Additive quark model and multiple production of hadrons

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A review of the current state of the quasinuclear quark model and of its application to hadron-hadron and hadron-nucleus collisions at high energies is presented. Particular attention is devoted to multiple production of secondary particles. It is shown that such processes can be used as a basis for a detailed verification of the hypotheses of quark additivity and quark statistics. Comparison of calculations performed in this model with experimental data provides information on the quark hadronization mechanism.

CONTENTS

1. INTRODUCTION

There is now no doubt about the quark-gluon structure of hadrons. The production of quark and gluon jets has, in fact, been observed in e^+e^--annihilation processes and in deep inelastic collisions. It is currently believed that quantum chromodynamics—the theory of interaction between quarks and gluons—should be capable of describing hadronic matter both at large and small distances. It is commonly accepted that QCD should also provide an explanation of the confinement of quarks and gluons. Asymptotic freedom in QCD provides not only a theoretical justification for the parton model, but also enables us to calculate departures from it. The characteristics of a large number of strong-interaction processes can be calculated within the framework of QCD perturbation theory.

QCD-based phenomenology seems a reasonable way to proceed in the physics of soft processes, where perturbative methods are not directly applicable. This avenue seems sufficiently promising at the present time. It has led to considerable advances in many branches of hadron physics, above all, in the spectroscopy of hadrons (both ordinary, i.e., light, and charmed). The other area in which this approach has been very effective is the physics of soft hadron collisions at high energies, and also relativistic nuclear physics.

The phenomenology of soft processes is essentially based on the concept of the "dressed quark". This provides a bridge between modern ideas and the quark model of the 1960's,[1–4] and enables us to understand its successes. The standard assumption is the hypothesis of small dimensions of dressed quarks as compared with the dimensions of hadrons. One can then speak of the quasinuclear structure of hadrons:[1)] hadrons consist of two (mesons) or three (baryons) dressed quarks separated in space, similarly to the way in which light nuclei consist of nucleons. A dressed quark presents itself as a cluster of quark-gluon matter consisting of a valence quark, gluons, and a sea of quarks and antiquarks.[6–11] The hypothesis of relatively small dimensions of dressed quarks has turned out to be very fruitful in the analysis of soft processes.

Light-hadron spectroscopy[12,13] is based on the use of dressed quarks with masses $m_u \approx m_d \approx 300-360$ MeV, $m_s - m_u \approx 150$ MeV. The same quark masses can be used within the framework of the quasinuclear structure of hadrons to calculate the magnetic moments of baryons[2] and the radiative decay widths of vector mesons.[14–16] Although there have been some discrepancies between observations and predictions, there is, on the whole, reasonable agreement.

[1)]The phrase, "quasinuclear model," was put forward by Lipkin.[5]

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The quasinuclear structure of hadrons should lead to the additivity of hadron interaction processes at high energies (impulse approximation). When applied to hadron-hadron collisions, the additive model predicts the following ratio of total cross sections for NN- and πN-collisions, which is in agreement with experiment:[17,18]

$$\frac{\sigma_{\rm tot}^{\rm NN}}{\sigma_{\rm tot}^{\pi \rm N}}=\frac{3}{2}. \qquad (1)$$

Measurements of total cross sections for the interaction between strange hyperons and nucleons are also in reasonable agreement with the predictions of this model.

Very striking arguments in favor of the additive nature of dresed-quark interactions are provided by hadron-nucleus collisions.[19–30] The quasinuclear structure of hadrons can be used to calculate the ratio of the multiplicity of secondary hadrons in the central region for NA- and πA-collisions at high energies, and leads to the following result for $A \to \infty$:

$$\frac{\langle n_{\rm ch}\rangle_{\rm NA}}{\langle n_{\rm ch}\rangle_{\pi \rm A}}=\frac{3}{2}. \qquad (2)$$

Calculations of the ratio of secondary-particle yields in the region of fragmentation of the incident hadron are based exclusively on the values of the quark-nucleon cross section $\sigma_{\rm inel}^{\rm qN}=\sigma_{\rm inel}^{\rm NN}/3$ and the nucleon density distribution in nuclei. All the relationships obtained in this way[22,24,28] are in good agreement with experiment.

The question of the ratio of the size $r_{\rm q}$ of dressed quarks to the size $R_{\rm h}$ of hadrons is one of the most important for the quasinuclear picture that we are considering. Estimates of $r_{\rm q}$, deduced from various data on hadron-hadron collisions, give somewhat different values, but they all lie in the range[6,31–36]

$$\frac{1}{30}\leqslant\frac{r_{\rm q}^2}{R_{\rm h}^2}\leqslant\frac{1}{5}, \qquad (3)$$

i.e., all estimates indicate that the dimensions of dressed quarks are much smaller than those of hadrons.

The precision attained in the description of experimental data by the quark model is typically in the range 20–30%. Discrepancies of this order have actually been observed in a number of cases (for example, for the magnetic moments of $\Sigma-$ and Ξ hyperons[37,38] and for the ratio of total cross sections $\sigma_{\rm tot}^{\rm NN}/\sigma_{\rm tot}^{\pi \rm N}$). However, in other cases, the agreement is much better for a wide range of phenomena (for example, for the mass splitting in baryon multiplets). Here, one can only fully concur with the view[39] that "this area is full of riddles .... One of them is: why does the nonrelativistic quark model perform so well and why, on the other hand, sometimes it does not?"

Undoubtedly, the quark model based on the assumption of dressed quarks is, on the whole, in agreement with experiment if by this we mean an uncertainty of not more than 20%. However, it is also clear that, taken literally, the quasinuclear hypothesis cannot describe some (possibly very characteristic) details of hadron structure. The understanding of the reason for this would constitute a very important advance in the understanding of the physics of soft processes as well. The way forward is not only to continue theoretical studies of the reasons for (or against) the quasinuclear structure of hadrons, but also to extend the range of validity of the quark model. Investigations in new areas will undoubtedly lead to advances in the understanding of new aspects of the quark structure of hadrons.

The assumption of the quasinuclear structure of hadrons raises a number of problems, the principal of which is the extent to which the hypothesis of spatially separated dressed quarks is consistent with QCD. It is essential to be able to establish whether the existence of two characteristic hadron dimensions, namely, the hadron and dressed-quark radii, is consistent with QCD.

A number of possible reasons for the appearance of the second characteristic dimension in QCD has been discussed. It has been suggested (V. G. Gribov, private communication, 1979) that the small size of the dressed quark is due to the presence of relatively heavy glueballs[40] or of effective massive gluons of mass $M_{\rm g}$ (Refs. 41–43). The range of the effective interaction of the dressed quark due to t-channel heavy-gluon exchanges will then be of the order of $1/M_{\rm g}$.

Another possibility involves the existence of instanton-type QCD vacuum fluctuations. According to the model given in Ref. 36, the appearance of dressed quarks may be due to the presence of instantons with characteristic radius $r_{\rm q}$.

Although the reasons for the appearance of these two characteristic radii are still not completely clear, there are certain well-founded suspicions. The small radius of the dressed quark is probably due to the small effective range of the pure gluon interaction, whereas the radius of hadrons is due to the relatively large size of the region of quark interactions. The large glueball mass, or the large effective mass of gluons, is a manifestation of the same phenomenon, namely, the fact that the gluon confinement radius is appreciably smaller than the quark confinement radius.

The hypothesis of quasinuclear stucture of hadrons has substantially expanded its range of application in recent years. It has been augmented by the rules of quark statistics[44,45] and has provided a basis for the explanation of an extensive range of experimental data on multiple production at high energies with uncertainty characteristic for the quark model, i.e., of the order of 20%. One of the aims of the present review is to provide a systematic account of this approach, and briefly to compare it with alternative models of multiple production such as the recombination model,[46–49] dual topological unitarization,[50–55] and the fragmentation model developed at Lund[56,57] (a more complete bibliography may be found in the papers just cited; see also below).

Another aim of this review is to say something about the situation as a whole: to understand the extent to which the hypothesis of quasinuclear structure of hadrons is a reflection of reality, we must have a sufficiently clear appreciation of both the successes and difficulties of the model.

2. QUARK STRUCTURE OF HADRONS—DRESSED QUARKS

A. Magnetic moments of hadrons and radiative decays of vector mesons

Historically, the first serious success of the quasinuclear quark model was the calculation of the magnetic mo-

TABLE I. Magnetic moments of baryons (in nuclear magnetons).

Particle	Quark model[2]	Experimental value[58]
p	3	2.79
n	−2	−1.91
Λ	−0.67	−0.61±0.01
Σ⁺	2.89	2.36±0.01
Σ⁻	−1.11	−1.17±0.03
Ξ⁰	−1.56	−1.25±0.01
Ξ⁻	−0.56	−0.69±0.04

TABLE II. Values of $\sqrt{\Gamma(\mathrm{V}\to\mathrm{P}+\gamma)}$ (keV$^{1/2}$) for the decay of vector mesons.

Decay	Quark Model[14–16]	Experimental value[58]
$\omega\to\pi^0\gamma$	34.6	28.1±1.6
$\rho\to\pi\gamma$	11.0	8.2±0.4
$\rho^0\to\eta\gamma$	8.4	8.1±0.9
$\varphi\to\eta\gamma$	10.4	8.3±0.6
$K^{*+}\to K^+\gamma$	7.0	7.1±0.4
$K^{*0}\to K^0\gamma$	13.7	8.7±2.0

ments of baryons.[2]

Assuming that the magnetic moment of the dressed quark is equal to the standard Dirac value $\mu_q = e_q(2m_q)$, whereas the masses of nonstrange dressed quarks are 1/3 of the mass of the nucleon ($m = m_d = \frac{1}{3}m_N$), it is possible to calculate the proton and neutron magnetic moments in the "nonrelativistic approximation." The measured magnetic moment of the Λ-hyperon can be used to determine the mass of the strange quark ($m_s/m_u = 3/2$[2)]), after which the magnetic moments of the other strange hyperons can also be calculated. Table I compares the results obtained in this way with experimental data. In all cases, the agreement is typical for quark models: the discrepancy between theoretical and experimental values does not exceed 20–30%.

These discrepancies between the quark model predictions and experimental results have been discussed quite widely (see, for example, Refs. 37 and 38). There are different approaches to their interpretation and different views as to the status of the quark model at this point.

The same values of the quark magnetic moments can be used to calculate the radiative decay widths for vector mesons $\Gamma(\mathrm{V}\to\mathrm{P}+\gamma)$ (see Table II, which lists the values of $\sqrt{\Gamma(\mathrm{V}\to\mathrm{P}+\gamma)}$ which are proportional to the quark magnetic moments). The agreement with experiment is comparable with that in Table I. It is noticeable that the magnetic moments of quarks in mesons and baryons are equal, showing that dressed quarks in hadrons are relatively independent objects resembling quasiparticles.

B. Hadron masses

Traditionally, spectroscopy has been a copious source of information about the structure of particles. Hadrons have been no exception, and their spectroscopy has continued to provide information about their quark structure. The best description of the hadron mass splitting is based on the assumption of De Rujula, Georgi, and Glashow,[59] who proposed that the principal effect here was due to the short-range gluon exchange between the constituent (dressed) quarks. According to Ref. 59, the interaction between two quarks in a baryon or meson in the case of orbital angular momentum $L = 0$ has the form

$$\langle V_{\mathrm{Breit}}\rangle_{L=0} = A\cdot\frac{8\pi}{3}\,\frac{\sigma_1\cdot\sigma_2}{m_1m_2}\,|\,\psi(0)\,|^2, \tag{4}$$

where A is equal to $4\alpha_s/3$ for the meson (M) and $2\alpha_s/3$ for the baryon (B), α_s is the quark-gluon interaction constant, m_i and $\boldsymbol{\sigma}_i$ are the mass and the spin operator of the quark, and $|\psi(0)|^2$ is the square of the quark wave function at $\mathbf{r}_1 = \mathbf{r}_2$ (integrated with respect to $\mathbf{r}_3$ in the case of the baryon). Actually, the entire coefficient in front of $\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2/m_1m_2$ is a phenomenological parameter, but its sign is determined uniquely. The hadron mass splitting in the lowest S-wave SU(6) multiplets, due to the interaction (4), is compared with experiments in Table III. Good agreement has been achieved for $|\psi_M(0)|^2 = |\psi_B(0)|^2$, i.e., the probability of a "collision" between two quarks is the same in both the meson and the baryon.

It is possible to proceed further within the framework of the nonrelativistic quark model and assume that the baryon masses are described by the simple formula based on (4):

$$m_B = \sum_i m_i + b_B \sum_{i<j} \frac{\sigma_i\cdot\sigma_j}{m_im_j}, \tag{5}$$

where $b_B = (8\pi/3)A\,|\psi_B(0)|^2$. This formula is in good agreement with the masses of all baryons in the 56-plet (see Table IV). However, this formula is less satisfactory for mesons for which the theoretical values systematically exceed the experimental data by an amount of the order of 100 MeV. This is readily seen, for example, by comparing the quark-model relation $(1/2)\,[(1/4)m_\pi + (3/4)m_\rho] = (1/3)[(1/2)m_N + (1/2)m_\Delta]$ with experiment. The contributions of the interaction (4) to the right- and left-hand sides of this equation cancel out, and the result should be equal to the mass of the u- or d-quark. The experimental values of the left- and right-hand sides are, respectively, 303 and 363 MeV. The average masses of dressed quarks in mesons and baryons are thus found to be different, but the difference is not particularly large: as before, we see that the quark model frequently leads to a discrepancy of the order of 20% as compared with experiment.

A further advance in the description of the static properties of hadrons within the framework of the potential model of quarks was achieved in Refs. 60 and 61, where account was also taken of the hyperfine quark-quark interaction accompanying gluon exchange (interaction between the magnetic dipole moments of the quarks), and an additional oscillator-type term was introduced into the potential, which ensured the "nonescape" of quarks. This model has succeeded in providing a description of the masses and decay widths of excited baryons (70-plet with $L = 1$ and some higher mul-

[2)]This ratio of the masses of dressed quarks corresponds to the mass difference $m_s - m_u \sim 150$ MeV, which is in good agreement with the estimated mass difference between point u- and s-quarks in QCD.

TABLE III. Mass distribution in the lowest baryon and meson multiplets (MeV) due to the interaction (4) with $|\Psi_B(0)|^2 = |\Psi_M(0)|^2$, $m_u = m_d = 360$ MeV, $m_s/m_u = 3/2$, $b_M/2 - b_B = (8\pi/3) A_B |\Psi_B(0)|^2 = 1.6\cdot 10^6$ MeV.

	Calculation[59]	Experiment[58] (averaged over isomultiplets)		Calculation[59]	Experiment[58] (averaged over isomultiplets)
$m_\Delta - m_N$	300	295	$m_{\Xi^*} - m_\Xi$	200	217
$m_\Sigma - m_\Lambda$	68	77	$m_\rho - m_\pi$	600	632
$m_{\Sigma^*} - m_\Lambda$	267	274	$m_{K^*} - m_K$	400	398

tiplets), and in improving the description of the static properties of the lowest baryons (for example, the charge radius of neutrons).

The interaction (4) gives reasonable results for the mass splitting of charmed hadrons. Thus, it follows from (4) that

$$\frac{m_{D^*} - m_D}{m_\rho - m_\pi} \approx \frac{m_u}{m_c},$$

which is in reasonable agreement with experiment.

The static properties of hadrons are thus satisfactorily described by the quark model with the interaction given by (4), which may arise during exchange of effective "heavy gluons." The presence of this short-range interaction is an argument in favor of the fact that dressed quarks can be treated as real objects of small radius.

Another interesting result provided by hadron spectroscopy, which is important for the presentation given below, is that the quark functions are approximately the same for mesons and baryons: $|\Psi_M(0)|^2 \approx |\Psi_B(0)|^2$.

C. Interactions between hadrons at high energies and quark additivity

We now recall the well-known arguments in favor of the impulse approximation in hadron collisions at high energies, which confirm the hypothesis of the quasinuclear structure of hadrons. An important consequence of this approximation is the Levin-Frankfurt relation (1)[17,18] for the total cross sections. The diagrams of Fig. 1 lead not only to (1) but also to the relations for the differential πN- and NN-scattering cross sections:[62]

$$\frac{d\sigma(\pi p \to \pi p)}{dt} = |a_{qq}(t)|^2 F_p^2(t) F_\pi^2(t), \qquad (6)$$
$$\frac{d\sigma(pp \to pp)}{dt} = |a_{qq}(t)|^2 F_p^4(t),$$

where $F_p(t)$ and $F_\pi(t)$ are the proton and pion formfactors, respectively, and $a_{qq}(t)$ is the quark elastic scattering amplitude. For sufficiently high initial energies, it may be considered that the interaction amplitude is the same for quarks and antiquarks: $a_{qq}(t) = a_{q\bar{q}}(t) = a_{\bar{q}\bar{q}}(t)$.

The experimental status of (1) will be clear from an inspection of Fig. 2: The experimental and theoretical results agree to within 10–15%. The agreement deteriorates with increasing energy. The same conclusion follows form an analysis of the dispersion relations,[63] which shows that $\sigma_{tot}^{\pi p}$ increases more rapidly than σ_{tot}^{pp} in the region $p \sim 1000$ GeV/c (in which there are no direct measurements).

