# Magnetic superconductors

A. I. Buzdin, L. N. Bulaevskiĭ, M. L. Kulich, and S. V. Panyukov

*M. V. Lomonosov Moscow State University; P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow; Institute of Physics, Serbian Academy of Sciences, Belgrade, Socialist Federal Republic of Yugoslavia*

Usp. Fiz. Nauk **144**, 597–641 (December 1984)

Superconductivity and ferromagnetism are antagonistic types of ordering, and their mutual effects give rise to several interesting phenomena which have recently been studied in rare earth compounds. A theoretical analysis shows that while a ferromagnetic superconductor is a type II superconductor near the superconducting transition point $T_{c1}$, it becomes a type I superconductor near the ferromagnetic transition point $T_M$. A new theory derived for the case $T_M \ll T_{c1}$ predicts the formation of a transverse domain-like magnetic structure near $T_M$. In clean superconductors the electron spectrum is gapless. A change in the behavior from type II to type I upon cooling to $T_M$ has been observed experimentally in $ErRh_4B_4$. Experimental data on $ErRh_4B_4$, $HoMo_6S_8$, and $HoMo_6Se_8$ prove the existence of superconductivity and a magnetic ordering below $T_M$.

**TABLE OF CONTENTS**

1. Introduction ..... 927
2. Interaction of conduction electrons and magnetic moments ..... 932
   a) Structural features of magnetic superconductors and Hamiltonian of the magnetic and electron systems. b) Effects of the crystal field. c) Ruderman-Kittel-Kasuya-Yosida indirect exchange interaction of localized moments in the normal state. d) Electromagnetic (dipole-dipole) interaction in the normal state. e) Magnetic functional of a ferromagnet in the normal state.
3. Properties of ferromagnetic superconductors in the nonmagnetic superconducting phase (above $T_M$) ..... 935
   a) Role of magnetic scattering. b) The upper critical magnetic field. c) The lower critical magnetic field. d) Experimental data on the critical fields in $ErRh_4B_4$.
4. Structure of the coexistence phase in ferromagnetic superconductors ..... 939
a) General principles for calculating a functional for the coexistence phase. b) Coexistence phase near the point $T_M$ and critical fluctuations. c) Coexistence phase with a strong exchange field. d) Transition from the coexistence phase to the normal ferromagnetic phase. e) Superconducting properties of the coexistence phase. f) Role of spin-orbit scattering and $T$, $\theta_{ex}$ phase diagram.
5. Theoretical predictions regarding the coexistence phase of ferromagnetic superconductors and comparison with experimental results ..... 945
   a) Basic theoretical conclusions. b) Properties of $HoMo_6S_8$ and $HoMo_6Se_8$. c) Experimental data on $ErRh_4B_4$. d) Re-entrant superconductivity in ternary silicates, stannides, and pseudoternary compounds.
6. Conclusion ..... 951
References ..... 951

## 1. INTRODUCTION

Superconductivity and magnetism represent types of ordering which compete with each other, so that whether the two types of ordering can coexist in the same crystal is by no means a trivial question. There are two mechanisms for the mutual effects of the magnetic moments and the superconducting electrons: the electromagnetic and exchange mechanisms.

The electromagnetic mechanism for the suppression of superconducting pairing by the magnetic induction in a ferromagnet was studied by Ginzburg back in 1956, before the Bardeen-Cooper-Schrieffer (BCS) theory was worked out.[1] As the coexistence phase in a type I superconductor, Ginzburg studied the Meissner ferromagnetic superconducting state, in which the magnetic moment and the superconductivity order parameter are homogeneous throughout the sample (Fig. 1). In this state the magnetic induction in the sample is zero, since the magnetic field induced by the moments is cancelled by the superconducting currents which flow along the surface of the sample, in a layer with a thickness of the order of the London penetration depth $\lambda_L$. Noting that the induction of a ferromagnet is large in comparison with the critical field of a superconductor, Ginzburg[1]

 0038-5670/84/120927-27$01.80  

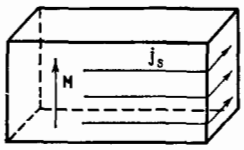


FIG. 1. Meissner superconducting ferromagnetic phase. The arrows show the directions of the screening currents $\mathbf{j}_s$ flowing along the surface of the sample and of the magnetization $\mathbf{M}$ in the sample.

and Zharkov[70] concluded that a coexistence phase was possible only in exceptional cases, when the effect of the magnetic induction was suppressed for some reason (e.g., in a thin film or in a state with a moment directed opposite to the external field). In ordinary bulk samples, in contrast, the onset of ferromagnetic order would essentially rule out any subsequent development of superconducting pairing.

The reciprocity of this antagonism between superconductivity and ferromagnetism in terms of the electromagnetic mechanism was established in some later studies by Blount and Varma,[25] Ferrel *et al.*,[26] and Matsumoto *et al.*[27] If, during cooling, the superconducting transition occurs at a temperature above that corresponding to the magnetic transition, the long-range part of the magnetic field, i.e., the long-range part of the magnetic dipole interaction of the moments, is screened by Meissner currents. The long-range part of the interaction of the moments always promotes the establishment of a ferromagnetic state, so that superconductivity prevents the formation of a ferromagnetic phase by lowering the temperature at which it would appear.

The second mechanism for the interaction between superconductivity and magnetism—the exchange (EX) mechanism—was proposed by Matthias, Suhl, and Corenzwit.[2] In a magnetically ordered state, the conduction electrons are acted upon by the exchange field of the magnetic moments. The spins of the electrons in a Cooper pair are in opposite directions; the exchange field tends to put them in the same direction, thereby preventing singlet superconducting pairing. This effect has been labeled the "paramagnetic effect" (see Ref. 4). Superconducting pairing is also suppressed by the exchange scattering of electrons by the moments, since this scattering always disrupts the singlet state of the Cooper pair. This scattering has been studied in detail by Abrikosov and Gor'kov[3] and has been termed "magnetic scattering"; it occurs in both a paramagnetic phase and a magnetically ordered phase.

Anderson and Suhl[7] showed that according to the exchange mechanism for the interaction between ferromagnetism and superconductivity the effects of the two would again be mutual, since the onset of magnetism is less likely from the energy standpoint in a superconducting state than in a normal state. The exchange interaction of localized moments and conduction electrons leads to an indirect interaction of the localized moments through conduction electrons; this is the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction. According to the RKKY interaction, the ferromagnetic-ordering temperature is determined by the paramagnetic susceptibility of the electron system. The contribution of the long-range part of the RKKY interaction, which always promotes a ferromagnetic ordering, is proportional to the electron paramagnetic susceptibility $\chi_e(\mathbf{q})$ at the wave vector $\mathbf{q}=0$. In a superconducting state the susceptibility $\chi_s(0)$ is lower because of the gap at the Fermi surface, and the transition to a ferromagnetic ordered phase will be suppressed.

It is clear that the mutual effects could be strong only in the case of ferromagnetism and superconductivity. In an antiferromagnet the average magnetic induction and the average exchange field are very small in a region with a size of the order of the superconducting correlation length, so that the mutual effects of superconductivity and antiferromagnetic ordering will be weak if the exchange scattering is weak. Working from arguments of this type, Baltensperger and Strassler[39] concluded that the coexistence of antiferromagnetism and superconductivity was possible, and later calculations[118,123–127] and experiments have confirmed this conclusion. In the case of ferromagnetism, in contrast, the resultant effects of the electro-magnetic and exchange mechanisms essentially rule out a simple coexistence; all that occurs is a first-order S-FN transition between a nonmagnetic superconducting (S) phase and a ferromagnetic normal (FN) phase (according to Gor'kov and Rusinov[5]), or there is a coexistence of superconductivity and a modified magnetic ordering. The possibility of this second situation was first predicted by Anderson and Suhl[7] in 1959 on the basis of the exchange mechanism. These arguments were subsequently extended in Refs. 25–27 to systems in which only the electromagnetic mechanism is operating.

We mentioned earlier that according to the exchange mechanism the paramagnetic susceptibility of electrons with a zero wave vector, $\chi_s(0)$, decreases in a superconductor, resulting in a suppression of the temperature of ferromagnetic ordering (Fig. 2). Anderson and Suhl noted, however, that the effect of Cooper pairing on the electron susceptibility $\chi_s(\mathbf{q})$ decreases with increasing wave vector $\mathbf{q}$, so they concluded that in the presence of superconductivity magnetism should arise not as a ferromagnetic order but as an inhomogeneous magnetic structure. Assuming that the magnetic transition temperature $T_M$ lies well below the superconducting critical temperature $T_{c1}$, Anderson and Suhl described $\chi_s(\mathbf{q})$ by the expression

$$\chi_s(\mathbf{q}) \sim 1 - \frac{\pi}{2\xi_0 q} - a^2 q^2, \tag{1.1}$$

which holds at $T=0$ and if $q\xi_0 \gg 1$. The quantity $a \sim k_F^{-1}$ in

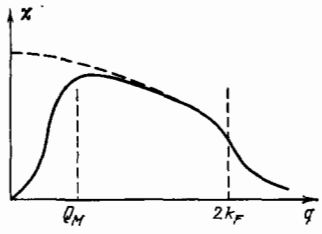


FIG. 2. Electron paramagnetic susceptibility $\chi_s$ as a function of the wave vector $q$. Solid line—in the superconducting state at $T \ll T_c$; $\chi_s$ reaches a maximum at $Q_M \approx (a^2\xi_0)^{-1/3}$. Dashed line—in the absence of superconductivity; the susceptibility reaches its maximum at $q=0$.

(1.1) is the magnetic rigidity (the magnetic correlation length), of the order of an atomic length, and $\xi_0$ is the superconducting correlation length. The temperature at which magnetic ordering with a wave vector $\mathbf{q}$ arises is proportional to $\chi_s(\mathbf{q})$. After determining the value of $\mathbf{q}$ corresponding to the maximum of $\chi_s(\mathbf{q})$, Anderson and Suhl found the wave vector of the inhomogeneous magnetic structure at the point of its appearance, $Q_M \approx (a^2\xi_0)^{-1/3}$; here $\xi_0^{-1} \ll Q_M \ll a^{-1}$, since $\xi_0 \gg a$. An inhomogeneous magnetic order with a wave vector $\mathbf{Q}$ in the interval $a^{-1} \gg Q \gg \xi_0^{-1}$ can coexist with superconductivity since it has only a slight effect on the superconductivity by virtue of the second of these inequalities, and at the same time it differs only slightly from a ferromagnetic state in terms of energy by virtue of the first inequality. Clearly, a situation of this sort is possible because the magnetic correlation length $a$ is small in comparison with the superconducting length $\xi_0$; of the two antagonistic types of order, that which is modified is that which has the smaller rigidity (or correlation length). In this case it is the magnetic order which changes, and Anderson and Suhl labeled the magnetic structure in the coexistence phase a "cryptoferromagnetic" structure.

According to the electromagnetic mechanism, the behavior of the magnetic order in the coexistence phase is analogous. Since a magnetic field which varies over space is screened less than a uniform field is, a nonuniform magnetic structure arises in the superconducting phase below $T_M$. Its wave vector $\mathbf{Q}$ corresponds to a minimum of the magnetic-nonuniformity energy and of the interaction of the superconductivity with the magnetic field induced by the localized moments:

$$|\mathbf{M}_q|^2 q^2 a^2 + \frac{|\mathbf{A}_q|^2}{4\pi\lambda_L^2}, \qquad (1.2)$$

where $\mathbf{M}_q$ and $\mathbf{A}_q$ are the Fourier transforms of the magnetization and of the vector potential of the magnetic induction. Assuming $q\lambda_L \gg 1$, we can write $i[\mathbf{q}\mathbf{A}_q] = 4\pi\mathbf{M}_q$, and after a minimization we find[25-27] $Q \approx (a\lambda_L)^{-1/2}$.

Anderson and Suhl[7] and some subsequent investigators[25-27,35,36] found the wave vector of a nonuniform magnetic structure at the point of its appearance, $T_M$. The subsequent theoretical effort was aimed at identifying the type of magnetic structure and the features of the superconducting characteristics of the coexistence phase in the temperature range in which the magnetization is not yet small. It turned out that the answers to the questions posed depend strongly on which type of interaction (exchange or electromagnetic) is considered. Working from the exchange mechanism for an

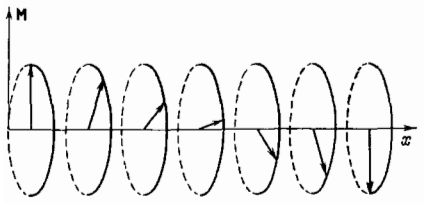


FIG. 3. Helicoidal magnetic order of the moments in the coexistence phase of isotropic ferromagnets. The arrows show the directions of the moments in the sample.

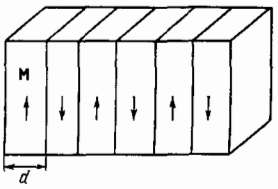


FIG. 4. Domain magnetic structure in the coexistence phase of uniaxial ferromagnets. The arrows show the directions of the magnetic moments in the domains.

isotropic system, Bulaevskii *et al.*[24] found that a helicoidal magnetic structure occurs in the coexistence phase (Fig. 3), while far from the point $T_M$ in a pure superconductor the superconductivity is gapless in nature. Upon cooling, a coexistence phase of this type survives even in strong exchange fields. It was clear, however, that the conclusion that the magnetic structure has a spiral nature applies only to the model isotropic system. A magnetic anisotropy localizes the change in the magnetization direction within domain walls and transforms the spiral structure into a domain structure (Fig. 4). The possibility that a domain structure could occur in a coexistence phase was first pointed out by Fulde and Ferrell.[77] They also found the period of this domain structure in weak exchange fields, i.e., near $T_M$.

According to the electromagnetic mechanism, a phase with a spiral structure is also established[26,27] below $T_M$, but, when the magnetic anisotropy is taken into account, a structure of a domain type has a lower energy.[9,28] Upon cooling there can be a transition from this phase into another coexistence phase—one with a lattice of spontaneous vortices[28-31,122] (Fig. 5)—or directly into a ferromagnetic normal phase, depending on the value of the Ginzburg-Landau parameter $\varkappa$. This research direction is reviewed by Tachiki[31] and Ishikawa[92]; see also the reviews by Fulde and Keller[111] and Izyumov and Skryabin.[134]

The coexistence phase was studied in Refs. 32–34, where the exchange and electromagnetic mechanisms were considered, as was a magnetic anisotropy in the region of strong depairing fields. This study was carried out on the basis of the microscopic BCS theory and Gor'kov's equations or Eilenberger's corresponding semiclassical equations. The results derived in those studies therefore give a complete quantitative description of real systems, including

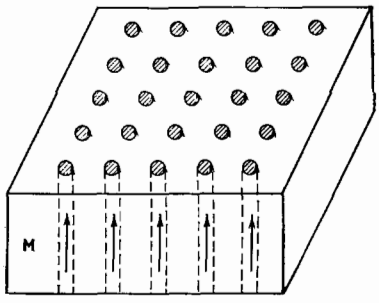


FIG. 5. Coexistence phase with spontaneous vortices. Arrows—directions of the moments in the sample; circles—superconducting vortex currents; hatched regions—normal bases of the vortices.

the most interesting systems, with a strong superconducting pairing near the magnetic transition, i.e., systems with $T_M \ll T_{c1}$.

It was found in Refs. 32–34 and also in some earlier studies[35–38] that the exchange mechanism dominates the formation of the coexistence phase even if its contribution ($\theta_{ex}$) to the ferromagnetic-ordering energy is small in comparison with the corresponding electromagnetic contribution, $\theta_{em}$ (per localized moment at $T = 0$). The reason is that the effect of the nonuniform magnetic field on the superconductivity weakens with increasing wave vector of the magnetic structure, **Q**, far more rapidly than does the effect of the nonuniform exchange field, and the difference between their effects on the superconductivity is characterized by the small parameter $\theta_{em}/\theta_{ex}(\lambda_L Q)^2$. We will see below that in a real compound the parameter of the electromagnetic mechanism, $\theta_{em}$, is comparable in magnitude to the exchange parameter $\theta_{ex}$, so that the large factor $(Q\lambda_L)^2 \approx \lambda_L^2/a\xi_0$ makes the structure of the coexistence phase depend primarily on the exchange mechanism; the role played by the electromagnetic mechanism becomes one of simply making the nonuniform magnetic structure a transverse structure and substantially reducing the magnetic fluctuations and their effect on the superconductivity above $T_M$.

Spin-orbit scattering suppresses the effect of the exchange field on superconductivity, increasing the electron paramagnetic susceptibility[119] $\chi_s^{(0)}$. Estimates show, however, that the complete suppression of the exchange mechanism and the dominance of the electromagnetic interaction become possible only in very dirty samples, in which the electron mean free path approaches the interatomic distance. Consequently, the theory of Refs. 32–34 unambiguously predicts, for ferromagnetic superconductors which are not very dirty, a one-dimensional transverse domain magnetic structure with a wave vector $Q \approx (a\xi_0)^{-1/2}$ in the coexistence phase, and it predicts that the superconductivity of this phase in clean crystals will be of a gapless nature in the range of strong exchange fields.

Less-definite conclusions about the properties of the coexistence phase and the conditions for its appearance can be drawn from experiments on magnetic superconductors. The first studies carried out by Matthais, Suhl, and Corenzwit[2] were devoted to superconductors with magnetic impurities—rare earth (RE) ions. These experiments yielded no definite information about the possible coexistence of magnetic long-range order and superconductivity, since the magnetic impurities in the samples in those expeirments could form clusters which would make the system magnetically inhomogeneous. It was also learned that the order of the localized moments in systems of this sort is a spin-glass order without any genuine long-range order (see the reviews by Maple[6] and Ishikawa[92]). The current stage of experimental study of magnetic superconductors began in roughly 1976 after the synthesis of ternary compounds with a regular lattice of rare earth elements, e.g., (RE)$Rh_4B_4$ and (RE)$Mo_6S_8$. Some of these compounds turned out to be superconducting, and neutron-scattering measurements revealed that the crystals go into a magnetically ordered state at low temperatures, $T_M < T_{c1}$ (Tables I and II). It was found that the appearance of antiferromagnetic order does not destroy superconductivity,[10–12] but the formation of ferromagnetic long-range order in $ErRh_4B_4$ and $HoMo_6S_8$ is accompanied by a transition from a superconducting state to a ferromagnetic normal state at the point[13–15] $T_{c2}$. Neutron-scattering measurements carried out by Lynn *et al.*[16–18] for polycrystalline $HoMo_6S_8$ samples, by Moncton *et al.*[19] for polycrystalline $ErRh_4B_4$ samples, and by Sinha *et al.*[20] for a single crystal of this compound showed that the magnetic ordering occurs at a temperature $T_M > T_{c2}$, and in the interval between $T_M$ and $T_{c2}$ there is a superconducting phase with a nonuniform magnetic structure. These experimental facts confirm the predictions of Anderson and Suhl. Table II lists

TABLE I. Basic properties of the ternary borides and chalcogenides.

| Compound | $T_{c1}$, K | $J_{eff}$ | $T_M$, K | $\bar{\theta}_{ex}$, K | $\theta_{em}$, K | Magnetic structure | $N(0)$ states / eV·spin·RE | Reference |
|---|---|---|---|---|---|---|---|---|
| $YRh_4B_4$ | 10.8 | — | — | — | — | — | 5.5 | 106, 112 |
| $LuRh_4B_4$ | 11.5 | — | — | — | — | — | 5.5 | 106, 112 |
| $SmRh_4B_4$ | 2.7 | 1/2 | 0.87 | 2.4 | | AF | | 57, 106, 107 |
| $HoRh_4B_4$ | — | 1/2 | 6.7 | 1.1 | 4.3 | F | | 106, 107, 112 |
| $ErRh_4B_4$ *) | 7.2 | | 0.2 | | 1.8 | AF | | 135, 136 |
| $TmRh_4B_4$ | 9.8 | | 0.4 | 0.3 | | AF, complicated | 5.13 | 57, 60, 106, 107, 112, 139 |
| $LuMo_6S_8$ | 2.2 | — | — | — | — | — | $\approx 3$ | 107 |
| $GdMo_6S_8$ | 1.4 | | 0.84 | 0.5 | 0.6 | AF | | 107, 108 |
| $TbMo_6S_8$ | 2.04 | | 1.05 | 0.32 | 0.9 | AF | | 107, 108 |
| $DyMo_6S_8$ | 2.05 | | 0.4 | 0.17 | 1.2 | AF | | 107, 108 |
| $ErMo_6S_8$ | 2.2 | | 0.2 | 0.06 | 0.2 | AF | | 107, 108 |
| $ErMo_6Se_8$ | 6.0 | | 1.1 | | | AF, complicated | | 107, 108 |
| $LuMo_6Se_8$ | 6.2 | — | — | — | — | — | $\approx 6.5$ | 107, 108 |
| $HoMo_6Se_8$ | 5.5 | 1/2 | 0.53 | 0.14**) | | F | $\approx 6.5$ | 151 |

*Modification with a body-centered structure.
**Estimate based on the difference betweeen the values of $T_{c1}$ for $LuMo_6Se_8$ and $HoMo_6Se_8$.

TABLE II. Basic properties of the returning superconductors $ErRh_4B_4$ and $HoMo_6S_8$.

| Compound | $N^{-1}(0)$, K·spin/RE | $v_F$, $10^7$ cm/s | $T_{c1}$, K | $\tilde{\Delta}_0$, K | $\lambda_L(0)$, Å | $\xi_0$, Å | $l$, Å | $\Delta_0$, K |
|---|---|---|---|---|---|---|---|---|
| $ErRh_4B_4$ | 1850 | 1.3 [a] | 8.7 | 10 [c] | 900 [a] | 210 [a] (‖a)<br>160° (‖c) | $\gg \xi_0$ | 15.5 |
| $HoMo_6S_8$ | 3600 | 1.8 [b] | 1.8 | 4 [c] | 1200 [d] | 1500 | 60 [e] | 3.2 |

| Compound | $H^*_{c2}(0)$, kOe | $n$, cm$^{-3}$ | $\mu(T=0)$, $[\mu_B]$ | $B(0)$, kOe | $h_0$, K | $\theta_{ex}$, K | $\tau_s^{-1}$, K | $\theta_{em}$, K |
|---|---|---|---|---|---|---|---|---|
| $ErRh_4B_4$ | 10 [a] | $10^{22}$ | 5.6 | 6.5 | 40 | 0.87 [h] | 3 [j] | 1.8 |
| $HoMo_6S_8$ | 4.8 [f] | $4\cdot10^{21}$ | 9.1 | 4.8 | 24 [g] | 0.15 [i] | 0.9 [j] | 1.3 |

| Compound | $T_M$, K | $d=\pi/Q$, Å | $\tilde{a}$, Å | $T_{c2}$, K | $S^2_{c2}$ | $(S^{(c)}_{c2})^2$ | $T^{(c)}_{c2}$, K |
|---|---|---|---|---|---|---|---|
| $ErRh_4B_4$ | 1.0 [k] | 45—50 [l] | 1 [m] | 0.8 [k] | — | — | — |
| $HoMo_6S_8$ | 0.74 [l] | 100 [n] | 2.5 [m] | 0.7 [l] | 0.35 [n] | 0.56 [n] | 0.62 [l] |

[a] According to Ref. 47; [b] Ref. 109; [c] $\tilde{\Delta}_0 = 1.76T_{c0}$; [d] from $N(0)$ and $v_F$; [e] from $H^*_{c2}(0)$ from Ref. 15 and $\xi_0$; [f] Ref. 15; [g] from $\theta_{ex}$; [h] $\theta_{ex} = h_0^2 n(0)$; [i] Ref. 108, $\theta_{ex} \approx \bar{\theta}_{ex}$; [j] $\tau_s^{-1} = 2\pi\theta_{ex}$; [k] Ref. 20; [l] Ref. 117; [m] from $Q$ and $\xi_0$; [n] Ref. 18.

the wave vector $Q$ of the nonuniform magnetic order along with data on $T_{c2}$, $T_M$, and $T_{c1}$. It was later found that the behavior of re-entrant superconductors, i.e., superconductors which exhibit a transition to a ferromagnetic normal phase upon cooling, is not universal. There is no coexistence phase in $ErRh_{1.1}Sn_{3.6}$ (Ref. 21), $Tm_2Fe_3Si_5$ (Refs. 15 and 122), or $Ho_{0.6}Er_{0.4}Rh_4B_4$ (Refs. 22 and 23); they convert directly from a nonmagnetic superconducting phase to a ferromagnetic normal phase at the first-order transition point $T_{c2}$.

