Broken symmetry and magnetoacoustic effects in ferro- and antiferromagnetics

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This review of some aspects of the magnetoacoustics of ferro- and antiferromagnetic materials has been written in connection with the 25th anniversary of the rise of this field of physics of magnetic phenomena. Primary attention is paid to relatively new problems that have not been reflected in the existing monographs and reviews. The topic is a group of linear magnetoacoustic effects that manifest spontaneous symmetry breaking caused by magnetic ordering in a system of two coupled fields: the magnetization field **M** (**r**) and the deformation field $u_{ij}(\mathbf{r})$. To some extent these effects are analogous to the Higgs effect in the theory of elementary particles (the Higgs mechanism of the origin of the mass of a particle) or the Meissner effect in the theory of superconductivity. A direct analog of the stated effects is the so-called magnetoelastic gap in the magnon spectrum, while an analog of an accompanying effect is the softening of the quasiacoustic modes interacting with it (up to the vanishing of the corresponding dynamic elastic moduli). However, a characteristic feature of such effects in crystalline (anisotropic) magnetic materials is that they are manifested mainly near points of magnetic (spin-reorientation) phase transitions. This review treats the coupled magnetoelastic waves in ferro- and antiferromagnetic materials of different types that show phase transitions with respect to temperature, magnetic field, or pressure.

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CONTENTS

1. INTRODUCTION

a) History of the problem

About 25 years ago (1956–1958) the first studies[1-3] appeared that predicted the existence of coupled magnetoelastic (magnon-phonon) waves in ferro- and antiferromagnetic materials. These studies actually opened up a new field in the physics of magnetically ordered materials ("magnetics")—magnetoacoustics, which has subsequently found a number of important applications (see, e.g., the reviews of Refs. 4, 5).

However, the present article is aimed at discussing a set of magnetoelastic (ME) effects that also pertain to this field, but have been discovered consideraly later and have been hardly reflected in the review (or monograph) literature. The need for such a treatment is further dictated by the fact that contradictory (and in some cases just plain wrong) views exist on these effects. At the same time, the situation here is that the stated effects are actually of general physical interest and have analogs in other fields of physics.

The study of the ME phenomena of interest to us began in 1963–1965 in Refs. 6–11. An effect was discovered[7,8] almost simultaneously and independently that has been called respectively in the Soviet and the Western literature the "magnetoelastic-gap"[6,10] or the "frozen-lattice" effect.[12-22] It was found in experiments on antiferromagnetic resonance (AFMR) in hematite (α-Fe_2O_3) that the resonance frequency ω_0 is described by the formula

$$\omega_0^2 = \omega_M^2 + \omega_{ME}^2. \tag{1.1}$$

Here the quantity ω_M corresponded to the AFMR frequency given by the theory existing at that time (and determined by the external magnetic field **H** and the magnetic anisotropy), while the extra term directly represented the discovered effect.

The very first studies[6-9] advanced the hypothesis that this contribution to the AFMR frequency involves spontaneous magnetostriction, which takes place in the ground (equilibrium) state of an antiferromagnet. The striction gives rise to elongation (or contraction) of the specimen in the direction of the antiferromagnetism vector, which creates an additional effective anisotropy field for the spin oscillations. It is important to stress that the effect under discussion involves the fact that these spontaneous lattice deformations do not follow the oscillations of the magnetization excited in AFMR. They are as though "frozen" (hence one of the names of the effect). It is important to distinguish this effect from the renormalization by magnetostriction of the magnetic-anisotropy constants, which are determined from static measurements. In contrast to the latter, the effect in which we are interested does not vanish,

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even in the limiting case of a completely isotropic medium, for which $\omega_M = 0$ (as $H \to 0$), and hence we have $\omega_0 = \omega_{ME}$. On the contrary, in this case the effect, not being masked by the anisotropy, is manifested in its purest form.

The theory of AFMR with account taken of spontaneous deformations in the ground state has been developed in Refs. 6 and 10. Here not only the stated nature of the effect in hematite was confirmed, but also some new predictions were made.

It was shown[10] that this ME effect is common to all ferro- and antiferromagnetic materials, while differing in different cases only in magnitude. It was found that the existence of "frozen" spontaneous deformation should be treated, not as a hypothesis, but as the result of systematic solution of the coupled equations of motion for homogeneous oscillations of the magnetization and deformation.

The estimates of Ref. 10 showed that the quantity ω_{ME} must have an anomalously large value for rare-earth ferromagnetic substances (Dy and Tb) having anisotropy of the "easy-plane" type, for which $\omega_{ME}/\gamma \sim 10^5$ Oe (γ is the magnetomechanical ratio). This prediction has been confirmed in experiments on inelastic scattering of neutrons by spin waves in these metals.[13,14] The point is that the ferro- (or antiferro-) magnetic resonance frequency ω_0 and also be defined (with certain exceptions involving demagnetizing fields of the specimen) as the minimum value of the frequency ω_k for the corresponding branch of spin waves in the limit of the wave vector $\mathbf{k} = 0$: $\omega_0 = \omega_{k|k=0}$. In other words, the frequency ω_0 amounts to the gap in the spin wave spectrum. As has been done in Refs. 13 and 14, this can be found by extrapolating the function $\omega_k \equiv \omega(\mathbf{k})$ to $k \to 0$. The quantity ω_{ME} gives the ME contribution to this gap. This is the source of the other name of the effect.

A lively discussion evolved at one time around the problem of the spin-wave frequency (as $\mathbf{k} \to 0$) in the rare-earth metals and their alloys on the topic of which of the models is correct here: the "frozen" or "free" lattice (frozen or free lattice model; see, e.g., Refs. 12–21). Echoes of this discussion are found even now.[22-24] The bases for this discussion were both purely subjective[1)] and objective, involving the fact that it had not been possible to describe the resonance in these metals from the standpoint of a "frozen lattice". Although it has had the last word, yet the problem of adequate description of the *UHF* properties of the rare-earth metallic ferromagnetic materials remains open to a considerable extent. We shall return again later to the concepts of the frozen and free lattice from the standpoint of coupled magnetoacoustic (magnon-phonon) waves.

Already there are many tens of studies examining theoretically or experimentally effects that are somehow related to taking into account the spontaneous deformations in the ground state. Besides hematite, which has been rewarding in this respect (in addition to Refs. 6–9, see also Refs. 25–33), these effects have been studied in detail in another rhombohedral antiferromagnetic material (also with weak ferromagnetism), $FeBO_3$,[27,31,34,35] in the cubic antiferromagnetic garnets,[36,37] and also in certain orthoferrites (antiferromagnetic compounds, often with weak ferromagnetism, of the type of $VFeO_3$) (see Refs. 38–42), and in "easy-axis"-type antiferromagnetic materials (MnF_2 and Cr_2O_3).[43-50] The difference has been discussed[23] between the magnetic anisotropy constants of cobalt manganese ferrite ($Co_{0.25}Mn_{0.75}Fe_2O_4$), as determined from static measurements and from ferromagnetic resonance (FMR), with the aim of detecting a "frozen-lattice" effect.

It has been found from these studies that the ME gap amounts to only one (perhaps not the most interesting) aspect of the phenomenon: it corresponds to only one point ($\mathbf{k} = 0$) of the spectrum of one of the branches of the unitary spectrum of coupled ME waves (specifically the quasimagnon branch). This gap arises from the influence of the elastic subsystem on the magnetic subsystem. Of course, there is the other side of the coin—the inverse influence of the spin oscillations on the branch of acoustic oscillations (phonons), which interacts with it. Both these coupled modes of oscillations (the quasimagnon $\omega_I(\mathbf{k})$ and the quasiacoustic mode $\omega_{II}(\mathbf{k})$) are shown schematically in Fig. 1 for an antiferromagnetic material in the simplest "pure" case in which $\omega_M = 0$ in (1.1), so that the magnon gap fully reduces to the magnetoelastic gap. In this case the interaction of the modes proves strongest. The dotted lines show the spectra without the interaction, while the solid lines show them with allowance for the ME interaction. We see that, in addition to the appearance of the gap ω_{ME} for $\omega_I(\mathbf{k})$, a strong deformation arises in the quasiacoustic branch (so that in the asymptotic when $\omega_{II} \ll \omega_{ME}$, i.e., for small enough values of $\mathbf{k}$, the dispersion law for this branch can even change from linear to quadratic). Experimentally this effect is manifested as a decrease in the velocity of sound with decreasing ω_M in (1.1). In various cases, this can be attained by varying the magnetic field, temperature, or pressure (see Refs. 25–27, 34, 39–41). Simultaneously with the decrease in the velocity of sound near the point $\omega_M = 0$, the damping can rise sharply.[42]

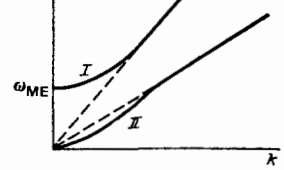

FIG. 1. Coupled ME waves in an isotropic antiferromagnetic material. The dotted lines represent the magnons and sound in the absence of ME interaction.

[1)]However amazing this may seem now, a number of physicists could not believe at all that a lattice can be at rest (frozen lattice) during homogeneous oscillations of the spins, rather than following them (free lattice).

b) Analogies and differences with the Meissner-Higgs effect. Role of phase transitions

Despite the great number of papers cited above and others in the field under discussion, at times there was a lack of core ideas reflecting the universality of the phenomena, as well as of bridges to other branches of physics that might, in particular, serve as analogies. To fill this gap, so as to stress the general physical aspects of the problem, is the fundamental goal of this article (see also Ref. 51).

The point is that, from the standpoint of interest to us, a magnetic material amounts to a system of two interacting fields (the magnetization **M** and the deformation u_{ij}) in which *spontaneous symmetry breaking* (SSB) occurs—ferro- or antiferromagnetic ordering. In this sense such a system is an analog of the Higgs model in elementary-particle theory.[52]

Analogies between the theories of elementary particles and of the solid state possess both cognitive and practical importance for their development. The concept of SSB, which arose in the theory of ferromagnetism, has been considerably developed and widely applied in the gauge theories of elementary particles. We shall speak here of the analogies with the Higgs effect, which plays a large role in these theories and which was discovered (amazing coincidence!) at about the same time as the effect of the ME gap (1964). However, one can treat on the same basis the analogies with the Meissner effect in the superconductivity model of Ginzburg and Landau,[53] since the Higgs model in turn is only a relativistic analog of this model (see, e.g., Ref. 54).

We have no opportunity to take up even briefly the models that yield a Meissner-Higgs effect. Yet perhaps there is no need of this, since a number of excellent articles on this topic has appeared in recent years in the journal Uspekhi Fizicheskikh Nauk (in addition to Ref. 54, see also Refs. 55 and 56). For us it is important to recall only the situation that gives rise to the Higgs model, in which initially massless particles described by the vector (e.g., electromagnetic) field A_μ ($\mu = 0, 1, 2, 3$) acquire a non-zero mass by the interaction of the vector with the complex scalar field $\phi(x)$ $= \phi_1 + i\phi_2$, which corresponds to charged bosons.

This occurs when spontaneous ordering (Bose condensation) arises in the ϕ subsystem. The ground state obtained here $\langle 0|\phi|0\rangle \equiv \phi_0 = \phi_{10} + i\phi_{20}$ proves, on the one hand, to be degenerate with respect to rotation of the "vector" (ϕ_{10}, ϕ_{20}) in the (ϕ_1, ϕ_2) plane. On the other hand, each given state having concrete values of ϕ_{10} and ϕ_{20} will no longer possess the circular rotational symmetry in this plane inherent in the initial Lagrangian for the field ϕ; such a rotation transforms one state (ϕ_{10}, ϕ_{20}) into any other (ϕ'_{10}, ϕ'_{20}) having the same energy. The symmetry of the ground state is broken with respect to that of the Lagrangian.

If we examine the excited states of the complete system of the fields ϕ and A_μ, then, upon taking into account the non-zero vacuum average ϕ_0, we must redefine the scalar field by setting $\phi = \phi_0 + \phi'$ (here we have $\langle 0|\phi'|0\rangle = 0$). Consequently the quadratic form in A_μ and ϕ' in the complete Lagrangian determining the spectrum of excitations will contain a term of the form $|\phi_0|^2 A_\mu^2$. The latter means that the particles corresponding to the field A_μ acquire a non-zero rest mass. In other words, their spectrum will contain an energy gap—a minimal energy below which these particles do not exist. This is the Higgs mechanism for the origin of mass in a system with SSB.

Now let us treat small coupled (via magnetostriction) oscillations of the magnetization **M** and the deformations

$$u_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

in an isotropic ferromagnetic material near its ground state. In addition to the spontaneous magnetization $\mathbf{M}_0$, we also take account in it of the spontaneous deformations $u_{ij}^{(0)}$ coupled with the latter, so that we have

$$\mathbf{M} = \mathbf{M}_0 + \Delta\mathbf{M} \text{ and } u_{ij} = u_{ij}^{(0)} + \Delta u_{ij}.$$

The general form of the ME interaction responsible for this coupling of **M** and u_{ij} (in the linear approximation in u_{ij}) will be given later [Eq. (2.1)].

In the absence of magnetic anisotropy, of an external magnetic field, and without taking account of the ME coupling, the energy of the oscillations of the vector **M** is determined by the exchange interaction, or more precisely, by its increase caused by the spatial inhomogeneity of the oscillations (i.e., oscillations with non-zero gradients of **M**). This increase in the exchange energy is proportional to $(\nabla\mathbf{M})^2$, which yields a gap-free spectrum with respect to the frequency $\omega_k \sim k^2$ for the spin waves (magnons). Taking account of the ME coupling gives rise (in the component of the energy quadratic in the oscillations) to terms of the form $u^{(0)}(\Delta\mathbf{M})^2$, which arise from the spontaneous deformation $u_{ij}^{(0)}$ in the ground state (homogeneous contraction or elongation of the specimen in the direction of its spontaneous magnetization $\mathbf{M}_0$). These terms play the role of an effective magnetic anisotropy for the oscillating component of the magnetization $\Delta\mathbf{M}$, and thus give rise to a non-zero gap in the spectrum of the quasi-magnon branch of the coupled ME waves, *even in a magnetically isotropic medium*. In this sense they are analogous to the cited term $|\phi_0|^2 A_\mu^2$, which gives rise to the non-zero mass of the particles of the vector field in the Higgs model.

