Nonlinear scattering of radio waves by metallic objects

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The article contains a systematized account of experimental and theoretical aspects of the nonlinear effect of the generation of harmonics and combination frequencies at metal-oxide-metal contacts at radio frequencies. The special features and the regularities of nonlinear scattering of radio waves by metallic objects containing these types of contacts are investigated.

CONTENTS

1. INTRODUCTION

In the early forties it was found[1] that when metallic structures are present in the electromagnetic field of powerful radio transmitters of communications radio stations nonlinear effects arise which consist of the appearance of scattered radio waves at combination frequencies and of harmonics which were absent in the radiation from the transmitters, and also of the appearance of mutual modulation of radio waves (cross modulation) similar to what takes place in the ionosphere in the case of the Luxembourg-Gor'kiĭ effect.[2]

Later in experiments on ocean vessels equipped with several sufficiently powerful radio transmitters it was found[3] that the principal source of this nonlinear scattering of radio waves are contacts of metallic parts of objects situated in the field of the radio transmitters (connections and points of attachment of wire antennas, parts of the ship's hull, threaded connections of metallic parts, etc.).

Such nonlinear scattering of radio waves led to unacceptably high level of interference in radio reception (intermodulation interference—IMI), frequently exceeding the level of useful signals in these frequency channels.[3]

The frequency f_x of IMI is determined by the relationship

$$f_x = mf_1 + nf_2 + \ldots + qf_r, \qquad (1)$$

where $f_1, f_2, \ldots, f_r$ are the transmitter frequencies, m, n, q are positive or negative integers (including zero), while the sum $|m| + |n| + \ldots + |q|$ denotes the order of the intermodulation interference (in the simplest case f_x is the frequency of a harmonic of appropriate order).

A characteristic feature of the observed IMI was that the most intense ones were interferences of the third order (with frequencies $2f_1 \pm f_2$ in the case of two transmitters), in the simplest case—interference at the frequency of the third harmonics ($3f_1$, $3f_2$).

Similar nonlinear effects were observed in receiving-transmitting circuits of multichannel communications systems using ultrahigh frequencies (UHF). Their source were contacts at the points of detachable connections of waveguide or coaxial transmission lines[4] metallic coiled cable sheaths[5], etc. Particularly harmful were the IMI at the combination frequencies $2f_1$-f_2 which fall into the frequency band being utilized when the frequencies f_1 and f_2 are sufficiently close. In insufficiently tightly constructed flange connections of waveguide circuits the level of these IMI could attain values of -55 dB/W (with transmitter power of the order of 1 W) i.e., it was quite high.[4]

At the same time it turned out that these nonlinear effects could also find useful applications. Thus, the generation of combination frequencies of the third order was used to discover hidden defects in linear passive radio components (resistors, capacitors, etc.) and at the points of contact, where it is difficult, if not impossible, to carry out direct measurements of contact resistance.[6] It was proposed to utilize the above nonlinear effect for detecting weakened connections of metallic parts, and also cracks in metallic parts and structures.[7] So-called nonlinear radar equipment capable of detecting metallic objects of artificial origin against a background of strong interfering reflection from the earth's surface was constructed utilizing the nonlinear effect described above.

The cases described above can be regarded as different manifestations of the effect of nonlinear scattering of radio waves by objects containing contacts of metallic surfaces.

At the present time for a number of reasons (cf., further below, Sec. 2) a complete understanding of the physical mechanisms of the passage of current through such contacts has not been achieved (cf., for example, Ref. 9). At the same time in the periodic literature in the USSR and abroad a number of papers has been published elucidating the features and regularities of nonlinear phenomena brought about by the presence of such contacts, both in connection with the aforementioned parasitic effects of IMI, and also in connection with their useful applications.

The present article examines the experimental and theoretical aspects of the aforementioned phenomenon of nonlinear scattering of radio waves.[1)]

[1)] We do not deal here with nonlinear effects associated with a nonlinear dependence of the vectors of the electric or magnetic polarization on the corresponding fields in the medium.[11]

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2. NONLINEAR PROPERTIES OF A METAL-OXIDE-METAL CONTACT

Contact between metallic surfaces usually takes place through a thin oxide film which in the majority of cases one can approximately regard as a dielectric or a high resistance semiconductor.[12,13]

In a number of investigations it has been shown theoretically and experimentally that in the case of a sufficiently small thickness of the dielectric film (less than a few tens of angstroms) the principal mechanism of transport of charge carriers across the contact of metals separated by such a film is the tunnel effect[14–20] (cf., the literature in Ref. 14).

In Ref. 17 expressions have been obtained for the volt-ampere characteristic (VAC) of the tunnel current through the contact taking into account the influence of the dielectric and electrical image forces on the magnitude and the shape of the potential barrier in the dielectric layer. In the case of low voltages across the contact (usually less than ~ 1 V) this VAC in the case of the same metal forming the contact can be approximated by the following nonlinear relationship:

$$i=\frac{v+\beta v^3}{R_0}\ ; \qquad (2)$$

here v is the voltage across the contact, i is the tunnel current through the contact, β is the nonlinearity coefficient of the VAC, $R_0 - \rho_0/a_T$ is the initial (at $v=0$) contact resistance, ρ_0 is the "specific" resistance of the tunnel contact, a_T is the area of the tunnel contact.

The quantities ρ_0 and β depend on the height of the barrier φ_0, the thickness of the dielectric film s, and also on the value of the relative dielectric permittivity ε of the dielectric medium. The magnitude of ρ_0 depends strongly on s and φ_0 (it changes by several orders of magnitude as s or φ_0 is doubled), while the nonlinearity coefficient β of the VAC changes but little and is of the order of 1 V^{-2} for typical values of s (10–30 Å) and φ_0 (1–2 V). With voltages across the contact greater than 1–1.5 V the current increases with increasing voltage much more rapidly than according to formula (2).

As is well known, the tunnel current depends only weakly on the temperature. Thus, for example, the tunnel current through the contact for a film thickness $s = 20$ Å and barrier height $\varphi_0 = 1.5$ V changes by less than 10% as the temperature T varies from absolute zero to 300 K.[19]

If the contact consists of dissimilar metals, then the VAC becomes asymmetric;[20] formula (2) in this case should be supplemented by a quadratic term (it determines the detecting properties of the metal-oxide-metal contact[21]).

