Heavy quarks

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Research on the physics of heavy quarks over the past five years is reviewed. The results are discussed from a unified point of view based on quantum chromodynamics.

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CONTENTS

INTRODUCTION

The study of heavy quarks is exceedingly important to high-energy physics today. Suffice it to say that the quark model itself was finally confirmed only after the discovery of heavy quarks. This discovery had a dramatic effect on all subsequent experimental and theoretical research. It stimulated rapid progress in such originally distinct directions as quantum chromodynamics and the model of the electroweak interaction. It is not by chance that the discovery of the first heavy quark (the charmed or c quark[1]) was rewarded with a Nobel Prize.[2]

The initial results on the c quark are reported in some other reviews,[3,4] with which we assume the reader is familiar. There is also some useful information in Refs. 5–12, where related topics are reviewed, but familiarity with these other reviews is not necessary for following the discussion below.

Many new results have been obtained over the past two or three years: The charmonium family has been substantially expanded, a heavier quark has been discovered (the "beauty," or b, quark[13]; see also Ref. 8), and direct evidence of gluons is being studied. It has accordingly become necessary to put all this information in systematic form and to discuss the most recent developments from a common point of view. In this review we work from a general theoretical picture which has now acquired some completely clean outlines: the picture based on quantum chromodynamics.

The first of the family of new particles, the J/ψ, was discovered only eight years ago,[1] but already that event seems like ancient history. A tremendous distance has been traveled from the pioneering paper by Appelquist and Politzer,[14] who deciphered the nature of charmonium, to the highly refined methods available today. It was initially believed that charmonium could very well prove to be the hydrogen atom of the physics of strong interactions; if so, then a significant part of hadron physics could be related to the spectroscopy of charmonium, just as molecular spectra were related to the spectrum of hydrogen.[15] In a sense, but not literally, this prediction has proved to be true.

We now have a quantitative description of all aspects of heavy quarkonium. The accuracy of the description varies, since we still lack a complete solution of the confinement problem. In general, we do understand how this system is constructed, and we understand it well enough that we can frequently use it as a probe for strong interactions in the same way that we use colliding e^+e^- beams. This is a unique probe since it yields direct information on the properties of the gluonic medium filling the physical vacuum, while the more conventional probes, γ^* and W, are associated with quarks. Furthermore, heavy quarkonium is an excellent arena for studying weak interactions, including their most fundamental manifestations (Higgs bosons, axions, ...; see Ref. 10, for example).

In speaking of a theory for heavy quarkonium we should bear in mind that this theory has not yet matured to a level comparable to the theory of the hydrogen atom. Several "standard" approaches have been worked

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out, but so far no one has been able to travel the entire distance from the fundamental Lagrangian of quantum chromodynamics to real experimental numbers (masses, widths, etc.) without appealing to some additional assumptions. The closest we come to a fundamental chromodynamics is the so-called sum-rule method based on a picture of a complex nonperturbative structure of the quantum-chromodynamics vacuum. Unfortunately, this method is not completely universal. The nonrelativistic potential model, which actually goes back to the pioneering paper by Appelquist and Politzer,[14] is extremely popular. On occasion, old recipes and models—bags, the quark—hadron duality, etc.—are used, but with new meaning. We will briefly describe the theoretical tools used to analyze heavy quarkonium, and we will discuss the specific facts. We will naturally be focusing on the recent results, and we will attempt to single out the key results.[1)] Singling out the key results is of course a subjective process. In particular, we will discuss the physics of e^+e^- collisions, while we will not take up such questions as the photoproduction, hadron-production, and neutrino-production of heavy quarks. The interested reader can consult some other reviews.[23,24,28] Regarding the purely theoretical side of the problem we might add that we will not stress the nonrelativistic potential model which has served as a basis for most of the papers on heavy quarks. Our reasoning is that some excellent reviews[29] have been written on this model by specialists in the field (see Ref. 30 for a summary of recent results).

In addition to some questions dealing directly with heavy quarkonium, we will discuss some related problems in which quarkonium might be said to be serving as a staging area for the theoretical assault.

We will begin the review with a summary of the basic experimental facts which have been obtained over the past five years (Section 1). Our intention is to furnish the reader that information which is important from the theoretical standpoint. These results are interpreted theoretically in Sections 2–5, which are the meat of this review. We will also take up such questions as the mass spectrum of the heavy particles, their leptonic and hadronic decays, and effects associated with the weak interaction. Section 5 discusses the applicability of the methods of quantum-chromodynamics perturbation theory to processes involving heavy quarks. The expected properties of toponium—a system constructed from a sixth and as yet undiscovered t quark—are discussed briefly in Section 6.

Finally, in Section 7 we list those directions in theoretical and experimental research which appear at the moment to be the most promising.

[1)]The experimental facts which we have used are based primarily on the results reported to the Lepton—Gamma Symposium in Bonn in 1981 and the Twenty-First International Conference on High-Energy Physics[25-27] (Paris, 1982). As a rule, we have cited in the text only those results which appeared too late to be included in the most recent addition of the PDG tables.[31]

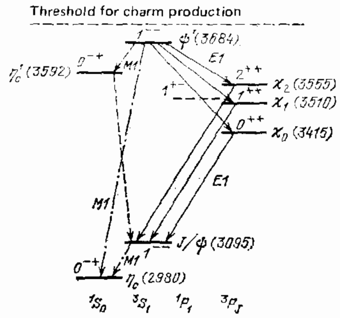

FIG. 1. Levels of charmonium and radiative transitions between these levels. Solid lines—E1 transitions; dot-dashed lines—observed M1 transitions; dashed lines—observed levels and transitions.

1. BASIC EXPERIMENTAL FACTS

a) c quarks

Over the six years which have elapsed since the publication of Vaĭnshteĭn's review[4] the experimental physics of c quarks has continued to develop rapidly. Research on e^+e^- collisions has taken the following basic directions: 1) detailed analysis of radiative transitions between levels of charmonium; 2) searches for missing levels (in particular,[2)] $1^1S_0, 2^1S_0, {}^1P_1, \ldots$); 3) study of the radiative decays of the J/ψ meson and a search for new hadronic states lying below J/ψ; 4) study of the properties of hadrons with unconcealed charm.

Important progress in research on the properties of charmonium can be credited to the use of the Crystal Ball (CB) neutral detector on the SPEAR installation at Stanford. The Crystal Ball is essentially a spherical shell of NaI(Tl) crystals which can measure γ rays highly accurately [the resolution in terms of the γ-ray energy E_γ is $\Delta E_\gamma/E_\gamma = 2.7\%/E_\gamma^{1/4}$ (the units are GeV), and the angular resolution is 1–2°]. The total statistical base acquired by the Crystal Ball is $\sim 2.2\cdot 10^6$ J/ψ mesons and $\sim 1.8\cdot 10^6$ ψ' mesons. The Crystal Ball has recently been shipped to Hamburg for measurements on the DORIS installation.

1) *Charmonium*

Figure 1 shows the known levels of charmonium lying below the threshold for $D\bar{D}$ production, along with the radiative transitions between these levels.

1.1) *ψ mesons.* Above the threshold for charm production in the $e^+e^- \to$ hadrons cross section the resonance ψ'' (3770) has been discovered. It is called a "D-meson factory" ($M_{\psi''} - 2M_{D^0} \approx 40$ MeV). Several other resonant structures have been observed beyond ψ''. The characteristics and tentative quantum numbers of these states are listed in Table I. For completeness we are also showing here the characteristics of the J/ψ and ψ' mesons. In particular, this table shows some precise measurements of their masses which were taken at

[2)]We are using the spectroscopic notation $(n_r+1)^{(2S+1)}L_J$, where n_r is the radial quantum number (the number of zeros in the radial wave function).

TABLE I. The properties of J/ψ, ψ′, and higher-lying vector states $c\bar{c}$ corresponding to resonances in the cross section $\sigma(e^+e^- \to \text{hadrons})$.

	Mass, MeV	$n^{2S+1}L_J$ *)	Γ, MeV	$\Gamma(\psi \to e^+e^-)$, keV
J/ψ	3096.93±0.09 **)	1^3S_1	0.063±0.009	4.6±0.4
ψ′	3686.0±0.10[32]	2^3S_1	0.215±0.040	2.05±0.2
ψ″	3770±3	1^3D_1	25±3	0.26±0.05
ψ(4030)	4030±5	3^3S_1	52±10	0.75±0.10
ψ(4160)	4159±20	2^3D_1	78±20	0.77±0.23
ψ(4415)	4415±6	4^3S_1	43±20	0.49±0.13

*)Here and below (Table VII) the quantum numbers are chosen from the potential-model calculations.[29,23] We note that $n = n_r + 1$. See footnote[2)].

**)Here and below, the particular group which carries out the measurement is cited only where its result is the most accurate or where there is a substantial discrepancy between the results reported by different groups. Otherwise, we are giving world-wide average experimental results.

Novosibirsk[32] through the use of a novel method involving a resonant depolarization of the $e^\pm$ beams.

1.2) *C-even states.* These states have been studied in the inclusive spectra of γ rays from the decays of ψ′ and J/ψ and also in the reactions $J/\psi \to \gamma +$ hadrons, $J/\psi \to 3\gamma$, $\psi' \to \gamma +$ hadrons, and $\psi' \to 2\gamma + J/\psi$. Five states have now been reliably established: three ^3P_J levels[33] (χ_0, χ_1, and χ_2) and two states of paracharmonium, $\eta_c(2980)$ (Refs. 34 and 35) and $\eta_c'(3592)$ (Ref. 36). The latter have been identified as 1^1S_0 and 2^1S_0 levels. Figure 2 shows the inclusive γ spectrum in the radiative decay of[16] ψ′.

1.3) *The decays* $\psi' \to J/\psi + \eta$ *and* $\psi' \to J/\psi + \pi^0$. An auxiliary but extremely interesting result which has emerged from research on cascade radiative transitions between ψ′ and J/ψ is the observation of the decays $\psi' \to J/\psi + \eta$ and $\psi' \to J/\psi + \pi^0$. The latter decay breaks the isotropic symmetry, and the ratio of the widths corresponding to these decays permits a "direct" measurement of the current masses of quarks.

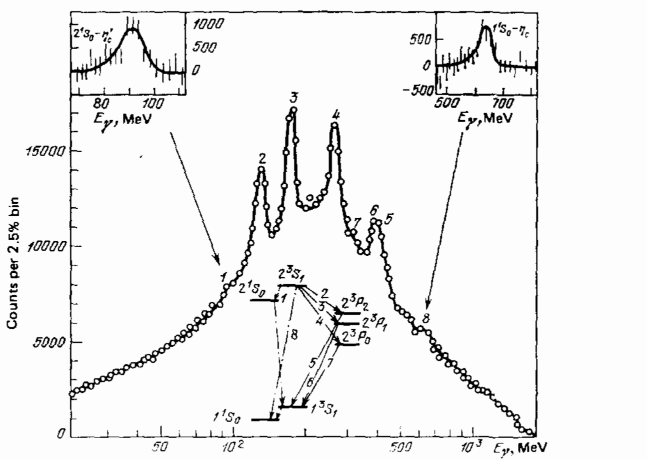

FIG. 2. Inclusive γ spectrum in the radiative decay of ψ′ as measured by the Crystal Ball group.[16] The observed peaks are compared with radiative transitions between the levels of charmonium. The distributions near the η_c and η_c' resonances are shown at the top (the background has been subtracted).

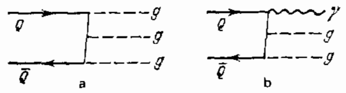

FIG. 3. Diagrams corresponding to the quarkonium transitions $Q\bar{Q} \to 3g$ (a) and $Q\bar{Q} \to \gamma gg$ (b).

The Crystal Ball group undertook an effort to detect the elusive $^1P_1$ ($J^{PC} = 1^{+-}$) level of charmonium from the decay $\psi' \to \pi^0\,(^1P_1)$ and the cascade[37] $\psi' \to \pi^0\,(^1P_1)$. They found the following limitations (95% confidence level) for the expected range of masses $M(^1P_1)$ [see Eq. (2.20)]:

$$\mathrm{BR}\,(\psi' \to \pi^0\,{}^1P_1) < 0.42\%,\ \mathrm{BR}\,(\psi' \to \pi^0\,{}^1P_1) \times \mathrm{BR}\,({}^1P_1 \to \gamma\eta_c) < 0.20\%\ (\text{with}\, M\,({}^1P_1) = 3.50 - 3.515\,\text{GeV}), \quad (1.1)$$

$$\mathrm{BR}\,(\psi' \to \pi^0\,{}^1P_1) < 0.55\%,\ \mathrm{BR}\,(\psi' \to \pi^0\,{}^1P_1)\,\mathrm{BR}\,({}^1P_1 \to \gamma\eta_c) < 0.14\%\ (\text{with}\, M\,({}^1P_1) = 3.515 - 3.525\,\text{GeV}). \quad (1.2)$$

2) *The decays* $J/\psi \to \gamma +$ *light hadrons*

This is a gluon-physics analog of the famous e^+e^- annihilation. In quark-gluon terms we would say

$$Q\bar{Q} \to gg\gamma,\ gg \to \text{light hadrons} \quad (1.3)$$

(the two-gluon system is in a colorless state).

In varying the energy of the γ ray we are simultaneously varying the invariant mass of the hadron system:

$$m^2_{\text{light hadr.}} = M^2(1-x),\quad x = \frac{2E_\gamma}{M}.$$

In lowest-order quantum-chromodynamics perturbation theory, it follows from a comparison of the diagrams in Fig. 3 that[38]

$$\delta_\gamma = \frac{\Gamma_{\gamma gg}}{\Gamma_{3g}} = \frac{36}{5}\,Q_q^2\,\frac{\alpha}{\alpha_s(m^2)}, \quad (1.4)$$

where Q_q is the electromagnetic charge of the quark. In this approximation the x spectrum of the γ rays rises essentially linearly over the entire range of x. In identifying $\Gamma_{\gamma gg}$ with $\Gamma \to (J/\psi \to \gamma + \text{light hadrons})$ we are assuming a gluon-hadron duality. Actually, this duality breaks down at large x (at small values of m^2_{had}). If we nevertheless assume $\Gamma_{\gamma gg} = \Gamma(J/\psi \to \gamma + \text{light hadrons})$ then the relative probability for the radiative transition of J/ψ into light hadrons is related to δ_γ by

$$\mathrm{BR}\,(J/\psi \to \gamma + \text{light hadrons}) = \frac{\delta_\gamma}{1+\delta_\gamma}\,[1 - (R+2)\,\mathrm{BR}\,(J/\psi \to e^+e^-) - \mathrm{BR}\,(J/\psi \to \gamma\eta_c)],$$

where

$$R = \sigma\,(e^+e^- \to \text{hadrons})/\sigma\,(e^+e^- \to \mu^+\mu^-). \quad (1.5)$$

With $\alpha_s = 0.18$ we find BR $(J/\psi \to \gamma + \text{light hadrons}) \approx 8\%$.

Experimentally, the yield of inclusive γ rays has been identified reliably only at $x \gtrsim 0.5$. At lower values of x, the subtraction of the contribution of two-γ decays of η and π^0 mesons from hadronic decays of J/ψ introduces large errors. It follows from the SPEAR data[39] that the total yield of γ rays is in reasonable agreement with (1.5), while the shape of the γ spectrum is sharply different from the perturbation-theory prediction[38] (Fig. 4a). We see from Fig. 4a that at the intermediate values $x = 0.5$–0.7 the ratio $(dN/dx)_{\text{expt}}/(dN/dx)_{\text{theo}}$ is significantly larger than 1, possibly because

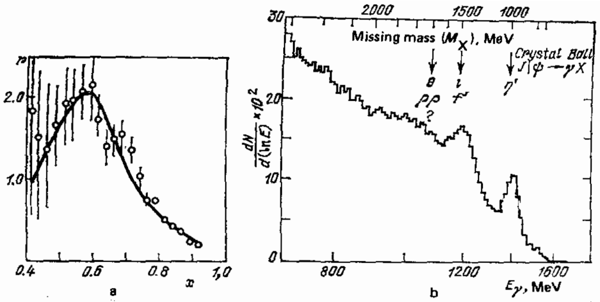

FIG. 4. a—Normalized inclusive spectrum of γ rays from the decay of J/ψ, $r=(dN/dx)_{\text{expt}}/(dN/dx)_{\text{(QCD, lowest order)}}$. The Breit-Wigner curve corresponds to the analysis incorporating a broad tensor resonance with a mass $M=2$ GeV and $\Gamma=0.6$ GeV. The experimental points are from Ref. 39. b—Spectrum of γ rays from the decay $J/\psi\to\gamma+$light hadrons in the E_γ range from 0.8 to 1.6 GeV (Ref. 25).

of resonant structures of some sort. At high values of x this ratio is less than 1. More-detailed measurements of the inclusive γ rays have shown, however, that this region is dotted with peaks corresponding to radiative transitions of J/ψ both to known mesons (π^0, η, η', f, f') and to new states[16,25,40] (Fig. 4b and Table II).

Two of these, ι and θ, are solidly established since they have been observed in different channels by two groups (Table III). In addition to these solidly established mesons, some other structures (possibly resonances) have been observed in J/ψ decays. We are thinking primarily here of the peak in the $J/\psi\to\gamma\rho^0\rho^0$ channel.[43] A fit with a Breit-Wigner curve yields[43]

$$M_{\rho\rho}=1650\pm 50\,\text{MeV},\ \Gamma_{\text{tot}}=200\pm 100\,\text{MeV},$$

and

$$BR\,(J/\psi\to\gamma\rho^0\rho^0,\ m_{\rho\rho}<2\text{GeV})=(1.25\pm 0.35\pm 0.4)\ 10^{-3}.$$

We do not rule out the possibility that this peak is a manifestation of the θ meson, and in this case $\rho\rho$ would be one of the important θ decay modes. The data available are inadequate to confirm or refute this hypothesis.

A broad enhancement has been observed[25] in the decay modes $J/\psi\to\gamma\eta\pi^+\pi^-, \gamma\eta\pi^0\pi^0$. A fit with a Breit-Wigner curve yields

$$M_{\eta\pi\pi}=1710\pm 45\,\text{MeV},\ \Gamma_{\text{tot}}=530\pm 110\,\text{MeV}.$$

The number of events in the peak is $\sim 5\cdot 10^2$, and the

TABLE II. Exclusive radiative decays of J/ψ.

Decay mode	BR ($\times 10^{-4}$)	Comments
$J/\psi\to\gamma\pi^0$	$(3.6\pm1.1\pm0.8)\cdot10^{-1}$	CB[41]
$J/\psi\to\gamma\eta'$	36 ± 5	
$J/\psi\to\gamma\eta$	8.6 ± 0.9	$\frac{BR\,(J/\psi\to\gamma\eta')}{BR\,(J/\psi\to\gamma\eta)}=4.7\pm0.6$, CB[41]
$J/\psi\to\gamma f\,(1270)$	15 ± 4	CB[42], $f\to\pi^0\pi^0$. $x=A_1/A_0=0.88\pm\pm0.13$; $y=A_2/A_0=0.04\pm0.19$ (A_λ are the amplitudes for the decay of f with the helicities $\lambda=0, 1, 2$)
$J/\psi\to\gamma f'\,(1515)$		
$f'\to K\bar{K}$	$(1.6\pm0.5\pm0.8)$	MARK-II [25]
$f'\to\eta\eta$	(0.9 ± 0.9)	CB [25]

TABLE III. Properties of the new states[25] ι and θ.

Property	MARK II group	Crystal Ball group
M_ι, MeV	1440^{+10}_{-15}	1440^{+20}_{-15}
$\Gamma\,(\iota\to$ anything) MeV	50^{+30}_{-20}	55^{+20}_{-30}
J^P	—	0^- ($\iota\to\delta\pi$ mode observed)
C	+	+
$BR\,(J/\psi\to\gamma\iota)\,BR\,(\iota\to K\bar{K}\pi)$	$(4.3\pm1.7)\cdot10^{-3}$	$(4.0\pm1.2)\cdot10^{-3}$
$BR\,(J/\psi\to\gamma\iota)\,BR\,(\iota\to\eta\pi\pi)$	—	$<2\cdot10^{-3}$ (90 % c. l.)
M_θ, MeV	1700 ± 20 ($\theta\to K^+K^-$ mode observed; two-resonance fit)	1670 ± 50 ($\theta\to2\eta$ mode observed; two-resonance fit)
$\Gamma\,(\theta\to$ anything), MeV	156 ± 30	160 ± 80
J^{PC}	2^{++}, 95 % c. l.	—
$BR\,(J/\psi\to\gamma\theta)\,BR\,(\theta\to\eta\eta)$	—	$(3.8\pm1.6)\cdot10^{-4}$
$BR\,(J/\psi\to\gamma\theta)\,BR\,(\theta\to K\bar{K})$	$(12.4\pm1.8\pm5.0)\cdot10^{-4}$	—
$BR\,(J/\psi\to\gamma\theta)\,BR\,(\theta\to\pi\pi)$	$2.4\cdot10^{-4}$ (90 %c. l.)	$<6\cdot10^{-4}$

corresponding relative probability is

$$BR\,(J/\psi\to\gamma\eta\pi^+\pi^-)=(3.5\pm0.2\pm0.7)\cdot10^{-3},$$
$$BR\,(J/\psi\to\gamma\eta\pi^0\pi^0)=(2.3\pm0.3\pm0.8)\cdot10^{-3}.$$

We might note that these numbers are comparable to the most intense of the previously identified radiative decays: $BR(J/\psi\to\gamma\eta')\sim4\cdot10^{-3}$ and $BR(J/\psi\to\gamma f)\sim1\cdot10^{-3}$).

Hadron reactions have yielded indications of other mesons which are candidates for glueballs. We will not take up those results; the interested reader is directed to Ref. 25 or the original papers.

In the literature the ι and θ mesons are presently regarded as serious candidates for glueballs, i.e., bound states of gluons. There are arguments for and against this interpretation, and some of them will be discussed in Section 2. At any rate it should be kept in mind that the low-lying pseudoscalar and tensor nonets are already completely filled [the isoscalar states $0^{-+}-\eta, \eta'$; $2^{++}-f(1270), f'(1515)$].