We note that, up to 1964, only the other quadratic terms of the ME coupling of the oscillations had been taken into account, namely those of the form $\mathbf{M}_0\Delta\mathbf{M}\Delta u$. The latter vanish as $k \to 0$ because the dynamic component (Δu_{ij}) of the tensor u_{ij} can only be inhomogeneous. Therefore they do not contribute to the ME gap. Henceforth we shall call the terms of the stated two types the *strictive* and *dynamic* terms, respectively.

Thus the ME gap in magnetic materials is just as generally physical in nature (actually at the level of the first principles of physics) as the Higgs effect, in the sense that both arise as the result of SSB in a system of

interacting fields.[2)] The ground state of a ME medium is degenerate with respect to rotation of the spontaneous magnetization $\mathbf{M}_0$ together with the deformations $u_{ij}^{(0)}$ created by it. In line with the Goldstone theorem, this degeneracy corresponds to gapless (acoustic) modes of coupled oscillations (of the type of the lower curve in Fig. 1). At the same time, the spontaneous deformations remove the degeneracy with respect to pure spin rotations, whereby the quasimagnon mode acquires the gap ω_{ME}. We note that the problem of the existence of an ME gap was first treated from the standpoint of symmetry in Ref. 57, while the present analogy was noted in Refs. 58 and 59.

The frequency ω_{ME} amounts to the frequency of uniform precession of $\mathbf{M}$ around the effective field of the magnetic anisotropy ($\mathbf{H}_{ME}$), which arises from the breaking of the initial spherical symmetry of the isotropic medium owing to the spontaneous ("frozen") deformations of the ground state (Fig. 2).

Further we note the importance in SSB effects of the fact that the ground states $(\mathbf{M}_0, u_{ij}^{(0)})$ in an isotropic magnetic material form a continuous degenerate set that corresponds to the continuous group of all rotations and reflections in three-dimensional space (supplemented by the exchange operation $\mathbf{M} \to -\mathbf{M}$) that leave the energy (thermodynamic potential) of this magnetic material is invariant. This is precisely why the magnon gap in an isotropic magnetic material completely reduces to the ME gap.

At the same time, the symmetry of crystalline magnetic materials is discrete, so that their degenerate states (corresponding to minimum energy and obtainable from one another by applying the discrete set of symmetry operations of the paramagnetic crystal) are separated by potential barriers. From the standpoint of magnetic properties this implies the existence of magnetic anisotropy in crystals, which gives rise to the "easy" and "hard" directions of magnetization of crystalline magnetic materials, and by itself leads to a nonzero gap in the magnon spectrum. Here the overall

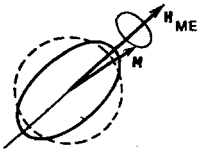

FIG. 2. The spontaneous deformations of an isotropic ferromagnetic material give rise to the effective magnetic-anisotropy field H_{ME} for homogeneous oscillations of the magnetization with respect to the ground state with these "frozen deformations".

[2)]However, the analogy ends with this, since the Higgs effect consists not only in the creation of mass in the vector field, but also in the disappearance of the massless component of the scalar field, which is "expended" in creating the extra degree of freedom of the vector field, which becomes massive. Of course, for ME oscillations the Goldstone (acoustic) modes do not vanish. In particular, this difference involves the fact that the Higgs model belongs to the *gauge* theories, which cannot be said of the model of a ME medium (in any case, for its commonly used form).

magnon gap in many cases is described by a two-term formula of the form of (1.1). It is not always a simple problem to separate these two terms experimentally. However, in the physics of the problem from the standpoint of SSB effects, one must perform this separation as follows. The first term (ω_M^2) reflects the symmetry of the equilibrium magnetic properties as determined by the symmetry group of the paramagnetic crystal (*unbroken* symmetry). Theoretically one can easily distinguish it from ω_0^2 as being the magnon gap calculated under the assumption of equilibrium coupling of the oscillations of the magnetic moments and the deformations ($\Delta\mathbf{M}$ and Δu_{ij}). Then the residual $\omega_{ME}^2 = \omega_0^2 - \omega_M^2$ will define in pure form the SSB effect, or ME gap.

The contribution ω_{ME}^2 to the gap ω_0^2 is very small in most cases. Under these conditions, the reverse effect of the spin waves on the elastic degrees of freedom is also small. Conveniently one can use the following ratio as a dimensionless parameter characterizing the effectiveness of ME coupling:

$$\zeta = \frac{\omega_{ME}^2}{\omega_0^2}. \tag{1.2}$$

In line with what we have said, as a rule under ordinary conditions, we have $\zeta \ll 1$.

And just here we must bring into the scene the magnetic phase transitions (PTs). We are here dealing with PTs of the "order-order" type, in which one ordered magnetic state transforms into another, e.g., by coherent rotation of all the magnetic moments (orientational PTs).

The special role of PTs in observing SSB effects consists of the fact that in a PT the above-mentioned potential barriers that separate the degenerate states vanish: at a second-order PT the initial state immediately adjoins another and transforms into it continuously, while at a first-order PT there are points at which the initial state loses stability. Here, in both cases the term ω_M^2 in (1.1) loses its positive definite character (which arises from the equilibrium nature of the ground state), so that at these PT points we have $\omega_M^2 = 0$, and hence, $\zeta = 1$.

Thus, near PT points the ME effects in which we are interested, that arise from SSB, must be maximal, and by analogy to the case of an isotropic magnetic material, not at all small. Therefore the ME spectra in Fig. 1 pertain not only to an isotropic antiferromagnetic substance, but also to the PT point of an anisotropic antiferromagnetic.

As we shall show in Sec. 2, a very important point is that the modes of coupled ME oscillations being discussed are of a type such that, as $k \to 0$, the magnetization in one of them (quasimagnon mode) oscillates with respect to "frozen" deformations, while in the other (quasiphonon mode) $\Delta\mathbf{M}$ and Δu oscillate in phase, accompanying one another in *quasiequilibrium* fashion. Some analogs are the optical and acoustic lattice vibrations. In this sense the dispute between the "frozen" and "free" lattice approximations is resolved as being (when $k \to 0$) frozen—for quasimagnons, and free—for quasiphonons. Yet for finite k, of course, neither of

the approximations fits.

Finally we note that the ME gap cannot be abolished by any agents acting on the magnetic material (temperature, a field, pressure, or even a change in concentration) until this agent abolishes the magnetic order itself that breaks the symmetry. (Similarly we cannot abolish the Meissner effect without abolishing the superconductivity.) This is precisely why a PT in an interaction parameter ensures this impossibility of abolishing the phenomenon.

In the simplest case we can see this even from the example of an isotropic ferromagnetic material (see Fig. 2): an attempt to diminish the ME gap by unilateral pressure, which eliminates the spontaneous deformations, or an external magnetic field that compensates the effective field H_{ME}, must immediately lead to an orientational PT (to a state of lower energy). After this the stated agent will no longer diminish the magnon gap, but increase it. Analogously, in an anisotropic magnetic material, an external agent that diminishes the magnon gap ω_0 must cause a PT as soon as the latter reaches its minimum value, which directly determines the magnitude of the ME gap ω_{ME} (Fig. 3).

The further aim of our article is to demonstrate in greater detail the general propositions formulated above on concrete examples of magnetic materials of different structures and for different PTs.

2. MAGNETOELASTIC WAVES IN FERROMAGNETIC MATERIALS

a) Overall formulation of the problem

The effects in which we are interested arise from the ME interaction. In the case of a ferromagnetic material it makes the following contribution to the energy density (see, e.g., Ref. 60, pp. 774-780):

$$E_{ME}(\mathbf{r}) = B_{ijnl} m_i m_j u_{nl}. \tag{2.1}$$

(As usual, one sums over the subscripts that occur twice, with each of the subscripts *ijnl* running through the three values: x, y, and z.) Here B_{ijnl} is the magnetoelastic-constant tensor; $\mathbf{m} = \mathbf{M}(\mathbf{r})/M_0$ is the unit vector of the local magnetization $\mathbf{M}(\mathbf{r})$, whose modulus is considered to be a conserved quantity: $\mathbf{M}^2(\mathbf{r}) = M_0^2$, so that $\mathbf{m}^2 = 1$. Finally,

$$u_{nl} = \frac{1}{2}\left(\frac{\partial u_n}{\partial x_l} + \frac{\partial u_l}{\partial x_n}\right)$$

is the deformation tensor, which is defined by a symmetric combination of the derivatives of the displacement $\mathbf{u}$.[3)]

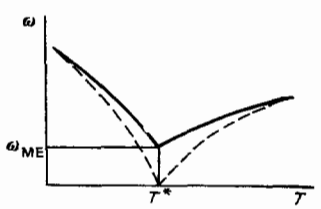

FIG. 3. The magnon gap $\omega_0 = \sqrt{\omega_M^2 + \omega_{ME}^2}$ as a function of the temperature near the magnetic phase-transition point T^* at which $\omega_M(T^*) = 0$. The function $\omega_M(T)$ (dotted line) determines the same gap in the absence of the SSB effect.

The overall energy density also includes the magnetic component $E_M(\mathbf{r})$ and the elastic component $E_E(\mathbf{r})$. The former can be written in the form of the following expansion in $\mathbf{m}$ and $\partial\mathbf{m}/\partial x_i$:

$$E_M(\mathbf{r}) = A_{ij}\frac{\partial \mathbf{m}}{\partial x_i}\frac{\partial \mathbf{m}}{\partial x_j} + K_{ij}^{(1)} m_i m_j + K_{ijnl}^{(2)} m_i m_j m_n m_l + \ldots - \mathbf{MH} - \frac{1}{2}\mathbf{MH}_M. \tag{2.2}$$

The first term (with the coefficients A_{ij}) amounts to the increase in exchange energy due to the spatial inhomogeneities of $\mathbf{m}$ with respect to directions ("inhomogeneous exchange"), while the following terms—the terms with $K^{(\nu)}$ ($\nu = 1, 2, \ldots$) are the energy of magnetic crystallographic anisotropy. The magnitude of the anisotropy constants $K^{(\nu)}$ usually decreases with increasing degree 2ν of the expansion (actually, in this article, the terms that we have written out will suffice). The latter two terms respectively represent the Zeeman energy in the external field $\mathbf{H}$ and the magnetostatic energy involved with the inhomogeneities of the magnetization $\mathbf{M}$ (including the energy of the demagnetizing fields of the specimen surface). The magnetostatic field $\mathbf{H}_M$ is the solution of the equations of magnetostatics:

$$\operatorname{div}(\mathbf{H}_M + 4\pi\mathbf{M}) = 0, \quad \operatorname{rot}\mathbf{H}_M = 0. \tag{2.3}$$

Finally, the elastic-energy density has the form

$$E_E = \frac{1}{2} C_{ijnl} u_{ij} u_{nl}. \tag{2.4}$$

Here C_{ijnl} is the elastic-modulus tensor.

The explicit form and the number of independent components of the material tensors B, A, K, and C are determined by the symmetry of the crystal.

The total energy (thermodynamic potential) is

$$\mathcal{E} = \int E(\mathbf{r})\,d\mathbf{r} \quad (d\mathbf{r} \equiv dx\,dy\,dz). \tag{2.5}$$

Here

$$E(\mathbf{r}) = E_M + E_{ME} + E_E, \tag{2.6}$$

determines both the equilibrium states (the values of $\mathbf{m} = \mathbf{m}^{(0)}$ and $u_{ij} = u_{ij}^{(0)}$ corresponding to the minimum of ξ) and the spectrum of small oscillations about these states:

$$\Delta\mathbf{m} = \mathbf{m} - \mathbf{m}^{(0)} \text{ and } \Delta u_{ij} = u_{ij} - u_{ij}^{(0)}. \tag{2.7}$$

If one is interested in the equilibrium magnetic properties of a ferromagnetic material (or even the dynamic properties, but those that conserve the equilibrium coupling between u_{ij} and $\mathbf{M}$), then, by expressing the equilibrium magnetostrictive deformations u_{ij} in terms of $\mathbf{M}$ from the condition $\partial E/\partial u_{ij} = 0$, and then substituting them into E_{ME} in (2.1) and E_E in (2.4), one can easily show (Ref. 60, p. 778, or Ref. 62, p. 57) that the total energy density in (2.6) in this case acquires the form of its magnetic component (2.2). The differences

[3)]For simplicity we shall not treat the antisymmetric component of the overall distortion tensor, which characterizes the inhomogeneous elastic displacements in a continuous medium.[61] Apparently, in most cases taking into account the terms corresponding to it introduces no substantial changes into the phenomena in which we are interested.

will consist only in that the tensor $K^{(2)}_{ijnl}$ will be renormalized, and will now include a magnetostrictive contribution (determined by combinations of tensors quadratic in B and reciprocal in C). What we have said reflects the fact that the symmetry of the equilibrium *macroscopic* magnetic properties ("symmetry of directions") will be the same after allowing for the spontaneous magnetostrictive deformations as when we do not allow for them. This is determined by the point-group symmetry of the paramagnetic crystal.[4)]

At the same time, the symmetry of each given ground state characterized by the equilibrium values $\mathbf{m}^{(0)}$ and $u^{(0)}_{ij}$ will be spontaneously broken even for $\mathbf{H}=0$. (For example, an isotropic medium will be described by the symmetry of an axial vector, while a cubic crystal magnetized along a cube edge acquires tetragonal symmetry.)

