FROM THE CURRENT LITERATURE

New aspects in 1/f noise studies

G. N. Bochkov and Yu. E. Kuzovlev

N. I. Lobachevskiĭ State University, Gor'kiĭ
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This article contains a brief review of the contemporary empirical pattern of $1/f$ noise in conducting systems and of attempts of its theoretical interpretation. It is noted that a spectrum of the form $\sim 1/f$ can be explained either as a consequence of the continuous hierarchy of macroscopically long relaxation times, or, on the contrary, as a result of absence of long-lived correlations and a characteristic time macroscale ("scaleless" $1/f$ noise). Theories of $1/f$ noise (fluctuation models of the number of charge carriers, temperature fluctuations, and "degradation" models) are traditionally directed toward the first type, and restrict the treatment to specific types of systems. The second type makes it possible to detect universal features of $1/f$ noise and, concealed in the formal fluctuation models of mobility carriers, it allows for a uniform and successful description of many situations. An explanation was recently suggested for the nature of mobility fluctuations, the diffusion coefficient, and other kinetic quantitites. It is noted that the current values of kinetic quantities are quite uncertain. Empirically this uncertainty is perceived in the form of "scaleless", flickering fluctuations of flowing kinetic quantitites. It also implies that real Brownian motion of carriers is characterized by non-Gaussian statistics. Analysis of the statistics leads to a spectrum of precisely $1/f$ type and to expressions for $1/f$ noise levels (depending only on microscopic space-time interaction scales of carriers with the medium) in satisfactory agreement with experiment.

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CONTENTS

1. INTRODUCTION

Electric $1/f$ noise was discovered by Johnson more than fifty years ago as an effect of slow fluctuations, flickers, being the emission capability of electron tube cathodes and of the tube current fluctuations generated by these flickers (the so-called "flicker effect"; whence another name of $1/f$ noise, flicker noise). The random flicker variations were superimposed on fast thermal (or shot) current fluctuations. The difference between these two components of the noise current is conveniently described by the spectral density $S_J(\omega)$, which demonstrates the frequency composition of noise and is also directly observable in experiments on noise characteristics. The quantity $S_J(\omega)d\omega/\pi$ is the portion of the mean square current fluctuation due to random variations with frequencies in the interval from ω to $\omega+d\omega$. For purely shot noise the spectral density S_J $=$const is constant over a wide range from vanishing frequency to frequencies that are reciprocals of the electron flight time. Because of flickers $S_J(\omega)$ increases at low frequencies approximately inversely proportionately to frequency, $S_J(\omega)\sim\omega^{-1}$. The low-frequency noise component that exhibits this curious behavior is what is called $1/f$ noise.

Following the flicker effect, $1/f$ noise was discovered in carbon resistors, and later in resistors of semiconducting and other materials, as well as in semiconducting devices and various elements of electron technology. $1/f$ fluctuations are well-known in biophysical systems (electric potential fluctuations, as well as rates of several chemical reactions). Also well-known are "nonelectric" $1/f$ noises, such as in fluctuations losses due to internal friction in quartz crystal resonantors (however, they are detected during the resonator operation in an electric generator circuit, leading to $1/f$ fluctuations in the frequency of a highly stable generator). Contemporary most sensitive technology, SQUIDs and other instruments based on Josephson junctions, is not free of $1/f$ noise: $1/f$ noise hinders the approach to the fundamental quantum sensitivity limit of measurements at low frequencies (kHz and below), though this limit is practically almost achieved in the high-frequency region.

The common property of all these systems (as well as of some enumerated below) is that $1/f$ noise emerges as a satellite of steady-state irreversible processes. Its contribution disappears from the fluctuation spectra of an electric current or potential, when the irreversible flows vanish and the system is brought into a thermodynamically equilibrium state. In other words, viewed from the outside $1/f$ noise appears to be the result of fluctuations of the dissipative parameters of the system, such as resistances, conductivities, etc., which determine the magnitude of irreversible flows for given nonequilibrium conditions. Generally the existence of such "parametric" low-frequency fluctuations is not at all surprising. As an example we consider resistor noise of a conducting material sample. It is understood that the dependence of the conductivity g on any macroscopic quantity must serve as source of conductivity fluctuations due to the ordinary thermodynamic fluctuations of this quantity. For example, the number of free carriers (conduction electrons and holes) directly affects the conductivity value g, and thereby leads to its fluctuations, reflecting the relatively small carrier number fluctuations. If a small potential difference x is applied to the sample, the current $J(t)$ can be written in the form $J(t)=\bar{J}+i(t)+x\delta g(t)$, where $\bar{J}=gx$ is the mean current, $i(t)$ is thermal white noise, and $\delta g(t)$ is the conductivity fluctuation. Due to the smallness of the fluctuations, the natural linear kinetic model of generation-recombination processes leads to an exponential law of fluctuation damping, with some spread in relaxation times τ_n for various processes, i.e., to a spectrum of the form

$$S_J(\omega)=2Tg+\bar{J}^2\sum_n a_n\frac{\tau_n}{1+(\omega\tau_n)^2}\,. \tag{1.1}$$

Here the first term corresponds to thermal white noise, whose spectrum is uniform up to frequencies reciprocal to the mean free path times of carriers, and in a weak nonequilibrium state (for a sufficiently small current $\bar{J}$) it can be determined from the Nyquist equation. The sum of Lorentzians describes "parametric", comparatively low-frequency, noise, whose power is $\sim\bar{J}^2$. This result is obtained within quite different conductivity fluctuation models.

According to simple theoretical concepts the spectrum of low-frequency noise, which is also called excess noise, should have the above property in a nonequilibrium state with the current. In reality, however, something totally different is observed. If the results of noise measurements at relatively high frequencies can be described by Eq. (1.1), then at low frequencies (starting, as a rule, with several kHz, and sometimes even over the whole frequency interval, where excess noise dominates over white noise) a $1/f$ type spectrum is observed

$$S_J(\omega)\propto\frac{\bar{J}^2}{\omega} \tag{1.2}$$

(or $S_J\sim\omega^{-\gamma}$ with an exponent $\gamma\approx1$). The physical reasons for this spectral behavior are generally quite unclear. This is the problem of $1/f$ noise.

We note that formally the spectrum (1.2) can be reduced to a sum of Lorentzians as in (1.1) by the simple mathematical operation:

$$\frac{1}{\omega}=\frac{2}{\pi}\int\limits_0^\infty\frac{\tau}{1+(\omega\tau)^2}\frac{d\tau}{\tau}.$$

It is tempting to interpret this possibility as an indication of the existence of a very large set of relaxation times τ (summed with a weight τ^{-1}). In most cases, however, there is insufficient justification for such a hypothesis, and one cannot provide reasonable physical explanation of the observed spectral behavior ω^{-1}, i.e., of the existence of $1/f$ noise.

The actual $1/f$ noise is always weak in the sense that it encompasses only a small portion of the mean square current fluctuation (the main contribution is due to white noise). However, it can exceed white noise by several orders of magnitude at low frequencies, and is the dominant damaging factor in the operation of instruments characterized by a high selectivity in frequency, i.e., by a long time constant. We note that, being parametric (multiplicative), $1/f$ noise "settles" on the harmonic signal, and is transferred to the neighborhood of the operating frequency of any instrument. It cannot be "filtered out", since a consequence of the ω^{-1} dependence is that even very low frequencies provide significant contributions to the noise. This is one of the reasons why it is practically very difficult to get rid of the interference of $1/f$ noise.

The problem of origin and mechanisms of electric $1/f$ noise has been discussed in the scientific and technical literature for a number of years, but even now it has been solved sufficiently. Such a solution assumes in the first place a clear knowledge of the principal physical nature of $1/f$ noise as a very common and, to all appearances, universal property of systems conducting an electric current (systems in which charge transfer occurs). It is precisely this part of the problem, so important from a theoretical point of view, where the situation is less clear, despite the massive accumulation of experimental information and the large effort of empirical generalizations.

This situation has extended the problem of $1/f$ noise from the narrowly applied "engineering" realm, converting to it a problem of general physical significance. The interest in recent years in the $1/f$ noise problem shown by specialists in statistical physics, physical kinetics, and fluctuation theory is not accidental. On the other hand, experimental studies of $1/f$ noise deepened and became more focused. As a result there appears an unexpected pattern of $1/f$ noise; material has been obtained, making it possible even now to attempt to "draw" it, though in crude, schematic form. Such attempts are made, and are necessary, since they create the theoretical basis for further studies. However, as a study of the literature shows, under contemporary conditions any attempt of theoretical rationalization of $1/f$ noise or a review of the problem necessarily carries the mark of personal experience and the authors' tendencies. The reason for this is not the shortage of experimental facts (certainly not the lack of abundance and diversity), but the ambiguity in their interpretation and the difficulty in distinguishing the most important from secondary, i.e., in final account, the absence of theoretical concepts of $1/f$ noise adequate to deal with the phenomenon. It is precisely the latter circumstance that dictated the direction of the present brief review. $1/f$ noise is an intriguing subject so far from fitting in the traditional scheme of concepts of fluctuation and kinetic processes in physical systems, that its dicussion may be interesting and useful to readers of many specialities dealing with the analysis of kinetics and fluctuations. The solution of the $1/f$ noise problem also implies, in our opinion, transition to a deeper understanding and more exact model description than presently available of the statistical structure of a variety of transport processes (which are the source noises). Accordingly we did not strive for a detailed enumeration of experimental work and or a summary of all theoretical ideas, put forward at various times for the explanation of $1/f$ noise. The topic of the present paper is to discuss the essence of the problem, results of several experiments (that appear the most important at the present time, and allow us to "feel" the nature of $1/f$ noise), and the shift occurring in recent years in theoretical studies, stressing the general aspects of the problem.

2. EMPIRICAL PATTERN OF $1/f$ NOISE

a) The most extensive recent review on $1/f$ noise was written by Hooge, Kleinpenning, and Vandamme.[1] We are primarily indebted to these authors for systematization (one can even say, unification) of the empirical pattern of $1/f$ noise. It is now commonly assumed that the basic qualitative and quantitative features of $1/f$ noise in uniform conducting media are described satisfactorily by the so-called "Hooge equation"

$$S_J(\omega) = \bar{J}^2 \frac{2\pi a}{\omega N}; \qquad (2.1)$$

where $S_J(\omega)$ is the spectral density of current fluctuations (from here on we denote by the symbol S the spectrum of the $1/f$ component of the total noise, leaving out the wideband component), $\bar{J}$ is the mean current value (the constant current through the conductor), N is the number of charge carriers in the "noisy" sample of the medium, a is a dimensionless quantity, sometimes called the Hooge constant, and $S(\omega)$ is the spectral density of the relative current fluctuations. The empirical equation for the spectral density of the voltage (emf) fluctuation at the sample has a similar form.

