase volume (Ref. 18, 19, 59, 76). A high background is also required for the explanation of the $K\bar{K}$ spectrum (Fig. 7b). However, this analysis is not valid when a search is made for traces of $qq\bar{q}\bar{q}$ states. We must try to explain the data by assuming a superallowed (strong) coupling of δ to $\pi\eta$ and $K\bar{K}$ channels. The narrow peak in the $\pi\eta$ mass spectrum must then be due to the influence of the $K\bar{K}$ channel threshold which lies next to the mass of the resonance, i.e., it is a kind of threshold feature accompanying the broad δ resonance. It turns out that the entire $\pi\eta$ mass spectrum between the threshold and 1.1 GeV can be reproduced qualitatively with the aid of a single δ resonance that is strongly coupled to $\pi\eta$ and $K\bar{K}$ (i.e., without introducing any background at all; see Fig. 7a).[22,23] This is assisted by finite width corrections to the δ propagator [see (10)–(12)], which illustrates the importance of these corrections in the case of large coupling constants.[18,21,22,23] For the $\pi\eta$ mass spectrum, we used the expression

$$\frac{dN}{dm_{\pi\eta}}=\frac{2N}{\pi}\frac{s\Gamma_{\delta\pi\eta}(s)}{|D_\delta(s)|^2},\qquad s=m^2_{\pi\eta}\tag{19}$$

(an analogous expression can also be used for the $K\bar{K}$ mass spectrum), while, for the coupling constants, we used the predictions of the four-quark model [in which $\delta^0=s\bar{s}(u\bar{u}-d\bar{d})/\sqrt{2}$]:

$$g_{\delta^-K^-K^0}=\sqrt{2}\,g_{\delta^0K^+K^-}=\frac{g_{\delta^-\pi^-\eta}}{\cos(\theta_q-\theta_{\eta\eta'})}=-\frac{g_{\delta^-\pi^-\eta'}}{\sin(\theta_q-\theta_{\eta\eta'})},\tag{20}$$

θ_q is the "ideal" mixing angle, and $\cos\theta_q=\sqrt{2/3}$, $\theta_q=35.3°$. When $\theta_q-\theta_{\eta\eta'}=45°(\theta_{\eta\eta'}=10°)$

$$\frac{g_{\delta^-K^-K^0}}{\sqrt{2}}=g_{\delta^0K^+K^-}\approx g_{\delta^-\pi^-\eta}\approx -g_{\delta^-\pi^-\eta'}.\tag{21}$$

Moreover, in the $qq\bar{q}\bar{q}$ model [see (1) and Table II]

$$g_{\delta^0K^+K^-}\cos\alpha=-g_{S^*K^+K^-}\quad or\quad g_{\delta^0K^+K^-}\approx -g_{S^*K^+K^-}.\tag{22}$$

We note that, in the fitting procedure, we also take into account the coupling of the δ to the $\pi\eta'$-channel. Table IV lists the values of the constant $g^2_{\delta^0K^+K^-}/4\pi$ and the partial width of the δ resonance calculated with the aid of this constant and (21). These correspond to the two curves in Fig. 7a. It is clear that all the intermediate variants are also possible, i.e., relation (22) is admissible (for values of $g_{S^*K^+K^-}$ see Sec. 5A). Thus, with the presently avialable statistics (see Fig. 7), it seems to us that we cannot conclude that the description of

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4)Formal agreement with the δ^0_0 data can readily be achieved by assuming, for example, that the background phase δ_B falls rapidly for $\sqrt{s}\gtrsim 1.3$ GeV. To ensure that the phase of the $\pi\pi\to K\bar{K}$ amplitude remains unaltered, we must also introduce the phase $\bar{\delta}_B$ [cf. (16)] with the restriction $\delta_B+\bar{\delta}_B\approx$ const. However, the nature of the rapid variation in δ_B is then unclear.

TABLE IV. Parameters of the δ resonance obtained with allowance for the relationships given by (21) in the analysis of the data from Ref. 59 (see also Refs. 22 and 23).

Fit No.	m_δ, MeV	$\frac{g^2_{\delta^0 K^+K^-}}{4\pi}$, $(GeV)^2$	$\Gamma_{\delta\pi\eta}$ for $s = 1\ (GeV)^2$ MeV	$\Gamma_{\delta K\bar{K}}$ for $s = 1.2\ (GeV)^2$ MeV
1 2	988 990	3 2.3	497 380	582 446

the $\pi\eta$ mass spectrum by a narrow δ resonance and a high noncoherent background is necessarily the only one. We emphasize that the δ-resonance data impose much more stringent limitations on the size of the superallowed coupling constant than, for example, the S* data.[21-23]

In the experiment reported in Ref. 59, the higher-order waves were not separated out in the $\pi\eta$ system. On the other hand, one would expect that the P-wave is very small, since it corresponds to exotic quantum numbers. The contribution of the D-wave A_2 meson must be taken into account near 1.3 GeV (see Fig. 7a). We note that, in the region up to 1.3 GeV, our fit of the dominant s-wave contribution to the $\pi\eta$ mass spectrum[59] can be appreciably improved by assuming that, above 1 GeV, the δ resonance is accompanied by some smooth s-wave contributions. Their "tails" will also appear in the region below 1 GeV. The δ resonance can then destructively interfere with them for $m_{\pi\eta} < m_\delta \approx 2m_K$ and give rise to appreciable constructive interference for $m_{\pi\eta} > 1.5$ GeV, which, obviously, improves the description of the data (see Sec. 8B in this connection).

There are very few data on the K^-K^0 mass spectrum (see Fig. 7b) in the process $K^-p \to \delta^- \Sigma^+_{1385} \to [(\pi^-\eta), (K^-K^0)] \Sigma^+_{1385}$ and, as noted in Ref. 22, these data are not very critcial to our discussion.