3) *Charmed hadrons*

3.1) D *mesons* ($c\bar{u}, c\bar{d}$). These mesons have now been studied quite thoroughly. There are two doublets: the spinless D^0 and D^+ and the vector D^{*0} and D^{*+}. Their properties are studied primarily in the resonance ψ'' ($\psi''\to D\bar{D}$ almost completely) and in the $\psi(4030)$ peak. The masses of the D mesons and of the basic nonleptonic decays are given in Ref. 31. The new data on D^* are summarized in Table IV. The average charged-particle multiplicities in the D decays are

$$\langle n_{\text{ch}}\rangle_{D^0}=2.46\pm0.14,\ \langle n_{\text{ch}}\rangle_{D^+}=2.16\pm0.16. \tag{1.6}$$

TABLE IV. Properties of the D^* mesons.[44]

	Decay mode	BR, %	Comment
D^{*0}	$D^0\pi^0$	47 ± 9	The emission of D is assumed to be isotropic with respect to the direction of motion of D^* in the reaction $e^+e^-\to D^*\bar{D}^*\to D+\ldots$
	$D^0\gamma$	53 ± 9	
D^{*+}	$D^0\pi^+$	44 ± 7	
	$D^+\pi^0$	28 ± 7	
	$D^+\gamma$	28 ± 10	

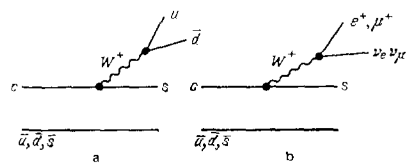

FIG. 5. Diagrams describing the decays of charmed mesons in the approximation of the decay of a free quark ($\Delta C = \Delta S$).

Measurements of the semileptonic decays of D mesons can yield some extremely important information on the decay $c \to se^+\nu_e$. Since we have $\Delta T = 0$ for this transition, we can expect the widths of the semileptonic decays of D^0 and D^+ to be essentially equal:

$$\Gamma(D^0 \to e^+ + \nu_e + X) = \Gamma(D^+ \to e^+ + \nu_e + X).$$

It then follows immediately that the lifetime ratio of the D^+ and D^0 mesons is

$$\frac{\tau_{D^+}}{\tau_{D^0}} = \frac{\mathrm{BR}(D^+ \to e^+ + \nu_e + X)}{\mathrm{BR}(D^0 \to e^+ + \nu_e + X)} . \tag{1.7}$$

The developments on the experimental side of this question have been quite dramatic. The value $\mathrm{BR}(D^+ \to e^+ + \nu_e + X)$ ($19^{+4}_{-3}\%$) found in the SLAC measurements is close to the value expected for the decay of a free c quark, ~1/5 (Fig. 5). At the same time, the same experiments revealed a much lower value for $\mathrm{BR}(D^0 \to e^+ + \nu_e + X)$, so that the ratio τ_{D^+}/τ_{D^0} has turned out to be large ($\tau_{D^+}/\tau_{D^0} > 4.3$ according to the DELCO data,[45] for example). Further evidence of a sort that the lifetimes of D^0 and D^+ differ substantially—by a factor of several units—comes from direct measurements in nuclear emulsions on proton accelerators (see Table V and Refs. 12, 23, and 24). The anomalous value observed for τ_{D^+}/τ_{D^0} stimulated a burst of theoretical speculation (see Ref. 46, for example). Skipping ahead a bit, we note that reasonable theoretical estimates of the ratio τ_{D^+}/τ_{D^0} yield numbers close to unity, no more than 1.5–2 in the extreme case. However, many theoreticians have faltered under the onslaught of experimental data and have proposed a large number of models which "explain" the anomalous value of τ_{D^+}/τ_{D^0} and which "predict" $\tau_{D^+}/\tau_{D^0} \sim 5$–10.

Since the beginning of 1982, the situation has apparently begun to return to normal, after the appearance of data from direct measurements of the lifetimes of D^0 and D^+ mesons carried out at SLAC in an experiment on photoproduction in a hydrogen bubble chamber at $E = 20$ GeV. The value[26] $\tau_{D^\pm} = (7.3^{+3.0}_{-2.5}) \cdot 10^{-13}$ s turned out to be in approximate agreement with emulsion data and with the value expected from the model for the decay of the free quark ($\tau_c = 6.5 \cdot 10^{-13}$ s). On the other hand, the value of $\tau_{D^0} = (7.7^{+3.5}_{-2.5}) \cdot 10^{-13}$ s found at SLAC is significantly higher than in previous measurements (see Table V).[26]

TABLE V. Lifetimes of charmed particles.[26]

Experiment	FNAL (ν), E531	CERN (πp), NA16	CERN (π), NA18	CERN (γ), NA1	SLAC (e^+e^-), MARK II	SLAC (γ), BC 72/73
Property $\tau(D^0)$, 10^{-13} s	$3.2^{+1.0}_{-0.7}$ (14)	$3.9^{+1.4}_{-0.9}$ (14)	$4.1^{+2.6}_{-1.3}$ (9)	—	$3.7^{+2.5}_{-1.5}$	$7.7^{+3.0}_{-2.5}$ (13)
$\tau(D^+)$, 10^{-13} s	$11.4^{+6.6}_{-4.4}$ (11)	$9.2^{+4.4}_{-2.5}$ (13)	$6.3^{+4.4}_{-2.3}$ (7)	$9.5^{+3.1}_{-1.9}$ (98)	—	$7.3^{+3.0}_{-2.5}$ (14)
$R = \frac{\tau(D^+)}{\tau(D^0)}$	$3.6^{+2.5}_{-1.5}$	$2.4^{+1.5}_{-0.8}$	1.5 ± 1	—	—	$0.9^{+0.7}_{-0.4}$
$\tau(F^+)$, 10^{-13} s	$2.0^{+1.8}_{-0.8}$ (3)	$1.9^{+1.7}_{-0.8}$ (3)	$4.4^{+5.0}_{-1.7}$ (5)	$5.0^{+5.0}_{-2.5}$ (5)	—	—
$\tau(\Lambda_c^+)$, 10^{-13} s	$2.3^{+1.0}_{-0.6}$ (8)	$1.9^{+1.4}_{-0.7}$ (4)	—	—	—	—

*The number of events is shown in parentheses.

We might also note that the semileptonic decays seem to be essentially exhausted by the three-particle modes $D \to K^*(890)$, $e\nu_e$ ($37 \pm 16\%$), and $D \to Ke\nu_e$ ($55 \pm 14\%$).[47]

3.2) *The* F *meson* ($c\bar s$). The situation is not settled here. The preliminary results reported by the DASP group,[48] which show evidence of the production in e^+e^- collisions of an F meson with a mass $M_F = 2030 \pm 60$ MeV and of an F^* meson with $M_{F^*} = 2140 \pm 60$ MeV (in the reactions $e^+e^- \to F\underset{\to\gamma+F}{F^*}$, $F^*\bar F^*$), have not been confirmed by other, more recent experiments with the Crystal Ball.[49] The existence of the F mesons has so far been solidly established only in experiments on proton accelerators ($M_F = 2020 \pm 15$ MeV). It has been asserted[12,26] that the emulsion data correspond to the value $\tau_F = (2.0^{+1.8}_{-0.8}) \cdot 10^{-13}$ s. More-recent measurements yield the larger number

$$\tau_{F^+} = (4-5) \cdot 10^{-13}\,\mathrm{s}, \tag{1.8}$$

although the error is very large (Table V). There is the hope that the experimental value will stabilize at the level in (1.8) as the statistical base is increased. The value $\tau_F \sim 2 \cdot 10^{-13}$ s, like the number given below for τ_{Λ_c}, seems a bit strange from the theoretical standpoint. As we will show in Section 3, the lifetimes of *all* charmed particles must agree with the simple-minded estimate $\tau_c \approx 6.5 \cdot 10^{-13}$ s within the preasymptotic corrections [~0(50%)].

3.3) *The charmed baryons* Λ_c (cud) and Σ_c (cdd, cud, cuu). Experiments on proton accelerators have yielded a variety of pieces of evidence for the production of Λ_c, and there are also indications that Σ_c^+ and Σ_c^{++} have been observed. From these measurements one finds[26] $\tau_{\Lambda_c} = (1.5-3) \cdot 10^{-13}$. Charmed baryons have been detected in e^+e^- collisions on the basis of the rapid increase in the inclusive yield of p, $\bar p$, Λ, and $\bar\Lambda$ at $W \approx 4.5$ GeV (Ref. 50). The Λ_c mass is 2282.2 ± 3.1 MeV; the relative decay probabilities are given in the tables of Ref. 31.

By studying the yields of inclusive electrons in e^+e^- annihilation at the energy $W = 4.5-6.8$ GeV (events containing baryons), the MARK-II group measured[51] the semileptonic decays of charmed baryons:

$$\mathrm{BR}(\Lambda_c \to e^+\nu_e X) = 4.5 \pm 1.7\%.$$

As mentioned above, the value $\mathrm{BR}(\Lambda_c \to e^+\nu_e X)$, like that of τ_{Λ_c}, seems to be too low by a factor of several units (see Section 3 for more details).

b) b quarks

A striking new event occurred in particle physics in May–June 1977. The CFS Collaboration, headed by

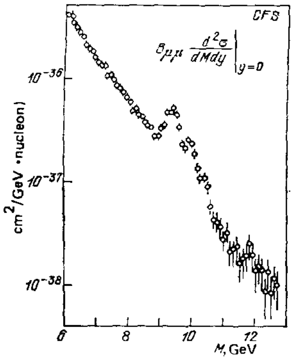

FIG. 6. Mass spectrum of $\mu^+\mu^-$ pairs measured in p–N collisions at $p=400$ GeV/c (Ref. 13).

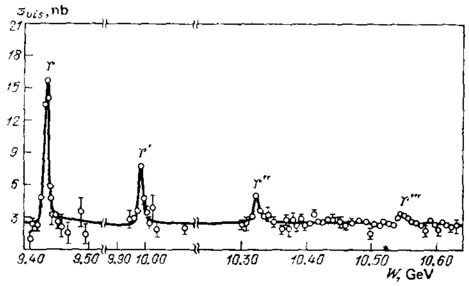

FIG. 7. Observed cross section for the annihilation $e^+e^- \to$ hadrons near the Υ resonances according to measurements by the CLEO group.[17]

L. M. Lederman, discovered a new family of heavy particles, with masses of the order of 10 GeV, in experiments on the proton accelerator at Fermilab.[13] These particles, which were named Υ mesons, were found from the $\mu^+\mu^-$ mass spectrum in proton-nucleus collisions (Fig. 6). This discovery withstood a genuine test by fire when a fire broke out in the experimental apparatus immediately after the first observations of Υ mesons. After everything was restored to working conditions, the results were found to be reproducible. A further increase in the statistical base led to the assertion that the data are described best under the assumption that there are three narrow peaks, corresponding to Υ, Υ', and Υ'' mesons with respective masses ~9.46 GeV, ~10 GeV, and ~10.4 GeV.

Immediately after the discovery of the Υ mesons, it was generally believed that we would be seeing a repetition of the events which unfolded with regard to the J/ψ, and we began to see manifestations of an even heavier quark, the b quark, with a mass $M_b \approx 4.8$ GeV and with the new quantum number of "beauty",[3)] which is conserved in strong interactions. Here the Υ mesons are particles with a hidden beauty; i.e., they correspond to 1^{--} levels of quarkonium, $b\bar{b}$ (bottonium).

Experiments on e^+e^- annihilation were necessary for definitive proof of the existence of this new quark and for a detailed study of its properties, as in the case of the J/ψ meson. The colliding e^+e^- beams available at the time did not have an energy sufficient for a search for Υ. However, immediately after the report of the CFS results, an effort was undertaken to modify the DORIS installation, and no later than late April or early May 1978 two groups, PLUTO[52] and DASP-2,[53] observed a narrow peak in the e^+e^--annihilation cross section corresponding to an Υ meson with a mass $M_\Upsilon \approx 9.46$ GeV. By the end of August 1978, experiments on DORIS had also revealed an Υ' with a mass $M_{\Upsilon'} \approx 10.02$ GeV (Ref. 54). The observed Υ and Υ' widths agreed well with the energy resolution calculated for DORIS, so that their intrinsic widths were extremely small ($\ll$20 MeV). This result was the strongest argument for interpreting the Υ mesons as bound states of new quarks.

[3)]In the literature we also find another name for this new quark: "bottom," i.e., the lower quark in the new quark doublet.

The Υ and Υ' resonances were also found in the CESR installation (at Cornell University) which came on line in 1979. Here again, another narrow resonance, Υ'', with $M_{\Upsilon''} \approx 10.35$ GeV, was observed[55,56]; finally, a broad Υ''' resonance[57,58] with $M_{\Upsilon'''} \approx 10.57$ GeV and a width Γ roughly twice the energy resolution of the CESR beams ($\Gamma = 14 \pm 15$ MeV) was observed. This width is more than two orders of magnitude greater than the intrinsic width of the Υ meson, so that there is no Zweig-Iizuki restriction on the decay of Υ'''. A further analysis showed that this resonance is an analog of $\psi(3770)$ and is a B-meson factory (i.e., a factory for mesons consisting of the quarks $b\bar{u}$ and $b\bar{d}$): $\Upsilon''' \to B\bar{B}$. No new states of $\tilde{\Upsilon}$ of any sort were found[18] between Υ'' and Υ''' [$\Gamma(\tilde{\Upsilon} \to e^+e^-) < 0.03$ keV]. Figure 7 illustrates the situation with the measured cross sections for $e^+e^- \to$ hadrons near the Υ resonances. We might note that the comparatively slight rise of the cross sections above the background observed at the Υ resonance—by a factor of only a few units—in comparison with the rise in the case of the J/ψ meson (where the ratio of the resonance to the background is about two orders of magnitude) stems from both the serious degradation of the energy resolution in the e^+e^- beams and the increase in the background, on the one hand, and the small charge of the new quark,[59] $Q_b = -1/3$, on the other. Measurements of the ratio R at energies $W = 10.40$–11.60 GeV, above the Υ''' resonance, have shown an increase by an average of $\overline{\Delta}_R \approx 0.2$ (the systematic errors essentially cancel out upon subtraction). Measurements of other characteristics of e^+e^- annihilation (the multiplicity, the shape of the hadron events, and the yields of direct leptons) also demonstrate this increase in the cross section. These results agree well with the suggestion that the charge of the new quark is $Q_b = -1/3$, while the theoretical prediction is $\overline{\Delta R}_{\rm theo} = 3Q_b^2 = 1/3$.

1) *Properties of the* Υ *resonances*

Figure 8 is a schematic diagram of the levels of $b\bar{b}$ quarkonium (or bottomium) and of the transitions expected between these levels. In contrast with charmonium, only the $\Upsilon(n^3S_1)$ levels[17] and the excited P levels,[25] (2^3P_J), have been reliably established at this point. This is not surprising, since the world of beauty particles is much younger, and furthermore the statistical base available is about two orders of magnitude smaller than that for charmonium.

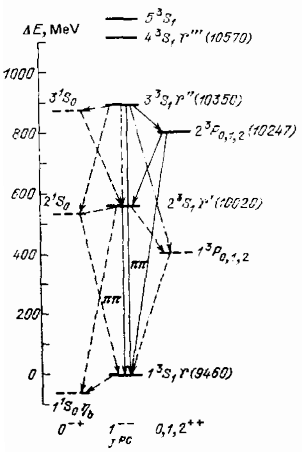

FIG. 8. Family of levels of $b\bar{b}$ quarkonium. Solid lines—observed levels or transitions; dashed lines—unobserved levels or transitions; dot-dashed lines—transition of Υ'' to the P ground level 1^3P_J. The observation of this transition is not an absolutely solid fact (see the text proper).

The characteristics presently known for the Υ resonances are listed in Tables VI and VII.

It follows from Table VI that the width of the direct decays $\Upsilon \to 3g \to$ hadrons,

$$\Gamma(\Upsilon \to 3g) \approx \Gamma_\Upsilon - (3 + R(W = 10\,\mathrm{GeV}))\Gamma(\Upsilon \to e^+e^-) \qquad (1.9)$$

is about $\Gamma(\Upsilon \to 3g) \approx 27.5^{+6}_{-5}$ keV, or much smaller than $\Gamma(J/\psi \to 3g) \approx 44$ keV. This fact indicates a decrease in the color constant α_s with increasing momentum transfer, and it is striking evidence in favor of asymptotic freedom (see Subsection 2b for more details).

In direct (nonelectromagnetic) decays of Υ, a detailed study has been made of the shape of the events in order to confirm their three-gluon nature, $\Upsilon \to 3g$ (for details see Refs. 11 and 63). In complete agreement with quantum chromodynamics, the distributions in eight separate topological characteristics in the decay of Υ agree well with the three-gluon model, while away from the resonance these distributions correspond best to the production of a quark-antiquark pair. The confirmation of the three-jet nature in the hadronic decays of Υ was historically the first direct evidence for gluons. Only

TABLE VI. Properties of the Υ meson[18,31*].

		Comments
M_Υ, MeV	9459.7 ± 0.6	Novosibirsk. Resonant-depolarization method[60]
$BR(\Upsilon \to \mu^+\mu^-)$, %	3.2 ± 0.4	$BR(\Upsilon\to e^+e^-) \approx BR(\Upsilon\to\mu^+\mu^-)$ is assumed
$\Gamma(\Upsilon \to e^+e^-)$, keV	1.18 ± 0.04	
Γ_Υ, keV	36.9 ± 4.8	
$\langle n_\Upsilon^{ch}\rangle$	7.9 ± 0.6 [61]	DASP-2 data. The multiplicities correspond to direct decays of the resonances. Away from a resonance, $\langle n_{off}^{ch}\rangle = 6.9 \pm 0.6$
$\langle n_\Upsilon^{p+\bar{p}}\rangle$	0.64 ± 0.16 (DASP-2) [61] 0.59 ± 0.05 (CLEO)	In the background $\langle n_{off}^{p+\bar{p}}\rangle = 0.27\pm0.02$ (CLEO)
$\langle n_\Upsilon^{\Lambda+\bar{\Lambda}}\rangle$	0.25 ± 0.03	In the background $\langle n_{off}^{\Lambda+\bar{\Lambda}}\rangle = 0.08\pm0.01$ (CLEO)

*) CLEO group. Preprints CLNS 82/536 and CLNS 82/547.

TABLE VII. Properties of the Υ', Υ'', and Υ''' resonances[18,62*].

	Υ'	Υ''	Υ'''
$M - M_\Upsilon$, MeV	559.5 ± 0.3 (CLEO)	890.7 ± 0.5 (CLEO)	1113.0 ± 1
$n^{2S+1}L_J$	2^3S_1	3^3S_1	4^3S_1
$\Gamma(\Upsilon(nS)\to e^+e^-)$, keV	0.54 ± 0.03	0.42 ± 0.03 (CLEO)	0.275 ± 0.06
$BR(\Upsilon(nS)\to e^+e^-)$, %	1.6 ± 1.0 (CLEO) 2.0 ± 0.4 **)	3.2 ± 1.6 (CLEO)	$(1.9\pm0.8)\cdot10^{-3}$
$\Gamma_{tot}(\Upsilon(nS))$, keV	27.3 ± 4.7	13.1 ± 6.6	
$\Gamma(\Upsilon(nS))\to 3_g$, keV	13.0 ± 3.4	8.7 ± 2.6	
$BR(\Upsilon(nS)\to\Upsilon+\pi^+\pi^-)$, %	19.2 ± 2.6	$4.9\pm0.9\pm0.5$	
$\langle n^{ch}\rangle$	$8.?3\pm0.26$		11.5 ± 0.4
$\langle n^{\Lambda+\bar{\Lambda}}\rangle$	0.31 ± 0.03	0.15 ± 0.03	0.006 ± 0.06
$\langle n^{p+\bar{p}}\rangle$	0.41 ± 0.08	0.47 ± 0.02	0.21 ± 0.15

*See the comment in Table VI.

**The total width $\Gamma_{\Upsilon'}$ is found from

$$\Gamma_{\Upsilon'} = \left[\frac{\Gamma(\Upsilon'\to e^+e^-)}{BR(\Upsilon\to\mu^+\mu^-)} + \Gamma(\Upsilon'\to\gamma P)\right](1 - BR(\Upsilon'\to\Upsilon\pi\pi))^{-1},$$

where $\Gamma(\Upsilon'\to\gamma P)$ is estimated by $\frac{\Gamma(\Upsilon'\to\gamma P)}{\Gamma(\psi'\to\gamma P)} \approx \left(\frac{Q_b}{Q_c}\right)^2 \frac{m_c^2}{m_b^2}$.

because of the inadquate energy of the gluon jets did this fact taken by itself fall short of being a sufficient basis for declaring the discovery of gluons. This declaration did follow the observation of bremsstrahlung gluons in the reaction $e^+e^- \to q\bar{q}g \to$ hadrons on the PETRA (at Hamburg) at $W \gtrsim 30$ GeV (see Ref. 11, for example).

The first attempts have been undertaken to determine the width of the radiative decay $\Upsilon \to gg\gamma \to \gamma +$ hadrons.[64] These measurements yield the extremely weak restriction $\Gamma(\Upsilon \to (gg\gamma)/\Gamma(\Upsilon \to 3g) < 27\%$ [cf. (1.4); $\delta_\gamma^\Upsilon \approx 3.5\%$].

Some extremely important characteristics of vector bottomium are the total and purely hadronic ("three-gluon") widths. While these characteristics have been measured directly for the Υ ground state, in the cases of Υ' and Υ'' we are forced to resort to a theoretical reconstruction.[18] Let us consider Υ' for definiteness. The total width is $\Gamma_{tot}(\Upsilon') = \Gamma(\Upsilon' \to 3g) + \Gamma(\Upsilon' \to \gamma \to \ldots) + \Gamma(\Upsilon' \to \pi\pi\Upsilon) + \Gamma(\Upsilon' \to \gamma + \text{P-level}) +$ negligibly small channels. Furthermore, for the first two terms we can obviously adopt

$$\Gamma(\Upsilon' \to 3g) + \Gamma(\Upsilon' \to \gamma \to \ldots) = \frac{\Gamma(\Upsilon' \to e^+e^-)}{BR(\Upsilon \to \mu^+\mu^-)},$$

and we can also use

$$\Gamma(\Upsilon' \to \pi\pi\Upsilon) = \Gamma_{tot}(\Upsilon')[BR(\Upsilon' \to \pi\pi\Upsilon)]^{-1}.$$

The width of the radiative E1 transitions to P levels has been estimated theoretically only in potential models or through a simple scaling from the corresponding numbers for ψ'. In the case of Υ'' there are direct experimental data for $BR(\Upsilon'' \to \gamma P)$, and, incidentally, these data agree well with model-based calculations (more on this below). We can thus reconstruct the total widths of Υ' and Υ'' and their three-gluon widths. The results are listed in Table VII.

Some recent measurements of the pion spectrum in the decays $\Upsilon' \to \Upsilon\pi^+\pi^-$ and $\Upsilon'' \to \Upsilon\pi^+\pi^-$ have some far-reaching consequences for the theory.[65] While the distribution in the invariant pion mass (for a total of 17 events) in the former decay is compatible with the corresponding distribution for $\psi' \to J/\psi\pi\pi$, in the second de-

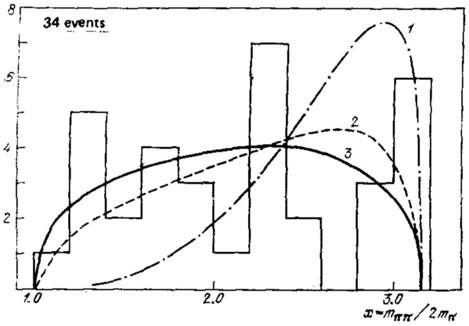

FIG. 9. Distribution in the invariant mass of pions in the decay[65] $\Upsilon'' \to \Upsilon\pi^+\pi^-$. 1—the corresponding distribution for $\psi' \to J/\psi\pi\pi$; 2—the phase volume, 3—the best fit.

cay we find a sharply different picture (Fig. 9). The $m_{\pi\pi}$ distribution is absolutely flat, and this result cannot be explained by standard theoretical arguments. We will discuss this question in more detail in Section 2.