The theory for the coexistence phase with a domain structure (a DS phase)[32–34] is in basic agreement with experimental data on the returning magnetic superconductor $HoMo_6S_8$. However, only polycrystalline samples of this compound are available, and it is not possible in this case to unambiguously determine the magnetic structure of the coexistence phase. Furthermore, the $HoMo_6S_8$ samples are dirty superconductors, and in them there is no gapless superconductivity in the coexistence phase. There has accordingly been particular interest in the re-entrant superconductor $ErRh_4B_4$, which has been synthesized in the form of single crystals[20] and deposited films.[136] Neutron-scattering measurements by Sinha *et al.*[20] with a single crystal and also some earlier measurements by Moncton *et al.*[19] with polycrystalline samples show that the volume of the $ErRh_4B_4$ sample occupied by the coexistence phase with the inhomogeneous magnetic structure is very small. This fact is at odds with the theoretical predictions of Refs. 24–34. The reason for the anomalous behavior of $ErRh_4B_4$ has yet to be identified, but the difference between the Mössbauer and neutron-scattering measurements of the moment in the low-temperature magnetic phase[110,140] (below $T_{c2}$) seems to be evidence of an irregular asperomagnetic order in the samples.[40,141] All the theoretical studies of the coexistence phase[24–34] have been carried out under the assumption of a regular magnetic subsystem. The conclusions reached there have not been applicable to compounds with a pronounced magnetic disorder, which would suppress the formation of a coherent inhomogeneous magnetic structure.

Experiments carried out on $ErRh_4B_4$ single crystals by Crabtree *et al.*[47] and Behroozi *et al.*[66] have shown that the properties of ferromagnetic superconductors at temperatures slightly above $T_M$ are no less interesting than those in the coexistence phase. Here, however, the distinctive behavior is seen only in the presence of an external magnetic field. As the temperature approaches $T_M$, the polarization of the localized moments in the magnetic field increases, and there is a corresponding increase in the role played by the exchange field of the localized moments, which—along with the magnetic induction—determines the conditions for the appearance of a superconducting nucleation center. As a result, because of the increase in the magnetic susceptibility of the localized moments as $T \to T_M$, the upper critical magnetic field $H_{c2}$ for the transition from the normal state to the superconducting state may decrease more rapidly than the thermodynamic field $H_c$, and the stage is set for a change in the type of superconductivity from type II near $T_{c1}$ to type I near $T_M$. In a clean superconductor, in contrast, an inhomogeneous superconducting state of the Larkin-Ovchinnikov-Fulde-Ferrel type occurs near $T_M$. Magnetic superconductors, in particular, $ErRh_4B_4$, thus present a unique opportunity for studying this interesting superconducting phase.

Finally, there is yet another interesting aspect to the question of the coexistence of magnetism and superconductivity. Matthias and Suhl[103] suggested back in 1960 that superconductivity might be preserved in a ferromagnetic normal phase near domain walls, where the magnetization reverses direction, and where the destructive effect of the exchange field is weakened. The question of the existence of superconducting domain walls has been taken up in several subsequent studies,[104,105,147,149] but it has yet to be finally

resolved.

In Section 2 of the present review we outline the basic information available on the structure and magnetic interactions in ternary compounds. In Section 3 we examine the behavior of ferromagnetic superconductors above $T_M$. In Section 4 we describe methods for calculating the coexistence phase, determine the roles played by the electromagnetic and exchange mechanisms in the formation of this phase, and find its basic characteristics. Section 5 compares theoretical predictions with experimental data on ferromagnetic superconductors. In the Conclusion we summarize the most interesting properties of magnetic superconductors and the problems which require further research.

## 2. INTERACTION OF CONDUCTION ELECTRONS AND MAGNETIC MOMENTS

### a) Structural features of magnetic superconductors and Hamiltonian of the magnetic and electron systems

About 15 classes of superconducting ternary compounds are now known,[41,43,106,116] and five of them contain systems in which a superconducting transition and a magnetic transition have been found. Among these classes of compounds are the ternary borides $(RE)(T)_4B_4$, where the transition metal T is Rh or Ir; chalcogenides of molybdenum and rare earth metals of the type $(RE)Mo_6(X)_8$ (X = S, Se); silicates of transition and rare earth metals of the type $(RE)_2(T)_3Si_5$ (T = Fe, Co) and $(RE)Rh_2Si_2$ (Ref. 116); and the ternary stannides $(RE)T_xSn_y$, where T = Rh, Os. The ternary borides exhibit a ferromagnetic and antiferromagnetic order (Table I); this class includes the re-entrant superconductor $ErRh_4B_4$ with a primitive tetragonal structure (see the review by Maple *et al.*[112]) and the antiferromagnetic superconductor $ErRh_4B_4$, which has the same composition but a different (body-centered) crystal structure.[135,136] (We will be using the notation $ErRh_4B_4$ to represent a ferromagnetic superconductor.) The class of chalcogenides also includes both types of magnetic behavior (see the review by Ishikawa *et al.*[108]). The class of silicates includes only one re-entrant ferromagnetic superconductor, $TmFe_3Si_5$ with $T_{c1} \approx 1.3$ K and $T_{c2} = T_M \approx 1.1$ K (Refs. 51 and 112). The rhodium and osmium stannides ($ErRh_{1.1}Sn_{3.6}$, $ErOs_xSn_y$, and $TmOs_xSn_y$) also exhibit re-entrant superconductivity with $T_{c1} \approx 1.2$ K and $T_{c2} = T_M \approx 0.6$ K, but neutron-scattering and specific-heat measurements show that compounds of this class do not have any genuine long-range magnetic order down to the lowest temperatures, $T \ll T_{c2}$ (Refs. 21, 52, 53, 112, and 113).

In addition to the regular ternary compounds, in the classes of borides and chalcogenides, there are a large number of pseudoternary compounds, among which there are re-entrant ferromagnetic superconductors and antiferromagnetic superconductors.[22,23,54–58,61] As in the stannides, the arrangement of the localized magnetic moments in these pseudoternary compounds is an irregular type.

In this review we will be discussing for the most part ternary compounds with a regular arrangement of magnetic moments, since it is only for such systems that we have an adequate theoretical description at this point.

The lattice of the ternary compounds $(RE)Rh_4B_4$ and

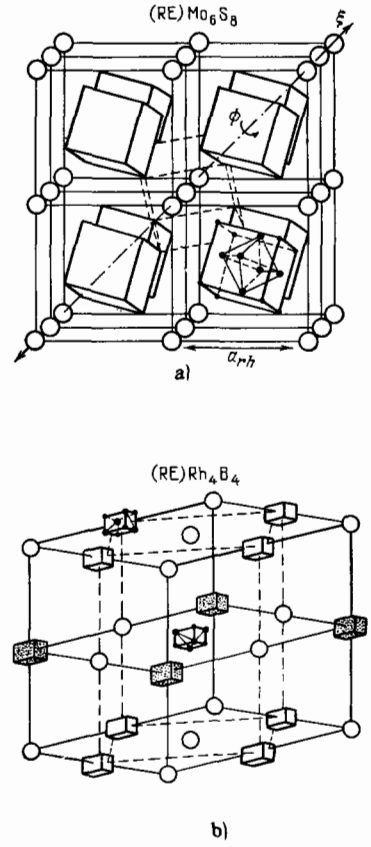


FIG. 6. Crystal structures of (a) $(RE)Mo_6S_8$ and (b) $(RE)Rh_4B_4$.

$(RE)Mo_6S_8$ is of such a structure that the Rh and B atoms (or Mo and X atoms, respectively) form clusters which are bound to each other well but which are separated by rather large distances from the rare earth atoms (Fig. 6). The magnetism is caused by the 4f electrons of the RE atoms, and the superconducting properties are caused primarily by the d and s electrons of the transition elements. The RE atoms are far apart (6.5 Å in the chalcogenides and 5.3 Å in the borides), so that there would presumably be essentially no direct exchange interaction of the localized moments. Calculations carried out on the band structure of ternary borides and chalogenides by Freeman and Jarlborg[109] reveal a significant transfer of electrons from the rare earth atoms to the clusters. As a result, the local density of conducting electron states at the RE atoms is low: 2–3% of the density at the Mo atoms in the chalogenides and about 15% of the density at the Rh atoms in the borides. The exchange interaction of the 4f electrons of the RE atoms and the conduction electrons is correspondingly weak. The cluster structure of the ternary borides and chalcogenides, with a significant transfer of electrons from the RE atoms to the clusters, therefore leads to a weak indirect RKKY interaction, low critical magnetic temperatures (of the order of 1 K), and, ultimately, the possible coexistence of magnetism and superconductivity.

We will be discussing only those compounds for which the levels of the 4f electrons of the RE atoms lie significantly below the Fermi level and whose magnetism can be de-

scribed by the model of localized moments.[1)] For such compounds we can use a description which allows us to single out a magnetic subsystem (regularly positioned localized moments **J** in the crystal field) and an electron subsystem. The interaction of these subsystems occurs through the exchange and electromagnetic interactions of the localized moments and the conduction electrons.

In the BCS model for superconducting pairing, the Hamiltonian of the complete system of localized moments and conduction electrons is

$$\mathcal{H} = \int d^3\mathbf{r} \Big[ \frac{1}{2m} \psi^+(\mathbf{r}) \Big( \mathbf{p} - \frac{e}{c} \mathbf{A} \Big)^2 \psi(\mathbf{r}) + \Delta(\mathbf{r}) \psi^+(\mathbf{r}) i\sigma_y \psi^+(\mathbf{r}) - \Delta^*(\mathbf{r}) \psi(\mathbf{r}) i\sigma_y \psi(\mathbf{r}) + \psi^+(\mathbf{r}) \Big[ \sum_i \tilde{\mathcal{J}}(\mathbf{r} - \mathbf{r}_i)(g-1) \hat{\mathbf{J}}_i + g\mu_B \mathbf{B} \Big] \sigma \psi(\mathbf{r}) + \frac{|\Delta(\mathbf{r})|^2}{\lambda} + \frac{\mathbf{B}^2}{8\pi} \Big] + \sum_i [-\mathbf{B}(\mathbf{r}_i) g\mu_B \hat{\mathbf{J}}_i + \mathcal{H}_{cr}(\hat{\mathbf{J}}_i)] + \mathcal{H}_{sc}, \quad (2.1)$$

$$\mathbf{B} = \mathrm{rot}\, \mathbf{A};$$

here $\Delta(\mathbf{r})$ is the superconducting order parameter for singlet pairing of electrons, $\psi(\mathbf{r})$ is a spinor, $\sigma$ is a Pauli matrix, **A** is the vector potential, $\lambda$ is the dimensionless parameter of the electron-phonon interaction, $\tilde{\mathcal{J}}(\mathbf{r})$ is the exchange integral, $g$ is the Landé factor, the operator $\hat{\mathbf{J}}_i$ represents the angular momentum at site $i$ with coordinate $\mathbf{r}_i$, the term $\mathcal{H}_{cr}(\mathbf{J}_i)$ reflects the effect of the crystal field on the localized moments, and $\mathcal{H}_{sc}$ reflects the scattering of electrons by nonmagnetic impurities. The electron part of the Hamiltonian describes the Cooper pairing of conduction electrons in the presence of the exchange field of the localized moments and the magnetic induction **B**.

For a given field **B**, Gibbs averaging with Hamiltonian (2.1) yields the Gibbs potential. The field **B** and the thermodynamic potential of the system are found by minimizing the Gibbs potential with respect to **B**.

In this section of the review we will be discussing the interaction of the localized moments in the normal (nonsuperconducting) phase.

### b) Effects of the crystal field

The crystal field partially lifts the degeneracy in terms of the directions of the angular momentum, giving rise to that system of levels for the RE ions which determines their magnetic behavior in crystal.[64,65] Measurements of the specific heat and Mössbauer studies[49,108,110,112] have shown that the crystal-field effects in these compounds are important and that the level splitting energies of an ion in the crystal field are usually far larger than the energy of its magnetic ordering. For this reason, the magnetic properties of most ternary compounds are determined by the lowest ion level in the crystal field. This level is usually doubly degenerate.

The effect of $\mathcal{H}_{cr}$ thus reduces to replacing the momentum operator $\hat{\mathbf{J}}_i$ in (2.1) by the effective-moment operator $J_{eff} = 1/2$ and introducing an anisotropy energy. There is reason to believe that it is precisely this simple situation, with $J_{eff} = 1/2$, which prevails in $HoMo_6S_8$ (Ref. 18), although the complete structure of levels of the $Ho^{3+}$ ion in this compound has not yet been established.

The situation in $ErRh_4B_4$ is slightly more complicated. The Hamiltonian of the crystal field for this compound has now been completely determined by Dunlap *et al.*[115,141] from measurements of the specific heat. The lowest levels are two doublets; the next two doublets are separated from the first two by an energy of 1.4 K, which is comparable to the temperature of the magnetic transition. All the other levels of the ion are significantly higher, and they are unimportant at temperatures $T \lesssim T_{c1}$ (see also Refs. 43, 44, and 110). The system of levels from the two closely spaced doublets cannot be described by introducing an effective moment, but in terms of its magnetic properties in the normal state the ideal $ErRh_4B_4$ crystal is a standard ferromagnet with a second-order transition at the Curie point $\theta$ and with an easy-plane anisotropy; the easy plane is the $a$, $b$ basis plane of the crystal.[40] In the single crystals which have been studied by Sinha *et al.*[20,47,48] the moment has been directed in all cases exclusively along one of the equivalent axes, apparently because of stresses in the sample.

### c) Ruderman-Kittel-Kasuya-Yosida indirect exchange interaction of localized moments in the normal state

The magnetic moments act through the exchange interaction to polarize the spins of the conduction electrons, and this effect gives rise to the Ruderman-Kittel-Kasuya-Yosida (RKKY) indirect exchange interaction of localized moments. In second order in the exchange interaction of the localized moments and the electrons in the normal state, we find from (2.1) an effective RKKY Hamiltonian for the moments (see Refs. 8 and 111, for example):

$$\mathcal{H}_{ex} = \frac{N}{2} \sum_{\mathbf{q}} n^2 (g-1)^2 \times \Big[ \sum_{\mathbf{G}} \mathcal{J}^2(\mathbf{q}+\mathbf{G}) \chi_e(\mathbf{q}+\mathbf{G}) - \frac{1}{N} \sum_{\mathbf{k}} \mathcal{J}^2(\mathbf{k}) \chi_e(\mathbf{k}) \Big] \hat{\mathbf{J}}_{\mathbf{q}} \hat{\mathbf{J}}_{-\mathbf{q}},$$

$$\hat{\mathbf{J}}_{\mathbf{q}} = \frac{1}{N} \sum_i \hat{\mathbf{J}}_i e^{i\mathbf{q}\mathbf{r}_i}, \quad \mathcal{J}(\mathbf{k}) = \int d^3\mathbf{r}\, \tilde{\mathcal{J}}(\mathbf{r}) e^{i\mathbf{k}\mathbf{r}}, \quad (2.2)$$

where $N$ is the total number of localized moments, the $\mathbf{r}_i$ are the coordinates of the localized moments, $\chi_e(\mathbf{k})$ is the paramagnetic susceptibility of the electron gas per localized moment, in units of $g_e^2 \mu_B^2$, the vectors **G** are the reciprocal-lattice vectors of the RE atoms, and $n$ is the density of localized moments. The sum over the vectors **G** allows for the discrete arrangement of the magnetic ions at the lattice sites; the sum over **q** goes over the first Brillouin zone of the

[1)] We are eliminating from consideration compounds of the type[62,134] $Y_9Co_7$, quasi-one-dimensional compounds of the type[97] $(TMTSF)_2PF_6$ and[116] $(RE)Rh_2Si_2$, with delocalized magnetic electrons. A theoretical approach to the description of systems of this type is set forth in Refs. 63 and 68.

lattice of localized moments. The last term in square brackets eliminates the self-effect of the moments; it can be discarded, since the corresponding contribution to the energy does not depend on the state of the magnetic system.

In the sum over $\mathbf{q}$ in (2.2), the germs with $\mathbf{G}=0$ and $q \ll G$ represent the long-wavelength part of the RKKY interaction. The contribution of this part to the energy of the ferromagnetic state at $T=0$ (per localized moment) can be written in the form $-\theta_{ex}$, where $\theta_{ex}=h_0^2\chi_e(0)$, a $h_0=\mathscr{J}(0)(g-1)n\langle\hat{J}_z\rangle$, and $\langle\hat{J}_z\rangle$ is the average value of the moment in the ferromagnetic normal state (the easy axis is the $z$ axis). In Hamiltonian (2.1) we find $\chi_e=N(0)$ is the density of electron states per localized moment.

The parameter $h_0$, the energy of an electron in the exchange field of the localized moments in the ferromagnetic normal state at $T=0$, determines the slight difference between the energies of the electrons of a Cooper pair, i.e., the paramagnetic effect.[4]

The quantity $\theta_{ex}=h_0^2N(0)$ is positive; i.e., the long-wave part of the exchange interaction always tends to establish ferromagnetism in the ground state. The contribution of this part of the exchange interaction to the Curie temperature $\theta$ (in the normal state) is also positive and proportional to $\theta_{ex}$. The superconductivity reduces the paramagnetic susceptibility of the electron gas, $\chi_e(\mathbf{q})$, at small wave vectors, $|\mathbf{q}|\lesssim\xi_0^{-1}$. It is this suppression of the long-wave part of the RKKY interaction which is illustrated in Fig. 2 (see also Refs. 137 and 91). That the electron paramagnetic susceptibility decreases below the superconducting transition in ternary borides has been confirmed experimentally by Kumagai and Fradin.[67] This decrease is manifested by an increase in the nuclear relaxation time below $T_c$ in the compound $Er_xY_{1-x}Rh_4B_4$ (the relaxation rate depends on the RKKY interaction of the localized moments of the rare earth Er ions).

Since superconductivity turns off the long-wave part of the RKKY interaction, the parameter $\theta_{ex}$ is a measure of the interaction energy of the superconducting and magnetic order within the framework of the exchange-interaction mechanism. Actually, $\theta_{ex}$ is the increase (per localized moment) in the energy of the ferromagnetic state in the presence of a superconducting screening of the RKKY interaction at $T=0$.

In the sum over $\mathbf{G}$ in (2.2), the terms with $\mathbf{G}\neq 0$ describe the short-wave part of the RKKY interaction, which is essentially unaffected by the superconductivity. We denote by $-\theta'_{ex}$ the contribution of the short-wave part to the energy of the ferromagnetic normal phase at $T=0$; the magnitude and sign of $\theta'_{ex}$ depend strongly on the electron band structure through the components of $\chi_e(\mathbf{q}+\mathbf{G})$ with $\mathbf{G}\neq 0$. In order of magnitude, we have $|\theta'_{ex}|\approx\theta_{ex}$.

#### d) Electromagnetic (dipole-dipole) interaction in the normal state

According to (2.1), the localized moments create a magnetic field $\mathbf{B}$, and the interaction through this field leads to a magnetic dipole-dipole interaction of the localized moments:

$$\mathscr{H}_{EM}=-g^2\mu_B^2\sum_{i\neq j}\left[\frac{1}{r_{ij}^3}\mathbf{J}_i\mathbf{J}_j-\frac{3}{r_{ij}^5}(\mathbf{r}_{ij}\mathbf{J}_i)(\mathbf{r}_{ij}\mathbf{J}_j)\right], \quad (2.3)$$

where $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$. The contribution of this interaction to the energy of the ferromagnetic state at $T=0$ can be characterized by the parameter $\theta_{em}=2\pi n\mu^2$, where $\mu=g\mu_B\langle\hat{J}_z\rangle_{T=0}$. The exact magnitude of this contribution and its sign depend strongly on the type of lattice of the localized moments. According to calculations by Redi and Anderson,[68] in a ternary chalcogenide only the magnetic-dipoel interaction should lead to antiferromagnetic ordering.

For ferromagnetic superconductors it is also convenient to single out the short-wave and long-wave contributions to the dipole-dipole interaction. Now, however, this separation can be carried out better in the coordinate representation than in the momentum representation.[69] We construct a sphere of radius $R$ around the $i$th dipole, and we choose $R$ to satisfy the conditions $G^{-1}\ll R\ll d\ll\lambda_L$, where $d$ is a scale dimension of the magnetic inhomogeneity under consideration (in the coexistence phase of ferromagnetic superconductors, $2d$ is the period of the magnetic structure). The interaction of all the moments within this sphere with the $i$th moment constitutes the short-wave part of the electromagnetic interaction, which is not subject to the influence of superconductivity at $R\ll\lambda_L$. We denote by $-\theta'_{em}$ the contribution of this part of the interaction to the energy of the ferromagnetic ground state; the sign of $\theta'_{em}$ depends on the type of lattice of the localized moments (for a cubic lattice we would have $\theta'_{em}=0$). The interaction of the $i$th dipole with the dipoles outside this sphere gives us the long-wave part of the electromagnetic interaction. This part was calculated in Ref. 69; the contribution of the long-wave part of the electromagnetic mechanism to the energy of the ferromagnetic normal state at $T=0$ is $-\theta_{em}/3$. The long-wave part of the electromagnetic interaction is similar to the long-wave of the exchange interaction in that it always promotes ferromagnetic ordering, and the total superconducting screening of the field of the magnetic moments by virtue of the Meissner effect raises the energy of the ferromagnetic state by an amount $\theta_{em}$. On the other hand, in the ferromagnetic state at $T=0$ the moments induce a magnetic field $B(0)=4\pi\mu n$, which suppresses the superconductivity because of an orbital effect.[4] The mutual effects of the magnetic ordering and the superconducting pairing can thus be characterized in the framework of the electromagnetic mechanism by the parameters $\theta_{em}$ and $B(0)$, which play the same roles as those played by the parameters $\theta_{ex}$ and $h_0$ for the exchange mechanism. Tables I and II show the parameters $\theta_{em}$ and $B(0)$ for certain compounds. The total contribution of all mechanisms to the energy of the ferromagnetic normal phase at $T=0$ si $-\theta_0$, where $\theta_0=\theta_{ex}+(\theta_{em}/3)+\theta'_{em}+\theta'_{ex}$.