Let us suppose that it will be demonstrated that $\Gamma_{\delta\pi\eta} < 100$ MeV. Does this totally preclude the possibility of the four-quark composition of the δ resonance? In principle, we also have to establish whether the coupling of the δ to $K\bar{K}$ is superallowed, since there must be some particular reasons why the constants $g_{\delta\pi\eta}$ are suppressed. There is definitely no conflict between this situation and any of the currently available data.[21,23]

One further remark is in order here. A few years ago, it was noted[18] that $\Gamma_{\delta\pi\eta}$ could be large (approximately 300 MeV) but, nevertheless, the narrow structure in the $\pi\eta$ spectrum[59] could be reproduced by taking into account the strong effect of the $K\bar{K}$ channel. It is therefore sometimes considered that our analysis is simply a refined variant of the observation made in Ref. 18. This is not the case. To describe the narrow δ peak in the $\pi\eta$ mass spectrum for $\Gamma_{\delta\pi\eta} \approx 300$ MeV, Flatte[18] had to abandon the SU (3) symmetry predictions [SU(3) yields $g^2_{\delta^- K^- K^0}/g^2_{\delta\pi\eta} = 3/2$; $\theta_{\eta\eta'} = 0$ in Ref. 18], and use the ratio $g^2_{\delta^- K^- K^0}/g^2_{\delta\pi\eta} \approx 4$ in the fitting procedure in order to achieve the maximum possible effect of the $K\bar{K}$ channel on the $\pi\eta$ mass spectrum. Both in Ref. 18 and in other analyses of the data,[59] it was necessary to introduce a large, smooth, and noncoherent (?) background under the

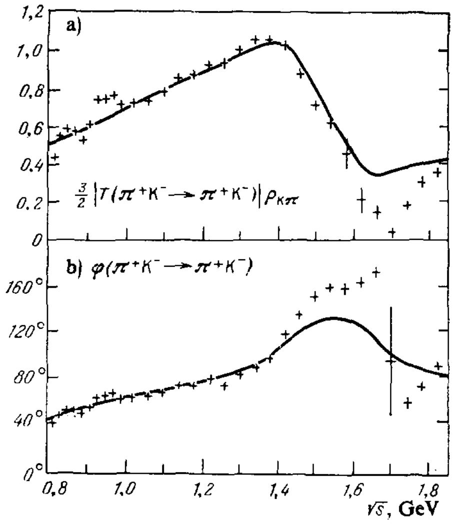

FIG. 8. Modulus (a) and phase (b) of the s-wave amplitude for the process $\pi^+K^- \to \pi^+K^-$. The points (crosses) are taken from Ref. 58. The theoretical curves are discussed in the text.

narrow δ peak in the $\pi\eta$ mass spectrum. On the other hand, we have shown[23,22] that the departure from SU (3) symmetry for large values of $\Gamma_{\delta\pi\eta}$ is not at all essential. In fact, the $\pi\eta$ mass spectrum[59] can be satisfactorily reproduced without abandoning the SU (3) symmetry by including the finite-width corrections in the δ-resonance propagator and abandoning, as already noted, the high noncoherent background in the description of the spectrum.

C. Scalar $\varkappa$ (1500) resonance

The basic data on the s-wave amplitude and phase in the $K\pi \to K\pi$ process in the broad energy range between 0.75 and 2.2 GeV were obtained relatively recently by the two groups[55,58] and have been discussed in a number of reviews and theoretical papers. (Refs. 1, 6, 9, 15, 56, and 57). Figure 8 shows typical experimental data on $|T(K^\pm \pi^\mp \to K^\pm \pi^\mp)|3\rho_{K\pi}/2 = |T_{1/2} + (1/2)T_{3/2}|$ and the phase $\varphi(T = e^{i\varphi}|T|)$, T_I is the amplitude corresponding to a particular isospin I. The amplitude $T_{1/2}$ is the dominant one and is responsible for the characteristic features in the behavior of $|T(K^\pm\pi^\mp \to K^\pm\pi^\mp)|$ and φ (Refs. 55–58). The modulus $|T|$ grows slowly and smoothly up to a maximum at about 1400 MeV, and then falls abruptly in the range 1400–1600 MeV. The behavior of the phase φ is also found to vary in this range. These properties are usually associated[1,11,15,55-58] with the s-wave resonance $\varkappa$ ($m_\varkappa = 1350$–1500 Mev, $I = 1/2$) that appears against a high continuous background.[15,55-58] The parameters of the resonance (total width, mass, elasticity, and partial widths) have not been established accurately, and only estimates are available at present.[1,11,55,58] As in the case of the $\pi\pi \to \pi\pi$, $K\bar{K}$ processes, here again, there are several possible solutions for the s wave (several sets of data) for $\sqrt{s} > 1.46$ GeV, as in Ref. 55, or for $\sqrt{s} > 1.8$ GeV, as in Ref. 58. They differ by the detailed behavior of $|T|$ and φ. It is found that the most acceptable solution (B) in Ref. 55 for the s wave with $I = 1/2$ violates the unitarity condition in the range 1.6–1.8 GeV. The most dramatic feature is that, in the last experiment,[58] it was possible to obtain with good statistics a unique solution up to $\sqrt{s}\approx 1.8$ GeV which, however, was found to violate the unitarity condition for the s wave with $I=1/2$ for $\sqrt{s}\approx 1.5$–1.8 GeV (Refs. 57 and 58). Thus, in the region that contains the most interesting resonance structure, one cannot perform a quantitative comparison between theoretical predictions and experimental data. For a correct extraction of the s wave in the region where $\sqrt{s}>1.4$ GeV (and to verify a number of unexpected features in the D and p waves; see Ref. 58), we must modify the procedure used to analyze the angular momentum data on the $K^-\pi^+$ system, obtained in Ref. 58 for the process $K^-p\to K^-\pi^+n$.

If, nevertheless, we try to draw a theoretical curve through the experimental points[58] (see Fig. 8), we find that, in the simplest parametrization, we must take into account a high elastic background and a sufficinetly broad $\varkappa$ resonance, coupled mainly to the $K\pi$ channel [see Eq. (15)]. We have also taken into account the coupling of the $\varkappa$ to the $K\eta$, $K\eta'$, $K^*\rho$, and $K^*\omega$ channels in accordance with the $qq\bar{q}\bar{q}$ model (see Tables I and II), assuming it to be a member of the ($\underline{9}^*$, 0^+) nonet. The fit to the experimental data with $m_\varkappa=1500$ MeV and $\Gamma_{\varkappa K\pi}(m_\varkappa^2)\approx 230$ MeV yields $g^2_{\varkappa K\pi}/4\pi\approx 1.6\,(\mathrm{GeV})^2$, which is greater by a factor of 1.5–1.8 than the prediction of the four-quark model:

$$g_{\varkappa K\pi}=\sqrt{\frac{3}{2}}\,g_{\varkappa^0K^+\pi^-}\approx 0.41g_{S^*K^+K^-}$$
$$\approx 0.41g_{\delta^0K^+K^-}\approx\sqrt{\frac{3}{2}}\,g_{\varepsilon\pi^+\pi^-},\qquad (23)$$

if we use the results of Sec. 5A for $g_{S^*K^+K^-}$. Of course, no important conclusions can be drawn from this because the data in the region of the resonance must be regarded as only qualitative in the "Pickwickian sense."