If we know the concrete form of the energy in (2.6) for a crystal being studied, we can write in the usual way (see, e.g., Ref. 63) the coupled equations of motion for $\mathbf{M}$ and u_{ij}:

$$\dot{\mathbf{M}}=\gamma\left[\mathbf{M}\,\frac{\delta E}{\delta \mathbf{M}}\right], \tag{2.8}$$

$$\rho\ddot{u}_i=\frac{\partial}{\partial x_j}\frac{\partial E}{\partial u_{ij}}\frac{1+\delta_{ij}}{2}\,. \tag{2.9}$$

(Here $\delta/\delta\mathbf{M}$ is the variational derivative, and $\delta_{ij}=1$ or 0 respectively for $i=j$ and $i\neq j$). Upon solving these equations in the linear approximation in the small oscillations of (2.7) about the ground state $(\mathbf{m}^{(0)}, u^{(0)}_{ij})$, we find as a result the spectrum of coupled ME waves. Our fundamental problem consists in studying these spectra for crystals of different symmetries in the vicinity of magnetic phase-transition points, where the effects of SSB are most strongly manifested.

However, we must bear in mind the fact that the pattern of the observed ME phenomena can depend substantially on the presence of dissipative processes. In particular, this pertains to the close neighborhood of PT points, where the damping of oscillations can increase strongly. To estimate the role of damping, we shall respectively add to the right-hand sides of the equations of motion: for the magnetization in (2.8), a relaxation term in the Gilbert form[64]:

$$-\frac{r}{M_0}[\dot{\mathbf{M}}\mathbf{M}] \tag{2.10}$$

(here r is a dimensionless relaxation parameter), while for the deformations in (2.9) we shall add a dissipative term of the form[65]

$$\frac{\partial\sigma'_{ij}}{\partial x_j}. \tag{2.11}$$

Here $\sigma'_{ij}=\eta_{ijnl}\dot{u}_{nl}$ is the dissipative stress tensor. (The viscosity tensor η_{ijnl} has the same symmetry as the elastic-modulus tensor C_{ijnl}.)

[4)]For magnetic materials having two or more sublattices and a corresponding number of magnetizations, one must take into account also the spatial position of the symmetry elements in the unit cell of the crystal, paying attention to which permutation of the sublattices is performed by each of the elements.[62]

b) Cubic ferromagnetic material

First let us examine the simplest case of a ferromagnetic material of cubic symmetry magnetized to saturation along one of the cube edges, e.g., [001] || Z. Here (and everywhere below for other crystals) we shall not write out again the energies (2.1)–(2.4) with account taken of the concrete form of the tensors B, A, K, and C. We shall only point out the minimal set of components of these tensors, which directly converts (2.1)–(2.4) into the traditionally employed expressions for the energy of crystals of the corresponding symmetry. We can easily see that all these tensors possess symmetry properties with respect to permutation of indices that allow these indices to be combined in pairs. As is usually done in the theory of elasticity, we introduce the following notation:

$$\begin{gathered}xx\equiv 1,\quad yy\equiv 2,\quad zz\equiv 3,\ yz=zy\equiv 4,\\ zx=xz\equiv 5,\quad xy=yx\equiv 6.\end{gathered} \tag{2.12}$$

(For example, we have $K^{(1)}_x=K^{(1)}_x\equiv K^{(1)}_5$, $C_{xyxy}=C_{yxyx}=C_{xyyx}=C_{yxxy}\equiv C_{66}$).

For crystals of cubic symmetry, in particular, we have

$$\left.\begin{aligned}&A_{ij}=A\delta_{ij},\quad K^{(1)}_{ij}=K_0\delta_{ij},\quad K^{(2)}_{12}=K^{(2)}_{21}\\ &\quad=K^{(2)}_{23}=K^{(2)}_{32}=K^{(2)}_{13}=K^{(2)}_{31}=K^{(2)}_{44}=K^{(2)}_{55}=K^{(2)}_{66}\equiv\frac{1}{6}K,\\ &\quad B_{11}=B_{22}=B_{33}\equiv B_1,\quad B_{44}=B_{55}=B_{66}=\frac{1}{2}B_2,\\ &C_{11}=C_{22}=C_{33},\quad C_{12}=C_{21}=C_{13}=C_{31}=C_{23}=C_{32},\\ &\quad C_{44}=C_{55}=C_{66}.\end{aligned}\right\} \tag{2.13}$$

We can assume the remaining components to be zero.

If we are interested in the equilibrium properties, then we find from the condition of minimum ξ in (2.5), with account taken of (2.2) and (2.4), that

$$\begin{gathered}m^{(0)}_i\equiv\alpha_i=\text{const},\\ u^{(0)}_{ii}=-\frac{B_1}{C_{11}-C_{12}}\left(\alpha_i^2-\frac{C_{12}}{C_{11}+2C_{12}}\right),\quad u^{(0)}_{ij}=-\frac{B_2}{2C_{44}}\alpha_i\alpha_j\quad(i\neq j).\end{gathered} \tag{2.14}$$

Upon substituting these values of $\mathbf{m}$ and u_{ij} into (2.1)–(2.6), we obtain the following expression for the thermodynamic-potential density under the conditions of equilibrium among them:

$$E_0=\text{const}+K^*(\alpha_x^2\alpha_y^2+\alpha_y^2\alpha_z^2+\alpha_z^2\alpha_x^2)-M_0\boldsymbol{\alpha}\mathbf{H},$$

Here we have

$$K^*=K+\frac{B_1^2}{C_{11}-C_{12}}-\frac{B_2^2}{2C_{44}}\,. \tag{2.15}$$

Thus, taking account of magnetostriction in equilibrium processes renormalizes the magnetic anisotropy constant: $K\to K^*$. Here K^* amounts precisely to the anisotropy constant that one measures in static experiments (from magnetization curves or from rotational moments). It also governs the orientational magnetic PT. Thus, if the constant K^* changes sign with varying temperature at some point T^*, then the axis of easy magnetization changes upon passing through this point: when $K^*>0$ in the absence of an external field, the vector $\boldsymbol{\alpha}$ lies along one of the edges of the cube, while when $K^*<0$, it lies along one of the body diagonals.

Now let us study coupled oscillations of $\Delta\mathbf{M}$ and Δu_{ij} about a state having $\boldsymbol{\alpha}\parallel\mathbf{H}\parallel\mathbf{Z}$, in which, according to (2.14) the spontaneous deformations are

$$u_{zz}^{(0)} \equiv U_l = -\frac{(C_{11}+C_{12})B_1}{(C_{11}-C_{12})(C_{11}+2C_{12})},$$
$$u_{yy}^{(0)} = u_{xx}^{(0)} \equiv U_t = \frac{C_{12}B_1}{(C_{11}-C_{12})(C_{11}+2C_{12})}, \quad u_{ij}^{(0)} = 0 \quad (i \neq j). \tag{2.16}$$

In the special case of ME waves propagating along the direction of magnetization (i.e., with a wave vector $\mathbf{k} \| \boldsymbol{\alpha} \| Z$), we obtain the following expressions from the system (2.8), (2.9):

$$(\omega_k \pm \omega)\, m_\pm(\mathbf{k}) + ik\gamma \frac{B_2}{M_0} u_\pm(\mathbf{k}) = 0, \tag{2.17}$$
$$\frac{ikB_2}{\rho} m_\pm(\mathbf{k}) + [\omega^2 - \omega_t^2(\mathbf{k})]\, u_\pm(\mathbf{k}) = 0,$$
$$[\omega_l(\mathbf{k}) - \omega]\, u_l(\mathbf{k}) = 0. \tag{2.18}$$

Here

$$\omega_k = \frac{\gamma}{M_0}[2K + 2B_1(U_t - U_l) + Ak^2] + \gamma H \tag{2.19}$$

is the spin-wave frequency with account taken of the spontaneous deformations of (2.16) in the ground state (i.e., with account taken of the strictive ME contribution). Further,

$$\omega_t(\mathbf{k}) = s_t k \quad \left(s_t^2 = \frac{C_{44}}{\rho}\right),$$
$$\omega_l(\mathbf{k}) = s_l k \quad \left(s_l^2 = \frac{C_{11}}{\rho}\right)$$

are respectively the frequencies of transverse and longitudinal elastic waves; and

$$m_\pm(\mathbf{k}) = m_x(\mathbf{k}) \pm i m_y(\mathbf{k}) \quad \text{и} \quad u_\pm(\mathbf{k}) = u_x(\mathbf{k}) \pm i u_y(\mathbf{k})$$

are the circularly polarized Fourier components of the oscillations of the magnetization and elastic displacements.

Equation (2.18) implies that a longitudinal elastic wave with the frequency $\omega_l = s_l k$ does not interact with the spin waves, while the two independent systems of equations of (2.17) give rise to two types of coupled ME waves:

dextropolarized waves ($u - iu_y = m_x - im_y = 0$) with frequencies determined by the dispersion equation

$$(\omega_k + \omega)(\omega_t^2 - \omega^2) - \zeta\omega_0\omega_t^2 = 0; \tag{2.20}$$

levopolarized waves ($u + iu_y = m_x + im_y = 0$) with the dispersion equation

$$(\omega_k - \omega)(\omega_t^2 - \omega^2) - \zeta\omega_0\omega_t^2 = 0. \tag{2.21}$$

In Eqs. (2.20) and (2.21) we have introduced the effective ME coupling parameter

$$\zeta = \frac{\gamma B_2^2}{C_{44}M_0\omega_0} \equiv \frac{\omega_{ME}}{\omega_0} \tag{2.22}$$

and the spin-wave gap

$$\omega_0 = \frac{2\gamma}{M_0}\left(K + \frac{B_1^2}{C_{11}-C_{12}}\right) + \gamma H. \tag{2.23}$$

The frequency $\omega = \omega_0$ is the only positively definite (and non-zero) solution of the dispersion equations (2.20) and (2.21) for homogeneous oscillations ($\mathbf{k} = 0$).

The term with B_1^2 in ω_0 amounts to the ME contribution to the gap arising from the spontaneous deformations, which break the initial cubic symmetry of the crystal.

It is important to stress that, as regards equilibrium (quasistatic) properties, a cubic ferromagnetic material remains cubic even after we allow for the spontaneous deformations of (2.16), which lead only to renormalization of the cubic anisotropy constant: $K \to K^*$ (2.15). At the same time, the symmetry of the crystal is broken with regard to dynamic properties: in the case being studied with $\mathbf{M} \| \mathbf{H} \| [001]$, the ground state has tetragonal symmetry (or more exactly, the symmetry of a tetragonal ferromagnetic material). The term with B_1^2 in (2.23), that we have pointed out, directly arises from this tetragonal increment to the anisotropy.[5)]

With a view to studying the region near the PT point, let us rewrite ω_0 in (2.23) with account taken of (2.15) as follows:

$$\omega_0 = \frac{2\gamma}{M_0}\left(K^* + \frac{B_2^2}{2C_{44}}\right) + \gamma H. \tag{2.24}$$

When $H = 0$, the state with $\mathbf{M}_0 \| [001]$ becomes unstable at the point where

$$K^* = 0 \quad \text{or} \quad K + \frac{B_1^2}{C_{11}-C_{12}} = \frac{B_2^2}{2C_{44}}. \tag{2.25}$$

As we have already noted, an orientational PT occurs when K^* changes sign: from the state $\mathbf{M}_0 \| [001]$, the ferromagnetic material goes over to the state $\mathbf{M}_0 \| [111]$. Immediately at the transition point we have

$$\omega_0 = \omega_{ME} = \frac{\gamma B_2^2}{M_0 C_{44}}. \tag{2.26}$$

This is the smallest spin-wave gap, which is characteristic of the PT point, and is the ME gap in its pure form.

We stress again that, precisely because of the broken symmetry of the ground state, the magnon gap does not vanish at the PT point. It is only because of it that the dynamics (spectrum) is determined not by the "thermodynamic" *cubic* anisotropy constant K^* of (2.15), which is responsible for the statics (stability), but by the "mechanical" *tetragonal* anisotropy constant $K^* + B_2^2/2C_{44}$. It makes no difference whether the term with B_1^2 is zero or not (and together with it, the strictive deformations in the ground state). Still the equilibrium cubic constant with allowance for deformations is not K, but K^*. And in the case $B_1 = 0$, the tetragonality is introduced into the dynamics by the absence in ω_0 of (2.23) of the term $-B_2^2/2C_{44x}$ that exists in K^* in (2.15).

An external magnetic field shifts the PT point, which at $H \neq 0$ will be determined by the condition

$$\frac{2K^*}{M_0} + H = 0. \tag{2.27}$$

Yet the magnitude of the gap at this point remains as before that in (2.26).

Now let us study coupled ME waves having frequencies

[5)] Sometimes it is said that a dynamic renormalization of the cubic anisotropy constant occurs here: $K \to K + B_1^2/(C_{11} - C_{12})$, which differs from the static renormalization. However, actually this dynamic increment to K involves lowering the symmetry of the crystal. The latter proves to be different, depending on the direction of the magnetization $\mathbf{M}$ with respect to the crystallographic axes: when $\mathbf{M}_0 \| [001]$ the ground state of the lattice is tetragonal, while when $\mathbf{M}_0 \| [111]$ it will be rhombohedral, and orthorhombic when $\mathbf{M}_0 \| [110]$, etc. In the formula for the frequency ω_0, the ME increment to K also proves to depend on the direction of $\mathbf{M}$. Thus it cannot be treated as resulting from renormalization of the cubic constant K.

determined by Eqs. (2.20) and (2.21) with $k \neq 0$. The second equation gives two positive roots and one negative root, while the roots of the first equation are obtained from those mentioned merely by a sign change. The latter implies that it suffices to study the solution of only one equation, e.g., (2.21). Here the two positive roots correspond to two ME circular waves with levopolarization (L-waves), while the third, or negative root, taken as the modulus, gives the frequency of a single dextropolarized wave (D-wave).