#### e) Magnetic functional of a ferromagnet in the normal state

To describe the coexistence phase in ferromagnetic superconductors we will need a functional of the magnetic subsystem which describes inhomogeneous states near the ferromagnetic ground state ("near the ground state" means that the wave vectors characterizing the inhomogeneity are small in comparison with $G$). This functional can be derived

easily in the simple approximation of a self-consistent field. In this approximation, we describe the magnetic system by the quantities $\mathbf{S}_i$, the average values of the moments at sites $i$. We use the normalized values $\mathbf{S}_i = \langle \hat{\mathbf{J}}_i \rangle / \langle \hat{\mathbf{J}}_z \rangle_{T=0}$, so that we have $S_{zi}(T=0) = 1$.

To find the magnetic free-energy functional it is sufficient to replace the operators $\hat{\mathbf{J}}_i$ in the moment interaction Hamiltonians (2.2) and (2.3) by their average values and to add to the resulting interaction energy the functional $F_0(\mathbf{S}_i, T)$ of the noninteracting moments, where crystal-field effects are taken into account.

We introduce separation into short- and long-wave parts in the interaction energy of the localized moments, and we introduce the Fourier components $\mathbf{S}_q$ of the quantities $\mathbf{S}_i$. The exchange interaction of the localized moments and the short-wave part of the electromagnetic interaction can be written as a functional which is quadratic in the quantities $\mathbf{S}_q$, with coefficients which are analytic in $\mathbf{q}$ as $\mathbf{q} \to 0$. The long-wave part of the electromagnetic interaction can be found directly from (2.1) by averaging the magnetization $\mathbf{S}(\mathbf{r})$ over a volume of radius $R$. The complete magnetic functional can then be written in the form

$$F_M(\mathbf{S}_i, T) = \sum_i F_0(\mathbf{S}_i, T) + \sum_q \left[ (-\theta_0 + \theta_{ex} a^2 q^2) \mathbf{S}_q \mathbf{S}_{-q} + \frac{1}{2} \theta_{em} q^{-2} (\mathbf{q}\mathbf{S}_q)(\mathbf{q}\mathbf{S}_{-q}) \right] + \sum_i \left[ -\mu \mathbf{H}_0 \mathbf{S}_i + \frac{1}{2} D (S_{xi}^2 + S_{yi}^2) \right], \quad (2.4)$$

where $\mathbf{H}_0$ is the external magnetic field, $D$ is the anisotropy parameter, and $F_0(\mathbf{S}_i, T)$ is the isotropic part of the functional of an isolated ion. For an ion with $J_{eff} = 1/2$ we have

$$F_0(S, T) = -T \int_0^S b_{1/2}(x)\, dx,$$

where $b_{1/2}(x)$ is the inverse Brillouin function. For two closely spaced low-lying doublets the functional of an isolated ion can be calculated numerically if the Hamiltonian $\mathscr{H}_{cr}(\hat{\mathbf{J}})$ is known.[40]

Let us examine the applicability of the self-consistent-field approximation for describing the magnetism of ternary compounds. In ordinary ferromagnets with a stong exchange interaction, the Curie temperature is $\theta \gg \theta_{em}$, and the fluctuational region in these materials is large because of the short-range nature of the direct or indirect (RKKY) interaction of the localized moments, i.e., because of the small magnetic rigidity $a$. In the compounds with $\theta \approx \theta_{em}$, under consideration here, however, the fluctuations are strongly suppressed by the long-range part of the electromagnetic interaction.[32,114] As a result, in the presence of uniaxial anisotropy the fluctuations are the same as in four-dimensional space, and they grow only logarithmically as the ferromagnetic point $\theta$ is approached.[71] Such fluctuations would not be noticeable in practice, and a uniaxial ferromagnet with $\theta \approx \theta_{em}$ could be described highly accurately by the self-consistent-field method. It is this behavior which has been observed[72,73] experimentally in $HoRh_4B_4$. Accordingly, to describe the ferromagnetic superconductors we will use the self-consistent-field approximation, bearing in mind that uniaxial anisotropy prevails in the $HoMo_6S_8$ crystals and in the $ErRh_4B_4$ samples which have been studied experimentally.

Furthermore, using the simple self-consistent-field approximation, we are ignoring spin waves. This simplification is legitimate in the case of pronounced anisotropy, in which there is a gap in the spin-wave spectrum.

## 3. PROPERTIES OF FERROMAGNETIC SUPERCONDUCTORS IN THE NONMAGNETIC SUPERCONDUCTING PHASE (ABOVE $T_M$)

### a) Role of magnetic scattering

We will first estimate the effect of magnetic scattering on the nonmagnetic superconducting phase $S$. We will see that this effect is constant and small over the entire temperature range $\theta < T < T_{c1}$ in systems with $\theta_{ex} \ll T_{c1}$. The imposition of a magnetic field above $T_M$ or a transition to a magnetically ordered state at $T < T_M$ reduces the exchange scattering. Consequently, over the entire temperature range in systems such as $ErRh_4B_4$ or $HoMo_6S_8$ with $\theta_{ex} \lesssim T_M \ll T_{c1}$ magnetic scattering can be taken into account simply through a renormalization of the parameter $\tilde{\Delta}_0$, which is a measure of the superconducting gap at $T = 0$ in the absence of localized moments.

The effect of exchange scattering on the transition temperature $T_{c1}$ can be found from the behavior of $T_c(x)$ in the series of compounds $(RE)_x Y_{1-x} Rh_4B_4$ or $(RE)_x Y_{1-x} Mo_6S_8$. Since $Y^{3+}$ is a nonmagnetic ion, the addition of magnetic RE ions leads to a decrease in $T_c$. According to the theory of Abrikosov and Gor'kov,[3] in the absence of a crystal field and far from the point $T_M$ we have

$$\frac{dT_c(x)}{dx} = -\frac{\pi^2}{2} N(0) \overline{\mathscr{J}^2(\mathbf{q})} (g-1)^2 nJ(J+1) = -\frac{\pi^2}{2} \bar{\theta}_{ex}, \quad (3.1)$$

where $\mathscr{J}^2(\mathbf{q})$ is the average value of $\mathscr{J}^2(\mathbf{q})$ over the change in the electron momentum on the Fermi surface.

Corresponding data on $T_c(x)$ in ternary borides are reported in Refs. 46, 49, and 112; data on ternary chalcogenides are reported in Refs. 50 and 108. Working from these results and (3.1), we have calculated the parameters $\bar{\theta}_{ex}$; the results are shown in Tables I and II. Below, working from the experimental data on $H_{c2}$, we find the parameter values $h_0 = 40$ K and $\theta_{ex} = 0.9$ K in $ErRh_4B_4$. Comparison of $\bar{\theta}_{ex}$ and $\theta_{ex}$ for this compound reveals only a small difference, and expression (3.1) is good enough for an order-of-magnitude estimate of $\theta_{ex}$. For the exchange-scattering times $\tau_s$ we have $\tau_s^{-1}(T_{c1}) = (dT_c/dx)(4/\pi) = 2\pi\bar{\theta}_{ex}$. From the data in Table II we find the critical temperature in the absence of localized moments to be $T_{c0} = 11.5$ K and 2.5 K for $ErRh_4B_4$ and $HoMo_6S_8$, respectively, and the dimensionless exchange-scattering parameter $x_s = [\tau_s(T_{c1})\Delta_0]^{-1}$ is small in these materials (0.15 and 0.25).

As the point of a second-order magnetic transition is approached, we know that the magnetic fluctuations grow, and this growth can in principle lead to a growth of $\tau_s^{-1}$. We therefore consider the behavior of $\tau_s^{-1}$ as $T \to \theta$. An expression for $\tau_s$ above $\theta$ has been derived by Rainer[120] with allowance for the interaction of the localized moments in the static approximation for magnetic fluctuations. When the crystal field is ignored, this expression can be written as

$$\tau_s^{-1} = N(0)(g-1)^2 \sum_{\mathbf{q}} g(\mathbf{q})\, \mathcal{J}^2(\mathbf{q}) \langle \hat{\mathbf{J}}_{\mathbf{q}} \hat{\mathbf{J}}_{-\mathbf{q}} \rangle,$$

$$g(\mathbf{q}) = (4\pi k_F^2 q)^{-1}, \quad q \gg \xi_0, \tag{3.2}$$

where the form factor $g(\mathbf{q})$ has the same meaning and the same $\mathbf{q}$ dependence as the difference between the electron paramagnetic susceptibility and normal phases, $\chi_s(\mathbf{q})$ and $\chi_n(\mathbf{q})$; i.e., $\chi_s(\mathbf{q}) - \chi_n(\mathbf{q}) = \pi 2 q \xi_0$ at $q \gg \xi_0^{-1}$ [see (1.1)]. The static approximation can be used for these compounds with $\theta \approx T_M \ll T_{c1}, \Delta_0$ since the magnon frequencies are small in comparison with the typical electron frequencies.

The correlation function $\langle \mathbf{J}_{\mathbf{q}} \mathbf{J}_{-\mathbf{q}} \rangle$ in (3.2) satisfies the sum rule

$$\sum_{\mathbf{q}} \langle \hat{\mathbf{J}}_{\mathbf{q}} \hat{\mathbf{J}}_{-\mathbf{q}} \rangle = J(J+1) \sum_{\mathbf{q}} 1, \tag{3.3}$$

where the sum over $\mathbf{q}$ is carried out over the first Brillouin zone of the lattice of localized moments.

It follows from (3.2) and (3.3) that as the temperature $T$ approaches the second-order transition point $\theta$ from above in a ferromagnetic superconductor the quantity $\tau_s^{-1}$ increases because of the increase in the correlation functions $\langle \hat{\mathbf{J}}_{\mathbf{q}} \hat{\mathbf{J}}_{-\mathbf{q}} \rangle$ with small values of $q$ [they make a large contribution because of the form factor $g(\mathbf{q})$]. In the Ornstein-Zernike approximation, ignoring the long-range part of the dipole interaction, we find $\langle \hat{J}_{z,\mathbf{q}} \hat{J}_{z,-\mathbf{q}} \rangle \sim (T - \theta + 2\theta_{ex} a^2 q^2)^{-1}$, and the quantity $\tau_s^{-1}$ increases logarithmically as[98] $T \to 0$. Incorporating the long-range part of the electromagnetic interaction erases this logarithmic growth for crystals with an easy-axis anisotropy. In this case we have

$$\langle \hat{J}_{z,\mathbf{q}} \hat{J}_{z,-\mathbf{q}} \rangle \sim (T - \theta + 2\theta_{ex} a^2 q^2 + \theta_{em} \cos^2 \varphi)^{-1}, \tag{3.4}$$

where $\varphi$ is the angle between $\mathbf{q}$ and the easy axis, and the correlation function $\langle \hat{\mathbf{J}}_{i\mathbf{q}} \hat{\mathbf{J}}_{i,-\mathbf{q}} \rangle$ with $i = x,y$ do not diverge at all as $T \to \theta$ because of the magnetic anisotropy. As a result we may conclude that the temperature dependence of $\tau_s^{-1}$ in the interval $(\theta, T_{c1})$ is very weak, and up to the point $\theta$ we can assume $\tau_s(T) \approx \tau_s(T_{c1})$. For compounds with a small parameter $x_s$ in the temperature region $T \ll T_{c1}$ we therefore assume $\Delta_0 = 1.76 T_{c1}$.

#### b) The upper critical magnetic field

In calculating the upper critical magnetic field we need to take into account the orbital effect of the magnetic induction $\mathbf{B} = \mathbf{H} + B(0)\mathbf{S}$ and the paramagnetic effect of the exchange field $\mathbf{h} = h_0 \mathbf{S}$. We will restrict the discussion here to compounds with $T_M \ll T_{c1}$, and we will assume that the magnetic scattering is taken into account through a renormalization of the parameter $\Delta_0$. Systems with approximately equal values of $T_M$ and $T_{c1}$ were studied in Ref. 148.

We note at the outset that there are two factors which distinguish ferromagnetic superconductors near $\theta$ from ordinary superconductors.

a) The magnetization of the localized moments in the magnetic field leads to a decrease in the thermodynamic critical field $H_c$ in ferromagnetic superconductors near $\theta$. In the presence of a field $\mathbf{H}$, the magnetic energy of the localized moments in the normal state decreases substantially from the value in the superconducting state, since in the latter the magnetic field in the sample is screened by the superconducting currents. As a result we have $H_c \to 0$ as $T \to \theta$, because of the increase in the susceptibility of the localized moments, $\chi_m(T)$, in accordance with the Curie-Weiss law as the Curie point $\theta$ is approached in the normal phase.

b) The exchange field $\mathbf{h}$ suppresses the formation of a superconducting nucleating region if $\mathbf{h}$ becomes comparable to $\Delta_0$. For a given $H$ as $T \to \theta$ we find a growth $h \sim H(T-\theta)^{-1}$. As a result of the paramagnetic effect of the exchange field, the upper critical field $H_{c2}$ for the appearance of a superconducting nucleating region also decreases as $T \to \theta$ in compounds with $h_0 \gg \Delta_0$. If the decay of $H_{c2}$ is more rapid than that of $H_c$, then we find a type I superconductor near $\theta$, although far from the point $\theta$ the orbital effect of the magnetic field is dominant, and here all magnetic superconductors are type II superconductors.

What is the effect of a field directed parallel to the easy axis? It is for this direction of the field that the susceptibility of the localized moments in the normal moments in the normal phase (N) increases without bound as $T \to \theta$, and all the characteristic features of the behavior of ferromagnetic superconductors in a magnetic field are manifested.

In the presence of the magnetic field, four types of superconducting states can exist: a Meissner phase (MS), a vortex state (VS), an inhomogeneous state (LOFF),[76,77] and an intermediate state (IS). In the first phase, the superconducting order parameter is homogeneous, and the field does not penetrate into the sample. In the VS phase, this order parameter depends on the coordinates $(x,y)$; the field is directed along the $z$ axis), and the field penetrates into the sample through the formation of a vortex lattice. In the LOFF phase the orbit parameter depends on all three coordinates and is periodic with a period of the order of $\xi_0$ at $T \ll T_{c1}$. As in the VS phase, there is apparently not a complete screening of the field in the LOFF state. The intermediate phase occurs in the case of a first-order superconducting transition in a magnetic field in samples with a demagnetizing factor $N_z \neq 0$. This phase is characterized by a layered or filamentary structure of alternating normal and superconducting regions.

The thermodynamic field $H_c$ for the N-MS transition is determined by equating the magnetic energy of the normal phase to the free energy of the superconductor. At $T \ll T_{c1}$, the latter energy is given by the expression $(-H_{c0}^2 + H_a^2)/8\pi n$, where $H_{c0} = \sqrt{4\pi N(0) \Delta_a^2 n}$, and $H_a$ is the applied field, which is related to the internal field by $H_a = H(1 + 4\pi n \chi_m N_z)$ in the temperature range in which the magnetization is proportional to the magnetic field. In

$$-\int_0^{H_a} dH M(H) = -\frac{1}{2} H^2 [\chi_m (1 + 4\pi N_z n \chi_m)].$$

As a result we find

$$H_a(T) = H_{c0} \{[1 + 4\pi n (N_z + 1)\chi_m](1 + 4\pi n N_z \chi_m)\}^{-1/2},$$

$$\chi_m = \frac{\tilde{\mu}^2}{T - \theta}. \tag{3.5}$$

We have $H_c \sim (T - \theta)$ for $N_z \neq 0$ and $H_c \sim \sqrt{T - \theta}$ for $N_z = 0$ at temperatures near (but not very near) $\theta$. Expression (3.5) was first derived for the case $N_z = 0$ by Machida.[150] Figure 7a shows the line $H_c(T)$; near $\theta$ this line

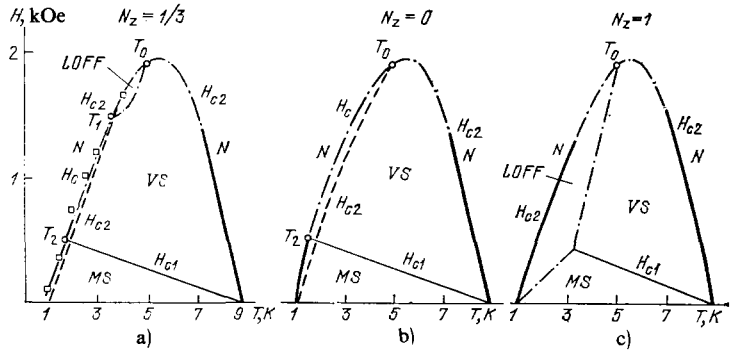


FIG. 7. $H$, $T$ phase diagram above $T_M$ for various $ErRh_4B_4$ samples. The demagnetizing factors are: a—$N_z = 1/3$; b—$N_z = 0$; c—$N_z = 1$. The field is directed along the **a** axis. MS—Meissner phase; VS—vortex phase; LOFF—inhomogeneous superconducting phase; $N$—normal phase.

separates the N and MS phases, while at higher temperatures it converts into the line separating the N and VS phases.

We now seek the second-order transition line $H_{c2}(T)$, i.e., the line at which a superconducting nucleating region forms, for clean superconductors. A procedure for calculating $H_{c2}$ with allowance for the orbital and paramagnetic effects of the magnetic field has been worked out by Gunther and Gruenberg,[79] whose results we will use.

The consequence of the orbital effect is determined by the critical field $H^*_{c2}(0)$ which would be observed in the absence of localized moments at $T = 0$. For compounds with $\theta \ll T_{c1}$, this quantity can be determined from data on $H_{c2}(T)$ near $T_{c1}$, where the magnetization of the localized moments has no effects on $H_{c2}(T)$. In this region we have $H^*_{c2}(T) \approx H_{c2}(T)$, and for clean superconductors we find $H^*_{c2}(T) = 0.72 T_c (dH_{c2}/dT)_{T=T_{c1}}$ from the Gor'kov theory.[74] At low temperature, $T \ll T_{c1}$, taking into account the paramagnetic effect of the exchange field, we find

$$H_{c2}(T) = \beta H^*_{c2}(0)\, \alpha^{-1} f(\alpha), \qquad \alpha = \frac{\beta \sqrt{2}\, H^*_{c2}(h_0 \chi_m(T))}{\mu \Delta_0}, \quad \beta = \frac{\sqrt{2}}{1 + 4\pi n \chi_m(T)}, \tag{3.6}$$

where the function $f(\alpha)$ was determined numerically in Ref. 79; at $\alpha > 1.8$, the LOFF phase occurs. If we have a small parameter $\rho = [\Delta_0 B(0)/h_0 H^*_{c2}(0)]^2 \ll 1$ (this is the situation in $HoMo_6S_8$ and $ErRh_4B_4$; see the estimates of the parameters in Table II), we find $\alpha \gg 1$ and $f(\alpha) \approx 1.07$ near $\theta$. For compounds with $\rho \ll 1$, which we will be discussing below, we then find from (3.6)

$$H_{c2}(T) = 0.75 \frac{\Delta_0 \mu}{h_0 \tilde{\mu}^2}(T - \theta), \quad T - \theta \ll \theta; \tag{3.7}$$

i.e., the orbital effect of the magnetic induction can be ignored at temperature near $\theta$.

It follows from (3.5) and (3.7) that the shape of the $(H,T)$ phase diagram depends on the relation between the parameters $\theta_{ex}$ and $\tilde{\theta}_{em} = 1.12\theta_{em} N_z(N_z + 1)$. If $\tilde{\theta}_{ex} > \tilde{\theta}_{em}$, we have $H_c(T) > H_{c2}$ near $\theta$, and there is a first-order N-MS or N-VS transition, depending on the relation between $H_c$ and the lower critical field $H_{c1}$, which separates the VS and MS phases. As a result, for $\theta_{ex} > \tilde{\theta}_{em}$ we find the phase diagrams in Figs. 7a and 7b. On these diagrams, $T_0$ is the tricritical point, at which the line $H_c(T)$ terminates; here $T_0 \leqslant 0.55 T_{c1}$. The point $T_1$ is the intersection of the curves of $H_c(T)$ and $H_{c2}(T)$, while $T_2$ is the intersection of the curves of $H_c(T)$ and $H_{c1}(T)$. In Fig. 7a we have the MS, VS, and LOFF phases, while the latter is not present in Fig. 7b. The intermediate phase is not seen on the $(H,T)$ diagrams, since for this phase the internal field is constant, equal to $H_c$. At $\theta_{ex} < \tilde{\theta}_{em}$ we have $H_{c2}(T) > H_c(T)$ near $\theta$, and in the region $\theta < T < T_0$ the N-LOFF transition occurs (Fig. 7c). Interestingly, since the relation between $H_c$ and $H_{c2}$ near $\theta$ depends strongly on the demagnetizing factor, all three types of phase diagrams can be produced in a given compound by changing the shape of the sample.

According to numerical calculations by Nakanishi and Maki,[78] in the absence of an orbital effect the N-LOFF transition is actually a first-order transition but close to a second-order transition. The incorporation of the orbital effect moves the transition closer to second order. As the electron mean free path decreases, the region with the LOFF contracts.[143] Sakai *et al.*[144] have calculated $H_{c2}$ for dirty ferromagnetic superconductors. For polycrystalline samples we need to allow for the anisotropy of the magnetic susceptibility and for the percolation nature of the transition in the sample because of the random orientation of crystals with respect to the applied magnetic field.[96] Corresponding calculations have been carried out only for $H_{c2}$ in dirty superconductors.[127,128]

### c) The lower critical magnetic field

We turn now to the effect of the localized moments on the screening of the magnetic field in the MS phase,and we seek the boundary between the MS and VS phases. Examining the changes in the superconducting and magnetic quantities over distances of the order of the penetration depth $\lambda \gg \xi$ in a magnetic field **H** parallel to the $z$ axis, we can write the field-dependent part of the free-energy function as

$$F = \int d^3\mathbf{r} \left[ n \left( \frac{\mu^2}{2\chi_m} + \theta_{em} \right) S_z^2(\mathbf{r}) + \frac{B^2(\mathbf{r})}{8\pi} - \mu B(\mathbf{r}) n S_z(\mathbf{r}) - \frac{B(\mathbf{r}) H}{4\pi} \right] + \sum_{\mathbf{q}} \left[ \frac{1}{2} Q_s(\mathbf{q})\, \mathbf{A}_{\mathbf{q}} \mathbf{A}_{-\mathbf{q}} + \theta_{ex} \frac{\chi_n(q) - \chi_s(q)}{\chi_n(0)} S_{z,\mathbf{q}} S_{z,-\mathbf{q}} \right], \tag{3.8}$$

where $Q_s(\mathbf{q})$ is the electromagnetic kernel of the superconductor. In the homogeneous superconducting phase at $\lambda \gg \xi$ we can use $\chi_s(q) = 0$ and restrict the analysis to the local approximation for the kernel, $Q_s(\mathbf{q}) = (4\pi\lambda_L^2)^{-1}$, where $\lambda_L$ is the ordinary London penetration depth, $\lambda_L^{-2}(0) = 8\pi N(0) n v_F^2 e^2/3c^2$. For a single vortex filament running

along the axis at the point $x, y = 0$, the functional (3.8) takes on the following form (per unit length of the vortex) after a minimization with respect to $S_z(\mathbf{r})$:

$$F = \int d^2\mathbf{r} \left\{ p \left[ \frac{B^2(\mathbf{r})}{8\pi} + \frac{1}{8\pi p \lambda_L^2} [\mathbf{A}(\mathbf{r}) - \nabla\varphi(\mathbf{r})]^2 - \frac{B(r) H}{4\pi} \right] \right\},$$
$$p = 1 - \theta_{em} \left( \theta_{em} + \theta_{ex} + \frac{\mu^2}{2\gamma_m} \right)^{-1}, \quad (3.9)$$

where $\varphi(\mathbf{r})$ is the phase of the superconducting order parameter, which changes by $2\pi$ when one goes around the vortex filament.