Along with tunnel emission among the number of the principal mechanisms of the passage of current across a metal-oxide-metal contact we also have the over-the-barrier (Schottky) emission, i.e., the external-electric-field-dependent thermal emission of electrons from a metal with an energy exceeding the barrier height.[14,20,22] The VAC of the thermal emission (Schottky) current is also nonlinear.

The ratio between the tunnel and the thermal emission currents depends on the barrier height, the thickness of the dielectric layer, the temperature and the applied voltage. In Ref. 20 a comparison is given of the calculated VAC of the thermal emission current and the tunnel current through a metal-dielectric-metal contact for typical values of the barrier height φ_0 and the thickness of the dielectric s for $T = 300$ K. For $s \lesssim 40$ Å the tunnel current dominates for all values of v. For $s \gtrsim 50$ Å for low voltages the thermal emission current dominates, while starting with a certain voltage across the contact the tunnel current predominates. The thermal emission current falls off sharply as the temperature is decreased.

In the presence of impurities in the dielectric layer charge transfer can occur due to the so-called hopping conductivity, i.e., by means of hopping of electrons between localized states situated deep within the forbidden band of the dielectric.[14,23]

In addition to the aforementioned mechanisms also other significant mechanisms are possible for charge transfer through thin oxide layers between metallic surfaces (cf., review articles in Ref. 14).

In actual metal structures the oxide layer separating the metal surfaces is usually a quite imperfect electrically "dirty" dielectric, which has a large number of impurities and defects, and has an inhomogeneous structure and a thickness which is different in different places. In a number of cases, as for example, in oxidation of iron, the oxide films may consist of several layers of different chemical composition.[13] Physical investigation of mechanisms of the passage of current through such "dirty" inhomogeneous oxide films are exceedingly difficult, as a result of which the physical processes occurring in them have received little study.

The complexity and multiplicity of phenomena in such type of contacts can be illustrated by the results of experiments on the measurement of VAC of contacts of steel in the temperature range of 77 K–300 K.[24,25] Depending on the point of contact of the same contact samples in the sphere-plane shape made of steel (steel 20) one could observe VAC of significantly differently types, shown in Fig. 1. For $T = 300$ K a typical VAC was one of the types shown in Fig. 1a (a "cubic" VAC), occasionally VAC were observed of the type of Fig. 1b (VAC of the type of an "integral").

At a temperature of 77 K for the majority of contact points an S-like VAC was observed with a falling section with negative resistance (Fig. 1c). As the resistance of the external circuit R_A was increased a stable relaxation type oscillation was observed (illuminated areas in Fig. 1d) with frequencies of 20–60 MHz; with a lower resistance by varying the voltage of the source one could observe a switchover effect (i.e., a sharp change of the voltage across the contact). As the temperature was raised to 300 K the S-like VAC, as a rule, was transformed into a "cubic" VAC (cf., Fig. 1a) and only extremely rarely did it retain its S-like shape with an unstable oscillation. In those cases when at 77 K the VAC had a "cubic" shape of Fig. 1a, the shape was retained also as T was raised to 300 K with, as a rule, an insignificant change in the absolute value of the current. One can assume that for these cases, judging by the weak temperature dependence of the VAC, the tunnel current predominated. The S-like VAC was also observed in contacts with a film of niobium oxide in Ref. 26; the mechanism responsible for the appearance of negative resistance was associated with the phenomenon of avalanche multiplication accompanying impact ioniza-

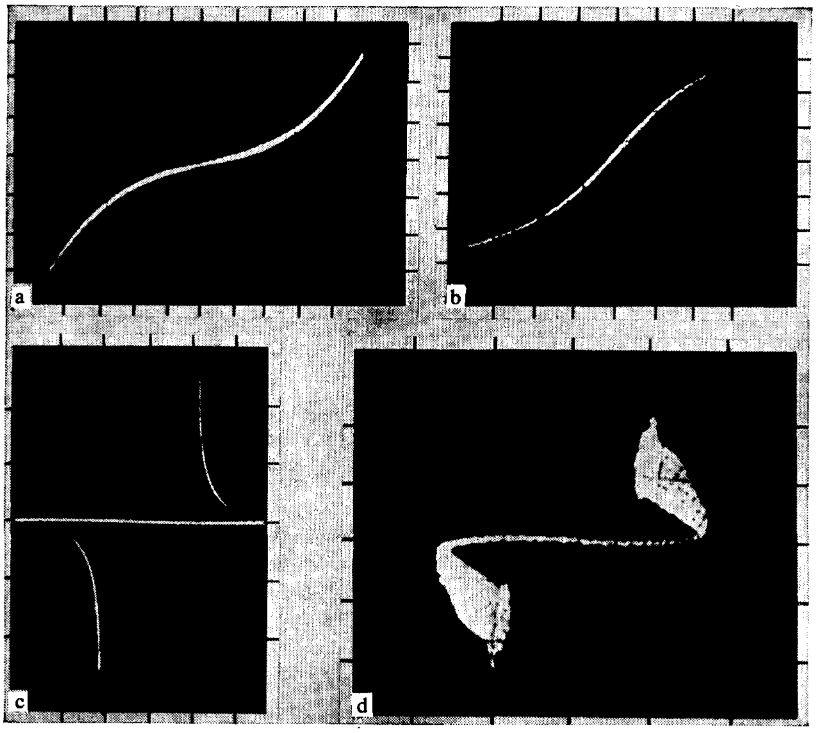

FIG. 1. Experimental VAC of a contact made of steel at $T = 77–300$ K. The contact is in the form of a sphere (radius 3 mm)-plane, polished, and then oxidized in air at 400°C for 8 hours. Scales along the axes: a) x-0.5 V/div, y-6 mA/div; b) x-1 V/div, y-50 mA/div; c) x-5 V/div, y-10 mA/div; resistance of the external circuit $R_A = 1$ kOhm; d) x-5 V/div, y-0.7 mA/div; resistance of the external circuit $R_A = 10$ kOhm.

