Dynamic equations of the theory of elasticity of metals

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In pure metals at low temperatures, when the mean free path of conduction electrons is very large, the electron contribution to elastic forces becomes nonlocal. The forces that appear in the equations of motion of the lattice are then functionals of the electron distribution. The dynamics of conduction electrons, and the influence of the external magnetic field and of the self-consistent electric fields then become important. In this review, we examine the equations of elasticity, using the general assumptions of the modern theory of metals, based on the model-independent macroscopic approach. A detailed discussion is given of the deformation potential and its symmetry properties, and of the role of directional symmetry of the magnetic field. The effective interaction between electrons and sound waves and the role of electric fields accompanying an elastic wave are discussed as examples.

CONTENTS

INTRODUCTION

In pure metals at low temperatures, when the mean free path of conduction electrons is very long and substantially exceeds not only the interatomic distances but also the characteristic scales of variation of elastic fields (for example, the wavelength of an acoustic wave in a metal), the electron contribution to elastic forces becomes nonlocal and requires special examination. Slow relaxation processes in the electron gas, which leaves the state of equilibrium when the lattice is deformed, transform the theory of elasticity into a kinetic theory in which one has to consider both the dynamic equation of motion of the lattice and the equation for the quasiparticle (conduction electron) distribution function (the density matrix, in the quantum-mechanical case). The forces that appear in the equations of motion of the lattice are found to be functionals of the electron distribution. The dispersion relation, and the external magnetic field that influences the electron dynamics, then play an essential role in the overall picture.

Finally, Maxwell's equations describe the self-consistent electromagnetic fields that arise in the case of nonequilibrium states of the electron subsystem and accompany the deformation of the lattice.

Under the above conditions, mechanical properties begin to depend on the resonance and relaxation parameters of the degenerate electron gas in the metal, such as the mean free path l, the orbital radius r in the magnetic field, the skin depth δ, the collision frequency ν, the cyclotron frequency Ω, and so on. These lengths are much greater than the interatomic separations (l, r, $\delta \gg a$) and the frequencies are much lower than the atomic and plasma frequencies (ν, $\Omega \ll \omega_{at}$, ω_p). This effect of conduction electrons was first discovered in experiments on the absorption of ultrasonic waves, and was successfully exploited in the diagnostics of the electron

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gas in metals.[1)]

However, for a long time, the theory was not self-consistent in the sense that absorption of ultrasonic waves was calculated from the rate of increase of entropy in the electron subsystem of the metal, and it was sufficient to consider the acoustic field as given. The self-consistent approach was introduced into the theory of free electrons by Steinberg,[16] and the description used in the presence of a magnetic field was equivalent to magnetic hydrodynamics, which was used by Alpher and Rubin[17] to examine the effect of the field on the velocity of sound. Electromagnetic excitation of sound in metals was considered in Ref. 18.

The transition to another range of problems (measurement of the dispersion of the velocity of sound, electromagnetic excitation of sound in metals by contactless methods, and studies of electronic retardation of dislocations, the effective interaction of electrons with sound, electronic rotation of the plane of polarization of sound, and so on) have stimulated interest in the self-consistent description and in the derivation and utilization of the equations of elasticity that are valid under the quite general assumptions underlying the modern theory of metals.[19–21] The present review is devoted to this range of topics.

On the other hand, such questions are related to the general problem of the electron-phonon interaction and lead to a relatively detailed discussion of its long-wave aspect.

The analysis of forces acting in a deformed metal has attracted a relatively large number of publications,[19–30] which we shall consider below, confining our attention to the model-independent macroscopic approach in the quasiclassical region.

Thus, for sufficiently long electron mean free paths, the transport equation for conduction electrons, the Maxwell equations, and the equations of the theory of elasticity constitute a closed set of equations. This set was put forward by Silin[19] (under certain restrictions on the topology of the Fermi surface in the collisionless region; cf. Sec. 5) and was derived by Kontorovich[20] for an arbitrary topology, and taking into account collisional momentum transfers. An analogous set of equations was introduced by Vlasov and Filippov.[21]

The structure of the present review is as follows. The equation of elasticity is obtained in Sec. 1, using only the law of conservation of momentum and the phenomenological form of the contribution of conduction electrons. The complete set of linearized equations, the derivation of which is given later (Sections 3 and 4), is introduced in Sec. 2 for convenience. Various ways of writing down the equations of elasticity are compared in Sec. 5. The equations for elastic waves with allowance for the contribution of conduction electrons and the fields excited by them are then written down (Sec. 7) in terms of the transport coefficients in the deformed metal (Sec. 6). Section 8 discusses the deformation potential and its symmetry properties (Sec. 9), including the role of directional symmetry of the magnetic field (Sec. 10). Sections 11–13 consider examples of the "effective" interaction between electrons and sound, and the role of electric fields that accompany the elastic waves.

1. CONDUCTION-ELECTRON TERMS IN THE ELASTIC FORCE

Quasiparticles, i.e., conduction electrons, in a deformed metal experience not only the Lorentz forces that include macroscopic electromagnetic fields, but also forces due to the electron-lattice interaction that results from the change in the microscopic atomic electric fields. In its turn, the lattice experiences the reaction of the electron gas, and this is responsible for the appearance in the equations of elasticity of terms that are averages over the local distribution of quasiparticles at a given point in the metal.

Forces on conduction electrons that are due to the deformed lattice are electromagnetic in character, but only the smooth macroscopic fields can be taken into account with the aid of the Maxwell equations.

It is very important to remember that the contribution of the microscopic atomic fields due to the lattice for deformations that vary over distances much greater than the interatomic separation (Fig. 1) can be taken into account in a very general form in terms of the change in the dispersion relation for conduction electrons[2]

$$\varepsilon(\mathbf{r}, \mathbf{p}, t) = \varepsilon_0(\mathbf{p}) + \delta\varepsilon(\mathbf{r}, \mathbf{p}, t), \tag{1.1}$$

so that a closed theory can thus be constructed in a purely phenomenological manner. Conduction electrons can than be looked upon, as before, as a perfect gas (Fermi liquid) of quasiparticles in the self-consistent electromagnetic and acoustic fields, and this enables us to exploit the well-established formalism of the electron theory of metals.[5,12] The most general and, probably, the simplest derivation of the contribution of conduction electrons to the force is to utilize the connection between the mean momentum of particles and quasiparticles,[20] first used by Landau in the theory of the Fermi liquid[31,32] to determine the connection between the effective mass of excitations in liquid He^3 and the mass of the helium atom. In our case, the quasiparticle distribution

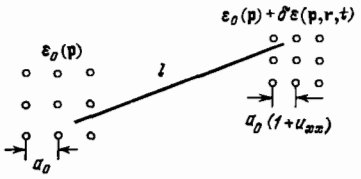

FIG. 1. The electron distribution is established over the mean free paths l, and the dispersion relation over the atomic distances a.

[1)]The absorption of ultrasonic waves in solids was first examined microscopically by Landau and Rumer[1] in the case of perfect dielectrics. Studies of the absorption of the ultrasonic waves in conductors were begun by Akhiezer[2] and extended to the case of strong spatial dispersion by Pippard[3] and by Akhiezer, Kaganov, and Lyubarskiĭ.[4] The scale of such studies has expanded considerably in connection with attempts to determine the energy spectrum and the associated studies (mainly by I. M. Lifshitz and his school) of the dynamics of quasiparticles with an arbitrary dispersion relation.[5] The discovery by Bommel of magnetoacoustic oscillations and the possibility noted by Pippard of using them to determine the parameters of the Fermi surface have stimulated intensive studies of the absorption of ultrasonic waves in a magnetic field (cf. references in the reviews given in Refs. 6–10). The latter became possible as a result of advances made in the study of galvanomagnetic and resonance phenomena in metals.[5,11–13] The general theory of absorption of ultrasonic waves in metals in a magnetic field was constructed by V. L. Gurevich[14] and requries a more precise formulation of the concept of the deformation potential introduced in Refs. 2 and 15. The essential role of electric fields excited by sound was elucidated by Gurevich.

function $f(\mathbf{r}, \mathbf{p}, t)$ is related to the density $n(\mathbf{r}, t)$ of free electrons by the normalization condition

$$\langle\langle f\rangle\rangle \equiv \frac{2}{h^3}\int d\mathbf{p}\, f(\mathbf{r}, \mathbf{p}, t) = n(\mathbf{r}, t). \tag{1.2}$$

The integral is evaluated within the confines of the Brillouin zone,[2] and summation over conduction bands is implied. As a consequence of the equation of continuity, the quasiparticle flux $\langle\langle(\partial\varepsilon/\partial\mathbf{p})f\rangle\rangle$ is equal to the particle (electron) flux. The electric (electron) current density is therefore given by

$$\mathbf{j}^{el} = e\left\langle\left\langle\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle\right\rangle, \tag{1.3}$$

and the mass flux density, i.e., the free-electron momentum density, can be expressed in terms of the distribution function and the quasiparticle dispersion relation, as follows (e and m are, respectively, the charge and mass of the free electron):

$$\pi^{el} = m\left\langle\left\langle\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle\right\rangle. \tag{1.4}$$

It is clear that the expressions for the densities $\mathbf{j}^{el}$ and π^{el} have the same degree of generality.

We note that crystals differ from liquid helium in that, since $\mathbf{p}$ is the quasimomentum, the difference $\langle\langle\mathbf{p}f\rangle\rangle - m\langle\langle(\partial\varepsilon/\partial\mathbf{p})f\rangle\rangle$ is not, in general, equal to zero, so that (1.4) does not lead to any general consequences for the effective mass of the excitations that would be analogous to those found by Landau for He^3.

It is convenient to express the momentum density in terms of the current density:

$$\pi^{el} = \frac{m}{e}\mathbf{j}^{el}. \tag{1.5}$$

Since, at the relatively low frequencies ($\omega \ll \omega_{pe}$) in which we are interested, the metal as a whole is electrically neutral, and recalling the local character of the deformational interaction noted above, the equation of motion assumes the form[20]

$$\frac{\partial\pi_i}{\partial t} = \frac{\partial}{\partial x_k}(\sigma_{ik}^{lat} + \sigma_{ik}^{el}) + \frac{1}{c}[\mathbf{j}, \mathbf{H}]_i, \tag{1.6}$$

where $\pi = \pi^{lat} + \pi^{el}$ is the total momentum density of the lattice ($\pi^{lat} = \rho_{lat}\dot{\mathbf{u}}$) and of the electrons, $\mathbf{j} = \mathbf{j}^{el} - en\dot{\mathbf{u}}$ is the current density, including the ion contribution, $\dot{\mathbf{u}}$ is the local deformation rate, and $\mathbf{u}$ is the displacement vector. The last term in this equation is obviously the average Lorentz force acting on the electrically neutral electron-ion system. The stress tensor contains electron terms that are averages involving the electron distribution function at a given point in the lattice at a given time (which, for reasons discussed in Secs. 2 and 3, will be denoted by f'). We now expand the expression to be averaged in powers of $\partial u_i/\partial x_k$:

$$\sigma_{ik}^{el} = \langle\langle\lambda_{ik}f'\rangle\rangle + \langle\langle\mu_{iklm}f'\rangle\rangle\frac{\partial u_l}{\partial x_m} + \ldots \tag{1.7}$$

Expressing ρ_{lat} in terms of the density of the metal $\rho = \rho_{lat} + mn$, and using the definitions of π^{lat}, π^{el}, and $\mathbf{j}$, we can rewrite (1.6) in the form

$$\rho\ddot{u}_i = f_i, \quad f_i = \frac{\partial\sigma_{ik}}{\partial x_k} + \frac{1}{c}[\mathbf{j}, \mathbf{H}]_i - \frac{m}{e}\frac{\partial j_i}{\partial t} + \frac{\partial}{\partial x_k}\langle\langle\lambda_{ik}f'\rangle\rangle, \tag{1.8}$$

where the electron stress tensor σ_{ik}^{el} contains the leading term of the expansion in u_{ik} and the remaining terms are included together with σ_{ik}^{lat} in σ_{ik}. The term $(m/e)\partial\mathbf{j}/\partial t$ that contains the free-electron mass is responsible for the Stewart-Tolman effect. We note that, in contrast to the terms describing the deformational interaction, this term appears in the equation in its exact form.[3] It follows from the local character of the deformational interaction (insofar as it applies to the moment of momentum)[33] that the tensor σ_{ik}^{el}, and with it $\lambda_{ik}(\mathbf{p})$, are symmetric (the latter because f' is arbitrary). This is the deformation potential tensor in terms of which (as we shall see in Sec. 3) we can express the change in the energy of an electron that accompanies the deformation of the lattice in the local coordinate frame, and f' is the quasiparticle distribution function in the same frame. The distribution function will be examined in detail in the subsequent sections. To elucidate the form of σ_{ik}^{el}, we can (in the case of elastic scattering) make use of the law of conservation of energy.[34] (see Sec. 4). In the case of arbitrary scattering, on the other hand, we can use for the same purpose the condition that the nonequilibrium free energy of our subsystem, which interacts with a thermostat, should be a minimum. The derivation is somewhat more laborious in this case, but the result is the same.[20] Finally, the standard definition of the force that involves the variation of internal energy with respect to displacement at constant entropy[20] again leads to the same result. We note that the total energy is defined in the laboratory system (the l system). On the other hand, it is convenient to perform all calculations in the comoving (local) set of coordinates (the c-system). It is in this system that periodicity is locally conserved and the dispersion relation can be introduced in a natural way for a conduction electron.

In the simplest situations, Eq. (1.8) admits of a direct generalization to the quantum-mechanical case[27,35] through the replacement

$$\langle\langle Af\rangle\rangle \equiv \frac{2}{h^3}\int d\mathbf{p}\, Af \to \mathrm{Sp}\,\hat{A}\hat{f}, \tag{1.9}$$

where $\hat{f}$ is the single-particle density matrix that satisfies the quantum-mechanical transport equation. It is also valid in the Fermi-liquid case (Ref. 31),[4] which is immediately clear if we examine the variational derivation since, by holding entropy constant, we also hold the distribution function constant during the variation procedure and, consequently, there cannot then be any difference as compared with the pure gas case. We note that Eq. (1.8)[5] is also suitable for investigating purely electronic nonlinear effects (for example, those connected with the resonant trapping of electrons by an elastic wave[38]).

Equation (1.8) can also be directly generalized to nonideal crystals containing dislocations[39] when the continuum description is used.[33,40]

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[2] In the laboratory frame, the integration is performed over the region into which the Brillouin zone is transformed in the course of lattice deformation; see Sec. 8.

[3] This term is often small, in which case it can, of course, be omitted. The above approach is, however, fundamentally important, since it has led to the appearance of that term in Eq. (1.8) and, at the same time, avoids several difficulties connected with the arbitrary nature of the electron dispersion relation (see Sec. 5).

[4] The dispersion relation is then a functional of f, which, in general, can lead, besides renormalization, to new oscillation branches at high frequencies[31,36,11] and may influence the analytic properties of the transport coefficients near singularities.[120]

[5] Or Eq. (1.6), if the expansion given by (1.7) is not possible (for example, this may occur when a superstructure appears or the degeneracy of the electron spectrum is removed by deformation).

2. LINEAR THEORY (REFERENCE SECTION)

In this section, we summarize the equations that will be derived later, and introduce the basic notation.

The deviation of the distribution of conduction electrons in metals from the local equilibrium distribution (including deviations due to the deformation of the crystal) leads to the appearance of considerable forces that are exerted by the electrons on the lattice. These forces can be expressed as functionals of the nonequilibrium part χ of the electron distribution function. One of these functionals is the total current density $\mathbf{j}$ and another is the density of the deformation force $\mathbf{f}^d$:

$$\mathbf{j} = -e\langle \mathbf{v}\chi\rangle, \quad f_i^d = -\frac{\partial}{\partial x_k}\langle \Lambda_{ik}\chi\rangle. \tag{2.1}$$

The closed set of equations consists of the equations of elasticity (including forces exerted by the electrons on the lattice) and the transport equations. It has the form

$$\rho_0 \ddot{u}_i = \lambda_{iklm}\frac{\partial u_{lm}}{\partial x_k} + D_i, \quad \mathbf{D} = \frac{1}{c}[\mathbf{j}, \mathbf{H}] - \frac{m}{e}\frac{\partial \mathbf{j}}{\partial t} + \mathbf{f}^d,$$

$$\left(\frac{d}{dt_0} + \hat{\nu}\right)\chi = g \equiv -e\mathbf{v}\tilde{\mathbf{E}} - \Lambda_{lk}\dot{u}_{lk},$$

$$\tilde{\mathbf{E}} = \mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}, \mathbf{H}] - \frac{m\ddot{\mathbf{u}}}{e}, \quad \Lambda_{lk}(\mathbf{p}) \equiv \lambda_{lk}(\mathbf{p}) - \frac{\langle \lambda_{lk}\rangle}{\langle 1\rangle}, \tag{2.2}$$

where the currents and fields are related by the Maxwell equations[6)]

$$\operatorname{curl}\operatorname{curl}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}, \tag{2.3}$$

and (2.1) are, in effect, the constitutive equations.

The angle brackets in (2.1) and (2.2) represent averaging over the undisturbed Fermi surface (FS) ($dS_{\mathbf{p}}$ is an element of the Fermi surface):

$$\langle\psi\rangle \equiv \frac{2}{\hbar^3}\int \frac{dS_{\mathbf{p}}}{v}\psi, \tag{2.4}$$

$\langle 1\rangle$ is the density of states on the FS, $\chi\partial f_0/\partial\varepsilon(\langle\chi\rangle = 0)$ is the deviation of the distribution function from the local equilibrium function (cf. Refs. 6, 20, and 22), $\hat{\nu}$ is the linearized collision operator,[7)] $e<0$ and m are the charge and mass of the electron, λ_{iklm} is the elasticity tensor of the lattice, which includes the equilibrium contribution due to conduction electrons, and

$$\frac{d}{dt_0} \equiv \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{\partial}{\partial \tau}, \quad \mathbf{v} = \frac{\partial \varepsilon}{\partial \mathbf{p}}, \tag{2.5}$$

where

$$\frac{\partial}{\partial \tau} \equiv \frac{e}{c}[\mathbf{v}, \mathbf{H}]\frac{\partial}{\partial \mathbf{p}},$$

and τ is the orbital period in the magnetic field.

The deformation potential $\lambda_{ik}(\mathbf{p})$ takes into account the change in the electron energy spectrum in the deformed lattice, which, in the linear approximation in the comoving c-system (indicated by primes), has the form

$$\varepsilon'(\mathbf{r}', \mathbf{p}', t) = \varepsilon_0(\mathbf{p}') + \lambda_{ih}(\mathbf{p}')u_{ih} - m\dot{\mathbf{u}}\mathbf{v}'. \tag{2.6}$$

The last term in (2.6) is connected with the noninertial nature of the reference frame and represents the d'Alembert force $m\ddot{\mathbf{u}}$ acting in this system, where m is the free-electron mass.[41]

The transformation to the c-system [see below, Eq. (3.3) and Sec. 8], in which the dispersion relation for conduction electrons is formulated, is an essential step in our phenomenological approach.[20] The idea of this transformation is due to L. D. Landau.[8)] In this way, we not only exclude the motion of the deformed lattice, but also "straighten it" (locally) to the required precision. This follows from the properties of the transformation (see Sec. 8), in which the Brillouin zone has a center of symmetry, and the dispersion relation is expressed in terms of the functions $\varepsilon_0(\mathbf{p}')$ and $\lambda_{ik}(\mathbf{p}')$, $\mathbf{v}'$ that are periodic in $\mathbf{p}'$, the period being equal to that of the reciprocal lattice. On the contrary, in the laboratory l-system, the electron Hamiltonian $\varepsilon(\mathbf{r}, \mathbf{p}, t)$ is given by[20]

$$\varepsilon(\mathbf{r}, \mathbf{p}, t) = \varepsilon_0(\mathbf{p}) + (\lambda_{ik} + p_i v_k)\frac{\partial u_i}{\partial x_k} + (p_i - mv_i)\dot{u}_i, \tag{2.7}$$

and is not equal to the electron energy. As expected for the deformed lattice, the two quantities are not periodic functions of the quasimomentum $\mathbf{p}$. The corresponding quantum-mechanical approach was developed by Gurevich, Lang, and Pavlov,[42,43] who obtained the microscopic expression for $\lambda_{ik}(\mathbf{p})$.