Allowance for the heavier resonances that may be present[58] and the background problem will be, as in the case of the $\pi\pi\to\pi\pi$ process, a subject for future theoretical investigations. However, the most immediate task is, of course, to obtain physically sensible and reliable data in the resonance region.

6. PHASE AMBIGUITY IN s-WAVE $\pi\pi$ SCATTERING ABOVE THE $K\bar{K}$ CHANNEL THRESHOLD FOR LARGE COUPLING CONSTANTS BETWEEN S*(980) AND $K\bar{K}$

In the detailed analysis of data on the processes $\pi\pi\to\pi\pi$ and $\pi\pi\to K\bar{K}$ near the $K\bar{K}$ threshold, we encountered a circumstance that seems to us to be particularly important.[71] We have found that admissible sets of theoretical fits (as a rule, the data can be fitted for a variety of parameter sets) can be divided into two qualitatively different classes. Some of the parameter sets lead to the elastic s-wave $\pi\pi$-scattering phase ($\delta_0^0=\delta_{res}+\delta_B$), whose behavior is in agreement with the generally accepted (standard) behavior of experimental data for δ_0^0 in the region of the $K\bar{K}$ threshold (see, for example, sets 1 and 3 and Fig. 2a, and Sec. 5A and Fig. 9b). At the same time, other sets, (for example, set 2 of Sec. 5A) could describe the data on δ_0^0 that are lower than the generally accepted values at 180° for $\sqrt{s}\geqslant 2m_K$ (see Figs. 2a and 9b). It was noted in Refs. 21, 22 and 71 that these nonstandard sets, and sets with the generally accepted behavior of the phase δ_0^0 can actually provide equally good description of both the existing data on $|T(\pi\pi\to K\bar{K})|$ (see Figs. 2–4) and, most importantly, the data on the normalized moments of spherical harmonics $\langle Y_l^0\rangle$ (see Fig. 9a), which determine the angular distributions of $\pi\pi$ mesons in reactions with single-pion exchange in the t-channel: $\pi^\pm N\to\pi^+\pi^-(N,\Delta)$. We emphasize that it is precisely the measured quantities $\langle Y_l^0\rangle$ that contain the primary information on the S* phenomenon,[27,30–33] while the data on the phase δ_0^0 are extracted from them by different data-processing procedures.[31–34,66] We recall that the most clearly defined manifestation of the S* phenomenon is the abrupt reduction in the magnitude of the moment $\langle Y_1^0\rangle$, which falls to zero at the threshold of the $K\bar{K}$ channel[27,30–32] (see Fig. 9a).

The question then is: is the usually reported behavior of the phase δ_0^0 correct?[12] The point is that this phase is deter-

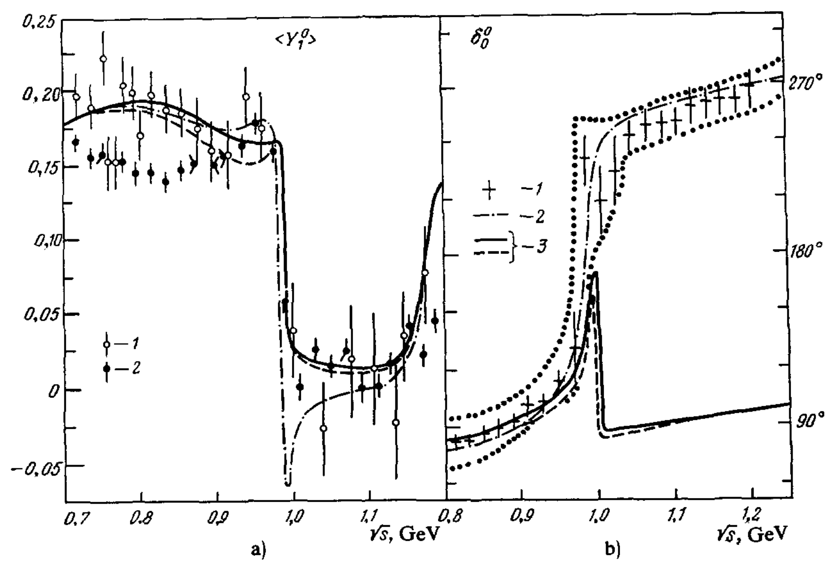

FIG. 9. (a) The moment $\langle Y_1^0\rangle$ as a function of $\sqrt{s}$ (1—data from Ref. 31 extrapolated to the pion pole; for the sake of completeness, we also show the nonextrapolated data (2) from Ref. 32, which are in good agreement with the nonextrapolated data from Ref. 31); (b) the phase δ_0^0 (1—Refs. 32 and 33); the dotted curve defines the band in which the other sets of data for the phase are confined (see Refs. 32 and 12); the other curves are taken from Ref. 71; they correspond to: 2—variant with "standard" behavior of the phase δ_0^0 in the region of the $K\bar{K}$ threshold (see Sec. 5A, Fig. 2a); 3—two close variants with "nonstandard" behavior of δ_0^0 above the $K\bar{K}$ threshold (variant 2; see Sec. 5A and Fig. 2a); see text for details; the parameters of the S* resonance in Ref. 71 are very close to those listed above in Table III.

mined at each point s to within $n\pi$, and the relative magnitude at two neighboring points is determined by continuity. In the present case, the phase changes abruptly (by roughly 100°) in a region 20–40 MeV wide near the $K\bar{K}$ threshold (see Fig. 9b), which has not been investigated in sufficient detail (by which we mean the use of a statistically adequate step in the invariant mass of the $\pi\pi$ system, of the order of 2–3 MeV). One cannot therefore say *a priori* that δ_0^0 lies in the first or third quadrant for $s \gtrsim 4m_K^2$. The generally accepted behavior of the phase[12] $s \gtrsim 4m_K^2$ is actually based on the assumption that the experimental data near the $K\bar{K}$ threshold are described by resonance formulas that are valid only for weak coupling between S* and $K\bar{K}$, i.e., without the finite-width corrections. On the other hand, the ambiguity in the behavior of δ_0^0 above the $K\bar{K}$ threshold is produced precisely by new possibilities in the description of resonance phenomena for large coupling constants between S* and $K\bar{K}$, which arise when the finite-width corrections are taken into account. They have all been described in some detail in Refs. 21, 22, and 71. The above ambiguity in the behavior of the elastic $\pi\pi$ scattering phase above the $K\bar{K}$ channel threshold can be removed only by an experimental procedure. This can be done by scanning the region of the $K\bar{K}$ threshold in the process $\pi\pi\to\pi\pi$ with a step of 2–3 MeV in an experiment with good statistics. Data on the phase of the $K\bar{K}\to K\bar{K}$ amplitude near the threshold would also assist in the solution of this problem. However, they are quite incomplete at the present time, and it is difficult to see any possible advances in this area at present. More accurate data on the phase of the amplitude for the $\pi\pi\to K\bar{K}$ process near the threshold (see Sec. 5A and Fig. 5) would also assist in resolving this difficulty. At present, the errors at the threshold are large, and all specific conclusions do not seem entirely reliable.