At moderately high energies, we can neglect the dimensions of the dressed quarks as compared with the dimensions of hadrons [i.e., the dependence of $a_{qq}(t)$ on t in (6)]. The dependence of $d\sigma/dt$ on t is then exclusively determined by the formfactors of the colliding particles. This behavior is, in fact, observed experimentally for $p_{lab} \approx 10$–20 GeV/c.

In the energy range accessible at the present time, the interaction amplitude of the stange quark is appreciably different from that for nonstrange quarks. This leads to the following relationships between the hyperon-nucleon scattering cross sections:

$$\sigma_{\Lambda N} - \sigma_{pN} \approx \sigma_{\Sigma N} - \sigma_{pN} \approx \frac{1}{2}(\sigma_{\Xi N} - \sigma_{pN}). \qquad (7)$$

Experimental data for $p_{lab} = 135$ GeV/c yield the following values:[64]

$$\sigma_{pp} - \sigma_{\Sigma^- p} = 4.3 \pm 0.3 \text{ mb},$$
$$\sigma_{pp} - \sigma_{\Xi^- p} = 9.1 \pm 0.3 \text{ mb},$$
$$\sigma_{pn} - \sigma_{\Sigma^- n} = 5.3 \pm 0.4 \text{ mb},$$
$$\sigma_{pn} - \sigma_{\Xi^- n} = 9.2 \pm 0.5 \text{ mb}.$$

They are in good agreement with (7).

TABLE IV. Masses of baryons in the S-wave 56-plet (MeV). The parameters in (5) are the same as in Table III.

Particle	Calculated from (5)	Experiment (averaged over isomultiplets)[58]	Particle	Calculated from (5)	Experiment (averaged over isomultiplets)[58]
N	930	937	Σ*	1377	1384
Δ	1230	1232	Ξ	1329	1318
Σ	1178	1193	Ξ*	1529	1533
Λ	1110	1116	Ω	1675	1672

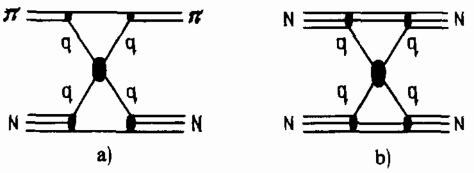

FIG. 1. Elastic pion-nucleon (a) and nucleon-nucleon (b) scattering in the quasinuclear quark model.

The size of the dressed quark estimated within the framework of the quasinuclear hadron-structure hypothesis enables us to verify the self-consistency of this hypothesis. The radius of the dressed quark can be estimated from the total hadron scattering cross section by expressing it in terms of the total quark-quark scattering cross section in accordance with the diagrams of Fig. 1. At moderately high energies, $\sigma_{tot}(qq) \approx \sigma_{tot}(NN)/9 \approx 4.5$ mb. Assuming that the total quark scattering cross section is determined by their geometric dimensions, $\sigma_{tot}(qq) \approx 2\pi(2r_q)^2$, we obtain[32] $r_q^2 \approx 0.5\ \mathrm{GeV}^{-2}$.

Another way of estimating the radius of the dressed quark is based on the elastic scattering of hadrons. The Regge parametrization of hadron amplitudes leads to the expression $|a_{qq}(t)|^2 = A \exp[2\alpha'_p t \ln(s/s_0)]$, for the elastic quark-quark amplitude, where A and s_0 are certain constants and α'_P is the slope of the Pomeranchuk trajectory. From the standpoint of the quark model, the quantity $\alpha'_P \ln(s/s_0)$ characterizes the sizes of the colliding quarks. If we suppose that $|a_{qq}(t)|^2$ [see (6)] determines the dimensions of the dressed quarks in the same way that $F_p(t)$ determines the size of the nucleon, we obtain $r_q^2 \approx 3\alpha'_P \ln(s/s_0)$. Experimental estimates show that $\alpha'_P \approx 0.1 - 0.3\ (\mathrm{GeV}/c)^{-2}$. If we adopt the common assumption that $s_0 \approx 1\ \mathrm{GeV}^2$, we find that $r_q^2/R_p^2 \approx 1/3$–$1/12$ for $s \approx 100$–$1000\ \mathrm{GeV}^2$. However, if we recall that s_0 actually characterizes the energy range $s > s_0$ dominated by amplitudes with Regge-pole exchange, the more probable value seems to be $s_0 \approx 20$–$50\ \mathrm{GeV}^2$. We then obtain $r_q^2/R_p^2 \approx 1/5$–$1/20$. This ratio should increase with increasing energy. The same Regge parametrization can be used to try to isolate the exponential dependence of the amplitude $a_{qq}(t)$ against the background of the power-type behavior of formfactors, directly from data on inelastic hadron scattering (this method was proposed in Ref. 65). Data on $d\sigma(pp \to pp)/dt$ and $d\sigma(\pi p \to \pi p)dt$ show that[32] $r_q^2/R_p^2 \approx 1/10$–$1/20$.

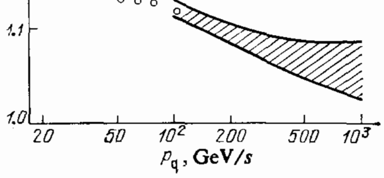

FIG. 2. Experimental status of (1): open circles—direct measurements; shaded area—predictions from Ref. 63 (p_q—quark momentum in the laboratory system, $p_q = p\pi/2 = pN/3$).

The value of r_q^2 estimated from the ratio of diffraction to absorption processes yields[35] $r_q^2 \approx 1.2$–$1.8\ \mathrm{GeV}^{-2}$, i.e., $r_q^2/R_p^2 \approx 1/10$.

It is clear that, although different estimates lead to different values for the radius of the dressed quark, it is found that $r_q^2 \ll R_p^2$ in all cases. This important result enables us to judge the degree of self-consistency of the hypothesis of quasinuclear hadron structure.

There are published estimates of the dimensions of dressed quarks, based on the analysis of data on hard processes, but these estimates are not as yet entirely reliable. Data on the production of lepton pairs with high effective masses, and on the production of hadrons with high transverse momenta, lead to values between 1/5 and 1 for r_q^2/R_p^2, depending on which model is used to analyze the data.[31,34,66] The model-independent estimate of r_q^2/R_p^2 obtained by comparing QCD calculations with data on the correlation function in deep inelastic collisions (fourth-order twist) shows that the more likely result is $r_q^2/R_p^2 \approx 1/5$[67].

Quasinuclear hadron structure presupposes the presence of corrections to the impulse approximation that are due to multiple interactions of quarks and the corresponding shadow effects. Analyses of elastic scattering data with allowance for these corrections have continued to be reported for many years (see Refs. 68–73 and the references therein). Allowance for multiple quark collisions leads to a characteristic behavior of the elastic differential hadron scattering cross section. As an example, Fig. 3 shows the elastic cross section $d\sigma(pp \to pp)/dt$, calculated with allowance for the shadow corrections. Arrow II in this figure shows the value of t at which the second diffraction minimum could appear. (Its absence was ascribed in Ref. 72 to the large real part of the amplitude for $|t| > 2\ \mathrm{GeV}^2$.)

However, the description of the elastic cross section by the quark model for $|t| > 0.5\ \mathrm{GeV}^2$ with multiple-scattering effects taken into account, in fact, be regarded as totally satisfactory. So far, it has not been possible to achieve a description of $d\sigma(pp \to pp)/dt$ for both small and large $|t|$. The im-

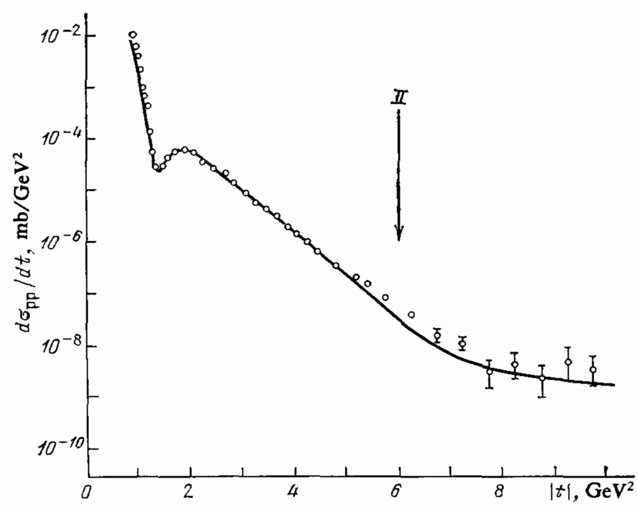

FIG. 3. Elastic pp-scattering for large $|t|$ (Ref. 74) in the quark model.[72]

possibility of this description within the framework of the usual quasinuclear model of hadrons has been noted before (P. E. Volkovitskiĭ, A. M. Lapidus, and K. A. Ter-Martirosyan, private communication, 1979). Shadow corrections of the usual type provide a larger contribution to the pp-scattering amplitude than to the πp-amplitude because of the larger number of quarks in the proton. Thus, allowance for the multiple scattering of quarks leads to a reduction in the magnitude of the ratio given by (1), i.e., $\sigma_{\rm tot}^{\rm pp}/\sigma_{\rm tot}^{\pi{\rm p}} < 3/2$, whereas the opposite situation prevails in experiment, namely, $\sigma_{\rm tot}^{\rm pp}/\sigma_{\rm tot}^{\pi{\rm p}} \approx 1.6$–$1.65$ (see Fig. 2). It is known from analyses of shadow corrections to scattering by deuterons that inclusion of inelastic screening at high energies leads to the appearance of antishadow corrections.[75,76] However, in the case of the deuteron, the inelastic contributions are always smaller than the elastic contributions. The large value of the ratio $\sigma_{\rm tot}^{\rm pp}/\sigma_{\rm tot}^{\pi{\rm p}}$ can be explained in the model[36] with "grainy" instanton structure of the QCD vacuum. This model involves strong quark-antiquark attraction in the state with $J^P = 0^-$. The presence with a 20–30% probability of a "passive" component in pseudoscalar mesons, with the quarks screening one another (and therefore interacting only weakly with other particles), could remove the contradiction. Definite conclusions with regard to multiple quark-quark collisions could be drawn from an analysis of diffraction scattering of hadrons by nuclei.[77] It is still not clear, however, whether the hypothesis of the "passive" component in pions agrees with experimental data because the amount of such data is still inadequate.

The principal conclusion that can be drawn from the analysis of data on hadron interactions at high energies is the same as before, namely, that the hypothesis of quasinuclear hadron structure is confirmed experimentally to within about 20%.

D. Multiple production of hadrons: the space-time picture

The quasinuclear hadron structure leads to a very characteristic space-time picture of multiple production of hadrons. The process proceeds in three stages that are well separated in time, namely, the QCD stage of quarks and gluons, the dressed quark stage, and the hadron stage. The possibility of this temporal separation of the three stages of the production process is based on the idea of production time that was introduced quite a long time ago[78–80] (see the discussion in the review paper of Ref. 29) and remains one of the cardinal features of the quark-parton hypothesis.[81,82] From the point of view of the hadron structure that we are considering here, the important point is that the formation of compound systems is also characterized by its own productions time.

As an example, let us consider the production of secondary particles in deep inelastic lepton-nucleon collisions. The virtual photon (or weakly interacting vector boson) interacts with one of the parton-quarks in the nucleon and ejects it. Two spectator quarks and the "wounded" dressed quark from which the parton has been ejected are left behind. The parton-quark traveling in the direction of the virtual photon produces a jet of new parton-quarks and gluons, which then transforms into a jet of dressed quarks and, finally, the dressed quarks coalesce into the hadrons observed experimentally. Let us estimate the lifetimes of the intermediate stages of the multiple production process, i.e., the parton-quark and gluon stage and the dressed-quark stage. The dressed-quark production time in the system in which it is at rest should be determined by its radius $r_{\rm q}$. If this quark has momentum p, the corresponding time must be of the order of $r_{\rm q}p/m$, where $m \approx 0.3$–0.4 GeV is the mass of the dressed quark. This is obviously the lifetime of the parton-quark and gluon cloud. (Naturally, the slow parton-quarks and gluons undergo "clusterization" into dressed quarks well before fast ones do.)

The lifetime of the cloud of dressed quarks is determined by the time they take to coalesce into hadrons. For composite systems, their production time is of the order of the time necessary for a "collision" between the constituents, i.e., of the order of $\tau_0 \sim R/v$, where R is the radius of the system and v is the average velocity of the constituents. Since in compound systems $v \sim 1/(mR)$, we have $\tau_0 \sim R^2 m$. Fast hadrons are thus produced from dressed quarks in a time of the order of $R^2 p$. This means that the lifetime of the cloud of dressed quarks is appreciably longer than their production time, i.e., the duration of the stage of parton-quarks and gluons [the corresponding numerical factor is $mR(R/r_{\rm q}) \sim 4$–5, which is considered parametrically large in the quasinuclear model]. It may then be supposed that the idea of soft decoloration and soft hadronization of dressed quarks is in reasonable agreement with the quasinuclear picture of hadron structure. In fact, a large number of new $q\bar{q}$ pairs is created during the lifetime of the dressed-quark cloud, and a degree of equilibrium is established in this cloud. This means that the probability that each quark will find a suitable partner for the creation of a hadron is determined by statistical factors, and there is no essential modification of the momentum distributions.

We now turn to multiple production in deep inelastic lepton-nucleon collisions. In the system in which the target nucleon is at rest, the spectator quarks are almost at rest so that, in accordance with the foregoing discussion, they should become hadronized by the nucleon. The "wounded" quark and the slow component of the parton-quarks and gluons produce one or two pairs with suitable color in a time of the order of $r_{\rm q}$ after which the spectator quarks g_j and g_k transform into a baryon B_{jk} (Fig. 4a), or into a meson M_j and baryon B_k (Fig. 4b), or into two mesons M_j and M_k (Fig. 4c).

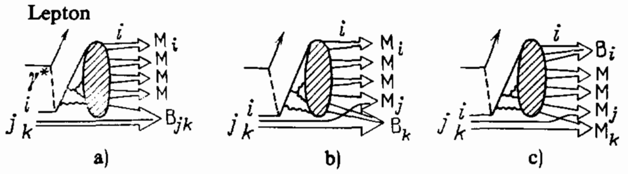

FIG. 4. Hadron production in deep inelastic collisions. The particle momenta in the laboratory system increase in the upward direction; the hadrons B_{jk}, B_k, M_j, M_k, are the slowest, and B_i, M_i the fastest. The hadronization process in this sytem proceeds in the upward direction. The first quarks to transform into hadrons are q_j, q_k, and the last is the quark q_i.

The newly produced dressed quarks do not, on average, carry any special quantum numbers (they are the constituent sea quarks or antiquarks, q or $\bar{q}$).

After the hadronization of spectator quarks, their color quantum numbers appear in the faster part of the jet. Next, the faster constituent sea quarks are hadronized in a time of the order of R_p^2, and the "unsuitable" quantum numbers are transferred upward along the multiperipheral comb. In the final stage, the region of current fragmentation is hadronized. Since the hadronization process occurs sequentially (in the upward direction in Fig. 4), the region of current fragmentation automatically contains quantum numbers ensuring the production of white hadron states. In this region, the constituent sea quarks unite with the constituent quark q_i which has the quantum numbers of the ejected parton-quark. The meson $M_i = q_i\bar{q}$ (see Figs. 4a and b) is produced when this is not accompanied by baryon charge transfer along the multiperipheral chain, whereas the baryon $B_i = q_i qq$ (see Fig. 4c) is produced when this transfer does occur.

The hadronization picture that we are considering is not invariant because the hadronization time of different parts of the jet will be different in different reference frames. This is a general property of all multiperipheral processes,[82] although the properties of the final physical states do not depend on the choice of the coordinate frame, i.e., covariance prevails.

Hadronization in e^+e^--annihilation occurs in an analogous manner. The virtual γ-ray produces a pair of point partons (quark and antiquark) that are emitted in opposite directions in the center-of-mass system. This pair creates new parton-quarks and gluons which then transform into dressed quarks. The latter combine into hadrons, and the final result is the appearance of two hadron jets emitted in opposite directions (if, of course, hard gluons capable of initiation of other jets are not emitted at earlier stages). As in the case of deep inelastic collisions, here again we have fragmentation hadrons (M_i and $\bar{B}_{\bar{i}}$ in Fig. 5) that carry the quantum numbers of the original parton-quarks, and sea hadrons (M and B) without special quantum numbers.

The mechanism of soft hadronization and soft decoloration is one of the most important features of the space-time picture that we are considering. Arguments supporting the presence of these mechanisms will emerge from the following description of multiple production processes. However, direct manifestations of these mechanisms can be seen in processes involving the diffractive dissociation of hadrons.

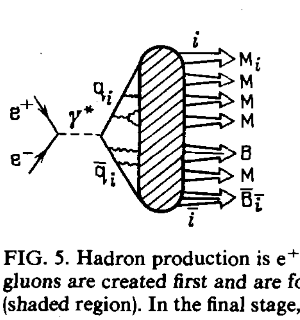

FIG. 5. Hadron production is e^+e^--annihilation. The QCD quarks and gluons are created first and are followed by the cloud of dressed quarks (shaded region). In the final stage, dressed quarks coalesce into hadrons.