Let us assume initially that the ME coupling parameter is $\zeta \ll 1$, which usually corresponds to states far enough from the PT point. In this case the dispersion curves are shown schematically in Fig. 4a, from which we see that the strongest interaction of the magnons with the transverse elastic waves occurs at the point of intersection of the original noninteracting branches of the spectrum, where we have

$$s_t k = \omega_k \approx \omega_0. \tag{2.28}$$

The approximate part of the equation means that hereinafter we assume that the following condition is satisfied:

$$A\omega_0 \ll s_t^2 \quad \text{or} \quad \omega_0\omega_E \ll \omega_D^2. \tag{2.29}$$

Here $\omega_E = A/a^2$ is the exchange frequency, and ω_D is a characteristic frequency of the order of the Debye frequency (a is the interatomic parameter). The relationship (2.28) is the condition for the so-called *magnetoacoustic resonance*.[1,3,63,66] The interaction removes the degeneracy, and splitting occurs at the intersection point $k = \omega_0/s_t \equiv k_0$ by the relative amount

$$\frac{\omega_I^{(\zeta)} - \omega_{II}^{(\zeta)}}{\omega_0}\Big|_{k=k_0} \approx \sqrt{2\zeta}. \tag{2.30}$$

The frequency shift of the dextropolarized wave at this same point is considerably smaller:

$$\frac{\omega_t - \omega_{III}^{(\zeta)}}{\omega_0}\Big|_{k=k_0} \approx \frac{1}{2}\zeta. \tag{2.31}$$

In the long-wavelength region of the spectrum where $\omega_t \ll \omega_0$, an approximate solution of Eq. (2.21) yields

$$\omega_I \approx \omega_0 + \omega_E (ak)^2 + \zeta \frac{\omega_D^2}{\omega_0} (ak)^2, \tag{2.32}$$

$$\omega_{II} \approx \omega_{III} \approx \omega_t \sqrt{1-\zeta}. \tag{2.33}$$

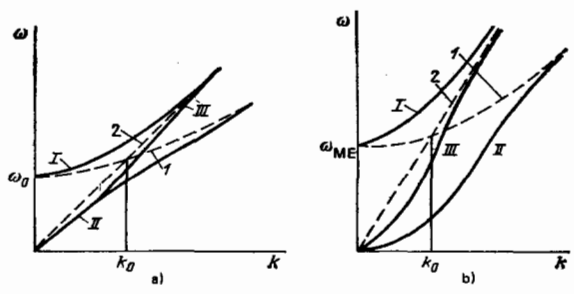

FIG. 4. Coupled ME waves with the wave vector $\mathbf{k} \parallel \mathbf{M}_0 \parallel [001]$ in a cubic ferromagnetic material. a) Far from the PT point ($\zeta \ll 1$); b) at the PT point ($\zeta = 1$). The dotted lines correspond to magnons (1) and transverse phonons (2) without taking into account the dynamic ME coupling between them, but with account taken of the spontaneous ("frozen") deformations in the ground state. Solid-lines-quasimagnons (I) and the quasiphonons (II, III).

The formula (2.33) determines the decrease in speed of quasisound (of both levo- and dextropolarization) or of the corresponding dynamic shear modulus caused by ME interaction:

$$\tilde{s}_t = s_t \sqrt{1-\zeta}, \quad \tilde{C}_{44} = C_{44}(1-\zeta). \tag{2.34}$$

Since, according to (2.22) and (2.23), the parameter ζ depends on the field H, the stated quantities also become functions of H. However, their strongest change occurs near the PT point, where $\zeta \to 1$. Here Eq. (2.34) no longer holds, and the solution of Eq. (2.21) for $\zeta = 1$ in the long-wavelength region $k \ll k_0$ yields the following frequencies:[6)]

$$\left.\begin{aligned} \omega_I &= \omega_{ME} + \frac{\omega_D^2}{\omega_{ME}}(ak)^2, \\ \omega_{II} &= \omega_E (ak)^2 \end{aligned}\right\} \quad \text{(L-waves)}, \tag{2.35, 2.36}$$

$$\omega_{III} = \frac{\omega_D^2}{\omega_{ME}}(ak)^2 \quad \text{(D-waves)}. \tag{2.37}$$

Thus, at the PT point the long-wavelength components of all three branches of the spectrum change their form most radically: for the two quasiacoustic branches the dispersion law becomes quadratic (with substantially different coefficients of k^2 for waves of the L- and D-types), while for the quasimagnon branch the coefficient of k^2 increases considerably (Fig. 4b).

We have introduced the terms "quasiacoustic" and "quasimagnon" only arbitrarily, starting with the absence or presence of a gap in the spectra, and taking into account the fact that, as $\mathbf{k} \to 0$, the frequency $\omega_I = \omega_{ME}$ corresponds to oscillations of the magnetization with "frozen" deformations. As soon as we go to a finite value of k, the arbitrariness of these names becomes evident: thus, for L-type waves the dispersion of quasimagnons in (2.35) is governed by the elastic forces (ω_D), while the dispersion of quasisound in (2.36) is governed by the exchange forces (ω_E).

In order to elucidate to what degree the oscillations pertaining to any of the branches of the spectrum affect the magnetic and elastic subsystems, one must find from Eqs. (2.17) the ratio for this branch of the corresponding amplitudes of oscillations $m_\pm$ and $u_\pm$. For the long-wavelength region ($k \ll k_0$) we have respectively for the three studied normal waves:

for quasimagnons[7)]

$$iku_- = \frac{\omega_t^2}{\omega_0^2}\frac{B_2}{C_{44}} m_-; \tag{2.38}$$

for quasiphonons

$$m_\pm = -\zeta \frac{C_{44}}{B_2}(iku_\pm). \tag{2.39}$$

In the former case (2.38) the elastic displacements u are relatively small, so that the name "quasimagnon branch" is to some degree justified, although as we have noted, its dispersion is no longer associated with

[6)] We recall that ω_{III} is the negative root of Eq. (2.21) with its modulus being used.

[7)] The intensity of the elastic oscillations can be conveniently characterized by the dimensionless quantity (ku), which defines the ratio of the displacement to the wavelength. We recall that $u_\pm$ are the dynamic components of the elastic displacements.

the exchange forces. As $\mathbf{k} \to 0$, the amplitude of the dynamic displacements u for this branch vanishes for a nonzero value of the quantity m_-, which corresponds to uniform precession of the magnetization about the state with broken symmetry.

Now let us turn to quasiphonons. According to (2.39), the amplitude of the relative oscillations of the magnetization for them (precession angle) is related as $\zeta \to 1$ to the amplitude of dynamic deformations by the large numerical coefficient C_{44}/B_2. Upon comparing (2.39) with (2.19), one can easily discern that this relationship is purely of an equilibrium type: the changes in the magnetization and the deformations for the quasiphonon modes (both of L- and D-types) are expressed in terms of one another in the same way as for equilibrium processes. What we have said allows us to understand this result as well: if one calculates the energy flux transported by these waves by the known formulas,[63] then it turns out that it is completely associated with the exchange terms in the thermodynamic potential of (2.2). Owing to the quasiequilibrium character, the fluxes associated with the elastic (2.4) and magnetoelastic (2.1) terms completely compensate one another.

The quasiequilibrium coupling between the oscillations Δu_{ij} and $\Delta\mathbf{M}$ for quasiphonons in the long-wavelength region $k \ll k_0$ (Fig. 4b) enables one to study the anomaly of the corresponding elastic modulus C_{44} near a phase transition point from a thermodynamic standpoint. Upon determining from the conditions $\partial E/\partial m = 0$ and $\partial E/\partial m_y = 0$ (with $m_z^2 = 1 - m_x^2 - m_y^2$) the changes in the magnetization caused by the shear deformations u_{xz} and u_{yz} (corresponding to the transverse circularly polarized waves in which we are interested), with account taken of the deformations (2.16) in the ground state, we have

$$m_x = -B_2\left(K + \frac{B_1^2}{C_{11}-C_{12}}\right)^{-1} u_{xz}, \quad m_y = -B_2\left(K + \frac{B_1^2}{C_{11}-C_{12}}\right)^{-1} u_{yz}. \tag{2.40}$$

Upon substituting these values of m_x and m_y into the thermodynamic potential E of (2.6), we can then find the elastic modulus renormalized with respect to E_M and E_{ME}:

$$\tilde{C}_{44} = \frac{1}{4}\frac{\partial^2 E}{\partial u_{xz}^2} = \frac{1}{4}\frac{\partial^2 E}{\partial u_{yz}^2}. \tag{2.41}$$

The result will be expressed by a formula of the form of (2.34), which corresponds to the vanishing of the modulus of $\tilde{C}_{44}$ as defined according to (2.41) at the PT point, where $\zeta = 1$.

Owing to the mixing of the oscillations $\Delta\mathbf{M}$ and Δu_{ij}, the quasiphonon mode can be excited either by sound or by a magnetic field. Here, whereas the deformations are "frozen" for the quasimagnon mode in the long-wavelength limit, on the contrary, for the quasiphonon mode the deformations follow the oscillations of the magnetization in quasiequilibrium fashion. In this sense the "frozen lattice" model holds for the quasimagnons, and the "free-lattice" model for the quasiphonons (although even for the latter the spontaneous deformations in the ground state also must be taken into account—this involves precisely the term with B_1^2 in the denominator of the formulas of (2.40)).

The results that we have presented can be applied in the special case to an isotropic ferromagnetic material if we set

$$K = 0, \quad B_1 = B_2 \text{ and } C_{11} - C_{12} = 2C_{44}. \tag{2.42}$$

We note that this state also can be treated as a PT point: dependent on the field H (for H = 0), the pressure, or the temperature (at values of these at which the isotropy is violated).

Finally we must bear in mind that damping has not at all been taken into account in the treatment of ME phenomena presented above [terms (2.10) and (2.11) in the equations of motion]. However, it is more appropriate to evaluate the role of damping using the example of ferromagnetic materials of the "easy-plane" type, for which the ME effects in which we are interested have actually been observed.

c) Ferromagnetic materials of the "easy-plane" (EP) type

As we have already noted, the real systems in which SSB effects have been observed, and in particular, the ME gap in the magnon spectrum, are uniaxial (e.g., hexagonal) ferromagnetic materials with magnetization lying in the basal plane, for which the magnetic anisotropy is rather small ("easy plane"). These ferromagnetic materials (EPFMs) include the rare-earth metals dysprosium and terbium at temperatures below certain critical temperatures Θ_{cr}, which are respectively equal to 87 K and 219 K. A first-order PT occurs at the temperature $T = \Theta_{cr}$, whereby for $T > \Theta_{cr}$ the ferromagnetic structure is converted into an antiferromagnetic helicoidal structure. However, the latter in fields H greater than a critical field $H_{cr}(T)$ that depends on the temperature again becomes ferromagnetic (see, e.g., Refs. 67, 12, 20, and 21). Important features of these ferromagnets are giant magnetostriction ($\Delta l/l \sim B/C \sim 10^{-3}$–$10^{-2}$) and a large uniaxial magnetic anisotropy $K_3^{(1)} \approx 6 \times 10^8$ erg/cm^3, which confines the magnetization to the basal plane.

The hexagonal anisotropy in the basal plane for single crystals of Dy and Tb, which determines the equilibrium direction of magnetization in this plane, also proves to be very high (at $T = 4.2$ K we have $K^{(3)} \approx 10^6$–10^7 erg/cm^3). Therefore we must take into account the corresponding terms in the anisotropy energy, although they are sixth order in the components of the magnetization. The giant magnetostriction can give rise to a considerable magnetostrictive contribution to $K^{(3)}$. For a systematic derivation of the latter, in the expansion of the magnetoelastic energy in powers of the deformations u_{ij}, we must take into account the second-order terms along with the linear terms, as in (2.1). This somewhat complicates the mathematics and the formulas that are derived for hexagonal ferromagnetic materials. But we are mainly interested in the qualitative features of the ME effects in EPFMs, which can be treated using the simpler example of ferromagnetic materials of tetragonal symmetry.

In a system of coordinates with the Z axis lying along the tetragonal axis and the X and Y axes along the edges

[100] and [010] of the basal face, only the following constants (with account taken of the notation (2.12) for pairs of indices) can remain as nonzero constants determining the energy in (2.6): $B_{11}=B_{22}$, $B_{21}=B_{12}$, B_{33}, $B_{44}=B_{55}$, B_{66}, $K_3^{(1)}\equiv K$, $K_{66}^{(2)}=K_{12}^{(2)}\equiv K_\square/12$, $C_{11}=C_{22}$, C_{33}, $C_{12}=C_{21}$, $C_{13}=C_{23}=C_{31}=C_{32}$, $C_{44}=C_{55}$, and C_{66}. [Everywhere hereinafter we shall describe the inhomogeneous exchange by a single constant A, as in the case of (2.13), thus neglecting the anisotropy of the tensor A_{ij}, which is inessential for our problems.]

For the sake of definiteness, let us study a saturation state with magnetization lying in the basal plane along the edge [100], so that $\mathbf{M}_0 \| \mathbf{H} \| \mathbf{X}$. Upon determining again the spontaneous deformations $u_{ij}^{(0)}$ in this state and calculating the spectrum of small oscillations $\Delta\mathbf{M}$ and Δu_{ij} about it, we obtain the following results.