The deformation force $\mathbf{f}^d$ given by (2.1) plays an important role in our theory. It first appeared in Silin's paper.[19] It is similar to the current $\mathbf{j}$ in that it has the character of an electron current and, as we shall show in the next section, it makes a substantial contribution to the Onsager relations. Because of the condition of electrical neutrality, discussed by Akhiezer, Kaganov, and Lyubarskiĭ,[4] it is convenient to replace λ_{ik} in (2.1) with the renormalized tensor Λ_{ik} [see (2.2)]. Both the force terms proportional to the current and $\mathbf{f}^d$ contain only that part of the force that is due to deviation from local equilibrium. The equilibrium part is included in the renormalization of the stress tensor σ_{ik}.

The force $\mathbf{D}$ that describes the contribution due to nonequilibrium electrons and electromagnetic fields has a simple physical interpretation. Let us write the deformation potential $\lambda_{ik}(\mathbf{p})$ in the form of two terms:

$$\lambda_{ik}(\mathbf{p}) = -mv_i v_k + L_{ik}(\mathbf{p})u_{ik}, \tag{2.8}$$

where the first term describes the momentum flux during the free motion of the electron and $L_{ik}(\mathbf{p})u_{ik}$ is the work done on the electron of quasimomentum $\mathbf{p}$ during the deformation of the lattice.[43] This enables us to give a simple interpretation to the individual terms in the force. Let us write (2.2) in the form

$$\rho_{\text{lat}}\ddot{u}_i = \lambda_{iklm}\frac{\partial u_{lm}}{\partial x_k} + D_i',$$

$$D_i' = \frac{1}{c}[\mathbf{j}, \mathbf{H}]_i - \frac{m}{e}\frac{\partial j_i}{\partial t} + \frac{\partial}{\partial x_k}\langle mv_i v_k\chi\rangle - \frac{\partial}{\partial x_k}\langle L_{ik}\chi\rangle. \tag{2.9}$$

The first term in D' is the momentum received by electrons and the lattice from the external field (with allowance for the fact that the system as a whole is electrically neutral) and the

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[6)] The Maxwell equations written in this form automatically take into account the fact that the metal, as a whole, is electrically neutral. In the dynamic case, this is equivalent to the condition $\operatorname{div}\mathbf{j} = 0 (\omega \neq 0)$.

[7)] In contrast to (1.8), where there were no such limitations, here we assume that the scatterers (for example, impurities) are dragged by the lattice.

[8)] It was put forward by Landau at a seminar at the Institute of Semiconductor Physics during a discussion of the first paper.[14] He subsequently wrote down the corresponding transformation formulas, and this is recorded in the notebook of I. M. Khalatnikov. These expressions have also been derived in other papers,[14,20] in which Landau's idea was employed.

remaining terms (with the opposite sign) describe the momentum carried off by electrons, so that the difference is equal to the momentum communicated to the lattice (per unit time per unit volume). In fact, $(m/e)\,(\partial \mathbf{j}^{el})/(\partial t)$ is the change in the momentum of electrons occupying a given volume element, the term $(\partial/\partial x_k)\langle m v_i v_k \chi\rangle$ is the momentum carried off during the free motion of electrons to other volume elements, and $(\partial/\partial x_k)\langle L_{ik}\chi\rangle$, is, according to the results of Lang and Pavlov mentioned above, [43] none other but the momentum communicated to the lattice when work is done upon it in the course of deformation.

The quantities λ^{ad}_{iklm}, $\lambda_{ik}(\mathbf{p})$, $\mu_{iklm}(\mathbf{p})$ can be determined by performing microscopic calculations of the elastic moduli of the metal in the adiabatic approximation (see Refs. 45 and 46 and the reviews in Refs. 47 and 48).[9)]

The term **D** in (2.2) is connected with the "nonadiabatic" contribution of conduction electrons. [Here, we have in mind the deviation from local equilibrium and not a departure from the adiabatic approximation upon which fundamental condition (1.1) is based, subject to obvious reservations relating to the contribution of the fields.]

Both the motion of the surface and the change in the direction of the normal must be taken into account in the boundary conditions for (2.2) when the conditions for the reflection of electrons from the metal surface are written down. These conditions assume the usual form[54–56] when they are written in the c-system in which the boundary is at rest.

3. TRANSPORT EQUATION, ENERGY, AND THE HAMILTON FUNCTION

The form of the transport equation for the quasiparticle (conduction electron) distribution function $f(\mathbf{r}, \mathbf{p}, t)$ in the deformed crystal was established by Akhiezer,[2] Akhiezer, Lyubarskiĭ, and Kaganov,[4] Gurevich,[14] and others in their investigations of the absorption of ultrasonic waves in metals. The transport equation was examined in our previous paper[20] with allowance for the Stewart-Tolman effect, and by Gurevich, Lang, and Pavlov,[42] who put forward a quantum-mechanical derivation. We shall follow Ref.20 and write the transport equation in the form

$$\frac{df}{dt} + \hat{\nu} f = 0, \tag{3.1}$$

where

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \frac{\partial \varepsilon}{\partial \mathbf{p}}\frac{\partial}{\partial \mathbf{r}} + \left(\mathbf{F} - \frac{\partial \varepsilon}{\partial \mathbf{r}}\right)\frac{\partial}{\partial \mathbf{p}} \tag{3.1'}$$

is the Stokes operator for the field derivative with respect to time, $\hat{\nu}$ is the collision operator, and **F** represents external forces in which we include the Lorentz force

$$\mathbf{F}_L = e\mathbf{E} + \frac{e}{c}\left[\frac{\partial \varepsilon}{\partial \mathbf{p}}, \mathbf{H}\right]. \tag{3.2}$$

The deformational interaction with the lattice is included in the Hamiltonian for the conduction electrons.

The dispersion relation, i.e., the Hamilton function for a conduction electron in the laboratory system, in which we have written the transport equation (3.1), can also be written in the form of (1.1), where $\delta\varepsilon(\mathbf{r}, \mathbf{p}, t)$ is the local change in the dispersion relation under deformation of the crystal. The explicit form of $\delta\varepsilon$ is given by (2.6) and (2.7), but some of the results can be obtained without specifying $\delta\varepsilon$.

We shall follow Landau's idea and the results reported in Refs. 14 and 20 to obtain an expression for the electron energy in the l-system. The transformation from the coordinate system K', moving together with the volume element dV' of the lattice, to the l-system can be performed with the aid of the point transformation

$$\mathbf{r} = \mathbf{r}' + \mathbf{u}(\mathbf{r}', t), \tag{3.3}$$

where $\mathbf{u}(\mathbf{r}', t)$ is the displacement of the lattice point $\mathbf{r}'$ at time t (Fig. 2). The canonical transformation with the generating function $\Phi = (\mathbf{r}' + \mathbf{u}(\mathbf{r}', t))\mathbf{p}$ between the old canonical variables $\mathbf{r}'$, $\mathbf{p}'$ and the new variables **r**, **p** yields, as usual,

$$\varepsilon'(\mathbf{r}', \mathbf{p}', t) = \varepsilon(\mathbf{r}, \mathbf{p}, t) - \mathbf{up}, \tag{3.4}$$

$$\mathbf{p}' = \mathbf{p} + \nabla(\mathbf{up}). \tag{3.5}$$

We note that, since the coordinate transformation given by (3.3) contains the time explicitly, and ε' is not a quadratic function of $\mathbf{p}'$, it turns out that, in accordance with well-established results in mechanics,[57] the new Hamilton function ε is not in general identical with the quasiparticle energy. Nevertheless, there is a simple connection[20] between its average over the quasiparticle distribution and the average electron energy. This relation can be obtained by multiplying (3.4) by the invariant of the canonical transformation $f\,dV\,d\mathbf{p} = f'\,dV'\,d\mathbf{p}'$ and then integrating with respect to the quasimomenta. From this, it follows that the energy of electrons in a volume element dV' in the K' system is

$$dE' = \langle\langle \varepsilon' f'\rangle\rangle\, dV' = \langle\langle \varepsilon f\rangle\rangle\, dV - \dot{\mathbf{u}}\,\langle\langle \mathbf{p} f\rangle\rangle\, dV. \tag{3.6}$$

On the other hand, we shall use the well-known energy identity

$$dE = dE' + \mathbf{u}\, d\mathbf{P}, \tag{3.7}$$

where dE is the average energy of the electrons in dV in the l-system and $d\mathbf{P}$ is the electron momentum in dV which, according to (1.4), is given by $d\mathbf{P} = \pi dV = m\langle\langle \partial\varepsilon/\partial\mathbf{p})f\rangle\rangle$. Eliminating $d\mathbf{P}$ and dE' from (3.7), we obtain the relationship between the mean energy and the mean Hamilton function in the l-system:[20]

$$dE = \langle\langle \varepsilon f\rangle\rangle\, dV - \dot{\mathbf{u}}\left\langle\left\langle\left(\mathbf{p} - m\frac{\partial \varepsilon}{\partial \mathbf{p}}\right) f\right\rangle\right\rangle dV. \tag{3.8}$$

When we derive the linearized equations, we shall take into account the fact[6,22,20] that the collision integral is made to vanish by the instantaneous equilibrium distribution

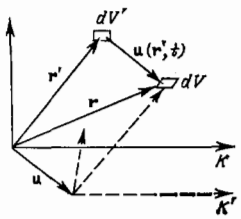

FIG. 2. Transformation from the laboratory frame K to the comoving frame K'.

[9)] This program has not as yet been implemented. Nor has the more realistic program of evaluating $\lambda_{ik}(\mathbf{p})$ for $\varepsilon = \varepsilon_f$ in the spirit of the pseudopotential theory and the Harrison model,[49] or the program of systematic measurements of $\lambda_{ik}(\mathbf{p})$ in analogy with measurements of the dispersion relation $\varepsilon_0(\mathbf{p})$ on the Fermi surface.[5,50] The deformation potential is not known as yet to any degree of detail for any good metal. For semimetals, see Refs. 51–53 (and Sec. 8).

function $f_0(\varepsilon - \mathbf{p}\dot{\mathbf{u}} - \delta\mu)$, where the Fermi distribution function is $f_0(\varepsilon) = \{\exp[(\varepsilon - \mu_0)/kT] + 1\}^{-1}$ (μ_0 is the chemical potential for the electrons, and the resultant distribution function will be sought in the form

$$f(\mathbf{r}, \mathbf{p}, t) = f_0(\varepsilon_0 + \delta\tilde{\varepsilon}) + \chi(\mathbf{r}, \mathbf{p}, t)\frac{\partial f_0}{\partial \varepsilon}, \tag{3.9}$$

where

$$\delta\tilde{\varepsilon} = \delta\varepsilon - \mathbf{p}\dot{\mathbf{u}} - \delta\mu, \tag{3.10}$$

i.e., $\delta\tilde{\varepsilon}$ includes both the change in the Hamilton function and in the electron chemical potential.

This procedure thus takes into account the changes in the electron density and the dispersion relation that follow without delay (to within microscopic time intervals) the deformation of the lattice. In the collisional region, this procedure corresponds to the complete entrainment (drag) of scattering centers by the moving lattice. In many cases, for example, in scattering by phonons, this assumption may not be valid *a priori*, in which case a more detailed analysis becomes necessary.

It is convenient to normalize χ by the condition

$$\langle\chi\rangle = 0, \tag{3.11}$$

which signifies the normalization of both the total and the instantaneous-equilibrium distribution function to the electron density $n(\mathbf{r}, t)$ at a given point in the lattice. It is readily seen that the current density $\mathbf{j}$ in (2.1) can be expressed in terms of χ and constitutes the sum of the electron current (1.3) and the ion current $\mathbf{j}^{\text{lat}} = -en\dot{\mathbf{u}}$. The electrical neutrality condition is

$$\operatorname{div}\mathbf{j} = -e\operatorname{div}\langle\mathbf{v}\chi\rangle = 0. \tag{3.12}$$

Using the equation of continuity for electrons, the explicit form of the deformational corrections $\delta\varepsilon$ to the dispersion relations (2.6) and (2.7) and the normalization condition, we obtain the expression for, $\delta\mu = \langle\lambda_{ik}\rangle u_{ik}/\langle 1\rangle$, the renormalization of the deformation potential (2.2), and the equation satisfied by the quantity χ given by (2.2).

4. CONSERVATION LAWS

To determine the force $\mathbf{f}$ in the equations of the theory of elasticity, given by (1.8), it is convenient to start with the conservation of the resultant momentum $\mathbf{P}$ of the system consisting of free electrons, the lattice, and the electromagnetic field. This conservation law is exact if we ignore momentum transfers to phonons (since, otherwise, the phonons would also have to be included in the system[58,59]):

$$\mathbf{P} = \int dV\left(m\left\langle\left\langle\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle\right\rangle + \mathbf{G} + \rho_{\text{lat}}\dot{\mathbf{u}}\right); \tag{4.1}$$

where the first term [cf. (1.4)] is the momentum of free electrons, expressed in terms of the dispersion relation and the quasiparticle distribution function, and $\mathbf{G}$ is the momentum density of the electromagnetic field.

For an arbitrary volume V, we have

$$\frac{dP_i}{dt} = \int_V dV\left(\rho_{\text{lat}}\ddot{u}_i + \frac{m}{e}\frac{\partial j_i^{\text{el}}}{\partial t} - \frac{1}{c}[\mathbf{j}, \mathbf{H}]_i - \frac{\partial T_{ik}}{\partial x_k}\right) = \int_S dS_k\psi_{ik}, \tag{4.2}$$

where S is the surface bounding the volume V, T_{ik} is the Maxwell stress tensor, and ψ_{ik} is a certain symmetric (because of conservation of momentum) tensor. Since the volume is arbitrary, this yields the following expression for the force:[20,30]

$$\rho\ddot{u}_i = \frac{1}{c}[\mathbf{j}, \mathbf{H}]_i - \frac{m}{e}\frac{\partial j_i}{\partial t} + \frac{\partial}{\partial x_k}(T_{ik} + \psi_{ik}). \tag{4.3}$$

To determine ψ_{ik} for a closed system, we can use the law of conservation of energy.[34] This can be written in the form

$$\mathscr{E} = \mathscr{E}_{\text{el}} + \mathscr{E}_{\text{em}} + \mathscr{E}_{\text{cla}}, \tag{4.4}$$

where the three terms correspond, respectively, to the electrons, the electromagnetic field, and the lattice. The expression is valid for elastic scattering of electrons in the c-system, which is valid with good accuracy for scattering by impurities dragged by the moving lattice. According to the transformation formulas, the electron energy $\mathscr{E}_{\text{el}}$ in the l-system can be written in terms of the spectrum and the distribution of electrons in the c-system,[10] as follows:

$$\mathscr{E}_{\text{el}} = \int dV'\left(\langle\langle\varepsilon' f'\rangle\rangle + \dot{\mathbf{u}}\left\langle\left\langle m\frac{\partial\varepsilon'}{\partial\mathbf{p}}f'\right\rangle\right\rangle + \frac{nm\dot{u}^2}{2}\right). \tag{4.5}$$

The condition for elastic scattering is

$$\left\langle\left\langle\varepsilon'\frac{df'}{dt}\right\rangle\right\rangle = \langle\delta\varepsilon'\hat{\nu}\chi\rangle = 0. \tag{4.6}$$

We thus arrive at the equation (4.6)

$$\frac{\partial\mathscr{E}}{\partial t} = \int dV\dot{u}_i\left(\rho\ddot{u}_i - \frac{\partial\sigma_{ik}}{\partial x_k} + \frac{m}{e}\frac{\partial j_i}{\partial t} - \frac{1}{c}[\mathbf{j}, \mathbf{H}]_i - \frac{\partial}{\partial x_k}\langle\langle\lambda_{ik}f'\rangle\rangle\right) - \int dV\frac{\partial q_k}{\partial x_k}, \tag{4.7}$$

and this, together with (4.3), leads to the equations of elasticity (1.8) and the expression for the energy flux density given by (4.16). If we were to start with (4.7) alone, the right-hand side of the equations of elasticity (1.8) would acquire additional terms of the form $[\dot{\mathbf{u}}, \boldsymbol{\Phi}]$ and $c_{kil}(\partial/\partial x_k)\dot{u}_l$, where $\boldsymbol{\Phi}$ is an arbitrary vector function of coordinates and c_{kil} is a constant tensor that is symmetric in the indices il. However, the law of conservation of momentum given by (4.2) shows that $\boldsymbol{\Phi} = 0$ and $c_{kil} = 0$.

When the system is placed in a thermostat and its energy is not conserved, the derivation of the equations can be based on the fact that the time derivative of the suitably defined free energy F of the nonequilibrium state should be a sign-definite (negative-definite) quantity[20] (Fig. 3). It is then sufficient to take the free energy in the form $F = \mathscr{E} - TS$, where $\mathscr{E}$ and S are, respectively, the energy and entropy of the nonequilibrium subsystem and T is the temperature of the thermostat. The free energy F is a minimum in the state

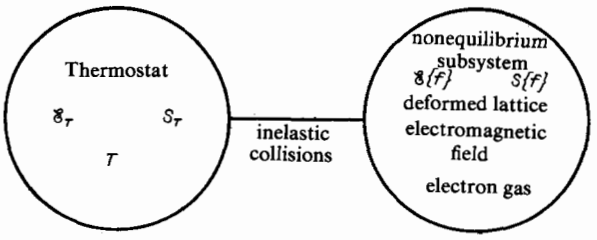

FIG. 3. In a subsystem that is not in equilibrium, the averages are functionals of the electron distribution f and the fields $\mathbf{u}$, $\mathbf{E}$, and $\mathbf{H}$.

[10] We emphasize particularly that only the process of the calculation is transformed to the comoving system; the "conservation law" (or the variational principle—see below) is formulated in the laboratory system.

of equilibrium, and its average time derivative $\overline{dF/dt}$ determines the power dissipation. This condition is sufficient to define ψ_{ik} in (4.3). Indeed, isolating in dF/dt the sign-definite dissipative term due to collisions, and also the surface term describing the energy flux, we must demand that the remaining volume terms should vanish, and this will give us the equation for ψ_{ik}. The final result is (1.8).

The expression for the momentum flux density Π_{ik} in the conservation law

$$\frac{\partial P_i}{\partial t} + \frac{\partial \Pi_{ik}}{\partial x_k} = 0, \tag{4.8}$$

assumes the form

$$\Pi_{ik} = T_{ik} - \sigma_{ik} + \langle \lambda_{ik} \chi \rangle. \tag{4.9}$$

It is clear that Π_{ik} is symmetric if we suppose that the Maxwell stress tensor T_{ik} and the renormalized stress tensor σ_{ik} are symmetric, in accordance with the conservation of momentum and the local nature of the deformational interaction.[11)]

The energy flux density $\mathbf{q}$ [see (4.7) and (6.1)] is given by

$$q_k = \left\langle \left\langle \frac{\partial \varepsilon}{\partial p_k} \varepsilon f \right\rangle \right\rangle + \frac{c}{4\pi} [\mathbf{E}, \mathbf{H}]_k - \dot{u}_i (\sigma_{ik} + \langle\langle \lambda_{ik} f \rangle\rangle). \tag{4.10}$$

5. ELECTRON LATTICE DRAG AND COLLISIONAL MOMENTUM TRANSFER

The equations of elasticity can be given a different form, depending on whether the transport equations for the mean momentum or the mean quasimomentum of conduction electrons is used.

The additional force acting on the lattice in metals can be represented by the force exerted by electrons (drag force f^{dr}) and the Lorentz force acting on the ions:

$$\rho_{\text{lat}} \ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - e n_0 \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{H}] \right)_i + f_i^{\text{dr}}. \tag{5.1}$$

Comparing (5.1) with (1.8) or (2.2), we obtain an explicit expression for the "drag force."

