The nature of spontaneous radiation

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The nature of spontaneous radiation is discussed in its technical and historical aspects. The point of view encountered in the literature according to which spontaneous radiation is induced radiation produced by the zero-point (vacuum) oscillations of the electromagnetic field is criticized. Spontaneous radiation is not a purely quantum effect since it occurs also in the classical region (and in this case it is described by classical electrodynamics).

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1. Spontaneous radiation, together with absorption and induced radiation, is among the fundamental and most elementary processes and, especially in optics, it is necessary to deal with it literally at each step. How to find the probability or intensity of spontaneous radiation, say, in the case of an atom in transition of an electron from some upper state to a lower state, is well known: it is necessary only to calculate some matrix element which depends on the wave functions of the upper and lower states.

But what is the nature of spontaneous radiation? What is it due to, and is it a quantum or a classical effect? If the reader knows the correct answer, which is rather trivial, let him nevertheless not hurry to accuse the author of the present note of trying to force a door which is already wide open. In order to be convinced of the existence of very widespread misunderstandings as to the nature of spontaneous radiation, it will probably be sufficient to ask one's neighbor. Before deciding to write this note I carried out such a survey, and with the same result which was recently reported in the literature.[1] Namely, a significant fraction of those asked believe that spontaneous radiation is the result of the existence of zero-point oscillations of the electromagnetic field, i.e., it is a quantum effect.

Nevertheless such an opinion is incorrect since the spontaneous radiation of light clearly exists in the classical theory and, generally speaking, cannot be considered a quantum phenomenon to a higher degree than absorption or induced radiation. The misunderstandings which one necessarily encounters in regard to the nature of spontaneous radiation are due to the course of the historical development of the quantum theory as a whole and of the quantum theory of radiation in particular. One can also mention somewhat more specifically the role of the fact that different methods are frequently used in the classical and quantum theory of radiation, and the problem is formulated differently. Therefore the classical nature of various results obtained by a quantum method (say, the method of perturbation theory) may turn out to be veiled.

This is why the following remarks may perhaps present a certain interest in historical and technical respects.

2. Quantum concepts, especially the concept of photons, made a place for themselves with exceptional difficulty.[2,3] It is sufficient to recall that the hypothesis introduced by Einstein in 1905[4] regarding quanta of light, which was developed by him[5] in 1916,[1)] was disputed, or perhaps to be more correct was not considered acceptable, in particular by Bohr, right up to 1925 (see Ref. 3).

Only the creation of quantum mechanics in 1925 and its generalization to the theory of radiation by Dirac in 1927[6] permitted a consistent and in principle clear discussion of radiation processes in atomic systems. However, for a considerable number of years the quantum theory of radiation remained little known and insufficiently understood. There are many explanations for this. Among these are the novelty and primarily the unusual nature of quantum mechanics itself and the fundamental changes in physics which it involved. A role was also played by the general situation characteristic of science 50 to 60 years ago: there were many fewer physicists (and especially theoretical physicists) than at present; exchange of information occurred as a whole much more slowly (there were comparatively few exceptions, although the most active and most outstanding physicists exchanged letters within Europe).

In addition, with respect to the quantum theory of radiation and the more general quantum electrodynamics, mastery of even simple aspects of the theory was hindered, apparently, by the appearance of basic difficulties associated with the appearance of infinite (divergent) expressions. The relativistic theory of the electron (the Dirac theory) which was developed in parallel with quantum electrodynamics also encountered difficutlies (such as the question of negative energies, the theory of holes, and so forth). All this has been discussed clearly and rather in detail in the recently published article by Weisskopf.[7] It is well known that it was possible to overcome these difficulties only at the end of the 1940s by creation of methods

[1)]In 1905 Einstein introduced the relation $E=\hbar\omega\equiv h\nu$ connecting the energy of light quanta E with the radiation frequency $\omega=2\pi\nu$. However, in the 1916 work light quanta already become full-fledged photons—they are endowed also with momentum $p=\hbar\omega/c$. Therefore one is justified, as in the review by Pais,[3] in not identifying the introduction of the concepts of light quanta and photons.

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of calculation which permits solution of quantum-electrodynamics problems in the higher approximations of perturbation theory.[8] The success achieved in quantum electrodynamics is colossal and is impressive on its own account,[7-9] without mentioning its significance for the contemporary development of all of quantum field theory (the theory of the electroweak interaction, quantum chromodynamics, and so forth).