A few words are in order here regarding the observation of the P levels of bottomium in E1 radiative transitions. The clearest result deals with the transition to excited P levels,

$$\Upsilon'' \to \gamma + 2^3P_J,$$

which has been studied in both the inclusive spectrum of photons, $\Upsilon'' \to \gamma + \ldots$, and the cascades[25] $\Upsilon'' \to \gamma + 2^3P_J \to 2\gamma + \Upsilon, 2\gamma + \Upsilon'$. From the statistical base available we can draw the following conclusions:

$$\left.\begin{aligned} &M\,(2\mathrm{P},\,b\bar{b}) \approx 10.250\ \mathrm{GeV}\ \ (\text{error} \lesssim 2\ \mathrm{MeV}),\\ &\mathrm{BR}\,(\Upsilon'' \to \gamma + 2^3P_J) = 34 \pm 3\%,\\ &\mathrm{BR}\,(\Upsilon'' \to \gamma + 2^3P_J)\,\mathrm{BR}\,(2^3P_J \to 2^3S_1 + \gamma) = 5.9 \pm 2.1\%,\\ &\mathrm{BR}(\Upsilon'' \to \gamma + 2^3P_J)\ \mathrm{BR}\ (2^3P_J \to 1^3S_1 + \gamma) = 3.6 \pm 1.2\%. \end{aligned}\right\} \tag{1.10}$$

In fact, it is even possible to resolve levels corresponding to different spins (Fig. 10).

The situation with regard to the P ground level of bottonium is more contradictory. The CUSB group studied the spectrum of inclusive γ rays in the decay of Υ'' and did not find a peak which they could associate with a transition to the 1P state. In contrast, experimentalists in the CLEO group assert that they see a signal corresponding to the exclusive cascade $\Upsilon'' \to \gamma_1 + (1^3P_J) \to \gamma_1 + \gamma_2 + \Upsilon \to \gamma_1\gamma_2\mu^+\mu^-$ with $E_{\gamma_1} \simeq 410$ MeV. If we assume that this signal does in fact reflect a transition

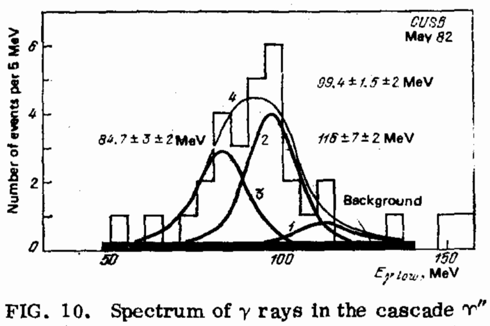

FIG. 10. Spectrum of γ rays in the cascade $\Upsilon'' \to \gamma + 2\,^3P_J \to \gamma\gamma + \Upsilon$ (from Ref. 25; the background has been subtracted). 1–3—The transitions $\Upsilon'' \to \gamma + 2\,^3P_J$, where $J = 0$, 1, 2; 4—resultant contribution of all transitions.

to the P ground state of bottomium, then we conclude

$$M\,(1^3P_J,\ b\bar{b}) = 9.93\ \mathrm{GeV},$$
$$\mathrm{BR}\,(\Upsilon'' \to \gamma 1^3P_J)\,\mathrm{BR}\,(1^3P_J \to \gamma\Upsilon) = 3.1 \pm 2.2\%.$$

This conclusion, however, would be premature from both the experimental and theoretical standpoints. The experimental "signal-to-noise ratio" is apprently too low for a statistically defensible assertion. For our purposes, of course, the theoretical argument is more important. According to the existing analysis of the quantum-chromodynamics sum rules the mass of the P ground level cannot be above 9.86 GeV. Most potential calculations in fact lead to predictions below 9.90 GeV. The matter is discussed in more detail in Section 2.

We also note that the CUSB group had previously studied radiative transitions to P levels, making use of the fact that, in contrast with $\Upsilon'('') \to 3g$, the $^3P_{0,2}$ levels decay preferentially to a two-gluon state. This circumstance should give rise to an admixture of two-jet events in Υ' and Υ'' by virtue of $\Upsilon\,(nS) \to \gamma + (n^3P_{0,2})$ cascades. Although there are clearly several ambiguities in a statistical approach of this type, we can still extract the following information[18]:

$$\begin{aligned} &\mathrm{BR}\,(\Upsilon\,(2S) \to \gamma + (1^3P_{0,2})) = 8 \pm 2\%,\\ &\mathrm{BR}\,(\Upsilon\,(3S) \to \gamma + (2^3P_{0,2})) = 20 \pm 3\%. \end{aligned} \tag{1.11}$$

The addition with equal weights of transitions to the $^3P_1$ levels leads to values of BR $[\Upsilon\,(nS) \to \gamma + (n^3P_J)]$ larger by a factor ~1.5. The second number in (1.11) is thus in excellent agreement with the direct measurement (1.10).

2) *Beauty hadrons*

"Beauty" mesons and baryons have one b quark among their constituents. The most accessible to study are the the B mesons ($\bar{B}^0 - b\bar{d}, B^-\ b\bar{u}$), since they are produced intensely at the Υ''' resonance. The CLEO group was recently the first to observe B mesons from a reconstruction of the following decay modes:

$$B^\pm \to \bar{D}^0\,(D^0)\,\pi^\pm,\ \ \bar{B}^0 \to D^0\pi^+\pi^-,$$
$$B^0\,(\bar{B}^0) \to D^{*\pm}\pi^\pm,\quad B^\pm \to D^{*\mp}\pi^\pm\pi^\pm.$$

All the relative probabilities are of the order of a few percent. In addition, there is a rich store of indirect information.

2.1) *The mass* M_B. This mass satisfies the inequality

$$M_{\Upsilon''} < 2M_B < M_{\Upsilon'''}.$$

It has been found experimentally[17,18] that the mode $\Upsilon''' \to B^*B^-$ accounts for only a small fraction of the total width of Υ''', so that the mode $\Upsilon''' \to B\bar{B}$ is the primary decay mode. The best present values for the masses of the B mesons are

$$\begin{aligned} &M_{B^0} = 5274.5 \pm 1.4\ \mathrm{MeV},\\ &M_{B^\pm} = 5274.0 \pm 2.1\ \mathrm{MeV}. \end{aligned} \tag{1.12}$$

2.2) *The yields of* K *mesons*. Weak decays of B mesons, which furnish the definitive evidence regarding the nature of the new quark b, were first discovered from a substantial increase in the yield of K mesons and inclusive leptons at the Υ''' resonance. Important information regarding the structure of a theory of the

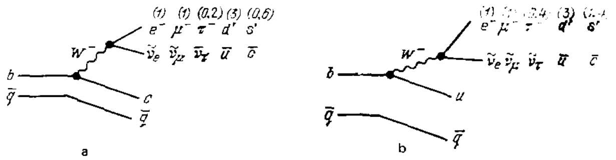

FIG. 11. Quark diagrams for the decays of the B meson in the approximation of the decay of a free quark. The ratios of the corresponding partial widths in $\Gamma\,(b \to e^- \bar{\nu}_e c\,(u))$ are shown in parentheses. These partial widths were calculated with allowance for the color factors and phase-volume effects.

weak interaction follows from these decays. All the data available, including the topological characteristics of hadronic events, indicate that a particle pair $B\bar{B}$ with a mass of about 5.3 GeV is formed and then decays. For the b quark, the transition $b \to \underset{\lfloor \to s}{c} + W^-$ is dominant[17,18] (Fig. 11a), as would be expected in the standard six-quark model. In particular, this decay should give rise to a large number of K mesons in the final state by virtue of the transition $B\bar{B} \to D\bar{D}X \to K\bar{K} + \ldots$ Monte Carlo calculations show that $\rho_K = [\sigma_{B\bar{B}}(K)]/[\sigma_{off}(K)]$, the ratio of the $(K^\pm + K^0)$ yields in $B\bar{B}$ events and in the nonresonant region, should be ≈ 1.8 for $b \to cW^-$ transitions (Fig. 11a) and $\rho_K \approx 1$ for $b \to uW^-$ transitions (Fig. 11b). From experiment[62] we have $(\rho_K)_{expt} = 1.9 \pm 0.3$.

That the transition $b \to uW^-$ is of minor importance is also demonstrated by the shape of the electron spectrum in the semileptonic decays $B \to e\nu_e X$.

2.3) *Semileptonic decays*. The electron yield increases sharply at the Υ''' resonance. By measuring the yield of electrons (or muons) one can learn about the semileptonic decays of B mesons.

The spectra of inclusive leptons and the multiplicity distributions in events involving leptons are consistent with the assumption that the three-particle decays $B \to D(D^*) l\nu_l$ play an important (and perhaps governing) role in the semileptonic decays of B.

The observed shape of the lepton spectra corresponds to the decay of a free b quark resulting in the production of a hadronic system $X = D + \ldots$ with a mass ~2 GeV. The possibility of an effective hadron mass $M_X \leq 1$ GeV has been ruled out by experiment. This result is a clear argument in favor of the dominance of the transition $b \to cW^-$, since in the $b \to uW^-$ case the mass of the X system could be significantly lower than 2 GeV.

Quantitatively, we have[26]

$$\mathrm{BR}\,(B \to e\nu_e X_u)/\mathrm{BR}\,(B \to e\nu_e X_c) \leqslant 0.093\ (90\%\ \text{c.l.}),$$

which means (see Section 4)

$$\left|\frac{U_{bu}}{U_{bc}}\right| \leqslant 0.21. \tag{1.13}$$

From the data of the Υ''' resonance we can determine the average multiplicities of charged particles (hadrons + leptons) in purely hadronic and semileptonic decays of the B meson[66]:

$$\langle n^{ch}\rangle_B^h = 6.3 \pm 0.2 \pm 0.2 \quad \langle n^{ch}\rangle_B^{s.l.} = 4.1 \pm 0.35 \pm 0.2, \quad \langle n^{ch}\rangle_B^{tot} = 5.75 \pm 0.1 \pm 0.2. \tag{1.14}$$

TABLE VIII. Relative probabilities for the decays[26] $B \to l\nu_l X$.

$\mathrm{BR}\,(B \to e\nu_e X)$	$\mathrm{BR}\,(B \to \mu\nu_\mu X)$
0.127±0.017±0.013 (CLEO)	0.122±0.017±0.031 (CLEO)
0.131±0.012±0.020 (CUSB)	0.15±0.035±0.035 (TASSO)
0.11±0.03±0.02 (MARK-II)	0.093±0.029±0.020 (MARK-J)
0.136±0.05±0.04 (TASSO)	

We see that the multiplicity in the B decay is high, so we see the reason for the difficulty in observing the individual exclusive channels.

The measured probabilities for the semileptonic decays of B are listed in Table VIII. They are consistent with each other within the experimental errors, and they are lower than the prediction for the case of the decay of a free b quark (Fig. 11), in which case we would expect $\mathrm{BR}\,(b \to c + e^- + \bar{\nu}_e) \approx 16\%$ when phase-volume effects are taken into account[4)] (see Ref. 67, for example). It may be that time will bring the experimental values of $\mathrm{BR}\,(b \to c + e^- + \bar{\nu}_e)$ into agreement with this number, especially for the $l = \mu$.

By measuring the yields of inclusive leptons with momenta higher than that permitted in the decay $B \to De\nu_e$, one could obtain unambiguous information about the $b \to uW^-$ transition.

Experiments on the mass spectrum of inclusive e^+e^- pairs have revealed a limitation on the decay $B \to \underset{\lfloor \to e^+e^-}{J/\psi} + X$ expected in the standard model[26]:

$$\mathrm{BR}\,(B \to J/\psi + X) \leqslant 1.4\%\ (90\%\ \text{c.l.}). \tag{1.15}$$

2.4) *Lifetime of the B meson*. The JADE group has found an upper limit on the lifetime of the B meson from experiments on PETRA[26,68]:

$$(\tau_B)_{expt} < 1.4 \cdot 10^{-12}\,\text{s}\ (95\,\text{c.l.}). \tag{1.16}$$

They obtained this limit from limits imposed on the B decay length according to events with inclusive muons. In this manner we get a new lower limit on the quark mixing angles (see Subsection 4a). We might note that on the basis of the information presently available on the mixing angles we could expect

$$\tau_B = 10^{-13} - 10^{-14}\,\text{s}. \tag{1.17}$$

2.5) *Nonstandard models*. All the data available on the decays of B mesons agree with the standard six-quark model for the weak interaction (see Subsection 4a). In addition, there are several clear experimental arguments[17,20] against the various nonstandard and exotic models.

Finally, the decays of B mesons rule out a possible existence of charged Higgs bosons $H^\pm$ with masses 2 GeV $< m_H < m_b$. Data on the inclusive yield of leptons and measurements of the energy carried off by charged hadrons in the decay of the B meson completely refute a dominant role of the cascades

$$b \to qH^-, \quad \underset{\lfloor\!\!\longrightarrow \tau^-\tilde{\nu}_\tau,\ c\tilde{s}}{} \tag{1.18}$$

4)The firming up of the nonleptonic Hamiltonian by hard gluons reduces this number slightly (to about 15%; see Ref. 7).

which would be expected in the case of an H^- in this mass interval.

2. THEORY OF HEAVY QUARKONIA

a) Spectrum

1) *Quarks and their masses*

We have seen that the existence of the two heavy quarks, c and b, has been solidly established. Their electric charges are 2/3 and −1/3, and they form the family of charmonium and bottonium, respectively. It is believed that there should be a third heavy quark, t, with a charge of 2/3, which has not yet been observed experimentally.

The most important characteristic of the heavy quarks is their mass. Because of quark confinement, we cannot weigh an isolated heavy quark in the sense that we can weigh, say, muons. It is nevertheless possible to introduce the concept of a so-called current quark, i.e., a quark devoid of its gluon cloud (more precisely, only the soft gluons are eliminated; the hard gluons give rise to logarithmic effects, which are easy to take into account).

The mass of a current quark depends on the normalization point and enters all calculations based on fundamental chromodynamics. This mass has been determined[69,70] from the quantum-chromodynamics sum rules for the c and b quarks:

$$m_c \approx 1.40\,\text{GeV},\ m_b \approx 4.80\,\text{GeV}. \qquad (2.1)$$

These numbers correspond to the so-called mass on the mass shell, which is a gauge-invariant quantity which can be determined well by perturbation theory. (A slightly lower value, $m_b \approx 4.71$ GeV, was derived in Ref. 71.) The Euclidean mass, which is also frequently mentioned in the literature,[69] depends on the gauge. In the Landau gauge, for example, we have

$$m(p^2=-m^2)=m(p^2=m^2)\left(1-\frac{2\alpha_s \ln 2}{\pi}+\ldots\right),$$

where the coefficient in front of α_s would change if we changed gauge. For charmed quarks we now have many independent estimates of the Euclidean mass.[69,72-74] All agree with each other and with (2.1):

$$m_c \approx 1.26\,\text{GeV} \quad (p^2=-m_c^2,\ \text{Landau gauge}). \qquad (2.2)$$

The situation regarding the b quarks is less happy. From the literature[74,71] we have m_b (Euclidean) ≈ 4.26 GeV, and this number appears to be too small: To be reconciled with estimate (2.1), it would have to be about 150 MeV larger. We see that further analysis is required here.

Investigators working with constituent quarks usually come up with much larger masses. This is not surprising, since the gluon cloud is being incorporated in the quarks in this case.

It should be noted that

$$M_{J/\psi} > 2m_c, \qquad (2.3)$$

but

$$M_\Upsilon < 2m_b,\ 2m_b - M_\Upsilon \approx 130\,\text{MeV}^{70}. \qquad (2.4)$$

While (2.3) is completely understandable, (2.4) may appear surprising. Confinement effects increase the mass of the resonance in comparison with twice the quark mass. But what happened in the $b\bar{b}$ system? In the Υ family the Coulomb attraction becomes numerically important and overcompensates for the positive mass shift caused by confinement forces.

As for the hypothetical t quark, its absence from the PETRA experiments[20,21] means

$$m_t > 18.3\,\text{GeV}. \qquad (2.5)$$

If we wish to reach an understanding of the quarkonium spectrum we must know the nature of the binding forces in addition to the quark masses. According to the present interpretation, quarks live in a complex medium: a nonperturbative quantum-chromodynamics vacuum which which is densely populated with long-lived fluctuations of the gluon field. These nonperturbative fluctuations make the energy density of the vacuum lower than that according to perturbation theory. If we now "inject" a $Q\bar{Q}$ pair into the vacuum, the color field of the quarks causes a slight freezing of the fluctuations in the vicinity,[75] which in turn leads to an effective attraction between the Q and $\bar{Q}$.

It is important to understand that in real systems such as J/ψ or Υ the attractive force is generally not described by a static potential.[76] The effect on the gluon medium can be reduced to a potential only if there is sufficient time for this medium to adjust to the (slow) motion of the quarks. In other words, a necessary condition for a potential is

$$\omega_{\text{quark}} \ll \omega_{\text{glue}}, \qquad (2.6)$$

where ω is a characteristic frequency. The characteristic frequencies in the charmonium and bottonium family are

$$\omega_{\text{quark}} \sim M_{\psi'} - M_\psi \sim M_{\Upsilon'} - M_\Upsilon \sim 0.6\ \text{GeV} \qquad (2.7)$$

and the characteristic frequencies of the gluon medium are approximately the same. In fact, the validity of the multipole expansion (more on this below) implies $\omega_{\text{glue}} < \omega_{\text{quark}}$. If this is the case, we can expect some large deviations from the potential picture, especially for charmonium. This expectation is supported in a sense by a recent analysis of various relativistic effects.[77] As the quark mass increases, the ordinary Coulomb forces associated with single-gluon exchange of course become progressively more important. In the limit $m_Q \to \infty$, the low-lying levels of quarkonium are purely Coulomb levels (Subsection 2.3).

2) *Gluon condensate*

The particular properties of the quantum-chromodynamics vacuum which are responsible for shaping the spectrum are still not completely understood. Certain crude characteristics are known. For example, the resultant effect of the long-wavelength gluon fluctuations is reflected by the vacuum matrix element of the square of the gluon field:

$$\langle \text{vac} \mid G^a_{\mu\nu} G^a_{\mu\nu} \mid \text{vac} \rangle \neq 0. \qquad (2.8)$$

On the other hand, this parameter reduces directly to the vacuum energy density[73]

$$\varepsilon_{vac} = \frac{1}{4} \langle vac \mid \theta_{\mu\mu} \mid vac \rangle \approx -\frac{9}{32} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \right\rangle, \tag{2.9}$$

where $\theta_{\mu\nu}$ is the energy-momentum tensor. Here we have made use of the circumstance that $\theta_{\mu\mu}$ is determined in quantum chromodynamics by the so-called triangle anomaly[78]:

$$\theta_{\mu\mu} = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_q m_q \bar{q} q \approx -\frac{9\alpha_s}{8\pi} G^a_{\mu\nu} G^a_{\mu\nu},$$

where $\beta(\alpha_s)$ is the Gell-Mann-Low function. On the other hand, the gluon condensate (2.8) plays a leading role in the physics of heavy quarkonium. Why? The $Q\bar{Q}$ pair forming the quarkonium level is in a colorless state, so that its coupling with the vacuum fields is a dipole coupling:

$$H_{int} = -\frac{1}{2} g (t_1^a - t_2^a) \mathbf{r} \mathbf{E}^a, \tag{2.10}$$

where $\mathbf{E}^a$ is the chromoelectric field, and the $t_{1,2}$ are the color SU(3) generators which act on the quark and antiquark indices, respectively. If we consider transitions between colorless states, the term of first order in H_{int} vanishes, and the leading role is assumed by the second-order term, proportional to

$$\langle \mathbf{E}^a \mathbf{E}^a \rangle = -\frac{1}{4} \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle,$$

plus the subsequent iterations.

The vacuum expectation value (2.8) was introduced in Ref. 79, where its value was derived from the sum rules for charmonium:

$$\left\langle vac \left| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \right| vac \right\rangle \approx 1.2 \cdot 10^{-2} \mathrm{GeV}^4. \tag{2.11}$$

More-recent papers,[70,71,80,81] based on similar principles but working from more extensive data, indicate that this vacuum expectation value may actually be 10–40% higher.

Comparing (2.11) with (2.9) we see that ε_{vac} is negative. This result is in full accordance with the circumstance that nonperturbative fluctuations should lower the vacuum energy density in a theory with confinement.

3) *Pre-Coulomb behavior*

In one particular case, the information embodied in (2.11) is sufficient to derive a correct and exhaustive theory for the levels of quarkonium. If the quark mass m is large enough, the quarks are bound primarily by Coulomb forces over distances of order k_n^{-1}, where k_n is defined by

$$k_n = \frac{m}{n} \frac{2}{3} \alpha_s(k_n) \tag{2.12}$$

(n is the principal quantum number, and m is the quark mass). At large values of k_n the orbital radius is small in comparison with the typical wavelength of the vacuum fluctuations, so that the following inequality holds:

$$| r D_\mu G_{\alpha\beta} | \ll | G_{\alpha\beta} |. \tag{2.13}$$

This inequality means that we can ignore all nonperturbative effects of higher order. The life of the quarks simplifies: They form a Coulomb system, but a system which is subject to a static external field. This field is (a) weak, (b) randomly oriented, and (c) chromoelectric [the corrections for the chromomagnetic field are smaller by two powers of $\alpha_s(k_n)$].

The problem formulated in this manner has an exact and elegant solution (based on an operator expansion); this solution was derived by Voloshin and Leutwyler.[76,82-84] They managed to find an analytic result for the level shifts:

$$M_{nl} = 2m - \frac{k_n^2}{m} \left(1 - \frac{m^2}{k_n^6} n^2 a_{nl} \left\langle \frac{\pi\alpha_s}{18} G^2 \right\rangle \right), \tag{2.14}$$

where l is the orbital angular momentum, and a_{nl} is a known coefficient function of order unity, say $a_{10} \approx 1.65$, $a_{20} \approx 1.78$, etc. (m is the so-called mass on the mass shell; see the discussion above). This expression can apparently be useful for the $t\bar{t}$ phenomenology. It is even more important from the theoretical standpoint, since it gives us a quantitative answer to the question of what are the quark masses at which the Coulomb picture is established.