7. MIXING OF THE S*(980) AND δ⁰(980) RESONANCES AS A THRESHOLD PHENOMENON

The mixing of the resonances $S^*(I=0)$ and $\delta^0(I=1)$ with different isotopic spins could be ignored in our discussion above because, for example, in the $\pi\pi\to K\bar{K}$ process, the effect is confined to a very narrow region (about 10 MeV) at the threshold for the creation of K^+K^- and $K^0\bar{K}^0$, and produce minor effects, whereas, in the $\pi\pi\to\pi\pi$ process, it can be totally ignored because it appears only in the second order in the $S^*-\delta^0$ transition. There are, however, other processes in which fine effects associated with the breaking of isotopic invariance, and due to $S^*-\delta^0$ mixing, can be more appreciable. They can be studied in the processes[77,78]

$$\pi^{\pm}N\to(S^*,\ \delta^0)+(N,\ \Delta)\to\pi^0\eta+(N,\ \Delta),$$

$$(K^-,\ \bar{K}^0)+N\to(S^*,\ \delta^0)$$

$$+(\Lambda,\ \Sigma,\ \Sigma_{1385})\to\begin{cases}\pi^+\pi^-+(\Lambda,\ \Sigma,\ \Sigma_{1385}),\\ \pi^0\eta+(\Lambda,\ \Sigma,\ \Sigma_{1385}),\end{cases}$$

$$D(1285,\ I^G(J^P)=0^+(1^+))\to\delta^0\pi^0\to3\pi,$$

$$\bar{p}n_{(\text{at rest})}\to(\pi^-,\ \rho^-)\,S^*\to(\pi^-,\ \rho^-)\,\pi^0\eta,$$

although this is not an easy task. The most suitable processes

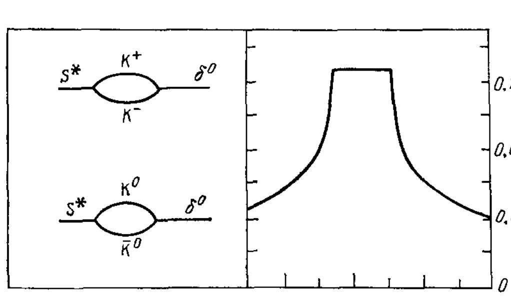

FIG. 10. (a) Diagrams responsible for $S^*-\delta^0$ mixing; (b) the function $|\rho_{K^+}-\rho_{K_0}|$ [see (24) and (25)].

are $\pi^{\pm}N\to(S^*,\delta^0)+(N,\Delta)\to(\pi^0\eta)+(N,\Delta)$, in which $S^*-\delta^0$ mixing provides us with the possibility of single-pion exchange in the t-channel, which leads to an enhancement of the $S^*-\delta^0$ interference effect that can be observed both in the mass spectrum of the $\pi^0\eta$ system and in the differential cross section $(d\sigma/d|t|)$ at $|t|<0.1\ (\text{GeV}/c)^2$. Moreover, from the theoretical point of view, there is a cardinal difference between $S^*-\delta^0$ mixing and, for example, the well-known $\rho^0-\omega$ mixing, and is therefore of independent interest. The mixing of the scalar S* and δ^0 resonances is examined in detail in Refs. 77 and 78. Here, we shall consider only the fundamental aspects of this phenomenon.

Because the masses of the S* and δ^0 resonances are close to the $K\bar{K}$ threshold, and because they are strongly coupled to the $K\bar{K}$ channel, the $S^*-\delta^0$ transition must be governed to a considerable extend by the K^+K^- and $K^0\bar{K}^0$ intermediate states (see Fig. 10a). In the region between the K^+K^- and $K^0\bar{K}^0$ thresholds, which is 8 MeV wide, the amplitude for the $S^*-\delta^0$ transition $M_{S^*\delta}$ is a quantity of the first order in the "electromagnetic" interaction, i.e., $\sim\sqrt{\alpha}$ and not α, as one would expect on the basis of general considerations.[5)] The amplitude $M_{S^*\delta}$ falls sharply outside this region, and tends to a value of the order of α. Actually, the sum of the diagrams shown in Fig. 10a converges and yields the following contribution to the $S^*-\delta^0$ transition amplitude near the K^+K^- and $K^0\bar{K}^0$ thresholds:[77]

$$\langle S^*\mid T\mid\delta^0\rangle=\sqrt{s}\,M_{S^*\delta}$$

$$=\frac{g_{S^*K^+K^-}g_{\delta^0K^+K^-}}{16\pi}\Big[i\,(\rho_{K^0}-\rho_{K^+})-\frac{1}{\pi}(\rho_{K^0}^2-\rho_{K^+}^2)+o\,(\rho_{K^0}^4-\rho_{K^+}^4)\Big],\qquad(24)$$

which we have written as a series in ρ_{K^0} and ρ_{K^+}; $\rho_K=\sqrt{1-(4m_K^2/s)}$.

Below the corresponding threshold, $\rho_K=i|\rho_K|=i\sqrt{-\rho_K^2}$. We shall refer to $|M_{S^*\delta}|$ as the "mass" of the

5)It is now clear that both the mass difference in isotopic multiplets and the mixing parameters for particles with different isotopic spins (such as $\rho^0-\omega$, $\pi^0-\eta$, $\Sigma^0-\Lambda$) are largely due to the difference between the masses of the light u and d quarks and not to the electromagnetic interaction. The quantity α that appears in our discussion is therefore an effective parameter representing isotopic symmetry breaking, and is only fortuitously equal to the electromagnetic fine-structure constant $\alpha=1/137$.