Returning to the early period of the development of the quantum theory of radiation, we can illustrate the lag which we have mentioned in understanding it in the example of the influence of Fermi's article[10] published in 1932. Although this occurred fine years after the appearance of Dirac's work,[6] only the Fermi article permitted many to understand the quantum theory of radiation. Thus, as is stated by the editor's preface to the Russian translation of the Fermi article,[10] it was called the "rose-colored bible" (from the color of the cover of the Reviews of Modern Physics). Later the well known physicist Bethe in 1955 remarked (see Ref. 7): "Probably many of you, like me, first became acquanited with field theory by reading Fermi's remarkable article." This review article (lectures) was actually written exceptionally clearly and simply, on the basis of the so-called Hamiltonian method, which permits direct transition from a classical treatment to a quantum treatment. This approach was continued in Heitler's book[11] published in 1936 and in Russian translation in 1940. The appearance of this book played in our country (and probably throughout the world) a major positive role.[2)]

In contemporary courses in quantum electrodynamics[8] the Hamiltonian method has given place to more refined procedures and presentations. Nevertheless, we are convinced that the Hamiltonian method retains a certain value even today both in the procedural aspect and from the point of view of applications to the electrodynamics of condensed media.[12]

3. We shall further characterize the situation in the quantum theory of radiation in the 1930s to some extent below, but we shall now turn directly to the question of spontaneous radiation. Before the creation of quantum mechanics, in the framework of the old quantum theory[2] the major stumbling block was the description (not to mention the understanding) of the processes of emission and absorption of light by a quantized system (an atom). In the absence of a systematic microscopic theory, the only approach was in some sense a phenomenological discussion involving use of emission and absorption probabilities. This was done by Einstein,[5] who explicitly mentioned an analogy with the description of radioactive decay.

For convenience we shall briefly recall some appropriate expressions related to transitions in a system (an atom) between the states n and m with energies E_n and $E_m > E_n$; for simplicity, as in the work of Einstein,[5] the radiation is assumed isotropic and unpolarized, and to be propagating in vacuum (generalization to the case of presence of a medium and with inclusion of various polarizations of the normal waves can be found, for example, in Chapter 10 of Ref. 12).

The probability of spontaneous radiation—transition of an atom from state m to state n with emission of one photon with energy $\hbar\omega = E_m - E_n$—in a time dt is

$$dW_{sp} = A_m^n \, dt, \tag{1}$$

so that the number of atoms in state m as the result only of spontaneous radiation would change according to the law of radioactive decay $N_m(t) = N_m(0)\exp(-A_m^n t)$. For induced radiation[3)] and absorption (the transition $n \to m$) in a field of radiation with density $\rho(\omega) = I(\omega)/c$ ($I(\omega) = 2\cdot 4\pi I_{k\sigma}(\omega)$ is the total spectral intensity of radiation, where $I_{k\sigma}(\omega)$ is the spectral intensity of radiation with one polarization, per unit solid angle) we have

$$dW_{ind} = B_m^n \rho \, dt, \quad dW_{abs} = B_n^m \rho \, dt. \tag{2}$$

In a state of dynamic equilibrium at temperature T in a gas of the atoms under discussion, the equality

$$e^{-E_n/kT} B_n^m \rho = e^{-E_m/kT} (B_m^n \rho + A_m^n), \tag{3}$$

must hold since the concentrations of atoms in states n and m are determined by the factors $e^{-E_n/kT}$ and $e^{-E_m/kT}$ (for simplicity we shall assume that the statistical weights of the states n and m are equal to unity). Taking into account that with increase of T the density of equilibrium thermal radiation $\rho(\omega, T)$ rises without limit, from Eq. (3) we arrive at the first of the Einstein relations:

$$B_n^m = B_m^n. \tag{4}$$

Then from Eq. (3) for the equilibrium radiation we obtain the formula

$$\rho(\omega, T) = \frac{A_m^n / B_m^n}{e^{E_m - E_n/kT} - 1} = \frac{A_m^n / B_m^n}{e^{\hbar\omega/kT} - 1}, \tag{5}$$

where we have used the relation $E_m - E_n = \hbar\omega$. In the classical limit $kT \gg \hbar\omega$ one can use the Rayleigh-Jeans formula

$$\rho(\omega, T) = \frac{\omega^2}{\pi^2 c^3} kT, \tag{6}$$

which follows immediately from the expression

$$\frac{8\pi k^2 \, dk}{(2\pi)^3} = \frac{\omega^2 \, d\omega}{\pi^2 c^3}$$

for the number of states (the factor $8\pi = 2\cdot 4\pi$ is due to allowance for all directions of propagation of light and for the presence of two possible polarizations) and from the classical result that kT is equal to the average energy of each "field oscillator".