tion—a reversible breakdown.[26,14]

It was shown in Refs. 24, 25 that the "cubic" VAC of Fig. 1a typical for 300 K independently of its shape at 77 K, when recorded at 300 K for several specified samples of contacts is described with a high degree of accuracy by formula (2) when the voltage across the contact is less than 1–1.5 V. As the contact pressure is varied over quite a wide range (from 0 to 2 kg force) the nonlinearity coefficient β of the VAC did not vary (for the contacts that were investigated it was of the order of magnitude 1 V^{-2}), in contrast to the resistance R_0 and the capacitance C of the contact which depend strongly on the pressure. When the voltage across the contact exceeds ~ 1.5 V the VAC is much steeper than given by (2), and with a further increase of the voltage the regime becomes unstable, and in the majority of cases an irreversible breakdown of the contact occurs. The relationship (2) was obtained for voltages $< 1–1.5$ V also for contacts of other metals by the authors of Refs. 24, 25 and in Ref. 27.

A number of features of the nonlinear effect of the production of harmonics and combination frequencies in metal contacts at radio frequencies has been studied experimentally in Refs. 24, 25, 28, 29, 30.

In Refs. 28–30 measurements were made of the level of intermodulation interferences which were produced by contacts included in the coaxial line of an UHF circuit. The experiments were carried out using approximately the same methodology in the ranges ~ 3 GHz,[28] ~ 4 GHz,[29] ~ 1.5 GHz[30] by measuring the power p_{nonlin}' of the IMI of the third $(2f_1\text{-}f_2)$ and of the fifth $(3f_1\text{-}2f_2)$ orders originating in a contact in the course of passage through it of UHF currents of frequencies close to f_1 and f_2 with a known power P_{fund} which is the same for both frequencies. Attention was devoted mainly to the contacts of the type sphere (radius from 1 to 10 mm)-plane, since other shapes of contacts (plane, cone-plane, etc.) turned out to be less convenient for experimentation.

The principal results of the experiments carried out in Refs. 28–30 consist of the following.

It was reliably established that the cause of the IMI that were investigated was the oxide film on the metallic surfaces forming the contact.

The intensity of the IMI diminishes as the pressure on the contact is increased. For a definite sufficiently great critical pressure the magnitude of which depends on the kind of metal, the thickness and the strength of the oxide, and also on the cleanliness in conditioning the surface, the IMI falls below the threshold of sensitivity of the apparatus as a result of the mechanical damage to the oxide and the establishment at that point of a true metal-metal contact. For this reason in the case of contacts made of oxygen-free copper, oxygen-free electrically polished nickel, beryllium copper, etc., which are electrically relatively "clean" contacts, i.e., they have a very thin and fragile oxide film, the level of the IMI was negligibly small at a pressure of the order of 100 g force (when the radius of the spherical contact was of the order of 1 mm). Under the same experimental conditions a high level of IMI (of the order of -60 dB/W at a power level of P_{fund} in the line of 1 W) was observed in the case of similar contacts made of steel, of stainless steel, aluminum and other materials for which the oxide film is sufficiently strong.

As the cleanliness of conditioning of the contact surfaces was increased the level of the IMI increased, while the magnitude of the critical pressure decreased.

In the case of contacts galvanically covered by gold or silver the level of the IMI fell below the threshold of sensitivity of the apparatus already at a contact pressure 10 g force.

When a dielectric film (teflon and other similar materials) of sufficient thickness was introduced between the metallic surfaces in contact the level of the IMI fell below the sensitivity of the apparatus.[29] This approach can be used to diminish the level of the IMI from flange connections in waveguide circuits. Another method consists of a considerable increase in the force pressing the waveguide flanges together.

In the case of all the contacts that have been investigated the following relationship was observed for the dependence of the power of the nonlinear response of the third order P_{nonlin} on the power P_{fund} of first harmonics that passes through the contact in the line:[28–30]

$$P_{\text{nonlin}} = kP_{\text{fund}}^{m}, \tag{3}$$

where the power index m is on the average equal to 3.

The typical experimental dependence of P_{nonlin} on P_{fund} plotted logarithmically (in dB) is shown[28] in Fig. 2. It is seen to be a straight line with a slope equal to 3 (the scatter of the experimental data on the magnitude of m in Ref. 28 is $\Delta m = \pm 0.1$).

The intensity of the third order IMI is considerably higher than of the fifth order IMI.

Comparing the results of the above experiments performed by different investigators in different frequency ranges at ultra high frequencies (1.5–6 GHz) we can note the strong frequency dependence of the intensity of IMI.[30] Thus, as the frequency was increased by a factor of two from 1.5 GHz[30] to 3 GHz[29] the level of the IMI decreased, as a rule, by more than 20 dB.

No theoretical analysis of the problem concerning the intensity of the IMI was given in the papers to which reference was made above.

A different relationship, more general than that described by formula (3), was observed by the authors of Refs. 24, 25 in carrying out experiments with metallic contacts at lower frequencies.

Figure 3 shows on a logarithmic scale (in dB) the dependence of the relative level of the amplitude of the voltage of

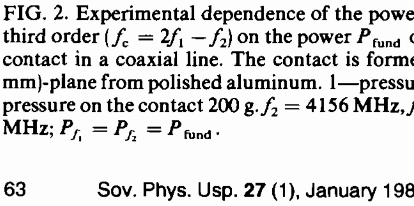

FIG. 2. Experimental dependence of the power P_{nonlin} of the IMI of the third order ($f_c = 2f_1 - f_2$) on the power P_{fund} of the first harmonics for a contact in a coaxial line. The contact is formed by a sphere (radius 1.5 mm)-plane from polished aluminum. 1—pressure on the contact 50 g, 2—pressure on the contact 200 g. $f_2 = 4156$ MHz, $f_1 = 4021$ MHz, $f_c = 3886$ MHz; $P_{f_1} = P_{f_2} = P_{\text{fund}}$.