$S^* - \delta^0$ transition. The "resonance" behavior of (24) is clearly illustrated in Fig. 10b. In the region between the K^+K^- and $K^0\bar{K}^0$ thresholds, the modulus of the first term in (24) is

$$|\sqrt{s}\,M_{S^*\delta}| \approx \left|\frac{g_{S^*K^+K^-}g_{\delta^0K^+K^-}}{16\pi}\right| \sqrt{\frac{m_{K^0}^2 - m_{K^+}^2}{m_{K^0}^2}}$$

$$\approx \left|\frac{g_{S^*K^+K^-}g_{\delta^0K^+K^-}}{16\pi}\right| 0{,}127. \qquad (25)$$

We emphasize that it is theoretically uniquely determined by the coupling constants between S^* and δ^0 and the $K\bar{K}$ channel, through the unitarity condition. Other intermediate states provide, generally speaking, a contribution of the same order as the second term in (24), i.e., lower by an order of magnitude. Thus, if the coupling of the S^* and δ^0 mesons with the $K\bar{K}$ channel is not suppressed dynamically, the $S^* \to \delta^0$ transition is almost entirely determined by the first term in (24). From the theoretical point of view, the "mass" of the $S^* - \delta^0$ transition should be large in the four-quark model of the S^* and δ resonances because the constants $g_{S^*K^+K^-}$ and $g_{\delta^0K^+K^-}$ are large in this model.

When the corresponding fitting procedure is applied to the constants $g_{S^*K^+K^-}$ and $g_{\delta^0K^+K^-}$, we obtain[77,78]

$$|M_{S^*\delta}| \approx 20-50\,\text{MeV}. \qquad (26)$$

We note that the known "masses" of the $\rho^0 - \omega$, $\pi^0 - \eta$, and $\Sigma^0 - \Lambda$ transitions are respectively equal to 2–3.5, 5, and 1 MeV.

Since $M_{S^*\delta}$ is large between K^+K^- and $K^0\bar{K}^0$ thresholds, we must, in general, take into account all the orders in $S^* - \delta^0$ mixing in the amplitudes for processes with the participation of the S^* and δ^0 resonances.[77] For example, the cross section for the s-wave process $\pi^+\pi^- \to \pi^0\eta$, which is forbidden by G-parity, can be written in the form

$$\sigma(\pi^+\pi^- \to \pi^0\eta) = \frac{16\pi}{\rho_{\pi\pi}^2}\Gamma_{S^*\pi^+\pi^-}\Gamma_{\delta^0\pi^0\eta}\left|\frac{\sqrt{s}\,M_{S^*\delta}}{D_{S^*}(s)\,D_\delta(s) - sM_{S^*\delta}^2}\right|^2 \qquad (27)$$

if we assume that it occurs only as a result of $S^* - \delta^0$ mixing ($\pi^+\pi^- \to S^* \to \delta^0 \to \pi^0\eta$). Between the thresholds for K^+K^- and $K^0\bar{K}^0$ creation we have

$$\sigma(\pi^+\pi^- \to \pi^0\eta) \approx 0.25\ R_1R_2\ \text{mb}, \qquad (28)$$

where $R_1 = g_{S^*K^+K^-}^2/g_{S^*\pi^+\pi^-}^2$, $R_2 = g_{\delta_0K^+K^-}^2/g_{\delta^0\pi^0\eta}^2$. When (27) is used in this estimate, we assume that $m_\delta \approx m_{S^*} = 2m_K$, and retain in D_{S^*} and D_δ only the quantities $\sqrt{s}\Gamma_{S^*\pi\pi}$ and $\sqrt{s}\Gamma_{\delta\pi\eta}$. Moreover, we have neglected $sM_{S^*\delta}^2$ in the denominator, which can be done if $R_1 \times R_2 \lesssim 5 (R_i \lesssim \sqrt{5})$. Suppose, for example, that $R_1 \times R_2 \approx 4$, which is actually possible.[78] We then have $\sigma(\pi^+\pi^- \to \pi^0\eta) \approx 1$ mb. This is too high for a "forbidden" process. For example, the unitary limit for the s-wave process $\pi^+\pi^- \to \pi^0\pi^0$ at $\sqrt{s} \approx 2m_K$ is 4.76 mb. Outside the region $2m_{K^+} \leqslant \sqrt{s} \leqslant 2m_{K^0}$, the cross section $\sigma(\pi^+\pi^- \to \pi^0\eta)$ is found to fall rapidly, so that a resonance peak 10–15 MeV wide appears in the process.

8. DISCUSSION

A. Törnqvist analysis for scalar mesons

In the above discussion, we produced a number of indirect pieces of evidence in favor of the $qq\bar{q}\bar{q}$ origin of the S^* and δ resonances. However, as we have emphasized, the most important argument in favor of the unusual nature of the S^* and δ resonances (possibly, of $qq\bar{q}\bar{q}$ origin) is the simple fact that the masses of the S^* and δ are degenerate despite the considerable coupling between S^* and the $K\bar{K}$ channel as compared with the $\pi\pi$ channel. On the other hand, in our view, no one has succeeded in explaining the properties of the S^* in a natural manner by assuming it to be a "normal" $q\bar{q}$ resonance. In particular, we disagree with the view expressed in Ref. 1 that the problem of scalar mesons was solved in Ref. 9, i.e., that they can be described as ordinary $q\bar{q}$ mesons with a large admixture of the $qq\bar{q}\bar{q}$ component in the form of virtual two-meson states, without introducing a special 0^+ $qq\bar{q}\bar{q}$ nonet. In an interesting, in our view, paper,[9] Törnqvist clearly demonstrated the importance of taking into account the interaction in the final state of scalar mesons. It was also shown that the mass spectrum of the 0^+ resonances, which is unusual for a two-quark system, can be explained with the aid of unitary corrections by assuming for the coupling constants on the mass surface some very unusual departures from the quark-counting relations. This can hardly be regarded as a solution of the scalar meson problem. Our view is that the problem has merely been transferred elsewhere. It i s assumed in Ref. 9 that the quark-counting rules are satisfied at the thresholds of the decay channels:

$$g_{Rab}(s) = \gamma_{Rab}F(K_{\text{cm}}) = \gamma_{Rab}e^{-(K_{\text{cm}}/K_{\text{cut off}})^2}, \qquad (29)$$