First let the wave vector $\mathbf{k}$ also lie along the X axis (i.e., along the equilibrium magnetization $\mathbf{M}_0$). In this case the dispersion equation of the coupled ME waves has the form

$$(\omega^2-\omega_l^2)\,[(\omega^2-\omega_k^2)\,(\omega^2-\omega_{t1}^2)\,(\omega^2-\omega_{t2}^2)-\zeta_{6k}\omega_k^2\omega_{t1}^2\,(\omega^2-\omega_{t2}^2) -\zeta_{4k}\omega_k^2\omega_{t2}^2\,(\omega^2-\omega_{t1}^2)-\zeta_{4k}\zeta_{6k}\omega_k^2\omega_{t1}^2\omega_{t2}^2]=0; \tag{2.43}$$

here

$$\omega_k=\gamma\sqrt{\left(\frac{A}{M_0}k^2+H+H_A+H_{ME4}\right)\left(\frac{A}{M_0}k^2+H+H_\square+H_{ME6}\right)} \tag{2.44}$$

is the frequency of the spin waves without taking into account the dynamic ME coupling (yet allowing for the spontaneous deformations in the ground state), while

$$\begin{aligned}\omega_l&=s_l k\left(\mathbf{e}_l\|\mathrm{X},\ s_l=\sqrt{\frac{C_{11}}{\rho}}\right),\\ \omega_{t1}&=s_{t1}k\left(\mathbf{e}_{t1}\|\mathrm{Y},\ s_{t1}=\sqrt{\frac{C_{66}}{\rho}}\right),\\ \omega_{t2}&=s_{t2}k\left(\mathbf{e}_{t2}\|\mathrm{Z},\ s_{t2}=\sqrt{\frac{C_{44}}{\rho}}\right)\end{aligned} \tag{2.45}$$

are the frequencies of longitudinal and transverse acoustic waves when unperturbed by the ME interaction. The directions of the polarization vectors and the velocities of these waves are indicated in the parentheses. In (2.44) we have introduced the effective static magnetic-anisotropy fields H_A and $H_\square$ as renormalized with allowance for magnetostriction, and also the ME fields H_{ME4} and H_{ME6}:

$$H_A=\frac{2K^*}{M_0}, \tag{2.46}$$

$$H_\square=\frac{K_\square^*}{M_0}=M_0^{-1}\left(K_\square+\frac{2B_{11}^2}{C_{11}-C_{12}}-\frac{4B_{66}^2}{C_{66}}\right), \tag{2.47}$$

$$H_{ME4}=\frac{4B_{44}^2}{C_{44}M_0},\quad H_{ME6}=\frac{4B_{66}^2}{C_{66}M_0}. \tag{2.48}$$

In Eqs. (2.46) and (2.47) we have written out the explicit form of the anisotropy constants renormalized by magnetostriction ("starred") only for the constant K^*, which determines the anisotropy in the basal plane, since the phase transition for the EPFM in which we are interested involves precisely this constant. In contrast to a cubic crystal, two parameters play a role here that determine the ME coupling of the waves:

$$\zeta_{4k}=H_{ME4}\left(\frac{A}{M_0}k^2+H+H_A+H_{ME4}\right)^{-1}, \tag{2.49}$$

$$\zeta_{6k}=H_{ME6}\left(\frac{A}{M_0}k^2+H+H_\square+H_{ME6}\right)^{-1}. \tag{2.50}$$

Let us present directly the solutions of (2.43) for the normal waves in the long-wavelength approximation. Namely, when

$$k\ll\frac{\omega_0}{s_{t1}}\ \text{and}\ \frac{\omega_0}{s_{t2}} \tag{2.51}$$

we obtain

$$\omega_I^2=\omega_k^2+\zeta_{6k}\omega_{t1}^2+\zeta_{4k}\omega_{t2}^2, \tag{2.52}$$

$$\omega_{II}\equiv\omega_l=s_l k\quad(\mathbf{e}_{II}\|\mathbf{k}), \tag{2.53}$$

$$\omega_{III}=s_{t1}k\sqrt{1-\zeta_{6k}}\quad(\mathbf{e}_{III}\|\mathrm{Y}), \tag{2.54}$$

$$\omega_{IV}=s_{t2}k\sqrt{1-\zeta_{4k}}\quad(\mathbf{e}_{IV}\|\mathrm{Z}). \tag{2.55}$$

The condition of minimal total energy implies that a uniaxial (tetragonal in this case) ferromagnetic material is an EPFM if the effective uniaxial anisotropy field H_A in (2.46) satisfies the inequality

$$H_A>0. \tag{2.56}$$

Here, as usual, we have $H_A\gg H_{ME4}$, and hence, $\zeta_{4k}\ll 1$, while the magnon gap is

$$\omega_0\equiv\omega_k|_{k=0}\approx\gamma\sqrt{(H+H_A)\,(H+H_\square+H_{ME6})}. \tag{2.57}$$

Consequently it turns out that a transverse acoustic wave at the frequency $\omega_{IV}\approx\omega_{t2}$ actually does not interact with magnons to the given approximation, just like a longitudinal wave at the frequency $\omega_{II}=\omega_l$. Only the transverse waves polarized linearly along Y can experience strong coupling with magnons. For these waves the corresponding coupling parameter $\zeta_6\equiv\zeta_{6k|k=0}$ can be of the order of unity with a sufficiently small anisotropy in the basal plane ($H_\square$).

We note that an orientational PT occurs upon sign change of $H_\square$ (depending on the temperature for $H=0$): when $H_\square>0$, the easy axis is an edge of the basal square (X and Y axes), while when $H_\square<0$ it is a diagonal. Here, as in a cubic ferromagnetic material, the maximal ME coupling with $\zeta_6\to 1$ will occur at the very transition point where $H_\square=0$. An analogous transition with ζ_6 going to unity can arise also from an external field. Thus, in the case being studied with $\mathbf{H}\|\mathbf{X}$, if $H_\square<0$ (so that the stated axis is the hard axis in the basal plane) the field-dependent PT point is $H=|H_\square|$.

All that we have said above about the ME effects for a cubic crystal near PT points as $\zeta\to 1$ holds also for an EPFM (minimal nonzero ME gap for quasimagnons, decrease in the velocity of sound of the magnon-associated acoustic branch until its dispersion law changes from linear to quadratic). However, a feature of the latter case is that the large uniaxial magnetic anisotropy (the field H_A) here can substantially increase the ME gap ω_{ME}, as happens for the rare-earth metals Dy and Tb mentioned above. Concomitantly the frequency range is increased in which the long-wavelength approximation holds [Eqs. (2.52)–(2.55)]. According to these formulas, in this case at the PT point, i.e., when $H=|H_\square|$, we have the following frequencies of the coupled ME waves respectively for quasimagnons and quasiphonons:

$$\omega_I\approx\omega_{ME}+\frac{\omega_E\omega_A+\omega_D^2}{2\omega_{ME}}(ak)^2 \tag{2.58}$$

and

$$\omega_{III}\approx\sqrt{\frac{\omega_E\omega_A\omega_D^2}{\omega_{ME}^2}}(ak)^2. \tag{2.59}$$

Here the ME gap is

$$\omega_{ME} \equiv \omega_0^{(min)} = \gamma \sqrt{(H_A + |H_\square| + H_{ME4})\, H_{ME6}}\,. \qquad (2.60)$$

Also $\omega_E = \gamma A/M_0 a^2$ and $\omega_A = \gamma H_A$ are the effective frequencies of the exchange interaction and the magnetic anisotropy.

The orientation of the wave vector $\mathbf{k} \parallel \mathbf{M}_0$ that we have been studying up to now (both in cubic and in tetragonal crystals) is the most favorable one for ME coupling of waves. The point is that in this case, as Eqs. (2.3) imply, the dipole (magnetostatic) contribution to the frequency of the spin waves vanishes. In order to elucidate the role of the dipole field, let us also present the approximate expressions for the frequencies of the long-wavelength [i.e., under the condition (2.51)] ME oscillations with the wave vector $\mathbf{k}$ lying in the basal plane at an arbitrary angle ϕ_k to the magnetization $\mathbf{M}_0 \parallel \mathbf{H} \parallel \mathbf{X}$:

$$\omega_I \approx \omega_k = \gamma \sqrt{\left(\frac{A}{M_0} k^2 + H + H_A + H_{ME4}\right)} \times \sqrt{\left(\frac{A}{M_0} k^2 + H + H_\square + 4\pi M_0 \sin^2 \varphi_k + H_{ME6}\right)}, \qquad (2.61)$$

$$\omega_{II,\,IV} = k \left\{C_{11} + \tilde{C}_{66} \mp [(C_{11} - \tilde{C}_{66})^2 - \sin^2 2\varphi_k (C_{11} + C_{12})(C_{11} - C_{12} - 2\tilde{C}_{66})]^{1/2}\right\}^{1/2} \frac{1}{\sqrt{2\rho}} \qquad (2.62)$$

$$(\mathbf{e}_{II} \parallel \mathbf{k},\ \mathbf{e}_{IV} \perp \mathbf{k},\ \mathbf{Z}),$$

$$\omega_{III} = k \sqrt{C_{44} \sin^2 \varphi_k + \tilde{C}_{44} \cos^2 \varphi_k}\, \frac{1}{\sqrt{\rho}} \quad (\mathbf{e}_{III} \parallel \mathbf{Z}). \qquad (2.63)$$

Here we have

$$\tilde{C}_{44} = C_{44}(1 - \zeta_{4k}), \quad \tilde{C}_{66} = C_{66}[1 - \zeta_6(\varphi_k)],$$
$$\zeta_6(\varphi_k) = \frac{\omega_{ME6}^2}{\omega_k^2} = H_{ME6}\left(\frac{Ak^2}{M_0} + H + H_\square + 4\pi M_0 \sin^2 \varphi_k + H_{ME6}\right)^{-1}. \qquad (2.64)$$

[Here ζ_{4k} is given by the former expression (2.49)]. Thus, when $\phi_k \neq 0$, all three acoustic branches of the oscillations (including the longitudinal elastic waves) prove to be coupled with the magnons. However, here the effectiveness of ME coupling is diminished by the dipole interaction, since the parameter of this coupling $\zeta_6(\phi_k) < 1$, even at the PT point (i.e., when $H + H_\square = 0$). And when $\phi_k = \pi/2$,

$$\zeta_6\left(\frac{\pi}{2}\right)\Big|_{k=0} = \frac{H_{ME6}}{4\pi M_0 + H_{ME6}} \qquad (2.65)$$

has a magnitude that is usually small in comparison with unity (perhaps except for ferromagnetic materials with giant magnetostriction).

We note that the formulas (2.52)–(2.55) or (2.61)–(2.63) given above for the spectrum of ME oscillations can also be applied for a ferromagnetic material of the "easy-axis" type (when $H_A < 0$, so that the basal plane is the plane of hard magnetization) whenever the magnitude of the field H (as before directed along the X axis) satisfies the condition $H > |H_A|$. Then the point $H + H_A = 0$ is the PT point with $\zeta_4 \to 1$.

Up to now we have not taken into account the damping of the magnetoelastic oscillations. At the same time, as we have already noted, it increases strongly near the PT point, and then the above-described pattern of phenomena can be altered substantially. For example, the damping can prove to be so strong that it no longer makes sense to speak of quasiacoustic waves with an altered dispersion law (from linear to quadratic).

Taking into account the corresponding relaxation terms (2.10) and (2.11) in the equations of motion (2.8) and (2.9), we find, in particular, the damping for the quasiacoustic mode of (2.54), which interacts most strongly with the spin waves. It is convenient here to define the damping coefficient as the ratio of the imaginary and real components of the complex wave vector $\tilde{k} = k + i\Gamma$:

$$\varkappa = \frac{\Gamma}{k} \equiv \frac{\lambda}{2\pi\delta}. \qquad (2.66)$$

Here $\lambda = 2\pi/k$ is the wavelength, and $\delta = 1/\Gamma$ is the effective damping length for the mode being studied.

A calculation for the long-wavelength region (2.51) yields

$$\varkappa \approx \frac{\omega}{2(1 - \zeta_{6k})}\left(\frac{\eta_{66}}{C_{66}} + \zeta_6 \frac{\tau}{\omega_0}\right). \qquad (2.67)$$

For the PT point (i.e., as $\zeta_6 \to 1$), we obtain the following expression from Eq. (2.67) taking (2.59) into account and assuming, in view of the smallness of $\omega_0^{(min)} = \omega_{ME}$, that the fundamental contribution to $\varkappa$ arises from magnetic damping:

$$\varkappa = \frac{1}{2} \tau \frac{\omega_D}{\sqrt{\omega_E \omega_A}}. \qquad (2.68)$$

Although usually $\tau \ll 1$, nevertheless, depending on the relationship of the other parameters in (2.68), the damping coefficient $\varkappa$ can be either smaller or larger than (or of the order of) unity. In the latter cases, quasiacoustic waves at the frequency ω_{III} (2.59) are absent. We note that a large uniaxial anisotropy constant K favors a decrease in $\varkappa$. Perhaps this occurs in the rare-earth EPFMs (Dy and Tb).