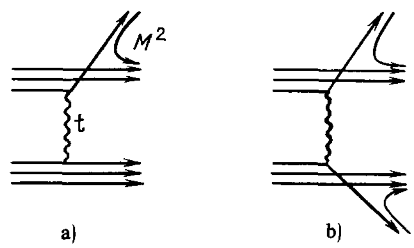

FIG. 6. Diffractive dissociation of one (a) and both (b) colliding nucleons in the quasinuclear quark model.

Diffractive dissociation occurs as a result of elastic scattering of quarks and the subsequent creation of one or more quark-antiquark pairs[83,84] (Fig. 6). When the creation of a new quark-antiquark pair occurs "softly," the momentum transferred to the nucleon that has remained intact in the process shown in Fig. 6a should be equal with sufficient accuracy to the momentum transferred to the quark in the other nucleon, i.e., the meson created by it.[84] For weakly bound compound systems (in the limit of zero binding energy), the relation between the square of the effective mass of the disintegrating system and the square of the momentum p transferred to the nucleon should be $M^2 = M_0^2 + 2|t|$, where M_0 is some effective mass of the system of dissociated quarks. This equation is not literally satisfied in real situations, but it may be expected that the diffractive dissociation spectrum $d^2\sigma/dM^2dt$ should contain a peak for a fixed p when $M^2 = M_0^2 + 2|t|$. Experimental data confirm this shift of the peak in $d^2\sigma/dM^2dt$ spectra (Figs. 7 and 8).

Soft hadronization should lead to a whole series of other correlation effects in diffractive dissociation, the observation of which would be of undoubted interest.

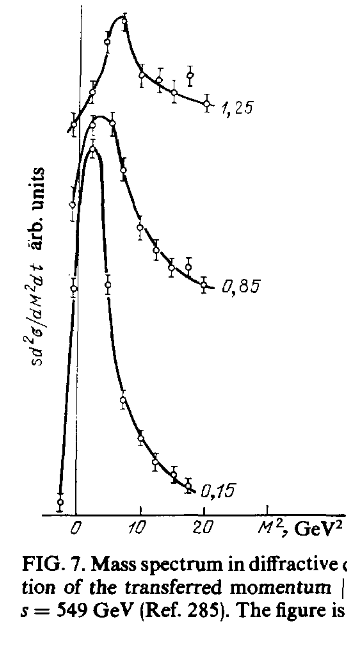

FIG. 7. Mass spectrum in diffractive dissociation of the nucleon as a function of the transferred momentum $|t| = 0.15$, 0.85, and 1.25 GeV² for $s = 549$ GeV (Ref. 285). The figure is taken from Ref. 84.

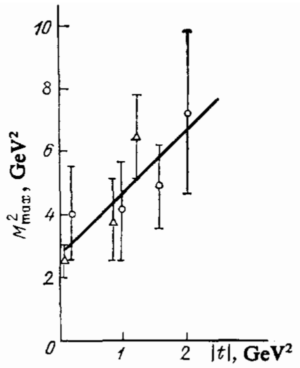

FIG. 8. Position of the peak in the mass spectrum of diffractive dissociation of the nucleon as a function of the transferred momentum.

E. Structure of multiple production in hadron-hadron collisions

When the quasinuclear hadron structure is taken into account, the process of multiple production in hadron-hadron collisions becomes the process of quark production in quark-quark collisions[86] (Fig. 9). The distinguishing feature of this structure of hadron collisions is the spectator mechanism of production of secondary fragmentation particles. Each of the dressed quarks carries about 1/3 of the momentum in the incident proton, and about 1/2 of the momentum in the meson, so that hadrons containing these spectator quarks are responsible for the fastest part of the secondary-particle spectrum.

Let us examine in greater detail the composition of secondary particles in nucleon-nucleon and meson-meson collisions.

Suppose that the incident baryon B_{ijk} consists of three dressed quarks i, j, and k, where quark k interacts whereas quarks i and j are spectators. Two possible cases then arise. Quark k may be scattered elastically but may produce diffractive dissociation of the target. Quarks i, j, and k then again combine into the original baryon B_{ijk} (with $x \gtrsim 0.9$), i.e., we have inelastic diffractive scattering $B_{ijk}N \to B_{ijk}X$, the probability Δ of which is 10–12% (Fig. 10a). In the other case (Fig. 10b–d), the quark k interacts in a truly inelastic fashion with probability $1 - \Delta$, and a sufficient number of quark-antiquark pairs is produced. The final quarks include

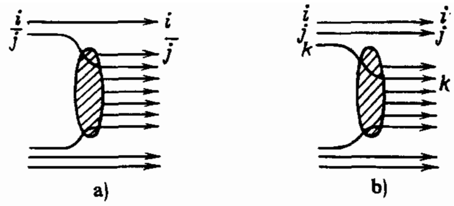

FIG. 9. Elastic pion-nucleon (a) and nucleon-nucleon (b) collisions in the quasinuclear quark model.

one special quark k' with the quantum numbers of the initial quark k (quark k' will henceforth be referred to as a tagged quark).

The spectator quarks i, j and the tagged quark k' transform into hadrons by coalescing with the newly created (sea) dressed quarks. Soft hadronization and soft decoloration then presuppose that no small quantities arise in the capture processes of Figs. 10b–d. The momenta of the secondary fragmentation hadrons are then largely determined by the momenta of the spectator quarks. The baryons B_{ij} (see Fig. 10b), whose composition includes two spectator quarks, fill the region $x \sim 2/3$. The baryons B_i, B_j and mesons M_i, M_j with one spectator quark (Figs. 10c and d) fall into the region $x \sim 1/3$. Hadrons whose composition includes the tagged quark, B_k and M_k (see Figs. 10b–d) usually have $x \sim 1/6$–$1/12$.

Finally, the slowest part of the spectrum is due to hadrons that do not contain quarks with special quantum numbers. These sea hadrons dominate the region $x \lesssim 0.1$–0.05.

Thus, in general, the baryon B_{ijk} can transform into different secondary particles:

$$B_{ijk} \to \Delta B_{ijk} + \tilde{\xi}_1 B_{ij} + \tilde{\xi}_2 (B_i + B_j) + \tilde{\xi}_3 (M_i + M_j) + \tilde{\xi}_4 B_k + \tilde{\xi}_5 M_k + \text{sea hadrons}\,(M, B, \bar{B}). \quad (8)$$

The coefficients in this expression are the transition probabilities (or average multiplicities). They are related by conditions that conserve the total probability in the hadronization of spectator quarks ($\Delta + \tilde{\xi}_1 + \tilde{\xi}_2 + \tilde{\xi}_3 = 1$) and the tagged quark ($\Delta + \tilde{\xi}_4 + \tilde{\xi}_5 = 1$), and the conservation of baryon charge ($\Delta + 2\tilde{\xi}_2 + \tilde{\xi}_1 + \tilde{\xi}_4 = 1$). The final result for the in-

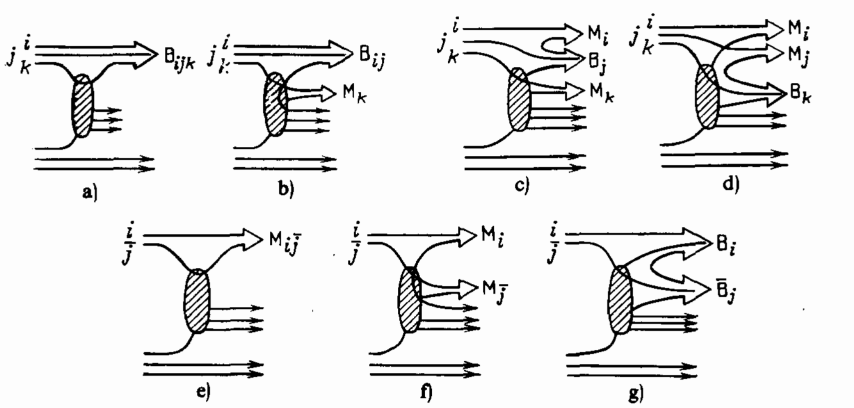

FIG. 10. Creation of secondary particles in inelastic nucleon-nucleon (a–d) and meson-nucleon (e–g) collisions.

elastic interaction of the baryon then is

$$\left.\begin{aligned}
B_{ijk} \to \Delta B_{ijk} + & \quad x \geqslant 0,9, \\
+(1-\Delta)\,\xi_1 B_{ij} + & \quad x \sim 2/3, \\
+(1-\Delta)\,[\xi_2 (B_i + B_j) + (1-\xi_1-\xi_2)(M_i + M_j)] + & \quad x \sim 1/3, \\
+(1-\Delta)\,[(1-\xi_1-2\xi_2)\,B_k + (\xi_1 + 2\xi_2)\,M_k] + & \quad x \sim 1/6-1/12, \\
+N(s)\,[dM + B + \bar{B}] & \quad x \leqslant 0,1.
\end{aligned}\right\} \qquad (9)$$

The last two terms describe the contribution of the sea particles, which is determined by the two parameters N and d describing the mean multiplicity in the central region, and the relationship between the secondary mesons and baryons in this region.

Let us now consider the case of a beam of π- or K-mesons consisting of quark i and antiquark $\bar{j}$. In this situation, a total of only four types of secondary hadron can be produced (see Figs. 10d–g), namely, diffractive $M_{i\bar{j}}$, spectator ($x \sim 1/2$), tagged ($x \sim 1/4$–$1/8$), and sea hadrons. The probabilities of these transitions are determined by coefficients $\delta \simeq 0.08$–0.1 (contribution of diffractive dissociation) and η, which determine the ratio of baryons to mesons among the spectator and tagged hadrons. The result is

$$\left.\begin{aligned}
M_{i\bar{j}} \to \delta M_{i\bar{j}} + & \quad x \geqslant 0,9, \\
+\tfrac{1-\delta}{2}\,[\eta\,(B_i + \bar{B}_{\bar{j}}) + (1-\eta)\,(M_i + M_{\bar{j}})] + & \quad x \sim \tfrac{1}{2}, \\
+\tfrac{1-\delta}{2}\,[\eta\,(B_i + \bar{B}_{\bar{j}}) + (1-\eta)\,(M_i + M_{\bar{j}})] + & \quad x \sim \tfrac{1}{4} - \tfrac{1}{8}, \\
+N(s)\,(dM + B + \bar{B}). & \quad x \leqslant 0,15.
\end{aligned}\right\} \qquad (10)$$

We note that the quasinuclear hadron structure leads to an interesting consequence for the spectra of secondary particles in meson-nucleon collisions. The spectra of secondary mesons in meson-nucleon collisions are not symmetric in the center-of-mass system of the colliding hadrons, since the quarks in the meson and nucleon carry different fractions of total momentum [this can be established by comparing (9) with (10)]. However, symmetry should be reestablished in the center-of-mass system of the quark-quark interaction. This restoration of symmetry has been confirmed experimentally (see, for example, Refs. 87–89 and Fig. 11).

Hadron production is frequently treated as a direct transition (recombinational[46–49,90–95] or fragmentational[50,51,96]) of parton-quarks into hadrons. In this approach, it is possible (with some additional assumptions) to describe the creation of secondary pions and kaons, but the analysis of fragmentation baryons encounters some difficulties that are a reflection of the fact that such models give rise to the problem of the role of gluons in the hadron production process. This problem does not arise when multiple creation is described in the language of dressed (constituent) quarks. Hadronization during the dressed-quark stage has become a popular topic in both recombination[10] and fragmentation[56,57,97] models.

F. Hadron-nucleus collisions

Inelastic hadron-nucleus collisions at high energies constitute[19–30] an important way of verifying the spectator mechanism of hadron fragmentation. The specific beam fragmentation picture that arises in the hadron-nucleus collision at sufficiently high energies is undistorted by purely nuclear effects: as explained before, the secondary-hadron production time is proportional to the hadron momenta, so that fast fragmentation hadrons are formed outside the nu-

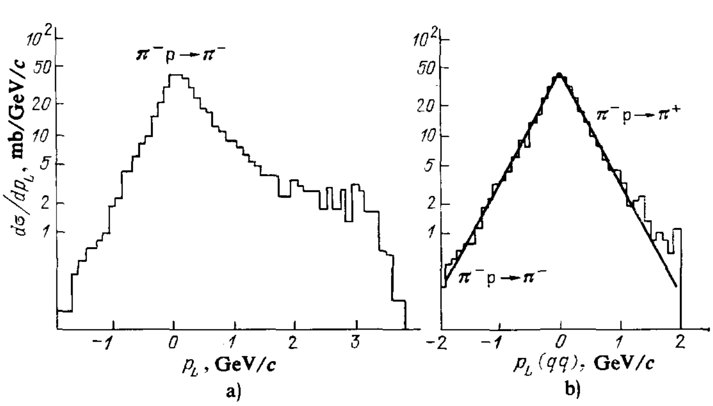

FIG. 11. Distribution of secondary π^--mesons created in π^--p collisions at 25 GeV/c (Ref. 87) over longitudinal momenta in the π^-p center-of-mass system (a) and in the center-of-mass system of the colliding quarks (b).

cleus and cannot therefore give rise to secondary interactions.

Secondary interaction effects can be used to obtain information on the dressed-quark production time which is traditionally written in the form[98] $\tau = p/\mu^2$. Estimates based on analyses of experimental data yield $\mu^2 \lesssim 0.3$ GeV^2 (Ref. 99), $\mu^2 \simeq 0.5$ GeV^2 (Ref. 100), and $\mu^2 = 0.7$ GeV^2 (Ref. 24). We recall that, in Section 4, we found that $\mu^2 = m_q/r_q \sim 0.2$–0.4 GeV^2, which is in reasonable agreement with the first two of these estimates.

A detailed discussion of hadron-nucleus interactions is given in the review paper of Ref. 29. Here, we shall confine our attention to results that reflect most clearly the quasinuclear quark structure of hadrons.

In a hadron-hadron encounter, only one pair of quarks collides in the impulse approximation: one each from the incident particle and the target. When a hadron collides with a heavy nucleus, its constituents can interact with different nucleons in the nucleus independently of one another, so that we have interactions involving the participation of several quarks in the fast particle. In the case of real nuclei (even for $A \sim 200$), some of the constituent quarks may pass through the nucleus without interaction. The number of such quarks determines the multiplicity of the fragmentation hadrons for large values of x.

Three different processes are possible in the baryon-nucleus collision, namely, one quark interacts and two pass through the nucleus (Fig. 12a), two quarks interact and one passes through the nucleus (Fig. 12b), and all three quarks interact (Fig. 12c). In the meson-nucleus collision, either one (Fig. 12d) or both (Fig. 12e) constituent quarks in the beam can interact.

The probabilities of these processes can be calculated for a given distribution of nuclear matter and given cross section for the inelastic interaction between quark and nucleon:

$$\sigma_q \equiv \sigma_{\text{inel}}^{qN} \approx \frac{1}{3}\sigma_{\text{inel}}^{NN} \approx \frac{1}{2}\sigma_{\text{inel}}^{\pi N} \approx 10 \text{ mb}. \qquad (11)$$

The probability of these processes can be written in the form[22]

$$V_k^h = \frac{n!}{(n-k)!\,k!\,\sigma_{\text{prod}}^{hA}} \int d^2\mathbf{b}\, e^{-(n-k)\sigma_q T(\mathbf{b})} \left(1 - e^{-\sigma_q T(\mathbf{b})}\right)^k, \qquad (12)$$

where k is the number of interacting quarks and h is the initial hadron consisting of n quarks ($n = 2$ for mesons and $n = 3$ for baryons). The function $T(b)$ can be expressed in

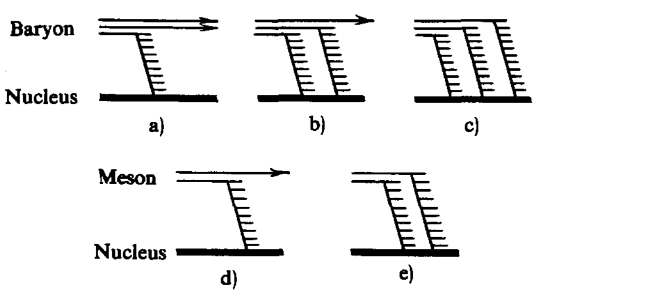

FIG. 12. Inelastic collision of baryons (a–c) and mesons (d–e) with nuclei in the quasinuclear quark model.

terms of the nucleon distribution desity in the nucleus:

$$T(\mathbf{b}) = A \int_{-\infty}^{\infty} dz\rho(\mathbf{b}, z), \quad 4\pi \int_{-\infty}^{\infty} dr\rho(r)\, r^2 = 1 \qquad (13)$$

($r = \sqrt{b^2 + z^2}$). The cross sections $\sigma_{\text{prod}}^{hA}$ have the same significance for hadron-nucleus collisions as the inelastic cross section for hadron-hadron collisions, and are obtained from the normalization condition $\Sigma_i V_i^h(A) = 1$:

$$\sigma_{\text{prod}}^{hA} = \int d^2b\, (1 - e^{-n\sigma_q T(b)}). \qquad (14)$$

The probabilities $V_i^h(A)$ calculated from the nuclear density functions found from eA collisions[101] for p, π, and K beams are given in Refs 22 and 102. In light nuclei, the dominant processes are those involving the interaction of one of the constituent quarks (Figs. 12a and d), but even for Be the probability of the process of Fig. 12b with two interacting quarks is not small (about 25%). For $A \sim 100$, the probabilities of all three proton fragmentation processes are of the same order.