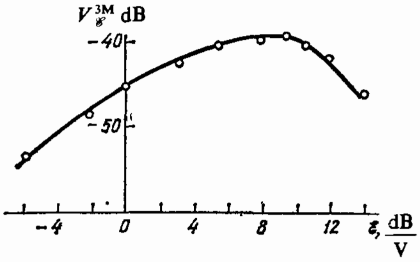

FIG. 3. Experimental dependence of the relative level of the third harmonic at the contact on the amplitude of the emf in the electrical circuit. The contact is made of steel (steel 20) in the form of a sphere (radius 3 mm)-plane, polished, and then oxidized in air at 400°C for 8 hours; the resistance of the external circuit is $R_A = 200$ Ohm; $R_0 = 100$ Ohm; $\beta = 1.75$ V^{-2}.

the third harmonic across the contact $V_{3M}/\mathscr{E}$ on the amplitude $\mathscr{E}_{\text{emf}}$ of the first harmonic in the electrical circuit containing a contact of oxidized steel surfaces, an audio frequency generator and a resistor connected in series.[24] This dependence corresponds to the relation given by (3) only at a low level of the emf $\mathscr{E}$ (< 1 V) of the generator at the frequency of the first harmonic. At a certain level of $\mathscr{E}$ saturation occurs (a broad maximum of the quantity $V_{3M}/\mathscr{E}$ appears, and with further increase of $\mathscr{E}$ the relative level of the third harmonic diminishes.)

A similar regularity was obtained in Refs. 24, 25 at frequencies of the order of 300 MGz (cf., below, Sec. 3). The theoretical explanation of these results, and also of the frequency properties of the effect of nonlinear scattering of radio waves is discussed further in Sec. 3.

3. SOME REGULARITIES IN NONLINEAR SCATTERING OF RADIO WAVES INDUCED BY CONTACTS OF METALLIC SURFACES

Let us examine the electrical circuit shown in Fig. 4 which describes phenomenologically the behavior of a metal-oxide-metal contact in an electric circuit and to which, as is shown below, one can reduce a number of important cases of nonlinear scattering of radio waves.

We shall assume that the nonlinear contact K has a volt-ampere characteristic, which, in accordance with the experimental data, is described by formula (2) when the voltage across the contact is $v < 1$–1.5 V, the voltage v across the contact (Fig. 4) satisfies in this case the differential equation

$$\frac{1}{\omega_A}\frac{dv}{dt} + v + \frac{a\beta}{1+a}v^3 = \frac{e(t)}{1+a}, \tag{4}$$

where we have used the notation

$$a = \frac{R_A}{R_0}, \quad \omega_A = 2\pi f_A = \frac{1}{C\left(\frac{R_A R_0}{R_A + R_0}\right)}.$$

We shall call the frequency f_A the critical frequency for the

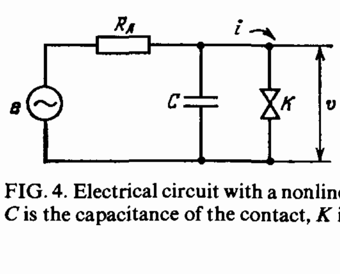

FIG. 4. Electrical circuit with a nonlinear contact. $e(t)$ is the external emf, C is the capacitance of the contact, K is a nonlinear contact.

contact.

We specify the *emf* setting it initially equal to

$$e(t) = \mathscr{E} \sin 2\pi f t. \quad (5)$$

The nonlinear differential equation (4) is a special case of an Abelian equation of the first kind and in the general case cannot be solved in quadratures.

However, employing the usual methods of the theory of nonlinear oscillations,[31] one can fairly simply obtain an approximate solution of the equations (4), (5) and to determine with sufficient accuracy the nonlinear response of the circuit mentioned above containing a nonlinear contact.

From the circuit of Fig. 4 and formulas (2) and (5) it follows that for $R_A = 0$ the current through the nonlinear contact contains only the first and the third harmonics of the frequency f. Therefore it is natural to assume that also in the case $R_A \neq 0$ these harmonics will be the predominant ones in the nonlinear response and that the approximate solution should be sought in the form

$$v \approx v_1 + v_3, \quad (6)$$

where v_1 is the voltage of the first harmonic of amplitude V_{1M}, and v_3 is the voltage of the third harmonic with the amplitude V_{3M}. We shall also assume that the amplitude V_{3M} is somewhat smaller than V_{1M} so that one can neglect terms of order $(V_{3M}/V_{1M})^2$ compared to unity.

In accordance with the quasilinear method of the nonlinear theory of oscillations,[31] we determine from formula (2) the resistance of the contact for the first harmonic (neglecting terms of order V_{3M}^2/V_{1M}^2):[32]

$$R_1 = \frac{V_{1M}}{J_{1M}} = \frac{R_0}{1 + (3/4)\beta V_{1M}^2}. \quad (7)$$

Further describing, in accordance with the quasilinear method of Ref. 31, the linear part of the circuit of Fig. 4 by appropriate impedances and complex amplitudes and using (7) as the characteristic of the contact one can obtain the following algebraic equation for the determination of the amplitude V_{1M} of the voltage across the contact:[32]

$$k^2 (V_{1M}^2)^3 + 2k (V_{1M}^2)^2 + V_{1M}^2 \left[1 + \left(\frac{f}{f_A}\right)^2\right] = \left(\frac{\mathscr{E}}{1+a}\right)^2, \quad (8)$$

where we have used the notation

$$k = \frac{3a\beta}{4(1+a)}.$$

Similarly from (6) and (2) one can obtain in the same approximation the formula for the determination of the desired amplitude of the third harmonic—the nonlinear response of the circuit of Fig. 4:

$$V_{3M}^2 = \frac{\left[\frac{a\beta}{4(1+a)} V_{1M}^3\right]^2}{\left[1 + \frac{3a\beta}{2(1+a)} V_{1M}^2\right]^2 + 9\left(\frac{f}{f_A}\right)^2}. \quad (9)$$