We now rewrite the electron dispersion relation (2.7) in the laboratory coordinate frame K in the form

$$\varepsilon(\mathbf{r}, \mathbf{p}, t) = \varepsilon_0(\mathbf{p}) + (\lambda_{ik} + m v_i v_k) u_{ik} + \left(p_i - m v_i \right) \left(\dot{u}_i + v_k \frac{\partial u_i}{\partial x_k} \right). \tag{5.2}$$

From this it is clear immediately that the limiting transition to free electrons decoupled from the lattice corresponds to ε, $\varepsilon_0 \to p^2/2m$, $\mathbf{p} \to m\mathbf{v}$ and, accordingly, $\lambda_{ik} \to -m v_i v_k$.[12)] The deformation force[20] $\mathbf{f}^{\text{d}}$ then tends to $\nabla_k \langle m v_k \chi \rangle$, i.e., to the term found in Ref. 28 in the microscopic derivation of the drag force without taking into account the deformational interaction.[13)]

In the microscopic approach, the collisional term appears naturally in $\mathbf{f}^{\text{dr}}$. It is therefore of interest to rewrite (5.1) so that it explicitly contains the electric fields acting on the lattice and the momentum (quasimomentum) transferred by electrons to the lattice as a result of collisions. We start with the quasimomentum transfer equation, which we obtain by multiplying the transport equation (2.2) by $\mathbf{p}$ and integrating it over the Fermi surface. This yields

$$\frac{\partial}{\partial t} \langle p_i \chi \rangle + \frac{\partial}{\partial x_k} \langle p_i v_k \chi \rangle + \langle p_i \hat{\nu} \chi \rangle = e \tilde{E}_k \langle p_i v_k \rangle - \frac{1}{c} [\mathbf{j}, \mathbf{H}]_i + \frac{e}{c} \varepsilon_{klm} H_m \oint_{\dot{\Sigma}} d\Sigma_k p_i v_l \frac{\partial f_0}{\partial \varepsilon} \chi. \tag{5.3}$$

The last term in this equation is the integral over the surface of the Brillouin zone and, if the Fermi surface intersects its faces, the integral will, in general, be nonzero. For closed surfaces, it is possible to choose the fundamental cell so that its faces are not intersected by the Fermi surface (Figs. 4 and 5). We then have $\langle p_i v_k \rangle = (n_- - n_+)\delta_{ik}$, where n_- is the electron concentration in the electron bands and n_+ is the concentration of holes. The transport equation then assumes the form

$$\frac{\partial}{\partial t} \langle p_i \chi \rangle + \frac{\partial}{\partial x_k} \langle p_i v_k \chi \rangle + \langle p_i \hat{\nu} \chi \rangle = -e E_i (n_- - n_+) - \frac{1}{c} [\mathbf{j}, \mathbf{H}]_i. \tag{5.4}$$

Correspondingly, for closed Fermi surfaces, the equation of elasticity (2.2) can be rewritten in the following form with the aid of (5.4):

$$\rho_{\text{lat}} \ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - e \tilde{E}_i (n_- - n_+) - \frac{\partial}{\partial x_k} \langle (\lambda_{ik} + p_i v_k) \chi \rangle - \langle p_i \hat{\nu} \chi \rangle - \frac{\partial}{\partial t} \langle (p_i - m v_i) \chi \rangle. \tag{5.5}$$

The collisional term with quasimomentum transfer $\langle \mathbf{p} \hat{\nu} \chi \rangle$ is often the leading term in (5.5). There has been considerable activity in this connection ("electron wind" theory; see the bibliography in Refs. 29 and 30). When $n_+ = 0$, i.e., for purely electronic groups, the expression given by (5.5) for the force acting on the lattice becomes identical with Eq. (4.1) in Ref. 20. In the collisional region, and if we neglect the Steward-Tolman term $(\partial/\partial t)\langle (\mathbf{p} - m\mathbf{v})\chi \rangle$, the latter equation becomes identical with the expression for the force proposed by Silin,[19] who was the first to introduce the deformation force into the equations of elasticity:

$$\rho_{\text{lat}} \ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - e n_0 \left(\mathbf{E} + \frac{1}{c} [\dot{\mathbf{u}}, \mathbf{H}] \right)_i + \frac{\partial}{\partial x_k} \langle\langle \mathscr{L}_{ik} f \rangle\rangle, \tag{5.6}$$

where $n_0 = \langle\langle f \rangle\rangle$ is the total density of conduction electrons and $\mathscr{L}_{ik}$ is the deformation potential in the laboratory frame.

For open surfaces we have

$$\langle p_i v_k \rangle = n_0 \delta_{ik} - \oint d\Sigma_k p_i f_0, \tag{5.7}$$

where the integral in the second term is evaluated over the surface of the Brillouin zone and n_0 is the total number of

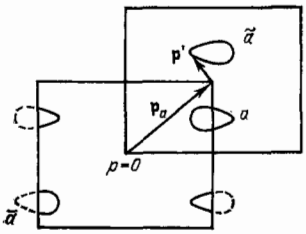

FIG. 4. Choice of a cell that does not intersect the Fermi surface by faces of the Brillouin zone.

[11)]The electron distribution χ is, in turn, determined by the fields $\tilde{\mathbf{E}}$ and $\dot{\mathbf{u}}$ even for mean free paths $l \gg a$, which is indeed a reflection of the nonlocality introduced by electrons.

[12)]The idea of this derivation is due to V. L. Gurevich; see also Chap. 8.

[13)]See also the discussion of this question, given in Ref. 30.

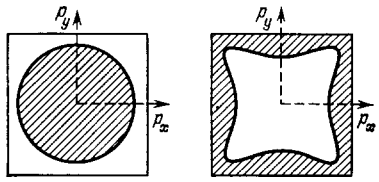

FIG. 5. Electron and hole zones. Shaded areas occupied by electrons.

conduction electrons per unit volume (Fig. 6). In this case, the last term in (5.5) is also nonzero, where the integral is evaluated over the belt corresponding to the section of the open Fermi surface by the faces of the Brillouin zone. The contribution due to the surface of the Brillouin zone is connected with momentum transfer to the lattice as the electron moving in the force field crosses the face of the zone.[14] The equation of elasticity written in this form for arbitrary topology is then found to assume a very complicated form:

$$\rho_{\text{lat}}\,\ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - e\,\langle p_i v_k\rangle\,\tilde{E}_k - \frac{\partial}{\partial x_k}\,\langle(\lambda_{ik} + p_i v_k)\,\chi\rangle - \langle p_i \hat{v}\chi\rangle - \frac{\partial}{\partial t}\,\langle(p_i - m v_i)\,\chi\rangle + \frac{e}{c}\,\varepsilon_{klm} H_m \oint \mathrm{d}\Sigma_k p_i v_l \chi\,\frac{\partial f_0}{\partial \varepsilon}, \tag{5.8}$$

where ε_{klm} is the antisymmetric unit tensor. The effective charge density of the lattice is then anisotropic and is represented by the tensor $e\langle p_i v_k\rangle$. For $n_+ = n_-$, the latter is found to vanish. For hole groups, the contribution to the effective charge density is negative, which was first pointed out by Fiks.[29]

When the faces of the Brillouin zone that are intersected by the Fermi surface are parallel to a direction chosen as z, and the latter has a sufficiently high degree of symmetry,[15] then[61]

$$\langle p_i v_k\rangle = \delta_{ik}\,(n_0 - \xi N), \tag{5.9}$$

where N is the number of states in the Brillouin zone and ξ is the number of "hole" zones in which states lying near the faces that are not intersected by the Fermi surface are occupied. For closed surfaces, the difference $n_0 - \xi N$ becomes equal to $n_- - n_+$.

Instead of (5.5), we can use the transport equation for the electron momentum. If we use it to replace the term

$$-\frac{m}{e}\frac{\partial \mathbf{j}}{\partial t} \equiv \frac{\partial}{\partial t}\,\langle m\mathbf{v}\chi\rangle$$

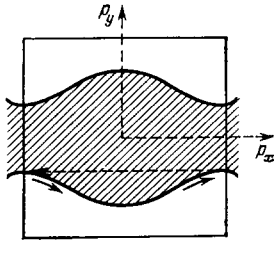

FIG. 6. Motion in a field in the case of an open Fermi surface.

[14]Either by changing its energy (term including the electric field) or by moving over an open orbit in a magnetic field (last term in (5.3)].

[15]For example, the presence of two vertical symmetry planes will suffice. This case occurs in the simple hexagonal lattice.

in (2.2), we obtain[20]

$$\rho_{\text{lat}}\,\ddot{u}_i = \frac{\partial \sigma_{ik}}{\partial x_k} - e\,\langle m v_i v_k\rangle\,\tilde{E}_k - \frac{\partial}{\partial x_k}\,\langle(\lambda_{ik} + m v_i v_k)\,\chi\rangle - \langle m v_i \hat{v}\chi\rangle - \frac{e}{c}\,\varepsilon_{klj} H_j \left\langle\left(\delta_{ik} - m\frac{\partial^2 \varepsilon}{\partial p_i\,\partial p_k}\right) v_l \chi\right\rangle. \tag{5.10}$$

Here again, we see the emergence of the effective tensor charge density of the lattice $-e\langle m v_i v_k\rangle$, which is sign-definite, but the tensor properties (anisotropy) appear even for closed Fermi surfaces. Thus, the concept of the effective charge is not invariant and has a definite meaning only in the context of the particular transport equation (for the quasimomentum or momentum) that is being used.

We thus see that, despite the definite advantages of writing equations in the form of (5.8) or (5.10), this procedure is in general too complicated. In the presentation given below, we shall use the most general and simplest form of Eqs. (1.8) and (2.2).

6. ELECTROACOUSTIC TRANSPORT COEFFICIENTS

The dynamic moduli of elasticity of a metal that will be obtained below are conveniently expressed in terms of the electroacoustic transport coefficients satisfying the Onsager symmetry relations.

To derive these relations, it is convenient to transform the mean dissipated energy, which is equal to the rate of reduction in the free energy of the system (see Sec. 4), i.e.,

$$-\frac{\partial \overline{F}}{\partial t} = \int \mathrm{d}V\,\overline{\langle \chi\hat{v}\chi\rangle} + \int \mathrm{d}\mathbf{S}\,\overline{\mathbf{q}}, \tag{6.1}$$

by using the transport equation (2.2) to substitute for $\hat{v}\chi$ [the flux through the surface, i.e., the second term in (6.1) is now omitted]. The result is

$$-\frac{\partial \overline{F}}{\partial t} = \int \mathrm{d}V\,\overline{\langle \chi g\rangle}, \tag{6.2}$$

where the bar represents time averaging. We have used the fact that, in the identity

$$\langle \chi\hat{v}\chi\rangle \equiv \langle \chi g\rangle - \frac{\partial}{\partial t}\left\langle\frac{\chi^2}{2}\right\rangle - \nabla\langle \chi\mathbf{v}\chi\rangle - \left\langle\frac{\partial}{\partial \tau}\frac{\chi^2}{2}\right\rangle \tag{6.3}$$

the second term vanishes when the time average is taken, the third term drops out after integration over the volume, and the last term is identically zero.

Substituting for g from (2.2) into (6.2), we obtain

$$-\frac{\partial \overline{F}}{\partial t} = \int \mathrm{d}V\,\overline{(\mathbf{j}\tilde{\mathbf{E}} + \sigma^{\text{el}}_{ik}\dot{u}_{ik})}, \tag{6.4}$$

where $\sigma^{\text{el}}_{ik} = \langle \Lambda_{ik}\chi\rangle$ is the nonadiabatic part of the electron stress tensor. In the present case of volume dissipation (the flux through the surface must be taken into account when the boundary of the metal provides an essential contribution[16]), it is convenient to transform to vector quantities, in which case, we integrate the second term by parts and omit the surface integral:

$$-\frac{\partial \overline{F}}{\partial t} = \int \mathrm{d}V\,\overline{(\mathbf{j}\tilde{\mathbf{E}} - \mathbf{f}^{\text{d}}\dot{\mathbf{u}})}. \tag{6.5}$$

It is clear from (6.4) and (6.5) that the change in free energy is connected with work done per unit time on the charges by

[16]This is important for Rayleigh waves and for thin metal films.[62]

the effective field and the deformation field.

We now consider the response of the system to a periodic perturbation of frequency ω. Since the system is in a thermostat, we know[63] that the average rate of change of the energy of the system is equal to the rate of change of its free energy. We thus arrive at Eq. (6.5), which must be used to determine the generalized forces and the coordinates in order to apply the general symmetry relations for the transport coefficients:[63]

$$-\frac{\partial F}{\partial t}=\sum_a \int \mathrm{d}\mathbf{r}\, x_a(\mathbf{r})\,\dot{f}_a(\mathbf{r}),$$

$$x_a(\mathbf{r})=\sum_b \int \mathrm{d}\mathbf{r}'\,\alpha_{ab}(\mathbf{r},\mathbf{r}')\,f_b(\mathbf{r}'). \tag{6.6}$$

It follows from (6.5)–(6.6) that, if we take $\tilde{\mathbf{E}}(\mathbf{r})$ and $\mathbf{u}(\mathbf{r})$, as the generalized forces, the corresponding generalized coordinates will be the currents $\mathbf{j}(\mathbf{r})/i\omega$ and $-\mathbf{f}^{\mathrm{d}}(\mathbf{r})$:

$$f_a(\mathbf{r})\to\tilde{\mathbf{E}}(\mathbf{r}),\quad \mathbf{u}(\mathbf{r}),$$
$$x_a(\mathbf{r})\to\frac{\mathbf{j}(r)}{i\omega},\quad -\mathbf{f}^{\mathrm{d}}(\mathbf{r}). \tag{6.7}$$

The role of the index, labeling x and f, is now assumed by the combination of the discrete index a and the continuous index $\mathbf{r}$. The symmetry relations for the transport coefficients will now be written for the homogeneous medium, where

$$\alpha_{ab}(\mathbf{r}-\mathbf{r}',\mathbf{H})=\alpha_{ba}(\mathbf{r}'-\mathbf{r},\mathbf{H}), \tag{6.8}$$

or, in terms of spatial Fourier components,

$$\alpha_{ab}(\mathbf{k},\mathbf{H})=\alpha_{ba}(-\mathbf{k},-\mathbf{H}). \tag{6.9}$$

Turning now to the constitutive equations that follow from (2.1) and (6.7), i.e.,

$$j_i=\sigma_{il}\tilde{E}_l+\partial_{il}u_l,\quad f_i^{\mathrm{d}}=c_{il}\tilde{E}_l+b_{il}u_l, \tag{6.10}$$

we see that the tensors $\hat{\sigma}$, $\hat{\partial}$, $\hat{c}$ and $\hat{b}$ must satisfy symmetry relations that follow[20] from (6.6)–(6.9):

$$\sigma_{il}(\mathbf{k},\mathbf{H})=\sigma_{li}(-\mathbf{k},-\mathbf{H}),$$
$$b_{il}(\mathbf{k},\mathbf{H})=b_{li}(-\mathbf{k},-\mathbf{H}),$$
$$\partial_{il}(\mathbf{k},\mathbf{H})=-i\omega c_{li}(-\mathbf{k},-\mathbf{H}); \tag{6.11}$$

where σ_{il} is the conductivity,[17] ∂_{il} is the "deformation conductivity," and c_{il} and b_{il} characterize changes in the elastic moduli due to the nonequilibrium nature of the electron gas (the former due to the field and the latter directly due to the deformational interaction). In particular, the relations given by (6.11) can be used together with the known expression for the current (its "deformation" part, described by the tensor ∂_{il}) to construct the "field" part of the deformation force, determined by c_{il}. The transformation for $\dot{u}_{lm}$ to u has enabled us to avoid the introduction of constitutive tensors of rank three.

We now introduce Green's operator $\hat{R}(\mathbf{p})$ for the transport equation and its symmetric and antisymmetric parts:

$$\hat{R}(\mathbf{p})=\left(\frac{\mathrm{d}}{\mathrm{d}t}+\hat{\nu}\right)^{-1},\quad \hat{R}^{s,a}=\frac{1}{2}[\hat{R}(\mathbf{p})\pm\hat{R}(-\mathbf{p})]. \tag{6.12}$$

The formal solution of the transport equation is

$$\chi=\chi^s+\chi^a=\hat{R}g,$$
$$\chi^a=-\hat{R}^s e\mathbf{v}\tilde{E}-\hat{R}^a\Lambda_{lm}\dot{u}_{lm},\quad \chi^s=-\hat{R}^a e\mathbf{v}\tilde{\mathbf{E}}-\hat{R}^s\Lambda_{lm}\dot{u}_{lm}. \tag{6.13}$$

The current and the deformation force are expressed, respectively, in terms of the antisymmetric (χ^a) and symmetric (χ^s) parts of the distribution function. Thus, the constitutive tensors are

$$\sigma_{il}=e^2\langle v_i R^s v_l\rangle,\ \partial_{il}=e\omega k_j\langle v_i R^a\Lambda_{lj}\rangle,$$
$$c_{il}=iek_j\langle\Lambda_{ij}R^a v_l\rangle,\quad b_{il}=i\omega k_j k_t\langle\Lambda_{ij}R^s\Lambda_{lt}\rangle. \tag{6.14}$$

The form of the transport equation (2.2) shows that $R(-\mathbf{k},\mathbf{p})=R(\mathbf{k},-\mathbf{p})$ when the collision integral is even in $\mathbf{p}$. Hence according to (6.14), it follows that, in this case, $\hat{\sigma}$, $\hat{\partial}$, $\hat{c}$ and $\hat{b}$ are even functions of $\mathbf{k}$ and, correspondingly, we obtain the symmetry relations in the form

$$\sigma_{il}(\mathbf{k},\mathbf{H})=\sigma_{li}(\mathbf{k},-\mathbf{H}),\quad b_{il}(\mathbf{k},\mathbf{H})=b_{li}(\mathbf{k},-\mathbf{H}),$$
$$\partial_{il}(\mathbf{k},\mathbf{H})=-i\omega c_{li}(\mathbf{k},-\mathbf{H}). \tag{6.15}$$

In particular, (6.15) is valid if there is a center of inversion. In the simplest case, where $\mathbf{H}=0$ and there is strong spatial dispersion ($\mathscr{P}$ represents the principal value), we have

$$R^s=\pi\delta(\mathbf{kv}),\quad R^a=-i\frac{\mathscr{P}}{\mathbf{kv}}. \tag{6.16}$$

The symmetry relations can then be readily verified directly because R is the multiplication operator.

7. DYNAMIC ELASTIC MODULI OF A METAL

We shall now obtain the equation describing the propagation of acoustic waves in metals (including the case where an external magnetic field is present).[19–21,64–66] We shall start with the equations of the theory of elasticity, given by (2.2). The force $\mathbf{D}$ describes the contribution due to nonequilibrium electrons and electromagnetic fields, and can be expressed in terms of the current and the deformation force.

If the source of the fields is the acoustic wave excited in the crystal, then by solving Maxwell's equations we can express the electric field $\tilde{\mathbf{E}}$ and, correspondingly, $\mathbf{j}$ and $\mathbf{f}^{\mathrm{d}}$ in terms of the deformations:

$$\tilde{E}_k=\hat{G}_{kl}u_l,\quad j_i=\hat{j}_{il}u_l,\quad f_i^{\mathrm{d}}=\hat{f}_{il}u_l. \tag{7.1}$$

The operator $\hat{G}_{kl}$ is related to the Green's tensor of the Maxwell equations. It is given below by (7.17).

The equation for the displacement $\mathbf{u}$ is

$$\rho\ddot{u}_i=\lambda_{iklm}\frac{\partial^2 u_l}{\partial x_k\,\partial x_m}+\hat{D}_{il}u_l,$$
$$\hat{D}_{il}=\left(\frac{1}{c}\varepsilon_{imn}H_n-\frac{m}{e}\delta_{im}\frac{\partial}{\partial t}\right)\hat{j}_{ml}+\hat{f}_{il}. \tag{7.2}$$

In an unbounded metal, the Fourier components of the current density $\mathbf{j}$ and the deformation force $\mathbf{f}^{\mathrm{d}}$ can be expressed in terms of the Fourier components of the field and the displacement with the aid of the constitutive equations (6.10) that contain the electroacoustic transport coefficients (the subscripts $\mathbf{k}$ and ω will be omitted):

$$\tilde{E}_i=G_{ih}u_h,\quad j_i=(\sigma_{ik}G_{kl}+\partial_{il})u_l,\quad f_i^{\mathrm{d}}=(c_{ik}G_{kl}+b_{il})u_l. \tag{7.3}$$

The transport coefficients satisfy the symmetry relations given by (6.6), so that, in addition to $\hat{c}$, it is convenient

[17] We shall use the standard notation σ_{ik} both for the stress tensor and for the conductivity, since these two quantities never appear at the same time.

to introduce the quantity

$$\tilde{\partial}_{ki}(\mathbf{k}, \mathbf{H}) \equiv \partial_{ik}(-\mathbf{k}, -\mathbf{H}) = -i\omega c_{ki}(\mathbf{k}, \mathbf{H}). \quad (7.4)$$

The tensor

$$\Delta_{kl} = \frac{m}{e}(i\omega\delta_{kl} - \varepsilon_{klm}\omega_m^c), \quad \omega^c = \frac{e\mathbf{H}}{mc} \quad (7.5)$$

can then be used to rewrite the Fourier components of **D** and the effective field **E**, as follows:

$$D_i = \Delta_{ik} j_k + f_i^d, \quad \tilde{E}_k = E_k - i\omega\Delta_{kl}u_l; \quad (7.6)$$

where

$$\tilde{\mathbf{E}} = \mathbf{E} + \frac{1}{c}[\dot{\mathbf{u}}, \mathbf{H}] - \frac{m}{e}\ddot{\mathbf{u}}$$

is the effective field, and the field **E** satisfying Maxwell's equations includes the gradient of the chemical potential.