A minimization of (3.9) under this condition yields an equation for $B(\mathbf{r})$. Its solution in the Fourier representation is

$$B_q = \frac{\Phi_0}{1 + pq^2\lambda_L^2}, \quad \Phi_0 = \frac{\pi\hbar c}{e}. \quad (3.10)$$

It follows from (3.10) that the magnetic field is quantized in the usual way, since the total flux is $B_{q=0} = \Phi_0$. It can also be seen from (3.9) that the effective field penetration depth $\lambda$ is reduced in proportion to $\sqrt{p}$ by the magnetization of the localized moments; i.e., $\lambda = \lambda_L \sqrt{p}$. The term in square brackets in (3.9) gives us the vortex energy per unit length after we substitute the solution (3.10). We see from (3.9) that this energy can be found from the standard expression[149] by replacing $\lambda_L$ by $\lambda$ and by multiplying by $p$. As a result we find the critical field for the apearance of vortices to be

$$H_{c1} = \frac{\Phi_0}{4\pi\lambda_L^2} \ln \frac{\lambda_L \sqrt{p}}{\xi}; \quad (3.11)$$

i.e., the factor $\sqrt{p}$ in the expression for $H_{c1}$ appears only in the logarithm, and the temperature dependence of $H_{c1}$ between $T_{c1}$ and $T_M$ turns out to be essentially the same as in an ordinary superconductor without localized moments.

Similar results on the change in the penetration depth and the value of $H_{c1}$ in ferromagnetic superconductors were derived by Sakai *et al.*[144] In Ref. 144, however,the calculation of the parameter $p$ ignored the superconducting screening of the exchange interaction, since the last term in the functional (3.8) was omitted. The expression for $p$ in Ref. 144 is thus a resultant expression, whose denominator lacks the term $\theta_{ex}$, and the limits $p \to 0, \lambda \to 0$, and $T \to \theta$ have been taken. Incorporating the term $\theta_{ex}$ leads to almost the standard temperature dependence of $\lambda$ in systems with $\theta_{ex} \approx \theta_{em}$. A change in the nature of the transition in a magnetic field near $\theta$ had been predicted earlier by Tachiki *et al.*[30,31] on the basis of a purely electromagnetic interaction of the electrons and localized moments. The reason for this change was identified as a change in the nature of the interaction of the vortex filaments due to a sharp decrease in $\lambda$ as $T \to \theta$ and a corrsponding decrease inthe Ginzburg-Landau parameter $\varkappa$ at $\theta_{ex} = 0$.

We wish to emphasize that the change in the type of transition near the point $\theta$ in ferromagnetic superconductors with a strong exchange interaction is not due to a change in the Ginzburg-Landau parameter $\varkappa = \lambda/\xi$. This parameter remains essentially constant as the temperature is lowered from $T_{c1}$ to $\theta$ in systems with $\theta_{ex} \approx \theta_{em}$. The change in the nature of the transition is a consequence of the decrease in $H_{c2}$ due to the paramagnetic effect of the exchange field, which is completely unrelated to the parameter $\varkappa$.

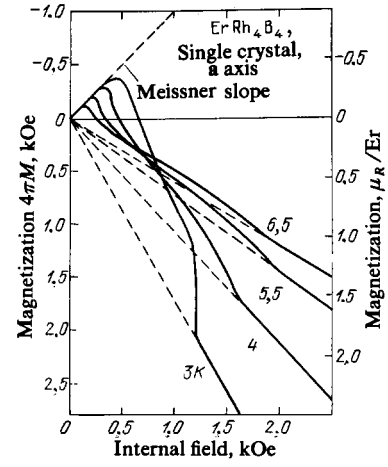


FIG. 8. Magnetization as a function of the internal field along the **a** axis in an $ErRh_4B_4$ single crystal at $T > 3$ K (Ref. 66).

### d) Experimental data on the critical fields in $ErRh_4B_4$

Figure 8 shows the results of measurements of the magnetization as a function of the internal field $H$ along the **a** axis carried out by Behroozi *et al.*[66] In an $ErRH_4B_4$ single crystal at various temperatures (the sample was spherical). In weak fields, up to $H_{c1}$, the Meissner regime prevails. When the field is raised above $H_{c1}$, vortices form in the sample, and the magnetization switches from diamagnetic to paramagnetic. In fields above the critical field the superconductivity disappears, and the susceptibility in the normal region above 3 K is described by the Curie-Weiss law. As the temperature is lowered, the field interval with the vortex phase (VS) contracts, and the jump in the magnetization at the transition to the normal state increases. Figure 9 shows the temperature dependence of the critical fields along the **a** axis and also the **c** axis.[47]

We first note that the field along the **c** axis is determined exclusively by the orbital effect, since the susceptibility of the localized moments along the **c** axis is very small. The experimental value $H_{c2}^{(c)}(T_M)/T_{c1}(dH_{c2}^{(c)}/dT) = 0.66$ agrees

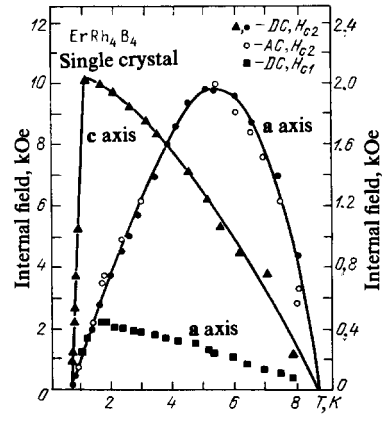


FIG. 9. Upper critical magnetic fields along the **a** and **c** axes in an $ErRh_4B_4$ single crystal as functions of the temperature.[47] Also shown here is the temperature dependence of $H_{c1}$ along the **a** axis; the dependence of $H_{c1}$ along the **c** axis is similar.

well with Gor'kov's theory, confirming the theoretical conclusion that the dependence of the exchange-scattering parameter $\tau_s^{-1}$ is weak in the interval from $\theta$ to $T_{c1}$. The behavior of $H_{c2}^{(a)}$ near $T_{c1}$ is also in agreement with the theoretical predictions for the orbital critical field $H_{c2}^{*}(T)$. Working from these data and taking into account the anisotropy of the correlation length, we find $\xi_0^{(a)} = 210$ Å, $\xi_0^{(c)} = 160$ Å, $v_F^{(a)} = 1.4 \cdot 10^7$ cm/s, and $v_F^{(c)} = 1.1 \cdot 10^7$ cm/s.

The data on $H_{c1}^{(a)}(T)$ reveal a standard temperature dependence $H_{c1}^{*}(T)$ up to the intersection with the curve of the upper critical field; i.e., this $ErRh_4B_4$ sample corresponds to the case in Fig. 7a or Fig. 7b. It follows from Fig. 8 that we have $\lambda_L^{(a)}(0) = 900$ Å, and this estimate yields a value for $v_F$ which is roughly the same as that calculated above from the data on $H_{c2}$ near $T_{c1}$. The agreement of these estimates shows that $ErRh_4B_4$ is a pure superconductor.

Let us examine the behavior of the upper critical field along the a axis in the region below $T_0 \leqslant 0.55 T_{c1} = 4.8$ K. Since the point $T = 4$ K lies near $T_0$, the experimental value of the critical field at this point is the same as $H_{c2}$ or at least close to it. Using expression (3.6), we find $h_0 = 40$ K, $\rho = 0.06$, $\theta_{ex} = 0.9$ K, and $\alpha(4\text{ K}) = 5.5$. Near this point we have the LOFF phase, since the value of $\alpha$ exceeds 1.8 (Ref. 79). The complete $H_{c2}(T)$ dependence is shown by the dashed line in Fig. 7a. The experimental points (the squares) lie above this line at $T \leqslant 3$ K and agree with it at $T > 3$ K. Since the transition to the superconducting state in a field is accompanied by a jump in the magnetization below 3 K, we may assume $H_c(T) > H_{c2}(T)$ below 3 K [we cannot use (3.5) here, since the moment depends on the field in a nonlinear way in this region]. Figure 7a is thus a phase diagram in the $H$, $T$ plane for a spherical $ErRh_4B_4$ sample. The continuity of the moment as a function of the applied field $H_a$ which is observed experimentally suggests an intermediate state below 3 K; this intermediate state has not yet been observed directly.

Figures 7b and 7c show suggested phase diagrams for $ErRh_4B_4$ for $N_z = 0$ and $N_z = 1$. It can be seen from Fig. 7c that a plate-shaped sample with an easy axis directed perpendicular to the plane of the plate makes it possible to observe the LOFF phase in the interval from 1 to 4 K and, in particular, to find the lines which separate the LOFF phase and the MS and VS phases.

In concluding this section we wish to point out that the experimental behavior of the upper critical magnetic field in $ErRh_4B_4$ clearly indicates a dominant role for the exchange mechanism in the disruption of the superconductivity under the influence of the localized moments near $T_M$. In this compound we have $H_{c2}^{*}(0) > B(0)$, and at $h_0 = 0$ as $H \to 0$ the transition S-FN cannot occur.

## 4. STRUCTURE OF THE COEXISTENCE PHASE IN FERROMAGNETIC SUPERCONDUCTORS

### a) General principles for calculating a functional for the coexistence phase

In the coexistence phase, the superconducting parameter for singlet pairing of electrons, $\Delta(\mathbf{r})$, and the average values of the moments at the sites are nonzero. To determine these parameters we have calculated the functional of the Gibbs free energy $F\{\Delta(\mathbf{r}), \mathbf{S}_i, \mathbf{A}(\mathbf{r})\}$, and we have found the order parameters by minimizing this functional with respect to $\Delta$, $\mathbf{S}_i$, and $\mathbf{A}$. We write the complete functional of the system as the sum of three functionals:

$$F\{\Delta(\mathbf{r}),\ \mathbf{S}_i,\ \mathbf{A}(\mathbf{r})\} = F_M\{\mathbf{S}_i,\ \mathbf{A}(\mathbf{r})\} + F_s\{\Delta(\mathbf{r})\} + F_{int}\{\Delta(\mathbf{r}),\ \mathbf{S}_i,\ \mathbf{A}(\mathbf{r})\}, \quad (4.1)$$

where $F_M$ is the functional of the magnetic subsystem in the normal state, $F_s$ is the functional of the superconducting subsystem in the absence of localized moments, and the positive definite functional $F_{int}$ describes the mutual effects of the superconducting and magnetic order due to the electromagnetic and exchange interactions.

We turn now to compounds with $\tau_s^{-1} \ll \Delta_0$. In this case the magnetic scattering is taken into account by renormalizing $\Delta_0$, and to describe the magnetic subsystem we use the simple self-consistent-field approximation, incorporating in $F_{int}$ the effect of the exchange field $\mathbf{h}(\mathbf{r})$ and of the magnetic induction $\mathbf{B}(\mathbf{r})$. The effect of the components of these fields with wave vectors of the order of $G$ on the superconductivity can be ignored in comparison with the effect of components with $q \ll G$. To describe the superconductivity we can thus use a continuum description of $\mathbf{S}(\mathbf{r})$ for the localized moments. In a system with a substantial anisotropy we may assume, in examining the effect of the localized moments on the superconductivity, that the moments are directed exclusively along the easy axis ($z$).

To find the superconducting part of the functional (4.1), we use the semiclassical Eilenberger equations.[81] We switch from the Gor'kov functions $G(\mathbf{r}, \mathbf{r}')$ and $F(\mathbf{r}, \mathbf{r}')$ to Eilenberger functions integrated over the energy available:

$$g(\mathbf{v},\ \mathbf{r}) = i \int \frac{d\xi}{2\pi} \int d\mathbf{r}' G_{--}\left(\mathbf{r} + \frac{\mathbf{r}'}{2},\ \mathbf{r} - \frac{\mathbf{r}'}{2}\right) e^{-i\mathbf{p}\mathbf{r}'},$$
$$f^{\sigma}(\mathbf{v},\ \mathbf{r}) = \sigma \int \frac{d\xi}{2\pi} \int d\mathbf{r}' F_{\sigma,-\sigma}\left(\mathbf{r} + \frac{\mathbf{r}'}{2},\ \mathbf{r} - \frac{\mathbf{r}'}{2}\right) e^{-i\mathbf{p}\mathbf{r}'}, \quad (4.2)$$

where the spin quantization axis is the $z$ axis, and the sign $\sigma = \pm$ specifies the spin direction. From Hamiltonian (2.1) we then find the equations

$$\left[\tilde{\omega} + ih(r) + i\frac{e}{c}\mathbf{v}\mathbf{A} + \frac{1}{2}\mathbf{v}\nabla\right] f^{-}(\mathbf{v},\ \mathbf{r}) = \tilde{\Delta}^{-}(\mathbf{r})\, g(\mathbf{v},\ \mathbf{r}), \quad (4.3a)$$

$$\left[\tilde{\omega} + ih(\mathbf{r}) + i\frac{e}{c}\mathbf{v}\mathbf{A} - \frac{1}{2}\mathbf{v}\nabla\right] f^{+}(\mathbf{v},\ \mathbf{r}) = \tilde{\Delta}^{+}(\mathbf{r})\, g(\mathbf{v},\ \mathbf{r}), \quad (4.3b)$$

$$g^2(\mathbf{v},\ \mathbf{r}) + f^{-}(\mathbf{v},\ \mathbf{r})\, f^{+}(\mathbf{v},\ \mathbf{r}) = 1, \quad (4.3c)$$

$$\tilde{\Delta}^{-}(\mathbf{r}) = \Delta(\mathbf{r}) + \frac{1}{2\tau}\bar{f}^{-}(\mathbf{r}), \quad \tilde{\Delta}^{+}(\mathbf{r}) = \Delta^{*}(\mathbf{r}) + \frac{1}{2\tau}\bar{f}^{+}(\mathbf{r}),$$
$$\tilde{\omega}(\mathbf{r}) = \omega + \frac{1}{2\tau}\bar{g}(\mathbf{r}), \quad \bar{f}^{\pm}(\mathbf{r}) = \int \frac{do}{4\pi} f^{\pm}(\mathbf{v},\ \mathbf{r}), \quad \bar{g}(\mathbf{r}) = \int \frac{o}{4\pi} g(\mathbf{v},\ \mathbf{r}), \quad (4.3d)$$

$$\Delta(\mathbf{r}) = \lambda \int_0^{\omega_D} d\omega\, \bar{f}^{-}(\mathbf{r}), \quad (4.3e)$$

where $\tau$ is the scale time for electron scattering by charged impurities, and the integration over the angle $o$ is an integra-

tion over the direction of the velocity v on the Fermi surface.

Equations (4.3) can be derived by minimizing the superconducting functional for the thermodynamic potential which was found by Eilenberger. This functional is

$$F\{f^-, f^+, g, \Delta\} = \int d^3\mathbf{r}\left[\frac{|\Delta(\mathbf{r})|^2}{\lambda} - 2\pi T\sum_{\omega}\int\frac{d\sigma}{4\pi} I\right], \tag{4.4}$$

$$I = \frac{1}{2}[f^-\Delta^*(\mathbf{r}) + f^+\Delta^-(\mathbf{r})] + g\left[\omega + ih(\mathbf{r}) + i\frac{e}{c}\mathbf{vA} + \frac{1}{4}\mathbf{v}\nabla\ln(f^+/f^-)\right] - |\omega| + \frac{1}{4\tau}(\overline{f^-}\overline{f^+} + \overline{g^2} - 1).$$

Using the solution of Eqs. (4.3), we can express $f^{\pm}$ and $g$ in terms of the order parameter $\Delta(\mathbf{r})$ and also the fields $h(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$. Substituting the resulting functions $f^{\pm}$ and $g$ into (4.4), we ultimately find the superconducitng functional $F_{\text{int}} + F_s$.

Equations (4.3) immediately yield an estimate of the relative roles of the exchange and electromagnetic mechanisms in the suppression of Cooper pairing of electrons. It can be seen from (4.3) that the effect of the vector potential depends on the direction of the electron's velocity, while the effect of the exchange field does not depend on the velocity, since this field affects only the spin variables. Consequently, $F_{\text{int}}$ is a functional of only the squares of these fields, i.e., of $\mathbf{A}^2$ and $h^2(\mathbf{r})$. This conclusion means that the relative roles of the exchange and electromagnetic mechanisms are determined by the parameter $\rho = F_{\text{int}}^{(EM)}/F_{\text{int}}^{(EX)} \approx e^2v_F^2A^2/c^2h^2$. For an order-of-magnitude estimate of $A$, we ignore superconducting screening. We can then write $\mathbf{B}(\mathbf{r}) = B(0)\mathbf{S}(\mathbf{r})$, and in the momentum representation we have $A_q(\mathbf{q}) = B(0)\times S_q/q$ and $h_q = h_0S_q$. For a magnetic structure with a scale wave vector $\mathbf{Q}$ we have $\rho \approx e^2v_F^2B^2(0)/c^2h_0^2Q^2 \approx 3\theta_{\text{em}}/\theta_{\text{ex}}(\lambda_L Q)^2$. Since the condition $\rho \ll 1$ holds in real compounds, we will ignore the electromagnetic contribution to $F_{\text{int}}$ below. The electromagnetic contribution to the magnetic functional $F_M$, in contrast, must be taken into account. Here it determines the Curie temperature and makes any magnetic structure other than a transverse structure unfavorable from the energy standpoint.

### b) Coexistence phase near the point $T_M$ and critical fluctuations

It is a simple matter to calculate the total functional near the point at which the magnetic structure appears, where we have $t = (\theta - T)/\theta \ll 1$. Here the quantities $S_q$ are small, and the functional $F_{\text{int}}$ can be found by perturbation theory, while expression (2.4) can be used for $F_M$. For harmonics with wave vectors $\mathbf{q}$ in the region $l \gg q^{-1}, \xi_0 \gg q^{-1}$ the total functional, quadratic in the magnetization $S_q$, is

$$F\{S_{zq}\} = -\frac{1}{2}N(0)\Delta_0^2 + F_M + F_{\text{int}}, \tag{4.5a}$$

$$F_M = \sum_q S_{zq}S_{z,-q}\left[\theta_{ex}(a^2q^2 + \alpha t) + \frac{1}{2}\theta_{em}\frac{q_z^2}{q^2}\right],$$

$$\alpha = \frac{\theta}{2\theta_{ex}}, \tag{4.5b}$$

$$F_{\text{int}} = \sum_q S_{zq}S_{z,-q}\theta_{ex}\frac{\chi_n(q) - \chi_s(q)}{\chi_n(0)} = \sum_q S_{zq}S_{z,-q}\frac{\pi\Delta_0}{2v_Fq}\theta_{ex}. \tag{4.5c}$$

From (4.5) we see that at $T_M$ a sinusoidal magnetization $S_z(\mathbf{r}) \sim \sin(\mathbf{Q}_M\cdot\mathbf{r})$ arises, where $Q_M = (\pi^2\Delta_0/4a^2v_F)^{1/3}$ (a result found by Anderson and Suhl), and the temperature $T_M$ differs from $\theta$ by a small quantity of the order of $a^2Q_M^2\theta$. It follows from (4.5) that the direction of $\mathbf{Q}$ must satisfy the condition that the magnetic structure is transverse; i.e., $Q_{Mz} = 0$. The direction of $\mathbf{Q}_M$ in the $x, y$ plane is determined by the anisotropy of the parameters $a^2$ and $v_F$, and it cannot be predicted for either $HoMo_6S_8$ or $ErRh_4B_4$, since the anisotropy of the magnetic rigidity is not known. In the self-consistent field approach the phase transition in the inhomogeneous magnetic state at the point $T_M$ is a second-order transition in dirty superconductors and also in clean superconductors with a sufficiently small value of $h_0$: $h_0 \lesssim \Delta_0(\xi_0/a)^{1/3}$ (Ref. 24). In principle, the nature of the transition to the inhomogeneous state may be changed by critical magnetic fluctuations if the direction of $\mathbf{Q}$ is not set by the anisotropy. This situation was analyzed in detail by Brazovskiĭ and Dzyaloshinskiĭ[86,87] and, in application to isotropic ferromagnetic superconductors, by Schuh and Grewe[88] and Kleinert.[89] For real anisotropic compounds, however, the direction of $\mathbf{Q}$ is fixed in the ordered phase, and critical fluctuations cannot change the type of transition. Furthermore, as we have already mentioned, the dipole–dipole interaction makes these fluctuations small, and they can be ignored everywhere except in a very small neighborhood of the point $T_M$. It can be seen from (4.5b) that at $t \gg a^2Q_M^2$ the fluctuations are purely ferrromagnetic. The fluctuations become strong only at $t \lesssim t_G \approx a^2Q_M^2$, and here an inhomogeneous magnetic order forms. The Ginzburg–Levanyuk parameter, which determines the region of strong fluctuations, is very small, $t_G \approx (a/\xi_0)^{2/3}$ in the clean case and $t_G \approx a/(\xi_0 l)^{1/2}$ in a very dirty superconductor.

As the temperature is lowered below $T_M$, the quantities $S_q$ increase. The approximations which we have used for $F_M$ and $F_{\text{int}}$ break down when the conditions $a^2q^2 \gg S^2 \sim t$ and $h_0S \ll \Delta_0$, respectively, are violated.

In the temperature interval $\Delta_0^2/h_0^2 \gg t \gg a^2Q^2$ ($Q$ is a scale wave vector of the magnetic structure) we can again use approximation (4.5c) for $F_{\text{int}}$, but terms of higher order in $S$ must be incorporated in the functional $F_M$. Because of these terms, a change in the modulus of $\mathbf{S}_i$ is unfavorable from the energy standpoint, and as the temperature is lowered the sinusoidal solution should convert into a domain structure. The problem which we are discussing here is completely analogous to that of calculating the domain structure in a normal ferromagnetic plate with a moment directed perpendicular to the plane of the plate. The function $F_{\text{int}}$ corresponds to the energy of the scattering magnetic field if we replace $\pi^2\Delta_0\theta_{ex}/2v_F = \pi\theta_{ex}/2\xi_0$ by $\theta_{em}/L$, where $L$ is the plate thickness.[90] In either case, the optimum structure would be a transverse one-dimensional magnetic structure, since a further breakup of the structure would lower $F_{\text{int}}$ by only an insignificant amount, while the energy of the domain

walls would increase in proportion to their area. For a two-dimensional checkerboard structure, for example, with a domain thickness $d$ we would have $F_{int}^{(2d)}/F_{int}^{(1d)} = 0.62$, while the surface energy of the walls would double.