The calculated probabilities $V_i^h(A)$ can be used to determine the multiplicity ratios for different regions of hadron-nucleus collisions.

When interacting dressed quarks produce secondary particles independently, the multiplicity in the central region for the processes of Figs. 12b and e will be greater by a factor of two than in the processes of Figs. 12a and d, whereas, for the process of Fig. 12c, it will be greater by a factor of three. We then find that the multiplicity ratio for secondary particles in pA and πA collisions in the central region is given by

$$R_0\left(\frac{pA}{\pi A}\right) = \frac{\langle n\rangle_{pA}}{\langle n\rangle_{\pi A}} = \frac{V_1^p + 2V_2^p + 3V_3^p}{V_1^\pi + 2V_2^\pi} = \frac{3}{2}\,\frac{\sigma_{\text{prod}}^{\pi A}}{\sigma_{\text{prod}}^{pA}}. \qquad (15)$$

For heavy nuclei, $\sigma_{\text{prod}}^{\pi A}/\sigma_{\text{prod}}^{pA} \approx 1$, and we obtain (2). Figure 13 shows a comparison between $R_0(pA/\pi A)$ and experimental data.

The production of fast secondary particles with $x \sim 1/2$ in pion-nucleus and pion-nucleon collisions is determined by the same process, involving the interaction of one quark from the incident pion (Figs. 9a and 12d). Hence, the multiplicity ratio for secondary fragmentation particles in πA and

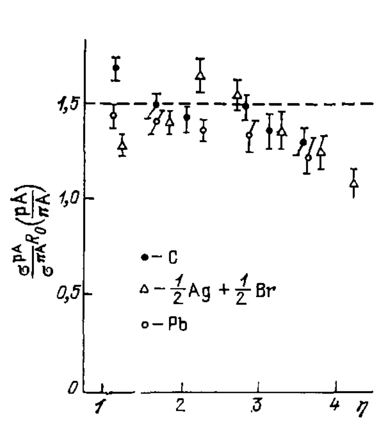

FIG. 13. Experimental verification[21] of (15): broken line—*prediction* of the quasinuclear model.

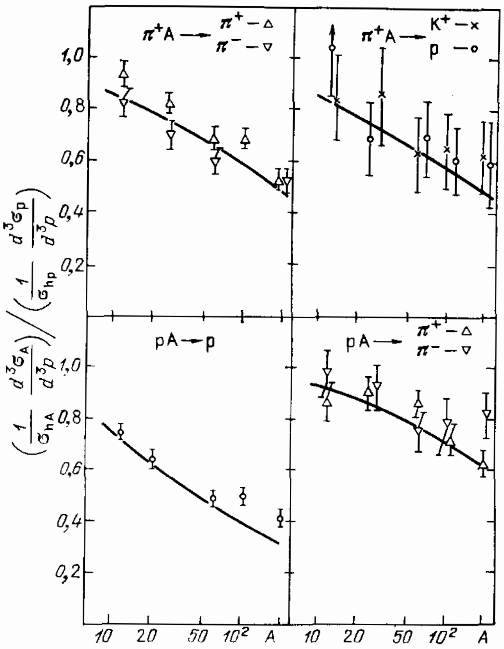

FIG. 14. Comparison of the predictions of the quasinuclear model with experimental data[103] at 100 GeV/c . $p_\perp = 0.3$ GeV/c. $\pi^+A \to \pi^\pm X$:$x = 0.5$; $\pi^+A \to K^+X$ and $\pi^+A \to pX$:$x = 0.5$; pA→pX:$x = 0.7$; pA→$\pi^\pm$ X:$x = 0.3$.

πp collisions for $x \sim 1/2$ is independent of x and given by[22]

$$R_{1/2}\left(\frac{\pi A}{\pi p}\right) = \left(\frac{1}{\sigma^{\pi A}_{\text{prod}}}\frac{d^3\sigma_{\pi A\to h}}{d^3p}\right)\left(\frac{1}{\sigma^{\pi p}_{\text{inel}}}\frac{d^3\sigma_{\pi p\to h}}{d^3p}\right)^{-1} = V^{\pi}_1(A). \quad (16)$$

For a beam of nucleons, the production of secondary nucleons with $x \sim 2/3$ is also found to proceed preferentially through the processes of Figs. 9b and 12a. On the other hand, the process of Fig. 12b may also contribute to the production on nuclei of mesons with $x \sim 1/3$. The result is[22]

$$R_{2/3}\left(\frac{pA}{pp}\right) = \left(\frac{1}{\sigma^{pA}_{\text{prod}}}\frac{d^3\sigma_{pA\to B}}{d^3p}\right)\left(\frac{1}{\sigma^{pp}_{\text{inel}}}\frac{d^3\sigma_{pp\to B}}{d^3p}\right)^{-1} = V^p_1(A),$$
$$R_{1/3}\left(\frac{pA}{pp}\right) = \left(\frac{1}{\sigma^{pA}_{\text{prod}}}\frac{d^3\sigma_{pA\to M}}{d^3p}\right)\left(\frac{1}{\sigma^{pp}_{\text{inel}}}\frac{d^3\sigma_{pp\to M}}{d^3p}\right)^{-1} = V^p_1(A) + \frac{4}{5}V^p_2(A). \quad (17)$$

Figure 14 demonstrates the agreement of (16) and (17) with experimental data.

So far, we have confined our attention of spectra with $x \lesssim 0.8$. However, processes of the form p + A→p + X and $\pi + A \to \pi + X$ for $x \gtrsim 0.90$–0.95 provide us with an interesting possibility of examining multiple elastic scattering of dressed quarks.

For low transferred momenta q^2, single scattering of quarks dominates the interaction with nuclei (Fig. 15a). As q^2 increases, the second (Figs. 15b and c), third (Figs. 15d–f), and higher-order interactions become appreciable as well. For example, in pPb-collisions, the differential cross section $d\sigma/dq^2$ contains the following contributions even for $q^2 = 1$

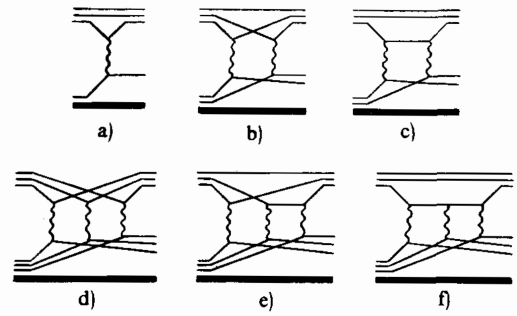

FIG. 15. Single (a), double (b,c), and triple (d–e) interactions contributing to the secondary-proton production cross section for $x \sim 1$ in pA collisions.

GeV^2: single scattering 18%, double scattering about 37%, triple scattering about 25%, quadruple scattering about 13%, and so on.

The cross sections $d\sigma(pA \to pX)/dq^2$ and $d\sigma(\pi A \to \pi X)/dq^2$ were calculated in Ref. 77. (Interference between diagrams of the same order, i.e., for example, the diagrams of Figs. 15d–f, was not taken into account in the earlier paper.[27]) Experimental studies of these processes would be particularly desirable: we recall that the contribution of elastic multiple scattering of dressed quarks in hadron-hadron collisions remains unclear because of the incorrect sign of the departure from (1).

G. Violation of scale invariance in the hadron fragmentation region at high energies

In the additive quark model, the rise in the total hadron cross section (or the cross section for the quark-quark interaction) should lead to characteristic changes in the production of fragmentation particles, i.e., to the breaking of scale invariance for $x \gtrsim 0.1$. In fact, the increase in the total cross section is accompanied by an increase in the Glauber shadow corrections to the impulse approximation and, as it turns out (in accordance with the AGK rules[104]), in the probability of interaction of two or three pairs of quarks at once in inelastic collisions as well. This leads to a reduction in the mean number of spectator quarks and, consequently, to a breaking of scale invariance.

The magnitude of this departure in pp-collisions was calculated in Refs. 105 and 106 within the framework of the Glauber quark interaction picture. The results of this calculation are shown in Fig. 16. Scale invariance breaking in hadron-nucleus collisions[30] is more important (cf. formulas in the previous section). Figure 16 also shows the estimated variation in the multiplicity of secondary baryons with $x \sim 2/3$, ($B_{2/3}$) for ^{14}N and Pb.

As the energy increases, there is an increase in the slope of the diffraction cones in inelastic scattering (both pp and πp) and this means that there is an increase in the dimensions of the dressed quarks. At ultrahigh energies, the dimensions of the dressed quarks may become comparable with the dimensions of the hadrons themselves, i.e., the parton-quark and gluon clouds in the interior of hadrons completely overlap.

When the interaction between these clouds is strong

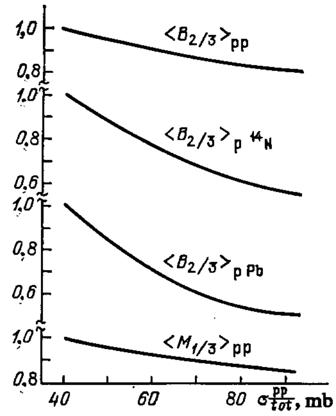

FIG. 16. Quasinuclear model prediction of the change in the multiplicity of secondary baryons with $x\sim 2/3$, $B_{2/3}$, and mesons with $x\sim 1/3$, $M_{1/3}$ in pp-, p^{14}N-, and pPb-collisions as a function of the total cross section σ^{pp}_{tot}.

enough, they eventually form a single parton cloud. When this object collides with a nucleon or nuclear target, it completely disintegrates into partons without the formation of spectator quarks. Strong violation of scale invariance is therefore expected in the fragmentation region at ultrahigh energies.[107] It may be considered that, for $x \gtrsim 0.1$, the dependence of the inclusive spectra on x in hadron collisions is then roughly the same as in e^+e^- annihilation.

The effects of violation of scale invariance in the central region have been examined[108] within the framework of the formalism of the supercritical pomeron and the DTU scheme. The spectra near $x\simeq 0$ and their variation with energy at the ISR and Collider energies can be satisfactorily described in this way. The breakdown of scaling in the fragmentation region is also predicted. However, these predictions are not unambiguous and depend on the way in which the energy is divided when cylindrical diagrams are cut.

Violations of scale invariance in the beam fragmentation region have not as yet been confirmed by accelerator experiments. Cosmic-ray data appear to indicate (see, for example, Refs. 109–112) the softening of inclusive spectra, which is in qualitative agreement with the quark-model predictions. The corresponding calculations were performed in Ref. 112 with allowance for the predicted scaling violation.

3. QUARK STATISTICS

A. Relationships between the yields of different secondary particles

The rules of quark statistics[44,45,113] enable us to calculate the probability of production of different hadrons in multiple production processes. These rules are essentially based on the quasinuclear structure of hadrons in which mesons and baryons are weakly bound systems of dressed quarks.

Let us examine the rules of quark statistics by considering the example of secondary hadrons produced in a quark jet. Suppose that some particular dressed quark q_i in a sea of quark-antiquark pairs q, $\bar{q}$ is transformed into a meson $M_i = q_i q$ or a baryon $B_i = q_i qq$ (Figs. 17a and b). The inclusive

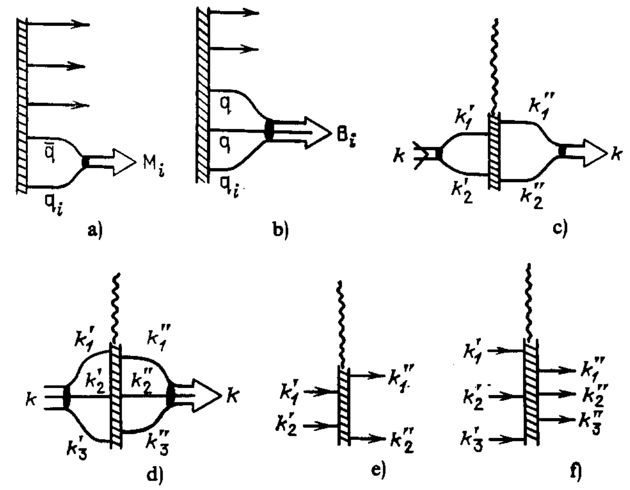

FIG. 17. Diagrams for the amplitudes (a,b) and inclusive cross sections (c,d) for the creation of a meson M_i or baryon B_i from a cloud of dressed quarks. Diagrams e and f correspond to the distribution functions F_n [see (16)].

cross sections for the production of M_i and B_i are determined by the diagram of Figs. 17c and d, and are given by ($h = M_i$ or B_i):

$$\frac{d\sigma_h(x,\mathbf{k}_\perp)}{dx\,d\mathbf{k}_\perp}=\int\prod_{i=1}^{n}\frac{dx'_i}{x'_i}\,d\mathbf{k}'_{i\perp}\delta\Big(\sum_{i=1}^{n}x'_i-x\Big)\,\delta\Big(\sum_{i=1}^{n}\mathbf{k}'_{i\perp}-\mathbf{k}_\perp\Big)$$
$$\times\frac{dx''_i}{x''_i}\,d\mathbf{k}''_{i\perp}\delta\Big(\sum_{i=1}^{n}x''_i-x\Big)\,\delta\Big(\sum_{i=1}^{n}\mathbf{k}''_{i\perp}-\mathbf{k}_\perp\Big)$$
$$\times F_n(x'_1,\mathbf{k}'_{1\perp},\ldots,x'_n,\mathbf{k}'_{n\perp}\mid x''_1,\mathbf{k}''_{1\perp},\ldots,$$
$$x''_n,\mathbf{k}''_{n\perp})\times\psi^*_h(s'_n)\,\psi_h(s''_n);\qquad(16')$$

where F_n are the distribution functions for the quark and anti-quark ($n = 2$) or three quarks ($n = 3$). These functions depend on the fractions of the total momentum of the jet carried off by quarks (x'_i and x''_i) and the transverse momenta of the quarks ($\mathbf{k}'_{i\perp}$ and $\mathbf{k}''_{i\perp}$; $\psi_h(s)$ are the meson or baryon wave functions. For simplicity, we shall suppose that the wave functions depend only on the total invariant energy of the quarks $s_h = (k_1 + \cdots + k_n)^2$ (more complicated situations that are possible in the system of three quarks do not affect the final result) and are normalized to the invariant phase volume $\delta^4(\Sigma^n_{i=1}k_i - k)\Pi^n_{i=1}(d^3\mathbf{k}_i/\mathbf{k}_{i0})$. In the system in which the hadron executes rapid motion, we have $s_n = x\Sigma^n_{i=1}(m^2_{iq} + k^2_{i\perp})/x_i - k^2_\perp$. In weakly-bound compound systems, $\psi_h(s)$ decreases rapidly with increasing s. The radius R^2_h of the compound system is a measure of the rate of decrease of ψ_h and is relatively large on the scale of the characteristic hadron quantities. All the values of x'_i and x''_i in (16), and also those of $\mathbf{k}'_{i\perp}$ and $\mathbf{k}''_{i\perp}$ should be comparable in magnitude. In other words, constituent quarks with low energies of relative motion combine into the hadron.

This has a number of important consequences. The interaction of dressed quarks at low relative energies exhibits SU(6) symmetry. It is well known that this means that the dependence of the wave functions ψ_h on s_n is the same for all members of a given SU(6) multiplet. Moreover, it also means that, in the approximation of exact SU(6) symmetry, the distribution functions F_n with comparable $x_i', x_i'', \mathbf{k}_{i\perp}', \mathbf{k}_{i\perp}''$ do not depend on quark quantum numbers such as spin, isospin, and strangeness. Hence, the inclusive cross sections for the production of all the mesons M_i or baryons B_i belonging to the same SU(6) multiplet should be the same. This leads to the following result (to be specific, we assume that $q_i = u$):

$$\frac{d\sigma_{\rho^+}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\left(\frac{d\sigma_{\pi^+}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\right)^{-1} = \frac{d\sigma_{K^{*+}}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\left(\frac{d\sigma_{K^+}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\right)^{-1} = 3,$$
$$\frac{d\sigma_{\rho^0}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\left(\frac{d\sigma_{\omega}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\right)^{-1} = 1, \quad (17')$$

and [in the approximation of exact SU(6) symmetry!]

$$\frac{d\sigma_{K^{*+}}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\left(\frac{d\sigma_{\rho^+}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\right)^{-1} = 1. \quad (18)$$

The ratio of inclusive cross sections at each value of x and $\mathbf{k}_\perp$ is therefore equal to the ratio of the statistical weights of the particles in the SU(6) multiplet.

In reality, SU(6) symmetry is broken. However, it is known that, in the quark model, the wave functions of hadrons—members of the same SU(6) multiplet)—can be assumed to be equal with sufficient precision even for broken SU(6) symmetry.[4] We shall employ the simple assumption that symmetry-breaking between u-, d-, and s-quarks can be described by introducing a suppression factor λ into the distribution of the strange sea quarks. The parameter λ lies in the range $0 \leqslant \lambda \leqslant 1$, where $\lambda = 1$ corresponds to exact symmetry (estimates based on the analysis of experimental data[114,115] yield $\lambda \simeq 0.2$–0.3).