The point is that FMR is observed[17,18] in these ferromagnetic materials with the magnetization along the hard direction of the basal plane in the UHF region (namely, at $\omega \approx 10^{11}$ s^{-1}). Moreover, according to (2.60) the minimum value of this frequency (for $H = |H_\square|$) must be at least an order of magnitude larger (since $H_A \approx 10^6$ Oe and $H_{ME} \approx 10^4$ Oe). We can assume[20] that in the cited experiments the UHF field actually excites the quasiacoustic waves ω_{III} (2.54) with a wavelength of the order of the depth of the skin effect. In direct excitation of these waves by ultrasound in terbium, it was possible to obtain[22] a decrease in the effective dynamic modulus $\tilde{C}_{66}$ [Eq. (2.64)] by 50% near the PT point. (A more detailed test of this formula was hindered by the impossibility of maintaining acoustic contact with the specimen as the PT point was approached).[8)]

3. ANTIFERROMAGNETIC MATERIALS

a) General status

The antiferromagnetic state is described by the relative antiferromagnetism vectors

[8)] One can easily find $\tilde{C}_{66}$ at the PT point itself:

$$\tilde{C}_{66} = C_{66} \frac{\omega \sqrt{\omega_E \omega_A}}{\omega_D \omega_{ME}}.$$

$$\mathbf{l}=\frac{\mathbf{M}_1-\mathbf{M}_2}{2M_0} \tag{3.1}$$

and the resultant magnetization

$$\mathbf{m}=\frac{\mathbf{M}_1+\mathbf{M}_2}{2M_0}. \tag{3.2}$$

The moduli of the magnetizations of the sublattices $\mathbf{M}_1$ and $\mathbf{M}_2$ are usually assumed to be conserved quantities: $\mathbf{M}_1^2 = \mathbf{M}_2^2 = M_0^2$, which is equivalent to the relationships

$$\mathbf{m}^2 + \mathbf{l}^2 = 1, \quad \mathbf{ml} = 0. \tag{3.3}$$

In a state of equilibrium (and in the absence of a magnetic field H) we have $\mathbf{M}_1 = -\mathbf{M}_2$, and the vector $\mathbf{l}$ has the maximal value $|\mathbf{l}| = 1$, while $\mathbf{m} = 0$.

The magnetization $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$ that appears in oscillations of $\mathbf{M}_1$ and $\mathbf{M}_2$ (and also upon magnetization with an external field that is small in comparison with the exchange fields) is always so small that we can assume that

$$\mathbf{m}^2 \ll 1 \quad (\text{and hence, } \mathbf{l}^2 \approx 1). \tag{3.4}$$

In view of what we have said, when writing out the energy density of an antiferromagnetic material, including the magnetoelastic, magnetic, and elastic contributions, just as for a ferromagnetic material we must be guided by the following rules:

1) The magnetoelastic energy is written in the form of Eq. (2.1) with the substitution $\mathbf{m} \to \mathbf{l}$.

2) One obtains from (2.2) the terms for inhomogeneous exchange and magnetic anisotropy by the same substitution; here the terms for the Zeeman and the magnetostatic energy must be left unchanged, as in (2.2).

3) After this, the magnetic energy E_M must be supplemented with the term

$$\frac{1}{2} A_0 m^2 \tag{3.5}$$

—the so-called *homogeneous* exchange energy, which gives rise to the fact that the ground (homogeneous) state is antiferromagnetic with $\mathbf{m} = 0$ and $l \approx 1$ (for $A_0 > 0$).

4) In antiferromagnetic materials with weak ferromagnetism (in which in the ground state a spontaneous magnetization $\mathbf{m} \neq 0$ satisfying the condition (3.4) can exist along with $\mathbf{l}$) one must also take into account in the magnetic energy E_M terms of the form

$$d_{ij} m_i l_j. \tag{3.6}$$

Here the explicit form of the tensor d_{ij} is determined by the magnetic structure and the symmetry of the crystal.[68,62]

After determining the ground state $(\mathbf{l}^{(0)}, \mathbf{m}^{(0)}, u_{ij}^{(0)})$ from the condition of minimum total energy, we can then find the spectrum of coupled ME waves of the antiferromagnetic material by solving the equation (2.9) of the theory of elasticity simultaneously with the equations of motion of the form (2.8) for the magnetization $\mathbf{m}_\alpha$ ($\alpha = 1, 2$) of each of the sublattices.

b) Orthorhombic antiferromagnetic materials—orthoferrites

It is convenient to start the study of SSB effects in antiferromagnetic materials with the rare-earth orthoferrites—compounds of the type of $ErFeO_3$.[42] In the past decade the orthoferrites have been widely studied owing to the existence of weak ferromagnetism in them. For us they are interesting for their orientational PTs, near which effects due to SSB are manifested most clearly and consequently have been detected experimentally.[69,38-41]

In line with the orthorhombic symmetry of the orthoferrites (and with account taken of the rules formulated in the preceding subsection), we can describe their energy with:

the homogeneous and inhomogeneous exchange constants A_0 and A_1; the magnetic-anisotropy constants $K_1^{(1)}$, $K_3^{(1)}$, $K_{11}^{(2)}$, $K_{33}^{(2)}$, and $K_{13}^{(2)} = K_{31}^{(2)}$; the magnetoelastic constants B_{11}, B_{12}, B_{13}, B_{21}, B_{22}, B_{23}, B_{44}, B_{55}, and B_{66}; the elastic constants C_{11}, C_{12}, C_{13}, C_{21}, C_{22}, C_{23}, C_{44}, C_{55}, and C_{66}. In addition, the energy contains terms of the form[62]

$$d_{13} m_x l_z - d_{31} m_z l_x, \tag{3.7}$$

which are responsible for the weak ferromagnetism.

Our further discussion will deal with the concrete case of the orthoferrites $ErFeO_3$ and $TmFeO_3$, in which orientational magnetic PTs are observed: at low temperatures below some point T_1 (e.g., for erbium orthoferrite $T_1 = 88.5$ K), we find the antiferromagnetism vector $\mathbf{l} \| \mathbf{c}$ (Z axis), while at high temperatures above a point T_2 (for erbium orthoferrite $T_2 = 98.1$ K), we find the vector $\mathbf{l} \| \mathbf{a}$ (X axis). In the temperature interval $T_1 \leq T \leq T_2$ (Fig. 5), the vector $\mathbf{l}$ gradually rotates in the XZ plane from one of these directions to the other. At the same time, the spontaneous weak ferromagnetic moment $\mathbf{m}$, being perpendicular to $\mathbf{l}$, rotates in this same plane from $\mathbf{a}$ to $\mathbf{c}$, so that the angle θ between $\mathbf{m}$ and $\mathbf{Z}$ varies from $\pi/2$ to 0. We shall call the states indicated above the low-temperature, intermediate, and high-temperature states.

Study of the equilibrium states by minimizing the total energy of the orthoferrite (for $H = 0$) shows the following.

The rotation of the vectors $\mathbf{l}$ and $\mathbf{m}$ described above (see Fig. 5) without leaving the XZ plane indicates that the axis $\mathbf{Y} \| \mathbf{b}$ is the hardest axis for the vector $\mathbf{l}$. Approximately (neglecting fourth-order anisotropy, including the magnetostrictive contribution to it), the latter holds under the conditions $K_1^{(1)} < 0$ and $K_3^{(1)} < 0$.

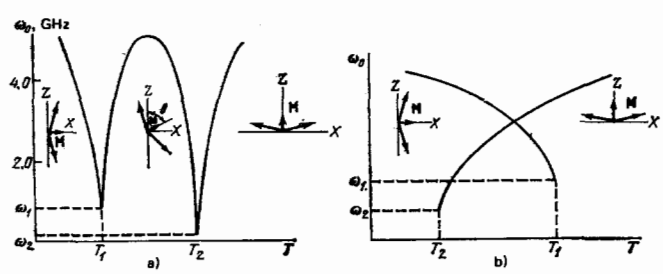

FIG. 5. Orinetational PTs and the temperature-dependence of the AFMR frequency (magnon gap for $\mathbf{k} \| \mathbf{Z}$) in orthoferrites. a) Second-order PT ($T_2 > T_1$); b) first-order PT ($T_2 < T_1$). Here we have $\omega_1 \equiv \omega_0\ (T_1) = \gamma\sqrt{H_E(H_{ME5} + H_M)}$, $\omega_2 \equiv \omega_0\ (T_2) = \omega_{ME5}$.

The orientational transition itself is governed by the temperature-dependence of the two effective magnetic-anisotropy constants:

$$K_{\rm I}=K_3^{(1)}-K_1^{(1)}+2K_{13}^{(2)}-2K_{11}^{(2)}+d_{31}(d_{21}-d_{13})\frac{1}{A_0}+K_{\rm I}^{\rm ME}, \quad (3.8)$$

$$K_{\rm II}=K_{11}^{(2)}-2K_{13}^{(2)}+K_{33}^{(2)}-(d_{31}-d_{13})^2\frac{1}{A_0}+K_{\rm II}^{\rm ME}. \quad (3.9)$$

The last terms in (3.8) and (3.9), whose explicit forms we shall not present, determine the magnetostrictive renormalization of the thermodynamic constants $K_{\rm I}$ and $K_{\rm II}$.[42]

The spins reorientation shown in Fig. 5a is realized in the case when $K_{\rm II}>0$. Here the three equilibrium states, whose boundaries are second-order PT points, correspond to the following conditions:

1) For the low-temperature state ($T\leqslant T_1$) with $\theta=\pi/2$ and $m=m_x\approx d_{13}/A_0$, we have

$$K_{\rm I}(T)\leqslant -2K_{\rm II}(T). \quad (3.9a)$$

2) For the intermediate state ($T_1\leqslant T\leqslant T_2$) with $\sin^2\theta=-K_{\rm I}/2K_{\rm II}$ we have

$$-2K_{\rm II}(T)\leqslant K_{\rm I}(T)\leqslant 0. \quad (3.9b)$$

3) For the high-temperature state ($T\geqslant T_2$) with $\theta=0$ and $m=m_z\approx d/A_0$, we have

$$K_{\rm I}\geqslant 0. \quad (3.9c)$$

In the other case when $K_{\rm II}<0$, the regions of existence of the low- and the high-temperature states overlap (Fig. 5b), so that $T_2<T_1$. Then the system can exist in two equilibrium states:

$$\theta=\pi/2\ (T\leqslant T_1),\ \text{when}\ K_{\rm I}(T)\leqslant -2K_{\rm II}(T); \quad (3.10a)$$

$$\theta=0\ (T\geqslant T_2),\ \text{when}\ K_{\rm I}(T)\geqslant 0. \quad (3.10b)$$

The transition from the one to the other state occurs by a first-order PT. The temperature T_1 and T_2 are the points of loss of stability.

Now let us present the results of calculating the coupled ME waves.

First we shall study a state with $\theta=0$ (which is realized under the conditions (3.9c) or (3.10b), respectively, for $K_{\rm II}>0$ or $K_{\rm II}<0$) in which waves are propagating with the wave vector $\mathbf{k}\parallel\mathbf{Z}\parallel\mathbf{m}$. In this case the interacting waves prove to be one of the two branches of the spin waves (namely, the lower-frequency branch, for which the oscillations of the vector $\mathbf{l}$ occur in the XZ plane) and one of the two transverse sound waves with polarization $\mathbf{e}_t\parallel\mathbf{X}\parallel\mathbf{a}$. The frequencies of these spin and sound waves (without taking into account the dynamic interaction between them) are respectively determined by the formulas

$$\omega_k=\gamma[H_{\rm E}(A_0M_0^{-1}k^2+K_{\rm I}M_0^{-1}+H_{\rm ME5})]^{1/2}. \quad (3.11)$$

$$\omega_t=\sqrt{C_{55}/\rho}\,k\equiv s_a k. \quad (3.12)$$

Here $H_{\rm E}=A_0/2M_0$ is the exchange field, and $H_{\rm ME5}=8B_{55}^2/M_0C_{55}$ is the ME field.

Further, if we take into account the dynamic ME coupling, we obtain the following expression for the frequencies of the coupled waves:

$$\omega_{\rm I,\,II}^2=\frac{1}{2}[\omega_k^2+\omega_t^2\pm\sqrt{(\omega_k^2-\omega_t^2)^2+4\omega_t^2\omega_{\rm ME5}^2}]. \quad (3.13)$$

Here $\omega_{\rm ME5}=\gamma\sqrt{H_{\rm E}H_{\rm ME5}}$ is the characteristic ME frequency, which corresponds to the ME field of the effective magnetic anisotropy $H_{\rm ME5}$. The essential point is that in antiferromagnetic materials this field enters into the frequency in a geometric-mean combination with the exchange field $H_{\rm E}$. This considerably enhances the ME gap (and other SSB effects) as compared with ferromagnetic materials. The plus sign in Eq. (3.13) corresponds to the branch having the gap

$$\omega_{\rm I}(k=0)=\gamma\sqrt{H_{\rm E}(K_{\rm I}M_0^{-1}+H_{\rm ME5})}. \quad (3.14)$$

Thus the latter can be viewed as the quasimagnon gap. The minus sign corresponds to the gapless branch $\omega_{\rm II}(k=0)=0$, so that the latter amounts to the quasiacoustic branch.

The term containing $K_{\rm I}$ in the quasimagnon gap in (3.14) corresponds to the orthorhombic anisotropy of the equilibrium state as renormalized by magnetostriction. The term containing $H_{\rm ME5}$, which does not fit within the framework of this renormalization, again is the SSB effect. As the temperature is lowered, at the spin-reorientation point T_2 where $K_{\rm I}=0$, the state with $\theta=0$ becomes quasidegenerate (when $K_{\rm II}>0$) or unstable (when $K_{\rm II}<0$). Here the quasimagnon gap of (3.14) has a minimum value, which is determined by the ME interaction:

$$\omega_0^{(\min)}=\omega_{\rm ME5}. \quad (3.15)$$

In the long-wavelength limit when $ak\ll\omega_0/\omega_E$ and $\omega_0/\omega_{\rm D}$, where $\omega_{\rm E}=\gamma\sqrt{A_0A}/M_0a$ is the exchange frequency (including the parameters of both homogeneous and inhomogeneous exchange), while $\omega_{\rm D}$ is again the Debye frequency, we obtain the following expression for the quasiacoustic mode from (3.13) to the accuracy of fourth-order terms in k:

$$\omega_{\rm II}^2\simeq\omega_{\rm D}^2(ak)^2(1-\zeta_5)+\zeta_5^2\frac{\omega_{\rm D}^2}{\omega_0^2}[\omega_{\rm E}^2-\omega_{\rm D}^2(1-\zeta_5)](ak)^4. \quad (3.16)$$

Here

$$\zeta_5=\frac{\omega_{\rm ME5}^2}{\omega_0^2} \quad (3.17)$$

is the dimensionless ME-coupling parameter. At the PT point itself, where $\zeta_5=1$, we have

$$\omega_{\rm II}\approx\frac{\omega_{\rm D}\omega_{\rm E}}{\omega_{\rm ME5}}(ak)^2. \quad (3.18)$$

Thus, again as we approach the PT point, the velocity of sound $\tilde{s}=\omega_{\rm II}/k$ (for the mode being studied with polarization $\mathbf{e}_t\parallel\mathbf{X}$) strongly declines until it vanishes as $k\to 0$ while $\zeta_5\to 1$.