For sufficiently weak signals, when the resistance of the contact R_1 from (7) can be approximately taken equal to R_0, we obtain

$$\left(\frac{V_{3M}}{\mathscr{E}}\right)^2 = \frac{(a\beta)^2 \cdot \mathscr{E}^4}{16(1+a)^8 \left[1 + \left(\frac{f}{f_A}\right)^2\right]^3 \left[1 + 9\left(\frac{f}{f_A}\right)^2\right]}. \quad (10)$$

For very strong signals we obtain from (8) and (9)[2)]

$$\frac{V_{3M}}{\mathscr{E}} \approx \frac{V_{1M}}{6\mathscr{E}} \approx \frac{1}{6} \sqrt[3]{\frac{4}{3a\beta\mathscr{E}^2}}. \quad (11)$$

Thus, the relationship (3) is valid only in the case of sufficiently weak signals. In the case of strong signals the relative level of the nonlinear response diminishes with increasing strength of the signal at the fundamental frequency. Qualitatively this is explained by the fact that the resistance of the contact for the first harmonic falls as the amplitude $\mathscr{E}$ of the emf increases, and as a result the growth of V_{1M} and V_{3M} to an ever increasing degree falls behind the growth of the emf. Using the same method one can obtain the formula for the nonlinear response at the combination frequencies $2f_1 \pm f_2$, when the emf represents the sum of oscillations at the two frequencies f_1 and f_2.

We now go on to consider the nonlinear scattering of radio waves induced by contacts of metallic surfaces.

When electromagnetic waves are propagated in open space the generally accepted measure of the intensity of their (linear) scattering by a given object is the effective scattering area (ESA) σ of this object, which is defined by the relationship[33]

$$\sigma = 4\pi r^2 \frac{\Pi_{rec}}{\Pi} \quad (12)$$

where Π_{rec} is the flux of the power (W/m^2) of the wave scattered by the object measured in the distant (Fraunhofer) zone at a distance r from the object, and Π is the flux of the power of the wave falling on the object.

It is natural also in the case of nonlinear scattering of radio waves by objects containing contacts of metallic surfaces to adopt for the measure of intensity of nonlinear scattering the magnitude of ESA by generalizing in an appropriate manner the relationship (12). Such a generalization can be carried out in several different ways.[7,9,32,34] In the further discussion we follow Ref. 32, where the nonlinear effective scattering area (NESA) is defined by formula (12) with the difference from the linear scattering being that in the case of nonlinear scattering Π_{rec} and Π refer to different frequencies: Π_{rec} is measured at the frequency of the scattered wave (the frequency of the harmonic or the combination frequency), while Π is the total flux of power of waves irradiating the object, if the object is irradiated by waves of several frequencies. In contrast to linear scattering in the case of nonlinear scattering of radio waves NESA σ_n depends on the flux Π of the power of irradiation.

The theoretical calculation of the ESA of more or less complex objects in the case of linear scattering of radio waves is not simple[35] and has been carried through with sufficient rigor only for objects of the simplest geometric shape.[36] For complex objects the ESA is usually determined experimentally.

In the case of nonlinear scattering of radio waves it is much more complicated to obtain a value of the ESA theoretically than in the case of linear scattering.

A thin vibrator (dipole) containing a contact of metals with a nonlinear VAC (a nonlinear vibrator) represents the

2)With a certain combination of the parameters of the contact an increase in $\mathscr{E}$ can lead to a breakdown and destruction of the oxide film ("fritting"[12]), before the relationship (11) is attained.

simplest model for a calculation of a nonlinear ESA. Just as an investigation of the Hertz vibrator (dipole) enables one to elucidate the principal regularities of emission and linear scattering of radio waves, an investigation of the nonlinear vibrator gives one the possibility of establishing the principal regularities of nonlinear scattering of radio waves. At the same time one can represent in the form of a nonlinear vibrator a number of real metal objects elongated in one direction, in particular the wire antennas described in Ref. 3 which produced intense nonlinear interference.

An important advantage of a nonlinear vibrator is also the fact that due to its simplicity when used in an experiment one can control sufficiently accurately the parameters of the nonlinear contact and in connection with that compare the theoretical and the experimental data.

In Refs. 24, 25 results are given of the measurements of NESA of a nonlinear vibrator which are compared with the results of a theory developed in Ref. 32. Since the theoretical and experimental results obtained for a nonlinear vibrator also characterize the general regularities of the phenomenon described above of nonlinear scattering of radio waves we shall consider them in somewhat greater detail.

If a thin nonlinear vibrator (Fig. 5) situated in free space is irradiated by an electromagnetic wave then the emf induced in the vibrator will produce in it a current of that frequency, which, in passing through the contact K of two small metallic objects included in the vibrator circuit will produce a fall of voltage at the contact. This voltage, as a result of the nonlinearity of the VAC of the contact, will induce in the vibrator currents of the harmonics. In the case when a monochromatic wave acts on the vibrator this will principally be the third harmonic. Correspondingly in the field of the waves scattered by the vibrator in addition to oscillations at the frequency of the incident wave there will appear oscillations at the frequency of the third harmonic (if the vibrator is irradiated by waves of several frequencies, then also scattered waves at combination frequencies of the third order will appear; this case is discussed subsequently).

When the nonlinear response of such a vibrator is being calculated we shall describe, in accordance with the quasi-linear method of Ref. 31, the linear part of the vibrator-contact system by an appropriate impedance, and we shall describe the contact—in calculating the current and the voltage of the first harmonic—by its resistance (7). To simplify the calculation we assume that the dimensions of the bodies in contact are much smaller than the length of the vibrator.

We shall also assume that the electric vector of the incident wave is parallel to the vibrator and that the frequency of the incident wave f is equal to the resonance frequency of a thin half-wave vibrator without a contact, i.e., $f = c/\lambda = c/4l$ (this assumption is not a fundamental one and is introduced only for the greater clarity of the derivations). For this frequency the resistance of the vibrator reduced to the points

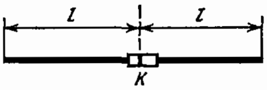

FIG. 5. Nonlinear vibrator.

of inclusion of the contact is purely active and is equal to the radiation resistance R_A. For a thin vibrator with the resonance frequency f the frequency $3f$ is also resonant and, therefore, the resistance of the vibrator at the frequency $3f$ reduced to the points of inclusion of the contact is purely active and is equal to the radiation resistance R_{A_3} at this frequency. In view of the insignificant difference in the values of R_A and R_{A_3} $R_{A_3} \times 73$ ohms, $R_{A_3} \approx 90$ ohms[37]) in order to simplify the final formulas we shall take $R_{A_3} \approx R_A$.