In our presentation below, we shall use the notation $a_{il}\varkappa_l \equiv a_\varkappa$, where $\varkappa = \mathbf{k}/k$, and a_{il} is an arbitrary tensor.

Let p and q be the principal axes of the contracted elastic tensor of the lattice $\lambda_{i\varkappa l\varkappa}$. Let η_p be the eigenvalues of this tensor in units of ρs^2, where s is a certain characteristic velocity of sound. We then have

$$\eta_{\bar{p}}\delta_{pq} \equiv \frac{\lambda_{p\varkappa q\varkappa}}{\rho s^2}. \quad (7.7)$$

The bar over an index indicates that summation is not performed over the index.

In terms of the principal axes p and q and in dimensionless variables, the equation for the Fourier components of the displacement, given by (7.2), assumes the form

$$[(\eta_{\bar{p}} - \zeta)\delta_{pq} + d_{pq}]u_q = 0; \quad (7.8)$$

where

$$\zeta \equiv \left(\frac{\omega}{ks}\right)^2, \quad d_{il} = -\frac{D_{il}}{\rho s^2 k^2}, \quad (7.9)$$

i.e., ξ is the dimensionless square of the phase velocity of the wave and $d_{ik}u_k$ is the (dimensionless) Fourier component of the force exerted on the lattice by the conduction electrons and the electromagnetic fields.

The expression for the (contracted) dynamic elastic moduli d_{pq} can be obtained by substituting the expression for the tensor G_{ik} given by (7.17) into (7.6) and (7.3). The result[20] can be written in the compact form:[66]

$$d_{pq} = \sum_{J=1}^{\mathrm{VI}} d_{pq}^{(J)},$$

$$d_{pq}^{\mathrm{I}} = -\frac{b_{pq}}{\rho s^2 k^2}, \quad d_{pq}^{\mathrm{II}} = \frac{c_{p\varkappa}\partial_{\varkappa q}}{\rho s^2 k^2 \sigma_{\varkappa\varkappa}}, \quad d_{pq}^{\mathrm{III}} = -\frac{k^2c^2}{4\pi}\frac{\Delta_{p\beta}\Delta_{\beta q}}{\rho s^2 k^2},$$

$$d_{pq}^{\mathrm{IV}} = \frac{c_{p\beta}^{\perp}\rho_{\beta\gamma}\partial_{\gamma q}^{\perp}}{\rho s^2 k^2}, \quad d_{pq}^{\mathrm{V}} = \frac{i\omega}{\rho s^2 k^2}\left(\frac{k^2c^2}{4\pi\omega}\right)^2 \Delta_{p\beta}\rho_{\beta\gamma}\Delta_{\gamma q},$$

$$d_{pq}^{\mathrm{VI}} = -\frac{ic^2}{4\pi\omega\rho s^2}(\Delta_{p\beta}\rho_{\beta\gamma}\partial_{\gamma q}^{\perp} - \tilde{\partial}_{p\beta}^{\perp}\rho_{\beta\gamma}\Delta_{\gamma q}). \quad (7.10)$$

The indices $\alpha, \beta, \gamma \ldots$ indicate projections along axes perpendicular to the wave vector **k**, $\varkappa$ represents a projection along the wave vector, the symbol $\perp$ represents renormalization that arises when longitudinal electric fields are eliminated (with the aid of the neutrality condition $j_\varkappa = 0$), $\rho_{\alpha\beta}$ are the components of the effective transverse resistivity tensor, $\hat{I}$ is the unit operator, and $\tilde{\partial}$ is defined in (7.4):

$$\hat{\rho} \equiv \left(\hat{\sigma}^{\perp} + i\frac{k^2c^2}{4\pi\omega}\hat{I}\right)^{-1}, \quad \sigma_{\alpha\beta}^{\perp} = \sigma_{\alpha\beta} - \frac{\sigma_{\alpha\varkappa}\sigma_{\varkappa\beta}}{\sigma_{\varkappa\varkappa}},$$

$$\partial_{\alpha l}^{\perp} = \partial_{\alpha l} - \frac{\sigma_{\alpha\varkappa}\partial_{\varkappa l}}{\sigma_{\varkappa\varkappa}}, \quad c_{l\alpha}^{\perp} = c_{l\alpha} - \frac{c_{l\varkappa}\sigma_{\varkappa\alpha}}{\sigma_{\varkappa\varkappa}}. \quad (7.11)$$

It is sometimes convenient to combine the terms d^{III} and d^{V} by using the definition of $\hat{\rho}$ given by (7.11):

$$d_{pq}^{\mathrm{III}} + d_{pq}^{\mathrm{V}} = -\frac{c^2}{4\pi\rho s^2}\Delta_{p\alpha}\rho_{\alpha\beta}\sigma_{\beta\gamma}^{\perp}\Delta_{\gamma q}. \quad (7.12)$$

The expressions given by (7.8)–(7.12) are a consequence of only the phenomenological relationships given by (7.10) and (7.3) and the Maxwell equations. In the quasiclassical case, when the electron gas is described by the transport equation (2.2), the transport coefficients in (7.10) can be evaluated with the aid of (6.14), whereas, in the quantum-mechanical case (including quantizing magnetic fields), they can be obtained from (1.9).

Let us now briefly consider[66] the structure of d_{il}. The individual terms in the expression for d_{il} differ by the transport coefficients they contain, and have different physical interpretations. The first three terms ($d^{\mathrm{I}}, d^{\mathrm{II}}$ and d^{III}) do not contain the transverse resistivity, whereas the last three terms $d^{\mathrm{IV}}, d^{\mathrm{V}}$ and d^{VI} are proportional to $\rho_{\alpha\beta}$ and are due to the transverse electric fields that arise during the propagation of sound waves. The term d^{I} describes purely deformational effects. Neither the longitudinal nor the transverse fields contribute to it. It corresponds to work performed by the component of the force $\mathbf{f}^d$ that is proportional to the deformation alone. The corresponding power is $\dot{\mathbf{u}}\mathbf{f}^d$, where $\mathbf{f}^d$ must be interpreted as being only the second term in (6.10). The term d^{II} represents the renormalization of the deformational term due to longitudinal electric fields. The term d^{III} is Hermitian and describes the variation in the velocity of sound due to inductive effects in an infinitely conducting medium.[60] It includes inertial contributions connected with the Stewart-Tolman effect. This term is universal: it does not contain transport coefficients. Relative to the x, y, z axes, where the field lies along the z axis, we have

$$d_{pq}^{\mathrm{III}} = -\frac{H^2}{4\pi\rho s^2}\begin{pmatrix} 1 & i\frac{\omega}{\omega_c} & 0 \\ -i\frac{\omega}{\omega_c} & 1 & 0 \\ 0 & 0 & \left(\frac{\omega}{\omega_c}\right)^2 \end{pmatrix}. \quad (7.13)$$

The term d^{IV} describes the contribution of transverse deformational currents and represents the renormalization of the deformational interaction due to transverse electric fields. It is particularly important for transverse sound. Correspondingly, d^{V} describes the effect of finite conductivity on the inductive terms, i.e., renormalization of the term d^{III} due to transverse fields. In some cases, the separation into these terms becomes purely arbitrary. Finally, the last term (d^{VI}) describes cross effects when either the work $\dot{\mathbf{u}}\mathbf{f}^d$ per unit time is performed by the component of the deformation force that is proportional to the electric field, or the current in the expression for the work done by the field ($\mathbf{j}\tilde{\mathbf{E}}$) is of deformational origin.

We shall now reproduce the tensor G_{ik} as well. It can be found from the electrical neutrality condition $j_\varkappa = 0$ by expressing the longitudinal effective field in terms of the transverse field:

$$\tilde{E}_\varkappa = -\frac{\tilde{E}_\alpha\sigma_{\varkappa\alpha}}{\sigma_{\varkappa\varkappa}} - \frac{\partial_{\varkappa l}u_l}{\sigma_{\varkappa\varkappa}}. \quad (7.14)$$

Elimination of the longitudinal field leads to the renormalization of the transverse conductivity tensors (7.11). The transverse field is found from the Maxwell equations (2.3), in

which the inhomogeneity is the deformation current $\mathbf{j}^{\mathrm{d}}$. It is then convenient to introduce the quantity $\rho_{\alpha\beta}$ from (7.11):

$$j_\alpha = \sigma^{\perp}_{\alpha\beta}\widetilde{E}_\beta + j^{\mathrm{d}}_\alpha, \quad \widetilde{E}_\alpha = -\rho_{\alpha\beta}j^{\mathrm{d}}_\beta + \frac{1}{c}[\dot{\mathbf{u}}, \mathbf{H}]_\alpha - \frac{m}{e}\ddot{u}_\alpha. \tag{7.15}$$

The current $\mathbf{j}^{\mathrm{d}}$ is given by

$$j^{\mathrm{d}}_\alpha = \sigma^{\perp}_{\alpha\beta}\left(\frac{1}{c}[\dot{\mathbf{u}}, \mathbf{H}]_\beta - \frac{m}{e}\ddot{u}_\beta\right) + \partial^{\perp}_{\alpha l}u_l. \tag{7.16}$$

Using the above relationships, we obtain the following expression for the Green's tensor[20] G_{ik}:

$$G_{iq} = -\left(\delta_{i\alpha} - \delta_{i\varkappa}\frac{\sigma_{\varkappa\alpha}}{\sigma_{\varkappa\varkappa}}\right)\rho_{\alpha\beta}\left(\frac{k^2c^2}{4\pi}\Delta_{\beta q} + \partial^{\perp}_{\beta q}\right) - \delta_{i\varkappa}\frac{\partial_{\varkappa q}}{\sigma_{\varkappa\varkappa}}, \tag{7.17}$$

which was used in the derivation of (7.10).

The dispersion relation that follows from (7.8) determines the (dimensionless) phase velocities $\sqrt{\zeta}$ of the elastic waves,[18] where the dynamic moduli d_{pq} given by (7.10) include the contribution due to nonequilibrium electrons and excited electromagnetic fields, and also the effect of the external magnetic field

$$\mathrm{Det}\,|\,(\eta_{\bar{p}} - \zeta)\,\delta_{pq} + d_{pq}\,| = 0. \tag{7.18}$$

As a rule, the interaction between electrons and the sound waves is ineffective (in Pippard's terminology[6]) because of the large difference between the velocities which, in turn, reflects the difference between the electron and ion masses: $m/M \sim s^2/v_{\mathrm{at}}^2 \sim 10^{-3} - 10^{-5}$. Because of this, $|d_{pq}| \ll 1$, and, for example, in the absence of degeneracy, the eigenvalues ζ are given by

$$\zeta_i = \eta_i + d_{\bar{i}\bar{i}} \quad (\eta_1 \neq \eta_2 \neq \eta_3). \tag{7.19}$$

[There are, however, possible situations where both an effective interaction is present (see below) and d_{pq} are not so small (they have "singularities"), in which case we must solve Eq. (7.8).] The phase velocity of the sound wave is obtained from $(\omega/k)_i = s\sqrt{\zeta_i}$, which yields the following order-of-magnitude estimate:

$$\frac{\Delta s}{s} \sim \mathrm{Re}\, d_{p\bar{p}}, \quad \frac{\gamma}{\omega} \sim \mathrm{Im}\, d_{p\bar{p}}. \tag{7.20}$$

Electronic rotation of the plane of polarization of sound occursin the degenerate case (this is investigated in detail for $\mathbf{k}\|\mathbf{H}$ in Refs. 21 and 64).

8. DEFORMATION POTENTIAL

The deformation potential tensor $\lambda_{ik}(\mathbf{p})$, introduced by Akhiezer,[2] is an individual characteristic of a conduction electron in the deformed crystal. The change in the energy of an electron $\lambda_{ik}(\mathbf{p})u_{ik}(\mathbf{r},t)$ during the deformation of the lattice[19] is related to this tensor [see (2.6)–(2.7) and below]. This characteristic of the electron can be determined either from microscopic theory or from experiment. So far, we know very little about it, which contrasts with the unperturbed dispersion relation at the Fermi level, which has been investigated in detail for most metals.

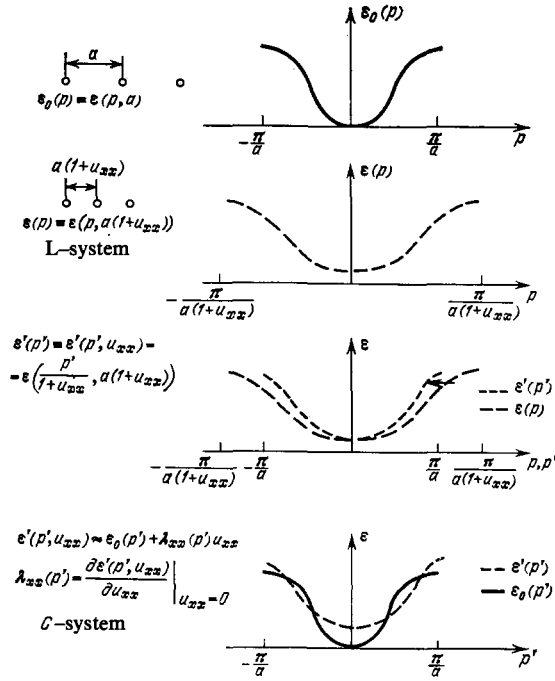

FIG. 7. Electron dispersion relation in the laboratory and the comoving systems of coordinates (introduction of deformation potential).

In contrast to the universal electric charge e and the spin, the "elastic charge" $\lambda_{ik}(\mathbf{p})$, taken together with the dispersion relation $\varepsilon_0(\mathbf{p})$, is the individual identifying characteristic of a quasiparticle that depends on its position on the Fermi surface.

As already noted, the dispersion relation is established over distances of the order of a few atomic distances (over which the periodicity of the lattice can already be distinguished) and, correspondingly, over atomic time intervals. Over such time and distance intervals, the lattice in the deformed crystal may be regarded as locally periodic, but with somewhat modified periods (and, correspondingly, modified symmetry), so that we may introduce the idea of the local dispersion relation $\varepsilon(\mathbf{r}, \mathbf{p}, t)$. This approach corresponds to the adiabatic approximation, in which the electron energy is determined for fixed positions of the ions.[71]

A consistent phenomenological introduction of the dispersion relation for a weakly deformed lattice is possible, as noted by Landau (see footnote 8), in the comoving coordinate frame. The transformation $\mathbf{r}' + \mathbf{u}(\mathbf{r}', t) = \mathbf{r}$ to the c-system[14,20] (see Fig. 2) can be used to eliminate the motion of the given volume element dV and, by changing the scale, retain the systematics of states of the undeformed crystal.

Indeed, consider the homogeneous and stationary deformation $u_{ik} = \mathrm{const}$ (for simplicity, assume that, initially, there is no rotation), for which the period $\mathbf{a}_0$ becomes $\mathbf{a}_0 + \delta\mathbf{a}$, where $\delta a_i = a_k u_{ik}$. The electron has a band spectrum both in the undeformed and the deformed crystal ($\varepsilon_0(\mathbf{p}) \equiv \varepsilon(\mathbf{p}; \mathbf{a}_0)$ and $\varepsilon(\mathbf{p}) \equiv \varepsilon(\mathbf{p}; \mathbf{a}_0 + \delta\mathbf{a})$, respectively). The dispersion relations $\varepsilon_0(\mathbf{p})$ and $\varepsilon(\mathbf{p})$ differ from one another not only by the explicit dependence on the lattice parameters $\mathbf{a}$ (through the transparency of the potential barriers, and so

[18] This expression determines the velocity also of other, i.e., electromagnetic and "electronic" Bose spectral branches, which, under certain definite conditions, can be present in the metal and appear as singularities in the moduli d_{pq} in the accepted form of writing them.[35,67–69]

[19] The simplest form of modulation of the electron energy was considered as far back as 1935 by Titeika.[70]

on), but also by the change in the region in which the quasimomentum **p** is defined, because of the change in the size (and shape) of the Brillouin zone. Thus, in the one-dimensional case, the quasimomentum in the undeformed crystal runs through the values

$$p^0_{(l)}=\frac{2\pi l}{Na_0};\quad l=-\frac{N}{2},\ -\frac{N}{2}+1,\ \ldots,\ \frac{N}{2};$$
$$-\frac{\pi}{a_0}\leqslant p^0\leqslant\frac{\pi}{a_0}, \tag{8.1}$$

whereas, in the deformed crystal, we have

$$p_{(l)}=\frac{2\pi l}{Na_0(1+u_{xx})},\quad -\frac{\pi}{a_0(1+u_{xx})}\leqslant p\leqslant\frac{\pi}{a_0(1+u_{xx})}. \tag{8.2}$$

At the same time, although the systematics of states with respect to l remains unaltered, the systematics with respect to the quasimomentum **p** undergoes a substantial change (Fig. 5). Transformation to the c-system by substituting **p**(**p**'), **a**(**a**') into $\varepsilon(\mathbf{p},\mathbf{a})$ will obviously conserve the systematics of states with respect to quasimomentum that is found in the undeformed crystal:

$$p'=p(1+u_{xx}),\quad p'_{(l)}=\frac{2\pi l}{Na_0},\quad -\frac{\pi}{a_0}\leqslant p'\leqslant\frac{\pi}{a_0}. \tag{8.3}$$

In the c-system, the dispersion relation assumes the form

$$\varepsilon'(p';u_{xx})\equiv\varepsilon\left(\frac{p'}{1+u_{xx}};a(1+u_{xx})\right). \tag{8.4}$$

The unperturbed dispersion relation can also be looked upon as given in the c-system ($a_0\equiv a'$). When the relative deformation is small, the deformation potential can be defined by (in the absence of degeneracy)

$$\varepsilon'(\mathbf{p}';u_{lk})=\varepsilon(\mathbf{p}(\mathbf{p}');\mathbf{a}(\mathbf{a}')),\quad \lambda_{ik}(\mathbf{p}')=\frac{\partial\varepsilon'(\mathbf{p}';u_{ik})}{\partial u_{ik}}\Big|_{u_{ik}=0}. \tag{8.5}$$

As noted in Ref. 72, the utilization of the c-system in the case of homogeneous stationary deformation enables us to use perturbation theory with unperturbed "boundary" conditions (periodic conditions).