Thus, when the quark wave functions vary slowly, the relationships given by (17) remain valid even in the real case of broken SU(6) symmetry, whereas (18) becomes

$$\frac{d\sigma_{K^{*+}}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\left(\frac{d\sigma_{\rho^+}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp}\right)^{-1} = \lambda. \quad (19)$$

To obtain the relationships between the meson and baryon yields, we turn to the integral given by (16). Since $\psi_h(s)$ decreases rapidly with increasing s, we can substitute $x_1' = x_1'' = x/n$ and $\mathbf{k}_{1\perp} = \mathbf{k}_{1\perp}'' = \mathbf{k}_\perp/n$ in the more slowly-varying function $F_n(x_1', \mathbf{k}_{1\perp}'' \ldots | x_1'', \mathbf{k}_{1\perp} \ldots)$ and take this function outside the integral sign. The integrand is determined entirely by the quark wave functions and, by virtue of Lorentz invariance, does not depend on x and $\mathbf{k}_\perp$ (this is immediately clear if we transform to the frame in which the created hadron is at rest and $x = 0$, $\mathbf{k}_\perp = 0$). For inclusive cross sections for the production of the meson M_i or baryon B_i, we thus have

$$\frac{d\sigma_h(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp} = \text{const} \cdot F_n\left(\frac{x}{n}, \frac{\mathbf{k}_\perp}{n}, \ldots\right). \quad (20)$$

The relative energies of dressed quarks in the cloud are determined by characteristic masses much greater than $1/R_h$. There is therefore a small probability that the cloud will immediately contain quarks with the sufficiently low relative momenta required by (20). As a rule, the rapidity difference between neighboring quarks will be much greater. This situation (see subsection 2D) can be interpreted as the decay of the excited system consisting of quarks and neighboring quarks into a meson M_i plus the remaining quarks, or the baryon B_i plus the remaining quarks. The quark q_i creates suitable partners for itself, and soft hadronization results from this. It is then natural to expect that the distributions

$$F_2\left(\frac{x}{2}, \frac{\mathbf{k}_\perp}{2}, \ldots \middle| \frac{x}{2}, \frac{\mathbf{k}_\perp}{2} \ldots\right)$$
$$\text{and } F_3\left(\frac{x}{3}, \frac{\mathbf{k}_\perp}{3}, \ldots \middle| \frac{x}{3}, \frac{\mathbf{k}_\perp}{3} \ldots\right)$$

should be roughly the same, i.e., proportional to some function $F_{q_i}(x, \mathbf{k}_\perp)$, that, somewhat arbitrarily, can be referred to as the initial distribution of the quark q_i in the cloud. In that case,

$$\frac{d\sigma_{M_i}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp} = w_1 F_{q_i}(x, \mathbf{k}_\perp),$$
$$\frac{d\sigma_{B_i}(x, \mathbf{k}_\perp)}{dx\, d\mathbf{k}_\perp} = w_2 F_{q_i}(x, \mathbf{k}_\perp). \quad (21)$$

The assumption of quark statistics means that the creation of quarks and antiquarks in the cloud of dressed quarks is uncorrelated, i.e., in each event, a quark q_i can interact with probability 1/2 with a quark, and with equal probability with an antiquark. Hence, $w_1 = C/2$ (C is a normalizing factor) and $w_2 = (1/2)(1/2)C$. The factor C is found from the normalization conditions ($w_1 + w_2$ is the total probability of hadronization of the quark, i.e., $w_1 + w_2 = 1$). We then have

$$w_1 = \frac{2}{3}, \quad w_2 = \frac{1}{3}. \quad (22)$$

According to the rules of quark statistics, the total probability that the quark q_i will undergo a transition to a baryon B_i is thus seen to be 1/3, i.e., it is equal to the baryon charge of the quark. Moreover, quark statistics predicts a universal momentum distribution (21) for B_i and M_i. The two results are confirmed by experimental data (see below).

We must now show how the relationships of quark statistics can be obtained by purely combinational considerations.[44,45] The combinatorial approach does not fully reveal the physics of the process, but it is much simpler.

It will be convenient to use the concept of "mean statistical quark" or "an ensemble," the definition of which relies on the fundamental proposition of quark statistics, namely, that sea quarks do not, on average, possess any special quantum numbers:

$$q = \frac{1}{2(2+\lambda)} u^\uparrow + \frac{1}{2(2+\lambda)} u^\downarrow + \frac{1}{2(2+\lambda)} d^\uparrow + \frac{1}{2(2+\lambda)} d^\downarrow + \frac{\lambda}{2(2+\lambda)} s^\uparrow + \frac{\lambda}{2(2+\lambda)} s^\downarrow. \quad (23)$$

The numerical coefficient in front of the quark symbol on the right-hand side of (23) gives the probability that a sea quark q is the particular given quark.

We now return to the question of hadronization of a given quark q_i and the sea of quark-antiquark pairs $(q, \bar{q})$. Let us arrange all the quarks in the order of increasing rapidity and assume, for simplicity, that the nearest neighbors in this sequence combine into hadrons. To be specific, we shall suppose that the quark q_i is the slowest and hence the hadronization process begins with it. The nearest neighbor of quark q_i can be, with equal probability, a quark q or antiquark $\bar{q}$. In the first stage, the quark q_i can combine with probability 1/2 with the antiquark $\bar{q}$ and form a meson ensemble $M_i = q_i\bar{q}$ or, with the same probability, an ensemble of "diquarks" $q_i q$ (we shall refer to M_i, $q_i q$, and other analogous ensembles as mesons, diquarks, and so on). In the first case, the hadronization of the quark q_i is complete whereas, in the second, we must consider the subsequent stages. The next neighbor may also be, with equal probability, either a quark q or antiquark $\bar{q}$. Hence, the diquark $q_i q$ can, with equal probability, combine with a quark into the baryon B_i or with an antiquark to form the system $q_i q\bar{q}$. According to the rule that we have adopted, nearest neighbors in the rapidity sequence transform into the sea meson $M = q\bar{q}$, and the quark q_i remains in the cloud of unhadronized quarks. This stage may be written in the form

$$\begin{aligned}
&q_i\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right)\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right)\\
&\to\left(\tfrac{1}{2}q_iq+\tfrac{1}{2}q_i\bar{q}\right)\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right)\\
&\to\left(\tfrac{1}{2}q_iq+\tfrac{1}{2}M_i\right)\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right)\to\left(\tfrac{1}{4}q_iqq+\tfrac{1}{4}q_iq\bar{q}\right)\\
&+\tfrac{1}{2}M_i\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right)\\
&\to\tfrac{1}{4}B_i+\tfrac{1}{2}M_{q_i}+\tfrac{1}{2}M_i\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right).
\end{aligned}\tag{24}$$

The numerical coefficients in this expression are the probabilities. We see that, even at this stage of hadronization, the probabilities of creation of M_i and B_i are in the ratio of 2:1, in accordance with (22). To use this method to examine the hadronization of all quarks, we must write down the long chain

$$q_i\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right)\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right)\ldots\left(\tfrac{1}{2}q+\tfrac{1}{2}\bar{q}\right)\ldots\tag{25}$$

and successively collect quarks into hadrons, for example, from left to right. When the chain is long enough, the average multiplicities of the resulting hadrons turn out to be

$$\langle n_{M_i}\rangle=\frac{2}{3},\qquad \langle n_{B_i}\rangle=\frac{1}{3},\tag{26}$$

$$\langle n_M\rangle=6N+\frac{1}{3},\quad \langle n_{\bar{B}}\rangle=\langle n_B\rangle=N.\tag{27}$$

The results given by (26) are identical with (22). The multiplicities of the sea mesons $M = q\bar{q}$, baryons $B = qqq$, and antibaryons $\bar{B} = \bar{q}\bar{q}\bar{q}$ are characterized by N which, in turn, is determined by the length of the chain and depends on the total energy of the jet. In practical calculations, N can be expressed, for example, in terms of the mean multiplicity of all the charged particles $\langle n_{ch}\rangle$. When N is large enough, the ratio of the multiplicities of sea mesons and baryons turns out to be the same as in Ref. 44:

$$\langle n_M\rangle:\langle n_B\rangle:\langle n_{\bar{B}}\rangle=6:1:1.\tag{28}$$

Another form of (26) and (27) is also used:

$$\begin{aligned}
&q_i+(q,\ \bar{q})\text{sea}\to\tfrac{1}{3}B_i+\tfrac{2}{3}M_i\\
&\quad+\left(6N+\tfrac{1}{3}\right)M+NB+N\bar{B}.
\end{aligned}\tag{29}$$

The coefficients on the right-hand side of (29) are obviously the mean multiplicities of the created hadrons.

Specific numerical estimates performed within the framework of the rules of quark statistics[116,117] show that the ratio $\langle n_M\rangle/\langle n_B\rangle$ approaches very slowly the asymptotic limit (28) as multiplicity increases, whereas $\langle n_{B_i}\rangle/\langle n_{M_i}\rangle$ rapidly approaches the value 1/2.

The mean multiplicities of secondary particles produced during the hadronization of the diquark $q_i q_j$ in the sea of quarks can be calculated in an analogous manner. The result corresponding to (29) is[113]

$$\begin{aligned}
&q_iq_j+(q,\ \bar{q})\text{sea}\to\tfrac{1}{2}B_{ij}+\tfrac{1}{12}(B_i+B_j)\\
&+\tfrac{5}{12}(M_i+M_j)+\left(6N+\tfrac{1}{6}\right)M+NB+N\bar{B},
\end{aligned}\tag{30}$$

where $B_{ij} = q_i q_j q$.

Before we turn to the quantitative description of multiple processes, we must take one further step, namely, decipher the specific hadron composition of meson and baryon ensembles.

B. Composition of hadron ensembles B_{ij}, B_i, B and M_i, M

The phrase "sea quark," in quark statistics, is understood to mean an average over an ensemble of u-, d-, and s-quarks [see (23)]. Accordingly, the "baryons" B_{ij}, B_i, B and "mesons" M_i M in (29) and (30) are also averages over certain ensembles of real baryons and mesons. Before we can use the relationships of quark statistics and compare them with experimental data, we must first determine which particular real hadrons saturate these ensembles.

Quark statistics enables us to calculate the relative production probabilities for different hadrons belonging to the same SU(6) multiplet. However, the relationship between the production probabilities for different multiplets cannot be calculated within the framework of quark statistics: they are distributed by the dynamic mechanism of hadronization of the cloud of dressed quarks, i.e., by the properties of the function F_n in (16). These relationships must therefore be determined experimentally.

It was assumed in Ref. 44 that multiple processes were dominated by the production of the lowest SU(6) multiplets with zero angular momentum, namely, the meson 1 + 35-plet ($J^P = 0^-, 1^-$) and baryon 56-plet ($J^P = 1/2^+, 3/2^+$). It is now clear that this is a relatively crude approximation: the production of the meson P-wave ($L = 1$) multiplet ($J^P = 0^+, 1^+, 1^+, 2^+$) accounts[118,119] for up to 30% of the total meson yield. It may be expected that the creation of the P-wave ($L = 2$) mesons provides a contribution of 5–10% (see Refs. 118 and 119). Baryon creation data are much more scanty but, as will be seen later, the creation of the baryon 70-plet ($J^P = 5/2^-, 3/2^-, 3/2^-, 3/2^-, 3/2^-, 1/2^-, 1/2^-, 1/2^-, 1/2^-$) can also give rise to a substantial contribution. We shall therefore examine the general case where the ensembles B_{ij}, B_i, B and M_i, M are superpositions of baryon and meson ensembles of different SU(6) multiplets.

The properties of the dressed-quark cloud in the region of fragmentational creation will be different from those in the central region. The multiplet composition of the meson M_i, M and baryon B_{ij}, B_i, B ensembles will then also be different. The SU(6) multiplet expansions can then be written in the form

$$\mathrm{M}=\sum_{L}\alpha_0^{\mathrm{M}}(L)\,\mathrm{M}(L),\qquad \mathrm{M}_i=\sum_{L}\alpha_1^{\mathrm{M}}(L)\,\mathrm{M}_i(L),\tag{31}$$

where different multiplets are characterized by the quark orbital angular momentum L (this quantum number uniquely characterizes the lowest multiplet). The coefficients $\alpha_0^{\mathrm{M}}(L)$ and $\alpha_1^{\mathrm{M}}(L)$ determine the relative contribution of different multiplets and satisfy the normalization condition $\Sigma\alpha_I^{\mathrm{M}}(L)=1(I=0,1)$.

An analogous expansion is available for the baryon ensembles:

$$\mathrm{B}_{ij}=\sum_{L}\alpha_2^{\mathrm{B}}(L)\,\mathrm{B}_{ij}(L),\qquad \mathrm{B}_i=\sum_{L}\alpha_1^{\mathrm{B}}(L)\,\mathrm{B}_i(L),$$

$$\mathrm{B}=\sum_{L}\alpha_0^{\mathrm{B}}(L)\,\mathrm{B}(L),\tag{32}$$

with the normalization condition $\Sigma_L\,\alpha_I^{\mathrm{B}}(L)=1(I=0,1,2)$.

We must now examine the hadron composition in individual multiplets. We shall consider in detail the creation of mesons and baryons with $L=0$.
vidual multiplets. We shall consider in detail the creation of mesons and baryons with $L=0$.

The "meson" M(0) is, as already mentioned, an average over the ensemble of pairs of sea quarks and antiquarks in S-wave states:

$$\mathrm{M}(0)=(\mathrm{q}\bar{\mathrm{q}})_{L=0}=\sum_{k,\,l}\frac{\mathcal{L}_k\mathcal{L}_l}{4(2+\lambda)^2}\,\mathrm{q}_k\bar{\mathrm{q}}_l.\tag{33}$$

The subscripts k, l label the type of quark and its spin component: u↑,u↓,d↑,d↓,s↑,s↓. For nonstrange quarks $\mathcal{L}_k=1(\mathrm{q}_k=\mathrm{u,d})$ and $\mathcal{L}_k=\lambda$ and for strange quarks $(\mathrm{q}_k=\mathrm{s})$. The factor $\mathcal{L}_k\mathcal{L}_l/4(2+\lambda)^2$ is the probability that the state $(\mathrm{q}_k\bar{\mathrm{q}}_l)_{L=0}$ will appear in the ensemble M(0). To transform in (33) from the states $(\mathrm{q}_k\bar{\mathrm{q}}_l)_{L=0}$ to the states of real mesons (π, ρ, K, and so on), we must calculate the probabilities with which these mesons appear in the state $(\mathrm{q}_k\bar{\mathrm{q}}_l)_{L=0}$. This means that the wave function $|\mathrm{q}_k\bar{\mathrm{q}}_l\rangle_{L=0}$ must be expanded in terms of the wave functions of the mesons π,ρ, K, and so on. The squares of the coefficients of this expansion give the required probabilities. For example,

$$(\mathrm{u}^{\uparrow}\bar{\mathrm{u}}^{\uparrow})_{L=0}=\frac{1}{2}\,\rho_{\uparrow}^0+\frac{1}{2}\,\omega_{\uparrow},\quad(\mathrm{u}^{\uparrow}\bar{\mathrm{d}}^{\downarrow})_{L=0}=\frac{1}{2}\,\pi^{+}+\frac{1}{2}\,\rho_0^{+}.\tag{34}$$

The numerical coefficients on the right-hand sides of (34) determine the probability of finding a given meson in the corresponding state $(\mathrm{q}_k\bar{\mathrm{q}}_l)_{L=0}$. Let (0) $\mathrm{h_M}$ represent the mesons in the S-wave 1 + 35-plet, so that (33) and (34) yield the following expansion for the "sea meson" M(0) in terms of real particles:

$$\mathrm{M}(0)=\sum_{\mathrm{h}}\mu_{\mathrm{h}(0)}\mathrm{h}_{\mathrm{M}(0)}.\tag{35}$$

The probability coefficients $\mu_{\mathrm{h}(L)}$ are given in Refs. 113, 120 and 121 (for $L=0,1$).

The expansion of $\mathrm{M}_i(0)$, M(1), $\mathrm{M}_i(1)$ in terms of real hadrons can be performed in a similar manner. So far, we have ignored the color degrees of freedom in our discussion of mesons. The point is that the symmetry properties of the meson wave functions with respect to the color and SU(6) indices are completely independent. This is not so in the case of baryons. The fully antisymmetric wave function of a system of three quarks can have mixed symmetry with respect to both color and SU(6). However, such states do not correspond to real baryons because they are not color singlets. Hence, the expansion of the ensembles B_{ij}, B_i, and B over the states of real hadrons requires the explicit allowance for the quark color quantum numbers.