The expansion parameter in (2.14) is evidently the combination

$$2 \frac{m^2}{k_n^6} n^2 \left\langle \frac{\pi\alpha_s}{18} G^2 \right\rangle,$$

which reaches a value of the order of unity for $b\bar{b}$. [We are thinking of the ground state; $k_1(b\bar{b}) \approx 0.96$ GeV at $\alpha_s(1\ \mathrm{GeV}) = 0.3$, i.e., $\Lambda_{\overline{MS}} = 100$ MeV]. For the lighter quarks the bonding force has nothing in common with the Coulomb interaction, and the latter is negligibly weak. In contrast, the heavier quarks form almost perfect Coulomb levels with very slight deviations. The family of Υ mesons falls somewhere in between: The Coulomb terms compete with the nonperturbative terms.

It is also instructive to consider the n dependence, which is very strong. Even the first excited level of $b\bar{b}$ is a completely non-Coulomb level. For $n = 2$ the lower boundary of the Coulomb region shifts towards $m \gtrsim 20$ GeV. At this quark mass, the number of excited levels below the continuum threshold is quite large,[85]

$$N \approx 2 \sqrt{\frac{m_t}{m_c}} \gg 7-8,$$

and we can observe the entire spectrum of dynamic scenarios in the same $Q\bar{Q}$ family.

The result for $\Gamma(1^3S_1 \to e^+e^-)$ is of much practical interest:

$$\Gamma = \Gamma_{Coul} \left| 1 + \frac{A_Z}{A_\gamma} \right|^2 \left(1 + \frac{m^2}{k_1^6} 4.93 \left\langle \frac{\pi\alpha_s}{18} G^2 \right\rangle \right). \tag{2.15}$$

Here we have given the expression derived by Voloshin; Leutwyler's expression is slightly different. Here Γ_{Coul} is the purely Coulomb width,

$$\Gamma_{Coul} = 4\pi (Q_q)^2 \frac{\alpha^2}{m^2} \frac{k_1^3}{\pi} \left(1 - \frac{16\alpha_s(m)}{3\pi}\right), \tag{2.16}$$

Q_q is the quark charge, and the correction factor $|1 + (A_Z/A_\gamma)|^2$ corresponds to the Z-boson contribution.[86,87] This factor is discussed in more detail in Section 6. Figure 12 shows a curve of the reduced width

$$\Gamma_{red} \equiv \Gamma(1^3S_1 \to e^+e^-) \left(Q_q^2 \left| 1 + \frac{A_Z}{A_\gamma} \right|^2 \right)^{-1}. \tag{2.17}$$

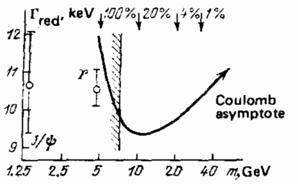

FIG. 12. Theoretical prediction of the reduced leptonic width (2.17) as a function of the quark mass. Experimental points for J/ψ and Υ are shown for comparison. The numbers at the top are the relative magnitudes of the $\langle G^2\rangle$ correction.

This curve is surprisingly flat at $m \gtrsim 10$ GeV, where the result can be believed.

Interestingly, the spin splittings can be evaluated by the same approach. The splitting in the ground state ($n=1$), for example is[88]

$$M(1^3S_1) - M(1^1S_0) = \frac{32\pi}{9}\frac{\alpha_s(m)}{m^2}\,|\psi_1(0)|^2 + \left\langle\frac{\pi\alpha_s}{18}G^2\right\rangle(4mk_1^2)^{-1}\cdot\frac{688}{153}, \quad (2.18)$$

where $\psi_1(0)$ is the wave function, which also contains two different terms,

$$|\psi_1(0)|^2 = \frac{k_1^3}{\pi}\left(1 + 4.93\,\frac{m^2}{k_1^6}\left\langle\frac{\pi\alpha_s}{18}G^2\right\rangle\right). \quad (2.19)$$

Substituting into (2.19) the mass of the b quark, $m_b = 4.8$ GeV, we find approximately 90 MeV. Unfortunately, at this mass the expansion parameter is poor, of the order of unity, and there is no reason to believe that the nonperturbative terms of higher order are negligible.

It was mentioned in Ref. 89 that the primary effect is simply a renormalization of $|\psi_1(0)|^2$, while the other nonperturbative contributions have an upper limit ($\lesssim 5$ MeV for $b\bar{b}$) under certain reasonable assumptions. On the other hand, $|\psi_1(0)|^2$ is known phenomenologically from $\Gamma(\Upsilon \to \mu^+\mu^-)$. We thus find

$$M_\Upsilon - M_{\eta_b} \approx 8\Gamma_{\mu\mu}(\Upsilon)\frac{\alpha_s(m_b)}{\alpha^2}\left(1 + 6{,}1\,\frac{\alpha_s(m_b)}{\pi}\right) + \left\langle\frac{\pi\alpha_s}{18}G^2\right\rangle(4mk_1^2)^{-1}\frac{688}{153} \approx 36\ \text{MeV}, \quad (2.20)$$

where we have also included the corrections $O(\alpha_s)$ found in Ref. 88. It is not surprising that this expression gives a reasonable result for J/ψ ($\sim 60 + 30 \approx 90$ MeV), although it cannot, of course, be used for charmonium.

It is perfectly clear that at this small mass difference an experimental search for η_b would be extremely difficult although definitely worthwhile.

Expressions analogous to (2.18) for the hyperfine splitting of the 1P and 2P levels were derived in Ref. 90. Unfortunately, as we go from $L=0$ to $L=1$ we find that the range of applicability of the method shifts rapidly toward larger masses. To save space we will not reproduce here the lengthy analytic expressions derived for

$$\Delta_1 = M(^1P_1) - M(^3P_0),\quad \Delta_2 = M(^1P_1) - M(^3P_1),\quad \Delta_3 = M(^3P_2) - M(^1P_1)$$

in Ref. 90. The approximation becomes valid at $m \gtrsim 40$ GeV, and for quarks with masses of 40–50 GeV occupying 1P levels we have $\Delta_1 \approx 9$ MeV, $\Delta_2 \approx 2$ MeV, and $\Delta_3 \approx 3$ MeV.

4) *Charmonium and bottonium*

The size of charmonium and bottonium is too large for direct use of the technique described above. An extremely powerful alternative approach is to use the quantum-chromodynamics sum rules, which yield accurate predictions for the lowest-lying levels with various quantum numbers in terms of fundamental parameters.

Let us consider, for example, the vector channel in charmonium. The spectral density R_c is defined in the standard way:

$$R_c = \sigma(e^+e^- \to \text{charm})\,[\sigma(e^+e^- \to \mu^+\mu^-)]^{-1},$$

where $\sigma(e^+e^- \to \text{charm})$ includes J/ψ, higher-lying resonances, and the charmed continuum. We know that, as it exists today, quantum chromodynamics cannot resolve the individual resonance structures; it predicts only an average cross section. We are thus forced to take a formal step backward from the situation discussed above. Instead of a specific level we now consider a weighted sum over many levels. If the weighting function is sharp enough, however, the sum may be essentially saturated by the lowest-lying state, and we will arrive at a (quari-) theory for such states.

In other words, everything depends on the method which we use to evaluate the integrals $\int R_c(s)f(s)\,ds$ with steep weighting functions (of the power-law or exponential type). The first systematic study of such integrals within the framework of quantum chromodynamics was undertaken in Refs. 91–93. Recent years have seen substantial progress in this direction. At present, the quantum-chromodynamics sum rules have been tested not only in heavy quarkonium but also in such problems as the spectrum of light mesons[73] and baryons,[94] the form factors at intermediate values of Q^2 (Ref. 95), three-particle coupling constants (e.g., $\rho\omega\pi$),[96] etc.

The basic theoretical steps are quite sample. We begin with a two-point function with suitable quantum numbers. For example, to analyze R_c we choose

$$\Pi_{\mu\nu} = i\int dx\,e^{iqx}\langle 0|\,T\{\bar{c}\gamma_\mu c(x),\ \bar{c}\gamma_\nu c(0)\}\,|0\rangle. \quad (2.21)$$

Figure 13(a) shows the simplest Feynman diagram contributing to $\Pi_{\mu\nu}$. Because of the much-celebrated asymptotic freedom, this diagram is the only one which survives in the deep Euclidean region of Q^2. The trivial smooth behavior of $R_c(s)$ at large s is unambiguously in accordance with asymptotic freedom.

When we go from the Euclidean region into the physical region, interactions which introduce an additional scale mass become important. Correspondingly, the

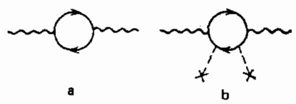

FIG. 13. a—Lowest-order diagram for the correlation function (2.21) (the solid lines represent heavy quarks and the wavy lines currents); b—coupling of the quarks with the vacuum fields (the dashed lines represent gluons).

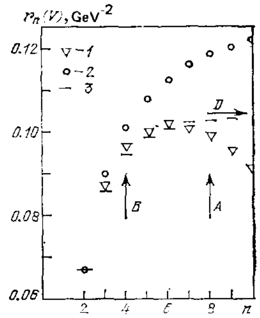

FIG. 14. Ratio of moments as a function of η [see (2.22) for a definition]. The arrow A marks the 20% level of the correction to $\langle G^2\rangle$; the arrow B marks the boundary between the regions of small and large experimental uncertainties (the uncertainty to the right of this arrow is $\lesssim 1\%$); the arrow D indicates the asymptotic value of r_n; 1—with powerlaw corrections; 2—without power-law corrections; 3—experimental.

smooth $R_c(s)$ curve becomes less smooth at lower values of s, and resonant structures arise.

Figure 13(b) shows the interaction which comes into play first and which is the most important. The crosses on the gluon lines mean that they are nonperturbative, and the diagram reduces to the vacuum expectation value (2.8), multiplied by a known function of Q^2 (Refs. 72–74 and 79).

On the other hand, $\Pi_{\mu\nu}$ can be expressed in terms of R_c through the use of a general dispersion relation. The properties of the resonances are thus related to the fundamental vacuum parameters.

We will not discuss "technical details." The interested reader is directed to the original papers.[71-74] To illustrate the characteristic features of the method, we reporduce a diagram from one of the first papers[73] (Fig. 14). This diagram gives us the ratio of moments

$$r_n = \int \frac{R_c(s)}{s^{n+1}}\,ds \cdot \left[\int \frac{R_c(s)}{s^n}\,ds\right]^{-1} = \frac{n^2-1}{n^2+(3/2)n}\,\frac{1}{4m_c^2}\left[1-\frac{(6n+14)\,n\,(n+1)\,(n+2)}{(2n+3)\,(2n+5)}\left\langle\frac{4}{9}\pi\alpha_s G^2\right\rangle\frac{1}{(4m_c^2)^2}+\dots\right] \quad (2.22)$$

as a function of n. At large values of n, all the contributions disappear except that of J/ψ, and we have $r_n \to M_\psi^{-2}$. In the theory as it exists today, of course, we cannot let n go to infinity mathematically, since the nonperturbative corrections "blow up." At $n=5-6$ (these numbers are already quite large from the physical standpoint), however, the term proportional to $\langle G^2\rangle$ is still under control. At these values of n, on the other hand, the J/ψ contribution exceeds 95%. The J/ψ mass is thus expressed in terms of the quark mass with an accuracy up to 1%.

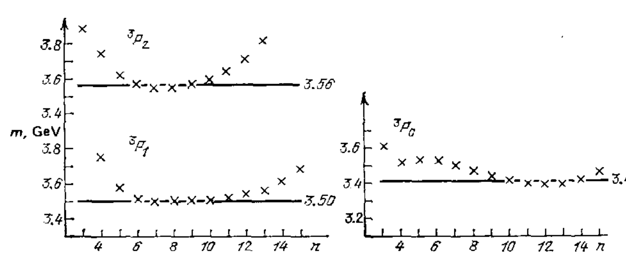

FIG. 15. Masses of the P levels of charmonium according to the quantum-chromodynamics sum rules, from Ref. 74.

Historically, the problem was inverted: The quark mass and $\langle G^2\rangle$ were adjusted to reproduce $M_{J/\psi}$. Armed with these parameters, we can generate some unambiguous predictions of the masses of the lowest-lying states with other quantum numbers. Recent results[74] derived for the P levels of charmonium are shown in Fig. 15. In all cases there is a stability plateau (related to the technical improvements of Ref. 74). The position of this plateau is in excellent agreement with the experimental masses. Reinders *et al.*[74] also predict the position of the relatively inaccessible $^1P_1$ level:

$$M(1^1P_1) = 3.51 \pm 0.01\ \text{GeV}. \quad (2.23)$$

This seems to be the most accurate and most reliable estimate available today, and apparently it will be confirmed after this level is discovered.[5)]

There are perhaps many who still remember the dramatic story of the η_c particle. It was first found in an inaccurate position (2.83 GeV), while the value found from the sum rules is[6)] (Ref. 72)

$$m_{\eta_c} = 3.00 \pm 0.03\ \text{GeV}.$$

The discovery of the 2.98-GeV state at Stanford[34,35] was therefore one of the greatest successes of quantum chromodynamics.

A corresponding analysis for the bottonium family is difficult to carry out, because the Coulomb interaction must be considered. All these difficulties were overcome, one by one, in Refs. 70 and 98, where a nonrelativistic version of the Borel technique was used. In this approach, the 1^- channel is considered first, and an accurate mass for the b quark is extracted (this mass has already been cited). The next step is to study the more complicated problem of the 1S-1P splitting. The final result for $M(1P)$ is[70]

$$M(1P,\ b\bar{b}) = 9.83 \pm 0.03\ \text{GeV}. \quad (2.24)$$

A similar value, 9.80 GeV, was derived by Bertlmann by a slightly different method.[71] Interestingly, the various potential models yield a wide range of predictions here, from 9.86 to 9.94 (with 9.90 being the most typical value; more on this below).

Hyperfine splitting in bottonium has also been studied by the sum-rule method; the following result has been

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[5)]We recall (Subsection 1a) that the state $^1P_1$ ($J^{PC}=1^{+-}$) cannot be observed neither as a resonance in a direct channel for e^+e^- annihilation nor in the decay $\psi' \to \gamma\, ^1P_1$. The cascade $\psi' \to \gamma\chi_2 \to \gamma\gamma\, ^1P_1$ is expected to be suppressed because of the small difference between the χ_2 and $^1P_1$ masses. So far, search for the transition[37] $\psi' \to \pi^0 P_1$ has been unsuccessful. Furthermore, the upper limit on $BR(\psi' \to \pi^0 P_1)$ is slightly lower than the existing, albeit rather crude, theoretical estimates.[37] The hope for discovering the $^1P_1$ level is presently pinned on the cascade $\psi' \to \gamma\eta_c' \to \gamma\gamma\, ^1P_1$.

[6)]X (2.83) immediately came under suspicion because of the large width expected for $J/\psi \to \gamma X$ on the basis of the simple estimate for the M1 transition.[97]

found[99]:

$$M_{\Upsilon} - M_{\eta_b} \approx 30 \text{ MeV}. \tag{2.25}$$

Here the gluon condensate plays an extremely modest role, $\lesssim 5$ MeV. Result (2.25) agrees very well with estimate (2.20) and is thus reliable.

Literally the same method was subsequently used in Ref. 74, but the result found there was $\Delta M \approx 60$ MeV. The apparent reason why this number is too high is that Coulomb corrections were ignored in Ref. 74.

The quantum-chromodynamics sum rules are insensitive to the positions of the radial excitations. As for the splitting of the ${}^3P_J$ levels, it can, in principle, be found by means of the sum rules, but this has not yet been done. In both cases we are thus forced to resort to a potential model. The bottonium family is analyzed exhaustively in Ref. 77, where relativistic corrections $\sim(v/c)^2$ are taken into account. That paper is essentially based on the potential of Eichten *et al.*[29] (a "funnel" potential). A Breit-Fermi Hamiltonian is constructed to take the spin dependence into account; the singularity at the origin associated with the Coulomb force is artifically smoothed out over distances $\lesssim 1/m_Q$. The level positions and the eigenfunctions are determined through a numerical solution of the equation with the Breit-Fermi Hamiltonian. McClary and Byers believe that the linear part of the potential is a Lorentz scalar: Only in this case can the spectrum in charmonium be reproduced. All the adjustable parameters are determined on the basis of charmonium, where the theoretical results agree with experiment within 20–30 MeV. The next step was to generate predictions for bottonium (Table IX). The agreement with experiment is very good in the cases of the 2^3S_1 and 2^3P_J levels. We can apparently expect roughly the same accuracy (10–20 MeV) for the 1P levels, whose center of gravity is at 9.90 GeV, according to Ref. 77.

The latter result and, especially, the sum-rule prediction (2.24) are strong arguments for doubting the observation of $\Upsilon'' \to (1P, b\bar{b}) + \gamma$ with $E_\gamma \approx 410$ MeV (the corresponding mass of the 1P level is 9.93 GeV; see Section 1).

The calculations of Ref. 77 exaggerate the 1^3S_1-1^1S_0 hyperfine splitting, yielding a result about three times

TABLE IX. $b\bar{b}$ bound states in the potential model of Ref. 77. Shown for each level is its position (in MeV) with respect to $\Upsilon(1\,{}^3S_1)$.

Level	Nonrelativistic approximation	With $(v/c)^2$ correction	Experimental[25,62]	Level	Nonrelativistic approximation	With $(v/c)^2$ correction	Experimental[25,62]
$3\,{}^3D_1$	1278	1231		$2\,{}^3P_1$	838	801	790±5
$4\,{}^3S_1$	1204	1160	1113±1	$2\,{}^3P_0$		761	773±8
$4\,{}^1S_0$		1133		$2\,{}^1P_1$		809	
$3\,{}^3P_2$		1091		$1\,{}^3D_1$	734	710	
$3\,{}^3P_1$	1120	1074		$2\,{}^3S_1$	585	560	560±0.3
$3\,{}^3P_0$		1037		$2\,{}^1S_0$		520	
$3\,{}^1P_1$		1081		$1\,{}^3P_2$		478	
$2\,{}^3D_1$	1026	991		$1\,{}^3P_1$	483	456	
$3\,{}^3S_1$	928	895	890.7±0.5	$1\,{}^3P_0$		407	
$3\,{}^1S_0$		864		$1\,{}^1P_1$		465	
$2\,{}^3P_2$		820	804±5	$1\,{}^1S_0$	0	−101	

the correct result [cf. (2.20)]. McClary and Byers[77] point out their predictions for the hyperfine splittings are not reliable, since they are sensitive to the particular method used to "smear out" the singularity at the origin. As usual in potential models, it is not possible to reproduce the lepton widths of the $n\,{}^3S_1$ levels (more on this below).

b) Leptonic and photonic decays

1) *Leptonic widths*

The same sum rules which have proved so useful in spectroscopy yield the leptonic widths of the ground levels. All the relativistic effects, the renormalizations, etc., are automatically taken into account. The result for J/ψ was known several years ago.[69,72,73] It is in excellent agreement with experiment, and further comments would seem superfluous. The situation with Υ is more "lively." Because of the large Coulomb factors which arise in the sum rules, the probability for this decay is very sensitive to the value of α_s. The theory requires[70]

$$\Gamma(\Upsilon \to \mu^+\mu^-) = 1.15 \pm 0.20 \text{ keV}$$

and

$$\alpha_s(1\,\text{GeV}) = 0.3 \pm 0.03, \tag{2.26}$$

which corresponds to $\Lambda_{\overline{MS}} \approx 100$–$150$ MeV. It is not possible to go beyond these limits, at least if our basic arguments are correct.

The experimental result for $\Gamma(\Upsilon \to \mu^+\mu^-)$ is (Table VI) $\Gamma(\Upsilon \to \mu^+\mu^-) = 1.17 \pm 0.05$ keV.

As for the quark-gluon coupling constant, we find that (2.26) again fits in well with the data available. The extensive results from deep inelastic lepton-photon and photon-photon scattering, the properties of three-jet events in e^+e^- annihilation, and several other characteristics (see Ref. 100, for example, for details) yield the following worldwide average[7)] for $\Lambda_{\overline{MS}}$:

$$\Lambda_{\overline{MS}} = 160^{+100}_{-80} \text{ MeV}. \tag{2.27}$$

This value is also in agreement with the value $BR(\Upsilon \to \mu^+\mu^-) = 3.3 \pm 0.5\%$ (discussed in detail in Ref. 99). It should be recalled that the quantum-chromodynamics sum rules[73] have never allowed values of α_s substantially larger than ~ 0.3 (just a few years ago it was still believed that $\Lambda \approx 500$–700 MeV).

In principle, the sum-rule method can be used to determine other widths, e.g., $\Gamma(\eta_c \to 2\gamma)$. This problem is slightly more complicated technically than in the case of the electron widths, since instead of the two-point functions it becomes necessary to analyze three-point functions of the type

$$\langle 0 | T\{\bar{c}(0)\gamma_5 c(0),\ \ j_\mu^{em}(x),\ \ j_\nu^{em}(y)\} | 0 \rangle. \tag{2.28}$$

For a more reliable prediction it is of course necessary to consider both the effects of the gluon condensate and ordinary gluon exchanges. The first steps in this direction were taken in Refs. 102, where the terms

[7)]See Ref. 101 for a determination of the parameter Λ_{MS}.

TABLE X. The ratios $\Gamma(n\,^3S_1 \to e^+e^-)/\Gamma(1\,^3S_1 \to e^+e^-)$ in the potential model.

Level	Buchmuller and Tye[29]	Martin[103]	Experimental[18]
$2S_1$, Υ'	0.44	0.51	0.46 ± 0.02
$3S_1$, Υ''	0.32	0.35	0.34 ± 0.02
$4S_1$, Υ'''	0.26	0.27	0.23 ± 0.02

$\sim G^2$ in the three-point function (2.28) were found. We see that the nonperturbative effects are important, and the $\eta_c \to 2\gamma$ width turns out to be slightly smaller (about 4.5 keV according to Ref. 102a) than in the simple-minded nonrelativistic model, which yields 6–6.5 keV. The value 4.5 keV is literally almost the same as the old estimate found[69] from the sum rules in their primitive form (i.e., without the gluon condensate). Of course, if the gluon condensate is omitted, it is not possible to evaluate the accuracy of the prediction reliably. The results of Ref. 102 make it possible to achieve the same degree of reliability as in the problem of the electron widths.

As mentioned earlier, the sum rules are insensitive to the radial excitations, and they do not give either their masses or their coupling constants with a satisfactory accuracy. The potential models are also unsatisfactory for estimating $|\psi(0)|^2$, which is the quantity determining $\Gamma(n^3S_1 \to e^+e^-)$ and $\Gamma(n^1S_0 \to 2\gamma)$. In particular, the values derived in Ref. 77 for $|\psi(0)|^2$ for Υ, Υ', and Υ'' are systematically twice as high as the experimental values. Furthermore, according to Ref. 77 the hyperfine interaction makes $|\psi(0)|^2$ for 1^1S_0 3–4 *times larger* (!) than for $1\,^3S_1$—a result in sharp contradiction of the sum-rule predictions (see the preceding paragraph). In the ratios $\Gamma(n^3S_1 \to e^+e^-)/\Gamma(1^2S_1 \to e^+e^-)$, however, the nonpotential effects apparently cancel out (Table X).