In the region of ohmic (linear) conductivity, to which, strictly speaking, Eq. (2.1) applies, the spectra of relative $1/f$ fluctuations of the current $J(t)$ and the voltage $V(t)$ are identical:

$$\frac{1}{\bar{V}^2} S_V(\omega) = \frac{1}{\bar{J}^2} S_J = (\omega) = S(\omega),$$

where $S(\omega)$ is independent of $\bar{J}$, $\bar{V}$. This equality is established experimentally, and must not be considered as obvious. It makes it possible to treat phenomenologically and describe the $1/f$ fluctuations of current and voltage (in the ohmic regime) as a result of fluctuations of resistance R or conductivity g, with the same "relative spectrum"

$$R^{-2} S_R(\omega) = g^{-2} S_g(\omega) = S(\omega) = 2\pi a (N\omega)^{-1}.$$

We agree further also to call $1/f$ noise flicker noise, flicker fluctuations, etc. These terms will also be applied in a wider sense to noise with a spectral shape $\sim\omega^{-\gamma}$, $\gamma > 0$, at low frequencies; the term "$1/f$ noise" will be used by us to underline proximity of the index γ to unity. Most often one does observe noise with an index $\gamma \approx 1$, while often the deviation of γ from unity does not exceed the experimental error throughout many frequency decades.

b) A striking and distinctive feature of flicker noise is that the increase in its spectral density with decreasing frequency occurs down to the lowest achievable measured frequencies $\sim 10^{-6}$–10^{-7} Hz, and no tendency to saturation is revealed.[1,2] Not less striking is the fact that this property of flicker noise, as also generally its spectral composition, is independent of the geometric dimensions of the system. The absence of saturation even at very low frequencies implies that the physical processes corresponding to flicker noise either have no characteristic upper time scale, or if they have one, then it is unusually large, larger than 10^6–10^7 sec. If a theory assumes the latter possibility, it must explain the physical significance of such a long time interval, and at the same time—a continuous set of shorter time scales. More appealing esthetically is the former possibility: the spectrum of flicker noise increases without bound with decreasing frequency.

Formally this behavior of the spectrum can be interpreted in two ways. Firstly, as the result of the existence of arbitrarily slow fluctuation processes of an infinitely wide set of time scales. Secondly, as the result of absence of slow processes, and correspondingly the absence of other time scales, except for some "microscopically" small scale. In different words, the flicker spectrum can describe both multiscale and "scaleless" fluctuation processes. The second possibility, paradoxical at first glance and difficult to understand intuitively, deserves special attention. More important is the response to the problem of the physical nature of scaleless fluctuations.

c) The level of $1/f$ noise is determined by the dimensionless quantity a in the empirical Eq. (2.1). It is noticeable that this quantity, as a rule, is independent of the number of charge carriers in the sample, N. Thus, the relative $1/f$ current fluctuations (as well as the voltage and resistance fluctuations) are inversely proportional to N, and the absolute fluctuations are proportional to N (since $\bar{J} \sim N$). The fact indicates, as is naturally assumed, that separate carriers provide statistically independent contributions to $1/f$ noise. In this case it could be expected that different regions of a conducting medium provide uncorrelated contributions to the total noise of the sample. Direct experimental verification shows that this is indeed so. It is striking, therefore, that fluctuation processes responsible for $1/f$ noise seem to be spatially localized, despite their "slowness". However, between sufficiently small and close (neighboring) regions there must be some correlation, there must exist a "correlation length of $1/f$ noise" for a given medium. Estimates of this length for several semiconductors gave a value smaller than 1 micron.[2]

Another remarkable experimental fact is the presence of some natural measure of $1/f$ noise, which typically corresponds in order of magnitude to the "Hooge constant" $a \sim 10^{-3}$. For many intrinsic (weakly alloyed) semiconductors[1] $a \sim 10^{-3}-10^{-2}$ (in an original paper[3] Hooge suggested the value $a \approx 2 \cdot 10^{-3}$), and a is usually independent of the sample temperature T. At sufficiently high temperatures values of $a \sim 10^{-3}-10^{-2}$ are also characteristic of metals, but here a depends on the temperature T, and can reach values[4] $a \sim 10^{-1}$. It is important to note that the levels of $1/f$ noise in a solid and a liquid metal are of the same order of magnitude. Thus, experiments with gallium gave $a \sim 2 \cdot 10^{-3}$ in the temperature range $T = 77-600$ K, while in the case of mercury at room temperature we also have[1] $a \approx 2 \cdot 10^{-3}$.

At the same time there exists a whole range of substantial deviations of the $1/f$ noise level from the "standard" (corresponding to $a \sim 10^{-3}$). For example, in indium antimonide InSb values $a \sim 10^{-5}-10^{-4}$ are observed[2,5] (though the "standard" value was given in Ref. 1). The noise is much lower in a strongly alloyed semiconductor than in an intrinsic one. According to Vandamme and Hooge, the following approximate dependence is satisfied $a \approx a_0(\mu/\mu_0)^2$, where $a_0 = 2 \cdot 10^{-3}$, μ is the carrier mobility, and μ_0 is their mobility in a pure material.[1] The semimetal bismuth does not follow the more or less common rules. In experiments with bismuth the quantity a is small, $a \sim 10^{-7}-10^{-4}$, and, besides, it depends significantly, as does N, on temperature, so that the combination a/N is approximately constant for a given sample, and $a/N \sim 10^{-2}/N_{at}$, where N_{at} is the number of atoms in the sample (see Ref. 8; different data, however, were given in Ref. 1). This clearly implies strong correlation between the various carriers in Bi in the process of flicker fluctuations.

On the other hand, in liquid electrolytes the $1/f$ noise level is much larger than "standard"; here[2] $a \sim 10$.

Noise is also anomalously high in a different type of inhomogeneous (disordered) media, granular, granular films, matrix composites, and so on (very thin fibers and films must also belong to inhomogeneous media). The enhancement of $1/f$ noise in inhomogeneous media is probably not related to additional noise sources; it can be explained, superficially and in general outline, by the fact that the main contribution to the conductivity and to noise is provided only by a small part N_{eff}($N_{eff} \ll N$) of the charge carriers (since not all carriers are located in positions of equal importance). In that case $S(\omega)$ in Eq. (2.1) must be replaced by $2\pi a/\omega N_{eff}$. For example, for a medium consisting of small metallic grains in a dielectric matrix it was found in Ref. 9 that N_{eff} was approximately half the number of grains. Comparison of the relative $1/f$ noise with the equation $S(\omega) = 2\pi a/\omega N_{eff}$ gave $a \approx 6 \cdot 10^{-3}$, i.e., the noise had an almost "standard" Hooge level. At the same time it can be said that in (1) $a \approx 6 \cdot 10^{-3} N/N_{eff}$, which is anomalously large. The measured index γ differed from unity, $\gamma = 1.10 \pm 0.03$ (comparison with the Hooge equation makes sense as long as $(\omega_2/\omega_1)^{|\gamma-1|} \sim 1$, where ω_1 and ω_2 are the boundaries of the frequency range under consideration).

d) The pattern of flicker noise in metals (metallic films and filaments) changes substantially during the transition from low to high temperatures, with the boundary temperature being approximately equal to the Debye temperature T_D. The first detailed experiments with metals were carried out by Voss and Clarke[6], and by Clarke and Hsiang[7] primarily at low temperatures (these studies are reviewed in Ref. 10). It was observed that the larger the temperature coefficient of the sample resistance, the higher is the relative noise level. This dependence is distinctly seen near the transition temperature to the superconducting state, since here the resistance temperature coefficient $\beta = |(T/R)\mathrm{d}R/\mathrm{d}T|$ is large. It was noted that the level of flicker noise, and sometimes the shape of its spectrum depend on the quality of thermal contact between the investigated sample and its surroundings (the sublayer, the method of attachments the medium in which the sample is immersed). Though the observed values of the index γ are often substantially larger than unity (reaching the value of 1.4), on the whole the experimental results were satisfactorily described by the empirical equation

$$S(\omega) = \frac{2\pi\beta^2}{\omega\,[3+2\ln(L/h)]\,C}, \qquad (2.2)$$

where C is the sample specific heat ($C < 3N_{at}$), and L and h are the sample length and width, respectively.

The influence of β and of heat exchange with the surroundings were interpreted by Voss and Clarke as an indication that flicker noise occurs due to ordinary thermodynamic fluctuations of the sample temperature. This conclusion was confirmed by the fact that flicker noise (in sharp distinction from semiconductors) displayed spatial correlations, several times stronger in extent and frequency dependence than would be the case for temperature fluctuations. Also the noise level corresponded here to the "temperature" hypothesis. However, the same authors[6] noted that the available phenomenological theory of temperature fluctuations does not make it possible to explain the observed spectra of flicker noise. It gives indices $\gamma = 1/2$ or $\gamma = 3/2$ (depending on the system geometry) at sufficiently high frequencies, but it predicts spectral saturation at frequencies $\omega < D_T/L^2$, where L is the largest sample size, and D_T is its temperature conductivity. And yet a spectrum close to $1/\omega$ is often observed also at significantly lower frequencies. Often, but not always; thus, in experiments with freely suspended metallic films in vacuum kept at the superconducting transition temperature, the spectrum became saturated[11] at $\omega < D_T/L^2$. At the same time in the case of normal films on a sublayer the spectrum continues growing (approximately according to the $1/\omega$ law).

The effect of heat exchange with the surroundings (sublayer) on $1/f$ noise in metallic films was especially investigated in Ref. 4 over a wide temperature range (from ≈50 K to 600 K). It seems that at low temperatures ($T < T_D$) the results are generally close to those of Voss and Clarke,[6] but for high T the $1/f$ noise loses both the spatial correlations and the dependence on β, and besides, it is unaffected by thermal contact with the surroundings. In this region the noise level corresponds to Eq. (2.1) with $a \sim 10^{-2}-10^{-1}$. The differential slope (the index γ) depends on frequency and on T, and varies within the limits $0.8 < \gamma < 1.2$. When the temperature is lowered from 400 K for Ag and 500 K for Cu the intensity of this noise first decreases quickly, and then settles at an approximately nonvarying level, substantially dependent on the sublayer thermal conductivity (material) and on β. Indeed, the better the thermal exchange with the sublayer, the lower is the noise level! Spatial correlations appear in the given region.