Let us now consider the case of arbitrary deformation. Since **a** represents vectors connecting closely spaced lattice points ($a_i\equiv dx_i$), we have for such points[33]

$$a_i=a'_i+\frac{\partial u_i}{\partial x_k}a'_k. \tag{8.6}$$

The basis vectors of the lattice $\mathbf{a}^{(l)}$ ($l=1,2,3$) transform in the same way. The transformation law for the reciprocal lattice vectors

$$b_l=b'_i-\frac{\partial u_k}{\partial x_l}b'_k, \tag{8.7}$$

that transform in the same way as the p_i, follows from the condition $\mathbf{a}\cdot\mathbf{b}=m$ and, hence, $a_i\delta b_i=-b_k\delta a_k$. Since energy is unaffected by the rotation of the given volume element (as a whole), the dispersion relation is a function of rotational invariants that consist of **p** and the ordinary and reciprocal lattice vectors: p^2, **pa**, **pb**, $\mathbf{a}^{(l)}$ $\mathbf{a}^{(m)}$, $\mathbf{b}^{(l)}$ $\mathbf{b}^{(m)}$. In contrast to the other scalar products, the product **p**·**a** is then invariant also under the general deformation (3.3) as well, where, to within linear terms, we have

$$p^2=(p')^2-2p'_ip'_ku_{ik},\quad \mathbf{pb}=\mathbf{p}'\mathbf{b}'-(p'_ib'_k+p'_kb'_i)u_{ik},$$
$$\mathbf{a}^{(l)}\mathbf{a}^{(m)}=\mathbf{a}'^{(l)}\mathbf{a}'^{(m)}+(a'^{(l)}_ia'^{(m)}_k+a'^{(l)}_ka'^{(m)}_i)u_{ik}. \tag{8.8}$$

Using these properties, we find that, for strong coupling and with the dispersion relation[20]

$$\varepsilon_0(\mathbf{p},\{\mathbf{a}\})=\varepsilon_0+\alpha\{\mathbf{a}\}+\sum_n\beta_n\{\mathbf{a}\}\exp\frac{i}{\hbar}\mathbf{pa}_n, \tag{8.9}$$

we have

$$\lambda_{ik}=\frac{1}{2}\sum_{l=1}^{3}\Big[\left(a_i^{(l)}\frac{\partial\alpha}{\partial a_k^{(l)}}+a_k^{(l)}\frac{\partial\alpha}{\partial a_i^{(l)}}\right)+\sum_n\left(a_i^{(l)}\frac{\partial\beta_n}{\partial a_k^{(l)}}+a_k^{(l)}\frac{\partial\beta_n}{\partial a_i^{(l)}}\right)\exp\frac{i}{\hbar}\mathbf{pa}_n\Big], \tag{8.10}$$

where the sum in (8.9) is evaluated over neighbors in the nearest coordination spheres. Only the **p** dependent part of $\lambda_{ik}(\mathbf{p})$ is of interest because the constant cancels out in the renormalized tensor $\lambda_{ik}(\mathbf{p})$. Thus, in the cubic lattice with nearest-neighbor interaction, this part of $\lambda_{ik}(\mathbf{p})$ is given by

$$\lambda_{ik}(p)=\sum_{l=1}^{3}\left(a_i^{(l)}\frac{\partial\beta}{\partial a_k^{(l)}}+a_k^{(l)}\frac{\partial\beta}{\partial a_i^{(l)}}\right)\left(\cos\frac{p_xa}{\hbar}+\cos\frac{p_ya}{\hbar}+\cos\frac{p_za}{\hbar}\right). \tag{8.11}$$

Hence, it is clear, in particular, that the known plane isoenergetic surfaces, that are obtained in the strong-coupling model by annulling the sum (square lattice) or the product (bcc[73]) of cosines, correspond to $\Lambda_{ik}(\mathbf{p})=0$.

For weak coupling, well away from points of degeneracy, for which

$$\varepsilon_0(\mathbf{p})=E_0(\mathbf{p})+\sum_{\mathbf{b}}\frac{|V_{\mathbf{b}}|^2}{E_0(\mathbf{p})-E_0(\mathbf{p}+\hbar\mathbf{b})},\quad E_0(\mathbf{p})\equiv\frac{p^2}{2m}, \tag{8.12}$$

we find on substituting **p**(**p**'), **b**(**b**'), $V(u_{ik})$ and expanding in powers of u_{ik}:

$$\lambda_{il}(\mathbf{p})=-\frac{p_ip_l}{m}+\sum_{\mathbf{b}}\frac{2|V_{\mathbf{b}}|^2}{E_0(\mathbf{p})-E_0(\mathbf{p}+\hbar\mathbf{b})}\left(\left|\frac{V_{\mathbf{b},il}}{V_{\mathbf{b}}}\right|+\frac{p_ib_l+p_lb_i+\hbar b_ib_l}{2\mathbf{pb}+\hbar b^2}\right), \tag{8.13}$$

where $V_{\mathbf{b},il}$ is the derivative of the unperturbed potential with respect to u_{il}, and the leading term mv_iv_l appears as a result of the transformation of $p^2/2m$.

In the region of quasimomenta, where $\varepsilon_0(\mathbf{p})=(p^2/2m^*)$, the deformation potential can be expressed in terms of the effective mass[30] and its derivatives with respect to the lattice parameters

$$\lambda_{ik}(\mathbf{p})=-m^*v_iv_k+\frac{p^2}{4}\sum_{l=1}^{3}\left(a_i^{(l)}\frac{\partial}{\partial a_k^{(l)}}+a_k^{(l)}\frac{\partial}{\partial a_i^{(l)}}\right)\frac{1}{m^*\{\mathbf{a}\}}. \tag{8.14}$$

The expression for the tensor $\lambda_{ik}(\mathbf{p})$ in terms of the Bloch matrix elements are given in the literature.[42,43]

Thus, to obtain the deformation potential, it is sufficient to examine homogeneous deformation. Both lattices (original and deformed) are then perfectly periodic and each has its own quasiparticle dispersion relation. The deformed lattice corresponds to the l-system. Transformation [according to (3.3)] to the coordinate system in which the periods are undisturbed is equivalent to the transformation to the c-system. In the dispersion relation for the deformed lat-

[20]The expressions given by (8.9) and (8.10) may be looked upon as quite general if one supposes that (8.9) is the Fourier expansion of the dispersion relation and the sum over n in (8.9) is evaluated over all the sites of the Bravais lattice.

tice, this corresponds to the substitutions for $\mathbf{p}(\mathbf{p}')$ and $\mathbf{a}(\mathbf{a}')$ given by (8.6), with the result that, in the perturbed dispersion relation, we then have a dependence on the components u_{ik}, and the first term in the expansion in terms of these components determines $\lambda_{ik}(\mathbf{p})$ in (8.5).

9. SYMMETRY PROPERTIES

The deformation potential $\lambda_{ik}(\mathbf{p})$ is a tensor function of quasimomentum that is invariant under transformations g of the point group G of the crystal:[71,74]

$$g\hat{\lambda}(g^{-1}\mathbf{p})\,g^{-1} = \hat{\lambda}(\mathbf{p}), \quad \hat{\lambda}(-\mathbf{p}) = \hat{\lambda}(\mathbf{p}). \tag{9.1}$$

The parity λ is a consequence of symmetry under time reversal. In the isotropic case, it is clear that $\lambda_{ik}(\mathbf{p})$ can be written in the form

$$\lambda_{ik}(\mathbf{p}) = \lambda_1(\varepsilon)\,\delta_{ik} + \lambda_2(\varepsilon)\,\frac{p_i p_k}{p^2}. \tag{9.2}$$

In the absence of a magnetic field, (9.1) relates the values of the components of the tensor $\hat{\lambda}(\mathbf{p})$ at different points in the Brillouin zone (that enter the star of the given vector $\mathbf{p}$). This does not impose any essential restrictions on the components of $\hat{\lambda}$. It is only at individual symmetric points (lines, planes) of the Brillouin zone that the components of $\hat{\lambda}(\mathbf{p})$ are found to be related by symmetry transformations at the given point. Consider the group $G_{\mathbf{p}} \subset G$, whose elements $g_{\mathbf{p}} \in G_{\mathbf{p}}$ form an invariant vector $\mathbf{p}$ that occupies a symmetric position in the zone; $g_{\mathbf{p}}\mathbf{p} = \mathbf{p}$. The tensor $\lambda_{ik}(\mathbf{p})$ then transforms as a constant tensor:

$$g_{\mathbf{p}}\hat{\lambda}(\mathbf{p})\,g_{\mathbf{p}}^{-1} = \hat{\lambda}(\mathbf{p}). \tag{9.3}$$

This means that the components of $\lambda(\mathbf{p})$ become related at symmetric points, and some of the components must vanish.

Table I lists groups $G_{\mathbf{p}}$ (symmetry groups that have as an invariant the vector component along which we take the z axis) and gives the selection rules for the components $\lambda_{xy}(p_z)$ and $\lambda_{xz}(p_z)$ (clearly, $\lambda_{zz}(p_z)$ is not zero). However, in metals (as opposed to semiconductors), the individual points do not play an essential role because whole regions of the Fermi surface provide a contribution. Symmetry properties do not, therefore, play a dominant role. The situation is quite different in a strong magnetic field.

We now turn to the symmetry properties of averages evaluated over the orbital period in a magnetic field.[66] In terms of the variables ε, p_z, and the orbital angle $\varphi = \Omega\tau$, we have

$$\hat{\lambda}(\mathbf{p}) = \hat{\lambda}(\varepsilon, p_z, \varphi), \tag{9.4}$$

and

$$\bar{\hat{\lambda}} = \bar{\hat{\lambda}}(\varepsilon, p_z), \tag{9.5}$$

if the section (in the given zone) is simply connected. Let us now consider the subgroup of the symmetry transformations g_H that do not affect the component of the quasimomentum in the direction of the field: $g_H p_z = p_z$. Under the group G_H of these transformations, the quantities $\bar{\lambda}_{ik}$ behave as the components of a constant tensor:

$$g_H\bar{\lambda}(\varepsilon, p_z)\,g_H^{-1} = \bar{\lambda}(\varepsilon, p_z). \tag{9.6}$$

The only nonzero components are then those that are invariants of the group G_H. The latter is determined by the symmetry of the direction Oz of the magnetic field in the crystal. Since the asymptotic behavior of the transport coefficients in a strong field depends on averages over the orbital period in the field, symmetry often plays a dominant role.

Let us begin by considering the case $Ox \| \mathbf{k}\perp\mathbf{H} \| Oz$. When we determine the asymptotic behavior that governs the absorption of transverse sound in strong fields in accordance with (10.5), the leading terms are determined by $\bar{\Lambda}_{x\alpha}(\alpha = y, z)$, and, for $\bar{\Lambda}_{x\alpha} = 0$, the higher-order terms in the expansion in $1/H$ contain the averages $\overline{v_x\psi_{x\alpha}} \equiv -m_*^{-1}\overline{p_y\Lambda_{x\alpha}}$, $\overline{v_x\psi_{x\alpha}^2}$, $\overline{\psi_{x\alpha}^2\nu^*}(\partial\psi_{x\alpha}/\partial\varphi \equiv \Lambda_{x\alpha})$.

For any symmetry, $\psi_{x\alpha}^2\nu^* \neq 0$, which gives rise to the terms $\sim d_{xx}^{\mathrm{I}}|r/l\,\cdot|^2$ in the asymptotic behavior of $d_{\alpha\alpha}^{\mathrm{I}}$. The average $\overline{v_x\psi_{x\alpha}}$ produces the terms $\sim d_{xx}^{\mathrm{I}}(kr)^2$ in $d_{\alpha\alpha}^{\mathrm{I}}$. Let us examine when they can be different from zero. The groups G_H which are of interest to us are groups that admit the existence of the invariant vector (p_z): C_1, C_s, C_n, C_{nv}. The quantity $\overline{p_y\Lambda_{xz}}$ transforms as the x, y component of an antisymmetric tensor (with the exception of the isotropic case, for which $\Lambda_{xz} \sim p_x p_z$). It can therefore be nonzero either because of its symmetric part (groups C_s and C_{2v} that do not contain the m_y plane), or because of the antisymmetric part that transforms as the z component of the angular momentum (groups C_n). The selection rules for $\overline{p_y\Lambda_{xz}}$ can be found in an analogous manner. The results are collected together in Table II. The selection rules for $\overline{v_x\psi_{x\alpha}^2}$ will be found by recalling that $\partial/\partial\tau$ in (2.5) transforms as the z component of angular momentum and, consequently, ψ_{xz} transforms as the y component of a vector, whereas ψ_{xy}—as its square. Hence, it follows that $\overline{v_x\psi_{x\alpha}^2} \neq 0$ for $G_H = C_{3v}$. For this symmetry of the field direction, $d_{\alpha\alpha}^{\mathrm{I}} \sim d_{xx}^{\mathrm{I}} kr|r/l\,\cdot|$ for $\bar{\Lambda}_{x\alpha} = 0$).

An essentially different picture arises when the sections are multiply connected and transform one into the other un-

TABLE I. Selection rule for the off-diagonal components of the deformation potential for symmetric directions (z axis) of quasimomentum in the Brillouin zone.

G_{p_z}	C_1	C_2	C_s			C_{2v}		C_{3v}	C_{4v}	C_{6v}
			m_x *	m_y	m_ξ **	$2m_x m_y$	$2m_\xi m_\eta$			
$\Lambda_{xy}(p_z)$	$\neq 0$	$\neq 0$	0	0	$\neq 0$	0	$\neq 0$	0	0	0
$\Lambda_{xz}(p_z)$	$\neq 0$	0	$\neq 0$	0	$\neq 0$	0	0	0	0	0

*m_x (m_y)—mirror plane containing the OX (OY) axis.
**m_ξ (m_η)—mirror plane not containing the OX (and OY) axis.

TABLE II. Selection rules for the deformation potential components averaged over the rotation angle for symmetric directions of the magnetic field in the crystal.

$G_{\mathbf{H}} \rightarrow$	C_1	C_2	C_s			C_{2v}		C_{3v}		C_{4v}	C_{6v}
			m_x	m_y	m_ξ	$2m_x m_y$	$2m_\xi m_\eta$	$3m_y$	$3m_\xi$		
$\overline{\Lambda_{xy}}$	$\neq 0$	$\neq 0$	0	0	$\neq 0$	0	$\neq 0$	0	0	0	0
$\overline{p_y \Lambda_{xy}}$	—	—	$\neq 0$	0	—	0	—	0	$\neq 0$	0	0
$\overline{\Lambda_{xz}}$	$\neq 0$	0	$\neq 0$	0	$\neq 0$	0	0	0	0	0	0
$\overline{p_y \Lambda_{xz}}$	—	$\neq 0$	—	0	—	0	$\neq 0$	0	0	0	0

der the transformations of the group G_H. What we have said above then applies only to the sum $\Sigma_a \bar{\Lambda}^a_{ik}$. On the other hand, each of the averages $\bar{\Lambda}^a_{ik}$ will, in general, be different from zero. Because of this, the leading term in the expansion for d^1_{aa} will not vanish even when the direction of the magnetic field in the crystal has a high degree of symmetry.

$$\sum_a \overline{\Lambda_{\alpha x} R^s \Lambda^a_{\alpha x}} \simeq \sum_a \frac{(\bar{\Lambda}^a_{\alpha x})^2}{\bar{\nu}^{*2}}. \tag{9.7}$$

For $\mathbf{k}\|\mathbf{H}\|Oz$, the "selection rules" for $\bar{\Lambda}_{xz}$, which determines the leading term in the deformational interaction of transverse sound (x is now the direction of polarization, z the direction of propagation and of the magnetic field), are the same as for $\mathbf{k}\perp\mathbf{H}\|\mathbf{u}$ (where x was the direction of propagation, z the direction of polarization and of the magnetic field; see Table II). However, in the directions in which $\bar{\Lambda}_{xz} = 0$, the next term in the expansion for $\mathbf{k}\|\mathbf{H}\perp\mathbf{u}$ is determined by the average $\overline{v_z \psi^2_{xz}}$, and, in contrast to the situation where $\mathbf{k}\perp\mathbf{H}\|\mathbf{u}$, the relative change in the velocity of transverse sound relative to longitudinal sound in all these symmetric directions is $\sim(kr)^2$.

The situation where deformation leads to the removal of degeneracy and to the splitting of bands, which occurs, in particular, in nonlinear effects during sound propagation,[38] constitutes a special case.

10. TRANSPORT COEFFICIENTS IN A STRONG MAGNETIC FIELD

In a strong magnetic field, the asymptotic behavior of the transport coefficients can be found under quite general assumptions about the electron spectrum.

We start with the microscopic expression given by (6.14) for the transport coefficients in terms of the Green's operator R for the Fourier component of the transport equation (2.2). In the integrals over the Fermi surface $\langle A \rangle$, the evaluation of the averages $\bar{A}$ over the orbital angle $\varphi = \Omega\tau$ in the magnetic field[21)]

$$\langle A \rangle \equiv \frac{2}{h^3} \int \mathrm{d}p_z \cdot 2\pi m^* \bar{A}, \quad \bar{A} \equiv \frac{1}{2\pi} \int_0^{2\pi} A(\varphi)\, \mathrm{d}\varphi; \quad \langle A \rangle \equiv \langle \bar{A} \rangle \tag{10.1}$$

can be performed in a sufficiently general form. We now introduce the relaxation time $\nu^{-1}(\mathrm{p})$ by making the replacement $\hat{\nu}\chi \rightarrow \nu\chi$ in the transport equation (2.2). For an arbitrary periodic function $g(\tau)$, the quantity Rg identically satisfies the equation

$$\frac{\partial}{\partial\varphi} Rg + \frac{\alpha}{\Omega} Rg = \frac{g}{\Omega}, \quad \alpha \equiv i(\mathbf{kv} - \omega) + \nu, \tag{10.2}$$

where the explicit expression for Rg is

$$Rg = \frac{1}{\Omega(1 - \exp(-2\pi\bar{\alpha}/\Omega))} \int_{\varphi - 2\pi}^{\varphi} \mathrm{d}\varphi_1 g(\varphi_1) \exp \frac{1}{\Omega} \int_{\varphi}^{\varphi_1} \mathrm{d}\varphi_2 \alpha(\varphi_2). \tag{10.3}$$

We shall confine our attention to closed sections through the Fermi surface. In a strong magnetic field, it is convenient in the evaluation of the leading terms of the expansion in $1/H$ to use the following identities that follow from (10.2) and (10.3) for periodic r and g:

$$\overline{\frac{\partial r}{\partial\varphi} Rg} = \frac{1}{\Omega}(-\overline{rg} + \overline{r\alpha Rg}), \quad \overline{gR\frac{\partial r}{\partial\varphi}} = \frac{1}{\Omega}(\overline{gr} - \overline{gRr\alpha}), \tag{10.4}$$

and the relation

$$\overline{rRg} \approx \frac{\overline{rg}}{\bar{\alpha}} + O\left(\frac{1}{\Omega}\right). \tag{10.5}$$

A number of exact and approximate equations that is useful in evaluating the average over the orbital period in a strong magnetic field is given in Ref. 66, (B1)–(B7). They readily yield the expansion for the transport coefficients in a strong magnetic field.

Let us initially confine our attention to the case $\mathbf{k}\perp\mathbf{H}$:

$$\mathbf{H}(0, 0, H), \quad \mathbf{k}(k, 0, 0), \quad v_x = m_*^{-1} \frac{\partial p_y}{\partial\varphi}, \tag{10.6}$$

$$v_y = -m_*^{-1} \frac{\partial p_x}{\partial\varphi}, \quad \bar{\alpha} = \bar{\nu}^*, \quad \nu^* = \nu - i\omega \tag{10.7}$$

and consider the leading term in the expansion for the coefficients b_{il} that determine the deformation term d^1_{il} in (7.10):

$$b_{il} = i\omega k^2 \overline{\langle \Lambda_{ix} R^s \Lambda_{lx} \rangle}, \quad \overline{\Lambda_{ix} R^s \Lambda_{lx}} \simeq \frac{\overline{\Lambda_{ix}} \cdot \overline{\Lambda_{lx}}}{\bar{\nu}^*} + O\left(\frac{1}{\Omega}\right). \tag{10.8}$$

When the z axis, which is also the direction of the magnetic field, is not a symmetry axis of the crystal, then, according to (9.6), we have $\bar{\Lambda}_{ix} \neq 0$, which leads to the following estimate[22)] for the matrix b_{il}:

$$b_{il} \sim i\omega k^2 n_0 \varepsilon / \nu^* \quad (n = 1), \tag{10.9}$$

[21)]The angle brackets also include summation over zones (the index is omitted) and over all singly-connected sections of a given zone (index a):

$$\langle A \rangle = \frac{2}{h^3} \sum \int dp_z \sum_a 2\pi m^*_a \bar{A}^a. \tag{10.1'}$$

[22)]We assume in making estimates that we are dealing with a good metal, in which the number of electrons per atom is of the order of unity and, accordingly, the only characteristic energy is the atomic energy $1 \sim \varepsilon \sim mv^2 \sim Ms^2$, where v and m are the Fermi velocity and electron mass, and M is the ion mass.

where the order of the z axis is indicated in parentheses. When **H** lies along a twofold axis, we have

$$b_{\mu\nu} \sim \frac{i\omega k^2 n_0 e}{\nu^*}, \quad b_{zz} \sim i\omega k^2 n_0 e \frac{1}{\nu^*}\left[\left(\frac{r}{l}\right)^2 + (kr)^2\right] \quad (n=2), \tag{10.10}$$

where $r \equiv v/\Omega$ is the twist radius and $l = v/\nu^*$ is the (complex) mean free path. For cases where **H** lies along a higher order axis, see Table II. The asymptotic form of the conductivity tensor has frequently been calculated[5,12,67,69] and the low-symmetry case is examined in Ref. 66 for $\mathbf{k} \perp \mathbf{H}$, $r/l \ll kr \ll 1$.