To determine the parameters of the magnetic structure in the coexistence phase (the DS phase) we can replace $F_M$ in (4.5b) by the functional for the transverse one-dimensional domain structure with a period $d = \pi/Q$ and a magnetization $S$ inside the domains:

$$F_M(S, Q, T) = F_0(S, T) - \theta_0 S^2 + \frac{\eta(S, T)Q}{\pi n}, \qquad (4.6)$$

where $\eta(S, T)$ is the surface energy density of a domain wall.[84] For compounds with a pronounced anisotropy a reversal of the $S_i$ can occur only as a result of the vanishing of the $S_i$ inside a wall (rotation of a moment is unfavorable from the energy standpoint), and a linear domain wall forms[85]:

$$\begin{aligned} &S_z(x) = S\,\mathrm{th}\,(x\sqrt{t}/a), \quad S_x = S_y = 0, \\ &\eta(S, T) = n\theta_{ex}S^2\tilde{a}(T), \quad \tilde{a}(T) = \frac{2\sqrt{2}}{3}a\sqrt{t}, \end{aligned} \qquad (4.7)$$

where $a\sqrt{t}$ is the wall thickness. Using (4.6) and (4.7) for $F_M$ along with $F_{int}$, we can determine the equilibrium parameters $S$ and $\mathbf{Q}$ of the magnetic structure from (4.5) as a function of the temperature, minimizing $F_M + F_{int}$ (Ref. 77). For the domain structure we have

$$S_z(\mathbf{r}) = \frac{4S}{\pi}\sum_{k=0}^{\infty}\frac{\sin(2k+1)\mathbf{Qr}}{2k+1} \qquad (4.8)$$

substituting the corresponding harmonics $S_q$ in $F_{int}$, we find $Q(T) = [4.2/\xi_0\tilde{a}(T)]^{1/2} \sim t^{1/4}$. The inhomogeneous magnetic structure assumes a domain nature of $Q^{-1} \gg a/\sqrt{t}$, and the temperature interval in which it forms is determined by the same small quantity as the one which characterizes the region of strong fluctuations, $t_G \approx (a/\xi_0)^{2/3}$. In practice, the region of the sinusoidal solution in the coexistence phase is negligibly small. The direction of $\mathbf{Q}$ in the $x, y$ plane depends on the anisotropy of the surface energy $\eta$ and also $v_F$. At this point we do not have enough information to predict this direction.

At which values of the anisotropy parameter $D$ does the spiral structure transform into a domain structure? At $D \ll \theta$, the moment S changes direction in a domain wall but remains the same in magnitude. This rotating solution with a wall width of the order of $a(\theta/D)^{1/2}$ and a surface energy $(\theta D)^{1/2}S^2$ converts continuously into a spiral structure as $D \to 0$. The domain structure corresponds to the case in which the wall thickness is small in comparison with the domain thickness $Q^{-1}$; this condition holds at $D/\theta \gg (q/\xi_0) \gtrsim 10^{-2}$. A very slight anisotropy is thus sufficient for the formation of a domain structure.

Upon further cooling the exchange field increases, and at $t \gtrsim \Delta_0^2/h_0^2$ the magnetic order begins to cause a significant suppression of Cooper pairing. The strong exchange field must now be taken into account in the calculation of the superconducting part of the functional $F_s + F_{int}$. It becomes a simpler matter to solve Eilenberger's equations (4.3) in this case because this field must vary rapidly over space in the coexistence phase. For typical wave vectors of the inhomogeneous structure, $Q \gg \xi_0^{-1}$, the superconducting order parameter is approximately homogeneous over space, and an analytic solution for the superconducting subsystem can be found by a perturbation theory in the small parameter $(Q\xi_0)^{-1}$ even in the region of a strong exchange field.

### c) Coexistence phase with a strong exchange field

In the case of a dirty superconductor with $Q\xi_0 \gg 1$ and $(h_q\tau)^2 \ll 1$ we are dealing with a diffusive regime of the motion of the Cooper pairs, since these inequalities guarantee an effective averaging of the exchange field over a distance $\xi_0$ and a conversion of the electron motion to an isotropic motion in the presence of the anisotropic inhomogeneous magnetic structure (because of scattering by impurities[32]). The spatial and angular dependence of the functions $g, f^{\pm}$, and $\Delta$ is weak, and the corresponding variable increments can be calculated by a perturbation theory in the small parameters $(q\xi_0)^{-1}$ and $(h\tau)$, for an arbitrary value of $h/\Delta$. Equations (4.3) are solved in this case by expanding the functions $g(\mathbf{v}, \mathbf{r})$, $f^{\pm}(\mathbf{v}, \mathbf{r})$, and $h(\mathbf{r})$ in a Fourier series in the coordinate $\mathbf{r}$; the harmonics with wave vectors $\mathbf{q} = k\mathbf{Q} \neq 0$ are small, proportional to the parameters $(q\xi_0)^{-1}$ and $h\tau$. As a result we find the functionals

$$\left.\begin{aligned} &F_s(\Delta) = -\frac{1}{2}N(0)\Delta^2 \ln\frac{e\Delta_0^2}{\Lambda^2}, \quad \tau_m^{-1} = \sum_q \frac{\pi h_q h_{-q}}{v_F q} L_1(ql), \\ &F_{int}(\Delta, \tau_m) = N(0)\left(\frac{\pi\Delta}{2\tau_m} - \frac{1}{3\tau_m^2}\right), \quad \tau_m\Delta \gtrsim 1, \\ &L_1(y) = \frac{2y\,\mathrm{arctg}\,y}{\pi(y - \mathrm{arctg}\,y)}, \end{aligned}\right\} \qquad (4.9)$$

where $\Delta$ is the superconducting order parameter averaged over the volume. We calculated an expression for $F_{int}$ in the case $\tau_m\Delta \gtrsim 1$ since only this case occurs in the coexistence phase.

It can be seen from the resulting expression that the effect of the exchange field with harmonics $q \gg \xi_0^{-1}$ on the superconductivity in a rather dirty crystal is described by the parameter $\tau_m^{-1}$. Comparison of expression (4.9) with the corresponding expression for a superconductor with magnetic impurities[3,82] shows that the effect of the exchange field on the average of the superconducting order parameter over the volume is equivalent to the effect of magnetic impurities, for which $\tau_m$ is the scale time for the exchange scattering of an electron with spin flips. The condition $\tau_m\Delta \gtrsim 1$ corresponds to the gap regime of superconductivity with magnetic impurities. Under the condition $h_q \ll \Delta$ the functional $F_{int}$ in (4.9) converts into the functional (4.5c), which we used previously. We now know the complete functional $F$ as a function of the parameters $\Delta$ and $S(\mathbf{r})$. The one-dimensional domain structure is again optimal since the parameter $(\Delta\tau_m)^{-1}$ is quite small (no greater than 0.68) in the region with the DS phase, and the dependence of $F_{int}$ on $S(\mathbf{r})$ differs only inconsequentially from the corresponding dependence in (4.5c). For a transverse one-dimensional domain structure we finally find the function $F(\Delta, \mathbf{Q}, s)$, whose minimization determines the corresponding equlibrium parameters of the DS phase.

Considering a clean superconductor,[34] we assume

$\Delta\tau \gg 1$ and $q\xi_0 \gg 1$. We find $F_{\text{int}}$ for a one-dimensional domain structure in the region of strong exchange fields,[34] $h \gg \Delta$. In the calculation we also make use of the small parameter $h/v_F Q$, and we again see that the conditions $\Delta$, $h \ll v_F Q$ are satisfied throughout the region of the DS phase. In the solution of Eqs. (4.3) with $\tau^{-1} = 0$ the functions $g$ and $\Delta$ can be assumed independent of the coordinate in a first approximation. The function $f^-(\mathbf{v}, \mathbf{r})$ can then be found easily, and we obtain a self-consistency equation for the average value of $\Delta$ over a domain:

$$\Delta \ln \frac{\Delta}{\Delta_0} = \int_0^\infty d\omega \int \frac{d\sigma}{4\pi} \left( \bar{f}^- - \frac{\Delta}{\sqrt{\Delta^2+\omega^2}} \right), \qquad (4.10a)$$

$$\bar{f}^- = \left[ \frac{e^{(\omega - ih)t_1}}{\omega - ih} - \frac{e^{-(\omega+ih)t_1}}{\omega + ih} \right] \frac{\alpha + i\beta}{\sqrt{1+\alpha^2+\beta^2}}, \qquad (4.10b)$$

$$\alpha + i\beta = (e^{2\omega t_1} - 1)^{-1} \int_0^{t_1} dt \exp\left\{ \int_0^t [\omega + ih(t')]\, dt' \right\}, \qquad (4.10c)$$

$$t = \frac{2x}{v_x}, \qquad t_1 = \frac{2d}{v_x},$$

where $\bar{f}$ is the average value of $f^-(\mathbf{r})$ over a domain, and we have $h(x) = +h$ or $-h$ inside the domains; the $x$ axis runs along the direction of $\mathbf{Q}$. It remains to integrate over the angles and frequencies on the right side of (4.10a). The expression in square brackets is nonzero in a narrow angular interval $\cos\theta \leqslant \gamma$, where $\gamma = \pi h / v_F Q$. In this angular interval, the electrons move in a strong exchange field, $h \gg \Delta$. For other directions the exchange field oscillates rapidly along the electron trajectory, the average exchange field is zero over the correlation length, and the function $\bar{f}$ here is the same as the function $f$ for an ordinary superconductor with $h = 0$, i.e., with $\Delta(\omega^2 + \Delta^2)^{-1/2}$.

In the angular interval $\cos\theta \lesssim \gamma$ the integral over $\omega$ diverges logarithmically at frequencies $\omega \ll h$ as $\Delta \to 0$. We accordingly partition the range of the integration over $\omega$ into the two intervals $(0, c)$ and $(c, +\infty)$, where $\Delta \ll c \ll h$. In the first of these intervals, the expression for $\bar{f}$ can be simplified by virtue of the condition $\omega \ll h$. Here

$$\bar{f} = \frac{\Delta K^2(\mu)}{\sqrt{\omega^2 + \Delta^2 K^2(\mu)}}, \qquad g = \frac{\omega}{\sqrt{\omega^2 + \Delta^2 K^2(\mu)}},$$

$$K(\mu) = \frac{v_F \mu}{hd} \sin \frac{hd}{v_F \mu}, \qquad (4.11)$$

where $\mu = \cos\theta$. In the region $(c, +\infty)$ the quantity $(1 + \alpha^2 + \beta^2)^{1/2}$ can be replaced by unity. Integrating over $\omega$, we find a self-consistency equation for $\Delta$, which can be used to reconstruct the functional $F_{int}$ in the following form:

$$F_{\text{int}} = N(0) \frac{\pi^2 h \Delta^2}{3 v_F Q} \ln \frac{Ch\sqrt{e}}{\Delta},$$

$$C = 0.88, \quad e = 2.718, \quad h \gg \Delta. \qquad (4.12)$$

The inhomogeneous part of $\Delta(x)$ can be treated by perturbation theory, with $\gamma$ as the small parameter. The corrsponding contribution to $F_{\text{int}}$ contains the small parameter $\gamma^2$. Figure 10 shows the functional dependence $\Delta(x)$ in the DS phase. The superconducting order parameter reaches a maximum at the domain walls, since near the walls the electrons move

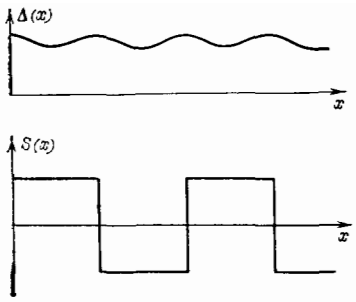


FIG. 10. The superconducting order parameter and the magnetization in the DS phase as functions of the coordinate $x$ (along the direction of $\mathbf{Q}$).

in an exchange field of varying direction. The modulation of $\Delta(x)$ in a clean superconductor is small, proportional to the parameter $\gamma$, while that in a dirty superconductor is proportional to the parameter $(h\tau)$. The intermediate-frequency case was studied in Ref. 83. It is not difficult to see that again in the case of a clean superconductor the conclusion that the one-dimensional structure is optimal remains in force, since $F_{\text{int}}$ again changes only insignificantly when we go to more complex structures.[34]

Inhomogeneous magnetic structures with $Q \gg \xi_0^{-1}$ were discussed above. It is not difficult to see that the coexistence phases with slowly varying magnetic structure (with $Q \lesssim \xi_0^{-1}$) have an energy higher than that of the DS phase if $\theta_{ex} > \theta_{em}(a/\xi_0)$. In these "slow" structures the mutual effects of the superconductivity and the magnetic order cause an energy increase by an amount $\approx \theta_{ex} S^2$ (because of the screening of the long-range part of the exchange interaction), while in the DS phase the increase is $F_{\text{int}} \approx \theta_{ex} S^2 (Qa)$ and $Qa \approx (a\theta_{ex}/\xi_0 \theta_{em})^{1/2}$. The Meissner (MS) and vortex (VS) coexistence phases are thus impossible in real ferromagnetic compounds.

Let us examine the applicability of the semiclassical Eilenberger equation for describing superconductivity in the presence of an inhomogeneous magnetic structure. These equations apply if the exchange or magnetic field varies slowly over the electron wavelength, i.e., if $Q^2/m \ll (\mathbf{v}\cdot\mathbf{Q})$. Accordingly, when applied to a domain structure these equations give an accurate description of only the angular interval $\cos\theta \gg Q/k_F$. In the case of a dirty superconductor or weak fields, we find a description accuracy of order $Q/k_F \approx \sqrt{a/\xi_0}$, since here all the angles are identically important in the derivation of self-consistency equations (4.3e). In clean superconductors with $h \gg \Delta$ the accuracy of the description is $Q/\gamma k_F \approx v_F^2 Q^2/h\varepsilon_F \approx (\Delta/h)^2$, since here the suppression of the superconducting order parameter comes primarily from angles of the order of $\gamma$.

Let us find the equilibrium values of the parameters $S(T)$, $\Delta(T)$, and $Q(T)$ in strong exchange fields, using the functional (4.6) for $F_M$ and taking $F_s$ and $F_{\text{int}}$ from (4.9) and (4.12).

The quantity $S(T)$ is determined primarily by the magnetic functional $F_M$, since the decrease in it, $\delta S$, caused by the superconductivity far from the point $T_M$ is small, proportional to $F_{\text{int}}/\theta \approx Qa$. For dirty superconductors with $\sqrt{a\xi_0} \ll l \ll v_F/h_0$ the functional dependence $S(T)$ is analogous

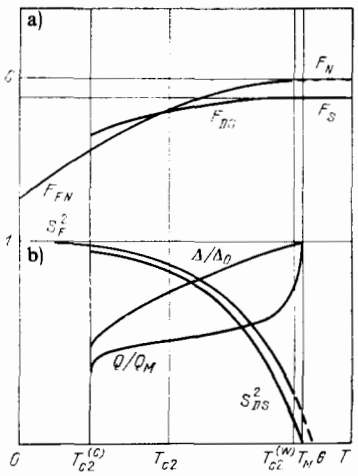


FIG. 11. a—Temperature dependence of the free energy, $F(T)$, of the N, S, DS, and FN phases; b—temperature dependence of the parameters $S^2(T)$, $\Delta(T)$, and $Q(T)$ of the DS and FN phases (schematic). A standard ferromagnetic behavior with a second-order magnetic transition in the absence of superconductivity is assumed.

to that in the absence of superconductivity, but the critical temperature is shifted by an amount $\delta\theta = -1.4\,(\xi_0 Q\alpha)^{-1}$, i.e., $\delta S \approx S\sqrt{a/\xi_0}$. In a clean superconductor the change in $S$ caused by the superconductivity is even smaller.

The increase in the exchange field $h = h_0 S(T)$ upon cooling leads to decreases in $\Delta(T)$ and $Q(T)$. The behavior of $S(T)$, $\Delta(T)$, and $Q(T)$ is sketched in Fig. 11.

At the point $T_{c2}$, of the first-order DS-FN transition, we have

$$Q_{c2} = \sqrt{\frac{3{,}1}{\xi_0 \tilde{a}(T_{c2})}},$$

$$\Delta(T_{c2}) = 0.83\Delta_0, \quad \sqrt{\tilde{a}\xi_0} \ll l \ll \frac{v_F}{h_0}, \tag{4.13a}$$

$$Q_{c2} = \sqrt{\frac{1{,}17}{\xi_0 \tilde{a}(T_{c2})}}\sqrt{\frac{\Delta_0}{h_0 S(T_{c2})}\ln\frac{Ch_0 S(T_{c2})\sqrt{e}}{\Delta_0}},$$

$$\Delta(T_{c2}) = 0.61\Delta_0, \quad l \gg \xi_0. \tag{4.13b}$$

The expressions which we have derived for $\delta S$ and $Q$ show that the relative deformation of the ferromagnetic structure caused by the superconductivity reduces to a change in the wave vector of the structure from zero to $Q \approx \sqrt{a/\xi_0}G$ and to a change in the magnetization by an amount $\delta S \approx \sqrt{a/\xi_0}S$. The effect of the superconductivity on the magnetic order can thus be characterized by the small parameter $\sqrt{a/\xi_0}$. On the other hand, the magnetic structure changes the superconducting order parameter $\Delta(T)$ more substantially, reducing it by a factor of about two at temperatures near $T_{c2}$. In contrast with the magnetic order, however, the superconducting order parameter remains essentially homogeneous over space. This striking qualitative difference betweeen the behavior of magnetic order and that of superconducting order in a coexistence phase stems from the difference in their correlation lengths ($a \ll \xi_0$).

### d) Transition from the coexistence phase to the normal ferromagnetic phase

The DS-FN first-order transition occurs at the point $T_{c2}$, where the free energies of these phases become equal. Figure 11 sketches the temperature dependence of the free energies of the S, DS, and FN phases. At the point $T_{c2}$ the following relations hold:

$$\frac{S_{c2}^2}{Q_{c2}} = \frac{0.077\Delta_0 v_F}{h_0^2}, \quad \sqrt{a\xi_0} \ll l \ll \frac{v_F}{h_0}, \tag{4.14a}$$

$$\frac{S_{c2}}{Q_{c2}}\ln\frac{Ch_0 S_{c2}\sqrt{l}}{\Delta_0} = \frac{3v_F}{2\pi^2 h_0}, \quad l \gg \xi_0. \tag{4.14b}$$

We see from (4.14) that $h_{c2} \gg \Delta_0$, i.e., that the transition to the FN phase occurs in the region of the strong exchange field, and the regime of a strong field is characteristic of essentially the entire region of the DS phase, except for a small neighborhood of the point $T_M$. The jump in $S$ at the point $T_{c2}$, from $S$ to $S + \delta S$, upon cooling leads to the release of a latent heat of the order of the superconducting condensation energy. Using (4.14) and the expressions for $Q_{c2}$, we find from the inequality $S \leqslant 1$ the condition $h < h_{0c}$; under this condition, the DS phase remains stable down to absolute zero. The critical value is $h_{0c} = 0.44\Delta_0[\xi_0/\tilde{a}(0)]^{1/3}$ for $l \gg \xi_0$, and it decreases with decreasing $l$. Except in the case of a very dirty superconductor, the condition $h_{0c} > \Delta_0$ holds.

The supercooling temperature of the DS phase, $T_{c2}^{(c)}$, is determined from the condition that the functional not have a minimum with respect to $\Delta$. In a dirty superconductor ($v_F/h_0 \gg l \gg \sqrt{\tilde{a}\xi_0}$) we find $S_{c2}^{(c)} = 1.26S_{c2}$, while in a clean crystal we find $S_{c2}^{(c)} = 1.35S_{c2}$. The region in which the supercooling of the DS phase occurs is therefore quite large; it may span the entire temperature interval below $T_{c2}$.

In the region $T_{c2}^{(c)} < T < T_{c2}$, an activation energy is required for the appearance of a critical nucleating region for the DS-FN transition. This energy is large in comparision with the temperature, since the minimum size of the critical nucleating region (a normal region with ferromagnetic order without domain walls) must exceed $v_F/h$. Below $T_{c2}^{(c)}$, the superconductivity vanishes without an activation energy, but here again the complete transition to the equilibrium FN phase may occur slowly, since it reduces to the disappearance of a large number of nonequilibrium domain walls, and the pinning of these walls at inhomogeneities of the crystal delays the transition.

The superheating temperature of the FN phase, $T_{c2}^{(W)}$, is determined by the condition for the appearance of an infinitesimally small superconducting nucleating region in the ferromagnetic phase. In a sample with domains,the minimum temperature is the temperature at which a nucleating region appears near a domain wall[103–105,147]; the corresponding temperature was found in Ref. 129. For compounds with $h_0 \gg \Delta_0$ the point $T_{c2}^{(W)}$ essentially coincides with $T_M$. An estimate of the formation energy of a critical nucleating region of the DS phase in the FN phase at temperatures below $T_{c2}^{(W)}$ yields a very large value, of the order of $\varepsilon_F^2/\Delta_0 \gg T_M$, and it is essentially impossible for the DS phase to appear during heating from the FN phase anywhere up to the point $T_{c2}^{(W)}$.

In compounds with $h_0 \gg \Delta_0$ the DS phase thus arises from the $S$ phase upon cooling below the point $T_M$ and dis-

appears at the point of the first-order transition $T_{c2}$, being replaced by the FN phase. During heating the FN phase persists essentially to the point $T_{c2}^{(W)} \approx T_M$, and only here does it convert into the DS phase. Consequently, the behavior of the system during heating from the FN phase in compounds with $h_0 \gg \Delta_0$ gives us information about the properties of the magnetic system in the absence of superconductivity.

#### e) Superconducting properties of the coexistence phase

In a dirty superconductor with $(h\tau)^2 \ll 1$ the energy gap in the spectrum of one-particle excitations is present throughout the region in which the DS phase exists. In this case the effect of the magnetic ordering reduces to one of simply reducing the superconducting order parameter $\Delta(T)$ and causing an additional decrease in the gap $\varepsilon_g$ in the quasiparticle spectrum, as in a superconductor with magnetic impurities.[3,82] This suppression of the gap by the magnetic order increases with cooling and is determined by

$$\varepsilon_g = \Delta(T)\{1 - [\Delta(T)\tau_m]^{-2/3}\}^{3/2}, \tag{4.15}$$

where the parameter $\tau_m$ is given by (4.9). At the point $T_{c2}$ we find $\varepsilon_g = 0.49\Delta(T) \approx 0.41\Delta_0$; at the supercooling point for the DS phase, $T_{c2}^{(c)}$, we have $\varepsilon_g = 0.11\Delta(T) \approx 0.064\Delta_0$.

In a clean superconductor, in the region with $h_0 S(T) \ll \Delta(T)$ (near $T_M$), there is also a gap in the quasiparticle spectrum, with a size of about $\Delta_0$. In the region $h_0 S(T) \gg \Delta(T)$ the superconductivity in the DS phase becomes gapless (as in the case of a spiral ordering of the moments), since the electrons moving perpendicular to **Q** sense a strong exchange field, constant over space, which disrupts their pairing. We turn now to the quasiparticle state density $\rho(E)$ in the gapless regime in the pure DS phase.[34]

The function $\rho(E)$ with $E \ll \Delta(T)$ is determined from the function $g(\omega)$ given by expression (4.11) by replacing $\omega$ by $iE$, integrating over the directions of **v**, and singling out the real part. From (4.11) we see that the quantity $\Delta(T) \times K(\cos\theta)$ is the gap for electrons which are moving at an angle $\theta$ from the **Q** axis. This gap does not exist for the infinite number of directions of **v** defined by the condition $\cos\theta = 0$ and by the condition for Bragg reflection of the Cooper pairs from the periodic structure of the exchange field. In the exchange field $h$ the Cooper pairs have a resultant momentum $k_x = 2h/v_F$, and Bragg reflection occurs at $k_x = 2mQ$, where $m$ is an integer. Because of this reflection, the pairs move in the same domain as the pairs with $\theta = \pi/2$. The functions $\bar{f}^{\pm}$ are zero for these directions, and the superconductivity is gapless.

At energies $E \ll \Delta(T)$ the quasiparticle density is determined by the gapless bands on the Fermi surface, and a summation of the contributions of these bands yields

$$\begin{aligned} \rho(E) &= 0, \quad h_0 S(T) \ll \Delta(T), \\ \rho(E) &= \gamma N(0) \frac{E}{\Delta} \ln \frac{4\Delta(T)}{\pi E}, \quad h_0 S(T) \gg \Delta(T). \end{aligned} \tag{4.16}$$

In the pure DS phase the state density $\rho(E)$ is finite for all $E$; it reaches a maximum at $E \approx \Delta(T)$ and $\rho(\Delta(T)) = \sqrt{3} N(0)/2\gamma$. Upon cooling below $T_M$, the transition to the gapless regime occurs at temperatures for which the conditons $S(T) \approx \Delta_0/h_0$ holds. Figure 12 shows the schematic behavior of $\rho(E)$ for temperatures above and below $T_M$.