Comparing Eqs. (5) and (6), we obtain the second Einstein relation:

$$A_m^n = \frac{\hbar\omega^3}{\pi^2 c^3} B_m^n, \tag{7}$$

[2)] The third edition of the book was published in 1954 (in Russian translation in 1956). Heitler is a representative of an older generation of theoretical physicists and naturally leans toward old methods (or in any case does not disavow them for reasons of fashion). Perhaps just for this reason, a comparison of the first and third editions[11] permits one to *see for himself the progress in the quantum theory of radiation during the corresponding two decades.*

[3)] For spontaneous and induced radiation Einstein used the *respective terms Ausstrahlung and Einstrahlung.*

also the Planck formula[4]:

$$\rho(\omega, T) = \frac{\omega^2}{\pi^2 c^3} \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}. \tag{8}$$

Use of the Einstein coefficients A_m^n, B_m^n, and B_n^m and of the relations (4) and (7) between them marked an important step forward, but the problem of calculating the coefficients themselves remained open. If we disregard the case of the harmonic oscillator and the results of application of the correspondence principle (which consists, roughly speaking, of the applicability of classical theory in the region of large quantum numbers), calculation of the coefficients A_m^n and B_m^n became possible only with the creation of quantum mechanics.

4. The Schrödinger equation for a charge e with mass m located in an "external" electromagnetic field described by potentials φ and $\mathbf{A}$ has the form

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi, \quad \hat{H} = \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2 + V + e\varphi, \tag{9}$$

where $\mathbf{p} = -i\hbar\nabla$ is the momentum operator and V is the potential energy; of course, the division of the energy into parts V and $e\varphi$ is rather arbitrary and, for example, in the hydrogen atom the energy $V = -e^2/r$ is equal to the same term $e\varphi$ in the potential field of the proton $\varphi_p = -e/r$ (r is the distance between the electron and the proton, which here is assumed to be fixed).

For the "unperturbed problem" we have

$$\hat{H} = \hat{H}_0 = \frac{\hat{p}^2}{2m} + V,$$

and the corresponding wave functions Ψ and energy values E_n determined by the equation $H_0\Psi_n = E_n\Psi_n$ can be considered known. Then, if the external electromagnetic field is the field of a sufficiently weak electromagnetic wave propagating in vacuum, the first approximation of perturbation theory is justified for discussion of absorption and induced radiation of light by an atom. Here, as is clear from Eq. (9), the interaction energy operator is

$$\hat{H}_{\text{int}} = \hat{H} - \hat{H}_0 \approx -\frac{e}{mc}\hat{\mathbf{p}}\mathbf{A} = \frac{ie\hbar}{mc}\mathbf{A}\nabla. \tag{10}$$

Here we have used the gauge in which $\operatorname{div}\mathbf{A} = 0$ and $\varphi = 0$ (in this way in Eq. (10) we have neglected only the interaction $(e^2/2mc^2)A^2$, which is usually completely justified). Assuming that the incident radiation has a continuous spectrum in the region of the transition frequency $\omega = (E_m - E_n)/\hbar$, where the spectral intensity of the radiation is $I_{\mathbf{k}\mathbf{e}}(\omega) = I/8\pi$ (see above and also Section 44 of Ref. 8), we can calculate the probability $W_{\mathbf{k}\mathbf{e}}$ of absorption (the probability of the transition $n \to m$) per unit time. The corresponding calculation is carried out in detail, for example in Section 35 of Schiff's book[13] and in one form or another also in many other texts (see for example Refs. 8, 11, 12, 14); there is no need or special justification for reproducing it here. The result (per unit solid angle) is as follows:

$$W_{\mathbf{k}\mathbf{e}} = \frac{4\pi^2 e^2}{m^2 c \omega^2} I_{\mathbf{k}\mathbf{e}}(\omega) \left| \int \Psi_m^* e^{i\mathbf{k}\mathbf{r}} (\mathbf{e}\nabla) \Psi_n \, d\mathbf{r} \right|^2, \tag{11}$$

where $\mathbf{e}$ is the unit vector of the polarization of the radiation (not to be confused with the charge e!), which is characterized by a wave vector $\mathbf{k}$, where $k = \omega/c$.

In the dipole approximation

$$\left| \int \Psi_m^* e^{i\mathbf{k}\mathbf{r}} (\mathbf{e}\nabla) \Psi_n \, d\mathbf{r} \right|^2 = \frac{m^2}{\hbar^2} \omega^2 \left| \int \Psi_m^* (\mathbf{e}\mathbf{r}) \Psi_n \, d\mathbf{r} \right|^2;$$

therefore for isotropically distributed atoms or with transition subsequently to isotropic radiation, the square of the matrix element in (11) must be replaced by

$$\frac{m^2\omega^2}{3\hbar^2} |\mathbf{r}_{mn}|^2, \quad |\mathbf{r}_{mn}|^2 = \left| \int \Psi_m^* x \Psi_n \, d\mathbf{r} \right|^2 + \left| \int \Psi_m^* y \Psi_n \, d\mathbf{r} \right|^2 + \left| \int \Psi_m^* z \Psi_n \, d\mathbf{r} \right|^2.$$