2) *Other methods and other directions*

The potential model has already been mentioned in connection with the various aspects of the physics of quarkonium. Another frequent approach is to use the traditional local duality,[104] which states that

$$\int_{s_0-\Delta s}^{s_0+\Delta s} \sigma_{phys}(s)\,ds = \int_{s_0-\Delta s}^{s_0+\Delta s} \sigma_{bare\ quark}(s)\,ds. \qquad (2.29)$$

We should not, however, demand more of these models than they can realistically provide.

It is important to realize that the potential describing the spectrum of charmonium and bottonium is none other than an *effective* potential. A genuine static energy might be manifested in highly excited levels (slightly below the continuum threshold). Here the distances between levels are small, and the quark frequencies are much lower than the characteristics frequencies of the gluon medium. In other words, in this case we are completely justified from the theoretical standpoint in speaking in terms of a potential.

The potential model is irreplaceable for rough estimates, and it gives us a good overall picture. However, it cannot (and does not have to) answer all the detailed questions, such as those regarding hyperfine splittings, the exact determination of the probabilities for leptonic decays, etc.

The simple-minded duality relations of the type in (2.29) are usually used to extract constants coupling the mesons with the various currents. We know that the amplitudes

$$\langle 0|\bar{c}\gamma_\mu c|J/\psi\rangle, \quad \langle 0|\bar{c}\gamma_\mu c|\psi'\rangle, \ldots$$

can be reproduced well in this way.

The origin of duality is completely transparent in the semiclassical treatment of the Schrödinger equation.[105] Duality is actually more general in nature and is explained on the basis that the interaction is turned off at short range. Let us consider a virtual photon with a high energy E, which converts into a $Q\bar{Q}$ pair. This conversion occurs at a distance of order E^{-1}, and its probability is proportional to $\sigma(e^+e^- \to \text{bare quarks})$. Only at much larger distances, $\sim E/\Lambda^2$, are nonconfinement effects completely in force. They serve as a large box which makes the spectrum discrete. The sum over close-lying discrete levels evidently reproduces $\sigma(e^+e^- \to \text{bare quarks})$ within terms of order E^{-k} $(k>0)$.

We thus see that local duality for highly excited states is a rather trivial fact. That local duality holds for the lowest-lying J/ψ state is more surprising, and it has been shown by the quantum-chromodynamics sum rules to be a consequence of the particular structure of the vacuum. We might note that by no means all the currents of the lowest-lying states obey local duality.

With increasing quark mass, the accuracy of (2.29) worsens. In the purely Coulomb situation, the Coulomb poles have to be added "manually" to the right side of (2.29), so that the procedure becomes almost meaningless.

It can be seen that the standard duality also fails for mesons with a high spin,[106] $J \geq 3$.

In many cases, quantum chromodynamics proposes a refined version of the simple equation (2.29):

$$\int_{\text{threshold}}^{\infty} \sigma_{\text{phys}}(s)\,f(s)\,ds = \int_{4m_c^2}^{\infty} \sigma_{\text{quark}}(s)\,f(s)\,ds \qquad (2.30)$$

with specific weighting functions $f(s) \sim s^{-n}$. Equations of this type are based on asymptotic freedom and dispersion relations.[93,69] The masses of the resonances and the position of the continuum threshold are inserted "manually"; the coupling constants of the resonances are the desired outcome. Khodjamirian[107] has studied the radiative transitions $\psi' \to \chi\gamma$ and $\chi \to J/\psi\gamma$, introducing the three-point functions

$$\int e^{-i(kx+qy)} \langle 0|T\{j_1(0)\,j_\mu^{em}(x)\,j_2(y)\}|0\rangle\,dx\,dy, \qquad (2.31)$$

where j_1 and j_2 are external currents with suitable quantum numbers, say $j_1 = \bar{c}c$ and $j_\mu^{em} = \bar{c}\gamma_\mu c$. In the Euclidean region there are two alternative expressions for matrix element (2.30) (Fig. 16), so that we have an overdetermined system of equations. Approximate solutions of these equations are shown in Table XI.

Shown for comparison in this table are the numbers

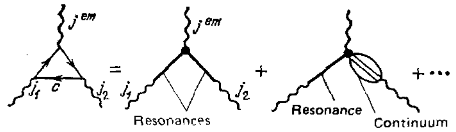

FIG. 16. Diagram representation of the quantum-chromodynamics duality relations.

found in the potential model.[77] The relativistic correction[77] is particularly large in $\psi' \to \chi_{0,1}\gamma$ transitions.

3) *Gamma transitions*

These transitions play a leading role, opening up the rich world of C-even levels of charmonium. Regarding the electric dipole transitions, nothing dramatic has occurred over the last few years in this theoretical arena. The results obtained in the potential models, by the dispersion approach, and from the nonrelativistic sum rules[108] (Thomas–Reiche–Kuhn, etc.) are coexisting peacefully in expectation of future development. Some of these results are shown in Tables XI and XII, which also show experimental data for comparison. We might note here that according to data obtained by the Crystal Ball group the quantity $\Gamma(\psi' \to \gamma\chi_J)_{\rm expt}/E_\gamma^3(2J+1)$ is roughly the same for all the χ_J states, specifically,[109] 1.00 ± 0.07, 1.05 ± 0.08, 1.37 ± 0.09 for $J = 0, 1, 2$ respectively. The corresponding data from the CUSB group for the bottonium family are[25]

$$\Gamma(\Upsilon'' \to 2\,{}^3P_J + \gamma)\,[E_\gamma^3(2J+1)]^{-1} = 1.03 \pm 0.5\,(J=0),\quad 1\,(J=1),\quad 0.95 \pm 0.3\,(J=2). \tag{2.31}$$

Behavior of this type fits in naturally with the nonrelativistic model of quarkonium. It should be recalled, however, that the relativistic corrections are large, especially in charmonium, so that we may be dealing with nothing more than a coincidence.

A few words are in order regarding M1 transitions. The allowed decays of the type $J/\psi \to \eta_c\gamma$ should be described very accurately by the very simple expression

$$\Gamma(1\,{}^3S_1 \to 1\,{}^1S_0 + \gamma) = \frac{16}{3}\mu^2\omega^3, \tag{2.32}$$

where μ is the Durac magnetic moment [μ = (the quark charge)$\cdot\sqrt{\alpha}/2m$]. On occasion we see the arbitrary assumption that the c and b quarks can have large anomalous magnetic moments; this assumption would of course invalidate (2.32). This assumption is incorrect. The derivation of (2.32) is controlled by the theory; furthermore, the corrections to it are calculable and are small. It can be shown that[110,111]

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{2}{9}\,\frac{\Gamma(\eta_c \to 2\gamma)}{\Gamma(J/\psi \to e^+e^-)}\,\alpha\,\frac{M_\psi^4}{M_{\eta_c}^3}\left(1 - \frac{M_{\eta_c}^2}{M_\psi^2}\right)^3(1 - 0.2\alpha_s), \tag{2.33}$$

TABLE XI. Radiative decays in charmonium (Γ, keV).

Decay mode	Ref. 107	Potential model[77]: Nonrelativistic approximation	Potential model[77]: With $\sim(v/c)^2$ corrections	Experimental[18,109]
$\psi' \to \chi_0\gamma$	8	45	19	21 ± 4
$\psi' \to \chi_1\gamma$	31	40	31	19 ± 4
$\psi' \to \chi_2\gamma$	31	27	27	17 ± 4
$\chi_0 \to J/\psi\gamma$	108	121	128	100 ± 40
$\chi_1 \to J/\psi\gamma$	160	250	270	< 700
$\chi_2 \to J/\psi\gamma$	136	362	347	330 ± 165

TABLE XII. Radiative transitions in bottonium.[17]

Transition	Relativistic value of $\langle r\rangle$, F	J	$\langle r\rangle$, F, with $(v/c)^2$ correction	$\Gamma_{\rm rad}$, keV	E_γ, keV	Experimental relative probability for the radiative transition
$3\,{}^3P_J \to 3\,{}^3S_1$	0.47	0	0.55	8.0	141	
		1	0.48	12	177	
		2	0.44	14	194	
$3\,{}^1P_J \to 3\,{}^1S_0$			0.39	15	215	
$3\,{}^3P_J \to 2\,{}^3S_1$	0.064	0	0.026	0.69	466	
		1	0.055	3.9	501	
		2	0.067	6.1	518	
$3\,{}^1P_1 \to 2\,{}^1S_0$			0.069	7.7	546	
$3\,{}^3S_1 \to 2\,{}^3P_J$	−0.52	0	−0.42	1.3 } BR (3S → 2P) = 29 ± 5 %	133	BR ($3\,{}^3S_1 \to 2\,{}^3P_J$) (34 ± 3 %)[25]
		1	−0.49	1.9	94	
		2	−0.53	1.8	75	
$3\,{}^1S_0 \to 2\,{}^1P_1$			−0.54	1.3	55	
$2\,{}^3P_J \to 2\,{}^3S_1$	0.34	0	0.39	12	199	
		1	0.35	16	238	
		2	0.32	17	257	
$2\,{}^1P_1 \to 2\,{}^1S_0$			0.28	18	285	
$2\,{}^3P_J \to 1\,{}^3S_1$	0.051	0	0.022	2.1	733	
		1	0.043	9.3	770	
		2	0.053	15	787	
$2\,{}^1P_1 \to 1\,{}^1S_0$			0.054	21	870	
$2\,{}^3S_1 \to 1\,{}^3P_J$	−0.33	0	−0.27	0.78 } BR (2S → 1P) = 10 ± 2 %	152	BR ($2\,{}^3S_1 \to 1\,{}^3P_J$) = 12 ± 3 %[18]
		1	−0.31	1.01	103	
		2	−0.33	0.96	82	
$2\,{}^1S_0 \to 1\,{}^1P_1$			−0.34	0.53	55	
$1\,{}^3P_J \to 1\,{}^3S_1$	0.19	0	0.20	26	399	
		1	0.20	36	446	
		2	0.20	39	467	
$1\,{}^1P_1 \to 1\,{}^1S_0$			0.16	46	550	

where

$$\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-) = 3Q_q^2\left[1 + O\left(\alpha_s, \frac{\mu_{\rm nonpert}}{m}\right)\right], \tag{2.34}$$

with a similar expression for Υ. It is easy to see why there are no large corrections to the magnetic moment in transitions of the type $J/\psi \to \eta_c\gamma$. Let us consider the amplitude for $\eta_c \to 2\gamma$, and let us write it in the form of a dispersion integral over one of the γ rays. Here J/ψ plays a dominant role. The other states are separated by a large "hole," $\delta^2 = 2M\Delta M$ (where $\Delta M \sim M\psi' - M\psi$), and their contribution is of the order of $\alpha_s(\delta^2)$. We thus find (2.33), and the corrections are determined by short distances.[110,111]

It has been naively expected that the ratio $\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-)$ would be approximately 4/3, so that we should have $\Gamma(J/\psi \to \eta_c\gamma) \approx 2.5$ keV. The perturbative corrections tend to reduce the ratio $\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-)$, giving us $(4/3)\cdot 1.12$ instead of 4/3, but the nonperturbative effects work in the opposite direction (see Subsection 1.1).

The minimum value allowed by the existing theory[102] is $\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-) = 0.8$. Here we would have $\Gamma(\eta_c \to 2\gamma) \approx 4$ keV and $\Gamma(J/\psi \to \eta_c\gamma) \approx 1.5$ keV. According to Ref. 25, on the other hand, we have $\Gamma(J/\psi \to \eta_c\gamma)_{\rm expt} = 0.76^{+0.23}_{-0.20}$ keV. The experimental result lies three standard deviations below the theoretical prediction, which seems absolutely reliable. In order to reproduce the value of 0.8 keV theoretically we would have to assume that $\Gamma(\eta_c \to 2\gamma)$ is half as large as we presently expect. We note that if $\Gamma(\eta_c \approx 2\gamma \approx 2$ keV we immediately run into trouble in other places[69]; in particular, we find monstrous violations of the Appelquist-Politzer recipe, which are completely beyond explanation.

In the $b\bar{b}$ family, the small mass difference $M_\Upsilon - M_{\eta b}$ hinders a search for the decay. The numbers are genuinely pessimistic:

$\Gamma(\Upsilon \to \eta_b\gamma) \sim 2$ eV and $\mathrm{BR}(\Upsilon \to \eta_b\gamma) \sim 10^{-4}$.

Unfortunately, it is no easier to reach η_b by starting from Υ'.

Decays of the type $\Upsilon' \to \eta_b\gamma$ and $\psi' \to \eta_c\gamma$ are forbidden in the nonrelativistic limit. The decay of ψ' has been seen experimentally with a width[16,35] $\Gamma(\psi' \to \eta_c\gamma) = 0.6 \pm 0.2$ keV. The deviations from the nonrelativistic approximation must therefore be important. What does the theory have to say about all this?

It has recently been argued that[111] the transition $\psi' \to \eta_c\gamma$ results primarily from an admixture of gluons in the ψ' wave function. The argument runs as follows: If local duality prevails, then the amplitude

$$A(\gamma \to \psi')\, A(\psi' \to \eta_c\gamma) \tag{2.35}$$

can be replaced, in the dual sense, by

$$A(\gamma \to \text{quarks, gluons}) \cdot A(\text{quarks, gluons} \to \gamma). \tag{2.36}$$

The latter product can of course be calculated easily, so that we can in turn find $A(\psi' \to \eta_c\gamma)$. The theoretical result is compatible with $\Gamma(\psi' \to \eta_c\gamma)_{\text{expt}}$. More importantly, the numerically greatest contribution to (2.36) comes from the intermediate state $c\bar{c}g$. It is natural to interpret this circumstance as a gluon admixture in ψ'. At the same time, a rather large width $\Gamma(\psi'' \to \eta_c\gamma)$ is predicted:

$$\Gamma(\psi'' \to \eta_c\gamma) \sim 1 \text{ keV}. \tag{2.37}$$

This width is far larger than in the standard potential models, where the transition $\psi'' \to \eta_c\gamma$ is strongly forbidden. Unfortunately, the latter estimate is not amenable to a simple experimental test, since the corresponding relative probability is less than $5 \cdot 10^{-5}$.

For Υ' the decay probability is suppressed in comparison with that of $\psi' \to \eta_c\gamma$ by at least the following factor:

$$\frac{1}{4}\left(\frac{M_\psi}{M_\Upsilon}\right)^2 \left(\frac{\alpha_s(\Upsilon)}{\alpha_s(\psi)}\right)^2 \cdot 0.8 \sim \frac{1}{100}. \tag{2.38}$$

Here the ratio of coupling constants is a measure of the gluon admixture, while the 0.8 reflects the phase volume. Combining this result with $\Gamma(\psi' \to \eta_c\gamma)_{\text{expt}}$, we find

$$\Gamma(\Upsilon' \to \eta_b\gamma)_{\text{theo}} \leqslant 8 \text{ eV}. \tag{2.39}$$

The corresponding relative probability is less than $5 \cdot 10^{-4}$.

c) Hadronic decays

1) *Heavy quarkonium and "old" hadrons*

The questions which we have discussed up to this point deal primarily with heavy quarks and their relationships with the surrounding vacuum medium. We now turn to another fundamental aspect of the problem: the relationship between heavy quarkonium and "old" hadrons. Theoretical and experimental research in this field yields information on the structure of the $Q\bar{Q}$ system, glueballs, and the traditional "old" hadrons. In many cases this is unique information, unobtainable by other methods.

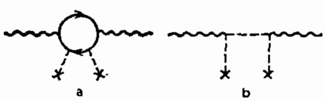

FIG. 17. Effects associated with the gluon condensate in the correlation functions of the quark and gluon currents. In the gluon case (b) we are dealing with Born diagrams. In the quark case (a), the diagrams necessarily contain loops. Each additional loop introduces a suppression ~$1/16\pi^2$.

2) *Inclusive hadronic decays*

The famous Appelquist-Politzer recipe[14] prescribes a calculation for the elementary processes

$$Q\bar{Q} \to 2g,\ 3g \quad \text{or} \quad q\bar{q}g$$

instead of a summation over a large number of exclusive channels. This remarkable invention is applicable, beyond any shadow of a doubt, to asymptotically heavy states $Q\bar{Q}$. Here, however, we are interested in charmonium and bottonium, and the various preasymptotic corrections in this case can be large.

The Appelquist–Politzer recipe presupposes an ideal gluon-hadron duality. For light quarks, 9 GeV^2 ($\approx M^2_{\eta_c}$), this is actually already the asymptotic region, where the hadronic cross section is equal to the quark cross section. Can the same be said for gluons?

The transition to the asymptotic regime is determined by nonperturbative effects, as could hardly be doubted today. The nonperturbative effects are radically different in the quark and gluon channels. The gluon currents are coupled to the vacuum fields much more strongly than the quark currents are (Fig. 17; see Ref. 112 for further details), and as a result the asymptotic regime begins for gluons at high energies.

To determine quantitatively just what this energy boundary is, is by no means a simple problem. Nevertheless, some estimates have been published. Novikov *et al.*[112] have shown that

$$(s_0)_{2\ \text{gluon}\ J^P=0^-} = 6-16 \text{ GeV}^2, \tag{2.40}$$

where s_0 is the boundary of the asymptotic region. If this result is correct, then the charmonium family is dangerously near the critical zone or in fact is inside it. It would thus not be surprising to find some (moderate) deviations from the perturbative equations for $c\bar{c}$ annihilation. On the other hand, $b\bar{b}$ annihilation should be described very accurately by these formulas.

Here is what experiment indicates: The Appelquist-Politzer recipe apparently holds for charmonium and bottonium in the 1^- and 2^+ channels but is violated for the C-even levels of charmonium with $J^P = 0^\pm$. An anomalously late onset of the asymptotic regime—precisely in spin-zero channels—has been predicted theoretically.[112] The reason lies in the very strong interaction of gluons with direct instantaon fluctuations in the vacuum, which would be possible only with a zero total spin.

We will first show that $\mathrm{BR}(\Upsilon \to \mu^+\mu^-)_{\text{expt}}$ confirms (among other things) the validity of the Appelquist-Politzer prescription.

Starting with $\Gamma(\Upsilon \to \mu^+\mu^-) = 1.17 \pm 0.05$ keV and $\mathrm{BR}(\Upsilon \to \mu^+\mu^-) = 3.3 \pm 0.5\%$. we find $\Gamma_{\text{tot}}(\Upsilon) = 35.5 \pm 7$ keV

TABLE XIII. Widths of the C-even levels of charmonium.

	η_c	η_c'	χ_0	χ_1	χ_2
Γ, MeV	12.4 ± 3.4	< 8 (95 % c. l.)	16.5 ± 1.5	$1.7^{+0.3}_{-1.7}$ 0.75 ± 0.3 *)	$2.1^{+1.0}_{-0.7}$ **)

*The relation $\Gamma(\chi_1 \to \gamma_1 J/\psi) = \Gamma(\chi_0 \to \gamma_0 J/\psi)\left(\frac{k_{\gamma_1}}{k_{\gamma_0}}\right)^3$ is assumed.

**The values of $\Gamma_{\chi J}$ in this table correspond to the inelusive decay $\psi' \to \gamma\chi J$; from the cascade $\psi' \to \gamma\chi_2 \to \gamma\gamma J/\psi \to \gamma\gamma l^+l^-$ we find $\Gamma_{\chi_2} = 4 \pm 2$ MeV (Ref. 109).

or $\Gamma_{\mathrm{direct.hadr}}(\Upsilon) = 27 \pm 6$ keV. We equate the hadronic and gluonic widths and use the formula

$$\frac{\Gamma_{3g}(\Upsilon)}{\Gamma_{\mu\mu}(\Upsilon)} = \frac{10}{81}\frac{\pi^2-9}{\pi}\frac{\alpha_s^3(m_b)}{\alpha^2 Q_b^2}\left[1 + (1.1 \pm 0.5)\frac{\alpha_s}{\pi}\right], \tag{2.41}$$

where the correction $O(\alpha_s)$ (in the $\overline{\mathrm{MS}}$ scheme) was found in Ref. 113. Hence we can determine the quark-gluon constant: $\alpha_s(m_b) = 0.156 \pm 0.013$. The standard renormalized-group formula yields $\alpha_s(m_c) = 0.210 \pm 0.028$ here, which in turn gives us

$$\frac{\Gamma_{3g}(J/\psi)}{\Gamma_{\mu\mu}(J/\psi)} = 15.5 \pm 6. \tag{2.42}$$

The agreement with the experimental value of 9.2 ± 2.4 is completely satisfactory.

On the other hand, we now know the hadronic widths for η_c and χ_0 (Table XIII). Their γ widths have been more or less fixed theoretically[69,110] (~4.5 and 4.5–5.5 keV, respectively), and we can compare the ratio of $\Gamma_{\mathrm{hadr}}(\eta_c)/\Gamma(\eta_c \to 2\gamma)$ and $\Gamma_{\mathrm{hadr}}(\chi_0)/\Gamma(\chi_0 \to 2\gamma)$ with

$$\Gamma(c\bar{c} \to 2g)/\Gamma(c\bar{c} \to 2\gamma). \tag{2.43}$$

A very ambitious effort has been undertaken to incorporate the first-order perturbative correction in ratios of the type in (2.43) (Refs. 113–115). In the case of η_c, for example, the following result has been found[114]:

$$\frac{\Gamma(c\bar{c} \to 2g)}{\Gamma(c\bar{c} \to 2\gamma)} = \frac{9}{8}\left(\frac{\alpha_s(m_c)}{\alpha}\right)^2\left(1 + 9{,}2\frac{\alpha_s}{\pi}\right), \tag{2.44}$$

where m_c is the quark mass, and the coefficient in front of α_s depends on the renormalization procedure [(2.44) corresponds to the $\overline{\mathrm{MS}}$ scheme].

For $\alpha_s(m_c) = 0.2$ the correction $O(\alpha_s)$ reaches 50%: large but not dangerous in the sense that the perturbation series is apparently still controlled by the theory and is not "blowing up."

Substituting the two-γ widths, we find $\Gamma_{\mathrm{hadr}}(\eta_c) \approx 6$ MeV and a corresponding result for χ_0: 6.2–7.6 MeV. The experimental data (Table XIII) systematically run twice as high as these values. We might note that if $\Gamma(\eta_c \to 2\gamma) \approx 2$ keV, as we conclude from $\Gamma(J/\psi \to \eta_c\gamma)$ (see the discussion above), then the discrepancy reaches a factor of four in the case of η_c—a possibility which is difficult to imagine.