Dutta, Eberhard, and Horn[4] hence draw the conclusion that there are two types of $1/f$ noise: "type A", related to thermal processes and basically determined by thermal exchange with the surroundings; and "type B", being an internal property of the metal, and independent of the surroundings (and, unlike "type A", not having noticeable spatial correlations). In silver "type B" predominates even at low temperatures. "Type A", however, cannot be reduced to temperature fluctuations, since its spectrum does not saturate for $\omega < D_T/L^2$ [the noise level measured in Ref. 4 is lower by approximately an order of magnitude than that following from Eq. (2.2)]. By the features enumerated above noise in Bi is also necessarily of "type A", since it has spatial correlations.[6]

At present it is difficult to predict how profound such a classification is. It can be understood, however, why there are two types of $1/f$ noise in metals (or, more precisely, two manifestations). Here the conduction electrons play a dual role: they transfer charge, and at the same time are, as is well known, principal heat carriers (the phonon part of the thermal conductivity is by far smaller than the electron part). The electric and thermal processes are intimately interlocked with each other. It is natural to assume that noise of "type A", to be called by us "temperature" noise, is due to Joule heating of the conductor by the current, and therefore is suppressed by sufficiently good heat transfer. This condition is always satisfied in semiconductors and dielectrics, since the number of (free) charge carriers acquiring energy in an elecric field is much smaller than the total number of degrees of freedom by which this energy is dissipated. In treating flicker fluctuations at frequency ω Joule heating must be regarded to be weak if the quantity $\eta = (R\bar{J}^2/CT)\cdot 2\pi/\omega$ is $\ll 1$, and strong if $\eta \sim 1$, when during a period $2\pi/\omega$ each degree of freedom acquires an additional energy of order T (temperature is everywhere expressed in energy units). Since $\eta = (4N/Ca)S_J(\omega)\bar{S}^{-1}$, where $S_J(\omega)$ is given by (2.1), $\bar{S} = 2T/R$ is the spectral density of thermodynamically equilibrium "white" current noise expressed by the Nyquist equation, $C \approx 3N_{at}$, and flicker noise can be observed only for $S_J(\omega) > \bar{S}$, the criterion formulated is equivalent to the condition $N/aN_{at} \ll 1$. In the case of metals this condition is not satisfied, and, consequently, the effects of both Joule heating and heat exchange with the surroundings become substantial. It is important to note that heat transfer to the surroundings occurs due to the motion of conduction electrons, and therefore "temperature" noise ("type A") must not differ in principle from noise of "type B".

A large quantity of experimental data has been accumulated on flicker noise by various inhomogeneous, but macroscopically ordered systems: semiconducting devices (p-n junctions, transistors), metal-semiconductor contacts, MOS structures, etc., including the regime of nonohmic (nonlinear) resistance. Studies in this field in our country are reflected, in particular, in papers of the recent All-Union conference on "Fluctuation effects in physical systems" (Vil'nyus, 1982).[12] Interesting information has been accumulated by now on the behavior of flicker noise in semiconductors in a strong electric field, when the carriers become hot, in a strong magnetic field, in the case of thermoelectric effects, etc.[1] These data confirm the reliability of the general features of $1/f$ noise enumerated above. The primary problem of the theory is to provide a similarly general simple and consistent explanation.

3. SOME FUNDAMENTAL PROBLEMS RELATED TO 1/f NOISE

a) Some differences of opinion may already be provoked by attempts to delineate the object of a theoretical study. Should one seek a single mechanism of flicker noise for various media and systems, or is it more correct to assume the existence of several kinds of flicker noise differing in the physical nature? There exist several opinions on this matter (see, for example, the reviews of Refs. 1, 2). In any case, at the present time we do not know of any single, reliably established and understood mechanism (or type) of flicker noise. Therefore it is natural to expect that some single, universal physical phenomenon is the underlying basis of the various manifestations of flicker noise ($1/f$ noise). This view does not contradict the possibility of existence of a whole range of specific mechanisms, if they are various manifestations of one and the same general feature.

b) The following analogy is useful with the thermodynamically equilibrium "white" current noise (or emf noise) in conducting systems which appears to be well understood. The specific microscopic mechanisms of white noise can be quite different. For example, it can be produced by quasi-free charged particles (conduction electrons, holes); tunnel or energy activated transitions of localized carriers in the case of jump conductivity; charge transfer by ions in a solution, etc., can have a different nature. In all cases, however, the cause of white electric noise is the random, Brownian motion of charged particles, which has two important properties, a fast random change in direction, and the possibility in principle of unlimited particle displacement (transfer of an arbitrarily large charge through the cross section of the conductor). It is understood that this universal nature of white noise does not change even if it becomes "colored" due to the reactive properties of the system. The geometrical size of the system is of no consequence. The Brownian nature of carrier motion in any case remains unchanged. This can be easily verified by considering random motion of carriers in an annular conductor (doughnut), a finite system in which unconfined motion is possible along the circle, and then mentally removing "the hole from the doughnut" and going over to a continuous finite sample of the medium. It is also undertsood that the disappearance of some particles and the creation of others (generation-recombination processes) does not alter the essence of the situation; in any event an arbitrarily large charge can be transferred across a given area inside the conductor. Thus, it can be asserted that the nature of electric (white) noise is determined by the Brownian motion of charge carriers.

c) Experimental facts produce a sufficient basis for the assumption that, despite the variety of quantitative characteristics of flicker noise and the variation of spectral shape (the index γ), we are dealing with an effect which is just as simple, general, and fundamental as white noise; the index $\gamma=1$ is characteristic of this fluctuation effect to the same extent that the index $\gamma=0$ is characteristic of white noise (and "colored" noise derived from it). This point of view does not exclude the existence of several types of flicker noise. It is very necessary to distinguish "nonequilibrium flicker noise", genetically related to thermodynamically nonequilibrium processes, from "equilibrium flicker noise" whose mechanisms are operative even in an ideally equilibrium system, although they are manifested in the spectrum of current or emf fluctuations only in the case of forced deviations from equilibrium against the background of a constant current. Nonequilibrium noise can, in turn, be caused both by an intrinsic internal lack of equilibrium of the system and by external perturbing actions.

Since nonequilibrium fluctuation processes can be (strictly speaking, always are) nonstationary, they can lead to an index $\gamma>1$ substantially differing from 1, whereas equilibrium noise (in the sense indicated above) must be stationary and have an index $\gamma\leq 1$. Unfortunately, this criterion of stationarity or nonstationarity, i.e., integrability or nonintegrability of the spectrum near zero frequency, applies only in a range of arbitrarily low frequencies. Actually the spectrum of flicker noise never has an ideal power shape, and is always known only up to frequencies not lower than the reciprocal observation time. Therefore, the experimentally observed γ in themselves do not, generally speaking, provide an indication of whether the noise is of an equilibrium or nonequilibrium nature. Such a conclusion can be drawn only on the basis of physical information on the system.

In the overwhelming majority of cases the observed indices $0.8\leq\gamma\leq 1.2$ do not deviate too strongly from unity,[2] the quantity $|\gamma-1|$ sometimes exceeds the experimental error, but the experiment itself is performed in a finite and bounded-from-below frequency interval. Unfortunately, even such a property of flicker noise, so important from the point of view of the theory, as the unbounded increase of its spectrum for $\omega\to 0$, cannot be completely verified due to the limited duration of the experiment. Strictly speaking, the same has to be said about the spectrum of ordinary white current noise; the verification of the statement on finiteness of its spectrum at vanishing frequency, i.e., on the presence of a finite (nonvanishing) conductivity at $\omega=0$, is possible only if an infinite time is available.

At the present time, among the investigators of flicker noise the opinion has become quite well established that the observed flicker fluctuations are in most cases of a thermodynamically equilibrium nature (i.e., they are equilibrium $1/f$ noise), although this can change if the system goes over into a nonequilibrium state (in an external field giving rise to the current). This opinion, based on many experiments, was clearly reflected in the reviews of Refs. 1, 2.

On the other hand, there exists and has a physical justification the theory of nonequilibrium, degradational origin of flicker noise (see, for example, Ref. 13). Indeed, no system is an ideally internally thermodynamically equilibrium one. For example, every semiconducting structure is a nonequilibrium one with respect to slow diffusion of alloyed impurities, as well as of externally introduced impurities, i.e., it is subject to diffusion degradation. This process has the nature of a nonstationary random process, which randomly "modulates" the various characteristics of the device including the resistance. Various structural defects (vacancies, dislocations) also diffuse.

If the characteristic diffusion time scale L^2/d (where L is the inhomogeneity size, and d is the diffusion coefficient of impurity atoms, vacancies, etc.) is much longer than the age of the structure, a nonstationary regime is realized. Simple analysis leads[13] to a spectrum of relative resistance fluctuations of the form $S(\omega)\sim\omega^{-3/2}$ for $\omega>d/L^2$. This result corresponds to independently and randomly moving atoms. The value

$\gamma = 3/2$ is, however, outside the values $0.9 \leq \gamma \leq 1.1$ observed in semiconducting structures. "Injection" into the region of interest to us of a group of impurity atoms (or vacancies, etc.) leads to the consequence that the number of atoms $\delta n(t)$ remaining inside the region after a time t decreases according to the law[14] $\delta n(t) \sim t^{-1/2}$. The Fourier transform of this response function is

$$\delta n(\omega) = \int_0^\infty e^{-i\omega t} \delta n(t)\, dt \propto (1-i)\, \omega^{-1/2}.$$

A random sequence of injections leads, obviously, to fluctuations of the number of impurity atoms $\delta n(t)$, whose spectral density is $\sim |\delta n(\omega)|^2 \sim 1/\omega$. This fact was also noted by Liu[15] (essentially this has been known for a long time[2]; on applying to an infinite RC-circuit, described by the diffusion equation, a white-noise current-source, we obtain voltage fluctuations with a $1/\omega$ spectrum[16]). One can imagine injection into the conducting region not of impurities and defects, but of thermal energy (phonons),[14,15] for example, heat flow from the metallic film to the sublayer, and in the opposite direction.