As a result ($\mathbf{d} = e\mathbf{r}$ is the dipole moment) we have

$$W_{\mathbf{k}\mathbf{e}} = \frac{4\pi^2}{3\hbar^2 c} I_{\mathbf{k}\mathbf{e}}(\omega) |\mathbf{d}_{mn}|^2. \tag{12}$$

The quantity of energy absorbed per unit time[5] is

$$S = \hbar\omega W_{\mathbf{k}\mathbf{e}} = \frac{4\pi^2\omega}{3\hbar c} |\mathbf{d}_{mn}|^2 I_{\mathbf{k}\mathbf{e}}(\omega) = \frac{4\pi^2}{3} \frac{e^2}{\hbar c} \omega |\mathbf{r}_{mn}|^2 I_{\mathbf{k}\mathbf{e}}(\omega). \tag{13}$$

For isotropic unpolarized radiation in Eq. (12) we can at once go over by multiplication by 8π to the probability W integrated over angles and summed over two possible polarizations. In other words, $W = 8\pi W_{\mathbf{k}\mathbf{e}}$, just as the total spectral intensity in this case is $I = 8\pi I_{\mathbf{k}\mathbf{e}}$. Therefore for W we shall retain Eq. (12) with replacement of $I_{\mathbf{k}\mathbf{e}}$ by I.

Calculation of the absorption probability W is obviously equivalent to finding the Einstein coefficient $B_n^m = B_m^n$, since $W = B_n^m \rho = B_n^m I/c$ [see Eq. (2)]. Consequently, according to Eq. (12) we have

$$B_n^m = \frac{4\pi^2}{3\hbar^2} |\mathbf{d}_{mn}|^2. \tag{14}$$

Similarly, of course, from (11) we obtain an expression for B_n^m in the more general case, not only in the dipole approximation.

5. In order to calculate the probability of spontaneous radiation "from first principles" it is necessary to quantize the electromagnetic field—to apply quantum theory not only to the atoms, but also to the radiation. However, if we use the Einstein relations (7) and (4), the answer is obtained immediately. Namely, in the dipole approximation (14), to which for simplicity we shall restrict the discussion, we have

$$W_{\text{sp}} = A_m^n = \frac{\hbar\omega^3}{\pi^2 c^3} B_n^m = \frac{4\omega^3}{3\hbar c^3} |\mathbf{d}_{mn}|^2. \tag{15}$$

The intensity (power) of the spontaneous radiation is

$$I_{\text{sp}} = \hbar\omega W_{\text{cn}} = \frac{4\omega^4}{3c^3} |\mathbf{d}_{mn}|^2. \tag{16}$$

These formulas (15) and (16) coincide, of course, with those obtained in quantum electrodynamics (see for ex-

[4]Actually in Ref. 5 Einstein only mentions the possibility of using the Rayleigh-Jeans formula (6) and uses the Wien displacement law $\rho(\nu, T) = \nu^3 f(\nu/T)$. From this law and Eq. (5) the relations $A_m^n = \alpha\nu^3$ and $E_m - E_n = h\nu$ are obtained and therefore also the Planck formula

$$\rho(\nu, T) = \frac{\alpha\nu^3}{e^{h\nu/kT} - 1}$$

with a still undetermined constant α. Since in Ref. 5 the frequency ν is used and not the angular frequency $\omega = 2\pi\nu$, the density ρ in Eq. (3) is the density $\rho(\nu)$ per interval $d\nu$, while $\rho(\omega)$ refers to an interval $d\omega$. Obviously $\rho(\nu) = 2\pi\rho(\omega)$ and the corresponding coefficients A' and B' are connected by the relation $A_m'^n = (8\pi h\nu^3/c^3) B_m'^n$ (more precisely $A_m'^n = A_m^n$ but $B_m'^n = B_m^n/2\pi$).

[5]Since we have omitted the derivation (actually we did not need it) we shall point out that Eqs. (12) and (13) are identical to Eqs. (35.23) and (17.19) respectively from Refs. 13 and 11.

ample Section 45 of Ref. 8 or Chapter 5.1 of Ref. 11).

Quantization of a free (pure) electromagnetic field in vacuum, as is well known, leads to the conclusion that each "field oscillator" with frequency ω_λ (the normal oscillation) has an energy

$$E_\lambda = \left(n_\lambda + \frac{1}{2}\right)\hbar\omega_\lambda, \tag{17}$$

where $n = 0, 1, 2, \ldots$ are integers which denote the number of photons in the state λ.

In the absence of photons $n_\lambda = 0$, but there remain the zero-point oscillations, which can be compared with the density of the zero-point energy

$$\rho_0(\omega) = \frac{\hbar\omega^3}{2\pi^2c^3}. \tag{18}$$

This expression is obviously obtained by multiplication of the zero-point energy $\hbar\omega/2$ of an individual oscillation (field oscillator) by the number of oscillations with all directions k and with both polarizations [per interval $d\omega$ this number is $\omega^2 d\omega/\pi^2c^3$, as we have already mentioned in connection with the derivation of Eq. (6)].