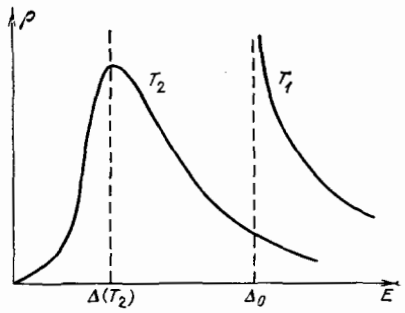


FIG. 12. Energy dependence of the quasiparticle state density in a clean ferromagnetic superconductor at different temperatures $T_1$ and $T_2$. The temperature $T_1$ lies in the interval $T_M < T < T_{c1}$, while $T_2$ lies in the interval $T_{c2} \lesssim T_2 < T_M$. The $\rho(E)$ behavior for $T_2$ in the DS phase corresponds to the gapless regime.

#### f) Role of spin-orbit scattering and $T, \theta_{ex}$ phase diagram

Up to this point we have ignored the spin-orbit scattering of electrons. This scattering mixes the singlet and triplet states of a Cooper pair, and the contribution of the triplet state causes the paramagnetic susceptibility of the electrons at a zero wave vector to be nonzero at $T = 0$. If there is a sufficiently intense spin-orbit scattering with a time $\tau_{so} \ll \Delta_0^{-1}$, the low-temperature susceptibility $\chi_s(0)$ approaches the normal susceptibility $\chi_n$ (Refs. 5 and 111), i.e.,

$$\frac{\chi_s(0)}{\chi_n} = 1 - \frac{\pi}{4}\tau_{so}\Delta_0, \quad \tau_{so}\Delta_0 \ll 1. \tag{4.17}$$

At very small values of $\tau_{so}$ the peak on the $\chi_s(\mathbf{q})$ curve at $q \neq 0$ may diappear; in this case, ferromagnetic ordering will occur in the model without an electromagnetic interaction below the point $T_M$. In this situation, the properties of the coexistence phase are actually determined exclusively by the electromagnetic mechanism. Let us estimate the value of $\tau_{so}$ above which the energy of the inhomogeneous DS coexistence phase is lower than that of the ferromagnetic superconducting phase. Clearly, the critical value of $\tau_{so}$ is reached only in the limit of a dirty superconductor. In this case, $F_{int}$ is determined by the difference $\chi_s(\mathbf{q}) - \chi_n(\mathbf{q})$, so that the dependence of $\chi_s(\mathbf{q})$ on $\tau_{so}$ is sufficient for the solution of this problem.

The quantity $\chi_s(\mathbf{q})$ has been calculated by Kaufman and Entin-Wohlman[119] for the case with spin-orbit scattering. This scattering does not affect $\chi_s(\mathbf{q})$ if $q^2 \gg (l l_{so})^{-1}$, where $l_{so} = v_F \tau_{so}$. Accordingly, for $Q^2 \gg (l_{so} l)^{-1}$ we can use the theory of the DS phase outlined above. In the very dirty limit for the DS phase we have $Q \approx (a\xi_0 l)^{-1/3}$, and our analysis holds for $l_{so}/l \gg (a\xi_0/l^2)^{2/3}$. The spin-orbit scattering is a relativistic effect, and $\tau_{so}$ contains a small parameter in addition to the ordinary-scattering time $\tau$. Consequently, the ratio $l_{so}/l$ must be at least $10^2$, and the spin-orbit scattering becomes important as $l$ approaches $a$.

We mentioned above that incorporating exchange scattering under the condition $\tau_s \Delta_0 \gg 1$ reduces to a renormalization of $\Delta_0$. We turn now to systems in which the conditions $\tau_s^{-1} \sim T_M \sim T_{c1}$ hold, so that we will be able to construct a

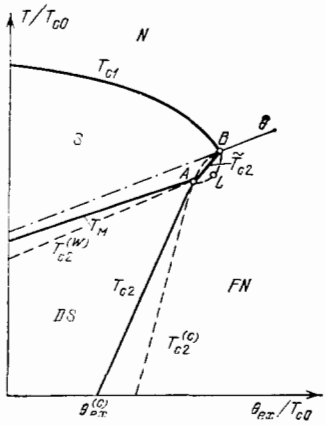


FIG. 13. Phase diagram of a ferromagnetic superconductor in the $T/T_{c0}$, $\theta_{ex}/T_{c0}$ plane (schematic). The solid lines separate the N, S, DS, and FN phases. The lines $T_{c1}$ and $T_M$ (at small values of $\theta_{ex}/T_{c0}$) represent second-order transitions, while the other lines represent first-order transitions. The dot-dashed line is the line of $\theta$; the dashed lines are supercooling and superheating lines for first-order transitions.

complete phase diagram of ferromagnetic superconductors in the $(T/T_{c0}, \theta_{ex}/T_{c0})$ plane.

In the region $(\theta_{ex}/T_{c0}) \ll 1$ (but with $\theta_{ex} \gg \theta\sqrt{a\xi_0}\lambda_L^{-1}$ the point at which the inhomogeneous state appears, $T_M$, lies above the point of the S-FN (first-order) phase transition, $\tilde{T}_{c2}$, and as the temperature is lowered we observe the succession of phase transitions N → S → DS → FN at $T_{c1}$, $T_M$ (a second-order transition), and $\tilde{T}_{c2}$ (first order). The last of these transitions may not occur if the ratio $\theta_{ex}/T_{c0}$ is quite small (Fig. 13). With increasing $\theta_{ex}/T_{c0}$ the line $\tilde{T}_{c2}$ approaches the line $T_M$, since the parameter $\Delta$ decreases at the point of the magnetic transition because of the increase in magnetic scattering and the thermal suppression of Cooper pairing. As a result, near the critical point the line of the first-order transition $\tilde{T}_{c2}$ should pass above the line $T_M$, and here we will observe the succession of phases N → S → FN upon cooling, without a coexistence phase.

The lines $T_M(\theta_{ex})$ and $\tilde{T}_{c2}(\theta_{ex})$ intersect very close to the intersection of the lines $T_{c1}$ and $\theta$. Figure 13 shows the vicinity of the tricritical point $B$.

A phase diagram was derived by Kaufman and Entin-Wohlman,[119] who did not consider the coexistence phase; they also calculated the temperature $\tilde{T}_{c2}$ of the S–FN transition. The superheating line of the FN phase was actually derived by Rainer,[120] who calculated the effect of magnetic scattering on $T_{c1}$, taking into account the magnetic correlations near $\theta$ (see also Ref. 111). When the long-range part of the electromagnetic interaction is ignored, this line passes slightly above $\theta$ because of the logarithmic divergence near $\theta$. Incorporating the magnetic dipole interaction causes the superheating line of the FN phase to drop below the line $\theta$. The supercooling line of the S phase is the line $T_M$.

On the complete phase diagram, shown in Fig. 13, there are two tricritical points, $A$ and $B$. The lines separating the S, DS, and FN phases meet at $A$; point $B$ is at the boundaries between the N, S, and FN phases. The Lifshitz point $L$ lies on the supercooling line of the S phase. Another important characteristic of a ferromagnetic superconductor, other than the parameter $\theta_{ex}/T_{c0}$, is the cleanliness of the crystal, i.e., the parameter $l/\sqrt{a\xi_0}$. As this parameter decreases, the region in which the DS phase exists becomes narrower, because the points $A$ and $\theta_{ex}^{(c)}$ approach the origin, and the region of the S-FN first-order phase transition expands.

We can now describe the complete picture of the mutual effects of the superconducting and magnetic order. Even under the condition $\theta_{ex}^{(c)} \ll T_{c1}$ the magnetism has a stronger effect as long as the condition $h_0 > \Delta_0$ holds. The reason is that the energy of the magnetic ordering is of the order of $\theta \sim h_0^2 N(0)$ (per localized moment), while the energy of superconducting condensation does not exceed $\Delta_0^2 N(0)/2$. In systems with $h_0 > h_{0c}$ the ferromagnetic ordering ultimately disrupts the superconductivity as the material is cooled, but at temperatures at which the magnetization is weak the magnetic order coexists with the superconductivity, transforming into a domain structure. In systems with $h_0 < h_{0c}$ the coexistence phase survives down to absolute zero. When $T_M$ and $T_{c1}$ are approximately equal, the coexistence becomes impossible because of the weakness of the Cooper pairing; here the two antagonistic types of order replace each other at the point of the first-order phase transition, $\tilde{T}_{c2}$.

In ferromagnetic superconductors with realistic values of the exchange parameter, the theory thus predicts only a single type of coexistence phase: a phase with a magnetic structure of the domain type. In superconducting weak ferromagnets[130,131] there can be other coexistence phases: one with spontaneous vortices and a Meissner ferromagnetic phase.[42] So far, weak ferromagnetism has not been observed in ternary superconducting compounds, but the body-centered phase of $ErRh_4B_4$ which has recently been discovered may turn out to be an antiferromagnet of this type.

## 5. THEORETICAL PREDICTIONS REGARDING THE COEXISTENCE PHASE OF FERROMAGNETIC SUPERCONDUCTORS AND COMPARISON WITH EXPERIMENTAL RESULTS

### a) Basic theoretical conclusions

*1) Phase transitions.* In compounds with $T_M/T_{c1} \ll 1$, a DS coexistence phase should be observed when the material is cooled below $T_M$. In compounds in which $T_M$ and $T_{c1}$ are nearly the same, the coexistence phase does not exist, and a first-order S–FN transition is observed during cooling (Fig. 13).

*2) Magnetic structure in the* DS *phase.* In the phase with the domain magnetic structure, (4.8), peaks $(2k+1)\mathbf{Q}$ should be observed in small-angle neutron scattering, where the values of $k$ are integers; peaks which are satellites of the ferromagnetic peak, $\mathbf{G} + (2k+1)\mathbf{Q}$, should also be observed in Bragg scattering. There are no ferromagnetic peaks in the DS phase. The equilibrium value of $\mathbf{Q}$ decreases slightly upon cooling; equilibrium with respect to the parameter $\mathbf{Q}$ may be established very slowly. The relative intensity of the $(2k+1)\mathbf{Q}$ peak in the small-angle scattering falls off with increasing $k$ in proportion to $(2k+1)^{-2}$ for an ideal domain structure in a single crystal, while in a polycrystalline sample it falls off in proportion to $(2k+1)^{-4}$. The possibility of experimentally observing the higher peaks in neturon scatter-

ing in the DS phase, however, seems extremely problematical. The defects of the magnetic subsystem (irregularities in the positions of the magnetic ions, local changes in the RKKY interaction due to impurities, etc.) cause a curvature of domain walls and corresponding deviations of the domain structure from the ideal structure shown in Fig. 4. According to the results found by Sham and Patton[153] and Imry and Ma,[154] in systems with dimensionality $d \leqslant 4$ magnetic defects disrupt the long-range order for an inhomogeneous magnetic structure which is incommensurable with the main ion lattice (the effect is analogous to that of charged impurities on a superstructure of the nature of a charged density wave). Distortions of the domain structure caused by defects are manifested experimentally as a slight weakening and broadening of the higher peaks in the neutron scattering. If the pinning of domain walls at the discrete ion lattice is ignored (according to the calculations in Ref. 40), the intensity distribution $I_k(\eta)$ of peak $k$ is described as a function of the detuning $\eta = |\mathbf{q} - \mathbf{G} - (2k+1)\mathbf{Q}|$ by the following curve for a single crystal:

$$I_k(\eta) = \frac{2\eta_k^3 I_0}{\pi(\eta^2+\eta_k^2)^2}, \quad \eta_k = (2k+1)^2\eta_0, \tag{5.1}$$

where the parameter $\eta_k$ determines the peak width.

*3) Superconductivity in the* DS *phase.* In a dirty superconductor with $(h\tau)^2 \ll 1$ there is an energy gap for single-particle excitations in the DS phase, and this gap drops rapidly upon cooling.

In a clean superconductor in the region of strong exchange fields, $h \gg \Delta$, the superconductivity is gapless (Fig. 12). The gapless nature of the spectrum and the shift of the peak in the quasiparticle state density upon cooling can be observed in tunnel experiments. In the gapless regime the equilibrium direction of Q can be changed by passing a superconducting current through a single crystal.[34]

*4) The* DS–FN *transition.* The domain walls induced by the superconductivity disappear at the transition point $T_{c2}$. This process should occur slowly. At the DS → FN transition the jump in the average moment $S$ at $T_{c2}$ is about $\sqrt{a/\xi_0}$, and it leads to the release of latent heat of the order of the energy of superconducting condensation. Upon heating, the FN phase persists in a metastable state up to the temperature at which an infinitesimally small superconducting nucleating region appears, $T_{c2}^{(W)}$. This temperature is very close to $T_M$ in systems with $h_0 \gg \Delta_0$. The temperature dependence of the intensity of the ferromagnetic peak $F$ and of its satellites $(2k+1)\mathbf{Q}$ expected theoretically upon the S–DS–FN transitions in a ferromagnetic superconductor with a regular magnetic subsystem is shown in Fig. 14. In real systems with defects, the higher peaks may not be present.

*5) Effect of a magnetic field on the* DS *phase.* A magnetic field $H < H_c$ directed along the easy axis lowers the temperature at which the DS phase appears, $T_M$, and shifts the point of the first-order transition, $T_{c2}$, to a higher temperature. If such a field penetrates into a superconductor in the DS phase (in a thin plate, for example), the value of $Q$ decreases, and even peaks appear ($2Q$, etc.). The DS phase is completely suppressed by a magnetic field parallel to the easy axis and above the critical value $H_c^{(w)} \approx \theta_{ex}\Delta_0/\mu h_0$. The effect of a field perpendicular to the easy axis on the DS phase is much weaker than the effect of a parallel field.[33,34,83]

### b) Properties of $HoMo_6S_8$ and $HoMo_6Se_8$

Neutron-scattering measurements for $HoMo_6S_8$ have been carried out with polycrystalline samples,[16–18,117] and this circumstance has seriously complicated comparison of the experimental results with the theoretical predictions.

In accordance with the conclusions reached in Subsection 4b, Lynn *et al.*[17,18] observed growth of ferromagnetic fluctuations in the small-angle neutron scattering upon cooling to 0.69 K. In the region $T < 0.69$ K, a neutron-scattering peak appeared near the wave vector 0.3 $Å^{-1}$. Figure 15 shows the neutron-scattering intensity in a plot against the scattering angle upon cooling to various temperatures.[18] These results clearly indicate the appearance of a nonuniform magnetic structure in the superconducting state. Below 0.65 K the nonuniform magnetic structure and the superconductivity disappear.

Figure 16 shows the intensity of neutron scattering through the angle corresponding to the wave vector

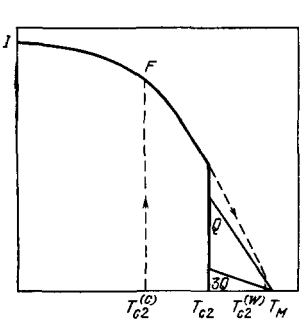


FIG. 14. Theoretical temperature dependence of the intensity of the ferromagnetic peak, $F$, and of the satellites, $(2k+1)\mathbf{Q}$. Solid lines—Equilibrium values; dashed lines—superheating curve of the FN phase and supercooling curve of the DS phase.

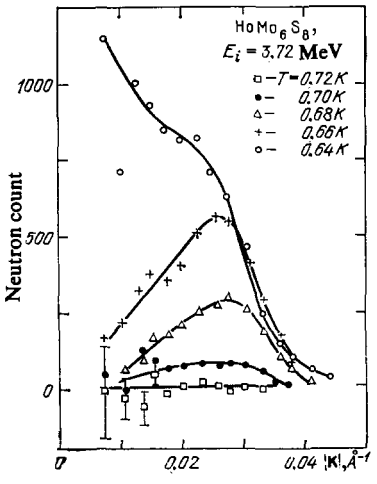


FIG. 15. Measurements of small-angle neutron scattering upon cooling near $T_{c2}$ in polycrystalline $HoMo_6S_8$ samples.[18] A peak is seen to appear with a wavelength of 230 Å below 0.72 K; this peak disappears at 0.64 K. At and below 0.64 K, the small-angle neutron scattering is anomalously intense.

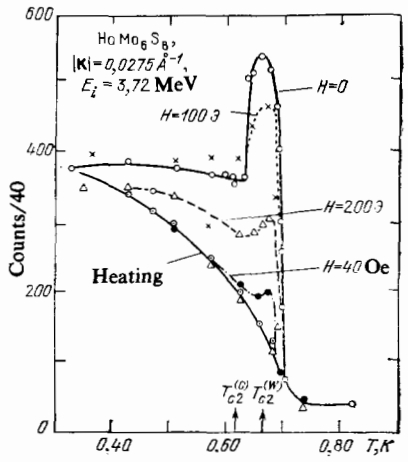


FIG. 16. Temperature dependence of the scattering with a wave vector $Q = 0.0275$ Å in $HoMo_6S_8$ (Ref. 18). During cooling, the intensity increases rapidly as an inhomogeneous magnetic structure appears. Below $T_{c2} = 0.65$ K, the scattering falls off as the FN phase appears. A magnetic field suppresses the inhomogeneous magnetic structure. Upon heating, the inhomogeneous structure is not observed. The superheating and supercooling points of the coexistence phase, $T_{c2}^{(w)}$ and $T_{c2}^{(c)}$, shown at the bottom were found from ac measurements of the susceptibility.

$Q = 0.0275$ Å$^{-1}$ as a function of the magnetic field and the temperature upon cooling and heating. The nonuiform structure does not appear during heating from the FN phase, while upon cooling this structure is suppressed essentially completely by a field above 400 Oe. Also shown in this figure are the superheating and supercooling temperatures of the superconducting phase, $T_{c2}^{(w)}$ and $T_{c2}^{(c)}$, according to measurements of the ac susceptibility in a field $H = 0$. Lynn *et al.*[18] pointed out that at temperatures near $T_{c2}^{(c)}, \approx 0.612$ K, determined from the susceptibility, the domain structure is metastable, since upon rapid cooling to 0.62 K the intensity of the peak at $Q = 0.027$ Å$^{-1}$ becomes twice that shown in Fig. 16; it then decreases to a level of 360 counts/min over a time interval of about an hour. At higher temperatures the structure is stable for at least a matter of days.

Lynn *et al.*[117] measured the neutron scattering at small

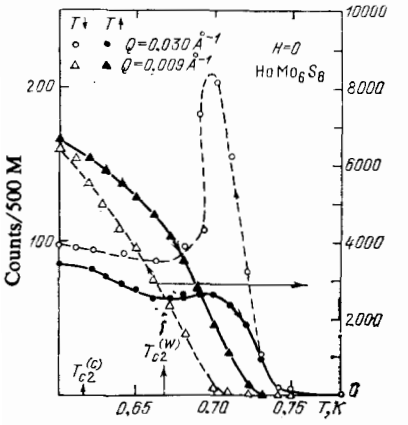


FIG. 17. Temperature dependence of the neutron scattering intensity for the modulated component and the ferromagnetic (0.009 Å$^{-1}$) component in $HoMo_6S_8$ (Ref. 117). During cooling to 0.7 K, only the nonuniform component is present.

angles corresponding to wave vectors $q = 0.009$ Å$^{-1}$ and $q = 0.030$ Å$^{-1}$. Their measurements showed that upon cooling from $T_M$ (0.74–0.75 K) to $T_{c2}$ (0.67–0.70 K) there is only a nonuniform structure with $Q = 0.030$ Å$^{-1}$, and ferromagnetic scattering ($q = 0.009$ Å$^{-1}$) appears only below 0.7 K (Fig. 17). It can also be seen from Fig. 17 that upon heating from the region of the FN phase the intensity of the peak at $Q = 0.030$ Å$^{-1}$ increases only negligibly above $T_{c2}$, while the ferromagnetic peak persists up to 0.72 K. Furthermore, this figure shows that upon cooling the volume occupied by the phase with the nonuniform structure decreases sharply below 0.69 K, but it persists along with the superconductivity down to $T_{c2}^{(c)}, \approx 0.62$ K in the metastable state. Consequently, during cooling the sample consists between 0.7 K and 0.62 K of a mixture of the FN phase and the superconducting phase with a nonuniform magnetic order; the time required for relaxation to equilibrium in the system is a matter of several hours.[117] The experimental data in Fig. 17 are in complete agreement with the theoretical predictions (Fig. 14). Figures 15–17 show the information on the main peak of the nonuniform magnetic structure. The higher peaks were not found in these measurements.

Figure 18 shows the results on Bragg scattering of neutrons for the (100) peak in $HoMo_6S_8$. In this figure we see an additional magnetic scattering below $T_M = 0.67$ K, which is proportional to the square of the average magnetic moment, $S^2(T)$. In Fig. 18 we see that this parameter has different values upon cooling and upon heating. The temperature dependence of the intensity upon heating is typical of a standard ferromagnet with an effective moment of 1/2 and a second-order transition at the Curie point.

All these experimental results show that the nonuniform structure has a transverse component (only this component is detected in the neutron scattering) and that upon cooling above $T_{c2}$ there is a Fourier component, in the structure with a wave vector $Q = 0.030$ Å$^{-1}$, while the ferromagnetic component and the $2Q$ component are missing. These results agree with the predictions regarding the domain structure of the coexistence phase (Subsection 5a2).

We turn now to the magnitude and temperature dependence of $Q$. It can be seen from Fig. 15 that the value of $Q$ is essentially independent of the temperature. We identify the value of the wave vector $Q = 0.30$ Å$^{-1}$ with the value of $Q_{c2}$.

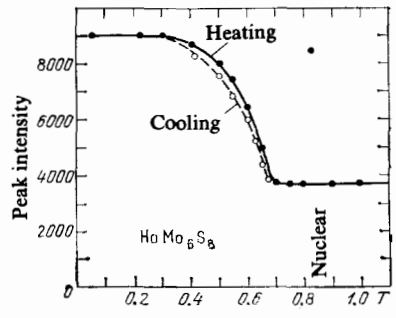


FIG. 18. Temperature dependence of the (100) peak in $HoMo_6S_8$ (Ref. 18). The intensity above $T_M = 0.67$ K is due to nuclear Bragg scattering. The additional scattering below $T_M$ is proportional to the square of the average magnetic moment.

We can then estimate $\tilde{a}$ with the help of expression (4.13a). Taking the estimate $v_F \approx 1.8 \cdot 10^7$ cm/s from the numerical calculations by Freeman and Jarlborg[109] for the band structure of ternary chalcogenides, we find $\xi_0 \approx 1.5 \cdot 10^{-5}$ cm. The experimental data of Ishikawa and Fischer[15] on $H_{c2}(T)$ near $T_{c1}$ yield $\xi \approx \sqrt{\xi_0 l} \approx 300$ Å and $l \approx 60$ Å. These estimates of $h_0$, $\xi_0$, and $l$ (Table II) show that these $HoMo_6S_8$ samples can be described by the theory for dirty superconductors with $Q^{-1} \ll l \ll v_F/h$. Using (4.13a), we find the reasonable value $\tilde{a}(T_{c2}) \approx 2.5$ Å. At the point $T_M$ we find $Q_M \approx 0.04$ Å$^{-1}$ from (4.5). We see that the decrease in $Q$ upon cooling should in fact be small, and it is difficult to detect experimentally since most of the change occurs near the point $T_M$, where the intensity of the $Q$ peak is low, and it is difficult to observe.