where $m_R\Gamma_{Rab} = g_{Rab}^2(K_{\text{cm}})^{2l+1}/\sqrt{s}$[9,79] and γ_{Rab} are the coupling constants of the resonance R that obey the quark-counting rules. The physical coupling constants g_{Rab} contain the formfactor $F(K_{\text{cm}})$ (Refs. 9 and 79) that violates the quark-counting rules on the mass surface of the resonance, K_{cm} is the momentum of particle a (or b) in the rest frame of the resonance R, and $K_{\text{cut off}} = 0.7$ GeV (Ref. 9). Here, we can immediately see the difficulty encountered by Törnqvist[9] in describing the nonet of tensor mesons (f, A_2, K^{**}, f′) which, in the $q\bar{q}$ model, are completely similar to the 0^+ mesons, since it is generally accepted that there is little (ls)-coupling (see, for example, Ref. 9). By using (29), we find that the ratio $\Gamma_{fK\bar{K}}/\Gamma_{f\pi\pi}$ is greater by a factor of 2.5 than the experimental result. On the other hand, simple quark counting, performed for the coupling constants on the mass surface [Eq. (29) without the formfactor], results in excellent agreement with experiment: $(\Gamma_{fK\bar{K}}/\Gamma_{f\pi\pi})_{\text{q.count}}/(\Gamma_{fK\bar{K}}/\Gamma_{f\pi\pi})_{\text{exp}} = 1.06 \pm 0.11$. Quark counting is also in good agreement with the data on $\Gamma_{A_2K\bar{K}}/\Gamma_{A_2\pi\eta}$. On the other hand, despite the considerable difference between the phase volumes of the final states in the $A_2 \to K\bar{K}$, $A_2 \to \pi\eta$ decays, the method used in Ref. 9 leads to a value of $\Gamma_{A_2K\bar{K}}/\Gamma_{A_2\pi\eta}$ that is greater by a factor of about five than the experimental value (for $K_{\text{cm}}(\pi\eta) = K_{\text{cm}}(K\bar{K})$, the predictions made in Ref. 9 would coincide with the results of quark counting on the mass surface).

Moreover, the analysis reported in Ref. 9 cannot be regarded as complete because the author of that paper has not examined the extensive data on the $\pi\pi\to K\bar{K}$ process (simultaneously with the process $\pi\pi\to\pi\pi$ in the region of $2m_K \sim 1.6$ GeV), which yield the greatest amount of information about the properties of the S* and ε resonances. Experience shows that this examination can yield many surprises. For example, it may turn out that, even in the scheme put forward in Ref. 9, in which a phenomenological formfactor is used [see (29)], it is not sufficient to include in the amplitude for $S^*\leftrightarrow\varepsilon$ only the contributions of intermediate pseudoscalar mesons $\pi\pi$, $K\bar{K}$, $\eta\eta$,..., as was done in Ref. 9, i.e., they must be augmented, for example, by introducing an additional constant [such as $C_{S^*\varepsilon}$; see (13)], which effectively takes into account the contributions of other intermediate states. This became clear in the course of our analysis.

B. The broad $(qq\bar{q}\bar{q})$ $\delta(980)$ resonance and the decays $\eta'\to\eta\pi\pi$; D, $E\to\eta\pi\pi$

The interpretation of the δ resonance in terms of the four-quark state, given in Sec. 5b, gave rise to objections in Ref. 10, where the following questions were examined: (1) the decay $\eta'\to\eta\pi\pi$ (2) the decays D, $E\to\delta\pi\to\eta\pi\pi$ and (3) the contributions of the "tadpoles" to the electromagnetic mass difference and mixing. It was concluded that the wide $qq\bar{q}\bar{q}$ resonance $\delta(980)$ was in conflict with experimental data.

It was not our aim, either in our previous papers[21–23] or here, to defend at all costs the $qq\bar{q}\bar{q}$ model. However, we must now examine arguments put forward by Bramon and Masso[10] and show that none of them is convincing and some are simply incorrect.[80]

(1) The $\eta'\to\eta\pi\pi$ decay.[6)] Bramon and Masso[10] assume that the $\eta'\to\eta\pi\pi$ decay occurs mainly through the intermediate δ meson: $\eta'\to\delta\pi\to\eta\pi\pi$. The experimental results is[1] $\Gamma_{\eta'\to\eta\pi\pi}\approx 180$ keV. If we take the value of $g^2_{\delta\pi\eta}/4\pi$ that corresponds to $\Gamma_{\delta\pi\eta}\approx 300$ MeV, and $g^2_{\delta\pi\eta'}\approx g^2_{\delta\pi\eta}$ as predicted by the $qq\bar{q}\bar{q}$ model (with $\eta-\eta'$ mixing angle $\theta_{\eta\eta'}\approx -10°$), then $\Gamma^{\text{theor}}_{\eta'\to\eta\pi\pi}$ turns out to be too high by roughly two orders of magnitude. What does this lead to? The immediate consequence is that the $qq\bar{q}\bar{q}$ model for the δ resonance is invalid. In precisely the same way, the assumption of the dominance of the δ resonance in this decay may also be incorrect. Having noted the conflict with the four-quark MIT-bag model, the authors of Ref. 10 ignore the fact that it is precisely in this model that one cannot ignore contributions other than those of the δ resonance to the $\eta'\to\eta\pi\pi$ decay. Indeed, the δ resonance in this model is accompanied by broad scalar resonances with isospin $I=1$ from other multiplets, namely, $C_\pi(\underline{36})$, $C^s_\pi(\underline{36})$, $C^s_\pi(\underline{9}^*)$, with masses of 1150, 1550, and 1800 MeV, respectively.[5] $C_\pi(\underline{36})$ is not coupled to the $K\bar{K}$ channel and, therefore, does not influence the description of the $K^-p\to[(\pi^-\eta),(K^-K^0)]\Sigma^+_{1385}$ process. The $C_\pi(36)$ (like the δ resonance) is coupled to the $\pi\eta$, $\pi\eta'$, and $K\bar{K}$ channels. Its contribution to the $\pi\eta$ mass spectrum in the $K^-p\to\pi^-\eta\times\Sigma^+_{1385}$ process in the region of the δ resonance takes the form of a low continuous coherent background, whose sign, according to the $qq\bar{q}\bar{q}$ model, is such that its inclusion improves the fit made in Refs. 22 and 23 (see the discussion at the end of Sec. 5B). The $\pi\eta$ mass spectra of the δ, C_π, and both C^s_π resonances differ from one another by the fact that the first of them shows a rapid rise near the $K\bar{K}$ threshold, but this feature is absent for the other two (since their masses lie above the $K\bar{K}$ threshold). However, the "tails" of all these resonances contribute to the $\eta'\to\eta\pi\pi$ decay, and must be taken into account on the same basis. To explore the kind of quantity that we are actually dealing with, we shall make literal use of the MIT bag predictions[5,70] for the masses and coupling constants of the δ, $C_\pi(\underline{36})$, $C^s_\pi(\underline{36})$, and $C^s_\pi(\underline{9}^*)$ resonances, neglecting their instability for the sake of simplicity. If we then consider the amplitude for the decay process