On the other hand, the Einstein relation (7) in combination with the definition of the probability of induced radiation $W_{ind} = B_m^n\rho(\omega)$ permits us to consider formally (at least in this step) the spontaneous radiation to be induced radiation under conditions in which the radiation energy density is

$$\rho_{sp}(\omega) = \frac{\hbar\omega^3}{\pi^2c^3}. \tag{19}$$

Comparison of Eqs. (18) and (19) immediately leads to the idea of considering spontaneous radiation as induced zero-point oscillations of the electromagnetic field. It is true that the density (18) is a factor of two smaller than the necessary density (19). In addition, as is well known,[11,12] the field can be quantized by completely legal means in such a way that $E_\lambda = n_\lambda\hbar\omega_\lambda$ and the zero-point energy of the field is equal to zero. However, the zero-point oscillations of the field themselves (like the zero-point oscillations of a mechanical harmonic oscillator, for example) of course exist without question—this is a direct consequence of the quantum theory.[6)] Therefore, such fine details as the factor 1/2 or even the possibility of equating to zero the energy of the zero-point oscillations in free space (without boundaries and charges) were considered unimportant from the point of view of the indicated interpretation of spontaneous radiation.

Without undertaking to judge how widespread was this interpretation in the past, very likely it was extremely popular. This is indicated by the review published in 1935 by Weisskopf,[15] who was at that time in the thick of the events—he took an active part in development of the quantum theory as this period.[7] In Ref. 15 he says the following (p. 635): "From quantum theory there follows the existence of so-called zero-point oscillations; for example, each oscillator in its lowest state is not completely at rest but always is moving about its equilibrium position. Therefore electromagnetic oscillations also can never cease completely. Thus, the quantum nature of the electromagnetic field has as its consequence zero-point oscillations of the field strength in the lowest energy state, in which there are no light quanta in space."

"The zero-point oscillations act on an electron in the same way as ordinary electrical oscillations. They can change the eigenstate of the electron, but only in a transition to a state with the lowest energy, since empty space can only take away energy, and not give it up. In this way spontaneous radiation arises as a direct consequence of the existence of these unique field strengths corresponding to zero-point oscillations. Thus, spontaneous radiation is induced radiation of light quanta produced by the zero-point oscillations of empty space."

The absence in Weisskopf's article[15] of any reservations or explanations forces us to think that at that time Weisskopf (and undoubtedly not only he) considered spontaneous radiation to be a consequence of the existence of zero-point oscillations of the field, and thereby to be a quantum effect. It is interesting (and it appears to us somewhat strange) that 45 years later (!) in the article of Ref. 7 Weisskopf writes approximately the same thing. After the formula for the probability of spontaneous radiation obtained by the methods of the quantum theory of radiation, he remarks that according to this formula "spontaneous emission appears as a forced emission caused by the zero-point oscillations of the electromagnetic field" and further: "The oscillatory nature of the radiation field therefore requires zero-point oscillations of the electromagnetic fields in the vacuum state, which is the state of lowest energy. The spontaneous emission process can be interpreted as a consequence of these oscillations."[7)] It is hard for me to believe that also at the present time Weisskopf considers spontaneous radiation to be due to zero-point oscillations, i.e., to be a quantum effect. The wording given (if we forget about the discrepancy of the factor of 1/2 mentioned above) can be understood also just as an indication of a certain formal interpretation. It is just in the latter spirit that Schiff expresses himself (see Ref. 13, Section 50): "From a formal point of view, we can say that the spontaneous emission probability is equal to the probability of emission that would be induced by the

[6)]We have in mind the fact that the field intensities are operators. Here even for a free field (i.e., in the absence of charges) the state with $n_\lambda = 0$ (the vacuum) is not an eigenstate of the field-strength operators. As a result, even with a quantization which provides equality to zero of the field energy for $n_\lambda = 0$, a certain "zero-point" fluctuation field necessarily is present (this means that the quantum-mechanical expectation values of a number of quantities such as the squares of the field strengths are different from zero). In the presence of charges a state with a given number of photons is not an eigenstate even of the transverse-field energy.

[7)]We give here our translation from the English version of the article,[7] since it is somewhat "softer" than the published Russian translation. (*Translator's note*—We have given the exact quotations from the Physics Today article.[7])

presence of one quantum in each state of the electromagnetic field. Now we have already seen that the smallest possible energy of the field corresponds to the presence of one-half quantum per state. This suggests that we regard the spontaneous emission as being induced by the zero-point oscillations of the electromagnetic field; note, however, that these oscillations are twice as effective in producing emissive transitions as are actual quanta, and are incapable of producing absorptive transitions."