We now turn to the hadronization of the meson and examine the decoloration mechanism during the coalescence of quarks and hadrons (Fig. 18). A quark q_i captures an antiquark $\bar{\mathrm{q}}$ from the cloud (or creates this antiquark). However, this does not guarantee the formation of a meson because, before the $\mathrm{q}_i\bar{\mathrm{q}}\to\mathrm{M}_i$ process can take place, the $\mathrm{q}_i\bar{\mathrm{q}}$ pair must be in a white state. Unless this is so, the colored excited state will decay and the quark q_i will again capture some antiquark $\bar{\mathrm{q}}$ (see Fig. 18b). When the pair $\mathrm{q}_i\bar{\mathrm{q}}$ is again colored, a new decay will occur, and so on. We consider (see Subsection D) that the time for the creation of new quarks is substantially shorter than the time of existence of the dressed-quark cloud, so that the above "decay" mechanism should ensure the transition of the $\mathrm{q}_i\bar{\mathrm{q}}$ system to the white state with 100% probability, which corresponds to the hypothesis of soft decoloration.

We now turn to baryons and, as an example, consider the decoloration process during the formation of $\mathrm{B}_{ik}(0)$.

The "baryon" $\mathrm{B}_{ik}(0)$ is an average over the ensemble of S-wave baryons containing the S-wave diquark $\{\mathrm{q}_i\mathrm{q}_k\}$ and the sea quark q [braces indicate that the wave function of the state under consideration is symmetrized in SU(6) indices]. It is clear that we obtain an incorrect result if we write for $\mathrm{B}_{ik}(0)$

$$(\{\mathrm{q}_i\mathrm{q}_k\}\,\mathrm{q})_{L=0}=\sum_{l}\frac{\mathcal{L}_l}{2(2+\lambda)}\,(\{\mathrm{q}_i\mathrm{q}_k\}\,\mathrm{q}_l)_{L=0}.\tag{36}$$

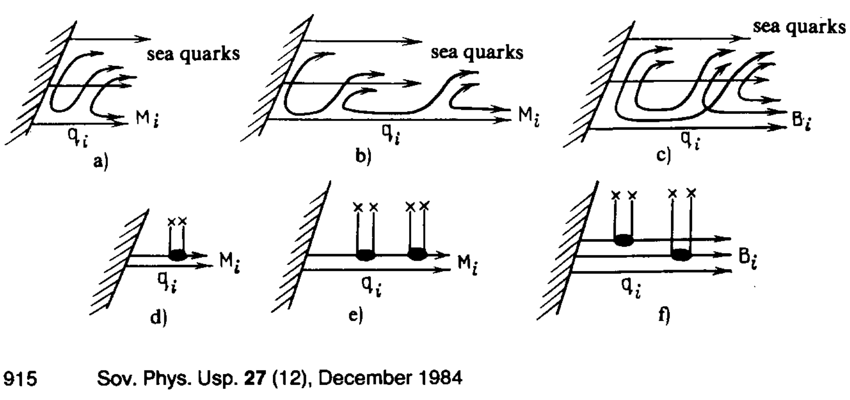

FIG. 18. Decay mechanism ensuring soft decoloration during the hadronization of quarks.

The state $(\{q_i q_k\} q_l)_L = 0$ is antisymmetric in l and the SU(6) antisymmetric states are colored. The ensemble (36) is thus seen to include both white and colored states.

We now separate the white and colored states in (36) in an explicit form (to be specific, we suppose that $q_i = q_k = u\uparrow$):

$$(\{u\uparrow u\uparrow\} q)_{L=0} = \frac{1}{2(2+\lambda)}\Big[\{u\uparrow u\uparrow u\uparrow\}_{L=0} + \frac{1}{3}\sum_{k\neq u\uparrow}\mathscr{L}_k\{u\uparrow u\uparrow q_k\}_{L=0} + \frac{2}{3}\sum_{k\neq u\uparrow}\mathscr{L}_k(u\uparrow u\uparrow q_k)_{70,\,L=0}\Big]. \tag{37}$$

The first two terms in brackets in (37) correspond to white states [completely SU(6) symmetric] and appear in the required "baryon" $B_{u\uparrow u\uparrow}(0)$. The last term, on the other hand, corresponds to colored states and we must examine its further interaction with the quark sea: the original quarks must transform into a white "baryon" with probability 1.

According to the decoloration decay mechanism, we must consider that the quark q_k in the color 70-plet does not "survive" but is lost to the quark sea, whereas the remaining diquark $u\uparrow u\uparrow$ again captures another sea quark. Iteration of this process (see Fig. 18c) results in the creation of only white states, and the ratio of creation probabilities is the same as in (37):

$$(\{u\uparrow u\uparrow\} q)_{L=0} \to \frac{3}{2(3+\lambda)}\Big[\{u\uparrow u\uparrow u\uparrow\}_{L=0} + \frac{1}{3}\sum_{k\neq u\uparrow}\mathscr{L}_k\{u\uparrow u\uparrow q_k\}_{L=0}\Big], \tag{38}$$

where the right-hand side gives the required ensemble $B_{u\uparrow u\uparrow}(0)$.

Decoloration in quark statistics is thus seen to be due to the multiple four-fermion interaction between a colored hadron and the dressed-quark condensate. In this language, the diagrams of Figs. 18a–c are rewritten as the diagrams of Figs. 18d–e. The decoloring interaction, i.e., the scattering of a given quark by the condensate, can be described by an effective Lagrangian of the form

$$G_{18}(k^2)\left(\bar\psi\frac{1}{\sqrt6}I_f\lambda_c\psi\right)\left\langle\bar\psi\frac{1}{\sqrt6}I_f\lambda_c\psi\right\rangle + G_{88}(k^2)\left(\bar\psi\frac{1}{2}\lambda_f\lambda_c\psi\right)\left\langle\bar\psi\frac{1}{2}\lambda_f\lambda_c\psi\right\rangle, \tag{39}$$

where the unit matrices I and Gell-Mann matrices (λ) act on the quark indices of the flavors (f) and colors (c). The averages over the condensate $\langle\psi\bar\Gamma_A\psi\rangle$ are "mesons" [color and flavor singlets and octets]. The equation $G_{18}(k^2) = G_{88}(k^2)$ corresponds to the rules of quark statistics (i.e., the capture of quarks from the condensate). $G_{18}(k^2)\neq 0$, $G_{88}(k^2) = 0$ corresponds to the exchange of gluons alone.

The foregoing analysis enables us to formulate a recipe for constructing baryon ensembles in the general case.[113] For "baryons" $B_{ik}(L)$, $B_i(L)$, and $B(L)$ this recipe is as follows. We construct ensembles $(\{q_i\bar q_k\}q)_L$, $(q_i qq)_L$ and $(qqq)_L$ that include white and color states. These ensembles are then allowed to retain only the white states, and normalization of the ensembles is restored by multiplying them by a common factor.

In particular, for the baryon $B_{ik}(0)$ discussed above, this procedure yields

$$B_{ik}(0) = \sum_l \frac{\mathscr{L}_l(1+\delta_{il}+\delta_{kl})}{4+\mathscr{L}_i+\mathscr{L}_k+2\lambda}\{q_i q_k q_l\}_{L=0}. \tag{40}$$

We recall that the numerical coefficients in this expression are the probabilities of finding the state $\{q_i q_k q_l\}_{L=0}$ in the ensemble $B_{ik}(0)$. The expansion of the state $\{q_i q_j q_l\}_{L=0}$ over the real-baryon ensemble can be performed by using the explicit form of the wave functions of the 56-plet. For example:

$$\{u\uparrow u\downarrow d\uparrow\}_{L=0} = \frac{1}{3}p + \frac{2}{3}\Delta^+,$$
$$\{u\uparrow d\downarrow s\uparrow\}_{L=0} = \frac{1}{2}\Lambda + \frac{1}{6}\Sigma^0 + \frac{1}{3}\Sigma^{*0}. \tag{41}$$

Apart from (40) and (41), this also yields

$$B_{ik}(0) = \sum_h \beta_{h(0)}(ik)\, h_{B(0)}, \tag{42}$$

where the coefficients $\beta_{h(0)}(ik)$ are the probabilities with which the corresponding real baryon of the 56-plet $h_{B(0)}$ appears in the ensemble $B_{ik}(0)$. The values of these coefficients are given in Refs. 113 and 121 which also list the coefficients $\beta_{h(0)}(i)$ and $\beta_{h(0)}$, for the ensembles $B_i(0)$ and $B(0)$:

$$B_i(0) = \sum_h \beta_{h(0)}(i)\, h_{B(0)}, \quad B(0) = \sum_h \beta_{h(0)} h_{B(0)}. \tag{43}$$

This expansion was obtained in Ref. 122 (see also Ref 121) for the case of baryon ensembles belonging to the 70-plet ($L = 1$).

In any realistic model of multiple production that takes into account isotopic symmetry of the strong interaction, secondary baryons should not be aligned in isotopic spin when the average over the initial-state isospin is taken. Actually, all directions in isotopic space are found to be equally probable after this average, and all isotopic states of secondary particles are created with equal probability. The decoloration decay mechanism examined above satisfies this requirement. At the same time, if the "colored baryon" is decolored by gluon exchange (which would correspond to a transition from the state $(u\uparrow u\uparrow q_k)_{L=0,70}$ to the state $\{u\uparrow u\uparrow q_k\}_{L=0}$ with $q_k \neq u\uparrow$, the final baryons would be isotopically aligned after averaging over the initial state (as found in Ref. 123). Thus the pure gluon decoloration mechanism leads to an unreasonable result in the case of baryons. We note, by the way, that secondary hadrons do not exhibit spin alignment in the above model with the SU(6) symmetric interaction between dressed quarks. It could appear, for example, as a result of a symmetry-breaking spin-orbital interaction between quarks.

Experimental data show that $\alpha_I^M(0)\approx 0.7$, $\alpha_I^M(1)\approx 0.3$, and $\alpha_I^M(2)$ is less than, or of the order of 0.1. The baryon production probabilities are less well known. To estimate the parameters $\alpha_I^B(0)$, $\alpha_I^B(1)$, governing the production of baryons belonging to the 56- and 70-plets, we can use the following simple considerations. The probability that a P-wave hadron will be produced is determined by the probability of finding in the dressed-quark cloud a P-wave sea quark. This probability (α_P) is characterized by the properties of the quark cloud itself, and it may be expected that it is independent of which sea particle, i.e., quark or antiquark, we consider. We then have

$$\alpha_2^{\mathrm{B}}(1) \approx \alpha_1^{\mathrm{M}}(1) \approx \alpha_0^{\mathrm{M}}(1) \approx \alpha_{\mathbf{P}},$$
$$\alpha_2^{\mathrm{B}}(0) \approx \alpha_1^{\mathrm{M}}(0) \approx \alpha_0^{\mathrm{M}}(0) \approx \alpha_{\mathrm{S}}, \qquad (44)$$

where we have neglected the production of higher-order resonances, so that $\alpha_{\mathrm{p}} + \alpha_{\mathrm{S}} = 1$.

For baryons containing two or three sea quarks, we have

$$\alpha_1^{\mathrm{B}}(0) \approx \alpha_0^{\mathrm{B}}(0) \approx \alpha_{\mathrm{S}}^2, \quad \alpha_1^{\mathrm{B}}(1) \approx \alpha_0^{\mathrm{B}}(1) \approx 2\alpha_{\mathrm{S}}\alpha_{\mathbf{P}}. \qquad (45)$$

When the probability of production of P-wave mesons is about 30%, the probability of production of the baryon 70-plet is shown by (44) and (45) to be 30% in the case of diquark fragmentation and 45% in the case of quark fragmentation or the production of a sea baryon.

C. Verification of the rules of quark statistics

Let us now examine the experimental status of the basic relationships in quark statistics.

Quark statistics predicts that the hadron creation probability is proportional to the number $2J + 1$ of its spin states. It is precisely this that leads to relationships of the form given by (17). However, experimental verification of such relationships requires the separation of the resonance decay products from directly created particles only for which this relation is valid. For example, the observed ρ/π ratio can change quite substantially when the decay products of some unidentified resonances are present among secondary pions. It is therefore more convenient to verify the 3:1 ratio for secondary K-mesons which multiply much more infrequently in resonance decays. We expect that, of all the strange secondary mesons, 75% have quark spin $sq\bar{q} = 1$ and only 25% have $sq\bar{q} = 0$. All particles with $sq\bar{q} = 1$ are resonances, so that 75% of secondary kaons should comprise the decay products of resonances with $sq\bar{q} = 1$ (if we neglect decays of nonstrange resonances into kaons).

The multiplet of mesons with $L = 1$ consists of four nonets with $J^P = 0^+, 1^+, 1^+$ and 2^+. The statistical weight of each of these nonets is proportional to $2J + 1$. This means that 5/12 of all the secondary particles belonging to the $L = 1$ multiplet should be tensor mesons (T), i.e., the total number of particles in this multiplet is (12/5)T. 75% of them are mesons with $sq\bar{q} = 1$. All vector mesons V with $L = 0$ also have $sq\bar{q} = 1$. The contribution of resonances with $sq\bar{q} = 1$ to the K-meson production cross section is shown by the rules of quark statistics to be V + (9/5)T and, if the contribution of multiplets with $L > 1$ is negligible, this should account for 75% of the total kaon production cross section. According to Ref. 124, the quantity V + (9/5)T [we shall represent it by K(1)] in K^-p-collisions at 32 GeV/c should assume the following values for the secondary kaons: $K^-(1) = 68 \pm 5\%$, $\bar{K}^0(1) = 71 \pm 4\%$, $K^0(1) = 65 \pm 13\%$, $K^+(1) = 78 \pm 19\%$. It is clear that the experimental value of K(1) is close to 75% in all cases.

The ρ^0 and ω yields should be equal as a consequence of the rules of quark statistics. This has now been generally accepted (see, for example, Ref. 125), but it must be understood that it is a direct consequence of the fact that the particle yields (i.e., the interaction of dressed quarks during hadronization) are independent of the isotopic spins of the quarks. Equal ρ^0 and ω yields look unnatural outside the framework of this independence. For example, in the multiperipheral model,[126] the ω-production probability was found to be much lower than the probability of ρ^0-meson production.

The hypothesis of quark statistics, as it relates to the equally probable combination of quarks and antiquarks during the hadronization process, ensures that the quark baryon number appears as the baryon production probability [see (26)]. Experimental verification of this result relies on measurements of the baryon production cross sections in quark jets. However, this gives rise to a difficulty in the separation of contributions due to fragmentation and sea baryons. The situation becomes simpler in the case of production in e^+e^--annihilation by a virtual γ-ray of a $c\bar{c}$ or $b\bar{b}$ pair. The creation of charmed quarks in the sea is suppressed and tagged c or b quarks are the only source of charmed hadrons.

In e^+e^--annihilation, there are two jets induced by a quark and antiquark, so that the total multiplicities of charmed mesons and baryons are given by $\langle n_{\mathrm{B}_{\bar{c}}}\rangle = \langle n_{\bar{\mathrm{B}}_{\bar{c}}}\rangle = 1/3$, $\langle n_{\mathrm{M}_{\bar{c}}}\rangle = \langle n_{\mathrm{M}_{\bar{c}}}\rangle = 2/3$. The multiplicity of secondary particles in e^+e^--annihilation is commonly measured in terms of $R(\mathrm{h}) = \sigma(\mathrm{h})/\sigma(\mu^+\mu^-)$. The charmed-baryon production cross section can be estimated from the increase in the total multiplicity of all baryons as we pass from energies below the charmed creation threshold to energies above the c-quark creation threshold. The measured[127] increase in the energy range $\sqrt{s} = 4.4 - 6.5$ GeV is 0.92 ± 0.11.

The result $R(\mathrm{B}_c + \bar{\mathrm{B}}_{\bar{c}}) = 8/9$ predicted by quark statistics is in good agreement with this increase in $R(\mathrm{B} + \bar{\mathrm{B}})$. We note that the increase in $R(\mathrm{B} + \bar{\mathrm{B}})$ that can be ascribed to the creation of c-quarks is estimated to be only 0.02–0.06. On the other hand, we must note that the total yield of baryons in e^+e^--annihilation at $\sqrt{s} = 6.5$ GeV is somewhat lower than that predicted by quark statistics with allowance for all the four types of quark: quark statistics gives $R(\mathrm{B} + \bar{\mathrm{B}}) = 20/9$, whereas the experimental result[127] is $R(\mathrm{B} + \bar{\mathrm{B}}) = 1.8 \pm 0.2$[127].

Quark statistics enables us to expand M_c and B_c in terms of hadrons belonging to the lowest S-wave multiplet, and to calculate the Λ_c and D yields with allowance for the decay of resonances. For $\lambda = 0.3$ we have[128]

Quark statistics	Experiment
$R(\Lambda_c^+ + \bar{\Lambda}_{\bar{c}}^-) = 0.62$,	$R(\Lambda_c^+ + \bar{\Lambda}_{\bar{c}}^-) = 0.52 \pm 0.13$[127],
$R(\mathrm{D}^+ + \mathrm{D}^-) = 0.40$,	$R(\mathrm{D}^+ + \mathrm{D}^-) = 0.75 \pm 0.31$[129],
$R(\mathrm{D}^0 + \bar{\mathrm{D}}^0) = 1.14$.	$R(\mathrm{D}^0 + \mathrm{D}^0) = 1.2 \pm 0.4$[129].

This may be regarded as completely satisfactory agreement. Data on the creation of charmed hadrons thus enable us to conclude that (26) and (29) are satisfied to within the 20–30% experimental uncertainty that is current at present.