$$\eta'\to[\delta+C_\pi(\underline{36})+C^s_\pi(\underline{36})+C^s_\pi(\underline{9}^*)]\,\pi\to\eta\pi\pi$$

we find that the hindrance factor for the contribution of the single δ resonance is

$$F=\left[1-1.50\left(\frac{m^2_\delta-s}{m^2_{C_\pi(\underline{36})}-s}\right)+0.75\left(\frac{m^2_\delta-s}{m^2_{C^s_\pi(\underline{36})}-s}\right)\right.$$
$$\left.+0.057\left(\frac{m^2_\delta-s}{m^2_{C^s_\pi(\underline{9}^*)}-s}\right)\right]\Bigg|_{\sqrt{s}=754\text{ MeV}}$$
$$\approx 1-1{,}28+0{,}26+0{,}01\approx -0.01, \qquad (30)$$

the value of which is reduced to the midpoint of the allowed interval for the invariant mass of the $\pi\eta$ system ($\sqrt{s}=754$ MeV) in the $\eta'\to\eta\pi\pi$ decay. We emphasize once again that the relative signs and magnitudes of the contributions in (30) are not the result of a fit to experimental data, but were taken in accordance with the MIT bag model[7)] (see Refs. 5, 70 and 7). Thus, it is clear that the MIT bag model does not provide a specific prediction for the $\eta'\to\eta\pi\pi$ decay although a reasonable value for $\Gamma_{\eta'\to\eta\pi\pi}$ can be readily obtained by slightly varying the masses of the resonances, taking into account their instability, possible variation in the relationships between the coupling constants, and so on.

The theoretical difficulties in describing $\Gamma_{\eta'\to\eta\pi\pi}$ decays in terms of the "primitive" $qq\bar{q}\bar{q}$ states of the MIT bag model are, of course, related in the first instance to resonances in the $\pi\pi$ system, $\eta'\to(\varepsilon+...)\eta\to\pi\pi\eta$. The difficulties here are the same as in the s-wave $\pi\pi\to\pi\pi$. The amplitude for this process is small at the threshold, although it could be quite high owing to the wide $\delta(650)$ $qq\bar{q}\bar{q}$ resonance.[5] However, it would be rather naive of us to seek here a contradiction with the MIT bag model in which, on the one hand, the requirements of chiral symmetry[82] are not taken into account and, on the other, the superallowed coupling of ε to $\pi\pi$ prevents us from looking upon it as a separate resonance (pole approximation). The ε meson, in fact, merges into the background and appears as a pole in the P-matrix.[6]

[6)]The remark on the quark structure of $\delta(980)$ made in Ref. 10 in connection with the $\eta'\to\eta\pi\pi$ decay was subsequently repeated in Ref. 81.

[7)]We recall that, in the simplest variant, the coupling between the lowest scalar four-quark states (states that are not excited in the orbital angular momentum or the radial quantum number) in the $\underline{9}$, $\underline{9}^*$, $\underline{36}$ U(3)-multiplets and the pseudoscalar (and vector) mesons, of which they "consist," is determined by the single constant g_0.[5]

(2) The decays D, E→δπ→ηππ. It is maintained in Ref. 10 that the qq$\bar{q}\bar{q}$ model for the δ resonance predicts that $R \equiv \Gamma(E\to\delta\pi)/\Gamma(D\to\delta\pi) \gg 1$ instead of $R_{exp} \lesssim 1$. Since the flavor structure of the δ meson is usually (and schematically) represented by $\delta^0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$, it seemed to the authors of Ref. 10 that the E meson, an almost pure s$\bar{s}$ state ($J^P = 1^+$), should readily decay into δπ, and the D meson [largely a $(u\bar{u} + d\bar{d})/\sqrt{2}$ state ($J^P = 1^+$)] should decay into δπ only as a consequence of a small s$\bar{s}$ admixture. However, this approach only *appears* to be natural. The conclusion would be valid if the coupling of the δ to the 1^+0^- system were Zweig-superallowed (as, for example, the coupling of the δ to πη, πη', K$\bar{K}$). The authors of Ref. 10 ignore the structure of the wave function of the four-quark δ meson which, in the MIT bag model, appears to consist of only the 0^- and 1^- pairs of q$\bar{q}$ mesons with which it has superallowed coupling.[5,7] The wave function of the δ has the form[70] (see Table I)

$$|\,\delta\rangle = |\,0.743PP - 0.041VV + 0.646\underline{V}\underline{V} - 0.169\underline{P}\underline{P}\rangle, \quad (31)$$

where P, V and $\underline{P}$, $\underline{V}$ are colorless 0^-, 1^- and colored 0^-, 1^- q$\bar{q}$ states. It does not contain the 1^+0^- component. The coupling between δ and Eπ and Dπ is accomplished in this case in the same order in α_s, for example, it is due to the diagrams shown in Fig. 11 and involving gluon exchange [because the $\underline{V}\underline{V}$ component is present in (31)]. These diagrams cannot, of course, be evaluated at present, but the main point is that there is no reason why the E meson should decay more readily into δπ than into the D meson. It is clear that $R_{exp} \lesssim 1$ does not present a challenge in any way to the four-quark MIT bag model.

(3) Bramon and Masso[10] have also discussed the possibility of using the δ^0 meson to explain the electromagnetic mass differences and mixing in the vector and pseudoscalar meson nonets in the "tadpole" model. It is now generally accepted, and the authors of Ref. 10 agree with this, too, that the best realization of the old "tadpole" mechanism is the SU(2) breaking scheme for the u and d quark masses. As far as we understand it, the contribution of the "tadpoles" to the electromagnetic mass differences and mixing was discussed in Ref. 10 in order to complete, so to speak, the range of objections to the broad (qq$\bar{q}\bar{q}$) δ resonance. We shall not pause to examine this question in detail and shall confine ourselves merely to noting that this argument is also due to a misunderstanding based on a naive utilization of the symbolic form of the wave function for the (qq$\bar{q}\bar{q}$) δ resonance. We emphasize that, had we wished to explain the electromagnetic mass differences and mixing of the qq$\bar{q}\bar{q}$ tadpoles, this would not have presented us with any fundamental difficulties in the MIT bag model (see Ref. 8 for further details).