In accordance with the foregoing discussion the electrical circuit of a nonlinear vibrator reduced to the points of inclusion of the contact coincides with the circuit of Fig. 4. The calculation of the nonlinear response of such a circuit has been carried out in the preceding discussion by the quasi-linear method of the nonlinear theory of oscillations and can be used directly for the determination of the nonlinear ESA σ_n of the nonlinear vibrator at the frequency of the third harmonic.

Indeed, the power P_n, scattered by the vibrator at this frequency, is, in accordance with the circuit of Fig. 4, equal to

$$P_n = \frac{V_{3M}^2}{2R_{A3}} \approx \frac{V_{3M}^2}{2R_A}. \tag{13}$$

The flux Π_{rec} of the scattered power at a distance r in the far zone is determined by the formula[37]

$$\Pi_{rec} = \frac{P_n D_3}{4\pi r^2}, \tag{14}$$

where D_3 is the coefficient of the directed action of the vibrator in the given direction.

Taking into account that the emf $\mathscr{E}$ induced in the vibrator is related to the flux of the irradiating power Π at the frequency of the first harmonic by the expression[37]

$$\mathscr{E}^2 = \frac{2\lambda^2}{\pi^2} W_0 \Pi, \tag{15}$$

where $W_0 = 120\pi$ ohms is the radiation resistance of free space, one can obtain from (12)–(15) the following expression for the NESA σ_n:

$$\frac{\sigma_n}{\lambda^2} = \frac{D_3 W_0}{\pi^2 R_A} \left(\frac{V_{3M}}{\mathscr{E}}\right)^2. \tag{16}$$

From (10), (16) we obtain the following expression for the NESA of the half-wave nonlinear vibrator in the case of weak signals:

$$\sigma_n = \frac{\beta^2 D_3 W_0^3 \lambda^6 R_A \Pi^2}{4\pi^6 R_0^2 \left(1+\frac{R_A}{R_0}\right)^8 \left[1+\left(\frac{f}{f_A}\right)^2\right]^3 \left[1+9\left(\frac{f}{f_A}\right)^2\right]}. \tag{17}$$

In the case of weak signals we have a quadratic dependence of the NESA (and a cubic dependence of the power P_{rec}, recorded by the receiver at the frequency $3f$) on the flux of the irradiating power Π.

From formula (17) follows the strong dependence of the value of NESA σ_n and the nonlinear power P_{rec} on the ratio f/f_A: for $f/f_A \gg 1$ in accordance with 8th power of this ratio. In the case of a sufficiently large flux Π of the irradiating power one can obtain from (11) and (16) the following approximate expressions for the NESA σ_n in the case of strong signals:

$$\sigma_n \approx \frac{D_3}{100 R_A} \sqrt[3]{\frac{\lambda^2 W_0}{\left(\beta \frac{R_A}{R_0}\right)^2 \Pi^2}}. \tag{18}$$

The dependence of σ_n on Π here is essentially different than in the case of weak signals: in the case of strong signals the NESA σ_n *decreases as the flux Π of the irradiating power is increased*. Consequently, at a certain intermediate value of Π the magnitude of σ_n passes through a maximum. We also note that for sufficiently strong signals σ_n is independent of f_A.

Figure 6 shows on a logarithmic scale (in dB) the calculated dependences of σ_n/λ^2 on the amplitude $\mathscr{E}$ of the emf induced in the vibrator for different values of the parameters $a = R_A/R_0$ and f/f_A.[32]

The linear rising section of the curves of Fig. 6 corresponds to the domain of weak signals, when the quadratic dependence of σ_n (and the cubic dependence of the power P_{sc}, recorded by the receiver) on Π is valid. It can be seen from Fig. 6 that as Π ($\mathscr{E}$) is increased saturation is observed (a broad plateau in the dependence $\sigma_n = f(\Pi)$) and a decrease of σ_n with a further increase in Π.[3)]

Just as before, in examining the circuit of Fig. 4 this is explained qualitatively by a decrease in the resistance R_1 of the contact for the first harmonic as $\mathscr{E}(\Pi)$ is increased.

As has been pointed out before, with an increase in v the current grows more steeply than in accordance with expression (2). This leads to still greater slowing down in the growth of the voltage (restriction on the voltage) with increasing $\mathscr{E}(\Pi)$, than in the case of a VAC of the cubic type given by (2). From the qualitative explanation given above it therefore follows that the magnitude of σ_n for large Π must decrease as Π increases somewhat more rapidly than in accordance with $1/\Pi^{2/3}$ (formula (18)), and, specifically, in accordance with a law intermediate between $1/\Pi^{2/3}$ and $1/\Pi$. (The law $\sigma_n \sim 1/\Pi$ corresponds to a complete limitation on the voltage across the contact.)

Formulas similar to (17) and (18) can be obtained also for NESA σ_{nc} at the combination frequency $f_c = 2f_1 - f_2$, if f_1 and f_2 are sufficiently close to the resonance frequency of the vibrator f, so that one can use the circuit of Fig. 4. The principal difference of the formula for σ_{nc} in the case of weak signals from the formula (17) at the frequency $3f$ consists of replacing the two frequency-dependent factors in the denominator of (17) by a single one: $[1 + (f/f_A)^2]^4$. Moreover, the coefficient of directed action D_3 at the frequency $3f$ is replaced by the corresponding quantity D_1 for the frequency f.

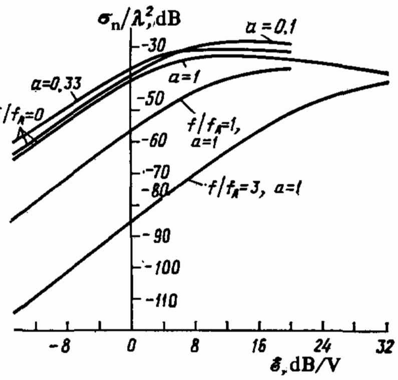

FIG. 6. Calculated dependence of the NESA of a half-wave nonlinear vibrator on the amplitude of the emf induced in it. Parameters: $a = R_A/R_0$, $\beta = 1\ V^{-2}$.