Now let us study waves in the same direction $\mathbf{k}\parallel\mathbf{Z}$ for the low-temperature state ($T\leqslant T_1$; see Fig. 5) in which $\theta=\pi/2$, i.e., $\mathbf{m}\perp\mathbf{k}$. The frequencies of the coupled ME waves in this state will be determined as before by an expression of the form of (3.13), but only with another value of the frequency of the magnons interacting with the sound:

$$\omega_k^2=\gamma^2H_{\rm E}[AM_0^{-1}k^2-(K_{\rm I}+2K_{11})M_0^{-1}+H_{\rm M}+H_{\rm ME5}]. \quad (3.19)$$

Here $H_{\rm M}=8\pi M_0d_{13}^2/A_0^2$ is the magnetostatic field associated with the oscillations of the spontaneous weak-ferromagnetic moment. Owing to the existence of $H_{\rm M}$ at the transition point T_1, where $K_{\rm I}+2K_{\rm II}=0$, we have the parameter $\zeta_5=H_{\rm ME5}/(H_{\rm ME5}+H_{\rm M})<1$ (in contrast to

the point T_2, at which $\zeta_5 = 1$). Thus the decrease in $\tilde{s}_a$ at the point T_1 will be limited by the dipole-dipole interaction. In general, this should be manifested in a smaller change in the velocity of transverse sound near T_1 than near T_2. Certain differences of this type have been observed experimentally[39-41] (Fig. 6). However, since the effects in which we are interested were relatively small ($\lesssim 1\%$) under the conditions of the experiments,[39-41] there are as yet hardly sufficient grounds for associating them with the influence of the dipole-dipole interaction.

We shall not present here the dispersion formulas for the intermediate state with $0 \leqslant \theta \leqslant \pi/2$, which is realized when $T_1 \leqslant T \leqslant T_2$ (the case $K_{II} > 0$), but refer the reader to the appropriate original study.[42] We note only that in this state the low-frequency spin-wave branch interacts not only with the transverse, but also with the longitudinal acoustic wave. Curve 3 in Fig. 6 shows the relative variation of the velocity of transverse sound with temperature in the intermediate state.

In order to elucidate the role of dissipation, we shall write out the final result of calculating[42] the velocity of transverse quasisound at the frequency $\omega \equiv \omega_{II}$ (3.18) and the damping coefficient for it as $\zeta_5 \to 1$ ($T \to T_2$) in the long-wavelength approximation, which corresponds to the condition $\omega \ll \omega_{ME5}$:

$$\tilde{s}_a \approx s_a \sqrt{\frac{\omega\omega_E}{\omega_{ME5}\omega_D}}(1 - 2\varkappa_t^2), \tag{3.20}$$

$$\varkappa_t \approx \left(\frac{r\gamma H_E}{\omega_{ME5}}\right)\frac{\omega_D}{\omega_E} \tag{3.21}$$

(again we have omitted the viscoelastic terms).

If the relaxation parameter r does not depend (or depends weakly) on the frequency, then the quantity $r\gamma H_E$ will determine the width of the AFMR line (in the given case at the frequency $\omega_0^{(min)} = \omega_{ME5}$). Consequently the first factor in (3.21) will constitute the relative magnitude of the width $\Delta\omega_r/\omega_0$ of the AFMR line. If the latter is small in comparison with unity, then the damping coefficient $\varkappa_t$, while reaching a maximum at the PT point T_2, may still have a rather small value, so that one might speak of the existence of quasiacoustic ME waves at this point. Here, as we have noted above [and as can now be seen from Eq. (3.20)], the velocity $\tilde{s}_a$ of this quasisound in the low-frequency region that we are studying ($\omega \ll \omega_{ME5}$) must sharply decline as we approach the point T_2 as compared with its value s_a far from this point.

The features of behavior that we have described of the velocity of sound near points of orientational PTs agree qualitatively with experiment in erbium and thulium orthoferrites.[38-42] Quantitative comparison requires further experiments, since the processing of the data in Refs. 39–41 was based on using formulas that do not take into account the ME gap in the quasimagnon spectrum and are thus inapplicable in the immediate vicinity of PT points. In particular, it is of interest to test the square-root dependence of the velocity $\tilde{s}_a$ on the frequency ω of quasisound predicted by Eq. (3.20).

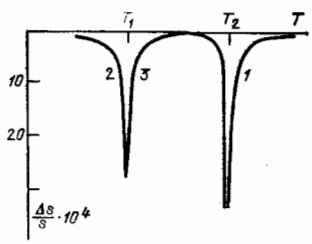

FIG. 6. Relative variation of the velocity of transverse sound with the temperature in $ErFeO_3$ for $\mathbf{k} \parallel Z$.

We should note that the regularities pointed out above for the AFMR frequency and the velocity of sound near points of orientational PTs actually must be manifested only in the immediate vicinity of these points. Since the decrease in the velocity of sound in the cited studies[39-41] amounted to no more than 1%, this probably meant that the authors did not succeed in approaching close enough to the PT points. A field-dependent PT would be more convenient, and in this respect is more stretched-out (see the next subsection c). Apparently an analogous situation is involved in the considerably larger magnitude (and even in the qualitative temperature course itself) of the AFMR frequency near the points T_1 and T_2 that was obtained experimentally in Ref. 70, as compared with its value as calculated directly for the points themselves. The point is that the authors of Ref. 70, as we see it, performed the measurements with an insufficiently small temperature increment. The results should also be very sensitive to the accuracy of positioning the crystal with respect to the direction of propagation of sound.

c) Antiferromagnetic materials of the "easy-plane" type. Pressure-dependent phase transition

As we have already noted, SSB effects are most noticeable and have been studied best in antiferromagnetic materials of the *EP* type, such as hematite (α-Fe_2O_3) and iron borate ($FeBO_3$). Variations of the velocity of sound of the order of 20% have been obtained in them.[25-28] Moreover, the effect of pressure (more exactly, unilateral stress) on the AFMR frequency has been studied experimentally[25,28] and theoretically[29,30] in hematite, and thus a pressure-dependent PT has actually been examined.

Crystals of hematite and iron borate have rhombohedral symmetry with a very small anisotropy in the basal *XY* plane, which we shall neglect for the sake of simplicity. Here it suffices to retain one term containing $K_3^{(1)}$ in the magnetic-anisotropy energy, while the magnetoelastic and elastic energies will be described by the same sets of constants as for an orthorhombic antiferromagnetic material (subsection 3b), upon which we must impose the extra conditions $B_{11} = B_{13} = B_{12} + 2B_{66}$, $B_{13} = B_{23}$, $B_{44} = B_{55}$, $C_{11} = C_{22} = C_{12} + 2C_{66}$, $C_{13} = C_{23}$, and $C_{44} = C_{55}$.

Moreover, in states of the EP type, these antiferromagnetic materials have a weak ferromagnetism, which arises from the energy in (3.6) of the form $d\,(l_x m_y - l_y m_x)$.

The state of a uniaxial antiferromagnetic material will be an EP-type state (so that at equilibrium the magnetization vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ will lie in the basal *XY*

plane) if we have the anisotropy constant $K_3^{(1)} > 0$. Let the directions $\mathbf{M}_1$ and $\mathbf{M}_2$ in this plane be given respectively by the azimuthal angles ϕ_1 and ϕ_2 (Fig. 7). Then, according to (3.1)–(3.3), the half-sum of these angles $\phi = (\phi_1 + \phi_2)/2$ determines the direction of the vector m, while their half-difference $\psi = (\phi_1 - \phi_2)/2$ determines the modulus of this vector: $m^2 = \cos^2\psi$.

Now let us assume that a magnetic field H and a mechanical stress $\sigma'_{xx} = -P$ are applied along the X direction (which, owing to the isotropy in the basal plane, can be chosen arbitrarily in it). The mechanical stress compresses the crystal when $P > 0$ and stretches it when $P < 0$.

In the presence of the stated unilateral pressure P, we must add to the total thermodynamic-potential density E of (2.6) a term of the form Pu_{xx}. Now let us introduce the effective pressure-associated field:

$$H_P = \frac{2B_{66}P}{C_{66}M_0}. \tag{3.22}$$

Then we can find from the minimum of E that the following two equilibrium states can exist in the system, depending on the magnitude of H_P:

1. At pressures for which $H_P \leqslant H_P^*$, where

$$H_P^* = \frac{H_d(H + H_d)}{H_E}$$

$$\left(H_d = \frac{d}{M_0},\ H_E = \frac{A_0}{M_0}\right), \tag{3.23}$$

the vector m is parallel to $\mathbf{H} \parallel \mathbf{X}$ ($\phi = 0$). Here we have $\Delta\psi = (\pi/2) - \psi = (H + H_d)/H_E$, and hence, $m = (H + H_d)/H_E$.

2. At pressures $P > 0$, for which $H_P > H_P^*$,

$$\cos\varphi = \frac{HH_d}{H_E(H_P - H_P^*) + HH_d} \text{ and } \Delta\psi = \frac{H_P}{H}\cos\varphi. \tag{3.24}$$

Hence, an orientational PT dependent on the pressure P occurs at the point $H_P = H_P^*$; as P increases further the vector m rotates from the direction $\mathbf{m} \parallel \mathbf{X}$ to the direction $\mathbf{m} \parallel \mathbf{Y}$ (reaching the latter when $H_P - H_P^* \gg HH_d/H_E$).

In the case $P < 0$ (stretch), only state 1 is realized.

We note that an analogous orientational phase transition can depend on the field H at a given pressure $P > 0$. Thus, we can pass through it in the reverse order with increasing H: at $H = 0$ we have $\mathbf{m} \parallel \mathbf{Y}$ according to (3.24). Then m rotates from $\phi = \pi/2$ to $\phi = 0$ in the interval

$$0 \leqslant H \leqslant H^* = \sqrt{\frac{H_d^2}{4} + H_E H_P} - \frac{H_d}{2}. \tag{3.25}$$

After this is achieved (owing to the overcoming of the anisotropy in the XY plane created by the unilateral pressure P), we return at $H \geqslant H^*$ to state 1 with $\mathbf{m} \parallel \mathbf{H}$.

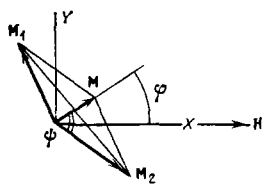

FIG. 7. Magnetizations of the sublattices in an antiferromagnetic material and the angles defining their configuration. Here ϕ defines the direction of the vector $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$, while ψ defines its modulus $M = 2M_0\cos\psi$.

Let us examine first coupled ME waves propagating along the field direction: $\mathbf{k} \parallel \mathbf{H} \parallel \mathbf{X}$.

One can show that also in this case the sound interacts most strongly with the low-frequency branch of the spin waves (corresponding to oscillation of the spins in the EP), whose frequency is respectively determined for the two above-mentioned states by the expressions:

1. $H_P \leqslant H_P^*$:

$$\omega_k^2 = \gamma^2 H_E (AM_0^{-1}k^2 + H_{ME6} + H_P^* - H_P). \tag{3.26}$$

2. $H_P \geqslant H_P^*$:

$$\omega_k^2 = \gamma^2 H_E \left[AM_0^{-1}k^2 + H_{ME6} + \frac{H_E H_P - H^2 + HH_d}{H_E H_P - H^2}(H_P - H_P^*) + H_M \sin^2\varphi \right]. \tag{3.27}$$

Here we have

$$H_M = \frac{8\pi M_0 (H + H_d)^2}{H_E^2}, \quad H_{ME6} = \frac{8B_{66}^2}{C_{66}M_0}.$$

In these expressions the field H_{ME6} takes into account the "frozen" spontaneous deformations, while H_P takes into account the deformations associated with the pressure P. Although in state 1 a compressive pressure ($P > 0$) decreases the spin-wave gap ω_0, the latter is not reduced to zero. This is because a phase transition from state 1 (Fig. 8a) to state 2 precedes this as P increases when the effective field H_P reaches the value H_P^* (3.23). With further increase in P (now in state 2), the gap again increases, and not only directly owing to the term containing H_P in (3.27), but also owing to the dipole term that appears when the angle ϕ between m and $\mathbf{H} \parallel \mathbf{k}$ becomes different from zero, in line with (3.24). Here the minimum gap, which is reached at the phase-transition point at $H_P = H_P^*$ (3.23) again is of ME type:

$$\omega_0^{\min} \equiv \omega_{ME6} = \gamma\sqrt{H_E H_{ME6}}. \tag{3.28}$$

Thus, actually the ME gap cannot be abolished by pressure as well.