6. This statement explains why the idea of relating spontaneous radiation to zero-point oscillations of the field arose, but it in no way proves the need or even the reasonableness of such an interpretation. In our opinion the interpretation is not only not necessary, but it is unreasonable.

Indeed, zero-point oscillations are a purely quantum effect which is not present in the classical theory (and which formally disappears as $\hbar \to 0$). Nevertheless spontaneous radiation clearly exists also in the classical theory and therefore is not a quantum effect. It is true that here one used a definition according to which a quantum effect is defined as one which is absent in the classical theory. However, how else are we to define a quantum effect? It is just on the basis of this idea that we consider to be quantum effects such phenomena as zero-point oscillations, the discreteness of atomic and molecular energy levels, the production of electron-positron pairs in an external field, Van der Waals forces between atoms which are in S states, the force of a Van der Waals type between plates at low temperatures separated by a vacuum gap, the discreteness of the energy of the electromagnetic field for each "normal mode" of oscillations in resonators, and so forth and so on. Usually quantum effects depend explicitly on the quantum constant $\hbar$ and disappear as $\hbar \to 0$. However, this is not obligatory. For example, superfluidity and superconductivity are quantum phenomena but they manifest themselves primarily not in the appearance of terms containing $\hbar$, but in a limitation of the class of possible motions (the condition of absence of vortices, or the London equation which generalizes it to the presence of a magnetic field).

What is spontaneous radiation? Obviously it is the radiation of a system which occurs in the absence of any external (with respect to the system) electromagnetic field. This definition is equally applicable in the classical and quantum regions. In Heitler's book, for example, he says the following (see Ref. 11, Section 17; p. 103 of the second edition): "The interaction between the atom and the radiation field can cause these radiative transitions even if, in the initial state, *no light quanta at all are present*. Supposing the atom to be excited in the initial state, then in the final state the number of light quanta will be increased from zero to some finite value. This process presents than a *spontaneous emission of light.*"

In the classical theory the situation is essentially exactly the same. If the radiation field at a given moment is equal to zero (in this case it is obviously not necessary to worry about zero-point oscillations of the field), but there is accelerated motion of a charge,[8] then at a subsequent time radiation will appear, the energy of which will increase with time. This is spontaneous radiation.

Spontaneous radiation appears because the state in which a mechanical subsystem (an atom, a moving charge, etc.) is at some level or is moving in a specified manner but the radiation field (and sometimes also the intrinsic electromagnetic field of the subsystem) is absent, is not a stationary eigenstate of the complete system (the mechanical subsystem + the electromagnetic field). Fermi understood all this very well, but obviously not everyone understood it, and therefore Fermi did not hesitate to explain the essence of the matter at the classroom level. Following the great physicist, we do not hesitate to give here the corresponding passage as a whole (see Ref. 10, Section 1).

"§1. *Fundamental concept*

Dirac's theory of radiation is based on a very simple idea; instead of considering an atom and the radiation field with which it interacts as two distinct systems, he treats them as a single system whose energy is the sum of three terms: one representing the energy of the atom, a second representing the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field."

"If we neglect this last term, the atom and the field could not affect each other in any way; that is, no radiation energy could be either emitted or absorbed by the atom. A very simple example will explain these relations. Let us consider a pendulum which corresponds to the atom, and an oscillating string in the neighborhood of the pendulum which represents the radiation field. If there is no connection between the pendulum and the string, the two systems vibrate quite independently of each other; the energy is in this case simply the sum of the energy of the pendulum and the energy of the string with no interaction term. To obtain a mechanical representation of this term, let us tie the mass M of the pendulum to a point A of the string by means of a very thin and elastic thread a. The effect of this thread is to perturb slightly the motion of the string and of the pendulum. Let us suppose for instance that at the time $t=0$, the string is in vibration and the pendulum is at rest. Through the elastic thread a the oscillating string transmits to the pendulum very slight forces having the same periods as the vibrations of the string. If these periods are different from the period of the pendulum, the amplitude of its vibrations remains always exceedingly small; but if a period of the string is equal to the period of the pendulum, there is resonance and the amplitude of vibration of the pendulum becomes considerable after a certain time. This process corresponds to the absorption of radiation by the atom."

[8]Here we have in mind motion of a charge in vacuum. In motion in a medium, a charge can radiate also in equilibrium motion (Cherenkov radiation and transition radiation; see Ref. 12).

"If we suppose, on the contrary, that at the time $t=0$ the pendulum is oscillating and the string is at rest, the inverse phenomenon occurs. The forces transmitted through the elastic thread from the pendulum to the string put the string in vibration; but only the harmonics of the string, whose frequencies are very near the frequency of the pendulum reach a considerable amplitude. This process corresponds to the emission of radiation by the atom."