Finally, in the tensor channel (i.e., for χ_2) the Appelquist-Politzer recipe works exceedingly well, giving us 1.7–2.3 MeV for $\Gamma_{\mathrm{hadr}}(\chi_2)$, while the experimental χ_2 width is $\Gamma(\chi_2)_{\mathrm{expt}} = 2.1 \pm 1$ MeV (Table XIII). We thus have an anomalous situation only for spin zero. As mentioned earlier, the possibility of this situation has been predicted theoretically.[112]

Turning now to the technical points, we will write down a few expressions[114,115] whose derivation required lengthy calculations:

$$\frac{B(0^{-+})}{B(0^{++})} = \begin{cases} 1 + 0.9\frac{\alpha_s}{\pi}(c\bar{c}) \\ 1 + 2.1\frac{\alpha_s}{\pi}(b\bar{b}), \end{cases} \quad \frac{B(2^{++})}{B(0^{++})} = \begin{cases} 1 + 6.5\frac{\alpha_s}{\pi}(c\bar{c}), \\ 1 + 4.0\frac{\alpha_s}{\pi}(b\bar{b}), \end{cases} \tag{2.45}$$

where $B(J^{PC}) = \Gamma(J^{PC} \to 2\gamma)/\Gamma(J^{PC} \to \mathrm{gluons})$. These ratios are convenient in that the result does not depend on the renormalization procedure. A result found recently[113] is of much practical importance:

$$\Gamma(Q\bar{Q},\ 1^- \to \text{gluons}) = \Gamma_0 \begin{cases} 1 - (3.8 \pm 0.5)\frac{\alpha_s}{\pi}(c\bar{c}), \\ 1 - (4.2 \pm 0.5)\frac{\alpha_s}{\pi}(b\bar{b}). \end{cases} \tag{2.46}$$

For further details and illustrative calculations we refer the reader to the paper by Barbieri *et al.*,[88] which gives a good review of the entire topic.

3) *Hadronic transitions between levels of quarkonium*

Decays of the type

$$\psi' \to J/\psi\pi\pi \quad \text{or} \quad \Upsilon' \to \eta\Upsilon \quad \text{etc.}, \tag{2.47}$$

serve as a probe of the gluon content of ordinary hadrons. Transitions of the type in (2.47) may be regarded as processes which occur in two steps: First there is an emission of soft gluons by heavy quarks at relatively short range, and then the gluons convert into heavy hadrons at a relatively long range. Since the dimension of quarkonium is small in comparison with that of the old hadrons, we can make systematic use of the well-known multipole expansion to describe the gluon emission.[116-119]

By working from nothing more than a factorization (and the symmetry properties of the transition amplitudes), one can find a set of predictions regarding the relative widths, e.g.,[118]

$$\left.\begin{array}{l} d\Gamma(2\,{}^3S_1 \to 1\,{}^3S_1 + 2\pi) = d\Gamma(2\,{}^1S_0 \to 1\,{}^1S_0 + 2\pi), \\ d\Gamma(1\,{}^3D_3 \to 1\,{}^3S_1 + 2\pi) = d\Gamma(1\,{}^1D_2 \to 1\,{}^1S_0 + 2\pi), \\ \cdots\cdots\cdots\cdots\cdots\cdots \end{array}\right\} \tag{2.48}$$

More intriguing is the unique opportunity for testing the low-energy theorems of quantum chromodynamics. Within the framework of the multipole expansion the following expressions hold[120]:

$$A(n_i\,{}^3S_1 \to n_f\,{}^3S_1 + \pi\pi) = C_1\langle 0|\mathbf{E}^a\mathbf{E}^a|\pi\pi\rangle + \text{higher multipoles}$$
$$A(n_i\,{}^3S_1 \to n_f\,{}^3S_1 + \eta) = C_2\langle 0|[\mathbf{E}^a\mathbf{H}^a]|\eta\rangle + \text{higher multipoles}$$

where $\mathbf{E}^a$ and H_a are the chromoelectric and chromomagnetic fields, respectively, and the coefficients C_1 and C_2 embody information about heavy quarkonium. These coefficients are proportional to each other, so that all the unknown factors cancel out from the ratio of amplitudes. Furthermore, by adopting some model or other for quarkonium we can find these factors explicitly.[121]

At first glance it would appear impossible to calculate such nontrivial matrix elements as

$$\langle 0|\mathbf{E}^a\mathbf{E}^a|\pi\pi\rangle, \quad \langle 0|[\mathbf{E}^a\mathbf{H}^a]|\eta\rangle,$$

which reflect a conversion of gluons into mesons at long range. Surprising though it may be, this can be done by working from only first principles. These matrix elements are related to the so-called triangle anomalies in the trace of the energy-momentum tensor and in the divergence of the axial current. The results are so attractive that a few examples are worth looking at. The matrix element $\langle 0|E^aE^a|\pi\pi\rangle$, for example, reduces to some combination of the following quantities[120,122]: $m^2_{\pi\pi}$, $b, \rho^G(\mu)$, $\alpha_s(\mu)$ (b is the first coefficient in the Gell-Mann–Low function, ρ^G is the fraction of the pion momentum corresponding to gluons, and μ is a normalization point, of the order of the reciprocal radius of quarkonium).

The ratio $\Gamma(\psi' \to J/\psi\pi\pi)/\Gamma(\psi' \to J/\psi\eta)$ was calculated in Ref. 120 and found to be in excellent agreement with experiment. The following is predicted for bottonium:

$$\frac{\Gamma(\Upsilon' \to \Upsilon\eta)}{\Gamma(\Upsilon' \to \Upsilon\pi\pi)} \approx 4\cdot 10^{-3}\left(\frac{M_{\Upsilon'}-M_{\Upsilon}-M_{\eta}}{10\ \mathrm{MeV}}\right)^3, \quad \frac{\Gamma(\Upsilon'' \to \Upsilon\eta)}{\Gamma(\Upsilon'' \to \Upsilon\pi\pi)} \approx 2\cdot 10^{-2}.$$

The shape of the pion spectrum can also be described theoretically[122]:

$$\frac{d\Gamma}{dq^2} \sim \left\{\left[q^2-\varkappa(\Delta M)^2\left(1+\frac{2m_\pi^2}{q^2}\right)\right]^2+\frac{\varkappa^2}{5}[(\Delta M)^2-q^2]^2\left(1-\frac{4m_\pi^2}{q^2}\right)^2\right\}, \tag{2.50}$$

where $q^2 \equiv m^2_{\pi\pi}$, $\Delta M = M(Q\bar{Q})_f - M(Q\bar{Q})_i$, $\varkappa = (b/6\pi)\alpha_s(\mu)\rho^G(\mu) \approx 0.2$ for charmonium. The second term, $\varkappa^2/5$, in (2.50) results from the D-wave contribution, so that we have a theoretical explanation for the suppression of the D wave ($\varkappa^2/5 \sim 1/125$). The value of $\varkappa$, we might note, is not universal: In $\Upsilon' \to \Upsilon\pi\pi$, this parameter should be smaller by a factor of about 1.5–2.

The dimension of Υ'' is quite large, so that the accuracy of the approach is slightly poorer in the transition $\Upsilon'' \to \Upsilon\pi\pi$. Nevertheless, we can expect $\varkappa \sim 1/5$. Ignoring small corrections in (2.51), we then find

$$\sqrt{\frac{d\Gamma}{dq^2}} \sim q^2 - Cm_\pi^2, \quad C = 3-4. \tag{2.51}$$

A linear increase with q^2 up to 0.3 GeV^2, in complete agreement with (2.51), has been observed in the decays $\psi' \to J/\psi\pi^+\pi^-$ and $\Upsilon' \to \Upsilon\pi^+\pi^-$. If the multipole expansion works, then we should find precisely the same behavior of $\sqrt{d\Gamma/dq^2}$ in this region of q^2 in the decay $\Upsilon'' \to \Upsilon\pi^+\pi^-$ also. We see from Fig. 9 that the experimental results tend instead to support $\sqrt{d\Gamma/dq^2} \approx$ const. It is difficult to explain this behavior of the pion spectrum theoretically. Voloshin has proposed one possible explanation: The pion spectrum in the decay $\Upsilon'' \to \Upsilon\pi\pi$ might be flat if near Υ'' there was a bottonium level of a molecular type, which we call X, with the quantum numbers $I = 1$ and $J^P = 1^+$. The decay would then proceed through the chain

$$\begin{array}{l} \Upsilon'' \to X_{\mathrm{virt}} + \pi, \\ \quad\ \ \llcorner\!\!\longrightarrow \Upsilon + \pi. \end{array}$$

Because of the proximity of the X pole, a multipole expansion would be ruled out. Does this explanation pass an experimental test? Time will tell.

For very heavy quarks forming Coulomb levels the quarkonium coefficients C_1 and C_2 in (2.49) can be calculated, so that the absolute widths are fixed unambiguously. The real c and b quarks, however, are not heavy enough, and we are forced to resort to models. Kuang and Yan[121] have offered one model. Possibly their most interesting result is an unexpected suppression of the $\Upsilon'' \to \Upsilon\pi\pi$ transition because of cancellation in C_1. It turns out that $\Gamma(\Upsilon'' \to \Upsilon\pi\pi)$ must be smaller than $\Gamma(\Upsilon' \to \Upsilon\pi\pi)$, despite the increased phase volume.

As discussed in Subsection 1b, we have the average value $\mathrm{BR}(\Upsilon'' \to \Upsilon\pi^+\pi^-) = 4.9 \pm 1.1\%$. In other words, we have $\Gamma(\Upsilon'' \to \Upsilon\pi^+\pi^-) \sim 1$ keV, which should be compared with $\Gamma(\Upsilon' \to \Upsilon\pi^+\pi^-)_{\mathrm{expt}} = 5.2 \pm 1.5$ keV. The suppression is obvious, and it shows that the model of Kuang and Yan[121] is not bad, at least at a qualitative level.

The most important conclusion which we would like to draw here is that the multipole expansion is applicable.

If the reader is not yet fully convinced of this by the facts that we have presented, we will add a few more words about $\Upsilon' \to \Upsilon\pi\pi$. According to the multipole expansion, we have

$$C_1 \sim \langle n_f\, {}^3S_1|(t_1^a - t_2^a)\, r_i G_{(8)}(e_n)\, r_i (t_1^a - t_2^a)|n_i\, {}^3S_1\rangle,$$

where $G_{(8)}$ is the nonrelativistic Green's function of a color-octet state. In other words, we would expect

$$\frac{\Gamma(\Upsilon' \to \Upsilon\pi\pi)}{\Gamma(\psi' \to J/\psi\pi\pi)} \approx \left(\frac{\langle r^2\rangle_{\Upsilon'}}{\langle r^2\rangle_{\psi'}}\right)^2 \approx \frac{1}{16}, \tag{2.52}$$

or $\Gamma(\Upsilon' \to \Upsilon\pi\pi) \approx 6.8$ keV. This expectation agrees very well with experimental data (as discussed above).

Hadronic transitions are very promising in yet another regard. The cascade

$$\begin{array}{l} \Upsilon' \to 1\,{}^1P_1 + \pi\pi \\ \quad\ \ \llcorner\!\!\longrightarrow 1\,{}^1S_0 + \gamma \end{array}$$

is the best place for detecting two inaccessible levels of $b\bar{b}$ at once: ${}^1P_1$ and ${}^1S_0$. The bottleneck in this chain is the first decay, for which the relative probability is small. According to Kuang and Yan,[121] this probability is about 1%, but further work will be required to shed the theoretical uncertainties of Ref. 121. Once the ${}^1P_1$ level has been reached, the η_b problem is solved: Nearly every other decay of ${}^1P_1$ is

$${}^1P_1 \to {}^1S_0 + \gamma.$$

The latter comment deals with decays involving a breaking of isotopic symmetry, $\psi' \to J/\psi\pi^0$ or $\Upsilon' \to \Upsilon\pi^0$. These decays furnish direct estimates of the masses of the current quarks. More precisely, the following theorem holds[123]:

$$\frac{\Gamma(\psi' \to J/\psi\pi^0)}{\Gamma(\psi' \to J/\psi\eta)} = \left(\frac{3\sqrt{3}}{4}\frac{m_d - m_u}{m_s}\right)^2 \left(\frac{|\mathbf{p}_\pi|}{|\mathbf{p}_\eta|}\right)^3 \left[1 + O\left(\frac{m_q}{\mu}\right)\right], \tag{2.53}$$

where m_q is the mass of the u, d, or s quark; and μ is a scale value for the strong interactions (several hundred MeV). A corresponding relation of course holds for Υ.

According to the Crystal Ball data,[33] the ratio of the widths on the left side of (2.53) is $(4.1 \pm 1.5)\cdot 10^{-2}$, from which we find

$$\frac{m_s}{m_d - m_u} = 29 \pm 6.$$

This result seems to be consistent with the standard values of the quark masses: $m_d = 7.5$ MeV, $m_u = 4$ MeV, and $m_s = 150$ MeV [here $m_s/(m_d - m_u) \approx 42$].

4) $J/\psi \to \gamma + light\ hadrons$, $\Upsilon \to \gamma + light\ hadrons$

As discussed in Subsection 1a 2, the observed shape of the γ spectrum in the decay $J/\psi \to \gamma$ + light hadrons differs from that predicted by the quark-gluon model (Fig. 3b), while the integral probabilities are approximately equal.

The primary questions here are as follows: Which states saturate the integral probability? At what value of $m^2_{\text{light hadr}}$ is the parton regime established? The last parameter—the boundary of the asymptotic region—is an important dynamic characteristic. We know from e^+e^- annihilation that for light hadrons this boundary lies near 1.5 GeV^2, but there are reasons to believe that this result is not of universal applicability, Novikov *et al.*[112] have presented arguments which move the boundary in the gluon spectrum upward along the energy scale, to $s_0 \gtrsim 6$ GeV^2 [see (2.40)]. If this is correct, the actual gluon continuum could hardly be studied in the radiative decays of J/ψ, and the study of this continuum would become a problem for Υ physics ($x < 0.94$).

On the other hand, the production of resonances can be studied far more easily by starting with J/ψ. It is usually assumed that the gg pair materializes in the form of a glueball, primarily a 2^+ glueball. This conclusion seems to be based on the perturbative analysis of Ref. 124. We wish to emphasize that in exclusive decays of the type

$$J/\psi \to \gamma + \text{meson} \tag{2.54}$$

the situation is by no means as simple. In the $O^{\pm}$ channels, the direct nonperturbative fluctuations effectively mix the quark and gluon degrees of freedom, so that the production of quark mesons is not suppressed at all.[112]

For 2^+, the nonperturbative mixing is slight, but there is another effect, which is frequently forgotten. The gluons are emitted in the course of an annihilation over a distance $\sim 1/m$. In other words, the gluon source reduces to $\theta^G_{\mu\nu}(m)$, where m designates the normalization point, and $\theta^G_{\mu\nu}$ is the gluon part of the energy-momentum tensor. On the other hand, the characteristic virtuality in the meson wave function is of order R^{-1}_{conf} (several hundred MeV), and we must take into account the evolution from m up to R^{-1}_{conf}. As a result we find the standard logarithmic mixing

$$\theta^G_{\mu\nu}(m) \Rightarrow \theta^G_{\mu\nu}(R^{-1}_{\text{conf}}) + \varepsilon\theta^q_{\mu\nu}(R^{-1}_{\text{conf}}), \tag{2.55}$$

where the mixing parameter ε is of the order of unity (Fig. 18). In particular, this result tells us why the classical quark meson f is produced in reaction (2.54) without any significant suppression.

Nevertheless, the final hadronic state in this decay is enriched by a variety of unusual states, beyond any doubt.

The states $\iota(1440)$ and $\theta(1640)$ are presently being discussed as candidates for blueballs (Table II). Let us discuss these states in turn. The arguments advanced for a gluonium interpretation of ι are as follows[16,125]: the fact that the lowest lying pseudoscalar quark nonet is full, the large probability for the J/ψ

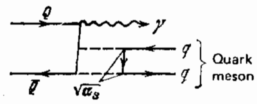

FIG. 18. Exclusive radiation annihilation of $J/\psi\,(\Upsilon)$. The additional factors of $\sqrt{\alpha_s}$ [cf. Fig. 3(b)] are offset by the large logarithm $\ln M^2R^{-2}_{\text{conf}}$, which arises from the integration over the loop.

$\to \iota(1440)$ transition (in comparison with ordinary mesons, other than η'), and the dominance of the decay channel $\iota \to \delta\pi$.

We find these arguments unconvincing (more on this below), and we are not inclined to consider ι to be a glueball, since the sum rules predict[112] that pseudoscalar gluonium should lie considerably higher, $M(0^-$, glue$) = 2$–2.5 GeV. If this is the case, the ι meson might be a radial excitation of η'.

The relatively large width $\Gamma(J/\psi \to \iota\gamma)$ seems natural according to standard duality:

$$\frac{\Gamma(J/\psi \to \iota\gamma)}{\Gamma(J/\psi \to \eta'\gamma)} = \left|\frac{\langle 0 | j_P | \iota\rangle}{\langle 0 | j_P | \eta'\rangle}\right|^2 \left(\frac{|\mathbf{p}_\iota|}{|\mathbf{p}_{\eta'}|}\right)^3, \tag{2.56}$$

where

$$j_P = \frac{3\alpha_s}{8\pi} G^a_{\mu\nu} G^a_{\alpha\beta}\varepsilon_{\mu\nu\alpha\beta}.$$

Introducing the gluon-current correlation function

$$P(q^2) = i\int d^4x e^{iqx}\langle 0 | T\{j_P(x),\ j_P(0)\} | 0\rangle,$$

we know that η' is the dual of the interval 0–5 GeV^2 in the corresponding spectral density,

$$|\langle 0 | j_P | \eta'\rangle|^2 = \frac{1}{\pi}\int_0^5 \operatorname{Im} P_0(s)\, ds, \quad \operatorname{Im} P_0(s) = \frac{9s^2}{8\pi^3}\alpha_s^2(s).$$

Even if the ι meson is the dual of the next interval of width 1 GeV^2 (i.e., if the duality interval stretches from 5 to 6 GeV^2), its residue in j_P would be approximately the same as that of η', and the ratio (2.56) would be approximately one.

As for $\theta(1640)$, the status of tensor gluonium for this meson would not be in contradiction of the sum rules of quantum chromodynamics. An estimate of $M(2^+$, glue$)$ yields values ranging from 1.3 to 2 GeV.

It should, of course, be noted, that a problem arises from the decays $\theta \to 2\pi$ and $\theta \to 2\eta$. If we assume that θ is a unitary singlet and that the amplitude for its decay is SU(3)-symmetric, we find

$$\Gamma(\theta \to \pi^+\pi^- + \pi^0\pi^0) \approx 4\cdot 3\Gamma(\theta \to \eta\eta), \tag{2.57}$$

where the factor of 4 on the right side stems from the phase volume and the D-wave nature of the decays. Experimentally we have $\Gamma(\theta \to 2\pi) \lesssim \Gamma(\theta \to 2\eta)$ (Table III). Both of the original suggestions which led to (2.57) have now actually been refuted. According to Ref. 126, the approximate equality of the θ and f′(1515) masses means that we cannot ignore the mixing of these mesons, and as a result we effectively have some mixture of strange quarks in the θ wave function. Furthermore, the amplitudes for the decays into a pair of Goldstone mesons do not obey $SU(3)_{\text{fl}}$. The $\theta_{\eta\eta}$ amplitude is enhanced by the significant admixture of gluons in the η

wave function; these gluons penetrate into η through the $\eta\eta'$ mixing. For the same reason, the decay $\theta \to \eta\eta'$ should be extremely important.[127]

It is in general an extremely complicated matter to establish the gluon nature of a meson. All the tests which have been proposed in the literature [(a) a small total width; (b) a large signal in $J/\psi \to \gamma X$; (c) a singlet according to $SU(3)_{fl}$ and a unitary symmetry of the decay amplitudes] have been negated by counterexamples.[25]

Among the other J/ψ and Υ decay modes we should mention $\gamma\eta$ and $\gamma\eta'$. The ratio of the corresponding widths reduces to[128]

$$\frac{\Gamma(J/\psi \to \eta'\gamma)}{\Gamma(J/\psi \to \eta\gamma)} = \left| \frac{\langle 0 \mid \alpha_s G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \mid \eta' \rangle}{\langle 0 \mid \alpha_s G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \mid \eta \rangle} \right|^2 \left(\frac{|p_{\eta'}|}{|p_\eta|} \right)^3.$$

While the denominator is determined by the symmetry properties exclusively, the numerator contains a great deal of nontrivial information about the relationship between the gluons and η'. Several models predict results[128-133] for $\langle 0 | \alpha_s G\tilde{G} | \eta' \rangle$. From the experimental Crystal Ball result (Table II)

$$\frac{\Gamma_{\eta'\gamma}}{\Gamma_{\eta\gamma}} = 4.7 \pm 0.6$$

we find

$$\left\langle 0 \left| \frac{3\alpha_s}{4\pi} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} \right| \eta' \right\rangle = M^2_\eta \ (120 - 140 \text{ MeV}), \tag{2.58}$$

in agreement with the quantum-chromodynamics estimates.[112]

3. UNCONCEALED-FLAVOR MESONS

Up to this point we have been discussing systems of the $Q\bar{Q}$ type, where Q is the heavy quark c, b, or t. The flavor of the quark in these mesons is a latent flavor, masked by the corresponding antiquark. It is perfectly obvious from the theoretical standpoint (and confirmed by experiment) that hadrons with an unconcealed flavor must exist. If we consider only those mesons which include a heavy quark, there are 12 essentially different combinations:

$$(c\bar{u},\ c\bar{d},\ c\bar{s}),\quad (b\bar{u},\ b\bar{d},\ b\bar{s}),\quad (t\bar{u},\ t\bar{d},\ t\bar{s}),\quad (c\bar{b},\ c\bar{t},\ b\bar{t}). \tag{3.1}$$

In the last three cases, where we are dealing with bound states of two heavy quarks, the theory of quarkonium does not differ substantially from the $Q\bar{Q}$ theory. Any question dealing with $c\bar{b}$ or $c\bar{t}$ is resolved to the same extent that it is resolved for $c\bar{c}$, and we learn essentially nothing new. The numerical estimates of course appear different, and a reader interested in some particular quantity is forced to go through the calculations again, using one of the methods described above (Section 2). We will not pursue this question in detail here; we simply note that the $b\bar{t}$ system appears to be a very curious one from the standpoint of electroweak effects.

In contrast, analysis of mesons of the $Q\bar{q}$ type can yield important new information about the structure of the quantum-chromodynamics vacuum. In the hierarchy of hadrons, these mesons occupy a place between the traditional old hadrons, on the one hand, and charmonium and bottonium, on the other. Their structure is simpler than that of light hadrons, since the heavy quark serves as a static center, rendering certain problems trivial. These mesons cannot, however, serve as a probe for studying effects which are inaccessible (or nearly so) in charmonium and bottonium, for example, the quark condensate in the vacuum.

The theoretical description of $Q\bar{q}$ has several distinguishing features. In general, which problems are interesting in connection with these mesons? First, there is the spectrum, or the splitting of levels with quantum numbers $J^P = 0^\pm, 1^\pm$, etc. We will see below that the lowest-lying are pseudoscalar mesons, giving us the situation to which we have long been accustomed. The D and B pseudoscalar mesons thus decay exclusively through the weak interaction. A calculation of their total widths brings together the weak decays of quarks and strong-interaction effects, which are particularly important for $c\bar{q}$. The simplicity of the asymptotic formulas ($m_Q \to \infty$) stands in contrast with the complexity of the preasymptotic corrections. The situation seems to be considerably clearer in the case of inclusive semileptonic decays. Finally, there is another interesting aspect of the problem: decays of the type $D(B) \to l\nu$ (where $l = e,\ \mu$, or τ). The quantum-chromodynamics sum rules can yield the constants for these decays, f_D (f_B), quite accurately.