In accordance with Subsection 4d, there is a significant hysteresis at the DS–FN transition: The FN phase persists nearly all the way to $T_M$, while the DS phase survives to $T_{c2}^{(c)} \approx 0.62$ K. For the temperatures $T_{c2}^{(c)} \approx 0.62$ K and $T_{c2} \approx 0.7$ K we find the relation $S_{c2}^{(c)}, = 1.26 S_{c2}$ from Fig. 17; this relation agrees with the theoretical prediction. The experimental data at the point $T_{c2}$ correspond approximately to the theoretical relation (4.14a): For the right side of (4.14a) we find the value 6 Å with $\theta_{ex} \approx \bar{\theta}_{ex} \approx 0.15$ K, while the left side is about 12 Å if we use the value $S_{c2}^{(c)} \approx 0.35$ from Fig. 18. Woolf *et al.*[121] have observed a very sharp peak in the specific heat near $T_{c2}$ which indicates a first-order transition at this point.

In accordance with Subsection 4e, the magnetic field reduces the temperature of the S–DS transition. Lynn *et al.*[117] have observed a corresponding decrease in the peak intensity with increasing field near $T_M$ (at $T = 0.735$ K). A suppression of the nonuniform magnetic structure in a magnetic field in $HoMo_6S_8$ below $T_M$ was observed in Refs. 16–18 and 117 (Fig. 16). The scattering intensity at the wave vector $Q$ falls off rapidly with increasing field above a field level of about 100–200 Oe. According to the theoretical results (Subsection 4e), the parallel critical field is about 100 Oe. Lynn *et al.*[117] also observed an anisotropy of the intensity of the $Q$ peak as a function of the orientation of the scattering wave vector **q** with respect to the field **H** (Fig. 19). This result is again in agreement with the predictions of Subsection 4e regarding the anisotropy of the behavior of the system in a magnetic field. In the arrangement **q**||**H**, there is a contribution to the neturon scattering from crystallites in which the arrangements **Q**||**H** prevails in the DS phase, and the magnetization (and the easy axis) are perpendicular to **Q** and **H** because of the transverse nature of the structure. In the case **q**⊥**H** there is a field component along the magnetization (and along the easy axis), and its effect on the coexistence phase is stronger than for a field directed along the hard direction. As a result, there is a more rapid destruction of the DS phase in those crystallites which contribute to the scattering for the case **q**⊥**H**.

Figure 19 shows an increase in the peak intensity upon the application of a weak magnetic field; only in the stronger fields does the intensity decrease in accordance with the theoretical prediction. The reason for the anomalous behavior of the $Q$ peak in weak fields is not clear.

We turn now to the properties of a second ferromagnetic superconductor from the family of ternary chalcogenides: the compound $HoMo_6Se_8$. Lynn *et al.*[151] have observed that polycrystalline samples of this compound become superconducting at $T_c = 5.5$ K, and the superconductivity persists down to the lowest temperatures reached experimentally, 0.04 K. At the point $T_M = 0.53$ K neutron scattering indicates the appearance of nonuniform magnetic order with a wave vector $Q(T)$ which decreases monotonically upon cooling, from $Q(T_M) = 0.09$ Å$^{-1}$ to $Q = 0.062$ Å$^{-1}$ (Ref. 151) at 0.4 K; then it remains essentially constant at $T < 0.4$ K. The ferromagnetic peak and the higher peaks were not observed. Measurements of the intensity of the $Q$ peak show that the magnetization reaches saturation at about 0.3 K.

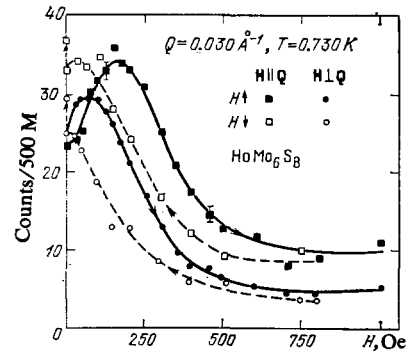


FIG. 19. Intensity of the peak at $Q = 0.03$ Å$^{-1}$ in the modulated phase of $HoMo_6S_8$ as a function of the magnetic field for parallel and perpendicular orientations of **Q** with respect to **H** (Ref. 117).

The most striking point here is that the superconductivity in $HoMo_6Se_8$ does not exhibit re-entrant behavior; i.e., the exchange field in this compound is lower than the critical value $h_{0c}$. It follows from (4.14) that $\theta_{ex}$ is then smaller than the corresponding critical value $\theta_{ex}^{(c)} = 0.077 \Delta_0 v_F N(0) Q(0)$ (we regard $HoMo_6Se_8$ as a dirty superconductor with $Q^{-1} \ll l \ll \xi_0$). Using the experimental values of $Q(0)$ and $T_c$ and the data in Table I, we find $\theta_{ex}^{(c)} = 0.34$ K, in comparison with the value $\bar{\theta}_{ex} \approx 0.14$ K (found from a comparison of $T_c$ for $LuMo_6Se_8$ and $HoMo_6Se_8$). Assuming $\theta_{ex} \lesssim \bar{\theta}_{ex}$, we see that the condition $\theta_{ex} < \theta_{ex}^{(c)}$ does in fact hold in $HoMo_6Se_8$. It follows from (4.9) that under the condition $0.6\theta_{ex}/\theta_{ex}^{(c)} \ll 1$ we can ignore the effect of magnetic order on superconductivity, and this condition is apparently satisfied in $HoMo_6Se_8$. The behavior of $Q(T)$ agrees well with the theoretical predictions. With decreasing temperature, the magntitude of $Q$ stops changing in that interval in which the magnetization is already increasing. This fact confirms the conclusion that magnetism has only a slight effect on superconductivity in $HoMo_6Se_8$, and it shows that the entire temperature dependence of $Q$ stems from the temperature dependence of the surface energy of a domain wall [see (4.7) and Ref. 152]. Lynn *et al.* observed a very long relaxation (of the order of 50 h) to equilibrium upon cooling below $T_M$. This result can be explained by arguing that the time required for the formation of an equilibrium number of domain walls is very long at such low temperatures.

In summary, the experimental data on $HoMo_6S_8$ and

$HoMo_6S_8$ are in basic agreement with the predictions of the theory of the DS phase.

### c) Experimental data on $ErRh_4B_4$

Measurements of the dc resistance and the ac magnetic susceptibility by Maple *et al.*[94] in polycrystalline $ErRh_4B_4$ samples demonstrate re-entrant behavior of superconductivity in this compound; the N–S transition at the point $T_{c1}$ is a second-order transition, while the transition to the normal state at $T_{c2}$ has a significant hysteresis and is undoubtedly a first-order transition. The magnetic nature of the transition at $T_{c2}$ was established from measurements of the specific heat by Woolf *et al.*[49]

The first neutron measurements by Moncton *et al.*[14] revealed a monotonic growth of the ferromagnetic Bragg peaks beginning at a temperature slightly above 1 K. Subsequent neutron studies[19], also with polycrystalline samples, revealed a peak at the wave vector $Q = 0.06$ Å$^{-1}$ in the small-angle scattering at temperatures between 1 K and 0.6 K; this peak was lower upon heating than upon cooling.

Figure 20 shows the results of the measurements of neutron scattering and of resistance by Sinha *et al.*[20] for a single crystal. Shown here is the temperature dependence of the intensity of the ferromagnetic $F$ peak, that of its satellite, and the resistance of the sample. The positions of the four satellites of the (101) ferromagnetic peak in the **b***, **c*** plane are determined by the vectors $\mathbf{Q} = \pm(0.042\mathbf{b}^* \pm 0.055\mathbf{c}^*)$ for the case in which the moment is directed along the **a** axis, i.e.,

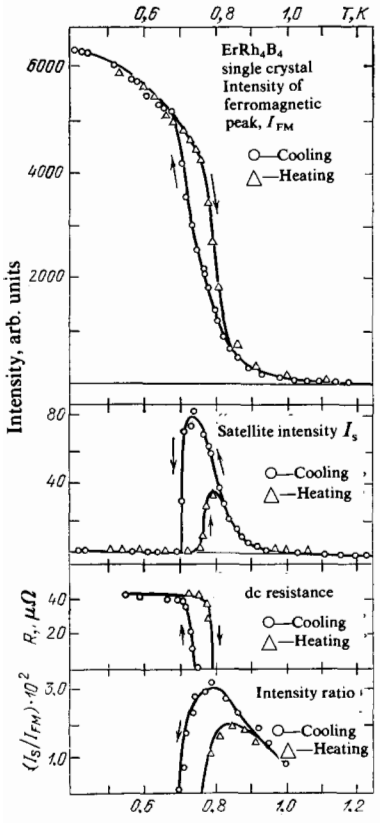


FIG. 20. Results of measurements of the neutron scattering and the dc resistance in an $ErRh_4B_4$ single crystal.[20]

for the case in which the nonuniform structure is a transverse structure. The intensities of two of the satellites are higher than those of the two others, indicating a one-dimensional nature of the nonuniform order. In some regions of a crystal there is a structure with a wave vector $\mathbf{Q} = \pm(0.042\mathbf{b}^*, 0.055\mathbf{c}^*)$, while in others there is a structure with the crystallographically equivalent vector $\pm(0.042\mathbf{b}^* - 0.055\mathbf{c}^*)$. A given structure is predominant because of stresses in the crystal. Higher satellites were not observed; i.e., their intensities were less than 2% of those of the main satellites. The value of $Q$ was essentially independent of the temperature.

The first point we note in Fig. 20 is that the satellites are observed at the same time as the ferromagnetic peak. Their total intensity is low in comparison with the intensity of the main peak, no more than 10% of the latter. According to measurements by Mook *et al.*,[145] there is no long-range ferromagnetic order at 1.1 K, and the ordering propagates only to regions 200 Å in size. A true long-range order is established at lower temperatures, and at 0.8 K the sizes of the magnetic regions exceed 10 000 Å.

Figure 20 shows data on the intensities of the $F$ and $Q$ peaks; these results are clearly at odds with the theoretical predictions in Fig. 14. We see that although a regular nonuniform magnetic structure does arise in $ErRh_4B_4$ at the transition from the nonmagnetic S phase to the normal magnetic FN phase, the volume of the sample in which this structure appears is so small as to be negligible in a first approximation.

The reasons for the suppression of the DS phase in $ErRh_4B_4$ have not yet been reliably established. Help with this puzzle may come from the experimental facts which indicate an anomalous magnetic behavior of the FN normal phase of the $ErRh_4B_4$ samples which have been studied. Shenoy *et al.*[140,110] pointed out a long time ago that the results of Mössbauer measurements of the moment in the FN phase are markedly different from the results of neutron measurements (Fig. 21). Shown in this figure, along with the results of the Mössbauer (top) and neutron (bottom) measurements, is a theoretical curve of the moment in the FN phase of the ideal $ErRh_4B_4$ crystal according to calculations[40] based on a well-known crystal-field Hamilton-

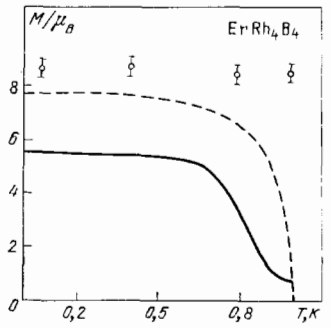


FIG. 21. Average value of the moment at the Er ion in $ErRh_4B_4$ as a function of the temperature. Top—data from Mössbauer studies[110]; center—theoretical curve for an ideal crystal[40]; bottom—data from neutron studies.[19]

ian.[115,141] The theoretical value of the saturation moment ($7.6\mu_B$) is close to the Mössbauer value ($8.3 \pm 0.2\mu_B$), but the neutron result ($5.6\mu_B$) is well below the expected value.

Fradin *et al.*[110] have noted that the difference between the results of the Mössbauer and neutron measurements is evidence for a disordered magnetization component of the system. It was suggested in Ref. 40 that the presence of a coherent component (along the **a** axis) and a disordered component (along the **b** axis) could be the result of the appearance of a randomly oriented easy axis in the basis plane because of defects of the crystal structure. A random anisotropy combined with the alternating-sign magnetic-dipole interaction and the RKKY interaction of the localized moments could lead to a magnetic order of the fan (asperomagnetic) type.[142] The sharp increase in the magnetization observed in $ErRh_4B_4$ at $T = 0.4$ K in Ref. 47, from $5.6\mu_B$ to $7\mu_B$, upon the imposition of a very weak magnetic field and also the slow onset of saturation, resulting in a value of $85.\mu_B$ in a field of 7 kOe, are characteristic of an asperomagnetic order.

According to the hypothesis of asperomagnetism in real $ErRh_4B_4$ crystals, the discrepancy between the theoretical predictions and experimental data can be explained on the basis that the irregularity of the magnetic order supresses the formation of a coherent nonuniform magnetic structure in the coexistence phase. The transition from the S phase to the FN phase then terminates in an increase in the asperomagnetic normal regions disseminated through the nonmagnetic S phase. On the whole, this transition is more reminiscent of the first-order S–Fn transition than of the S–DS–FN series of transitions.

Arguments along this line can explain the tunnel measurements[99–102] in $ErRh_4B_4$ and the measurements of the absorption of sound.[75,138] Umbach *et al.*[101] observed a rapid decay of the Josephson current below 1.4 K in a contact with $ErRh_4B_4$ and a splitting of the Fraunhofer peaks in a plot of the current against the magnetic field in the region 1.1–1.2 K. These two effects may be due to the appearance of ferromagnetic regions which suppress the Josephson current and which create an additional nonuniform field in the sample. The appearance of randomly positioned magnetic regions in the superconducting phase below 1.2 K makes the samples inhomogeneous for the propagation of ultrasound, possibly explaining the appearance of a broad peak between 1.4 and 0.9 K in the temperature dependence of the ultrasonic absorption.[75,138]

The observation of satellites in $ErRh_4B_4$ shows that the DS phase nevertheless does form. It apparently forms in that small part of the sample in which the crystal structure is nearly ideal. According to theoretical predictions this structure would be transverse and one-dimensional. The unsuccessful attempts by Mook *et al.*[145] to observe higher satellites do not rule out a domain nature of the magnetic structure, since the higher peaks would be very faint and broad in the case of an irregular domain structure [see (5.1)]. The wave-vector value $Q = 0.06$ Å$^{-1}$ leads to the reasonable estimate $\tilde{a} \approx 1$ Å from (4.14b).

The hypothesis of asperomagnetism thus leads to a qualitative explanation for the behavior of $ErRh_4B_4$ not only in the normal phase but also at the S–FN transition. If this hypothesis proves correct, we must conclude that even the $ErRh_4B_4$ single crystals which have been synthesized already are too irregular for a proper test of the theoretical predictions regarding the structure of the coexistence phase. A more nearly ideal behavior in single crystals may be achieved by imposing uniaxial pressure to single out a regular easy axis in the basis plane. We might note that the anomalous behavior of real $ErRh_4B_4$ samples seems to be due to a slight anisotropy in the easy plane.[40] This factor is not present in the uniaxial ferromagnetic $HoMo_6S_8$, and in this case small structure distortions do not lead to such a pronounced deviation of the magnetic structure from ferromagnetic. Accordingly, the synthesis of $HoMo_6S_8$ single crystals might reveal the complete picture of the behavior of ferromagnetic superconductors below $T_M$.

Table II lists the basic characteristics of $HoMo_6S_8$ and $ErRh_4B_4$.

#### d) Re-entrant superconductivity in ternary silicates, stannides, and pseudoternary compounds

We mentioned earlier that an S–FN first-order transition is observed in $TmFe_3Si_5$ at $T_{c2} \approx 1.1$ K ($T_{c1} = 1.3$ K). These results agree with the conclusion that there is no coexistence phase in compounds in which the values of $T_{c1}$ and $\theta$ are approximately equal (Subsection 4f).

A direct S–FN first-order transition has also been observed in ternary stannides and pseudoternary compounds. The interaction of the magnetic and superconducting orders in compounds of this type is exhibited most clearly by the system[80,95,132,145] $(Er_{1-x}Ho_x)Rh_4B_4$. The phase diagram of this alloy is shown in Fig. 22. The compound $HoRh_4B_4$ is a ferromagnet with an effective spin of 1/2 and with ordering along the **c** axis. For $x < 0.9$, re-entrant superconductivity is observed; at $x > 0.3$, the Ho moments are ordered along the **c** axis below $T_{c2}$, while at $x < 0.3$ the Er moments are ordered in the basis plane. In the region $0.3 < x < 0.9$ the transition at the point $T_{c2}$ is a sharp S–FN first-order transition; analysis of the neutron data of Woolf *et al.*[23] has revealed that the magnetization disappears abruptly at $T_{c2}$. Extrapolation to zero magnetization yields the Curie temperature $\theta$ in the absence of superconductivity; it is shown by the dashed line in Fig. 22 (at $x = 0.6$, the difference between $\theta$ and $T_{c2}$ is 0.2 K).

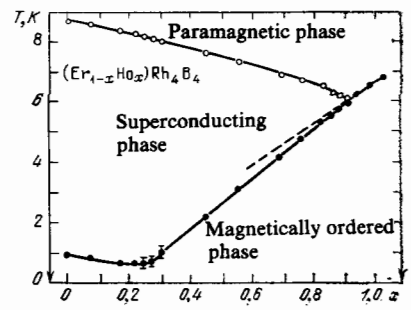


FIG. 22. Phase diagram of the pseudoternary compound $(Er_{1-x}Ho_x)Rh_4B_4$. The dashed curve shows the temperature of the magnetic transition, $\theta$, which would prevail in the absence of superconductivity.

The reason for the S–FN transition without the appearance of a coexistence phase in these compounds seems to consist of more than the proximity of $T_{c1}$ and $\theta$. It may be that, as in $ErRh_4B_4$, the disorder of the magnetic subsystem in irregular crystals suppresses the coherent nonuniform magnetic structure required for a coexistence phase. Measurements of the hyperfine fields at the $^{11}B$ nuclei by an NMR method by Kohara *et al.*[146] at 1.3 K do in fact reveal a marked scatter in the hyperfine fields in the compounds $Er_{1-x}Ho_xRh_4B_4$ with $x = 0.6$ and 0.8 in the presence of an external magnetic field because of a random direction of the Er moments and/or irregular position of the Ho moments. The behavior of compounds with an irregular magnetic subsystem is thus consistent with the suggestion that the magnetic disorder has a destructive effect on the coexistence phase.

## 6. CONCLUSION

Here is a list of the aspects of the physics of magnetic superconductors with localized magnetic moments which we find the most interesting.

1. All magnetic superconductors are type II superconductors near $T_{c1}$, but slightly above $T_M$ in ferromagnets and near $T_N$ in antiferromagnets they may be type I superconductors. The governing factors are the demagnetizing factor and the relation between the parameters of the electromagnetic and exchange interactions. This effect has been observed in $ErRh_4B_4$, and an intermediate state can be observed near 1 K in spherical samples of this compound.

2. In the same temperature interval in clean compounds, a phase with a nonuniform superconducting order parameter, of the LOFF type, can be produced in the presence of a parallel magnetic field. In ordinary superconductors this phase cannot be reached, and magnetic superconductors present a unique possibility for studying this phase experimentally. In particular, it should be observed in plate-shaped $ErRh_4B_4$ samples in a field perpendicular to the plate. As yet we do not have an exhaustive theoretical analysis of the LOFF phase.

3. In ferromagnetic superconductors with a regular magnetic subsystem and a magnetic point $T_M \ll T_{c1}$, a DS coexistence phase should be observed below $T_M$ with a nonuniform, one-dimensional, transverse magnetic structure of a domain type. In clean compounds the superconducting coexistence phase is of a gapless nature. Experimental data indicate the occurrence of the DS phase in $HoMo_6Se_8$, $HoMo_6S_8$, and $ErRh_4B_4$. In $ErRh_4B_4$, however, this phase is present in only a small fraction of the volume of the sample, apparently because of a magnetic disorder in real samples of this compound. The compound $HoMo_6S_8$ has been synthesized only in the form of polycrystalline samples and is a dirty superconductor.

4. Laser bombardment can suppress superconductivity partially or completely without having any serious effect on the magnetic order.[93] It thus becomes possible to study magnetic structure in the absence of superconductivity in a temperature range in which the coexistence phase exists, and it also becomes possible to act on the magnetic structure through the superconducting subsystem. Experiments of this type would make it possible to see directly how superconductivity affects magnetic order. We might note that the DS–FN transition can be observed by an optical method, since the reflection of light in the FN phase depends on the polarization, while there is no such dependence in the DS phase.

Several experimental and theoretical problems of the physics of magnetic superconductors await resolution.

First, there is the puzzling behavior of $ErRh_4B_4$ below $T_M$, although most researchers believe that this particular compound is presently playing a crucial role in research on the coexistence of superconductivity and ferromagnetism. The hypothesis of asperomagnetism in $ErRh_4B_4$ can explain the experimental facts, but the assumption of pronounced magnetic disorder in a rather regular single crystal seems implausible at first glance. In the near future we will undoubtedly see the answer to the question of the structure of the magnetic order in the FN phase of this compound from measurements of diffuse neutron scattering, from Mössbauer measurements in a magnetic field, and from NMR studies of the hyperfine fields at the $^{11}B$ nuclei, analogous to the measurements carried out by Kohara *et al.*[146] The application of uniaxial pressure to single crystals might also clarify the situation.

The $ErRh_4B_4$ problem seems to be intimately related to the effect of magnetic disorder on the coexistence phase of ferromagnetic superconductors. This problem has not yet been studied theoretically. Experimentally, it would be interesting to see measurements of small-angle neutron scattering in a series of compounds $Ho_{1-x}Y_xMo_6X_8$ with increasing values of $x$ (X = S, Se).

An intriguing question is the possible observation of localized superconductivity at domain walls in the normal phase of returning superconductors. An analysis[129] shows that an experimental seach for superconducting domain walls should be aimed primarily at irregular returning superconductors with an S–FN transitions without a coexistence phase. The compound $Ho_{1-x}Y_xMo_6S_8$ seems the most promising from this standpoint.

We wish to thank V. L. Ginzburg for interest in the problem, for reading the manuscript, and for critical comments. We also thank A. I. Larkin, D. A.Kirzhnits, Yu. V. Kopaev, I. O. Kulik, A. I. Rusinov, D. E. Khmel'nitskiĭ, D. I. Khomskiĭ, and E. F. Shender for useful discussions.

[1]V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **31**, 202 (1956) [Sov. Phys. JETP **4**, 153 (1957)].
[2]B. T. Matthias, H. Suhl, and E. Corenzwit, Phys. Rev. Lett. **1**, 92, 444 (1958).
[3]A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **39**, 1781 (1960) [Sov. Phys. JETP **12**, 1243 (1961)].
[4]D. St. James, G. Sarma, and E. J. Thomas, Type II Superconductivity, Pergamon Press, Oxford (1969) (Russ. Transl. Mir, M., 1970).
[5]L. P. Gor'kov and A. I. Rusinov, Zh. Eksp. Teor. Fiz. **46**, 1363 (1964) [Sov. Phys. JETP **19**, 922 (1964)].
[6]M. V. Maple, Appl. Phys. **9**, 179 (1976).
[7]P. W. Anderson and H. Suhl, Phys. Rev. **116**, 898 (1959).
[8]D. C. Mattis, The Theory of Magnetism, Harper and Row, New York (1965) (Russ. Transl. Mir, M., 1967, Ch. 7).
[9]U. Krey, Int. J. Magn. **3**, 65 (1973); **4**, 153 (1973).
[10]M. Ishikawa and Ø. Fischer, Solid State Commun. **24**, 747 (1977).