It was suggested by Miller[14] that one can thus explain the "temperature" flicker noise in metals, and as cause of "type B" noise one assumes vacancy creation on the surface and diffusion inside the metal. For the relative spectrum of "temperature" noise the following approximate expression is obtained

$$S(\omega) = \frac{\tau_0 u}{6hN_{\mathrm{at}}\omega}, \tag{3.1}$$

where N_{at} is the number of atoms in the film, h is the film thickness, u is the speed of sound, and τ_0 is the mean free time of thermal phonons in the sublayer material. For $h \approx 1000$ Å, which corresponds to experiments,[4] Eq. (3.1) gives a noise level correct in order of magnitude. A satisfactory estimate is also obtained for "type B" noise, with qualitative reproduction of its temperature dependence. It must be noted, however, that the presence of the film width in (3.1) (it reflects the surface, but not the bulk nature of the noise source in the given model) generally contradicts experiment and the empirical Eqs. (2.1), (2.2). Taking into account the finiteness of the system, the spectrum (3.1) must be saturated at frequencies $\omega < D_{\mathrm{T}}/L^2$ or $\omega < d/h^2$ (as was noted in Ref. 14). The physical cause of "collective" injections of diffusing quantities (number of atoms, vacancies, or thermal energy) remains unclear in the given model.[14,15] Speaking of thermal diffusion, this pattern is in obvious contradiction with the concept of bulk generation of fluctuating thermal fluxes.

It must be noted that models relating flicker noise with conductivity fluctuations due to diffusion of impurity atoms and structural units (defects) are restricted to describing the solid phase only. The rate of thermally activated diffusion processes depends on temperature exponentially, while the level (the quantity a in Eq. 1.1) and the spectrum of the noise at high temperatures do not exhibit a similar dependence, and noise in liquid metals, for example, does not differ strongly from noise in the solid phase.[1] This apparently implies that "diffusion" noise (which definitely is present) and $1/f$ noise are different effects, and "diffusion" noise is only part of the observed noise.

A similar statement can be always made on noise generated by the always present thermodynamic temperature fluctuations (modulating the conductivity). This form of low-frequency noise always exists, but is not identical to flicker noise. What is always observed is the result of "adding" various forms of noise, though it does not at all follow that this "addition" must be of a simple additive nature. This result could be described more correctly by a sum of a power law flicker term in the expression for the spectrum, and a number of Lorentzians (1.1), where τ_n, for example, is a set of temperature relaxation times. For $\beta \gg 1$ the noise due to the temperature fluctuations must dominate; this is the explanation of why, in the experiments of Ref. 11 in which the quantity β was $\gg 1$ spectral saturation was observed for $\omega < D_{\mathrm{T}}/L^2$. But generally noise in metals cannot be reduced to temperature fluctuations or nonequilibrium diffusion processes.

4. THERMODYNAMICALLY EQUILIBRIUM 1/*f* NOISE

a) A large part of the available models relates flicker noise with thermodynamically equilibrium fluctuation processes. If flicker noise has an equilibrium nature, and is not excited by current flow through the conductor it must be manifested somehow also in the equilibrium state in the absence of a constant current. It is easily suspected that the processes responsible for flicker noise must lead to low-frequency intensity fluctuations (the spectral power density) of equilibriun white noise $S(t)$. This is implied for instance by the Nyquist equation $\overline{S} = 2Tg$ (g is the conductivity), if $1/f$ noise is interpreted as a result of conductivity fluctuations, and this equation is applied to the fluctuating quantities: $S(t) = 2Tg(t)$. Voss and Clarke measured power fluctuations of white noise in equilibrium InSb and Nb films (without a current),[6] and indeed observed that they have a spectrum of $1/f$ type, while in InSb the level of relative power fluctuations was at low frequencies just about the same as the level of relative voltage fluctuations in a film with current.

b) A statistical analysis of the Voss and Clarke experiment was carried out by Nelkin and Tremblay.[17] They have measured the power fluctuations of white noise contained within the frequency band $(\omega_0 - \Delta\omega_0, \omega_0 + \Delta\omega_0)$, $\Delta\omega_0 \leq \omega_0$, and have averaged over the finite time $\tau > 1/\Delta\omega_0$ (the noise is passed through a filter with central frequency ω_0 and band $\Delta\omega_0$; the square of the output of the filter is averaged over time τ). The result of averaging, i.e., the flowing spectral density at frequency ω_0, $S(t) = S(t;\omega_0)$ changes randomly with time; in this case $\langle S(t)\rangle \equiv \overline{S} = 2Tg$. The angular brackets denote averaging over the statistical (equilibrium) ensemble.

If the current fluctuations $J(t)$ are considered in an equilibrium closed chain, $S(t)$ is a quadratic functional of $J(t)$. Consequently, the correlation function $\langle \delta S(t)\delta S(0)\rangle$, $\delta S(t) = S(t) - \overline{S}$ is expressed in terms of a fourth order current correlator, consisting of essentially two different contributions:

$$\langle J(t_1)J(t_2)J(t_3)J(t_4)\rangle = \langle J(t_1)J(t_2)\rangle\langle J(t_3)J(t_4)\rangle + \langle J(t_1)J(t_3)\rangle\langle J(t_2)J(t_4)\rangle + \langle J(t_1)J(t_4)\rangle\langle J(t_2)J(t_3)\rangle + \langle J(t_1),\ J(t_2),\ J(t_3),\ J(t_4)\rangle \quad (4.1)$$

[it was taken into account that in equilibrium $\langle J(t)\rangle = 0$]. The first contribution, the first three terms on the right hand side of (4.1), provides no information on flicker noise, since the equilibrium correlation current function $\langle J(t_1)J(t_2)\rangle$ decays rapidly at $|t_1 - t_2|\rangle \tau_\mu$, where τ_μ is a microscopically short correlation time of white noise. All information on the low-frequency fluctuations is contained in the last term of (4.1), the fourth current cumulant (the fully coupled part of the fourth moment), denoted in (4.1) by brackets with commas inside. Since the fourth cumulant characterizes a non-Gaussian fluctuating current, two important conclusions follow. Firstly, for correct statistical description of flicker noise it is in principle necessary to take into account the non-Gaussian nature of current fluctuations (and, naturally, the emf). Secondly, the Voss and Clarke experiment can be considered as experimental proof and measurement of the non-Gaussian nature of electric fluctuations. It is always the current or emf fluctuations that are measured, and not the resistance fluctuations (as is sometimes commonly assumed)!

Straightforward but awkward analysis shows that under the conditions $(\omega_0 + \Delta\omega_0)\tau_\mu \ll 1$, $\omega\tau \ll 1$ the spectrum of low-frequency intensity fluctuations depends on neither ω_0, $\Delta\omega_0$, nor τ. Thus, the low-frequency flicker fluctuations of the flowing power, contained within some frequency band of white noise, appear identical for any choice of band, as long as it does not cover the whole frequency spectrum up to τ_μ^{-1}. The total white noise power (integrated over all frequencies up to frequencies much larger than τ_μ^{-1}), i.e., the square of current or emf fluctuations, does not undergo flicker fluctuations.[8,17] We emphasize that this is important and significant in understanding the following.

This clearly indicates that a kinetic quantity, such as the power contained within a given frequency band, i.e., a quantity characterizing the long-time behavior of the system, undergoes $1/f$ fluctuations, but a dynamic quantity (square of current, fluctuation energy), localized in time does not.

The experimentally measured spectrum of power fluctuations contains, besides the flicker contribution, a constant background ("Gaussian") component, originating from the first three expansion terms of (4), and under the conditions mentioned it is

$$4\int_0^\infty \langle \delta S(t)\,\delta S(0)\rangle \cos\omega t\, dt = S_S(\omega) + \text{const}\cdot\frac{2\pi}{\Delta\omega_0}\bar{S}^2, \quad (4.2)$$

where $S_S(\omega)$ is the spectrum of low-frequency power fluctuations, subject to determination, and const is a quantity of the order of unity (depending on the filter parameters). It is natural to expect that for relative fluctuations the following equality is satisfied, at least approximately,

$$\frac{1}{\bar{S}^2} S_S(\omega) = S(\omega), \quad (4.3)$$

where $S(\omega)$ is equal to (2.1) (as was the case in experiments with InSb). The ratio of $S_S(\omega)$ to the second, background term in (4.2) then equals $\approx a\Delta\omega_0/\omega N$. (It is hence seen why this experiment is very difficult, requiring measurements on small samples and at low frequencies.)

c) Keeping in mind the conditions $\omega\tau < 1$, $(\omega_0 + \Delta\omega_0)\tau_\mu \ll 1$, one can simply relate $S_S(\omega)$ with the fourth current cumulant, considering $J(t)$ as Gaussian white noise (δ is a correlated random process) with randomly varying current statistical parameters (as a result of these changes the noise becomes non-Gaussian). In this phenomenological description[18]

$$\langle Q^2(t)\rangle \equiv \left\langle \left(\int_0^t J(t')\,dt'\right)^2\right\rangle = \bar{S}t, \quad (4.4)$$

$$\langle Q^{(4)}(t)\rangle \equiv \langle Q^4(t)\rangle - 3\langle Q^2(t)\rangle^2 = 3\iint_0^t K_S(t'-t'')\,dt'\,dt'', \quad (4.5)$$

where $Q(t) = \int_0^t J(t')dt'$ is the charge transferred through some cross section of a closed equilibrium chain during time $t \gg \tau_\mu$, $\langle Q^{(4)}(t)\rangle$ is the fourth-order charge cumulant, and $\langle S(t')S(t'')\rangle - \bar{S}^2 = K_S(t' - t'')$ is the correlation function of power fluctuations of "white" current noise. It hence follows that (for $t \gg \tau_\mu$)

$$K_S(t) = 2\iint_0^t \langle J(t),\ J(t'),\ J(t''),\ J(0)\rangle\, dt'\,dt''. \quad (4.6)$$

This result shows that information on equilibrium flicker noise can, in principle, be obtained by rigorous methods of statistical physics by analyzing equilibrium four-time correlators (cumulants). The spectrum $S_S(\omega)$ is related to (4.6) by the Fourier transform

$$S_S(\omega) = 4\int_0^\infty \cos\omega t K_S(t)\,dt.$$

d) Based on relation (4.6), one can very simply clarify the idea of scaleless flicker noise. Let $J(\omega)$ be the Fourier component of the current $J(t)$. We note that the spectrum of "white" noise $\bar{S} = \text{const}$, independent of frequency, does not change on stretching the frequency scale by a factor λ, and the charge scale by a factor $\lambda^{1/2}$. According to the Wiener-Khinchin theorem

$$\langle \sqrt{\lambda} J(\lambda\omega)\sqrt{\lambda} J(\lambda\omega')\rangle = 2\pi\bar{S}\lambda\delta(\lambda\omega + \lambda\omega') = 2\pi\bar{S}\delta(\omega + \omega') = \langle J(\omega)J(\omega')\rangle,$$

if $\bar{S} = \text{const}$. Consequently, the scale-transformed current $\lambda^{1/2}J(\lambda\omega)$ does not differ from $J(\omega)$ in the sense of quadratic statistical characteristics. The equilibrium current is the result of "fast" Brownian motion of charge carriers. We assume that this motion is not subject to effects of slow fluctuating processes, fluctuations of temperature, local medium structure, collective excitations, etc. (these effects always exist, but we neglect them). The random carrier motion has then only one, microscopically short, characteristic time scale τ_μ: the mean free time of flight (the momentum relaxation time) in the case of free carriers, the mean time between successive displacements in the case of jump conductivity. The large-scale structure of Brownian trajectories, if the latter is considered over time intervals $\Delta t \gg \tau_\mu$, cannot be sensitive to the microscale τ_μ. In other words, the Brownian motion must be scale-invariant, and at the same time the current $J(\omega)$ must be scale-invariant at low frequencies $\omega \ll \tau_\mu^{-1}$. This implies that the random processes $J(\omega)$ and $\lambda^{1/2}J(\lambda\omega)$ are statistically identical for $\omega\tau_\mu \ll 1$ in the sense of not only quadratic, but also higher correlations, i.e., in a statistical sense as a whole.