As we see from Fig. 8b, the dependence of ω_0 on the magnetic field H (at a given pressure P) has an analogous form with a minimum (3.28) at the phase-transition point $H = H^*$ (3.25). Curve 1 corresponds to the case $P = 0$ (when $H^* = 0$), and curve 2 to the case $P \neq 0$. The following result is highly characteristic: in the second case at a given frequency in the interval $\omega_{ME6} < \omega < \omega_{0|H=0}$, AFMR must occur at two values of the field H.[9] This appearance of an additional resonance upon

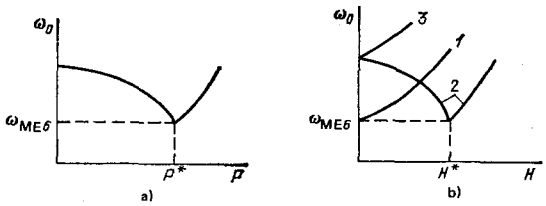

FIG. 8. The AFMR frequency as a function of the unilateral pressure $P > 0$ (compression) in the field $\mathbf{H} \parallel \mathbf{P}$ (a), and as a function of the field H (b). 1—$P = 0$; 2—$H_P > 0$, $\mathbf{H} \parallel \mathbf{P}$ or $H_P < 0$, $\mathbf{H} \perp \mathbf{P}$; 3—$H_P > 0$, $\mathbf{H} \perp \mathbf{P}$ or $H_P < 0$, $\mathbf{H} \parallel \mathbf{P}$.

[9] We should note that, strictly speaking, the frequency of homogeneous AFMR is not the limit $\omega_k|_{k\to 0}$ obtained from Eq. (3.27), since the latter corresponds to an infinite medium, whereas homogeneous AFMR is defined by the condition that the wavelength is large in comparison with the dimensions of the specimen. For a spherical specimen the AFMR frequency is obtained from ω_0 for $H_M = 0$.

applying unilateral pressure in the EP has been observed experimentally in Ref. 28.

One can represent the frequencies of the coupled ME waves for the two states ($H_P \leqslant H_P^*$ and $H_P > H_P^*$) by general formulas, which have the following form in the long-wavelength limit $k \ll \omega_0/s_t$:

$\omega_I^2 = \omega_k^2 + \zeta_{6k}\omega_t^2$—quasimagnon branch;

$\omega_{II} = \tilde{s}_t k$—transverse quasiacoustic branch polarized in the EP;

$\omega_{III} = \tilde{s}_l k$—longitudinal quasiacoustic branch;

Transverse sound polarized along the symmetry axis of the crystal interacts weakly with the magnons. Here we have introduced the notation

$$\tilde{s}_t^2 = \rho^{-1}(C_{11} - \zeta_{6k}C_{66}\sin^2 2\varphi), \quad \tilde{s}_l^2 = \rho^{-1}C_{66}(1-\cos^2 2\varphi), \quad \zeta_{6k} = \frac{\omega_{ME6}^2}{\omega_k^2}. \tag{3.29}$$

Here the angle ϕ is zero for state 1 ($H_P \leqslant H_P^*$) and is determined by Eq. (3.24) for state 2 ($H_P \geqslant H_P^*$). Correspondingly, the frequency ω_k is given by Eqs. (3.26) or (3.27).

The theoretical dependence on H of the velocity of the long-wavelength (as $k \to 0$) quasiacoustic waves in hematite in the presence of the unilateral pressure P is given in Fig. 9.[30] The experimental data[25-28] at $P = 0$ for the velocity of sound in α-Fe_2O_3 and $FeBO_3$ as a function of H fit well the theoretical curve $\tilde{s}(H)$.

At the PT point (with $H_P = H_P^*$ or $H = H^*$) where $\zeta_6 \to 1$ and $\tilde{s}_t$ (as $k \to 0$) vanishes, the dispersion law for ω_{II} again changes from linear to quadratic; here $\omega_{II}(k)$ takes on the form (3.18) with the substitution $\omega_{ME5} \to \omega_{ME6}$ and $C_{55} \to C_{66}$.

The formulas (3.20) and (3.21) for the velocity of quasisound and its damping at the phase-transition point for an orthorhombic crystal are also applicable in this case upon taking into account the indicated substitution.

Further, we note that qualitatively the properties of the ME waves for an EP-type antiferromagnetic material are analogous to those for an EP-type ferromagnetic material (Sec. 2c). The fundamental difference is that in the antiferromagnetic material all the effective magnetic-anisotropy fields (including H_{ME} and H_P) are intensified by the effective homogeneous-exchange field $H_E = A_0/M_0$, rather than by the uniaxial anisotropy field H_A. Consequently, for example, a formula of the form

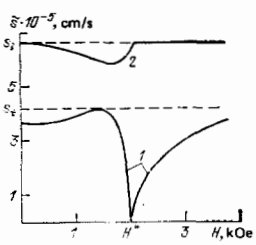

FIG. 9. Variation of the velocity of transverse (*1*) (polarized in the basal plane) and longitudinal (*2*) quasielastic waves in hematite with $\mathbf{L} \perp \mathbf{Z}$ and $\mathbf{k} \parallel \mathbf{H} \parallel \mathbf{P} \parallel \mathbf{X}$. $H_P > 0$. $P \approx 1$ kbar.

(3.18) (with the substitution $5 \to 6$) for ω_{II} in an antiferromagnetic material can be derived from the corresponding formula (2.59) for a ferromagnetic material by the simple substitution $H_A \to H_E$. Of course, we must bear in mind here the fact that for the ferromagnetic material we have allowed for the natural crystallographic anisotropy in the EP, which is described by the effective field $H_\square$, while here this anisotropy (the field H_P) is imposed by the external unilateral stress. Moreover, the exchange frequency ω_E is determined in different ways in the two cases.

The interrelation of the frequencies of the ME waves that we have noted for EP-type ferro- and antiferromagnetic materials is also conserved in the more general case in which the wave vector **k** makes some angle $\phi_k \neq 0$ with the magnetization $\mathbf{m} \parallel \mathbf{H} \parallel \mathbf{X}$ (we are referring to state 1). The corresponding formulas for an antiferromagnetic material in the long-wavelength limit are analogous to those for a ferromagnetic material (2.61)–(2.64), with only the distinction that here we have

$$\omega_k^2 = \gamma^2 H_\Delta [AM_0^{-1}k^2 + H_{ME6} + H_P^* - H_P + 8\pi M_0 (H + H_d)^2 H_E^{-1}\sin^2\varphi_k], \tag{3.30}$$

$\zeta_{4k} = 0$, and in $\zeta_6(\phi_k)$, the ME frequency ω_{ME6} is determined by Eq. (3.28).

In closing we must call attention to the important role of exchange intensification in observing SSB effects in antiferromagnetic materials. For example, let us compare ferromagnetic materials with EP-type antiferromagnetic materials. In both cases the magnitude of the effects is characterized by the ME coupling parameter ζ_6, which is determined, in line with (2.50) and (3.29), by the formulas

$$\text{(FM)} \quad \zeta_6 = H_{ME6}(H + H_\square + H_{ME6})^{-1}, \tag{3.31}$$

$$\text{(AFM)} \quad \zeta_6 = H_E H_{ME6}(H^2 + H_E H_{ME6})^{-1} \tag{3.32}$$

(assuming for simplicity that $H_P = H_d = 0$).

Of course, at the PT point itself (respectively with $H + H_\square = 0$ and $H = 0$), where the effects are maximal, we have $\zeta_6 = 1$ in both cases. However, in view of the smallness of the effective field H_{ME} (usually $H_{ME} \leqslant 1$ Oe), an experimentally important factor is the width of the region of variation of the field H in the vicinity of the PT within which ζ_6 declines to values $\zeta_6 \ll 1$ (at which SSB effects vanish).

For example, for $H_{ME} = 1$ Oe we find that a twofold decline in ζ_6 for a ferromagnetic material occurs when the field H deviates from the PT point by only 1 Oe. For well-known reasons, the observation of effects that arise in such a small field interval proves difficult (or totally impossible).

At the same time, for an antiferromagnetic material with the same value of H_{ME}, the exchange field H_E (equal to 10^6 Oe or more) causes the interval of variation of the field H in which ζ_6 declines by half to amount to 10^3 Oe. This explains why the SSB effects have been studied more in antiferro- than in ferromagnetic materials (apart from ferromagnetic materials with giant magnetostriction like Dy and Tb).

4. CONCLUDING REMARKS

In this article we have treated ME effects of broken symmetry in unlimited homogeneous (single-domain) magnetic materials, albeit only in a linear approximation (in the oscillation amplitude). We have shown that effects of interaction of spin and elastic waves that are relatively small under ordinary conditions are considerably enhanced and actually acquire a qualitatively new character near magnetic PT points.

Apparently from the practical standpoint studies of the quasiacoustic mode of coupled ME waves are of special interest: this is due not only to its pronounced weakening in the vicinity of the PT, but also to the fact that it is equally elastic and magnetic (and in particular, can be excited by a magnetic field). The latter situation and the potentiality of controlling the velocity of these waves with a magnetic field, elastic stresses, or the temperature must be of interest to specialists in electronics (for example, for purposes of designing delay lines regulated by the stated agents). Unfortunately, there are as yet very few experimental studies along this line. Such studies require sufficiently good single crystals (most likely antiferromagnetic ones) with a large magnetostriction, "convenient" magnetic PTs, and as small as possible a width of the AFMR line (the latter is dictated by the desire to diminish damping, which impedes the excitation and observation of quasisound).

Even greater prospects are promised by studies of magnetoacoustic effects that extend beyond the framework of the restrictions pointed out above. We must make some remarks on this topic.

First of all, nonlinear magnetoacoustic phenomena in magnetic crystals are of great interest (especially in antiferromagnetic materials, where again they are intensified by the exchange interaction). As has been shown,[32,33] the ME interaction renormalizes not only the velocity of sound or the harmonic elastic moduli (second-order), but also the third-order anharmonic elastic moduli: $\tilde{C}_3 = C_3 + \Delta C_3$. Here the ME contribution to the anharmonicity (ΔC_3) can exceed considerably the intrinsic strain contribution (C_3). Thus, it has been shown experimentally[71-73] that for thulium orthoferrite and for hematite $\Delta C_3/C_3$ can attain values of the order of 10^2–10^4. For this reason, even with relatively small deformations in the primary wave ($u_{ij} \sim 10^{-6}$), considerable nonlinear effects arise, such as parametric excitation of sound by sound, frequency-doubling of sound, acoustic detection, etc.[33] (For other SSB effects, see also Refs. 74–76).

Further, we note that everywhere above we have been dealing with effects caused by spontaneous breaking of *symmetry of directions* with a homogeneous spatial distribution of magnetic moments in the ground state of a ferro- or antiferromagnetic material. At the same time, magnetic materials contain highly mobile (with an unstable spatial distribution) inhomogeneous structures—domains and domain boundaries, which break the *translational symmetry*. Generally they correspond to inhomogeneous strictive deformations in the ground state. In magnetically soft (with a large magnetic susceptibility) magnetic materials, and also for certain special magnetic structures, e.g., an isolated domain boundary (or domain), the ground state of the system can be quasidegenerate with respect to displacements of the domain boundaries (or domains) together with the inhomogeneous deformations that "clothe" them. This will correspond to quasiacoustic (Goldstone) oscillations of the domain boundaries. At the same time, the breaking of translational symmetry in the presence of a continuously degenerate ground state can lead in addition under certain conditions to the existence of quasilocal (or resonance) modes of oscillations of the domain boundary with respect to the "frozen" inhomogeneous deformations created by it. This field of magnetoacoustics has arisen very recently and is in a stage of accelerated development (interacting with the problem of magnetoelastic solitons). Therefore we refer the reader to the original studies.[58-59,77-79]

Now we should turn our attention to the implicit assumption that we have adopted from the very outset that we are dealing with specimens of sufficiently large dimensions. The characteristic linear dimension L of a specimen affected by spontaneous deformations that determine the ME gap ω_{ME} must be large enough that the minimum frequency of elastic vibrations $\omega_L = sL^{-1}$ corresponding to it (which we can identify with the characteristic "decay frequency" of the spontaneous deformations) is small in comparison with the frequency of precession of the magnetization in the effective magnetic field of these deformations ω_{ME}. In other words, the following inequality must be satisfied to allow the existence of the effect of a finite ME gap in a finite specimen:

$$\omega_L < \omega_{ME}. \tag{4.1}$$

This is precisely the condition that the deformations are "frozen" for the quasimagnon mode as $k \to 0$.[58,77]

When we take into account the above-mentioned intensification effects for ω_{ME}, the condition (4.1) can practically always be satisfied, both for massive ferromagnetic materials (and all the more so for antiferromagnetic materials) and for thin films. The point is that in the latter case, according to the conditions of experiment, the characteristic dimension L is usually not the thickness of the film, but a dimension in its plane. Nevertheless, the condition (4.1) must be tested for each concrete design of an experiment.

But the condition (4.1) becomes especially important in treating ME oscillations of domain boundaries, since the role of L in this case is played by their effective thickness (the linear dimensions of the transition layer between domains). The inequality (4.1) actually is the condition for existence of the above-mentioned quasilocal ME oscillations of the domain boundary.

And finally another important remark. In treating ME phenomena near PT points, we have completely neglected fluctuations, although, as is known, the role played by the latter at these points can increase sharply. However, the possibility of neglecting this for a spin-reorientation-type PT, which was the main topic

above in our article, has been shown in a number of studies (see, e.g., Refs. 80 and 69).

In closing we note the close analogy between the anomalies of the dynamic elastic constants that exist in orientational magnetic PTs with which we have been dealing here and those in cooperative PTs of the Jahn-Teller type.[81] An even closer analogy exists between the phenomena that we have examined that are caused by SSB in magnetic materials and the corresponding effects in the case of structural transformations in ferroelectric materials (effects of pseudospin-phonon coupling; see, e.g., Ref. 82).

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