11. THE CONCEPT OF INEFFECTIVENESS AND SPECIAL CASES OF EFFECTIVE INTERACTION BETWEEN ELECTRONS AND SOUND

Because the velocity of sound is low ($s \sim 10^5$ cm/s) as compared with the velocity of electrons on the Fermi surface (for good metals, $v \sim 10^8$ cm/s, and, for semimetals, $v \sim 10^7$–10^8 cm/s), the electron-phonon interaction in the collisionless region is "ineffective" (in Pippard's terminology[6]). This is the reason why, as the frequency of sound increases, pure metals exhibit,[3,4] at low temperatures, a transition from viscous high-frequency absorption $\gamma/\omega \sim \omega/\nu$ to collisionless absorption $\gamma/\omega \sim s/v$ when the mean free path $l = v/\nu$ exceeds the wavelength of sound $\lambda = 2\pi/k$. The temperature dependence then disappears, since $l(T)$ in the expression for γ is replaced with λ. Collisionless absorption involves the participation of only electrons in a narrow "belt" $\mathbf{kv} = \omega$ that are in synchronism with the sound wave. Saturation of relative absorption γ/ω at the low level s/v is connected with the reduction in the width of the belt, and, correspondingly, in the number of resonant electrons, as the sound frequency increases. This can be demonstrated by considering the example of the main effect, namely, the pure deformational interaction [d^1 in (7.10)], and initially ignoring the contribution of electric fields excited by the sound waves (see Sec. 13).

Using the relaxation time approximation, and confining our attention to an elastic wave of given polarization, we can rewrite the dispersion relation (7.18) in the form

$$\left(\frac{\omega}{\omega_0}\right)^2 - 1 = -\frac{2i\omega}{\rho s^2 h^3}\int \frac{dS_p}{v}\frac{\Lambda^2}{i(\mathbf{kv}-\omega)+\nu}, \tag{11.1}$$

where $\Lambda \equiv \Lambda_{j\varkappa}$ is the component of the deformation potential, j is the polarization index, $\omega_0 = ks$ is the undisturbed frequency of the sound waves, and s is the sound velocity.

The right-hand side of (11.1) is small and the equation can be solved in accordance with the concept of ineffectiveness by replacing the frequency ω with the undisturbed frequency $\omega_0 = ks$. Hence, recalling that $R^s \sim 1/\nu$ for $kl \ll 1$ and $R^s \sim \pi\delta(\mathbf{kv})$ for $kl \ll 1$, we find in accordance with Refs. 3 and 4 and even earlier work on high-frequency absorption ($\omega\tau \gg 1$) in the "jelly" model:[75]

$$\frac{\Delta\omega}{\omega} \sim \frac{\Delta s}{s} + i\frac{\gamma}{\omega} \sim d^{\mathrm{I}}_{xx} \sim \begin{cases} \left(\frac{\omega}{\nu}\right)^2 + i\frac{\omega}{\nu} & kl \ll 1, \quad (11.2) \\ \left(\frac{s}{v}\right)^2 + i\frac{s}{v} & kl \gg 1. \quad (11/3) \end{cases}$$

In general, this result is not very sensitive to the geometry of the Fermi surface and, in particular, the absorption contains $s/v_{\mathrm{at}} \sim \sqrt{m/M}$, so that small groups provide the same contribution as large groups.[76]

However, on flat and cylindrical portions of the Fermi surface,[23] including local areas of flatness, i.e., the neighborhoods of points and lines on which the Gaussian curvature $\mathcal{K}(p)$ is zero (practically all metals with anisotropic Fermi surfaces have such points), the situation is quite different from that described above (Fig. 8). When the wave propagates along a plane or the axis of a cylinder, all electrons corresponding to these areas are simultaneously in synchronism with the wave.[78–80] The number of resonance electrons does not decrease (or does not decrease as rapidly) with increasing frequency (as on the belt), and the frequency increase continues up to the collision frequency renormalized to the fraction of the flat or cylindrical segment of the Fermi surface (Fig. 9). The individual points then provide a contribution that is comparable with the contribution of the remainder of the Fermi surface. We thus encounter cases of *effective* interaction between electrons and sound. There is a corresponding sharp angular dependence on the direction of **k**: absorption and dispersion increase in critical directions when all the electrons on a local plane or cylinder take part in the synchronous interaction with the wave. This situation is similar to that in a strong magnetic field **H** ($kr \ll 1$) for $\mathbf{k} \perp \mathbf{H}$ and closed Fermi-surface sections,[81,82] and also corresponds to the case of *effective* interaction. The strong dependence of absorption and dispersion on the direction of propagation is similar to the deviation effect[83–85,82] in a strong field.

This substantial contribution of isolated points and lines is connected with a property of the partial density of states. Let us illustrate this by considering the example of a cylindrical segment of the Fermi surface (of length Δp). When **k** is perpendicular to the axis of the cylinder (nonsingnular direction), the belt coincides with the generator, and the (deformational) absorption is

$$\frac{\gamma}{\omega} = \frac{2\pi}{h^3}\frac{\omega\Delta p}{\rho s^2}\int \frac{dp_x}{v}\Lambda^2\delta(kv_x - \omega) = \frac{\Delta p}{4\pi^2 h^3 \rho s}\frac{\Lambda^2}{v\,|\partial v_x/\partial p_x|_{v_x=\omega/k}}. \tag{11.4}$$

It is clear that, if the cross section of the cylinder contains a point p_c at which $(\partial v_x/\partial p_x)_c = 0$, then (11.4) shows that $\gamma \to \infty$ when the belt approaches this point.[24] The condition $\varepsilon_{xx} \equiv \partial v_x/\partial p_x = 0$ is a special case of the vanishing of the Gaussian curvature $\mathcal{K}(\mathrm{p}) = (1/\mathrm{v}^2)(\varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}^2)$. The inte-

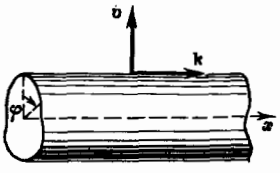

FIG. 8. Geometry of effective interaction on a cylindrical Fermi surface for $\omega\tau_{\mathrm{eff}} < 1$.

[23] The role of flat and cylindrical segments of the Fermi surface in the electron-phonon interaction and, in particular, in the Kohn singularity, was first investigated by Afanas'ev and Kagan.[77] Kaganov and Semenenko[76] have examined the role of singularities on the Fermi surface. The two approaches are combined here, since "singular" points are locally "planes" or "cylinders".

[24] The range of validity of these expressions will be established below.

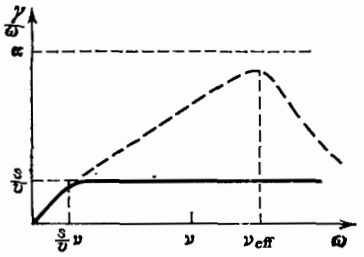

FIG. 9. Saturation of relative absorption in an arbitrary direction and its growth in the critical direction corresponding to the effective interaction.

gral in the dispersion relation given by (11.1) has a maximum when the pole $\mathbf{kv} = \omega$ lies in the neighborhood of a point (line) with zero curvature. Since $\mathcal{K} = \mathcal{K}_1 \cdot \mathcal{K}_2$ ($\mathcal{K}_1$, $\mathcal{K}_2$ are the principal curvatures of the surface), there are possible points of flatness at which both principal curvatures of the surface are zero, and parabolic points, at which only one curvature is zero (Fig. 10). Generally speaking, Fermi surfaces have lines of zero curvature. In the general case, for a given critical direction of k, the anomalous contribution is provided by one point on the line of zero curvature, i.e., the point of its intersection with the belt $\mathbf{kv} = \omega$.

The contribution of zero curvature points, and of finite areas of zero curvature, differs from ordinary high-frequency absorption in that it is very dependent on the mean free path and, therefore, exhibits a definite temperature dependence.[78-80] Fil', Denisenko, and Bezuglyĭ[87] were the first to report the discovery of these effects and the observation of the accompanying temperature dependence (Fig. 11). This approach also provides a natural explanation of previously observed anomalies[74,88,89] (Figs. 12 and 13; see also the paper by Suslov,[90] where it is shown, in particular, that the critical directions may have a high degree of symmetry in many metals; Fig. 14).

In the limit as $l \to \infty$, both absorption and dispersion exhibit singularities at the zero-curvature points. Their character, which gives us some information on the angular dependence for finite l, is indicated below. Finite collision frequency is taken into account in order to estimate the size of the effect and to investigate its frequency dependence.

We note that the introduction of a relaxation time cannot be justified[86] for finite areas of zero curvature (Fig. 15). This is possible at points of zero curvature because it is only a narrow region of $\mathbf{p}$ space—the "belt"—that provides a contribution to the interaction with sound, just as in the case of ordinary high-frequency absorption.

Let us now consider the contribution of the neighborhood of the zero-curvature point (line) to the integral (11.1), evaluated over all the Fermi surfaces:

$$\left(\frac{\Delta\omega}{\omega}\right)_{\text{sing}} = -G(\mathbf{p}_c) \int \frac{dS_p}{\mathbf{kv} - \omega - i\nu}, \tag{11.5}$$

where

$$G(\mathbf{p}_c) = \frac{2\omega}{\rho s^2 \hbar^3} \frac{\Lambda^2(\mathbf{p}_c)}{v(\mathbf{p}_c)}$$

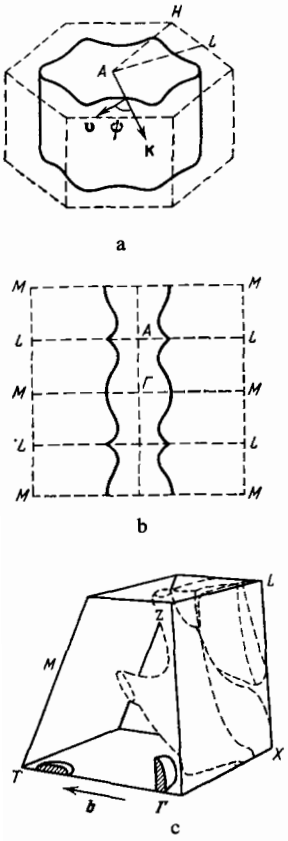

FIG. 10. Examples of Fermi surfaces with lines of flattening (dichalcogenide 2H-NbSe[112]), with parabolic lines (elements of the Fermi surface of gadolinium[113]), and, possibly, with areas of flattening (gallium[114]).

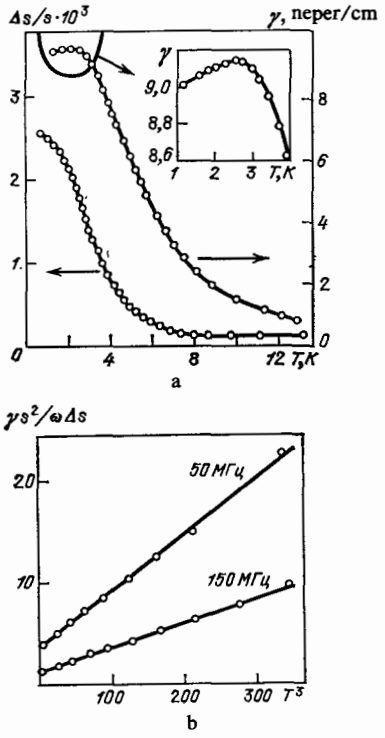

FIG. 11. a—Temperature dependence of absorption and dispersion of the velocity of longitudinal sound in pure Ga for the case of propagation along the **b** axis[87]; b—the function $\omega\tau \sim T^{-3}$ of Ref. 87, showing little flattening on the Fermi surface as compared with the thermal momentum of a phonon.[87,100]

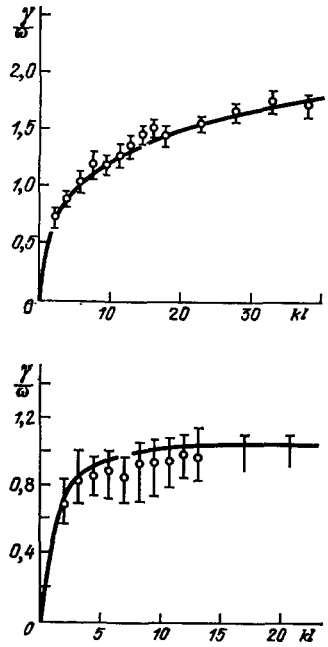

FIG. 12. Experimental data on the relative absorption of sound in copper[74]: a—[111] direction (setting in of saturation for $kl \sim 5$ can be seen), b—[100] direction (logarithmic rise up to $kl \sim 40$: after Suslov.

and the slowly-varying functions $A^2(\mathbf{p})$ and $v(\mathbf{p})$ have been taken outside the integral sign at the zero-curvature point $\mathbf{p}_c$.

An important point is that the expansion of $\mathbf{kv} - \omega$ in (11.5) near the zero-curvature point does not contain definite linear terms. Without pausing to perform the actual calculations (see Refs. 78–80), let us briefly consider the character of the resulting singularities (Fig. 16).

At an isolated point of flattening, any direction of $\mathbf{k}$ in the tangent plane (or, more precisely, on a cone at an angle s/v to it) will be a resonance direction. Absorption at the resonance $\mathbf{kv}_c = \omega$ has a logarithmic singularity of the form

$$\frac{\gamma}{\omega} \sim \frac{s}{v} \ln \left| \frac{\omega}{\mathbf{kv}_c - \omega} \right|,$$

and the velocity of sound undergoes a jump (X-type point):

$$\frac{\Delta s}{s} \sim \frac{s}{v} \theta \left(\frac{\mathbf{kv}_c - \omega}{\omega} \right),$$

or, on the contrary, absorption undergoes a discontinuity and the velocity of sound has a logarithmic singularity (O-type point).[79,80] A special direction will, however, appear if the point of flattening belongs to a parabolic line. When $\mathbf{k}$ is parallel to the plane touching it or perpendicular to it, the right-hand side of (11.5) and, with it, absorption and dispersion, may then have square-root-type singularities [see

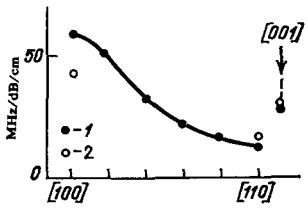

FIG. 13. Angular dependence of absorption in tin[88,89]. The maximum corresponds to the critical direction (after Suslov).

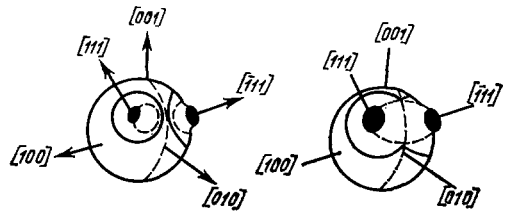

FIG. 14. Emergence of lines of parabolic points on to a symmetric direction in the course of an increase in interference between necks.[90]

(13.16)] of the form $\sim\sqrt{\omega/(\mathbf{kv}_c - \omega)}$, just as in the case of a line of flattening.

A finite l will smear out the singularities but, at high frequencies, this point alone is not sufficient. For example, for a line of flattening, absorption and dispersion (for $\omega \lesssim \nu$) have narrow maxima of angular width $1/kl \ll 1$, where the maximum values $(\gamma/\omega)_{\max}$, $(\Delta s/s)_{\max}$ increase with frequency as $\sqrt{\omega/\nu}$.

However, as $\omega \to \infty$, this singularity is only apparent. It corresponds only to the intermediate asymptotic behavior $\nu s/v \ll \omega \ll \nu_{\text{eff}}$. The dispersion relation (11.1) itself restricts this growth, which is replaced by a fall in absorption and by saturation of dispersion (Fig. 17). Equations (11.2)–(11.4), which correspond to perturbation theory in terms of $|\Delta\omega/\omega| \ll 1$ at high frequencies in the region of the angular resonance, are not valid. In this case, the difference $\mathbf{kv} - \omega$ in the denominator of (11.5) cannot be regarded as given, and we must find it from the dispersion relation which, in this case, has the following form in the resonance approximation($C = \pi G \Delta p \sqrt{2s/|\varepsilon_{xxx}|} \sim s/v$):

$$\omega - \omega_0 = -C\omega_0 \sqrt{\frac{\omega_0}{\mathbf{kv}_c - \omega - i\nu}} \begin{cases} i, & \varepsilon_{xxx} < 0, \\ 1, & \varepsilon_{xxx} > 0, \end{cases} \quad (11.6)$$

or

$$(\omega - \omega_0)^2 (\mathbf{kv}_c \sin\psi - \omega - i\nu) = \mp C^2 \omega_0^3. \quad (11.6')$$

It is clear that this is similar to the equation that describes the interaction between the sound wave ($\omega = \omega_0 \equiv ks$)

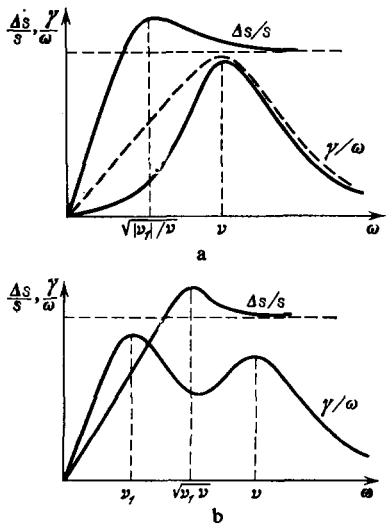

FIG. 15. Departures from the τ-approximation in the case of effective interaction for different ratios of frequencies of arrivals and departures (the region $\omega \to O$ is excluded).

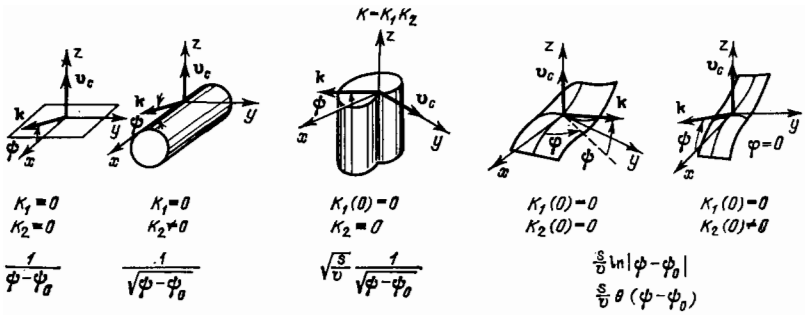

FIG. 16. Form of singularity for different local geometries.

and the "electron" wave ($\omega = \mathbf{kv}_c - i\nu$). The latter is lightly damped for $\omega\tau \gg 1$. It represents (for the $1/x$ singularity) a perturbation of the electron distribution (electron density) transported by an "electron beam" moving with velocity $\mathbf{v}_c$. Such waves are well known in the case of electron beams in vacuum and in plasma. Here, however, the separation of the "beam" from the entire ensemble of conduction electrons is due to the Fermi character of the distribution, and corresponds to flattening on the Fermi surface, which ensures that an appreciable number of conduction electrons ($\sim\sqrt{s/v}$) has velocities approaching v_c. The properties of the electron "wave" depend on the type of singularity.[97] By virtue of central symmetry, there are always two "opposing" beams with velocities $\pm\mathbf{v}_c$ but, for $\omega\tau \gg 1$, the sound wave is in resonance with only one of them.

The coupling between the electron wave and sound is due to either direct deformational interaction [as in (11.6)] or electric fields (see Sec. 13). For large $\omega\tau$, the sound waves are, in turn, coupled to the electron beam waves, which leads to the cessation of the viscous growth of absorption with frequency. We note that electron waves arise in the gas system and do not require any interaction between electrons.