[3)]In accordance with Ref. 9 in some unpublished experimental work on nonlinear contacts a linear dependence of the power P_3 of the third harmonic on P_{fund} was observed at high values of the power P_{fund} of the generator at the fundamental frequency. The author of Ref. 9 explains this by the linearization of the VAC of the contact for high voltages across it. We note that this corresponds to the independence of NESA σ_n from P_{fund} for high values of P_{fund}.

The differences in the formula for σ_{nc} in the case of strong signals from the corresponding formula (18) are less significant (fundamentally they amount to a replacement of D_3 by D_1).

In analogy with the foregoing it is easy to evaluate IMI for the case described in Sec. 2 when the nonlinear contact is included in the feeder circuit—a coaxial line loaded at both ends by its radiation resistance. Without reproducing the corresponding formulas we note that also in this case just as in the case of nonlinear scattering in open space, in the case of weak signals one obtains a quadratic dependence of the relative level of P_{nonlin}/P_{fund} and a cubic dependence of the absolute level of P_{nonlin} of the nonlinear response of the third order on the power P_{fund} at the fundamental frequencies, while in the case of strong signals one obtains a decrease of this relative level when P_{fund} is increased.

The material presented above is in agreement with the results of experiments carried out in Refs. 24, 25. On the basis of formula (17) an explanation is found also for the strong frequency dependence of IMI mentioned in Sec. 2 which was observed in experiments with nonlinear contacts in coaxial lines for different UHF ranges, and also for the effect of the contact pressure on the intensity of the IMI.

The fact that the authors of Refs. 28–30 have observed experimentally only the initial rising (linear in logarithmic coordinates) section of the dependence of P_{nonlin} on P_{fund} and did not observe the effect discussed above of the saturation and reduction of the relative level P_{nonlin}/P_{fund}, is explained by the circumstance that these authors were carrying out their experiments in the range of ultra high frequencies, when the inequality $f \gg f_A$ was certainly satisfied (in accordance with the measurements of Refs. 24, 25 the typical value of f_A of the contacts is of the order of 300–500 MHz). In such a situation, according to the theory of Ref. 32 and to the discussion given above (cf., Fig. 6), the saturation effect can be observed only for very large values of the transmitted power at the fundamental frequency, which were not attained in the case of experiments with continuous oscillations at ultra high frequencies.[28–30]

Figure 7 shows the results of Refs. 24, 25 concerning the experimental investigation of dependence of NESA σ_n of the nonlinear vibrator on the flux of the power Π.

Into the middle of a resonant (and therefore somewhat shortened) half-wave vibrator ($2l = 47$ cm) a steel contact such as described above (cf., Sec. 2, Fig. 3) was inserted, whose parameters (R_0, C, β), and also the pressure on the contact were controlled in the course of the experiments. With the aid of a field-measuring device the values of the flux Π of the wave incident on the vibrator at the fundamental frequency were measured, and also those of the flux Π_{rec} at the frequency of the third harmonic at a distance r in the far

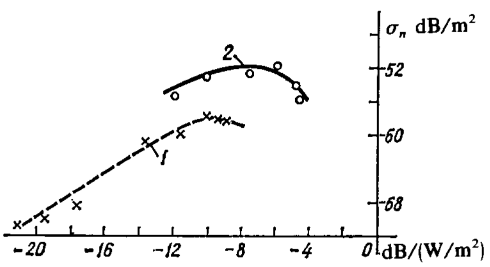

FIG. 7. Experimental dependence of the NESA of a half-wave nonlinear vibrator on the flux Π of the irradiating power. 1—vibrator with contact No. 1; 2—vibrator with contact No. 2.

zone of the vibrator. From measured values of Π and Π_{rec} the values of NESA σ_n were calculated using formula (12) for different values of Π.

In carrying out this kind of experiments on the investigation of nonlinear scattering an exceedingly high level of filtration and screening of the signal generator of the fundamental frequency is required: the presence of the third harmonic in its emission at a level higher than the allowable one can lead to considerable errors of measurement. A very high degree of filtration is required at the input to the field measuring device in order that the unavoidable parasitic induction of oscillations at the fundamental frequency would not give rise to nonlinear products in the input part of the receiver. It is also necessary that in the construction of the antennas of the oscillator and the receiver no nonlinear parasitic effects should arise.

The measurements that have been carried out[24,25] have shown that the theory developed in Ref. 32 describes with a satisfactory degree of accuracy the results of the experiments within the limits of the values of Π for which the approximation (2) is valid. Good quantitative agreement is obtained for weak signals. The effect of saturation and of a diminution of σ_n as Π is increased has been experimentally confirmed. However, the decrease in σ_n in the experiments begins at a lower value of Π than predicted by theory. This is explained by the fact that for large values of Π the approximation (2) used in the theoretical treatment is no longer valid.

The theoretical and experimental data reported above refer to the half-wave nonlinear vibrator ("resonant" vibrator)[4)]. It is not difficult to show that qualitatively analogous results must also be obtained in the case of a vibrator of length different from $\lambda/2$, only the decrease in σ_n must in this case begin at higher values of Π. It should be expected that also for more complex objects containing nonlinear contacts the regularities described above which were found for a nonlinear vibrator (transition of the quadratic dependence of σ_n on Π to the decrease of σ_n with increasing Π for large values of Π) will also be valid. Thus, on the basis of the principle of duality of the theory of gap antennas (the principle of A. A. Pistol'kors) we can assert that a gap vibrator in a metallic screen with a nonlinear contact has analogous characteristics for nonlinear scattering.