Imposing this requirement on the fourth current cumulant

$$\langle J(\omega_1),\ J(\omega_2),\ J(\omega_3),\ J(\omega_4)\rangle = 2\pi\delta\left(\sum_i \omega_i\right) B(\omega_1,\ \omega_2,\ \omega_3),$$

where B is the trispectrum of the current, it is easily verified that

$$B(\lambda\omega_1,\ \lambda\omega_2,\ \lambda\omega_3) = \frac{1}{\lambda} B(\omega_1,\ \omega_2,\ \omega_3),$$

and then concluded hence and from (9) that $S_S(\lambda\omega) = \lambda^{-1}S_S(\omega)$. Consequently, $S_S(\omega) \sim 1/\omega$ and the equilibrium $1/f$ noise emerges as an almost trivial, although not previously noted property of ordinary Brownian motion, as one of the manifestations of "white" noise. One thus obtains that the non-Gaussian equilibrium "white" noise (and real noise is always, of course, to some extent non-Gaussian) certainly possesses low-frequency intensity fluctuations with a $1/f$ type spectrum. These fluctuations are generated not by any specific slow processes, macroscopically long relaxation times, lifetimes, etc., but by the very same microscopic "fast" processes of interaction of the carrier with the medium which induce Brownian motion.

e) This point of view[18,19] reveals a pattern quite different from most other theoretical constructs, aimed at searching for $1/f$ noise mechanisms which would possess a wide hierarchy of time scales (see Refs. 20–25 and the detailed bibliography in Ref. 1). More interesting seem to be similar attempts from the opposite point of view. In this area interesting material for speculation is included in papers by Arecchi and coworkers,[26,27] where an example is found of "stochastic" behavior of a dynamic system, not having long proper time scales, but capable of "generating" a spectrum of $1/\omega$ type at low frequencies.

We have in mind the forced stochastic oscillations of a nonlinear oscillator:

$$\ddot{x} + k\dot{x} - x + 4x^3 = A\cos\omega_0 t. \tag{4.7}$$

For certain parameters A, ω_0, k two strange attractors are generated here, mutually mirror reflected on the $(x, \dot{x})$ plane. Stochastic oscillations occur in the attractive region of sometimes one, sometimes the other attractor (random intermittence); in this case the current mean value of $x(t)$ (averaged over a number of oscillations) is sometimes positive, sometimes negative, and the spectrum $S_x(\omega)$ contains at low frequencies flicker components with $\gamma \approx 1.1$–1.3.

We stress that the system is not retained long in any region. The presence of flicker fluctuations $x(t)$ implies that the total time $t_1(t)$ which the system spends, for example, in region 1 during the observation time t possesses anomalously large fluctuations. More precisely, $\langle t_1^2(t)\rangle - \langle t_1(t)\rangle^2 \propto t^{1+\gamma}$, since in the absence of intermittence this quantity increases as $\sim t$, i.e., considerably more slowly. The foregoing follows from the fact that $\int_0^t x(t')dt' \sim 2t_1(t) - t$.

The authors of Ref. 27 assume that this system is an example of realization of a universal nonlinear flicker noise mechanism. But we note that there exists a curious analogy between the behavior of system (4.7) at low frequencies and long times and the properties described above of the scaleless Brownian motion of the charge: the quantities $t_1(t)$ and $\int_0^t S(t')dt'$, on the average linearly increasing with time (as $t/2$ and $\overline{S}t$), both have an approximately quadratic increase of fluctuations with time. Indeed, due to the fact that $S_S(\omega) \sim \omega^{-1}$ the correlation function decreases logarithmically, and

$$\left\langle \left(\int_0^t S(t')\,dt' \right)^2 \right\rangle - (\overline{S}t)^2 \propto t^2 f(\ln t)$$

with some function $f(z)$ (in Section 6 we show that $f(z) \sim z^{-1}$). This implies that the possible fluctuation of the square of the charge transfered by diffusion increases with time as fast as the ensemble mean square. Taking the ratio of these two quantities, characterizing the fluctuating "rate" of charge diffusion during the time interval t, the possible variance of this ratio is almost independent of t (it decreases logarithmically with increasing t).

A similar statistical property in system (4.7) is attached to the random part of the time spent by the system in the attractive region 1, i.e., the random quantity $t_1(t)/t$. The following statistical pattern is, apparently, realized. Most probably, the quantity t_1/t attains a value close to the mean $\langle t_1\rangle/t = \frac{1}{2}$. There is, however, some probability that it will turn out to be anomalously large, for example, in the interval from 0.9 to 1. For $\gamma = 1$ the probability of this event does not decrease with increasing observation time t (while for $\gamma > 1$ it even increases). Thus, arbitrarily lengthy random deviations from the mean statistical regime of motion are possible. The system has no characteristic time scale which might be an upper bound on the duration of these deviations. This also implies that the quantity $x(t) \sim dt_1/dt$ has scaleless low-frequency fluctuations, and correspondingly a flicker spectrum (not related to long relaxation times). The correlation of two values of $x(t)$ far separated in time is due to the circumstance that both can belong, with finite probability, to the same prolonged deviation from the mean regime.

We summarize the main points of this section. Firstly, analysis of the Voss-Clarke experiment has shown that abundant information on $1/f$ noise can be obtained by studying the thermodynamically equilibrium system, a conductor without a current (theoretically this problem must be much simpler than the study of the nonequilibrium situation). Secondly, $1/f$ noise can be explained without recourse to special "slow" physical mechanisms (slowly evolving thermodynamic fluctuations), as an inherent statistical attribute of the dynamical nature of Brownian motion of charge carriers. Later on we consider this possibility in detail.

5. NUMBER OR MOBILITY FLUCTUATIONS OF CHARGE CARRIERS?

a) Two basic models are in competition as applied to semiconducting structures. The first, having a long history and a number of modifications, often called the McWhorter model, reduces the cause of flicker noise to fluctuations in the free carrier number due to: 1) their tunneling transition from the bulk of the conductor to long-lived surface states in the oxide layer, and 2) capture of carriers by structure inhomogeneities, having the nature of traps or assisting carrier transition into bulk bound states, including generation-recombination processes. In this model a wide set of time scales is generated by the exponentially strong dependence of bound state lifetimes on temperature and the width of potential barriers, while under quite general assumptions we obtain $a\sim 1/\omega$ law for the mobility fluctuation spectrum (the spectrum is, ultimately saturated at low frequencies). As a rule, however, exact calculations, allowing definitive quantitative comparison with experiment to be made, are not successful. This model is also dealt with in the literature (a review of the present state of the art is contained in Ref. 1; see also Ref. 28). As noted in Ref. 2, it is of a rather specialized nature, and cannot lay claim to a comprehensive treatment of flicker noise. Additional information is needed on the role of structural defects and traps. Such information might be provided by more detailed studies of liquid metals, amorphous metals, molten salts, i.e., systems in which there are no long-lived local structural features.

b) The second model assumes that the causes of flicker noise are mobility fluctuations of free carriers, possibly due to features of their scattering (but unrelated to fluctuations of local temperature and, generally, the local thermodynamic environment). This method was developed mostly by the authors of Ref. 1, which contains almost all the information on the achievements and problems of the model. It is assumed that the mobilities of the different carriers $\mu(t)$ fluctuate, statistically independently of each other, with a spectrum

$$S_\mu(\omega)=\mu^2\frac{2\pi a}{\omega}, \tag{5.1}$$

where μ is the mean value of the mobility, and the quantity a is of the order of $2\cdot 10^{-3}$. For a "distributed" description of the system this assumption is stated in the following form

$$S_\sigma(r,\ r';\ \omega)=\frac{2\pi a\sigma^2(r)}{\omega n(r)}\delta(r-r'), \tag{5.2}$$

where $S_\sigma(r, r';\omega)$ is the mutual spectral density of the conductivity fluctuation $\delta\sigma(r, t)$ at the points r and r', $\sigma(r)$ is the mean value of the specific conductivity at the point r, and $n(r)$ is the local concentration of charge carriers. A differentiated description is possible of the contribution to $\delta\sigma(r, t)$ of carriers with different energies and mobilities.[1]

It is remarkable that, starting from such assumptions, one is able to treat quite well, qualitatively and often even quantitatively, a number of situations: both homogeneous semiconductors and p-n junctions, transistors, point contacts, even in the nonohmic regime. In all cases sharp structural transitions, contacts, and surfaces are not in this theory sources of additional flicker noise, but occur in the capacity of "detectors" and "amplifiers" of bulk noise. Complicated processes occurring at structural junctions are "triggered" by flicker fluctuations of carrier flow from the bulk to the junction due to mobility fluctuations of the carriers. Therefore, the final equations are quite simple and contain a small number of microscopic parameters (remaining free of any macroscales). For example, the current fluctuations through a p-n junction in the white shot noise regime (when $\overline{S}=e\overline{J}$) are given by the expression

$$S_J(\omega)=e\bar{J}\frac{2\pi a}{4\omega\tau_{\rm l.b.}}, \tag{5.3}$$

where e is the elementary charge, and $\tau_{\rm l.b.}$ is the carrier lifetime in the junction base. This equation is in good agreement with experiment for $a\approx 10^{-3}$. The results of similar calculations are finding ever more experimental verifications (as reflected in Ref. 12).

On the other hand, it is well known that $1/f$ noise in semiconductors can depend substantially on the state of the surface particularly in the case of a thin sample, and this effect is not described quantitatively in the simple phenomenological model of mobility fluctuations. Within this model, however, the surface effect occurs quite naturally. Speaking of mobility fluctuations, under the latter one includes the result of random carrier motion during an extended time, long in comparison with the time interval τ_μ (see below), during which the carrier traverses a large region of the medium. During motions in a thin sample the surface must affect the statistical characteristics of "longitudinal" carrier diffusion, and thereby the mobility fluctuations. The diffusion length $\sim(D\tau_l)^{1/2}$ (τ_l is the lifetime) should serve as the characteristic thickness scale, and this is verified experimentally.