7. Not much remains for us to add. An understanding of the connection of the quantum theory of radiation (and of quantum electrodynamics in general) with the classical theory of radiation and classical electrodynamics was hindered to a significant degree by the difference in the formulation of the questions and problems and in the methods of their solution (we have already mentioned this above). For example, in the classical theory of radiation a problem is usually not formulated with initial conditions, and perturbation theory of the type used in the quantum theory of radiation is not used. When we are considering an electron moving uniformly in vacuum, we calculate the electromagnetic field dragged along by it; under stationary conditions (an electron moving with velocity $\mathbf{v}=\text{const}$ for all time, with $v<c$) there is no radiation field at all. However, in the quantum theory we usually consider a state in which there is an electron with momentum p (wave function $\Psi=c\exp[i(\mathbf{pr}/\hbar-E(p)t/\hbar)]$ and there is no electromagnetic field, in the sense that there are only zero-point oscillations of the field (the vacuum). However, such a state obviously is not an eigenstate for the system electron + field, and if we specify this state at $t=0$, then a uniformly moving electron will radiate (the energy of the field at $t>0$ is different from zero). Fock[16] called attention to this effect in 1934 (see also Ref. 17). However, as is clear from the above, there is no paradox here and in a certain approximation (neglecting recoil) results identical to those obtained in Ref. 17 in a quantum calculation follow from the classical theory.[18] Discussion of this question here in more detail is not well justified since it is considered completely in the easily available book of Ref. 12 (see Chapter 1).[9)] In Ref. 19, which is a continuation of Ref. 18, I discussed the question of the nature of spontaneous radiation and in particular I have criticized Weisskopf's opinion cited above.[15] These articles[18,19] were the first ones I ever wrote, and naturally the memory of them as a first love in theoretical physics stimulated me to a significant degree after four decades to write the present note. It is strange that a question which long ago seemed clear still brings forth any kind of discussion.

In connection with the problem of radiation by an electron of its own field[18] note that, both in classical theory and quantum theory (quantum electrodynamics and the quantum theory of other fields) one encounters extremely interesting problems in which the field entrained by the particle (its own field) is not in the stationary state.[20] We can suppose that the necessary attention has not been devoted, and obviously is not being de-

[9)] From discussion of this article it has become clear that some explanation is still appropriate here.

We shall represent the potential of the electromagnetic field in vacuum in the Coulomb gauge ($\operatorname{div}\mathbf{A}=0$) in the form

$$\mathbf{A}=\sum_{\lambda,\,i=1,2} q_{\lambda i}\mathbf{A}_{\lambda i},\ \mathbf{A}_{\lambda 1}=\sqrt{8\pi}c\mathbf{e}_\lambda\cos(\mathbf{k}_\lambda\mathbf{r}),\ \mathbf{A}_{\lambda 2}=\sqrt{8\pi}c\mathbf{e}_\lambda\sin(\mathbf{k}_\lambda\mathbf{r});$$

for simplicity here we have not explicity pointed out that for each λ there are two polarization vectors $\mathbf{e}_\lambda$ ($\mathbf{e}_\lambda\mathbf{k}_\lambda=0$, $e_\lambda=1$). Then from the equations of the field there follow (or more precisely one obtains as equivalent expressions) equations for $q_{\lambda i}$ ($\omega_\lambda^2=c^2k_\lambda^2$):

$$\ddot{q}_{\lambda i}+\omega_\lambda^2 q_{\lambda i}=\frac{e}{c}\mathbf{v}(t)\mathbf{A}_{\lambda i}(\mathbf{r}(t))=e\sqrt{8\pi}(\mathbf{e}_\lambda\mathbf{v}(t))\times\begin{cases}\cos(\mathbf{k}_\lambda\mathbf{r}(t)),\\ \sin(\mathbf{k}_\lambda\mathbf{r}(t));\end{cases}\tag{20}$$