The value of the NESA for a nonlinear vibrator is of the same order as the NESA for many metallic objects containing contacts: in accordance with experimental data (cf., for example, Ref. 9) the NESA for such objects lie within the limits 10^{-6}–10^{-9} m^2 with a power flux of 1 W/m^2. Measurements of the NESA of a nonlinear vibrator[24,25] were carried out using a resonant (half-wave) dipole (cf., Fig. 7), and, apparently, because of this the results obtained for the NESA of the vibrator are close to the upper limit of the values indicated above.

The existence of a dependence of the ESA on the flux of the irradiating power, which is characteristic for the nonlinear scattering of radio waves, significantly alters the qualitative and quantitative dependences between the quantities entering the formula for radar ranging in utilizing this phenomenon.[7,9] Assuming that the flux of irradiating power corresponds to the increasing branch of the dependence of σ_n on Π (cf., Figs. 6,7) one can for a certain range of variation of Π write this function approximately in the form of a power dependence:

$$\sigma_n = \sigma_{n0}\left(\frac{\Pi}{\Pi_0}\right)^{m-1}; \tag{19}$$

here Π_0 is the normalizing flux of power, for which $\sigma_n = \sigma_{no}$; the quantities σ_{no} and m are functions of Π that are "slowly-varying" from section to section. In the case of weak signals the power index m is as has been shown above, equal to three. In Ref. 9 it is pointed out that for many objects $m \approx 2.5$ for $\Pi \approx 1$ W/m^2.

On substituting (19) into the radar equation[33] the following expression is obtained for the power of the radio signal P_{rec}, received by a nonlinear radar receiver (it is possible to operate on the third harmonic or on the combination frequency $2f_1 - f_2$):

$$P_{rec} = \left(\frac{P_r D_1}{4\pi}\right)^m \frac{1}{r^{2m+2}} \frac{\sigma_{n0} A_{rec}}{\Pi_0^{m-1}}, \tag{20}$$

where P_r is the radiated power in the pulse in the case of pulse radar, D_1 is the amplification coefficient of the transmitting antenna at a radiated frequency f, A_{rec} is the effective area of the receiving antenna at the reception frequency ($3f$ or $f_c = 2f_1 - f_2$). For $m = 1$ one obtains from (20) the dependence characteristic for the usual (linear) radar operation[33], when $P_{rec} \sim r^4$.

In the case of nonlinear radar the dependence of P_{rec} on r is different. Thus, in the case of weak signals ($m = 3$) the received pulsed power of the signal P_{rec} is inversely proportional to the eighth power of r, while for $m = 2.5$ it is proportional to the seventh power of r.

[4)]In a collection of articles reporting papers presented at a symposium at the University of Washington, Seattle, U.S.A., "Nonlinear Electromagnetic Waves" (Russ. Transl. ed. A. A. Vedenov, Mir, M., 1983) in Ch. 11 the problem is investigated of a wire receiving antenna with a nonlinear load with an arbitrary VAC. The calculation of voltages in the case of a nonlinear load reduces to the solution of a nonlinear integral equation utilizing an expansion into a Volterra infinite series. In so doing the questions associated with cutting off the series are not analyzed rigorously. A description is also given of different numerical methods of solving the problem. In the case of a nonlinear load with a VAC defined by formula (2) of the present article results of a calculation are given (without an indication of the specific method by which they were carried out) in the form of a graph approximately corresponding to the well-known cubic dependence of the power of the third harmonic on the irradiating power (cf., Fig. 2 of the present article), which is characteristic for weak signals. The case of strong signals[32] was not analyzed in the quoted reference.

Using relation (20) it can be easily shown that maximum value of the power of the received signal P_f at the output of a matched filter is proportional to the quantity $E_{rad} P_p^{m-1}$, i.e., it is proportional not only to the energy of the emitted signal E_{rad}, as in the case of linear radar[3], but, in addition, it also depends *directly* on the pulsed power. Therefore in the case of nonlinear radar, in contrast to the linear case, the signal at the output of a matched filter can be increased for a fixed value of the energy E_{rad} by means of increasing the pulsed power P_p of the radiation. However, as P_p is increased the flux of the power Π, irradiating a nonlinearly scattering object can approach the value corresponding to the saturation of the dependence of σ_n on Π (cf., Figs. 6,7). On this section we have $m \approx 1$ and the relationship between the quantities in (20) will be the same as in the case of linear radar. With a further increase of P_p the increase of P_f continues to slow down and P_f tends to a constant value, in spite of an increase of P_p. One can achieve an increase in P_f in this case by increasing the energy E_{rad} of the emitted oscillations only by means of increasing the duration of the emitted pulse for a constant value of the pulsed power P_p.

A characteristic feature of nonlinear scattering of radio waves is the fact that the amplitude and the phase of a nonlinearly scattered wave in a number of cases have strong fluctuations. This is due to the modulation of the nonlinear response of the contacts of the metallic parts of an object which is determined by the influence of different kinds of small mechanical deformations of the object as a result of vibrations, etc. This process is of a random nature, but the corresponding correlation function may in a number of cases contain characteristics of the source of the mechanical deformations.[9]

The amplitude and phase modulation indicated above of the nonlinearly scattered wave should be distinguished from the modulation effect due to the contacts of metallic parts of an object being variable in time in the case of *linear* scattering of radio waves.[38] In the latter case we have a *linear* system with parameters variable in time and the intensity of the spectral components near the fundamental frequency associated with these modulation effects, as a rule, is greater than in the case of nonlinear scattering.

4. CONCLUSION

In this article we have examined the main specific features and regularities of nonlinear scattering of radio waves due to contacts of metallic surfaces. These regularities have been determined, principally, by the theoretical and experimental investigations of the scattering of radio waves by a nonlinear vibrator, i.e., by a vibrator containing a metal-oxide-metal contact. Such a nonlinear vibrator, in spite of its simplicity, gives a good description of the properties of many real objects.

At the same time it is useful, apparently, to enlarge the class of models which could be used for a comparison of the results of theoretical and experimental investigation of nonlinear scattering of radio waves. A detailed experimental investigation of the frequency properties of nonlinear scattering of radio waves brought about by contacts of metallic parts of objects would be of interest.

It is also very desirable to introduce greater clarity into the nature of the physical processes in real ("dirty") contacts of metallic surfaces of objects.

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