From summaries of the reviews of Refs. 1, 2 it may be seen that the problems concerning surface effects, relations between "bulk" and "surface" noises, and the choice between models of bulk noise remain as yet quite unclear (this particularly refers to extended MOS-structures[28]). It was noted in Ref. 2 that the models of carrier number and mobility fluctuations "overlap" to some extent, since, for example, the carrier delay at traps must lead to fluctuations of their effective mobility. But as applied to $1/f$ noise in a strong electric field, in the region of hot carriers the two models give distinguishable results, since they predict non-identical functional dependences of the noise levels on the field.[1] Special experiments with hot carriers clearly indicate in favor of the hypothesis of mobility fluctuations.[1,29,30] The same conclusion follows from measurement results of $1/f$ noise in a strong magnetic field in InSb (interestingly, in a field of the order of 20 kGauss the relative voltage fluctuations increase by a factor of 10^2–10^4).

6. BROWNIAN MOTION, A POSSIBLE UNIVERSAL SOURCE OF EQUILIBRIUM 1/f NOISE

a) The problem naturally arises of the nature of mobility fluctuations. So far the literature does not contain suggestions of any consistent explanation. Vandamme and Hooge have noted that the decrease in noise in strongly alloyed semiconductors can be explained by assuming that only that mobility part due to phonon scattering fluctuates, while the contribution to the mobility from impurity scattering does not undergo fluctuations. However, there is so far no convincing interpretation of fluctuations of the "phonon" part of the mobility.

A nontraditional explanation of mobility fluctuations can be provided on the basis of ideas of scaleless fluctuations, accompanying the Brownian motion of charge carriers.[18,19]

Attention must be turned to the fact that physical quantities such as the carrier diffusion coefficient D and the mobility $\mu(D=T\mu)$ associated with it, the white noise intensity $\bar{S}$ and the conductivity g associated with it ($\bar{S}=2Tg$) are by their physical meaning and mathematical definition nothing other than characteristics of the statistical ensemble of dynamically possible motions under given thermodynamic conditions. These are kinetic characteristics, requiring for their calculation both ensemble and time averaging, and for measurements—carrying out observations during a long time. This is clearly seen, for example, from the rigourous definition of the diffusion coefficient

$$D=\lim_{t\to\infty}\frac{1}{2t}\langle r^2(t)\rangle=\int_0^\infty\langle v(t)\,v(0)\rangle\,\mathrm{d}t,$$

where $v(t)$ is the velocity of a Brownian particle, $r(t)$ is its displacement after time t, generally following from the Green-Kubo equations. Consequently, D, μ, $\bar{S}$, and g cannot be considered as adequate and total characteristics of flowing random carrier motion, the specific Brownian trajectory $r(t)$ or $Q(t)$. For a specific, "live" carrier these characteristics do not exist, since it "does not know" the ensemble properties (and which moreover can quickly forget the past).

Differently stated, a real carrier (and a real Brownian trajectory) simply has no flowing diffusion coefficient or flowing mobility. Unlike dynamic quantitites, momentum, coordinates, or particle energy, these quantities do not actually exist; they only describe, and not exhaustively, the potential possibilities of its random motion. Similarly, a specific carrier or atom has no current temperature, since T is also an ensemble parameter. Moreover, unlike T, which has a dynamic prototype, the particle energy, D does not even possess a dynamic prototype; the "diffusion coefficient operator", the "mobility operator", etc., can not be defined sensibly.

b) Two conclusions can be drawn hence. Firstly, the current diffusion coefficient is in principle uncertain. This uncertainty is expressed by the fact that the "intensity", or "pace" of Brownian motion always undergoes natural fluctuations, which cannot be reduced to perturbations unconnected with Brownian motion. The particle interaction with the medium not only forces it into Brownian motion, but also constantly deflects this motion from some mean regime. Secondly, these natural fluctuations are not taken into account by the traditional "Gaussian" model of Brownian motion, since it identifies the concepts referring to the behavior of an isolated Brownian trajectory with the statistical description of the trajectory ensemble. Consequently, it is quite necessary to take into account non-Gaussian noise features.

It can be easily explained why "pace" fluctuations of Brownian motion (the diffusion coefficient) are characterized precisely by a flicker spectrum. The "pace," as well as D as applied to an ensemble, characterizes the behavior of Brownian trajectories over long time intervals $\Delta t\gg\tau_\mu$, during which the particle succeeds in completing many cycles of momentum and energy exchange with the medium, and totally forgets the past. Thus, the "pace" of diffusion is a distributed, "smeared out" in time characteristic of the motion, and therefore its variation does not affect the current ("instantaneous") dynamic and thermodynamic state of the system; thereby they do not lead to any opposite reactions. Further, since the "pace" is formed (created as a characteristic of the motion) over time intervals $\Delta t\gg\tau_\mu$ much longer than the kinetic time scales of random particle motions, then by necessity it lacks an intrinsic time scale. But this is what implies that pace fluctuations are inevitably scaleless, and possess a flicker spectrum at low frequencies. If, moreover, these pace variations are within the limitations imposed by the diffusion law $\langle r^2(t)\rangle=2Dt$, $\langle Q^2(t)\rangle=\bar{S}t$, the result is a spectrum of type $1/\omega$. Phenomenologically these must of course be perceived as fluctuations in mobility, diffusion coefficient, and white noise power. To describe them it is necessary (and, of course, sufficient) to treat only the dynamic quantities, $r(t)$, $J(t)$, $Q(t)$, etc.

c) Consider diffusion of a charge $Q(t)$ through some cross section of a thermodynamically equilibrium closed chain. The following physical requirements must be imposed on the "Brownian" random process $Q(t)$: 1) Charge transfer obeys the well-known diffusion law, i.e., equality (4.4) is exactly satisfied for all $t\gg\tau_\mu$. 2) For $t/\tau_\mu\to\infty$ $Q(t)$ asymptotically possesses the properties of a random process with independent increments. Physically this simply implies that the correlations of current $\dot{Q}(t)$ decay with time. 3) At large scales the statistical pattern of diffusion is independent of the detailed structure of microscopic interactions. Besides, there are no "slow" fluctuation processes influencing diffusion (this is possibly the most important idealization). In this case diffusion must possess some scale invariance. The type of the invariance is determined by condition 1). More precisely, the statistical pattern of diffusion remains unchanged under a simultaneous extension of the charge scale by a factor λ and of the time scale by a factor of λ^2 ($Q^2\sim t$). 4) The scale invariance breaks down at the characteristic microscales q_0, τ_0. It is clear that in every case $\tau_0\gtrsim\tau_\mu$. 5) The fluctuations of the charge $Q(t)$ are non-Gaussian. In fact, only the given condition, taking into account the actually existing non-Gaussian features, distinguishes the theory under discussion from the canonical diffusion model.

As shown by an uncomplicated mathematical analysis,[18,19] these requirements completely determine the structure and the most important properties of the characteristic charge function for $t \gg \tau_0$:

$$t^{-1}\ln\langle e^{iuQ(t)}\rangle = \bar{S}\int_{-\infty}^{\infty}\frac{(\cos uq-1)}{q^2}\frac{A(t)}{q_0+|q|}F\left(\frac{q^2}{2S't}\right)dq, \quad (6.1)$$

$$A(t) \equiv \left[2\int_0^{\infty}F\left(\frac{q^2}{2S't}\right)\frac{dq}{q_0+|q|}\right]^{-1} \approx \left(\ln\frac{t}{\tau_0}\right)^{-1}, \quad (6.2)$$

$$S' \equiv \frac{q_0^2}{\tau_0};$$

where iu is an arbitrary test parameter, and the function $F(z)$ satisfies the conditions

$$F(0)=1, \quad \int_0^{\infty}F(z)\,dz=1.$$

Knowing the characteristic function, we know all moments and cumulants of $Q(t)$; in particular, $\langle Q^{(4)}(t)\rangle = 2\bar{S}S't^2A(t)$. From Eqs. (4.4), (6.1), and (6.2) follows an expression for the correlation function and for the intensity fluctuation spectrum of white noise for $t \gg \tau_0$, $\omega\tau_0 \ll 1$:

$$K_S(t) = \frac{2}{3}\bar{S}S'\left(\ln\frac{t}{\tau_0}\right)^{-1}, \quad S_S(\omega) = \frac{4\pi S'\bar{S}}{3\omega}(\ln\omega\tau_0)^{-2}. \quad (6.3)$$

If the chain represents a sample (doughnut) of a statistically homogeneous equilibrium conducting medium of length L, then

$$\bar{S} = 2\left(\frac{l}{L}\right)^2 DN, \quad q_0 = \frac{e}{L}r_0.$$

The quantity r_0 determines the spatial boundary of the scale invariance region of Brownian motion, while τ_0 is its temporal boundary. In the given case (a model of statistically independent carriers) the relative intensity fluctuations are

$$\bar{S}^{-2}S_S(\omega) = \frac{2\pi a(\omega)}{\omega N}, \quad a(\omega) \equiv \frac{r_0^2}{3D\tau_0}(\ln\omega\tau_0)^{-2}. \quad (6.4)$$

d) Although the quantities r_0, τ_0 are characterized quite indistinctly, Eq. (6.4) leads to sensible specific estimates of the noise level (of course, in the given model one cannot expect to have estimates better than in order of magnitude). It is seen from (4.3) and (2.1) that $a(\omega)$ in (6.4) is the theoretical expression for the "Hooge constant" a.

1) Assume that the carrier loses the correlation between directions of motion already after a single "free flight" and exchanges momentum and energy with the heat both (inelastic scattering). Then $\tau_0 \approx \tau_\mu$ (τ_μ is the mean time of free flight), and r_0 is approximately the mean free path λ. In this simple case $r_0^2 \approx 2D\tau_0$. Choosing a typical value $\tau_0 \sim 10^{-12}$ sec for semiconductors, we find at the frequency $\omega/2\pi \sim 1$ Hz: $a(\omega) \approx 0.001$, in agreement with the value $a \approx 2\cdot 10^{-3}$ characteristic of many pure semiconductors.