D. Inclusive spectra and average multiplicities of secondary particles

The rules of quark statistics provide us with relationships not only for the total multiplicities of secondary particles, but also for the inclusive cross sections.

Let us begin by considering the inclusive cross sections of hadrons in quark jets. We shall start with the relationships for multiplicities given by (29) and by the expansions for the ensembles B_i, B and M_i, M [Eqs. (31), (32), (35), and so on]. The transformation from (29) to inclusive cross sections can be readily performed:

$$\frac{1}{\sigma}\frac{d\sigma}{dx}(q_i \to \text{hadron}) = \frac{1}{3}B_iF(x)+\frac{2}{3}M_i\Phi(x)+M\varphi(x)+Bf(x)+\bar{B}f(x), \qquad (46)$$

where $F(x)$ and $\Phi(x)$ are functions representing the transformation of a tagged quark into a baryon or meson, and $\varphi(x)$ and $f(x)$ are the hadron distribution functions. They are normalized as follows:

$$\int_0^1 dx\, F(x) = \int_0^1 dx\, \Phi(x) = 1, \quad \int_{\mu/\sqrt{s}}^1 dx\, \varphi(x) = 6N(s)+\frac{1}{3}, \quad \int_{\mu/\sqrt{s}}^1 dx\, f(x) = N(s), \qquad (47)$$

where μ is a characteristic hadron mass. The inclusive cross section in the form given by (47) is arbitrary to the extent that the multiplicity written in the form of (29) is arbitrary: to obtain particular spectra, we must use expansions of B_i, B, M_i, and M in terms of real hadrons.

It is important to emphasize that, according to the rules of quark statistics, the meson and baryon distribution functions are identical, i.e., $F(x) = \Phi(x)$ [see (21)]. However, this equality is valid only in the region of a typical multiple process for which $F(x)$ and $\Phi(x)$ are of the order of unity. As $x\to 1$, the functions $F(x)$ and $\Phi(x)$ are found to fall off rapidly [$F(x), \Phi(x) < 0.1$ for $x > 0.6$] and, in this region, the equality does not hold: according to Ref. 130, as $x\to 1$, $F(x)\sim(1-x)^3$ and $\Phi(x)\sim(1-x)$.

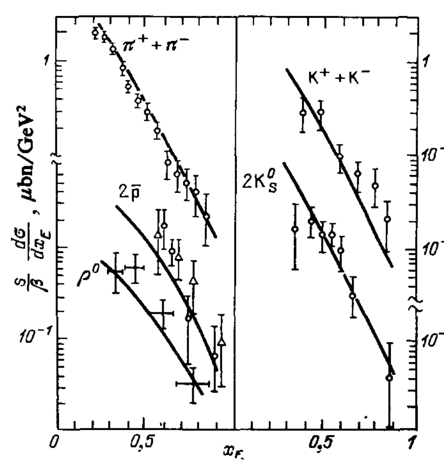

FIG. 19. Spectra of secondary particles from e^+e^--annihilation. Data taken from Refs. 131–133.

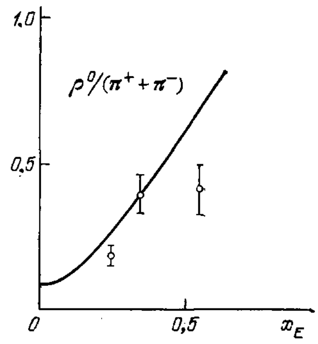

FIG. 20. The ratio $\rho^0/(\pi^+ + \pi^-)$ for e^+e^--annihilation for different values of x_E.

Figure 19 shows the inclusive cross sections for e^+e^--annihilation below the sea-quark production threshold as given by (47) for $0.2\leqslant x\leqslant 0.7$, where the spectra are actually determined by specifying a single function $F(x)\simeq\Phi(x)$.

Figure 20 shows the ratio of the ρ^0 to $\pi^+ + \pi^-$ yields in e^+e^--annihilation at higher energies[134] ($\sqrt{s} = 34$ GeV).

The spectra produced in hadron-hadron collisions have a more complicated structure. However, here again, we can use the same recipe for transforming from relationships for multiplicities [(9) and (10)] to inclusive cross sections. For transformations of the form proton $p_{ijk}\to$ secondary hadrons, we have

$$\begin{aligned}\frac{1}{\sigma_{\text{inel}}}\frac{d\sigma}{dx}(p\to\text{hadron}) &= \Delta B_{ijk}d_p(x) \\ &+\frac{1-\Delta}{3}[\xi_1B_{ij}D(x)+\xi_2(B_i+B_j)Q_p^B(x) \\ &+(1-\xi_1-\xi_2)(M_i+M_j)Q_p^M(x) \\ &+(1-\xi_1-2\xi_2)B_kF_p(x)+(\xi_1+2\xi_2)M_k\Phi_p(x) \\ &+(i\rightleftarrows k)+(j\rightleftarrows k)]+M\varphi_p(x)+Bf_p(x)+\bar{B}f_p(x).\end{aligned} \qquad (48)$$

According to (9), the normalizations are:

$$\int_{\mu/\sqrt{s}}^1 dx\, f_p(x) = N(s), \quad \int_{\mu/\sqrt{s}}^1 dx\, \varphi_p(x) = dN(s). \qquad (49)$$

and the remaining functions ($D, Q_p^B, Q_p^M, F_p, \Phi_p$) are normalized to unity. In the region in which these functions are large (of the order of unity), we must have

$$Q_p^B(x) = Q_p^M(x) \quad \text{for} \quad x\sim\frac{1}{3}, \qquad (50)$$

$$F_p(x) = \Phi_p(x) \quad \text{for} \quad x\leqslant\frac{1}{6}. \qquad (51)$$

In the region of fragmentation of the pion beam, the spectra are determined in an analogous manner:

$$\begin{aligned}\frac{1}{\sigma_{\text{inel}}}\frac{d\sigma}{dx}(\pi\to\text{hadron}) &= \delta M_{i\bar{j}}d_\pi(x)+\frac{1-\delta}{2}[\eta(B_i+\bar{B}_j)Q_\pi^B(x) \\ &+(1-\eta)(M_i+M_{\bar{j}})Q_\pi^M(x) \\ &+\eta(B_i+\bar{B}_j)F_\pi(x)+(1-\eta)(M_i+M_{\bar{j}})\Phi_\pi(x)] \\ &+M\varphi_\pi(x)+Bf_\pi(x)+\bar{B}f_\pi(x).\end{aligned} \qquad (52)$$

The normalization is the same as in the case of the proton beam. The following relationships must be satisfied:

$$\begin{aligned} Q_\pi^{\mathrm{B}}(x) &= Q_\pi^{\mathrm{M}}(x) \quad \text{for} \quad x \sim \frac{1}{2}, \\ F_\pi(x) &= \Phi_\pi(x) \quad \text{for} \quad x \leqslant \frac{1}{4}. \end{aligned} \tag{53}$$

Within the framework of the soft hadronization hypothesis, it is natural to try to relate the behavior of the functions $D(x)$ and $Q(x)$ to the distribution of dressed spectator quarks in a proton or meson. The spectator quarks create suitable partners for themselves during hadronization, i.e., they "shed" their momentum to sea quarks with which they combine into hadrons. We may therefore suppose that the x distribution of the fragmentation hadrons should approximately repeat the x distribution of the spectator quarks they contain. We shall assume that $Q^{\mathrm{M}}(x)$ and $Q^{\mathrm{B}}(x)$ are identical with the spectator-quark distribution and $D(x)$ is identical with the distribution of two spectator quarks (i.e., the "diquark"). The one-parameter approximation to the quark wave function of the incident hadrons[139] will provide us with sufficient precision for our purposes in the determination of $Q(x)$ and $D(x)$. The free parameter is uniquely determined by the mean square radius of the hadron (the values of $R_p^2, R_\pi^2, R_{\mathrm{K}}^2$ are given in Ref. 136).

The hypothesis of soft hadronization will also enable us to draw certain conclusions regarding the x distribution of mesons and baryons created during collisions between dressed quarks, i.e., tagged and sea hadrons. The cloud of created quarks is finally formed by multiple interaction of dressed quarks with one another. As a result, the multiperipheral comb (jet) of such quarks should not be very dependent on its origin. The fact that the x distributions of dressed quarks in such jets are universal does not, therefore, seem unexpected. We then have

$$\begin{aligned} F_{\mathrm{p}}(x) &= \int_x^1 \frac{dz}{z} Q_{\mathrm{p}}(z) F\left(\frac{x}{z}\right), \quad \Phi_{\mathrm{p}}(x) = \int_x^1 \frac{dz}{z} Q_{\mathrm{p}}(z) \Phi\left(\frac{x}{z}\right), \\ f_{\mathrm{p}}(x) &= \int_x^1 \frac{dz}{z} Q_{\mathrm{p}}(z) f\left(\frac{x}{z}\right), \quad \varphi_{\mathrm{p}}(x) = \int_x^1 \frac{dz}{z} Q_{\mathrm{p}}(z) \varphi\left(\frac{x}{z}\right), \end{aligned} \tag{54}$$

where the functions F, Φ and f, φ are determined by the distributions of tagged and sea hadrons in quark jets. The distribution functions for the meson beam are found in an analogous manner (with Q_{p} replaced with Q_π, Q_{K}).

Figures 21–23 demonstrate the description of the spectra of secondary particles obtained in this way. When $x \to 1$, there are often appreciable discrepancies between theory and experiment. Diffractive dissociation processes, which are usually described by three-Reggeon diagrams, play an appreciable role in this region. The hadron mass splitting in the multiplet leads to a difference between Regge trajectories, so that appreciable discrepancies as compared with quark statistics are expected in diffractive dissociation processes. In our calculations of the spectra, we have taken into account the three-pomeron contribution (PPP), but have ignored quasidiffractive contributions (PRR) due to exchanging a non-vacuum Reggeon (for example, A_2 with $\pi^- \to \rho^0$ and the subsequent decay $\rho^0 \to \pi^+\pi^-$). Hence, for example, the $\pi^- \to \pi^-$ spectrum is described quite well for $x > 0.8$, but a clear discrepancy is observed for the $\pi^- \to \pi^+$ spectrum.

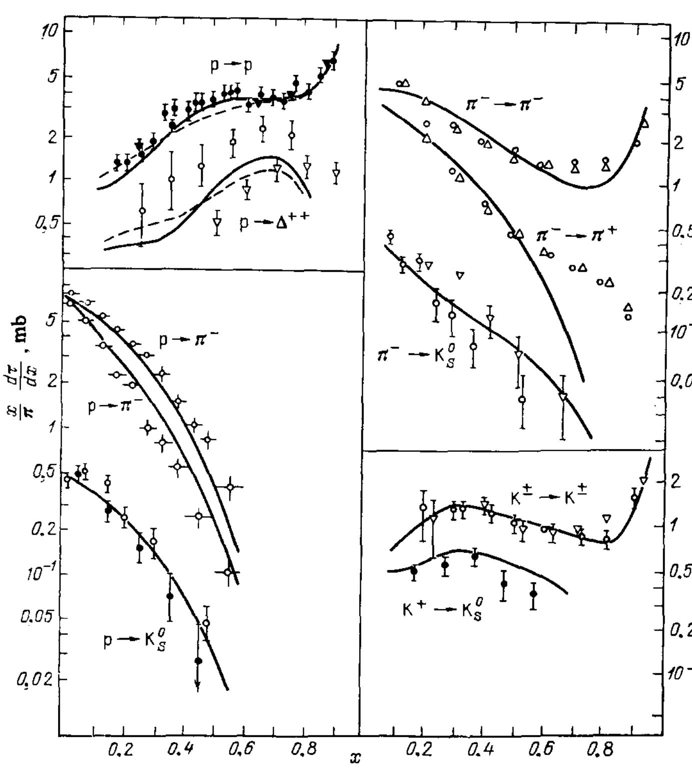

FIG. 21. Spectra of different secondary particles created in hadron-nucleon collisions and their description by the quasinuclear model. Data taken from Refs. 137–146. Broken lines show calculations with shadow corrections (see Subsection 3E). Here and henceforth, we give the τ-section.

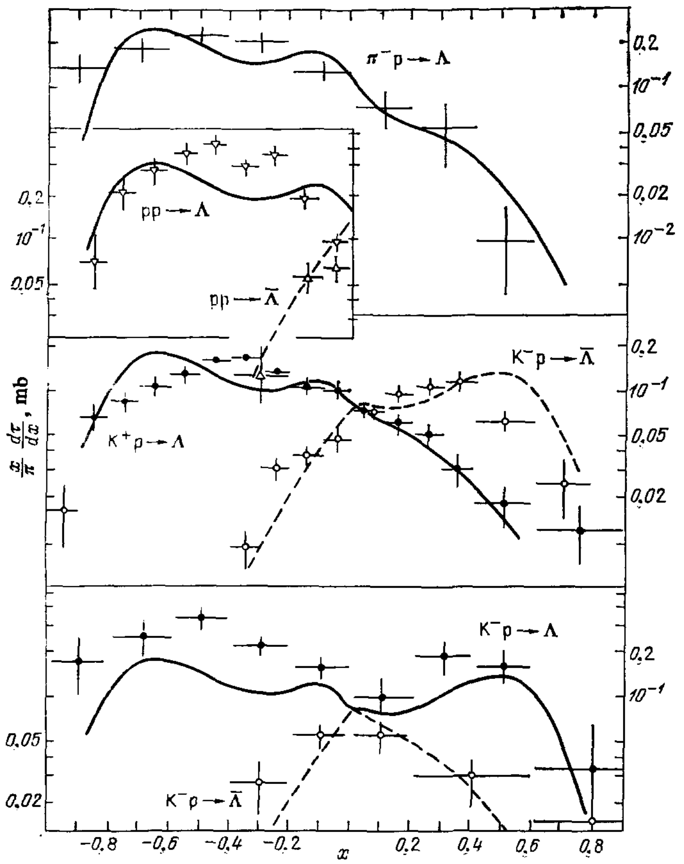

FIG. 22. The spectra of Λ- and $\bar{\Lambda}$-hyperons created on a nucleon target, as predicted by the quasinuclear model. Data taken from Refs. 145, 147, 148, and 149.

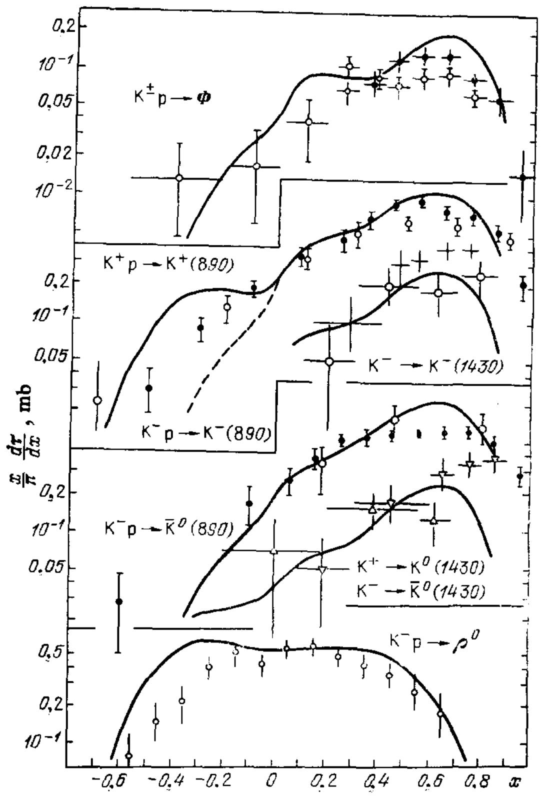

FIG. 23. Spectra of meson resonances created in K⁺p collisions (Refs. 150–155). The curves are calculated from the quasinuclear model.

Figure 24 shows the mean secondary-hadron multiplicities (references will be found in Ref. 156) and the corresponding predictions based on the rules of quark statistics.

The values of the parameters in (31), (49), and (52) are listed in Table V. Each of them can be found by normalizing to one of the experimental points. For example, $a_I^M(1) = 0.3$ is obtained from the ratio of the mean multiplicities of f- and ρ-mesons.

We note that the parameters ζ_1, ζ_2, η need not be the same as the values given by (29) and (30), which were calculated for the hadronization of an isolated special quark (diquark); several special quarks (spectator quarks and tagged quarks) are always present in hadron collisions. It is clear from Table V that the parameters ζ_1 and ζ_2 agree with the values (30) calculated from the rules of quark statistics. The parameter η is very different from the 1/3 given by (29) and confirmed by experimental data for hard processes (see Subsection C). This is probably an indication of a strong correlation between the spectator quark and the tagged antiquark (or vice versa) in meson-beam fragmentation processes.