The correction to the root $\omega = \omega_0$ for $C \lesssim 1$ and $\omega_0 \gg \nu_{\text{eff}} = \nu C^{-2/3}$ is shown by (11.6) to be (factors of the order of unity are omitted)

$$\left(\frac{\Delta\omega}{\omega}\right)\psi_{\max} \approx C^{2/3}\left(\pm 1 - i\,\frac{\nu_{\text{eff}}}{\omega}\right), \quad \psi_{\max} \sim \frac{s}{v} \qquad (11.7)$$

It is clear from (11.6) and (11.7) that absorption and the change in the velocity of sound reach the value $\sim(s/v)^{1/3}$ at the frequency $\omega \sim \nu_{\text{eff}} \sim \nu(v/s)^{1/3}$ and then, for $\omega \gg \nu_{\text{eff}}$, absorption decreases, whereas dispersion becomes saturated (see Fig. 17).

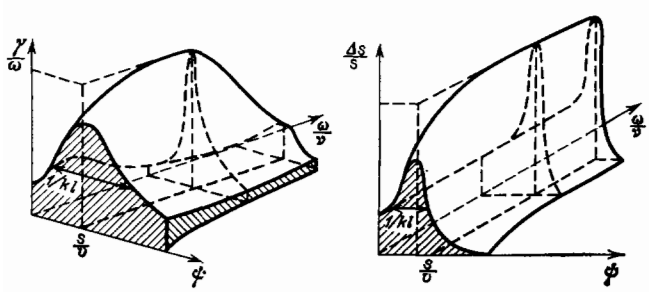

FIG. 17. Frequency and angular dependence of absorption and dispersion of the velocity of sound in the case of effective interaction for $\omega\tau_{\text{eff}} \sim 1$.

12. ABSORPTION AND DISPERSION OF LONGITUDINAL AND TRANSVERSE SOUND WAVES IN METALS IN A STRONG MAGNETIC FIELD

The role of the deformational interaction and of electric fields may be essentially different for purely longitudinal and purely transverse sound waves.

The interaction between transverse sound and electrons differs from longitudinal sound in a number of respects. They include the substantial contribution of transverse electric fields that arise during the propagation of (transverse) sound waves in metals (cf. Refs. 14–20), which is much greater than for longitudinal sound. Another feature is sensitivity to the anisotropy of the electron spectrum. The latter effect is connected with the fact that the deformational interaction between transverse sound and electrons includes a contribution due to the shear moduli of the electron gas (and not the compression moduli, as in the case of longitudinal sound), which vanish in the case of an isotropic spectrum. This effect is particularly well defined in the presence of a strong magnetic field, since the absorption of transverse sound is very dependent on the direction of this field in the crystal.[92,66]

The difference between the deformational interaction of electrons with longitudinal and transverse sound is formally reflected in the fact that, in the former case, it is determined by the diagonal and, in the latter, by the off-diagonal components of the deformation potential tensor $\Lambda_{ik}(\mathbf{p})$ (one of the indices indicates the direction of propagation and the other the polarization of the wave). Their averages over the orbital period[66] (which are responsible for the interaction with transverse sound in a strong field) may be equal to zero for a symmetric field direction (see Sec. 9), and this has an important effect on the frequency and field dependence of absorption (and dispersion) of transverse sound as compared with longitudinal sound.

In a strong magnetic field H and for $kr \ll 1$, the complex change in the frequency of sound for closed, simply connected sections of the Fermi surface can be expressed in terms of the averages over the orbital period in the magnetic field[66,80] as follows:

$$\frac{\Delta\omega}{\omega} = -\frac{\omega}{\rho s^2}\frac{4\pi}{h^3}\int m^*\,\mathrm{d}p_z\,\frac{\overline{\Lambda^2}}{k\bar{v}_z\sin\psi - \omega - i\nu}, \qquad (12.1)$$

where m^* is the cyclotron mass and $(\pi/2) = \psi$ is the angle between $\mathbf{K}$ and $\mathbf{H}$.

When $\mathbf{K} \perp \mathbf{H}$, this yields the estimate $\Delta\omega/\omega \sim i\omega/$

$(\nu - i\omega)$ for any kl and $\omega\tau$ (assuming that $\Lambda \sim \varepsilon \sim mv^2 \sim Ms^2$:

$$\frac{\gamma}{\omega} \sim \frac{\omega\nu}{\nu^2+\omega^2}, \quad \frac{\Delta s}{s} \sim \frac{\omega^2}{\nu^2+\omega^2}, \tag{12.2}$$

from which a possible dimensionless factor has been omitted; its order of magnitude will be indicated by ξ.

It is clear that, for $\omega \gtrsim \nu$, there will be a substantial renormalization of the velocity of the longitudinal sound, which was considered by Kulik[81] and found by Bezuglyĭ and Burma in gallium[82] [(Fig. 18), $\xi \sim 10^{-1}$]. Absorption for $kl \gg 1$, $\omega\tau \ll 1$ is much greater than collisionless absorption[25)] and is of the same order of magnitude as for weak dispersion in the absence of the field $\gamma/\omega \sim \omega/\nu$. When $\omega \sim \nu$, it reaches a maximum (of the order of ξ in very approximate estimates), i.e., we are dealing with the effective interaction between all the Fermi-surface electrons and sound. These results have a simple physical interpretation. The behavior of both absorption and dispersion corresponds to the contribution of second viscosity because of the presence of a slow process (collisions) in the electron subsystem, and is described by the Leontovich-Mandel'shtam-Knezer general theory (see Ref. 94). Indeed, the kinematic viscosity of the electron gas is $\eta(\text{el} \sim nmvl$ and, since the corresponding viscous force appears in the equation of motion of the lattice, the absorption coefficient is $\gamma \sim \eta_{\text{el}} k^2/\rho$. Since $Ms^2 \sim \varepsilon$, we arrive at the estimate given by (12.2).

Although the magnetic field does not appear in (12.2), it does play a very important role. As $\mathbf{H} \to \infty$, we have $r \to 0$ and the electron moves only in the direction of $\mathbf{H} \perp \mathbf{k}$. Collisionless absorption connected with Landau damping is then impossible, however high the frequency.[26)] Absorption is of collisional nature, also for $kl \gg 1$ (in contrast to the situation for $\mathbf{H} = 0$). As it moves along the magnetic field, the electron is also moving relative to the phase wave fronts of the acoustic field because of the propagation of the latter. Work is therefore done in the longitudinal sound wave against forces due to the electron pressure (averaged in the magnetic field), which is described by the component $\bar{\Lambda}_{xx} \neq 0$. Collisions lead to viscous absorption, and the average work done by the sound field on electrons becomes nonzero. For a purely deformational interaction in a strong field, the work done by the sound field on electrons is proportional to $\bar{\Lambda}^2_{ix}$, where i is the direction of displacement and x is the direction of propagation of sound, Thus,

$$d^{\text{I}}_{pq} \approx -\frac{i\omega}{\rho s^2}\left\langle \frac{\bar{\Lambda}_{px}\bar{\Lambda}_{qx}}{\bar{\nu}^*} \right\rangle \quad (n=1). \tag{12.3}$$

However, whereas for longitudinal sound the quantity $\bar{\Lambda}_{xx}$, that plays the role of partial electron pressure is always nonzero as $H \to \infty$, the quantities Λ_{xy} and Λ_{xz}, that describe the momentum flux as $H \to \infty$ in the case of transverse sound, are both zero, provided only that $\mathbf{H}$ is parallel to the symmetry axis of order greater than three for simply-connected sections of the Fermi surface (for further details, see Sec. 8 and Table II). If, on the other hand, the axis is a two-fold axis, then y-polarized transverse sound will behave as if it were longitudinal sound, whereas the absorption and dispersion of z-polarized transverse sound will exhibit spatial dispersion effects, i.e., the finite size of the twist radius over one wavelength will have an effect. When $kl \gg 1$, the asymptotic behavior of absorption and dispersion in such directions will have the form

$$d^{\text{I}}_{\alpha\bar{\alpha}} \sim \frac{i\omega}{\nu - i\omega}(kr)^2 \quad (n=2). \tag{12.4}$$

This effect of appreciable variation in the shear modulus was cofirmed experimentally in gallium by Bezuglyĭ *et al.*[95] (Figs. 20 and 21).

For a symmetric direction of the magnetic field in a crystal (see Table II), the electron shear moduli are connected with the finiteness of H and are due to incomplete averaging, i.e., the finiteness of the orbital period in the magnetic field as compared with the wavelength of the sound wave, the mean free time, and the displacement of the electron in one field period: kr, $|r/l_\bullet| \ll 1$.

A totally different result is obtained for multiply connected sections, when individual singly-connected portions are transformed into one another under symmetry transformations. The averages $\bar{\Lambda}^{\alpha}_{x\alpha}$ over each of the regions will, in general, be nonzero even for a high degree of symmetry of the direction of the magnetic field, and the asymptotic form of $d^{\text{I}}_{\alpha\bar{\alpha}}$ [cf. (9.7)] will be of the same order as d^{I}_{xx}. The result given by (12.4) is also sensitive to magnetic breakdown.

Let us now turn to the deflection effect that follows from (12.1) for $1 \gg |\sin\psi| \neq 0$. As already noted,[86] the angular singularities examined in Sec. 11 are its analogs in many respects. For a Fermi surface of general form, when the neighborhood of a reference point provides a contribution,

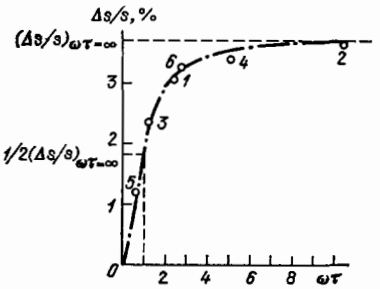

FIG. 18. Dispersion of sound velocity for $\mathbf{k} \perp \mathbf{H}$ in gallium for $kr \ll 1$ and $\mathbf{H} \| [001]$[82].

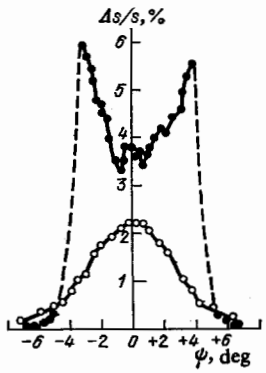

FIG. 19. Deflection effect in gallium for $\omega\tau = 1.2$ and $\omega\tau \approx 10.4$[82].

[25)] This point was apparently first noted by V. L. Gurevich;[14] see also Ref. 93.

[26)] We note, in this connection, that the condition $\mathbf{k} \perp \mathbf{H}$ is very critical because, even for deflections by an angle $\sim s/v$, the electrons are in resonance with the waves for $\omega\tau \gg 1$ (deflection effect[83,84]) (Fig. 19).

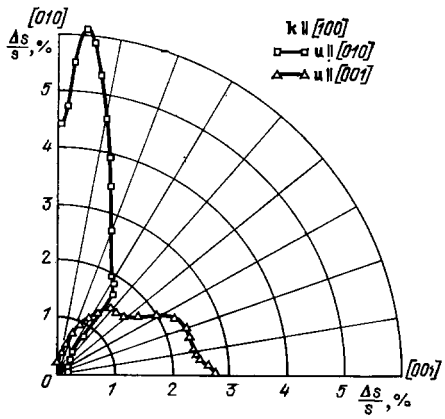

FIG. 20 Variation of the transverse sound velocity $\Delta s/s$ as a function of the direction of H in the (1,0,0) plane in pure gallium for different polarizations. $H = 15$ Oe, $\omega/2\pi = 50$ MHz, $T = 1.7$ K, $\mathbf{k} \perp \mathbf{H}$[95].

the singularity in the deflection effect is logarithmic[84,85] and, if $d\bar{v}_z/dp_z$ vanishes (for $p_z = p_{z0}$),[27] the singularity is of the root type.[86] Perturbation theory is not valid (for $\omega\tau \gg 1$) and, in the resonance approximation, the result for not too low values of $\sin\psi \gg A/kl$ is described (for the root-type singularity) by (11.6) and (11.7), in which the following substitutions must be introduced:

$$\nu \to \bar{\nu}, \quad kv_c \to k\bar{v}_z(p_{z0}) \sin\psi,$$
$$C \to C_{II} \sim 1, \quad \varepsilon_{xxx} \to \frac{\partial^2 \bar{v}_z}{\partial p_z^2}\Big|_{p_{z0}}. \tag{12.5}$$

The qualitative conclusions of Sec. 11 (see Fig. 14) can be correspondingly extended to this case. There are also differences that are reflected in the size of the effect ($C_H \sim 1$) and in the dependence of C_H on the angle of deflection, which leads to asymmetry, and also in the role that can be played by electric fields (see below—Sec. 13).

13. CONTRIBUTION OF ELECTRIC FIELDS TO THE INTERACTION OF ELECTRONS WITH SOUND

Gurevich[14] and Silin[19] drew attention to the fact that electric fields excited by a sound wave may provide a substantial contribution to the deformational electron absorption. It follows from the expression for the electron moduli (7.10) that the deformational interaction (just as the inductive interaction) is renormalized both by longitudinal and by transverse electric fields.[20,66] The contribution of transverse fields depends on the ratio of the wavelength of sound to the length of the electromagnetic wave (skin depth δ in the metal[28]), which is determined by the parameter $(k\delta)^2$.

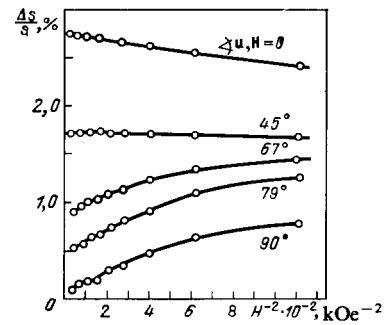

FIG. 21. The variation of $\Delta s/s$ for transverse sound with polarization $\mathbf{u} \| [001]$ as a function of H for different angles between u and $\mathbf{H}(\mathbf{k} \perp \mathbf{H})$.[95] Considerable variation in the contribution of electrons to the elastic shear moduli can be seen.

Self-consistent electric fields may act as the mechanism restricting the growth with frequency of absorption or dispersion at high frequencies in the case of effective interaction. The corresponding compensation of the deformational interaction by longitudinal electric fields was considered in Ref. 91 for a flat part of the Fermi surface with $\omega\tau \gg 1$. The compensating role of induced electric fields in the deflection effect was discussed in Ref. 96 for longitudinal sound. Screening of the resonance interaction by induced fields was found for transverse sound in Ref. 44.

We note that, for a local flattening of the Fermi surface, the contribution of electric fields in the case of longitudinal sound is usually unimportant up to very high and currently unattainable (or totally unrealistic) frequencies (see below) that are much higher (by a factor of v/s or $s/v \exp v/s$) than the collision frequencies.

Let us begin by considering the region of strong spatial dispersion $kl \gg 1$ with $H = 0$. Estimates of the transport coefficients for the usual ineffective interaction depend essentially on the quantity

$$(k\delta)^2 \equiv \frac{k^2c^2}{4\pi\sigma\omega}, \tag{13.1}$$

which appears in the effective resistivity tensor

$$\hat{\rho} = [\hat{\sigma}^{\perp} + i\sigma (k\delta)^2 \hat{I}]^{-1}. \tag{13.2}$$

By definition, $k\delta = \lambda_{em}/\lambda_s$, where $\lambda_{em} = 2\pi c/\sqrt{4\pi\sigma\omega}$ is the wavelength of the electromagnetic wave in the medium (with allowance for spatial dispersion) and $\lambda_s = 2\pi s/\omega$ is the wavelength of the sound wave. When these two lengths are very different, we obtain the well known estimate (11.3):

$$d_{pq} \sim \frac{s}{v}\left(i + \frac{s}{v}\right), \quad kl \gg 1. \tag{13.3}$$

When $k\delta \gg 1$, the contribution of electric fields to absorption can be neglected and, when $k\delta \ll 1$, the fields provide an important contribution[14] to absorption, which is comparable with the deformation contribution. Accordingly, for $k\delta \sim 1$ (i.e., for $\lambda_{em} \sim \lambda_s$, which corresponds to frequencies $\omega \sim 10^9$ c^{-1}), there is a transition from the linear frequency dependence of absorption for $k\delta \ll 1$ to a linear dependence (but with a different factor)[14] for $k\delta \gg 1$. Dispersion of the velocity of sound also should occur in this region, and the change in the velocity of sound should be of the same order as absorption.

Indeed, when $k\delta \sim 1$, we have

$$\rho_{\alpha\beta} \sim \sigma^{-1}(1 + i), \quad G_{il} \sim \frac{m}{e}\omega k v (1 + i). \tag{13.4}$$

This alone shows that, for $K\delta \sim 1$, electric fields provide a

[27] For example, for surfaces of revolution with H parallel to the axis, the orbit $p_z = p_{z_0}$ is a (parabolic) line of zero curvature on which $\partial v_z/\partial p_z = 0$ for any φ. Hence, it follows that $d\bar{v}_z/dp_z = 0$ (the curvature averaged over the orbital period is zero).

[28] Because of the nondissipative nature of the Hall current, and because of the presence of anomalous dispersion, which leads to positive permittivity near resonance transition frequencies, weakly-attenuated electromagnetic[67,68] and quantum-mechanical[35,69] waves can be present in metals in magnetic fields, and may strongly interact with sound.[69,11]

contribution of the same order to both absorption and dispersion.[20]

These estimates can be used to show that d_{pq} includes a contribution of only the deformation force $\mathbf{d}^{\mathrm{d}}$, and the contribution to it of longitudinal electric fields can be neglected, i.e., the important terms are d^{I} and d^{IV}:

$$d_{pq} = -\frac{1}{\rho k^2 s^2}(b_{pq} - c_{p\alpha}\rho_{\alpha\beta}\partial_{\beta q}). \tag{13.5}$$

At these frequencies, the estimate for d_{pq} is different from that given by (13.3):

$$d_{pq} \sim \frac{s}{v}(1+i) \quad (kl \gg 1, \quad k\delta \sim 1). \tag{13.6}$$

Simple formulas can be obtained for the velocity of sound and absorption in a broad range of frequencies, including the transition region corresponding to $k\delta \sim 1$.

$$\frac{\Delta s}{s} + i\frac{\gamma}{\omega} \sim d_{pp}$$

In the transverse resistivity, $\rho_{\alpha\beta}$, it is sufficient to retain only $\mathrm{Re}\,\sigma^{\perp}_{\alpha\beta} = \mathrm{Re}\,\sigma_{\alpha\beta}$. The final answer can be conveniently referred to the principal axes μ,ν of the tensor:

$$\rho_{\mu\nu} = \frac{\delta_{\mu\nu}}{\sigma_\mu + i\sigma(k\delta)^2}, \quad \mathrm{Re}\,\sigma_{\mu\nu} = \sigma_\mu\delta_{\mu\nu},$$
$$2\sigma_{1,2} = \mathrm{Sp}\,\hat{\sigma} \pm \sqrt{\mathrm{Sp}^2\hat{\sigma} - 4\,\mathrm{Det}\,\hat{\sigma}}, \quad \hat{\sigma} = \mathrm{Re}\,(\sigma_{\alpha\beta}). \tag{13.7}$$

According to (13.5), (13.7), and (6.14), we then have

$$d_{pq} = -\frac{i\omega}{\rho s^2}\left[\langle \Lambda_{p\varkappa} R^s \Lambda_{\varkappa q}\rangle - e^2 \sum_{\mu=1}^{2} \frac{\langle \Lambda_{p\varkappa} R^a v_\mu\rangle \langle v_\mu R^a \Lambda_{q\varkappa}\rangle}{\sigma_\mu + i(k^2c^2/4\pi\omega)}\right]. \tag{13.8}$$

Substituting the values of R^s, R^a and $\sigma_\mu = \pi e^2 k^{-1}\langle v^2_{\mu\delta}(v_x)\rangle$ into (13.8), we obtain the frequency dependence of absorption and dispersion[20] (Fig. 22).