2) Consider a metal at a high temperature $T \gtrsim T_D$. Assuming that scattering is inelastic, we can again put $r_0^2 \approx \lambda^2 \approx 1/3v_F^2\tau_\mu^2$, $\tau_0 \approx \tau_\mu$ (τ_μ is the momentum relaxation time). Since $D = (T/m^*)\tau_\mu$, we find from (6.4)

$$a(\omega) \approx \frac{2}{9}\frac{T_F}{T}(\ln\omega\tau_\mu)^{-2} \sim 3\cdot 10^{-2},$$

where T_F is the Fermi temperature. This estimate is also in satisfactory agreement with experiment. Compared with the preceding case of nondegenerate carriers, there appears here the additional factor $\sim T_F/T$. For $T \gg T_D$ one must take into account elastic electron scattering, due to which $\tau_0 \gg \tau_\mu$.

3) We select a Brownian particle in the liquid. For τ_0 it is necessary to take the relaxation time of the particle velocity τ_μ (there are no other time scales here). If the particle and liquid densities are identical, then $\tau_0 \approx \tau_\mu \approx 2R^2/9\nu$, where R is the radius, and ν is the kinematic viscosity. Introducing the mean free path in the given case makes no sense, since estimating λ from the ordinary kinetic relations gives a value much smaller than the intermolecular distance $b \approx 3\cdot 10^{-8}$ cm (even if R approaches b). Consequently, one must have $r_0 \approx b$. Since $D = T/(6\pi\rho\nu R)$, for values characteristic of water and $T = 300$ K we have

$$a(\omega) \approx \frac{9\pi b^2\nu^2\rho}{TR}(\ln\omega\tau_\mu)^{-2} \approx \frac{10^{-7}}{R},$$

where R is expressed in cm. For an ion with radius $R \sim b \approx 3\cdot 10^{-8}$ cm we hence find $a(\omega) \sim 3$, in good agreement with the well-known (see above) anomalously large value of the "Hooge constant" for electrolytes $a \sim 10$.

4) Finally, as one more example we take $1/f$ noise of a sharp structural transition—a p-n junction, a separation boundary of two media, a Josephson junction, etc. (in this category of situations one can also include electron emission noise). For definiteness we consider a Josephson junction. We assume that the transfer is realized by uncorrelated "elementary" acts. In each of which a charge e is transferred. Then we have in (6.1)–(6.3) $q_0 = e$. We note that $\bar{S} = ne^2$, where n is the mean number of charges (electrons) crossing the junction per unit time. For the spectrum of relative intensity fluctuations we have from (6.3)[18]

$$S(\omega) = \frac{2\pi a_0(\omega)}{\omega n\tau_0}, \quad a_0(\omega) = \frac{2}{3}(\ln\omega\tau_0)^{-2}. \quad (6.5)$$

The time τ_0 has the meaning of "electron lifetime at the junction" (duration of elementary transport process), and is determined in the given case by the tunneling probability through the dielectric layer. Further, n can be represented in the form $n = A/\tau_0 b^2$, where A is the area of a uniform junction, and b^2 is an area quantity, transmitting $1/\tau_0$ electrons per unit time, so that

$$S(\omega) = \frac{2\pi b^2}{\omega A}a_0(\omega).$$

Expression (6.5) must also be valid for relative fluctuations of a normal current through a junction in a nonequilibrium state, if $ex < T$ (x is the voltage at the junction).

The smallest possible value of b^2 equals $\approx n_{el}^{-2/3}$, where n_{el} is the electron concentration in the metal, i.e., the part of the junction area "taken by one electron"; consequently, $b^2 \gtrsim 6\cdot 10^{-16}$ cm^2. To estimate the logarithm we take $\tau_0 \sim 10^{-10}$ with a frequency ~ 1–10^4 Hz; then

$$S(\omega) \geqslant \frac{2\pi 10^{-18}}{\omega A},$$

where A is expressed in cm². Hence the ratio of contributions of $1/f$ noise and of white noise to the total contact noise equals

$$\xi=\frac{\bar{J}^2}{\bar{S}}S(\omega)\geqslant\frac{\bar{J}Re}{T}\frac{j}{2ef}10^{-18}\ \text{cm}^2,$$

where j is the normal current density, and $f=\omega/2\pi$. SQUIDs usually work in the regime where $\bar{J}ReT^{-1}\leq 1$, and j is of the order of the critical density of the excess current $j_c=J_cA^{-1}$. For $\bar{J}Re/T\approx 1/2$ and a characteristic value $j=10^3$ A/cm^2, $\xi=1000/f$ (Hz), which agrees with experiment for SQUIDs with these parameters.[34] The "intrinsic energy sensitivity of a SQUID"[34] in the low-frequency region is in units of $h\approx(J_c/8e)S(\omega)\approx 0.6j_cf^{-1}$ (where j_c is in A $\cdot$ cm^{-2}).

In conclusion it is necessary to note the following. The logarithmically slow damping of the correlation function (6.3) does not imply that the carriers retain for long memory of the past. The logarithmic law itself was obtained from the premise of absence of extended memory. This paradoxic situation is a reflection of the already underlined fact that the current values of kinetic parameters contain an uncertainty in principle. Therefore the extremely slow damping of the correlation function $K_S(t)$ cannot be interpreted literally, as a reflection of the kinetics of fluctuations, memory or aftereffect of the system or of long-lived thermodynamic correlations.

The quantity $K_S(t)$ represents a minimum uncertainty (mean square error), which cannot be exceeded, performing the measurements of a kinetic parameter (in the given case—white noise intensity) during a finite time t. The time behavior of $K_S(t)$ implies only that an individual Brownian trajectory of the system (realized in a specific effect or experiment) constantly finds with the passage of time new possibilities of departure from the "mean statistical" regime of motion, and this process never terminates and has no upper time scale, as is also the case for the dynamic motion underlying the Brownian motion. It can be said that $1/f$ noise has a dynamic nature, though it exists, as is the case for thermal white noise, only in association with the fact that the system is characterized by irreversible behavior (in deviating from equilibrium): it appears as a sort of a "tribute" that must be paid to the dynamics for the dissipative properties of the system.

We note that $1/f$ noise cannot be described by the Langevin equations (of Markovian or non-Markovian type). Any such equation contains dissipative parameters, the coefficients of friction, transport, etc., which are statistical ensemble parameters (spectral densities of "random" forces), but are treated as characteristics of the instantaneous dynamic state of the system. Thereby the Langevin equation is incapable of reproducing the "actual" individual trajectory, and in using it the $1/f$ noise is inevitably lost.

From the point of view presented in this section it has been possible to provide a quite general physical explanation of $1/f$ noise. The corresponding statistical model leads also to correct quantitative estimates, but, undoubtedly requires improvement on the basis of a microscopic analysis of the statistics of transport processes.

7. 1/f NOISE IN THE NONEQUILIBRIUM CURRENT STATE

The $1/f$ noise discussed above is in equilibrium in the most literal sense of this word: it exists in a thermodynamically equilibrium system, and can be observed without violating the equilibrium. In practice the noise is usually manifested and measured in the nonequilibrium, current, state, which is generated by including a source of given emf in the conducting circuit. In this state not only the current power of wide-band (white) component of noise, but also the current mean current value undergoes flicker fluctuations. Current fluctuations (current $1/f$ noise) has the same origin as equilibrium power fluctuations, i.e., current noise is not generated by the flowing mean dissipative current, but is "probed" by it (in this sense current noise may also be called "equilibrium"). It is natural to expect that in a weakly nonequilibrium state (sufficiently weak field) the current flicker noise directly reproduces the equilibrium flicker noise i.e., that Eq. (4.3) is satisfied, and the relative fluctuations of the current and of the intensity of equilibrium white noise are identical. This equality can be proved rigorously,[19] without recourse to any phenomenological model, by the so-called nonlinear fluctuations-dissipation relations (see, for example, Ref. 31). For the proof it is necessary to assume stationarity of the nonequilibrium current state.

We assume that at $t=0$ the "conductor plus surroundings" system is in equilibrium and has temperature T. At $t=0$ a source of constant emf $x=$ const is connected. The following exact and general relation, connecting the mean (over the ensemble) current value after inclusion of the perturbation $\langle J(t)\rangle_x$ with the cumulants of current fluctuations of second, fourth, and higher even orders:[19]

$$\langle J(t)\rangle_x=\frac{x}{T}\int_0^t\langle J(t),\ J(t')\rangle_x\,dt'-\frac{1}{6}\left(\frac{x}{T}\right)^3\int\int\int_0^t\langle J(t),\ J(t_1),\ J(t_2),\ J(t_3)\rangle_x\,dt_1\,dt_2\,dt_3+\ldots; \tag{7.1}$$

where the subscript x on the brackets reminds us of the nonequilibrium nature of the fluctuations; the multiple dots denote the contribution of the sixth and higher order cumulants, proportional to x^5 (for $x\to 0$), and $Q(t)=\int_0^tJ(t')dt'$ (in abbreviated notation (7.1) has the form

$$\langle Q(t)\rangle_x=\left\langle Q(t)\ \text{th}\left(\frac{x}{2T}Q(t)\right)\right\rangle_x,$$

where on the right hand side we have the "cumulant part" of the mean). We expand the mean current in a series in x: $\langle J(t)\rangle_x=g_1(t)x+g_3(t)x^3+\ldots$ Let the stationary (nonequilibrium) state be established at $t\gg\tau_\mu$, i.e., specifically: 1) the cubic response function behaves as $g_3(t)\to$ const; 2) the current correlation function depends only on the difference $t-t'$. Then

$$\langle J(t),\ J(t')\rangle_x=\langle J(t),\ J(t')\rangle_0+x^2K_g(t-t')+\ldots,$$

where $K_g(t)$ can be interpreted as the correlation function of the conductivity fluctuations $g(t)$. It is seen by considering the cubic terms of the expansion of (7.1) in x that for $t \gg \tau_\mu$

$$K_g(t) = \frac{1}{2T^2} \int\int_0^t \langle J(t),\ J(t_1),\ J(t_2),\ J(0)\rangle_0 \, dt_1 \, dt_2 \qquad (7.2)$$

(the zero implies equilibrium fluctuations); comparison of this equation with (4.6) shows that (for $\omega\tau_0 \ll 1$) $\times S_g(\omega) = (2T)^{-2} S_S(\omega)$, and in the weakly nonequilibrium state we have

$$S_J(\omega) = x^2 S_g(\omega) = J^2 \overline{S}^{-2} S_S(\omega) \quad \left(\overline{J} = gx = \frac{\overline{S}}{2T} x\right).$$

Thus, equality (4.3) is a necessary consequence of the "cubic" fluctuation-dissipation relations in the case of a stationary (weakly nonequilibrium) state. These relations (earlier studied by Efremov and also by Stratonovich) also provide a rigorous statistical expression (7.2) for such phenomenological characteristics as the correlation function of the conductivity fluctuations. Essentially, the fluctuation-dissipation relations prove the identity, in principle, of flicker current fluctuations with equilibrium intensity fluctuations.