[11]R. McCallum, D. C. Johnston, R. N. Shelton, and M. B. Maple, Solid State Commun. **24**, 391 (1977).
[12]H. C. Hamaker, L. D. Woolf, H. B. McKay, Z. Fisk, and M. B. Maple, Solid State Commun. **31**, 139 (1979).
[13]W. A. Fertig, D. C. Johnston, L. E. Doelong, R. W. McCallum, M. B. Maple, and B. T. Mattias, Phys. Rev. Lett. **38**, 387 (1977).
[14]D. E. Moncton, D. B. McWhan, J. Eckert, G. Shirane, and W. Thomlinson, Phys. Rev. Lett. **39**, 1164 (1977).
[15]M. Ishikawa and Ø. Fischer, Solid State Commun. **23**, 37 (1977).
[16]J. W. Lynn, A. Raggazoni, R. Pynn, and J. Jaffrin, J. Phys. Lett. **42**, L–45 (1981).
[17]J. W. Lynn, G. Shirane, W. Thomlinson, and R. N. Shelton, Phys. Rev. Lett. **46**, 368 (1981).
[18]J. W. Lynn, G. Shirane, W. Thomlinson, R. N. Shelton, and D. E. Moncton, Phys. Rev. **B24**, 3817 (1981).
[19]D. E. Moncton, D. B. McWhan, P. H. Schmidt, G. Shirane, W. Thomlinson, M. B. Maple, H. B. McKay, L. D. Woolf, Z. Fisk, and D. C. Johnston, Phys. Rev. Lett. **45**, 2060 (1981).
[20]S. K. Sinha, H. A. Mook, D. G. Hinks, and G. W. Crabtree, Phys. Rev. Lett. **48**, 950 (1982).
[21]K. Andres, J. P. Remeika, G. P. Espinosa, and A. S. Cooper, Phys. Rev. **B23**, 1179 (1981).
[22]B. Lachal, M. Ishikawa, A. Junod, and J. Muller, J. Low Temp. Phys. **46**, 467 (1982).
[23]L. D. Woolf, D. C. Johnston, H. A. Mook, W. C. Koehler, M. B. Maple, and Z. Fisk, Physica (Utrecht) **109–110**, 2045 (1982).
[24]L. N. Bulaevskii, A. I. Rusinov, and M. L. Kulic, Solid State Commun. **30**, 59 (1979); J. Low Temp. Phys. **39**, 256 (1980).
[25]E. I. Blount and C. M. Varma, Phys. Rev. Lett. **42**, 1079 (1979).
[26]R. A. Ferrel, J. K. Bhattaracharjee, and A. Bagchi, Phys. Rev. Lett. **43**, 154 (1979).
[27]H. Matsumoto, H. Umezawa, and M. Tachiki, Solid State Commun. **31**, 157 (1979).
[28]H. S. Creenside, E. I. Blount, and C. M. Varma, Phys. Rev. Lett. **46**, 49 (1981).
[29]C. G. Kuper, M. Revzen, and Z. Ron, Phys. Rev. Lett. **44**, 1454 (1980).
[30]M. Tachiki, H. Matsumoto, T. Koyama, and H. Umezawa, Solid State Commun. **34**, 19 (1980).
[31]M. Tachiki, Physica (Utrecht) **109–110**, 1699 (1982).
[32]L. N. Bulaevskiĭ, A. I. Buzdin, S. V. Panjukov, and M. L. Kulic, Solid State Commun. **44**, 1247 (1982); Phys. Rev. **B28**, 1370 (1983).
[33]L. N. Bulaevskiĭ, A. I. Buzdin, and S. V. Panjukov, Solid State Commun. **43**, 135 (1982); Zh. Eksp. Teor. Fiz. **83**, 768 (1982) [Sov. Phys. JETP **56**, 430 (1982)].
[34]L. N. Bulaevskiĭ and S. V. Panjukov, J. Low Temp. Phys. **52**, 137 (1983).
[35]A. M. Buzdin and L. N. Bulaevskiĭ, Zh. Eksp. Teor. Fiz. **79**, 1954 (1980) [Sov. Phys. JETP **52**, 987 (1980)].
[36]M. L. Kulic, Phys. Lett. **A81**, 359 (1981).
[37]L. N. Bulaevskiĭ, A. I. Buzdin, S. V. Panjukov, and A. I. Rusinov, Solid State Commun. **40**, 683 (1981).
[38]L. N. Bulaevskiĭ, A. I. Buzdin, S. V. Panjukov, and M. L. Kulic, Phys. Lett. **A89**, 93 (1982).
[39]W. Baltensperger and S. Strassler, Phys. Konden. Mater. **1**, 20 (1983).
[40]L. N. Bulaevskiĭ, A. I. Buzdin, M. L. Kulic, and S. V. Panjukov, J. Low Temp. Phys. in press (1985).
[41]R. N. Shelton, in: Proceedings of the Fourth Conference on Superconductivity in *d* and *f* Band Metals, Karlsruhe, 1982, p. 123.
[42]A. I. Buzdin, L. N. Bulaevskiĭ, and S. S. Krotov, Zh. Eksp. Teor. Fiz. **85**, 678 (1983) [Sov. Phys. JETP **58**, 395 (1983)]; Solid State Commun. **48**, 719 (1983).
[43]H. B. Kadousky, G.S. Knapp, J. S. Konvel,T. E. Klippert, and J. W. Downey, in: Ternary Compounds (eds. G. K.Shenoy, B. D. Dunlap, and F. Y. Fradin), North-Holland, Amsterdam, 1981, p. 151
[44]H. R. Ott, W. A. Fertig, D. C. Johnston, M. B. Maple, and B. T. Matthias, J. Low Temp. Phys. **33**, 159 (1979).
[45]F. Behroozi, G. W. Crabtree, S. A. Campbell, D. R. Snider, S. Scheider and M. Levy, J. Low Temp. Phys. **49**, 73 (1982).
[46]K. Okuda, Y. Nakakura, and K. Kadowaki, Solid State Commun. **32**, 185 (1979).
[47]G. W. Crabtree, F. Behroozi, S. A.Campbell, and D. G. Hinks, Phys. Rev. Lett. **49**, 1342 (1982).
[48]H. A. Mook, O. A. Pringle, S. Kawarazaki, S. K. Sinha, G. W. Crabtree, D. G. Hinks, M. B. Maple, Z. Fisk, D. C. Johnston, and L. D. Woolf, Proceedings of the Fourth Conference on Superconductivity in *d* and *f* Band Metals, Karlsruhe, 1982, p. 201.
[49]L. D. Woolf, D. C. Johnston, N. B. McKay, R. W. Callum, and M. B. Maple, J. Low Temp. Phys. **35**, 651 (1979).
[50]M. Ishikawa and J. Muller, Solid State Commun. **27**, 761 (1978).
[51]C. U. Segre and H. F. Braun, Phys. Lett. **A85**, 372 (1981).
[52]Z. Fisk, S. E. Lambert, M. B. Maple, J. P. Remeika, G. P. Espinosa, A. S. Cooper, H. Barz, and S. Oseroff, Solid State Commun. **41**, 63 (1982).
[53]J. M. Vandenberg, Mater. Res. Bull. **15**, 835 (1980).
[54]F. Acker and H. C. Ku, Phys. Rev. **B25**, 5962 (1982).
[55]K. N. Yang, S. E. Lambert, H. C. Hamaker, M. B. Maple, H. A. Mook, and H. C. Ku, Proceedings of the Fourth Conference on Superconductivity in *d* and *f* Brand Metals, Karlsruhe, 1982, p. 217.
[56]H. C. Hamaker, H. C. Ku, M. B. Maple,and H. A. Mook, Solid State Commun. **43**, 455 (1982).
[57]M. B. Maple, in: Proceedings of ICM'82, Kyoto, 1982.
[58]F. Acker, L. Schellenberg, and H. C. Ku, Proceedings of the Fourth Conference on Superconductivity in *d* and *f* Band Metals, Karlsruhe, 1982, p. 237.
[59]H. Adrian, R. Muller, and R. Behrle, Phys. Rev. **B26**, 2450 (1982).
[60]H. C. Hamaker, H. B. MacKay, M. S. Torikachvili, L. D. Woolf, and M. B. Maple, J. Low Temp. Phys. **44**, 533 (1981).
[61]H. C. Ku, S. E. Lambert, and M. B. Maple, Proceedings of the Fourth Conference on Superconductivity in *d* and *f* Band Metals, Karlsruhe, 1982, p. 231.
[62]B. V. B. Sarkissian, Proceedings of the Fourth Conference on Superconductivity in *d* and *f* Band Metals, Karlsruhe, 1982, p. 311; J. Appl. Phys. **53**, 8070 (1983).
[63]W. Wiethege, P. Entel, and B. Mühlschlegel, Z. Phys. **B47**, 35 (1982).
[64]H. B. MacKay, L. D. Woolf, M. B. Maple, and D. C. Johnston, J. Low Temp. Phys. **41**, 639 (1980).
[65]S. Maekawa and C. Y. Huang, in: Proceedings of the International Conference on Crystalline Field and Structure Effects in *f* Electron Systems, Philadelphia, 1979.
[66]F. Behroozi, G. W. Crabtree, S. A. Campbell, and D. G. Hinks, Phys. Rev. **B27**, 6849 (1983).
[67]K. Kumagai and F. Y. Fradin, Proceedings of the Fourth Conference on Superconductivity in *d* and *f* Band Metals, Karlsruhe, 1982, p. 227; Phys. Rev. **B17**, 2770 (1983).
[68]M. Redi and P. W. Anderson, Proc. Nat. Acad. Sci. USA **78**, 27 (1981).
[69]R. M. White, Quantum Theory of Magnetism, McGraw-Hill, New York, 1970 (Russ. Transl. Mir, M., 1972, Ch. 6).
[70]G. F. Zharkov, Zh. Eksp. Teor. Fiz. **34**, 412 (1958) [Sov. Phys. JETP **7**, 286 (1958)]; **37** 1784 (1959) [Sov. Phys. JETP **10**, 1259 (1960)].
[71]A. I. Larkin and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **56**, 2087 (1969) [Sov. Phys. JETP **29**, 1123 (1969)].
[72]F. Acker and H. C. Ku, J. Low Temp. Phys. **42**, 449 (1981).
[73]H. R. Ott, G. Keller, W. Odoni, L. D. Woolf, M. B. Maple, C. D. Johnston, and H. A. Mook, Phys. Rev. **B25**, 477 (1982).
[74]L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **34**, 735 (1958) [Sov. Phys. JETP **7**, 505 (1958)]; **36**, 1918 (1959) [Sov. Phys. JETP **9**, 1364 (1959)].
[75]S. C. Schneider, M. Levy, R. Chen, M. Tachiki, D. C. Johnston, and B. T. Matthias, Solid State Commun. **40**, 61 (1981).
[76]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **47**, 1136 (1964) [Sov. Phys. JETP **20**, 762 (1965)].
[77]P. Fulde and R. A. Ferrell, Phys. Rev. **A135**, 550 (1964).
[78]K. Nakanishi and K. Maki, Prog. Theor. Phys. **48**, 1059 (1972).
[79]L. W. Gruenberg and L. Gunther, Phys. Rev. Lett. **16**, 996 (1966).
[80]D. C. Johnston, W. A. Fertig, M. B. Maple, and B. T. Matthias, Solid State Commun. **26**, 141 (1978).
[81]G. Eilenberger, Z. Phys. **214**, 195 (1968).
[82]K. Maki, in: Superconductivity (ed. R. D. Parks), Vol. 2, New York, 1969, p. 1035.
[83]S. V. Panyukov, Fiz. Nizk. Temp. (1984) [*sic*].
[84]A. Hubert, Theorie der Domänenwände in geordnoten Medien, Springer-Verlag, Berlin, 1974 (Russ. Transl. Mir, M., 1977).
[85]L. N. Bulaevskiĭ and V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **45**, 772 (1963) [Sov. Phys. JETP **18**, 530 (1964)].
[86]S. A. Brazovskiĭ, Zh. Eksp. Teor. Fiz. **68**, 175 (1975) [Sov. Phys. JETP **41**, 84 (1975)].
[87]S. A. Brazovskiĭ, I. E. Dzyaloshinskiĭ, and B. T. Kukharenko, Zh. Eksp. Teor. Fiz. **70**, 2257 (1976) [Sov. Phys. JETP **43**, 1178 (1976)].
[88]B. Schuh and N. Grewe, Z. Phys. **B46**, 149 (1982).
[89]H. Kleinert, Phys. Lett. **A90**, 259 (1982).
[90]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred, Nauka, M., 1982, Ch. 5 [Engl. Transl. of earlier edition, Electrodynamics of Continuous Media, Pergamon Press, Oxford, 1960].
[91]W. A. Roshen and J. Ruvalds, Phys. Rev. **B28**, 1329 (1983).
[92]M. Ishikawa, Contemp. Phys. **23**, 443 (1982).
[93]D. N. Langenberg, in: Proceedings of Low Temp. LT-14, Vol. 5, North-Holland, Amsterdam, 1975, p. 223.

[94]M. B. Maple, H. C. Hamaker, L. D. Woolf, H. B. MacKay, Z. Fisk, W. Odoni, and H. R. Ott, in: Crystalline Electric Field and Structural Effects in $f$ Electron Systems (ed. J. E. Crow, R. P. Guertin, and T. W. Mihalisen), Plenum Press, New York, 1980, p. 533.
[95]H. A. Mook, W. C. Koehler, S. K. Sinha, G. W. Crabtree, D. G. Hinks, M. B. Maple, Z. Fisk, D. C. Johnston, L. D. Woolf, and H. C. Hamaker, J. Appl. Phys. **53**, 2614 (1982).
[96]S. Kirkpatric, Rev. Mod. Phys. **45**, 574 (1973).
[97]D. Jerome and H. J. Schulz, Adv. Phys. **31**, 299 (1982).
[98]K. Machida, J. Phys. Soc. Jpn. **50**, 2195 (1981); **51**, 1420 (1982); **52**, 1333 (1983).
[99]X. Kuwasawa, L. Rinderer, and B. T. Matthais, J. Low Temp. Phys. **37**, 179 (1981).
[100]V. Poppe, Physica (Utrecht) **B + C108**, 805 (1981).
[101]C. P. Umbach, L.-J. Lin, and A. M. Goldman, Proceedings of the Fourth Conference on Superconductivity in $d$ and $f$ Band Metals, Karlsruhe, 1982, p. 209.
[102]Y. Kuwasawa, S. Wrakano, and L. Rinderer, Phys. Lett. **A85**, 94 (1979).
[103]B.T. Matthais and H. Suhl, Phys. Rev. Lett. **4**, 51 (1960).
[104]Yu. V. Kopaev, Fiz. Tverd. Tela (Leningrad) **7**, 2907 (1965) [Sov.Phys. Solid State 7, 2360 (1966)].
[105]M. Tachiki, A. Kotani, H. Matsumoto, and U. Umezawa, Solid State Commun. **32**, 599 (1979).
[106]D. C. Johnston and H. F. Broun, in: Superconductivity in Ternary Compounds, II (eds. M. B. Maple and Ø. Fischer), Springer-Verlag, New York, 1982, p. 111 (Topics in Current Physics, Vol. 34).
[107]W. Thomlinson, G. Shirane, J. W. Lynn, and D. E. Moncton in: Superconductivity in Ternary Compounds, II (eds. M. B. Maple and Ø. Fischer), Springer-Verlag, New York, p. 229 (Topics in Current Physics, Vol. 34).
[108]M. Ishikawa, Ø. Fischer, and J. Muller, in: Superconductivity in Ternary Compounds, II (eds. M. B. Maple and Ø. Fischer), Springer-Verlag, New York, p. 143 (Topics in Current Physics, Vol. 34).
[109]A. J. Freeman and T. Jarlborg, in: Superconductivity in Ternary Compounds, II (eds. M. B. Maple and Ø. Fischer), Springer-Verlag, New York, p. 167 (Topics in Current Physics, Vol. 34).
[110]F. Y. Fradin, B. D. Dunlap, G. K. Shenoy, and C. W. Kimball, in: Superconductivity in Ternary Compounds, II (eds. M. B. Maple and Ø. Fischer), Springer-Verlag, New York, p. 201 (Topics in Current Physics, Vol. 34).
[111]P. Fulde and J. Keller, in: Superconductivity in Ternary Compounds, II (eds. M. B. Maple and Ø. Fischer), Springer-Verlag, New York, p. 249 (Topics in Current Physics, Vol. 34).
[112]M. B. Maple, H. C. Hamaker, and L. D. Woolf, in: Superconductivity in Ternary Compounds, II (eds. M. B. Maple and Ø. Fischer), Springer-Verlag, New York, p. 99 (Topics in Current Physics, Vol. 34).
[113]J. P. Remeika, G. P. Espinosa, A. S. Cooper, H. Barz, J. M. Rowell, D. B. McWhan, J. M. Vandenberg, D. E. Monoton, Z. Fisk, L. D. Woolf, H. C. Hamaker, M. B. Maple, G. Shirane, and W. Thomlinson, Solid State Commun. **34**, 923 (1980).
[114]P. W. Anderson, in: Ternary Compounds (eds. G. K. Shenoy, B. D. Dunlap, and F. Y. Fradin), North-Holland, Amsterdam, 1981, p. 139.
[115]B. D. Dunlap and D. Niarchos, Solid State Commun. **44**, 1577 (1982).
[116]I. Felner and I. Nowik, Solid State Commun. **47**, 831 (1983).
[117]J. W. Lynn, R. Pynn, J. Joffrin, J. L. Ragazzoni, and R. N. Shelton, Phys. Rev. **B27**, 581 (1983).
[118]K. Ishino and Y. Suzumura, Prog. Theor. Phys. **68**, 1776 (1982).
[119]M. Kaufman and O. Entin-Wohlman, Physica (Utrecht) **B + C84**, 77, 90 (1976).
[120]D. Rainer, Z. Phys. **252**, 174 (1972).
[121]L. D. Woolf, M. Tovar, H. C. Hamaker, and M. B. Maple, Phys. Lett. **A71**, 137 (1979).
[122]M. V. Jaric and M. Belic, Phys. Rev. Lett. **42**, 1015 (1979).
[123]A. I. Morozov, Fiz. Tverd. Tela (Leningrad) **22**, 3372 (1980) [Sov. Phys. Solid State **22**, 1974 (1980)].
[124]G. Zwicknage and P. Fulde, Z. Phys. **B43**, 23 (1981).
[125]M. I. Nass, K. Levin, and G. S. Grest, Phys. Rev. Lett. **46**, 614 (1981).
[126]T. V. Ranakrishnan and C. M. Varma, Phys. Rev. **B24**, 137 (1981).
[127]L. N. Bulaevskii, A. I. Buzdin, and M. L.Kulic, Solid State Commun. **41**, 309 (1982); Phys. Lett. **A85**, 161 (1981).
[128]A. I. Buzdin and L. N. Bulaevskiĭ, Fiz. Nizk. Temp. **6**, 1528 (1980) [Sov. J. Low Temp. Phys. **6**, 744 (1980)].
[129]A. I. Buzdin, L. N. Bulaevskiĭ, and S. V. Panyukov, Zh. Eksp. Teor. Fiz. **87**, 299 (1984) [Sov. Phys. JETP **60**, 162 (1984)].
[130]A. S. Borovik-Romanov, Lektsii po nizkotemperaturnomu magnetizmu (Lectures on Low-Temperature Magnetism), Novosibirsk, 1976.
[131]E. A. Turov, Fizicheskie svoĭstva magnitouporyadochennykh kristallov, Nauka, M., 1965 (Engl. Transl. of earlier edition, Physical Properties of Magnetically Ordered Crystals, Academic Press, N. Y., 1965).
[132]L. D. Woolf, Phys. Lett. **A93**, 419 (1983).
[133]M. B. Maple, H. C. Hamaker, D. C. Johnston, H. B. McKay, and L. D. Woolf, J. Less Common Met. **62**, 251 (1978).
[134]Yu. A. Izyumov and Yu. N. Skryabin, Fiz. Met. Metalloved. **49**, 903 (1980).
[135]H. Iwasaki, M. Isino, K.Tsunokuni, and Y. Muto, J. Magn. Magn. Mat. **31–34**, 519 (1983).
[136]K. E. Gray, J. Zasadzinski, R. Vaglio, and D. Hinks, Phys. Rev. **B27**, 4157 (1983).
[137]B. I. Kochelaev, A. R. Tagirov, and M. G. Khusainov, Zh. Eksp. Teor. Fiz. **76** 578 (1979) [Sov. Phys. JETP **49**, 291 (1979)].
[138]N.Toyota, S. B. Woods, and Y. Muto, Solid State Commun. **37**, 547 (1981).
[139]C. F. Majkrzak, S. K. Satija, G. Shirane, H. C. Hamaker, Z. Fisk, and M. B. Maple, Phys. Rev. **B27**, 2889 (1983).
[140]G. K. Shenoy, B. D. Dunlap, F. Y. Fradin, S. K. Sinha, C. W. Kimball, W. Potzel, F. Pröbst, and G. M. Kalvins, Phys. Rev. **B21**, 3886 (1980).
[141]H. B. Radouski, B. D. Dunlap. G. S. Knapp, and D. G. Niarchos, Phys. Rev. **B27**, 5526 (1983).
[142]J. M. D. Coey, J. Appl. Phys. **49**, 1646 (1978).
[143]L. G. Aslamazov, Zh. Eksp. Teor. Fiz. **55**, 1477 (1968) [Sov. Phys. JETP **28**, 773 (1969)].
[144]O. Sakai, M. Suzuki, S. Maekawa, M. Tachiki, G. W. Crabtree, and F. Behroozi, J. Phys. Soc. Jpn. **52**, 1341 (1983).
[145]H. A. Mook, O. A. Pringle, S. Kawarazaki, S. K. Sinha, G. W. Johnston, L. D. Woolf, and H. C. Hamaker, Physica (Utrecht) **B120**, 197 (1983).
[146]T. Kohara, Y. Kohori, K. Kumagai, and K. Asayama, Phys. Lett. **A96**, 425 (1983).
[147]M. L. Kulic, Phys. Lett. **A83**, 46 (1981).
[148]A. I. Buzdin, Zh. Eksp. Teor. Fiz. **87**, 906 (1984) [Sov. Phys. JETP **60**, No. 3 (1984)].
[149]P. G. de Gennes, Superconductivity of Metals and Alloys, Benjamin, New York (1966) (Russ. Transl. IL, Moscow, 1968).
[150]K. Machida, J. Low Temp. Phys. **53**, 405 (1983).
[151]J. W. Lynn, J. A. Gotaas, R. W. Erwin, R. A. Ferrell, J. K. Bhattacharjee, R. N. Shelton, and P. K. Lavins, Phys. Rev. Lett. **52**, 133 (1984).
[152]L. N. Bulaevskiĭ and S. V. Panjukov, Solid State Commun. **53**, 243 (1985).
[153]L. J. Sham and B. R. Patton, Phys. Rev. **B13**, 3151 (1975).
[154]Y. Imry and S.-K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

Translated by Dave Parsons