Other approaches to the multiple creation problem (see Refs. 157–159 and the references therein) have also been used to describe the mean multiplicities and spectra with allowance for the quark structure of hadrons. However, the creation of resonances is not described in the great majority of these papers, and their contribution to the spectra of long-lived particles is taken into account effectively (for example, on the basis of considerations of duality). An analysis of both

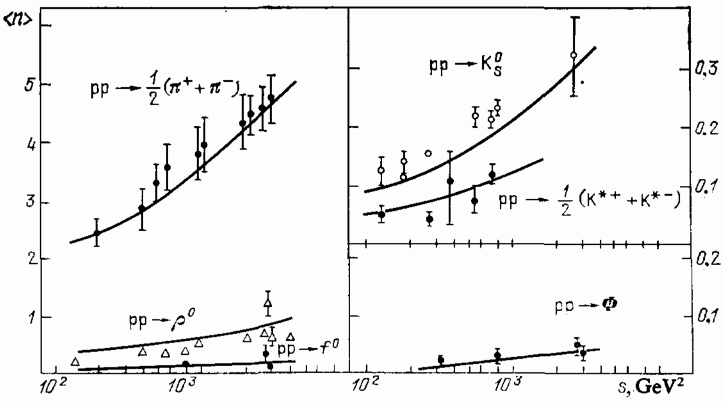

FIG. 24. Mean multiplicities of secondary particles created in pp collisions at high energy.

long-lived mesons and mesonic resonances is given in Ref. 160 within the framework of dual topological unitarization (DTU). The creation of pseudoscalar (P), vector (V), and tensor (T) mesons has been taken into account. By fitting the quark distribution function and the quark→meson function, it was possible to achieve a good description of the spectra of resonances (ρ, K^{*+}, Φ) in the entire range of x in K^+p and π^-p-collisions. However, a discrepancy between theory and experiment was found in the spectra of long-lived particles ($\pi^-p\to\pi^+$ and $pp\to\pi^-$ for $x=0$, and $\pi^-p\to\pi^-$ and $pp\to\pi^+$ for all x). It is possible that this discrepancy was due to the fact that the creation of other mesons with $L=1$ was not taken into account in Ref. 160 (see Table VI). The contribution of resonances with $J^P=1^+$ and $J^P=0^+$ to the formation of pion spectra is not small.

The description of the multiplicity spectra of both mesons and baryons has been carried out on the basis of the fragmentation model of the Lund group.[56,57] Here, the ratio of vector-to-pseudoscalar meson production probabilities was assumed to be a free parameter which was chosen to be V:P = 1-1 on the basis of ρ-meson production data for hard processes (the most accurate data correspond to large x). The creation of mesons with $L=1$ was not taken into account in the Lund model (Table VI). The creation of vector particles is satisfactorily described in this model (but not as well as by quark statistics; see Ref. 161). The description of secondary baryons in the Lund model and in quark statistics is performed at the same level of precision. We note that the rules for the formation of fragmentation baryons in the Lund model are effectively very similar to the rules of quark statistics. The creation of a baryon B_{ij} in the Lund model occurs with a probability of about 50% [$\xi_1=1/2$ in quark statistics; see (30)]. As in quark statistics, the ratio of the decuplet/octet creation probabilities is 2:1, and the SU(6) factors are taken into account for specific baryons. However, for the creation of baryons in the central region, the predictions of the Lund model are substantially different from those of quark statistics because the creation probabilities are assumed to be different for different diquarks in the Lund model.

E. Inclusive spectra in hadron-nucleus collisions

The description of hadron-hadron collisions can be used in conjunction with the quasinuclear structure of hadrons to predict uniquely the secondary-particle spectra in baryon-nucleus collisions for $x\gtrsim 0.1$. Extension to lower values of x is difficult. The point is that any model of multiple scattering gives rise to the question of the sharing of energy among secondary showers in the case of an inelastic interaction between the beam and several nucleons in the nucleus.[167,170] The quasinuclear model provides an unambiguous recipe for this sharing in the case of fragmentation secondary particles containing spectator quarks. On the other hand, when one initial quark undergoes several inelastic collisions, the question remains open, as before. Comparison with experiment and model estimates show that unambiguous predictions are obtained for $x\gtrsim 0.1$.

In this region, the inclusive cross section for an incident proton beam is

$$\begin{aligned}\frac{1}{\sigma_{\rm prod}(\mathrm{pA})}\frac{d\sigma}{dx}(\mathrm{pA}\to\mathrm{h}) &= \Delta(A)\,\mathrm{B}_{ijk}d_A(x)\\ &+\frac{1-\Delta(A)}{3}\Big[\frac{1}{2}V_1^{\mathrm p}(A)\,\mathrm{B}_{ij}D(x)+\Big(\frac{1}{6}V_1^{\mathrm p}(A)\\ &+\frac{1}{3}V_2^{\mathrm p}(A)\Big)\mathrm{B}_iQ_{\mathrm p}(x)+\Big(\frac{5}{6}V_1^{\mathrm p}(A)+\frac{2}{3}V_2^{\mathrm p}(A)\Big)\mathrm{M}_iQ_{\mathrm p}(x)\\ &+(V_1^{\mathrm p}(A)+2V_2^{\mathrm p}(A)+3V_3^{\mathrm p}(A))\\ &\times\Big(\frac{1}{3}\mathrm{B}_kF_{\mathrm p}(x)+\frac{2}{3}\mathrm{M}_k\Phi_{\mathrm p}(x)\Big)+(i\rightleftarrows k)+(j\rightleftarrows k)\Big]\\ &+(V_1^{\mathrm p}(A)+2V^{\mathrm p}(A)+3V_3^{\mathrm p}(A))\,[\mathrm{M}\varphi_{\mathrm p}(x)+\mathrm{B}f_{\mathrm p}(x)+\overline{\mathrm{B}}f_{\mathrm p}(x)],\end{aligned}\tag{55}$$

TABLE V. Values of parameters necessary for the calculation of inclusive spectra and mean multiplicities of secondary particles.

Parameter	d	ζ_1	ζ_2	η	Central region	Fragmentation region	$\alpha_I^M(1)$
Value	10	1/2	1/12	0,07	0,3	0,2	0,3

TABLE VI. Meson production probabilities in different schemes (in %).

J^P	0^- (P)	1^- (V)	2^+ (T)	1^+	1^+	0^+
Quark statistics	17.5	52.5	12.5	7.5	7.5	2.5
DTU[160]	29	55	16	0	0	0
Lund model[56]	50	50	0	0	0	0

where the V_1^p, V_2^p, V_3^p and Δ (A) are calculated (see Subsection 2F). We emphasize that the distribution of sea particles in quark-nucleus and in quark-nucleon interactions will be different.[162] Detailed examination of this situation from the standpoint of the quasinuclear structure of hadrons for $x \lesssim 0.1$ can be found in Refs. 29 and 30. All this is, however, relatively unimportant for $x \gtrsim 0.1$. We shall confine our attention to $x \gtrsim 0.1$.

For a meson beam, we have similarly,

$$\frac{1}{\sigma_{\text{prod}}(\pi A)} \frac{d\sigma}{dx}(\pi A \to h) = \delta_\pi(A) M_{i\bar{j}} d_A(x) + \frac{1-\delta_\pi(A)}{2} [0{,}07 V_1^\pi(A)(B_i + \bar{B}_{\bar{j}}) Q_\pi(x) + 0{,}93 V_1^\pi(A)(M_i + M_{\bar{j}}) Q_\pi(x) + 0{,}07 (V_1^\pi(A) + 2V_2^\pi(A))(B_i + \bar{B}_{\bar{j}}) F_\pi(x) + 0{,}93 (V_1^\pi(A) + 2V_2^\pi(A))(M_i + M_{\bar{j}}) \Phi_\pi(x)] + (V_1^\pi(A) + 2V_2^\pi(A))(M\varphi_\pi(x) + Bf_\pi(x) + \bar{B} f_\pi(x)). \quad (56)$$

Figures 25 and 26 show the spectra of secondary hadrons created in nuclear targets together with their description within the framework of the quasinuclear quark model.

Particle creation in hadron-nucleus collisions is described within the framework of the DTU scheme in Refs. 159, 163, and 164, using the multi-Reggeon exchange formalism for the hadron-nucleus amplitudes.[165–171] By cutting graphs with pomeron exchange (cylindrical diagrams in the language of the DTU scheme) we can calculate both the yield of fragmentation particles and the yield of particles in the central region. These calculations are sensitive to the way in which the initial energy is shared among the combs produced when pomerons are cut. In Ref. 159, the sharing of energy is controlled by the distribution of quarks in the incident proton in accordance with the fraction of momentum carried by them; it is assumed that the quark distribution can be obtained by extrapolating the shape of the Regge behavior for secondary particles (in this case, valence and sea quarks and diquarks) from the region $x = 0$ to the region of large x. This approach leads to a reasonable description of the pion spectra in pA collisions, but the calculated pion spectra obtained for πA-collisions with $x \gtrsim 0.5$ are very different from the experimental result (by factors of two-three).

It is important to remember that, in the quasinuclear model, all the rules for calculating the secondary-particle yields can be formulated in the same way as in DTU in the language of many-Reggeon amplitudes. The existence of fast particles (spectator quarks) in hadron-nucleus collisions will correspond to the specification of a particular method of energy sharing: fast secondary hadrons arise in the additive quark model from this type of sharing when one of the quarks retains its momentum with high probability.

The spectra for pA-collisions were calculated in Ref. 171 in the entire range of rapidity. The calculations were based on spectra for a nucleon target, which were appreciably different from the experimental spectra for small y. As noted in Ref. 172, the use of the experimental spectrum for pp-collisions in this type of calculation leads to poor agreement with data obtained in the fragmentation region of the target nucleus.

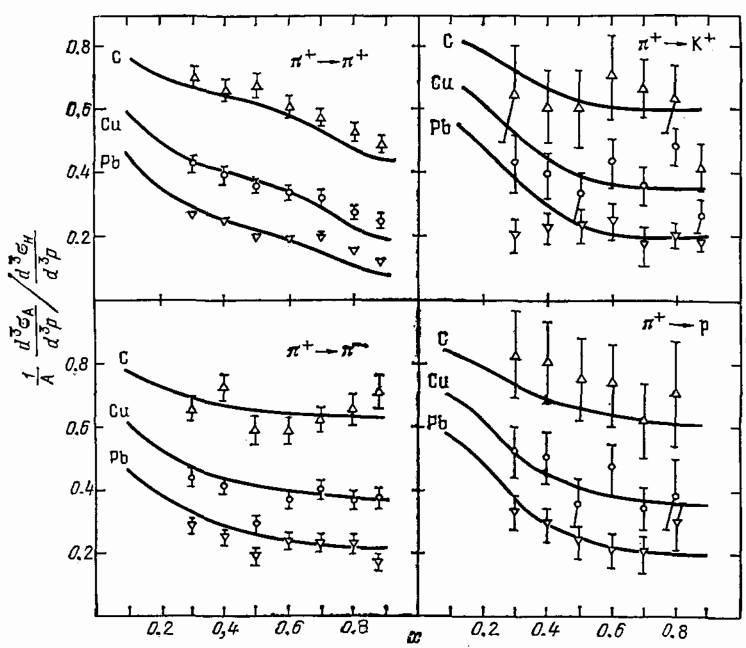

FIG. 25. Ratios of cross sections for the creation of secondary particles in π^+A and π^+p collisions[103] at 100 GeV/c ($p_\perp = 0.3$ GeV/c).

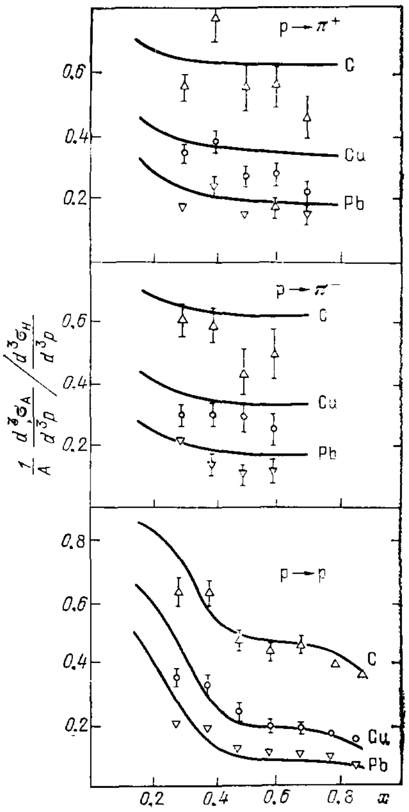

FIG. 26. Ratio of cross sections for the creation of secondary particles in pA and pp collisions[103] at 100 GeV/c ($p_\perp = 0.3$ GeV/c).

So far, we have discussed calculations of secondary-particle spectra within the framework of the impulse approximation. The calculated spectra become modified to some extent[173] when Glauber corrections for the interaction of several pairs of quarks are taken into account. The entire

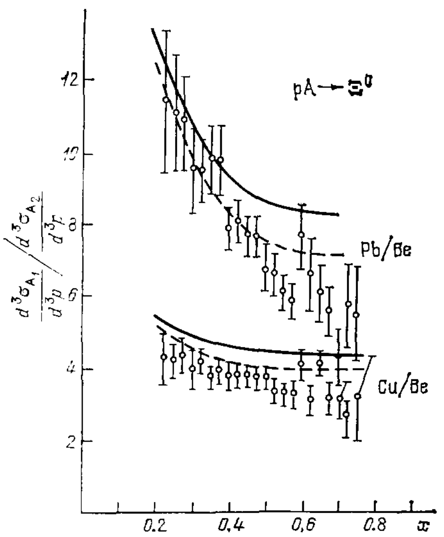

FIG. 27. Ratio of the Ξ^0-hyperon yield in pA collisions[174] at 300 GeV/c in the impulse approximation (solid curve) and with allowance for shadow corrections (broken curve).

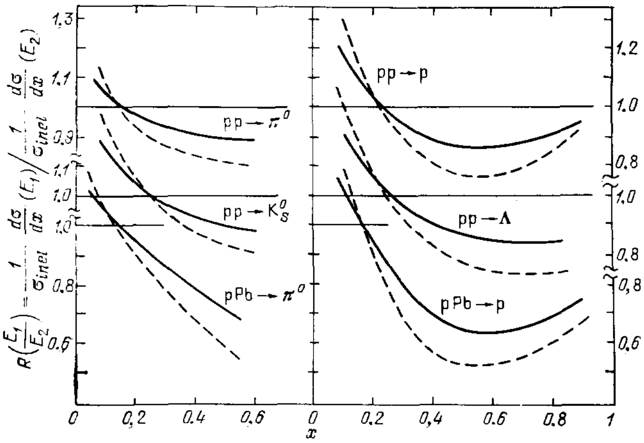

FIG. 28. Violation of scaling in secondary-particle spectra produced in pp and pPb collisions. The figure shows the ratio of spectra at energies for which $\sigma_{\rm tot}^{\rm pp} = 66$ mb (solid curves) and $\sigma_{\rm tot}^{\rm pp} = 93$ mb (broken curves) to spectra at $E \sim 100$ GeV.

difference is due to a reduction in the number of spectator quarks and an increase in the number of interacting quarks. As discussed in Subsection 2G, these quantities can be calculated within the framework of the Glauber approximation in a completely unambiguous way, so that calculations of the spectra with allowance for the quark shadow corrections do not require the introduction of any new parameters. It is clear from Figs. 21 and 27 that the changes in the spectra do not exceed 20–30%.

It was also mentioned in Subsection 2G that the increase in the total cross section should give rise to a breaking of scale invariance of the spectra in the fragmentation region when the quasinuclear quark model is employed. Such departures have been unambiguously calculated for different particles,[173] and the results are shown in Fig. 28. The increase in cross section also leads to an increase in multiplicity in the central region of the spectrum (because of the increase in the number of interacting quarks). Calculations show, however, that this effect is small (20–30% between ISR and SPS-collider energies). This means that the observed increase in multiplicity in the central part of the spectrum[175] is due to the increase in multiplicity in the quark-quark interaction itself.

4. CONCLUSION

The quasinuclear quark model can be successfully used to describe the multiple production of hadrons at high energies. The model has several attractive features: it is simple, almost free of empirical parameters, and its predictive power is considerable. It would appear that there are no experimental facts at present that are in serious disagreement (exceeding 20–30%) with the model.

Moreover, even from the most optimistic point of view, it is clear that only the first step has been taken in the direction of a description of multiple processes in terms of the quark structure of hadrons. We shall now try to list the basic problems that must be resolved if further progress along this route is to be achieved.

They can be divided into three groups, namely, those facing the experimenter, the phenomenology, and the theory.

We have shown that the quasinuclear model can be used to calculate the multiplicities and inclusive spectra of more than 200 secondary particles (54 mesons, 80 baryons, and 80 antibaryons) created during the interaction of pion, kaon, nucleon, and in principle, hyperon beams with nucleon or nuclear targets. Only a few percent of the resulting information can be compared with experiment. The first experimental task is therefore to measure the yields of the different meson and baryon resonances in hN- and hA-collisions. There is considerable interest in questions such as: is the quark model effective only to within 20–30%, as it is elsewhere, will there be systematic discrepancies between these predictions and experiment, and, if so, precisely where will they occur, and so on.

We have seen that there are other approaches to the description of multiple processes. From the phenomenological point of view, it is important to be able to understand the essential differences between the different models. It may well be that the differences between certain models are purely semantic. Moreover, different models have different ranges of validity.

Finally, theory should be able to tell us whether there are two dimensions (two confinement radii) in nature, and whether or not the additive quark model and quark statistics are merely fortuitously successful rules for calculations that do not reflect the true state of affairs.

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