On the other hand, it is understood that the characteristics of flicker noise must, generally speaking, be modified in an essentially nonequilibrium current state (in the region of nonohmic resistance). The nonstationarity of the current state (and the nonequilibrium) state is of course always, to some extent, nonstationary) must also be reflected in the properties of flicker noise. Therefore the changes involve not only the noise level, but also the spectral shape. One must not exclude even such a possibility as the dependence of the spectral index (at arbitrarily low frequencies) on the degree of conductor nonequilibrium, i.e., on x and on the conditions of noise measurements.

As a measure of nonequilibrium one can use the ratio of the magnitude of the work done by the field, dissipated in a typical "elementary" process of charge transfer, to the temperature of the heat bath or the intensity of thermal fluxes in the conductor, due to Joule heating and heat exchange with the surroundings. In real experiments with metals the degree of nonequilibrium is practically always low in the sense of the first criterion, but is often significant in the sense of the second criterion due to the high concentration of charge carriers and intense heating. In the case of semiconductors, on the other hand, the nonequilibrium is comparatively weak in the sense of the second criterion, but strong, in the region of hot carriers and nonohmic resistance, from the point of view of the first criterion.

Physically the nonstationarity of $1/f$ noise ($\gamma > 1$) in the state with a current is, in all probability, just the "thermal" nonequilibrium related to Joule heating[1)] (i.e., the second criterion is dominant). This is indicated by the fact that the reliably observed deviations of the index γ from unity are primarily characteristic of metals.[2)] It is at present difficult to say anything on the statistical mechanism of "thermal" nonstationarity of nonequilibrium $1/f$ noise. However, it is possible to assert that in the approximation quadratic in x, corresponding to "infinitely weak nonequilibrium", the index γ must remain equal to unity (not taking into account logarithmic corrections). This is seen from the relations given above with the fact taken into account that even in the absence of heat exchange between the conductor and the surroundings the cubic response $g_3(t)$ cannot increase with time faster than $\sim t$. Consequently, the effects of nonequilibrium, among them the deviation of γ from 1, are related to the sum of higher order "corrections" to the noise spectrum, proportional to $x^4, x^6, x^8, \ldots$

The simple formal-phenomenological method just described and the analysis of the current $1/f$ noise in the nonohmic regime (and in the stationary state) is based on the following assumptions: flicker fluctuations of the flowing mean current and the intensity of nonequilibrium white noise are due to fluctuations of some kinetic system parameter or number of parameters; the intensity of relative fluctuations of the parameter either remains the same as in equilibrium, or depends in a known way on the mean current (or mean power). As an example consider a semiconducting diode (p-n junction) in the shot noise regime for $xe/T \gg 1$, when $\overline{S} = e\overline{J}$, $\overline{J} = en$, where n is the mean number of carriers passing in the "direct" sense per unit time. Assuming the source of flicker fluctuations are n fluctuations, we have

$$S_J(\omega) = \overline{J}^2 \overline{S}^{-2} S_S(\omega).$$

Using then for $S_S(\omega)$ expression (6.3), with account taken of the dependence of $\overline{S}$ on x, leads[18] to an expression of the form (5.3), showing that the intensity of $1/f$ noise of a diode in the nonohmic regime is proportional to the carrier flow through the diode, as in the case for white noise intensity.

The dependence of the hot carrier current on the electric field E ($e = x/L$) is in many cases $\overline{J} = \varphi(\mu E e/v)$, where μ is the carrier mobility in a weak field, and v is a "nonkinetic" parameter (the phonon propagation velocity). Kleinpenning has shown that the $1/f$ noise of hot carriers can be correctly described by assuming that it is determined by flicker fluctuations of μ (independent of the field and uncorrelated for different carriers).[1,30] This assumption leads to the dependence

$$S_J(\omega) = \overline{J}^2 \frac{2\pi a}{\omega N} \left(\frac{\mu}{\overline{J}} \frac{\partial \overline{J}}{\partial \mu}\right)^2 = \overline{J}^2 \cdot \frac{2\pi a}{\omega N} \left(\frac{x}{\overline{J}} \frac{\partial \overline{J}}{\partial x}\right)^2,$$

[1)] This assumption makes it possible to obtain the following formal estimate of γ. The quality of heat exchange of a noisy conductor with the surroundings can be characterized by the reciprocal of the mean thermal relaxation time λ, and the heating intensity—by the time $t_0 = (\overline{J}x/CT)^{-1} \gg \lambda^{-1}$. The nonstationarity of noise must be expressed (in the region $\omega < \lambda$) by an additional dimensionless factor $\sim (\lambda/\omega)^\alpha$ in the noise spectrum; then $\gamma = 1 + \alpha$. At the same time this factor must be unimportant down to lower temperatures $\sim t_0^{-1}$. It hence follows that $\ln(\lambda t_0)^\alpha \sim 1$, $\alpha = \text{const} \cdot \ln^{-1}(CT\lambda/\overline{J}x)$, where const is a number of the order of unity. For experiments with metals typical values are $\lambda t_0 \approx 10-10^4$, corresponding to values $\gamma \approx 1.1-1.4$.

[2)] Thus, in a recent paper by D. M. Fleetwood, J. T. Masden, and N. Giordano, Phys. Rev. Lett. 50, 450 (1983), noise measurements of platinum filaments and films over a wide frequency range from 10^{-3} to 10^2 Hz yield $\gamma \approx 1.15$.

in good agreement with experiment, while voltage flicker fluctuations remain independent of field.

We note, finally, the following interesting result of Ref. 32. The theoretical treatment of a degenerate electron gas in a medium of immobile (infinitely heavy) scatterers in an external field (without taking into account the inter-electron interaction) showed that the nonequilibrium correction, quadratic in the voltage x, to the current correlation function does not contain flicker components. Thus, there is no $1/f$ noise during scattering by strictly immobile centers. This result is in agreement with the noted decrease in noise level in a strongly alloyed semiconductor.[1]

We note that in the system of Ref. 32 (the limiting case of a Lorentz gas) the electrons cannot transfer energy to the medium, since the scattering processes are ideally elastic. The energy relaxation time τ_e is infinitely long in this case (though the momentum relaxation time τ_m if finite). For a Brownian electron trajectory to acquire scale-invariant behavior it must develop during a time $\gg \tau_0$, and the latter in turn must exceed the maximum of the kinetic scales of the electron interactions with the medium τ_e, τ_m. In the limiting transition to immobile scatterers the scale τ_0 tends to infinity, and $1/f$ noise vanishes. Since the frequency band of $1/f$ noise is $<\tau^{-1}$, it is compressed without bound. Differently stated, the spectrum of $1/f$ noise degenerates in the limit to a δ-function at vanishing frequency, corresponding to "frozen" fluctuations. This transformation is accompanied, as is clear from (6.4), by a decrease in level of the $1/f$ noise component (the scale r_0 must remain finite, as is the case for the mean free path λ, although in every case $r_0 \gtrsim \lambda$); the white noise component, whose spectral width is $\sim\tau_m^{-1}$, remains without change.

There exists, thus, a number of useful theoretical results concerning $1/f$ noise in weakly nonequilibrium states, and simple, but formal prescriptions of calculating noise in several nonlinear systems. A consistent and general constructive approach to analyzing $1/f$ noise in substantially nonequilibrium systems has as yet not been suggested. Among the most important tasks of such an approach would be an explanation for and computation of indices γ larger than unity, corresponding to a nonstationary "in the fluctuation sense" (but not necessarily in the sense of mean current values over the ensemble) and nonergodic nonequilibrium current state. Naturally, the premises on which the theory of equilibrium $1/f$ noise discussed in the preceding section is based is insufficient for analyzing the nonstationary situation. For this it is necessary at least to introduce explicitly a treatment of energy dissipation and heat transfer processes (and their connections with charge transfer), and the flicker fluctuations accompanying these processes. This set of problems has not as yet been discussed in the literature.

8. CONCLUSION

As we have seen, the situation today in studies of $1/f$ noise does not make it possible to draw any conclusions in clearly stated terms. Nevertheless one can, in our opinion, draw the conclusion that experimental studies and theoretical work of recent years definitely lead to a pattern of "scaleless" $1/f$ noise, produced not by slowly flowing fluctuation processes, but by the "fast" Brownian motion itself of the charge carriers. The spectrum of scaleless $1/f$ noise is not saturated at low frequencies (although it is integrable in the case of equilibrium noise), but its intensity is determined only by the microsopic space-time scales of the carrier interaction with the medium. The formal model of mobility fluctuations which has demonstrated significant success is, essentially, a method of describing scaleless $1/f$ noise. It now appears possible to justify physically the development of this model. The puzzling mobility fluctuations with a spectrum of $1/f$ type turn out to be visible manifestation of the fundamental uncertainty of current mobility (as well as of the diffusion coefficient and of the general current values of kinetic quantities), an uncertainty due to the simple circumstance that the current kinetic quantities are not, in contrast to dynamic quantities, "presently" available in reality, but are in fact created in the observation process and during the running averaging (integration) over a long time. Thus, the scaleless $1/f$ noise occurs as a fundamental and general effect; it is generated, along with white noise, by any microscopic mechanism of charge transfer (though its statistical characteristics depend on the specific features of the mechanism). From this point of view it can be stated that Brownian motion itself is a universal source of $1/f$ noise.

What has been said does not imply, of course, that this source is unique. Whenever fluctuation processes occur in a system, which are characterized by a wide spread of relaxation times (or "lifetimes, memory times", etc.) they make a contribution, and, possibly, a decisive one in some cases, to the observed low-frequency noise and flicker noise. In the case of semiconducting structures, as already noted, it is not possible as yet to make a choice between "multiscale" (carrier number fluctuations) and "scaleless" (mobility fluctuations) noise. Not to be excluded is the possibility that both mechanisms are significant, and, despite their difference in principle, interact nontrivially with each other, without obeying an arithmetic law of addition of fluctuations.

A constructive conclusion that may be drawn is the necessity, along with the traditional search for "slow" mechanisms of $1/f$ noise, of a more detailed analysis of the microscopic statistical characteristics of charge transfer, so as, using rigorous methods of statistical mechanics and the achievements of physical kinetics, to perfect the pattern of $1/f$ noise (in particular, the mobility fluctuation model). There are reasons to hope that studies in this area also stimulate new directions of experimental studies both of $1/f$ noise itself and of the microscopic characteristics of the medium related to its parameters, and will enable one to approach the final solution of the $1/f$ noise problem.

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