We will discuss all these topics briefly. It should be noted that yet another direction, identified recently, is now developing rapidly, both experimentally and theoretically, becoming an industry all by itself. We have in mind the exclusive nonleptonic decays of D mesons (see Subsection 1a3). In principle, these decays can yield a rich store of information about the dynamics of quarks and gluons. Unfortunately, to attempt anything approaching an adequate discussion of this material would lead us far astray, so we will have to be content with some fragmentary comments.

a) Spectrum

The masses of mesons with unconcealed charm and beauty have been discussed in terms of the sum rules in Refs. 134 and 135. In contrast with charmonium and bottonium, the result depends dramatically on the quark condensate in vacuum:

$$\langle \text{vac} \mid \bar{\psi}\psi \mid \text{vac} \rangle \neq 0, \tag{3.2}$$

where $\psi = u$, d, or s. The existence of a quark condensate (3.2) has been known for many years,[136] but it has previously appeared only in pion physics.

Figure 19 shows the diagrams which are predominant in the current correlation function. As an example here we have chosen a current with the quark content ($c\bar{u}$). A cross on a quark or gluon line means, as usual, that

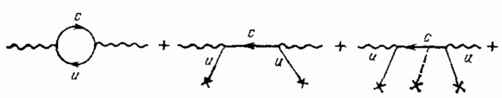

FIG. 19. Diagrams corresponding to the correlation function $i\int e^{ipx} d^4x \langle 0 \mid T\{\bar{c}\Gamma u(x),\ \bar{u}\Gamma c(0)\} \mid 0 \rangle$, where $\Gamma = 1,\ \gamma_5$, or γ_μ.

TABLE XIV. Masses of mesons containing the quark $b\bar{u}$ (or $b\bar{d}$). The theoretical uncertainty is about 100 MeV.

Quantum numbers J^{PC}	Mass, GeV	
	Ref. 134	Ref. 135
0^{-+}, 1^{--}	5.3	5.2
0^{++}, 1^{++}	6.1	6.0

the corresponding field is the vacuum field, i.e., forms a vacuum condensate.

In the limit $m_Q \to \infty$ the states differing only in the direction of the quark spin are obviously degenerate. In other words, $m(J^{PC}=0^{-+}) = m(J^{PC}=1^{--})$ and $m(J^{PC}=0^{++}) = m(J^{PC}=1^{++})$. With decreasing m_Q, a spin dependence arises. To get an idea of the magnitude of the effect, we review the results found by Reinders *et al.*[134] for the $b\bar{q}$ system. According to Ref. 134, the splitting of the pseudoscalar from the vector is about 50 MeV. A similar number is found for the splitting of the scalar and the axial vector.[8)] Both of these predictions still await experimental confirmation.

In the $c\bar{q}$ system we have some good data on D and D* mesons.[31] The experimental D*-D mass difference is about 140 MeV. Unfortunately, the analysis of the quantum-chromodynamics sum rules for this case has not yet been completed, and the only thing we have for comparison with this 140 MeV is the prediction of the nonrelativistic model.[137]

As for orbital excitations, we see a picture of unanimity in the quantum-chromodynamics theory. In both Refs. 134 and 135 it has been mentioned that there is a strikingly large quark-condensate effect, which induces large splittings between states with opposite parity, i.e., $\Delta m_0 = m(J^P=0^{-+}) - m(J^P=0^{++})$ and $\Delta m_1 = m(J^P=1^{--}) - m(J^P=1^{++})$. The results of Refs. 134 and 135 are summarized in Table XIV (the numbers have been rounded slightly). The technical details differ (a nonrelativistic Borel treatment in one case and a moment technique in the other), but the predictions agree well within the theoretical uncertainty, of the order of 100 MeV. The mass difference between the states with positive and negative parity is 0.8 GeV. In the $c\bar{q}$ system we should expect an even greater splitting Δm_0 and Δm_1. By way of comparison we recall that the potential model predicts only 0.5 GeV in this case.[137] Accordingly, the question of an experimental search for a scalar partner of the D meson appears extremely important.

We wish to emphasize again that the analysis based on the quantum-chromodynamics sum rules requires nothing in the way of a model: The fundamental vacuum parameters translate directly into the language of observable quantities.

8)Potential models[29] also predict a similar result, of about 50 MeV.

b) Constants of purely leptonic decays

Decays into an $l\nu$ pair are possible for mesons with the quark contents $c\bar{s}$ (the F meson), $c\bar{d}$ (the D^+ meson), and $b\bar{u}$ (the B^- meson). The second decay is suppressed in comparison with the first in the Cabibbo sense; the ratio of probabilities is proportional to $\tan^2\theta_C$ (more on this below). We know that the transition of a pseudoscalar particle into a left-hand neutrino and a charged lepton is proportional to the mass of the lepton for kinematic reasons, so that decays into a $\tau\nu$ pair are the most probable. For example,

$$\frac{\Gamma(D^+\to\tau^+\nu)}{\Gamma(D^+\to\mu^+\nu)} = \left(\frac{m_\tau}{m_\mu}\right)^2 \left[\frac{1-(m_\tau^2/m_D^2)}{1-(m_\mu^2/m_D^2)}\right]^2 \approx 2.5, \tag{3.3}$$

where the factor in square brackets reflects the difference in phase volumes. In the cases of F^+ and B^-, the dominance of τ becomes absolute.

The branching ratio BR $[D(B)\to\tau\nu]$ is determined by both the width of the transition to $\tau\nu$ and the total width of the decaying meson. Putting aside the question of the total width for the moment, we consider the absolute value of $\Gamma[D(B)\to\tau\nu]$.

It is convenient to introduce the constants f_D and f_B, which parametrize the corresponding amplitudes by analogy with the well-known f_π:

$$\langle D(B)|\bar{Q}\gamma_\mu\gamma_5 q|0\rangle = -if_{D(B)}p_\mu. \tag{3.4}$$

For the relative probabilities we then find

$$\begin{aligned} \mathrm{BR}(D^+\to\tau^+\nu) &= 2.5\cdot10^{-4}\left(\frac{f_D}{f_\pi}\right)^2, \\ \mathrm{BR}(F^+\to\tau^+\nu) &= 3.3\cdot10^{-2}\left(\frac{f_F}{f_\pi}\right)^2, \end{aligned} \tag{3.5}$$

where we have set the total width of D^+ to be $\Gamma_{tot}(D^+) = 1.25\cdot10^{12}$ s^{-1}, which is close to the experimental value (see Subsection 1a3.), and we have assumed $\Gamma_{tot}(F^+) \approx \Gamma_{tot}(D^+)$ (more on this below). We might also note here that by studying the secondary-particle distribution in the cascade

$$\begin{aligned} &F^+\to\tau^++\nu_\tau, \\ &\qquad\ \ \lfloor\!\longrightarrow \pi^+(\rho^+)+\bar{\nu}_\tau \end{aligned} \tag{3.6}$$

one can, in principle, find a limitation on the ν_τ mass.[138]

A key question here is just how greatly different the constants f_D and f_B are from $f_\pi \approx 133$ MeV [the constants f_D and f_F are equal within $O(10\%)$]. This question can be answered by analyzing the two-point functions induced by the current $\bar{Q}\gamma_\mu\gamma_5 q$ in the sum-rule method. We omit the details (the procedure is completely standard) and immediately write the result from Ref. 135:

$$f_D \approx 200 \text{ MeV}, \quad f_B \approx 140 \text{ MeV}. \tag{3.7}$$

Some similar sum rules were studied in Ref. 134, where a slightly larger value was found for f_B. That result can apparently be attributed to an underestimate of the role played by the continuum, whose threshold was put too high in Ref. 134. At any rate, these numbers are far smaller than those which were proposed back during the time of the theoretical speculation which was stimulated in an effort to explain the "D^+/D^0 anomaly." As mentioned in Section 1 (Subsection a3), there seems to be no need for such hypotheses.

The result in (3.7) agrees well with the phenomenological trend. We do in fact find $f_D/f_\pi \approx 1.5$. On the other hand, we know from experiment that increasing the mass of one of the quarks from 0 to 150 MeV (we have in mind the s quark) leads to an increase in the constant by a quarter:

$$\left(\frac{f_K}{f_\pi}\right)_{\text{expt}} \approx 1.27. \tag{3.8}$$

It is instructive to compare (3.7) with results based on other principles. The predictions[139] based on the simple bag model, for example, are several times larger. Further experiments will of course identify the correct model.

If we assume $f_D \approx f_F \approx 340$ MeV, we find

$$\mathrm{BR}\,(D^+ \to \tau\nu) \approx 6\cdot 10^{-4},\quad \mathrm{BR}\,(F^+ \to \tau^+\nu) \approx 0{,}07.$$

c) Total widths

If $m_Q \to \infty$, and if we are not interested in specific modes but instead carry out a summation over all modes, then the parton model is completely adequate. The inclusive probability for the $Q\bar{q}$ decay is determined simply by the probability for the weak transition of the Q quark into three light fermions, while the $\bar{q}$ quark plays the role of a passive spectator. The process is illustrated in Fig. 5, where we have $c \to su\bar{d}$, $s\mu^+\nu, se^+\nu$, if we ignore the Cabibbo-forbidden channels.

The virtual hard gluons (Fig. 20) slightly enhance the nonleptonic modes, but even at $m_c = 1.5$ GeV the effect is small, ~10% in the probability, and the effect becomes even weaker with increasing quark mass. The main point is that the logarithmic corrections for the hard gluons have been studied thoroughly, and they hold no surprises for us (see Ref. 7, for example).

If we ignore the logarithmic renormalizations altogether, we can find the lifetime of the c quark from the muon life-time by a simple scaling:

$$\tau_c = \frac{1}{5}\left(\frac{m_\mu}{m_c}\right)^5 \tau_\mu, \tag{3.9}$$

where the factor of 1/5 reflects the circumstance that the number of allowed modes is 5 ($se\nu, s\mu\nu, su_i\bar{d}_i$, where $i = 1, 2, 3$ is the color index). Here $\mathrm{BR}(D \to e\nu X) \approx 0.2$, in good agreement with experiment in the case of D^+.

Which quark mass should we substitute into this expression: the current mass or the constituent mass? Or perhaps the mass of the D meson? This question can be answered properly only if we know the preasymptotic nonperturbative corrections. In the asymptotic limit $m_c \to \infty$ the difference between the current and constituent masses of the c quark and the mass of the D meson would be insignificant, but back in the real world

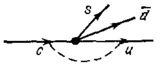

FIG. 20. Diagram of the decay $c \to su\bar{d}$ taking hard virtual gluons into account. Heavy point—weak Hamiltonian.

the corresponding numbers are 1.40, 1.55, and 1.86 GeV. The spread in the estimates for τ_c in two extreme cases is $(1.86/1.40)^5 \approx 5$. An even greater uncertainty would arise if we attempted to incorporate the mass of the s quark in the final state. In using the constituent quark, $m_s \approx 500$ MeV, we would reduce the accessible phase volume by a factor of two.

The sole reason for all this discussion has been to demonstrate the acute need for at least a rough understanding of the preasymptotic nonperturbative effects. These are the effects which convert the current mass, which is important at short range, into the constituent mass, and they cause the greatest deviations from the parton model. Unfortunately, we do not yet have a systematic analysis of effects of this sort based on quantum chromodynamics. We will briefly discuss here one of the corrections (the interference correction), which is formally suppressed by powers of $1/m_c$ but actually turns out to be of the order of 0.5 in D-meson decays because of the large numerical factors. We know quite accurately that some of the corrections do exist and may not be small, but the theoreticians do not know even the appropriate direction to take to estimate them.

This unsatisfactory theoretical situation has led to the suggestion that the asymptotic formulas be completely abandoned, at least in the case of the D mesons, and that Γ_{tot} be calculated by summing over all the accessible two-particle and quasi-two-particle modes.[140] The theoretical result is in reasonable agreement with the experimental lifetime, and, more significantly, $\Gamma_{\text{tot}}(D^+)$ and $\Gamma_{\text{tot}}(D^0)$ are roughly equal. For further details the reader is directed to the original paper.[140]

Nevertheless, such a radical step as abandoning the quark-parton formulas for D mesons seems unnecessary. Lacking a systematic theoretical framework, we can rely on our intuition, which suggests that it is the mass of the constituent c quark, $m_c = 1.55$ GeV, which should appear in (3.9), and the mass of the strange quark in the final state can be ignored. We then have

$$\Gamma^{-1}_{\text{tot}}(D^+) \approx \Gamma^{-1}_{\text{tot}}(D^0) \approx 6.5\cdot 10^{-13}\ \text{s}, \tag{3.10}$$

in excellent agreement with recent data[26] (see Subsection 1a). Inspired by this success, we might expect that the lifetime of the F meson is similar,[9)] $6.5\cdot 10^{-13}$ s. What is apparently happening is that the nonperturbative corrections with different phases interfere destructively, and their resultant effect reduces primarily to a renormalization of m_c.

In the case of the B mesons the question of which mass to substitute into an expression like (3.9) is totally unimportant, since the uncertainty is comparatively small. On the other hand, information about the quark mixing angles is still quite incomplete (Subsection 4a). As discussed in Section 1 (Subsection b2), experiment suggests that the dominant transition is of the b quark to a c quark, not a u quark. The relative probability

[9)]Since we are assuming that the nonleptonic decays of F are determined by the diagram in Fig. 5(a), the final state in the nonleptonic decays contains an $s\bar{s}$ pair and is thus rich in η, $K\bar{K}$, ...

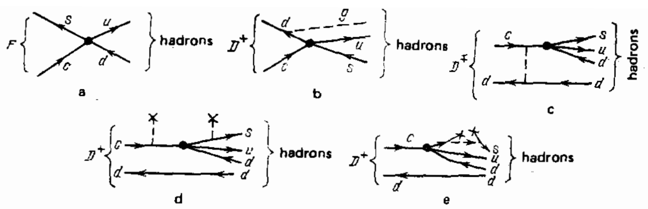

FIG. 21. Illustrative diagrams corresponding to corrections $\sim 1/m_q^2$ to the decays of heavy mesons.

for the semileptonic decay is $\mathrm{BR}(\mathrm{B}\to e\nu X)\approx 15$–$16\%$, depending on the particular assumptions regarding the masses m_c and m_b (Refs. 67 and 141).

d) Preasymptotic effects

As mentioned earlier, the lifetime of a meson in the limit $m_Q\to\infty$ agrees with the lifetime of the Q (Figs. 5, 11, and 20). There are a set of corrections of various natures which fall off in proportion to some power of $1/m_Q$. Some examples are shown in Fig. 21. Some of the diagrams depend on the flavor of the spectator [Figs. 21(a) and 21(b)]. It is this contribution which is responsible for the differences among the D^+, D^0, F, etc., lifetimes. In other diagrams [Figs. 21(c)–21(e)] the flavor of the spectator is unimportant. As always, a cross on a line represents an interaction with vacuum fields. The diagrams in Figs. 21(d) and 21(e) describe growth of the quark mass due to gluon and quark condensates—in intuitive language, a conversion of the current mass into constituent mass.

In this subsection of the paper we will concentrate on a very simple effect associated with the Pauli principle. We return to the diagram in Fig. 5, and we first assume that we are dealing with the D^+ meson. In integrating over the phase volume of the newly produced $\bar{d}$ we then unavoidably enter a region of momentum already occupied by the spectator $\bar{d}$ quark. The Pauli principle forbids two identical quarks from occupying the same position in phase space. Nothing of the sort occurs in the D^0 case, where the spectator quark is $\bar{u}$. The Pauli principle is of course in force only in a bounded part of the phase space, where the momenta of the $\bar{d}$ quarks are roughly the same, while the total phase volume increases rapidly with m_Q. In the specific case of the decays of the D mesons, however, this "bounded" region amounts to a substantial fraction of the entire phase volume.

Figure 22 shows some Feynman diagrams illustrating

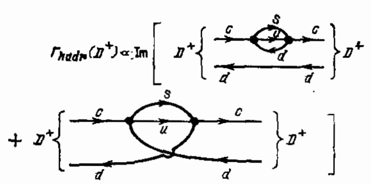

FIG. 22. Interference contribution to the hadronic width of the decay of the D^+ meson.

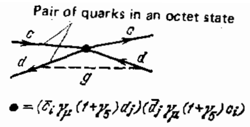

FIG. 23. Unfactorized contribution of four-fermion operators to the D-meson matrix element.

the interference contributions.[10)]

Omitting the details of the calculations, we write down the answer, which was derived several years ago by M. B. Voloshin and one of the present authors (M. Sh.):

$$\begin{aligned}\Gamma_{\mathrm{hadr}}(\mathrm{D}^+)=\frac{1}{m_\mathrm{D}}\Big\langle \mathrm{D}^+\Big|&\frac{G_F^2 m_c^5}{64\pi^3}\frac{1}{2}\bar{c}(1+\gamma_5)c\\&-\frac{G_F^2}{4\pi}m_c^2\Big\{\frac{1}{2}(C_+^2+C_-^2)[\bar{c}_i\gamma_\mu(1+\gamma_5)d_j][\bar{d}_j\gamma_\mu(1+\gamma_5)c_i]\\&+\frac{1}{2}(C_+^2-C_-^2)[\bar{c}\gamma_\mu(1+\gamma_5)d][\bar{d}\gamma_\mu(1+\gamma_5)c]\Big\}\Big|\mathrm{D}^+\Big\rangle,\end{aligned}\tag{3.11}$$

where the $C_\pm$ are known coefficients describing the renormalization of the operators

$$(\bar{c}s)_L(\bar{d}u)_L\pm(\bar{c}u)_L(\bar{d}s)_L$$

by hard gluons (see Ref. 7, for example).

Let us assume for a minute that the quarks do not have color degrees of freedom, and we can write $C_+=C_-=1$. Using

$$\langle \mathrm{D}^+|\bar{c}c|\mathrm{D}^+\rangle\approx 2m_\mathrm{D},\quad \langle \mathrm{D}^+|\bar{c}\gamma_\mu\gamma_5 d|0\rangle=-if_\mathrm{D}p_\mu,$$
$$\langle 0|\bar{d}\gamma_\mu\gamma_5 c|\mathrm{D}^+\rangle=if_\mathrm{D}p_\mu,$$

we would conclude that the interference term reduces the decay probability, in complete accordance with the Pauli principle:

$$\begin{aligned}\{\Gamma_{\mathrm{hadr}}(\mathrm{D}^+)\}_{\mathrm{colorless}}&\approx\frac{G_F^2m_c^5}{64\pi^3}-\frac{G_F^2}{4\pi}m_c^2\frac{1}{m_\mathrm{D}}\langle \mathrm{D}^+|(c\gamma_\mu\gamma_5 d)(\bar{c}\gamma_\mu\gamma_5 d)^+|\mathrm{D}^+\rangle\\&\approx\frac{G_F^2m_c^5}{64\pi^3}-\frac{G_F^2}{4\pi}m_c^2m_\mathrm{D}f_\mathrm{D}^2.\end{aligned}\tag{3.12}$$

It turns out that if we "turn on" the color indices the result is no longer unambiguous. The point is that now the two $\bar{d}$ quarks, even with identical momenta, can be in different color states, so that the interference can be both destructive and constructive. We should also take into account the circumstance that the coefficients $C_\pm$ are different from one[7]: $C_+^2\approx 1/\sqrt{2}$, $C_-^2\approx 2$.

As for the magnitude of the effect, we note that it depends on the values of the matrix elements of the four-fermion operators with respect to the D meson.

A simple-minded factorization (i.e., a saturation by the vacuum in the intermediate state) may not be very good from the quantitative standpoint, since the unfactorized contribution (Fig. 23) is of the order of the factorized contribution according to Ref. 143.

If we nevertheless use a factorization just to get a rough estimate, we find that the increment in $\Gamma_{\mathrm{hadr}}(\mathrm{D}^+)$ is positive (a constructive interference!):

$$\frac{\Delta\Gamma}{\Gamma_{\mathrm{parton}}}=16\pi^2\frac{f_\mathrm{D}^2}{m_c^2}\Big[\frac{1}{2}(C_-^2-C_+^2)-\frac{1}{6}(C_-^2+C_+^2)\Big]\approx\frac{16\pi^2}{3}\frac{f_\mathrm{D}^2}{m_c^2}.\tag{3.13}$$

[10)]Interference effects are analyzed in a similar spirit in Refs. 142, but we do not agree with the final results.

In the limit $m_c \to \infty$, we find $f_D^2/m_c^2 \to 0$. In the real world, this factor is $\sim 10^{-2}$, but this suppression is offset by the numerical factor ($16\pi^2/d \sim 50$. Where do such large numbers come from? The source is completely transparent: The parton mechanism corresponds to a three-particle decay of a quark, while the interference mechanism corresponds to a two-particle decay. The ratio of (dimensionless) phase volumes is of the order of $4\pi^2$. For the same reason, the annihilation diagram is generally enhanced (Fig. 21a). (Although this diagram is chirality-forbidden for pseudoscalar mesons.)

Let us summarize. A natural scale value for the preasymptotic power-law corrections, or at least some of them, is

$$4\pi^2 \frac{f_Q^2}{m_Q^2} \approx \begin{cases} 0.5 & \text{for F and D mesons,} \\ 0.05 & \text{for B mesons.} \end{cases}$$

In $c\bar{q}$ decays we could in principle expect significant deviations from the parton predictions ($\lesssim 100\%$). In ($b\bar{q}$) decays and, especially, ($t\bar{q}$) decays, the parton picture should hold very accurately.

e) Exclusive weak decays

With minor exceptions, the theoreticians have so far concentrated on the decays of D mesons. The situation is extremely complicated here. On the one hand, experiments reveal a large number of distinct modes, but the statistical and systematic errors of the measurements are still large.[31] The theoretical understanding, on the other hand, is, unfortunately, also inadequate. Although a rather large number of models have been proposed,[144,145] each contains some element of uncertainty, and the connection with first principles is not always clear. The various models are reviewed in Ref. 46.