here $\mathbf{r}(t)$ is the radius vector of a point particle (electron) with charge e, which is the only thing being discussed. In the case of a uniformly moving electron $\mathbf{v}=\mathbf{v}_0=\text{const}$, and in the right-hand side of Eqs. (20) there appear "forces" $e\sqrt{8\pi}(\mathbf{e}_\lambda\mathbf{v}_0)\cos(\mathbf{k}_\lambda\mathbf{v}_0 t)$ and $e\sqrt{8\pi}(e_\lambda\mathbf{v}_0)\sin(\mathbf{k}_\lambda\mathbf{v}_0 t)$. If at some (initial) moment of time $t=0$ the transverse electromagnetic field considered is equal to zero (in the classical case this is obviously possible), then all $\dot{q}_{\lambda i}=q_{\lambda i}=0$ (at $t=0$). The very existence of the charge e already means that the "forces"—the right-hand sides in the field equations (20)—are nonzero. Therefore it is obvious that at $t>0$ a field will appear (i.e., $q_{\lambda i}\neq 0$, $\dot{q}_{\lambda i}\neq 0$). In the case of a uniformly moving electron the "forces" are characterized by frequencies $\mathbf{k}_\lambda\mathbf{v}_0=(\omega_\lambda v_0/c)\cos\theta<\omega_\lambda$ (here θ is the angle between $\mathbf{v}_0$ and $\mathbf{k}_\lambda$; it has been taken into account that $v_0<c$). Therefore resonance between the frequency of the force and the eigenfrequency ω_λ of the field oscillators is impossible and in an established regime (formally at $t\to\infty$) radiation will not occur. The field which appears at $t>0$ is the entrained (intrinsic) transverse electromagnetic field of a uniformly moving electron, and also the radiation field which arises on instantaneous acceleration of the electron from a state of rest to a state with velocity $\mathbf{v}_0$ (the problem formulated with its initial conditions is obviously equivalent to the problem of finding the field just for change of the velocity of a charge from $\mathbf{v}=0$ to $\mathbf{v}=\mathbf{v}_0$ with the condition of absence of a field at $t\leq 0$).

It is appropriate to mention that with motion of a charge in a transparent medium with refractive index $n=\sqrt{\varepsilon}$ the field equations naturally retain the form (20), but in the right-hand sides a factor $1/n$ appears and, most important, now $\omega_\lambda^2=(c^2/n^2)k_\lambda^2$. Therefore for a uniformly moving charge the frequencies of the "force" $\mathbf{k}_\lambda\mathbf{v}_0=(\omega_\lambda v_0 n/c)\cos\theta$ can be equal to the eigenfrequency ω_λ. The condition for such resonance $\cos\theta=c/nv_0$ is just the condition for Cherenkov radiation (see Ref. 19 and Chapter 6 of Ref. 12).

In vacuum for a uniformly moving electron, as has been said, the resonance is impossible. However, for accelerated motion of a charge, in the frequency spectrum of the forces there are some frequencies ω_λ. The radiation with such resonance frequencies grows unboundedly with time (for a given motion of the charge, i.e., a given function $\mathbf{r}(t)$) and at sufficiently large t does not depend on the initial conditions. For example, in the case of a "mechanical" harmonic oscillator $\mathbf{r}(t)=\mathbf{a}_0\sin\omega_0 t$ in the dipole approximation, in which $a_0\ll\lambda_0/2\pi=c/\omega_0$, only electromagnetic waves with frequency $\omega_\lambda=\omega_0$ are radiated. This radiation is what is usually called spontaneous radiation.

A quantum calculation (of course with the unavoidable inclusion of the existence of zero-point oscillations of the field) for a uniformly moving charge in essence does not lead to any changes (the recoil associated with radiation can be assumed to be sufficiently small if the mass of the radiating particle is increased). For a mechanical oscillator, not to mention other radiators, the situation is, generally speaking, more complicated. However, this is due not to the zero-point oscillations of the field, but to the necessity, especially for low-lying levels, of taking into account the quantum nature of the radiator itself (see also below).

voted at the present time, to the corresponding group of questions, as a result of insufficiently general understanding of the fact that the field entrained by the particle is different from a set of free photons (or quanta of other free fields) and can be in a nonstationary state.

In conclusion a few more words regarding spontaneous radiation. Above I have persistently emphasized that spontaneous radiation exists already in the classical theory and in any case cannot be considered a purely quantum effect. However, it does not follow from this that one should conclude that spontaneous radiation is completely classical in nature in all cases. Description of atomic systems, zero-point oscillations of the field (their existence in the presence of boundaries leads to quite real and observable effects), states of a field with a small number of photons, and so forth, of course, is not done—and cannot be done without the quantum theory, and it is not necessary to attempt this (as frequently occurs). Obviously spontaneous radiation under quantum conditions (say, for a system with two levels) cannot be discussed in any systematic way classically (and in general it is not necessary). However, the same can be said of absorption, induced radiation, and many other processes. In particular, if the matrix element for spontaneous emission of one photon depends on the wave function of the vacuum (the state of the field oscillator in which all $n_\lambda = 0$), then, of course, we can also speak of absorption with the presence in the initial state of only one corresponding photon. However, no one has yet, it appears, proclaimed on this basis that the absorption of light is a purely quantum effect. To the same extent there is no reason to enter into a similar discussion for spontaneous radiation.

The stability of an atom in the ground state (the lowest state) and the absence of radiation in this state are unquestionably quantum phenomena. Therefore, it is possible in principle in some way to relate the stability of an atom (or system) in its ground state to the zero-point (vacuum) oscillations of the electromagnetic field as has been done by Faĭn.[1,21] However, I have not been able to see what such an approach gives that is new or useful. Apparently this is a matter of taste, and *de gustibus non disputandum* or, it is more correct to say that such arguments turn out to be fruitless.

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