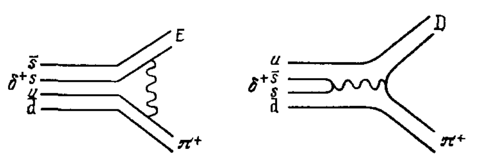

FIG. 11. Zweig-nonsuperallowed diagrams of the first order in α_s that are responsible for the coupling of the δ resonance to the Eπ and Dπ mesons. The wavy line corresponds to a gluon.

C. Where is the scalar q$\bar{q}$ nonet?

We must now say a few words about scalar mesons in the q$\bar{q}$ system. We shall suppose that four-quark interpretation of the "tabulated" scalar mesons is correct. This means that we have to face two questions: firstly, what has happened to the p-wave 0^{++}q$\bar{q}$ nonet, and, secondly, why is there no mixing between the q$\bar{q}$ and qq$\bar{q}\bar{q}$ scalar mesons?

The former question has two answers. Firstly, if the masses of the scalar q$\bar{q}$ mesons are high (more than 1.7 GeV as, for example, in Ref. 4), they simply cannot be seen in the two-particle channels, since there is practically no published work on the analysis of phase shifts and inelasticities in this region. Moreover, it is very probable that these resonances are highly inelastic in this case (i.e., they decay into multiparticle states), so that it is more efficacious to seek them in two-particle processes. Generally speaking, we note that any search for zero-spin particles with masses in the region of 2 GeV in hadronic reactions is an exceedingly difficult task. Secondly, if the masses of the scalar q$\bar{q}$ mesons lie in the range 1.2–1.7 GeV, they can also be highly inelastic (as, for example, the vector ρ'(1600)), and have not, therefore, been seen in two-particle processes.

Now a few words about mixing. If the masses in the q$\bar{q}$ nonet are large (greater than 1.7 GeV), the weak mixing between them and lighter qq$\bar{q}\bar{q}$ states in not too surprising. If, on the other hand, the masses in the q$\bar{q}$ nonet are roughly the same as in the qq$\bar{q}\bar{q}$ nonet, the absence of mixing is of course surprising. The question is: does it follow from the analysis given in Sec. 5 that the ε, ϰ are pure qq$\bar{q}\bar{q}$ states? The answer is, of course, that they are not. At the same time, it is difficult to imagine that the S* and δ mesons have a large admixture of the q$\bar{q}$ component. It is also not clear how one can simultaneously explain the degeneracy of the S* and δ masses and the suppression of coupling between S* and the ππ-channel.

We now note one further point. If the ε (1300-1400) resonance were a member of the "ideal" q$\bar{q}$ nonet with $(u\bar{u} + d\bar{d})/\sqrt{2}$ quark structure, it would not have a partner with $I = 1$ and a mass of 1300–1400 MeV (the $(u\bar{u} + d\bar{d})/\sqrt{2}$ state). Reported traces of this state[15,20,83] are, however, very unreliable, and the state is not included in particle tabulations.[1,11] At the same time, if ε(1300–1400), ϰ(1350–1500), and the recent[29] S*'(1770) resonances belong to the ($\underline{9}^*$, 0^+) qq$\bar{q}\bar{q}$ nonet, the discovery of the $I = 1$ resonance with a mass of about 1770 MeV width of 100–200 MeV as a partner of the S*'(1770) would be a very powerful argument in favor of the four-quark model. This resonance must be looked for in the mass spectra of the ρφ, K*$\bar{K}$*, K$\bar{K}$, πη, and πη' channels.

9. CONCLUSION

The above discussion seems to us to lead to the conclusion that the scalar S*, ε, δ, and ϰ resonances are very unusual and interesting objects, the nature of which is not as yet finally understood. We have also tried to show that the physics of scalar mesons has presented us with a number of questions that are of decisive importance for phenomenological models based on the ideas underlying the modern theory of strong interactions (color-trapping, quantum chromodynamics). To conclude, we list some of them.

(1) There is no doubt that, among the 0^+ resonances, there is one resonance, namely, S*(980), that has some very unusual properties. No matter what approach we take to the description of other scalar resonances, the degeneracy of the masses of S*(980) and δ (980) mesons, and the considerable coupling between S* and $K\bar{K}$ as compared with $\pi\pi$ is most naturally explained in terms of the four-quark structure of the S*.

(2) Although there are now extensive data on scalar mesons, they are still insufficient to enable us to choose reliably between different descriptions of them. Further efforts will be necessary in the region between 1.3 and 2 GeV (and, generally, near the $K\bar{K}$ threshold) in the $\pi\pi\to\pi\pi$, $\pi\pi\to K\bar{K}$ processes in order to obtain a clearer experimental picture of the behavior of the moduli and phases of the amplitudes for these processes. Theoretical (phenomenological) analyses of the data on scalar mesons cannot be regarded as impeccable either (some aspects of these data are essentially in a rudimentary state; for example, the question of the background is of this kind if we consider a nonformal description, i.e., data fitting).

(3) It is interesting to investigate new processes with the participation of the S*(980) and ε(1300–1400) resonances, for example, $\pi^+\pi^-\to\eta\eta$.

(4) Modern data require no new resonances in the range 1–1.6 GeV other than the tabulated data.[1]

(5) There is no indication that a scalar SU(3) singlet state (gluonium) exists in this region.

(6) Very interesting additional information on the quark composition of the S* and δ resonances can be expected from the $\gamma\gamma\to\pi\pi$ and $\gamma\gamma\to\pi^0\eta$ processes, respectively. Here, we face the very intriguing fact that may be connected with the unusual nature of the S* and δ resonances, namely, in contrast to the f and A_2 mesons, these resonances have not been seen in $\gamma\gamma$ collisions.[84–86] This is readily understood in terms of the $qq\bar{q}\bar{q}$ model. If the scalar S* and δ mesons are four-quark states from the ($\underset{\sim}{9}$, 0^+) MIT bag nonet, their coupling to the $\gamma\gamma$ should be highly suppressed.[7]

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