According to (13.8), the variation in the velocity of transverse sound passes through a maximum at

$$\omega^2 = \omega_\mu^2 \equiv (2\pi e)^2 s^3 c^{-2}\langle v_\mu^2 \delta(v_\varkappa)\rangle. \tag{13.9}$$

We now turn to the role of fields in the case of effective interaction,[29)] when the fraction of electrons on the Fermi surface that interacts synchronously with the sound wave (Sec. 11) is *not* relatively small (or is generally finite). According to (7.10), the dispersion relation for longitudinal sound with allowance for the renormalization of the deformational contribution by longitudinal fields ($d^{\mathrm{I}} + d^{\mathrm{II}} = d$) is

$$\left(\frac{\omega}{\omega_0}\right)^2 - 1 = -\frac{1}{\rho\omega^2}\left(b_{xx} - \frac{c_{xx}\partial_{xx}}{\sigma_{xx}}\right). \tag{13.10}$$

When $H = 0$, and if we introduce a relaxation time, we obtain[91]

$$\left(\frac{\omega}{\omega_0}\right)^2 - 1 = -\frac{\omega}{\rho s^2}\left[\left\langle \frac{\Lambda^2_{\varkappa\varkappa}}{\mathbf{kv} - \omega - i\nu}\right\rangle - \frac{\left\langle \frac{\Lambda_{\varkappa\varkappa} v_\varkappa}{\mathbf{kv} - \omega - i\nu}\right\rangle^2}{\left\langle \frac{v_\varkappa^2}{\mathbf{kv} - \omega - i\nu}\right\rangle}\right]. \tag{13.11}$$

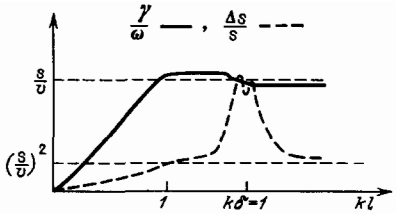

FIG. 22. Absorption[14] and dispersion[20] of transverse sound in the case of $\lambda_s \sim \lambda_{\mathrm{em}}$.

The contribution of fields in the region of the resonance $\mathbf{kv} = \omega$ depends in an essential manner on the local geometry of the resonance region on the Fermi surface.[97,91] At low frequencies $\omega \ll \nu$, this contribution is small, the viscous rise in absorption and dispersion takes place, and, as we have seen above, there is no transition to the collisionless case for $kl \sim 1$ under the conditions of effective interaction: the rise in absorption and dispersion due to the electron viscosity is found to continue.

However, at high frequencies, and provided only that the numerators in the integrals vary sufficiently slowly in the region of resonance and can be taken outside the integral sign, it is clear from (13.11) that the leading field and nonfield terms that increase with frequencies on the right-hand side of (13.11) will mutually cancel out. The rise in absorption with frequency is then replaced by a reduction, and the velocity of sound becomes saturated.

This compensation has a simple physical interpretation: it is connected with the electrical neutrality of the metal as a whole, which imposes the restriction on the current given by (3.10) ($j_x = 0$). Under the conditions we are considering here, when the currents are determined by the contribution due to electrons on a small segment of the Fermi surface with similar velocities, the longitudinal current can only vanish as a result of the mutual compensation of the field and deformational terms.[30)] In essence, a "single-point singularity" will not produce an appreciable deflection of electrons from the state of equilibrium without violating the electrical neutrality of the metal as a whole. However, this can be assured by the above cancellation but, since the singularity in the longitudinal conductivity σ_{xx} is strongly suppressed by the factor v_x^2, the cancellation sets in only for very large $\omega\tau$.

For the longitudinal conductivity, we have (with $\nu = \mathrm{const}$):

$$\frac{\sigma_{\varkappa\varkappa}}{e^2} = -\frac{i\omega^*}{k^2}\langle 1\rangle + \left(\frac{\omega^*}{k}\right)^2\langle R\rangle, \quad \omega^* \equiv \omega + i\nu. \tag{13.12}$$

The predominance of the singular part $\langle R\rangle^c$, which is necessary for the cancelation of the deformational terms by the field terms, occurs when

$$|\mathscr{R}| \gg 1, \quad \mathscr{R} \equiv \frac{\omega^*\langle R\rangle^c}{\langle 1\rangle}. \tag{13.13}$$

We now reproduce estimates of the singular part for different local geometries with $kv_c = \omega$, $\omega\tau \gg 1$ (to within phase factors and the angular dependence):

[29)]This part of our review was written in collaboration with N. S. Stepanova.

[30)]The infinities can cancel out as a result of the Fermi-liquid interaction (see Ref. 120).

$$\mathscr{R} \sim \begin{cases} \omega\tau - \text{plane}, \\ \sqrt{\omega\tau} - \text{cylinder}, \\ \frac{s}{v}\sqrt{kl} - \text{line of flattening}^{31)}, \\ \frac{s}{v}\ln kl - \text{point of flattening, parabolic point}. \end{cases} \tag{13.14}$$

In the case of the plane and the cylinder, $\mathscr{R}$ will also contain the factor ξ that is equal to the ratio of the area of the singular portion to the area of the entire Fermi surface. In principle, cancellation can be important for finite flat (or cylindrical) portions of the Fermi surface[91] for $\omega\tau \gtrsim 1/\xi$ but, even in this case, if $\xi \ll 1$, the resonance with the electron wave that leads to the above sound absorption maximum becomes important at much lower frequecies $\omega\tau \sim 1\sqrt{\xi}$, while the contribution of fields in the neighborhood of this maximum is still unimportant (Refs.. 78, 91, 47). (The contribution of fields that is less important for longitudinal waves becomes the determining factor for weakly-attenuated electron and electromagnetic waves near singular directions.[97])

For an arbitrary critial direction **k**, and if we take out the velocity $\mathbf{v}^c$ and the deformation potential $\hat{c}$ at the singular point, we obtain the following expressions for the transport coefficients (6.14) and (7.11) near the singularity:

$$\frac{\sigma_{\varkappa\beta}}{e^2} \to \frac{\omega^* v_\beta^c}{k}\langle R\rangle^c, \quad \frac{\sigma_{\alpha\beta}^\perp}{e^2} \to v_\alpha^c v_\beta^c \langle R\rangle^c 1^*, \quad \frac{b_{pq}}{i\omega k^2} \to \Lambda_{p\varkappa}^c \Lambda_{q\varkappa}^c \langle R\rangle^c \tag{13.15}$$

$$\frac{\partial_{\beta\eta}^\perp}{e\omega k} \to v_\beta^c \Lambda_{q\varkappa}^c \langle R\rangle^c 1^*, \quad \frac{c_{p\alpha}^\perp}{ie k} \to v_\alpha^c \Lambda_{p\varkappa}^c \langle R\rangle^c 1^*,$$
$$1^* \equiv 1 - \frac{1}{1 - i(\langle 1\rangle/\omega^* \langle R\rangle^c)} = \frac{1}{1 + i\mathscr{V}_t}. \tag{13.16}$$

Relative to axes 1, 2, where 1 lies in the **k**, $\mathbf{v}^c$ plane and 2 is perpendicular to it, we have ($v_2^c = 0$).

$$\hat{\sigma}^\perp \to \begin{pmatrix} \sigma_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{\rho} \to \begin{pmatrix} (\sigma_1 + ik^2c^2/4\pi\omega)^{-1} & 0 \\ 0 & -4\pi i\omega/k^2c^2 \end{pmatrix},$$
$$\sigma_1 \equiv e^2 (v_1^c)^2 \langle R\rangle^c 1^*. \tag{13.17}$$

Hence, for $\hat{d} = \hat{d}^{\mathrm{I}} + \hat{d}^{\mathrm{II}} + \hat{d}^{\mathrm{IV}}$, i.e., with allowance for the renormalization of the deformational interaction by both longitudinal and transverse fields, we have

$$d_{pq} \to -\frac{i\omega}{\rho s^2} \Lambda_{p\varkappa}^c \Lambda_{q\varkappa}^c \langle R\rangle^c 1^* 1_\perp^*, \tag{13.18}$$

where

$$1_\perp^* \equiv 1 - \frac{1}{1 + i(k\delta_c)^2} \equiv \left(1 - \frac{i}{(k\delta_c)^2}\right)^{-1}. \tag{13.19}$$

Here,

$$\delta_c = \frac{c}{\sqrt{4\pi\omega e^2 (v_1^c)^2 \langle R\rangle^c 1^*}} \tag{13.20}$$

lays the role of the characteristic scale of the transverse electromagnetic field near the singularity.

It is clear from (3.18) that $1^* \to 0$ as $\langle R\rangle^c \to \infty$, which corresponds to the above cancellation connected with longitudinal fields. At the same time, $\langle R\rangle^c 1^* \to -i\langle 1\rangle/\omega^*$ and

$$d_{pq} \to -\frac{1}{\rho s^2} \Lambda_{p\varkappa}^c \Lambda_{q\varkappa}^c \frac{\omega}{\omega^*} \frac{1}{1 - \frac{\omega}{\omega^*}\frac{1}{(k\delta_1)^2}}, \tag{13.21}$$

$$\delta_1^2 = \frac{c^2}{4\pi e^2 (v_1^c)^2 \langle 1\rangle}, \tag{13.22}$$

where $\delta_1 \sim 10^{-5}$ cm is the limiting value of δ_c.

In general, as $\langle R\rangle^c \to \infty$, the cancellation is assured by the longitudinal fields. However, cancellation due to transverse fields is possible and corresponds to the condition

$$k\delta_c \to 0, \quad 1_\perp^* \to i(k\delta_c)^2 \to 0, \tag{13.23}$$

which may correspond to the intermediate asymptotic behavior

$$(k\delta_1)^2 \ll |\mathscr{R}| \ll 1 \ll \omega\tau. \tag{13.24}$$

(The condition $\omega\tau \gg 1$ is connected with the single-point nature of the singularity.) In this case,

$$d_{pq} \to \frac{\Lambda_{p\varkappa}^c \Lambda_{q\varkappa}^c}{\rho s^2} \langle 1\rangle (k\delta_1)^2. \tag{13.25}$$

The conditions given by (13.24) require sufficiently high quality specimens with a mean free path of $l \gg \delta_0 v/s \sim 10^{-2}$ *cm*.

We have assumed so far that the antipodal point ($-\mathbf{p}_c$) lies outside resonance, and that there are no other resonance points for given **k** ("arbitrary" critical direction).

We shall now consider symmetric critical directions. The presence of point-symmetry elements on the Fermi surface ensures that points of zero curvature on the Fermi surface form a "star," i.e., a discrete or continuous (the latter for figures of revolution) set of points related by transformations $\hat{g}$ of the point group G of the Fermi surface (crystal):

$$\mathscr{K}(\mathbf{p}_c) = 0, \quad \mathscr{K}(\hat{g}\mathbf{p}_c) = 0 \quad (g\mathbf{p}_c \equiv \mathbf{p}_{qc}), \tag{13.26}$$

so that, for symmetric or nearly symmetric crystal directions, we must simultaneously take into account several points of zero curvature. It is shown in Ref. 90 that such situations are admissible; at any rate, the conditions $\mathbf{k}\cdot\mathbf{v}_c = 0$ can correspond to symmetric critical directions **k**. From the experimental point of view, symmetric directions are definitely preferable because, among other things, they enable us to operate with purely longitudinal or purely transvere sound.

The possible groups $G_{\mathbf{k}}$ (that leave the vector **k** invariant) are listed in Table I, in which **p** must be replaced with **k** and x, y with 1, 2 [see (13.17)].

The singular parts of the transport coefficients now contain sums over the equivalent points c' in (13.12) and (13.15). Taking $\langle R\rangle^c$ outside the summation sign, we see that $\sigma_{\varkappa\alpha} \sim \Sigma_{c'} v_\alpha^{c'}$ will vanish for an axially symmetric direction **k** and, hence, the tensors $\sigma_{\alpha\beta}, \partial_{\beta l}, c_{l\alpha}$ will not be renormalized in such cases (this is also clear from independent considerations).

For longitudinal sound, we obtain the same result as before, i.e., (13.18) but, instead of $\langle R\rangle^c$, we now have $\Sigma_{c'}\langle R\rangle^{c'} = N\langle R\rangle^c$, where N is the number of equivalent (for a given **k**) zero-curvature points:

[31)] For a transport line, when the expansion of $\mathbf{k}\cdot\mathbf{v}$ in the neighborhood of a zero-curvature point is one dimensional if the dependence on the secondcoordinate appears in the order n, then $\mathscr{R} \sim (s/v)(kl)^{(1/2)-(1/n)}$. In Ref. 97, $n = 3$ *and* $\mathscr{R} \sim (s/v)(kl)^{1/6}$.

$$d_{\varkappa\varkappa} = -\frac{i\omega}{\rho s^2} N \langle R\rangle^c (\Lambda^c_{\varkappa\varkappa})^2 1^*. \tag{13.27}$$

Relative to the axes μ, ν that diagonalize $d_{\alpha\beta}$, we have the following expression for transverse sound:

$$d_{\mu\bar{\mu}} = -\frac{i\omega}{\rho s^2}\langle R\rangle^c \left[\sum_{c'}(\Lambda^{c'}_{\mu\varkappa})^2 - \frac{\left(\sum_{c'} v^{c'}_{\mu}\Lambda^{c'}_{\mu\varkappa}\right)^2}{\sum_{c'}(v^{c'}_{\mu})^2 + i\frac{k^2c^2}{4\pi\omega e^2\langle R\rangle^c}}\right]. \tag{13.28}$$

The quantities $(\Lambda^{c'}_{\mu\varkappa})^2$, $v^{c'}_{\mu}\Lambda^{c'}_{\mu\varkappa}$ and $(v^{c'}_{\mu})^2$ can be transferred with the aid of (9.1) to one singular point. They then transform in the same way (as the square of the μ-component of a vector), so that

$$\sum_{c'}(\Lambda^{c'}_{\mu\varkappa})^2 = (\Lambda^c_{\mu\varkappa})^2 N', \quad \sum_{c'} v^{c'}_{\mu}\Lambda^{c'}_{\mu\varkappa} = v^c_{\mu}\Lambda^c_{\mu\varkappa}N',$$
$$\sum_{c'}(v^{c'}_{\mu})^2 = (v^c_{\mu})^2 N'. \tag{13.29}$$

Hence,

$$d_{\mu\bar{\mu}} = -\frac{i\omega}{\rho s^2} N' \langle R\rangle^c (\Lambda^c_{\mu\varkappa})^2 1^*_{\perp}, \tag{13.30}$$

where, in $1^*_{\perp}$ [see (13.19)] and in δ^2_c (13.20), we must replace $\langle R\rangle^c 1^*$ with $\langle R\rangle^c N'$. It is clear that, in longitudinal sound, renormalization is possible only by longitudinal fields, and, in transverse sound, it is only possible by transverse fields.

We note that, for the deflection effect in a strong magnetic field ($kr \ll 1$), the contribution of electric fields that corresponds to (13.15)–(13.25) is obtained by replacing the various quantities in these expressions with averages over the orbital period [in accordance with (10.5)]: $\Lambda^c \to \bar{\Lambda}^c$, $\mathbf{v}^c \to \bar{\mathbf{v}}^c$, $\langle R\rangle^c \to \langle 1/\bar{\alpha}\rangle^c$. Hence, relative to the axes 1, $2\perp\mathbf{k}$ such that $2\perp\mathbf{k}$, $\mathbf{H}(\bar{v}_2 = 0$, $\bar{v}_1 = v_z\cos\bar{\psi}$, $\bar{v}_{\varkappa} = \bar{v}_z\sin\psi$ in the case of closed sections), we obtain the following expression for the "single-point" singularity ($\omega\tau \gg 1$):

$$d_{pq} \to -\frac{i\omega}{\rho s^2}\bar{\Lambda}^c_{p\varkappa}\bar{\Lambda}^c_{q\varkappa}\left\langle\frac{1}{\bar{\alpha}}\right\rangle^c \bar{1}\cdot\bar{1}_{\perp}, \tag{13.31}$$

where

$$\bar{1} \equiv \frac{1}{1+i\mathscr{R}_H}, \quad \bar{1}_{\perp} \equiv 1 - \frac{1}{1+i(k\bar{\delta}_c)^2}, \quad \bar{\alpha} \equiv i(\mathbf{k}\bar{\mathbf{v}} - \bar{\omega}^*),$$
$$i\mathscr{R}_H \equiv \frac{1}{\langle 1\rangle}\left\langle\frac{i\omega^*}{\bar{\alpha}}\right\rangle \sim \begin{cases}\ln\omega\tau - \text{reference point}^{84,85},\\ \sqrt{\omega\tau} - \text{extremum point} \quad \bar{v}'_z = 0^{86}.\end{cases} \tag{13.32}$$

Longitudinal fields can lead to the cancellation of the singularity (cf. Ref. 71). We note, by the way, that when $\mathscr{R}_H$ contains the small factor $\xi \ll 1$, the resonance between longitudinal sound and the electron wave [see (11.7), (12.5)] sets in at such $\omega\tau (\sim \xi^{-3/2} \gg 1$ for $\bar{v}'_z = 0)$, for which longitudinal fields are still unimportant. The role of transverse fields for $\omega\tau \to \infty$ that lead to cancellation for $(k\delta)^2 \ll 1$ is discussed in Ref. 96 for longitudinal sound in an isotropic metal. Another consequence of (13.31) is that the deflection effect for transverse sound is very dependent on the orientation of the magnetic field relative to the symmetry elements of the crystal (see Table II).

CONCLUSIONS

The examples considered above, which are largely concerned with the dispersion of the velocity of sound[32] under the conditions of effective interaction with electrons, illustrate the convenience and efficacy of using the equations of elasticity for metals. We have not touched upon other natural applications of the equations in relation to the electromagnetic excitation of sound (the first papers in this field are those of Refs. 98 and 99 and a review is given in Ref. 100), the propagation of electron precursors of sound waves,[101–103] the analysis of the possibilities of electron-inertia experiments and measurements of electrical fields excited by sound in metals,[104,105] the computation of fields that appear near the boundaries of a deformed (for example, by gravitation) metal,[106] the equations of elasticity for superconductors,[107–109] and so on.

Our approach can be extended to ferromagnetic metals by taking into account the magnetization of conduction electrons by d-electrons (for the present state of this problem, and for references to it, see Ref. 119).

Equations (1.8) are also valid in the case of magnetic breakdown. However, the transport coefficients then contain weighted averages over segments of classical orbits with weights that are functions of the breakdown probability.[116] The form of the probability depends on the type of breakdown, i.e., whether it is stochastic[116,117] or coherent.[118] In the latter case, quantum interference between states on these segments becomes important and requires the density matrix for its description. Breakdown should influence the magnetoacoustic effects in a strong field discussed above, including the transverse propagation of sound, the deflection effect,[119] the behavior of the electron shear moduli, and so on. We particularly emphasize that the effects of the deformational interaction (discussed above and in Refs. 96 and 111) are fully within the range of applicability of the Born-Oppenheimer adiabatic approximation, which is not at all in conflict with the major electron renormalizations of the velocity of sound in metals that were obtained in individual cases.[33]

Actually, the energy of $\varepsilon_0 + \delta\varepsilon$ of an electron in a deformed metal, given by (1.1), is determined for fixed positions of the ions (8.5): $\varepsilon = \varepsilon(\mathbf{p}, u_{ik}(\mathbf{r}, t))$. The average electron energy, calculated with the nonequilibrium distribution function for the electron gas, appears in the equation of motion of the ions (equation of elasticity). Because the electron subsystem is not in equlibrium, this energy depends on the "slow" time (the frequency of elastic fields in the Fourier representation), on the magnetic field, and so on. It is not at

[32] These examples do not, of course, amount to a review of the dispersion and absorption of light in metals (see Refs. 6–10). In particular, references to experimental work are largely illustrative in character.

[33] We also note that the attempt made in Ref. 25 to simplify the derivation of the equations of the theory of elasticity in metals by varying the energy in the c-system cannot be regarded as successful because, actually, the authors of Ref. 25 varied the energy in the c-system with a fixed distribution function in the c-system and a fixed vector potential in the l-system, for which there is, of course, no *a priori* justification. The reason why their result was the same as the generally accepted result can be found in Ref. 20 (second paper).

all small in comparison with the pure lattice contribution, as is usually the case, and appears in the leading order of the adiabatic approximation. (For example, the "adiabatic" electron energy compensates the Coulomb repulsion between ions in a molecule.) It follows that the adiabatic contribution of conduction electrons to the velocity of sound in a metal also does not have to be small. It depends on the wave vector and frequency (dispersion) because the electron gas is not in a state of equilibrium. This leads to the necessity for a self-consistent description of elastic deformations with allowance for changes that occur in the electron gas in the metal. On the other hand, departures from the adiabatic approximation in the deformational interaction constitute corrections of the order of $\sqrt[4]{m/M}$, and so on, and their role in the phenomena that we are investigating is quite minor. Thus, cases of the effective interaction of electrons with sound fit completely into the framework of the adiabatic approximation, which corresponds to the introduction of the deformation potential and the validity of the equations of elasticity in metals [(1.8) and (2.2)].

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