To illustrate the nontrivial nature of the problems, which essentially convert into guesses regarding the quark-gluon dynamics, we will cite a single example, the ratio of widths

$$\frac{\Gamma(D^0 \to \bar{K}^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)}. \tag{3.14}$$

If we adopted a simple quark model, we would describe the amplitudes for these decays by the diagrams in Fig. 24. For the widths ratio we would then find

$$\frac{\Gamma_{\bar{K}^0\pi^0}}{\Gamma_{K^-\pi^+}} = \frac{1}{2}\cdot\frac{1}{9}\,\frac{(2C_+ - C_-)^2}{\left(\frac{2}{3}C_+ + C_-\right)^2}, \tag{3.15}$$

where the factor of 1/2 is isotopic, while the factor of 1/9 results from the color (the ud pair, converting into π^+, is in a color-white state from the very outset, and this state can be distinguished in the decay to $K^0\pi^0$ only by losing a factor three in the amplitude). The coefficients C_+ and C_-, associated with hard gluons, were

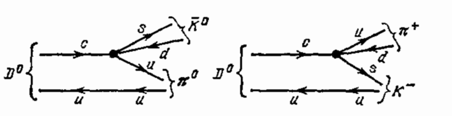

FIG. 24. Diagrams corresponding to the decays $D^0 \to \bar{K}^0\pi^0$ and $D^0 \to K^-\pi^+$ (in the approximation of the decay of a c quark).

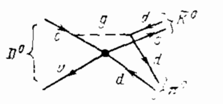

FIG. 25. Preasymptotic contribution to the decay $D^0 \to \pi^0\bar{K}^0$.

given in Ref. 7: $(2C_+ - C_-)^2/[(2/3)C_+ + C_-]^2 \approx 2\cdot 10^{-2}$. According to the simple quark model we would thus have $\Gamma_{\bar{K}^0\pi^0}/\Gamma_{K^-\pi^+} \sim 10^{-3}$. Experimentally, we find BR($D^0 \to \bar{K}^0\pi^0$) and BR($D^0 \to K^-\pi^+$) to be quantities of the same order of magnitude. The contrast with the theoretical expectation is striking. One possibility is that we should consider diagrams of the type in Fig. 25, which, according to Chernyak and Zhitnisky,[144] are numerically enhanced and resolve the situation.

If we still do not have a reliable theory for the exclusive weak decays of charmed particles, we might ask just how good our chances are of finding such a theory in the future. There are many problems in the field of the traditional strong interactions which have eluded solution for a long time. Our own opinion is that definite progress is extremely likely. Our optimism springs from the circumstance that we have a large parameter, the mass of the c quark, and the skillful use of this parameter might significantly improve the quality of the theoretical description.

4. HEAVY QUARKS AND WEAK INTERACTIONS

The decays of heavy quarks present an excellent opportunity for studying the structure of weak interactions. These decays not only permit us to study the standard six-quark model and quark mixing angles but also shed some light on the properties of such exotic entities as Higgs bosons and the axion.

In this section we will briefly discuss the information on the quark mixing parameters. This was the subject of a recent review,[5] where the reader can find a detailed discussion and an exhaustive bibliography. We will also discuss the decays of heavy hadrons which result in the production of Higgs bosons and the axion. Useful information on this topic is given in Refs. 10, 146, and 147. Finally, the weak neutral currents of heavy quarks are dealt with in a special subsection. We will have essentially nothing to say about the violation of CP invariance and the mixing of heavy neutral mesons; we direct the interested reader to Refs. 7 and 5.

a) Phenomenology of quark mixing

The discovery of the b quark (and of the third charged lepton, τ) led to a natural replacement of the Glashow-Illiopoulos-Maiani four-quark model[148] by the Kobayashi-Maskawa six-quark model[149] with three left-handed quark doublets,

$$\binom{u}{d'}_L,\ \binom{c}{s'}_L,\ \binom{t}{b'}_L,\quad q_L = \frac{1}{2}(1+\gamma_5)\,q \tag{4.1}$$

and the right-handed singlets

$$u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R,\quad q_R = \frac{1}{2}(1-\gamma_5)\,q; \tag{4.2}$$

here t is the sixth, as yet undiscovered, quark. At present, a lower limit $m_t > 18.3$ GeV is set on its mass.[20, 21]

The weak charged quark current j_μ can be written in general as

$$j_\mu = (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L, \tag{4.3}$$

where U is a 3×3 unitary matrix ($UU^+ = 1$). The matrix U is frequently taken in the Kobayashi-Maskawa parameterization[149] in terms of the three Euler rotation angles ($\theta_1, \theta_2, \theta_3$) in flavor space and the phase δ associated with the violation of CP invariance[11)]:

$$U = \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} \mathrm{d} & \mathrm{s} & \mathrm{b} \end{array} \\ \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - e^{i\delta} s_2 s_3 & c_1 c_2 s_3 + e^{i\delta} s_2 c_3 \\ s_1 s_2 & -c_1 s_2 c_3 - e^{i\delta} c_2 s_3 & -c_1 s_2 s_3 + e^{i\delta} c_2 c_3 \end{pmatrix} \end{array}, \tag{4.4}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$.

It follows from (4.3) and (4.4) that the experimental data on the various weak transitions can be used to determine the matrix elements U_{ik}.

1) U_{ud}. Experimentally, only the angle θ_1 is totally known. Since $U_{ud} = c_1$, θ_1 can be identified with the Cabibbo angle θ_C, which is measured in β decays which conserve strangeness:

$$c_1 = 0.9737 \pm 0.0025,\ s_1 = 0.2270^{+0.0104}_{-0.0110}. \tag{4.5}$$

2) U_{us}. It is simple to see that in the limit $\theta_2, \theta_3 \to 0$ the matrix U couples (d, s) quarks with (u, c) quarks, but the b quark is coupled only with the t quark. Consequently, the decays of the b quark discussed in Sections 1 (Subsection 1b) and 3 occur only at nonzero values of s_3 and/or s_2. Limitations on the angle θ_3 are imposed by data on the semileptonic decays of K mesons and hyperons in virtue of $U_{us} = s_1 c_3$: $|s_1 c_3| \simeq 0.219 \pm 0.002$. With θ_1 known, it follows that c_3 is approximately 1. With all the uncertainties taken into account,[46,150] we can write

$$|\sin\theta_3| = 0.28^{+0.21}_{-0.28}. \tag{4.6}$$

3) U_{ub}. The transition $b \to uW^-$ is determined by the matrix element $U_{ub} = s_1 s_3$. From the unitarity condition

$$|U_{ud}|^2 + |U_{us}|^2 + |U_{ub}|^2 = 1 \tag{4.7}$$

and from measured values of U_{ud} and $|U_{us}|$ it follows that $|U_{ub}|$ is small:

$$|U_{ub}| = 0.06^{+0.03}_{-0.06}, \tag{4.7a}$$

in complete agreement with all the data on the decays of B mesons (see Subsection 1b). As mentioned in Section 1, more accurate information on the fraction of $b \to uW^-$ decays cannot be found at the present level of experimental capabilities. With $\theta_3 = 0$, Cabibbo universality would hold exactly, and the phase factor $e^{i\delta}$ could be eliminated through a redefinition of the phase of the b quark. It should be noted that more-stringent limitations on the angle θ_3 would be of considerable interest from the standpoint of the CP-nonconservation problem. In particular, if experiments yielded $s_3 \ll s_1 \sim s_2$, then the phase δ might be of the order of 1. If $s_3 \sim s_2 \sim s_1$, on the other hand, we would have $\delta \sim 10^{-2}$.

4) U_{cd}. The matrix element $U_{cd} = -s_1 c_2$ can be determined quite accurately from an analysis of the K_L-K_S system (the virtual c quark contributes to the K^0-$\bar{K}^0$ transition). The following interval was found in Ref. 5:

$$0.19 \leqslant |U_{cd}| \leqslant 0.23, \tag{4.8}$$

Essentially the same restriction on $|U_{cd}|$ follows from data on the production of charm in neutrino reactions; more precisely, we mean the difference between cross sections

$$\sigma(\nu_\mu(d+s) \to \mu^- cX) - \sigma(\bar{\nu}_\mu(\bar{d}+\bar{s}) \to \mu^+ \bar{c}X).$$

Comparing (4.5) and (4.8), we see that c_2 is approximately one, and the angle θ_2 is small. Quantitatively,

$$|s_2| < 0.5.$$

5) U_{cs}. The most direct way to measure $U_{cs} = (c_1 c_2 c_3 - e^{i\delta} s_2 s_3)$ is to use the decays $D \to K^* e\nu_e, Ke\nu_e$. It should be recalled that until the t quark is found these decays are the best source of experimental information on the angle θ_2. Unfortunately, the experimental situation with regard to semileptonic exclusive decays of D is not yet completely settled. We can nevertheless use data on D^+, which are not in danger of being substantially revised. Analysis of the spectrum of electrons in the decay $D^+ \to e^+X$ shows that roughly half the events in $D^+ \to e^+X$ are associated with the mode $D^+ \to \bar{K}^0 e^+ \nu_e$. It follows that

$$\Gamma(D^+ \to \bar{K}^0 e^+ \nu_e) \approx \frac{1}{2}\Gamma(D^+ \to e^+X) = (1 \pm 0.5)\cdot 10^{11}\ \mathrm{s}^{-1}.$$

On the other hand, theory gives us

$$\Gamma(D^+ \to \bar{K}^0 e^+ \nu_e) \approx 1.5\cdot 10^{11} c^{-1} |f_+^{D\to K}(0)|^2 |U_{cs}|^2,$$

and if the form factor $f_+^{D\to K}(0)$ describing the transition $D \to K$ based on the vector current is approximately one, then

$$|U_{cs}| = 0.8 \pm 0.2.$$

We might assume $f_+^{D\to K}(0) \sim 0.5$, in which case we would have $|U_{cs}| \approx 1$. Analysis of the K_L-K_S system yields a similar restriction,[5] $0.8 < |U_{cs}| < 1.0$.

6) U_{cb}. It follows from the discussions in Subsection 1b2 that all the data available on the decays of B mesons confirm the hypothesis that

$$\left|\frac{U_{cb}}{U_{ub}}\right| \gg 1;$$

see also (1.13). Depending on the assumptions made regarding the phase δ, the information available on the matrix elements U_{cb} and U_{cs}, U_{cd} could be summarized in terms of joint restrictions on the angles θ_2 and θ_3 (see Ref. 5, for example).

As mentioned earlier (Subsection 1b2), the existing restriction on the lifetime of the B meson [Eq. (1.16)] furnishes a joint lower limit on the quark mixing angles. Indeed, under the assumption of decay of the free b quark (Fig. 11) we find, taking phase-volume effects in-

[11)] The signs of the phases are chosen in accordance with Ref. 7. We also recall that in the minimal SU(2)× U(1) model with a single doublet of Higgs bosons there is no natural way to introduce CP nonconservation in the case of a four-quark model.

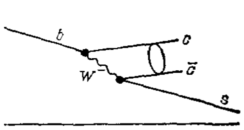

FIG. 26. Diagram for the decay $B \to J/\psi + X$.

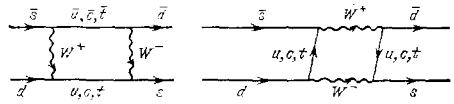

FIG. 27. Diagrams determining the mass difference $K_L^0 - K_S^0$.

to account, with $m_b = 4.8$ GeV, the following, for example[67,141]:

$$\tau_B \approx \frac{1}{3}\tau_\mu \left(\frac{m_\mu}{m_b}\right)^5 \frac{1}{(|U_{cb}|^2 + 2.5|U_{ub}|^2)} = \frac{3.7\cdot 10^{-15}\ c}{|U_{cb}|^2 + 2.5\,|U_{ub}|^2}. \qquad (4'.8)$$

According to (1.16), we should have

$$(|U_{cb}|^2 + 2.5\,|U_{ub}|^2) > 2.6\cdot 10^{-3}.$$

Combining this inequality with (1.13) and the condition for the unitarity of the Kobayashi-Maskawa matrix, we have

$$0.57 > |U_{cb}| > 0.05.$$

The dominant transition of the b quark, $b \to c + \underset{\mapsto \bar{u}d}{W^-}$, should give rise to charmed hadrons and to a comparatively large multiplicity of particles in the final state in the decay of the B meson, and these predictions are confirmed by experiment (Subsection 2b). In roughly 1/5 of the hadron cases we see the transition $b \to c + W^- \to c + \bar{c}s$; the $c\bar{c}$ system has a mass mainly smaller than $2M_D$, so that it undergoes a transition to a charmonium state (Fig. 26). Consequently, a comparatively high relative probability was expected for the transition[151] $B \to J/\psi + X$: $BR(B \to J/\psi + X \approx 1-3\%$. The 3% corresponds to the assumption that the production of a colorless bound $c\bar{c}$ state occurs with a unit probability.

The observation of this decay experimentally with a probability ~1% would be a further argument in favor of the dominance of the transition $b \to c$. Here it would be very interesting to study the exclusive modes, since there are arguments[151] which indicate that the final state should be a comparatively few-particle state, of the type $J/\psi + K\pi$.

The beauty baryon $\Lambda_b = bud$ might also have a significant (i.e., at the level of 1–2%) decay channel, $\Lambda_b \to J/\psi pK(\Lambda\pi)$.

7) $U_{td}s, b$). Direct experimental information on these matrix elements will, of course, have to await the observation of the t quark. The $t \to b$ transition should be predominant. Particularly interesting is $U_{td} = s_1 s_2$, from which we can determine just how small the angle θ_2 is. In the corresponding experiment it will be necessary to study the transition $t \to e^+\tilde{\nu}_e +$ pions.

The unitarity of the Kobayashi-Maskawa matrix imposes the restrictions

$$|U_{td}| < 0.13, \quad |U_{ts}| < 0.56, \quad 0.82 < |U_{tb}| < 0.99$$

An important restriction on the angle θ_2 follows from the theory for the contribution of t quarks to $K^0 \to \bar{K}^0$ transitions (Fig. 27). If this contribution to the mass difference $K_L^0 - K_S^0$ (the contribution is proportional to $m_t^2 s_2^4$) is not to be overly large in comparison with that of the c quark, we must assume that[152] $\mathrm{tg}\,\theta_2 \lesssim \sqrt{(m_c/m_t)} \lesssim 0.3$. On the other hand, since the contribution made by the t quark is important at $m_c \approx 1.40$ GeV, the angle θ_2 cannot be too small. This conclusion is supported by an analysis of CP nonconservation in the K^0-$\bar{K}^0$ system (see, for example Ref. 7).

Another parametrization of the mixing matrix, different from (4.4), is also frequently used. This parametrization involves three other angles θ, β, γ ($0 \leqslant \theta, \beta \leqslant \pi/2$, $-\pi/2 \leqslant \gamma$) and the new phase[153] $\delta' \leqslant \pi/2$:

$$U = \begin{pmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta \\ -c_\gamma c_\theta s_\beta e^{i\delta} & c_\gamma c_\theta - s_\gamma s_\beta s_\theta e^{i\delta} & s_\gamma c_\beta e^{i\delta} \\ -s_\beta c_\gamma c_\theta + s_\gamma s_\theta e^{-i\delta} & -c_\gamma s_\beta s_\theta - s_\gamma c_\theta e^{-i\delta} & c_\gamma c_\beta \end{pmatrix}; \qquad (4.9)$$

here $c_{\theta,\beta,\gamma} = \cos(\theta, \beta, \gamma)$, $s_{\theta,\beta,\gamma} = \sin(\theta, \beta, \gamma)$.

The angle θ is the Cabibbo angle θ_C, but at $\beta \neq 0$ the Cabibbo universality is disrupted:

$$\mathrm{tg}\,\theta = \frac{U_{us}}{U_{ud}}, \quad |U_{us}|^2 + |U_{ud}|^2 = \cos^2\beta < 1.$$

This parametrization is particularly convenient for describing the weak transitions of the b quark, since the angles β and γ are related directly to its decays:

$$\sin\beta = U_{ub}, \quad |\sin\gamma\cos\beta| \approx |U_{cb}|,$$
$$\mathrm{tg}\,\gamma = \left|\frac{U_{cb}}{U_{tb}}\right|. \qquad (4.10)$$

Since the Cabibbo universality is disrupted only slightly [cf. (4.7) and (4.7a)], the angle β must be small: $\beta \lesssim 6\cdot 10^{-2} \approx \theta/4$. Limitations on the angle γ arise, in particular, from data on $K^0 \to \bar{K}^0$ transitions. Figure 28 illustrates this situation with a summary of the existing limitations[68] on $\sin\beta$ and $|\sin\gamma|$, which follow from the universality of the weak coupling, the data on the mass difference $K_L^0 - K_S^0$, and measurements of the upper limit on τ_B [see Eqs. (1.16) and (4.10)]. The hatched regions show the values of $\sin\beta$ and $|\sin\gamma|$ which are presently ruled out by experiment; the lower left corner corresponds to values $\tau_B < 1.4\cdot 10^{-12}$ s (Ref. 68).

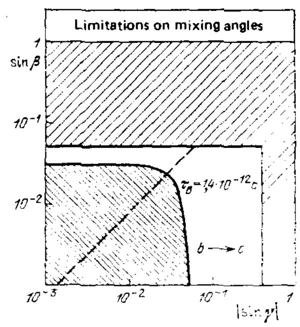

FIG. 28. Existing limitations on the quark mixing angles in the $\sin\beta$, $|\sin\gamma|$ plane.[68] The region at the lower left corner corresponds to $\tau_B < 1.4\cdot 10^{-12}$ s. The excluded regions are hatched. Values lying to the left of the dashed line are eliminated by the CLEO data on the K-meson yields in $B\bar{B}$ events.

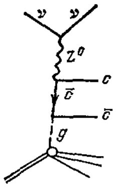

FIG. 29. Diagram describing the formation of a cc state in a neutrino reaction.

b) Weak neutral currents of heavy quarks

In the standard Glashow-Weinberg-Salam model,[9,154] the weak neutral quark current

$$(j^0_\mu)=\frac{1}{2}\sum_q (v_q\bar{q}\gamma^\mu q+a_q\bar{q}\gamma^\mu\gamma_5 q) \tag{4.11}$$

corresponds to the following values of the vector and axial constants:

$$\begin{aligned} &a_{u,c,t}=1, && v_{u,c,t}=1-\frac{8}{3}\sin^2\theta_W,\\ &a_{d,s,b}=-1=a_e, && v_{d,s,b}=-1+\frac{4}{3}\sin^2\theta_W, \end{aligned} \tag{4.12}$$

where θ_W is the Weinberg angle $[(\sin^2\theta_W)_{expt}=0.229 \pm 0.009]$.

The existing experimental data on the yields of J/ψ mesons in neutrino reactions (Fig. 29) and on the cross sections for hadron production in e^+e^- annihilation (Fig. 30) agree with the assumption of the standard model that the currents $\bar{c}c$ and $\bar{u}u$ have identical structures. Furthermore, the e^+e^--annihilation data rule out anomalously large constants in the $\bar{b}b$ neutral current.

Detailed information on the neutral currents of heavy quarks can be found in e^+e^- collisions in a study of the distributions in the polar angle ϑ and the azimuthal angle φ (in the case of polarized initial particles) at which the jets produced by these quarks are emitted (see Refs. 67 and 155, for example). In contrast with the case of light quarks, these jets can be identified completely unambiguously on the basis of the particular decays of the corresponding leader heavy hadrons—the so-called marker-quark method[67,155] (Subsection 5a). One obvious channel is the fragmentation of c quarks to D^* mesons, which can be seen extremely clearly and identified unambiguously in an experiment. Another approach is to detect the hard muons of a certain sign resulting from semileptonic decays of heavy particles. The yields of the c and b quarks can be separated reliably by a selection based on the momenta of the detected particles.

In particular, it would be very interesting to measure the forward-backward asymmetry for charm and beauty:

$$A^q_C=\frac{d\sigma_q(\vartheta)-d\sigma_q(\pi-\vartheta)}{d\sigma_q(\vartheta)+d\sigma_q(\pi-\vartheta)}, \tag{4.13}$$

from which we could find the parameter a_q.

At $W^2 \ll M_Z^2$ the quantity A^q_C is determined by the interference of the diagrams in Fig. 30 and has the value

$$A^q_C=\rho^q_C\frac{2\cos\vartheta}{1+\cos^2\vartheta} \tag{4.14}$$

where

$$\rho^q_C\approx 0{,}11\frac{a_e a_q}{Q_q}\left(\frac{W}{35\,\mathrm{GeV}}\right)^2\left(1-\frac{W^2}{M_Z^2}\right)^{-1} \tag{4.15}$$

[cf. (6.2) and (6.3)].

According to (4.15), the effect should be 1.5 times greater for c quarks, and 3 times greater for b quarks, than in the case $e^+e^-\to\mu^+\mu^-$. The asymmetry increases rapidly with the energy, and, according to calculations from more-accurate equations, at $W\approx 60$ GeV we have $\rho^b_C\approx -0.8$. Thereafter, the effect begins to fall off.[67] Precise measurements of A^q_C over a sufficiently broad energy range would make it possible to determine the mass M_Z from the propagator effect.

The constants v_q can be found in experiments with longitudinally polarized initial e^- and e^+. For example, for the electron helicity λ in the integral cross section for $e^+e^-\to\bar{q}q$ we find a P-odd correlation,

$$A^q_P=\frac{1}{\lambda}\frac{\sigma_q(\lambda)-\sigma_q}{\sigma_q}\approx -\frac{v_q}{a_q}\rho^q_C \quad (W^2\ll M_Z^2). \tag{4.16}$$

This correlation is particularly strong in the case of the b quark: At $W\approx 40$ GeV we have $A^b_P\approx 0.4$ ($A^c_P\approx 0.1$).

Skovpen' and Khriplovich[156] have discussed an elegant possibility for studying the interaction of the neutral axial electron current with the vector current of c and b quarks directly at the J/ψ and Υ peaks. The dependence of the total cross section for hadron production on the sign of the longitudinal polarization should give rise to a relative effect $A^b_P\approx 1.6\cdot 10^{-2}$ at the Υ resonance and $A^c_P\approx 4\cdot 10^{-4}$ at the J/ψ resonance. This effect is particularly marked in the case of the t quark, where (below Z^0) A^t_P becomes ~1.

Another possibility is to measure the longitudinal polarization of the final $\mu^+\mu^-$ ($\tau^+\tau^-$) at a narrow resonance.

c) Elusive-scalar-particle factory

The decays of hadrons containing heavy quarks represent one of the best sources of information on the various exotic entities which arise theoretically. Examples are the Higgs bosons (see Refs. 10 and 146, for example) and the axion (see Ref. 147, for example), for which the interaction constants for interactions with quarks are proportional to m_q.

The bare-bones standard model of the electroweak interaction (with a single doublet of Higgs particles)[154,9] contains the physical neutral boson H^0, while models containing several multiplets of scalar particles contain the physical charged particles $H^\pm$. The experimental search for the scalar bosons H is one of the foremost problems of physics today. However, it is not by

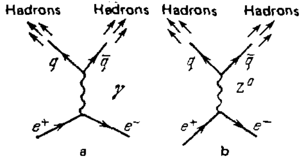

FIG. 30. Lowest-order diagrams for the process $e^+e^-\to q\bar{q}$.

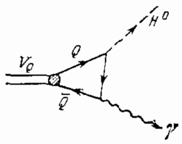

FIG. 31. Diagram describing the decay of vector quarkonium, $V_Q \to H^0 + \gamma$.

chance that these particles are called "elusive." On the one hand, the theory as it exists today does not tell us unambiguously the masses or structure of the Higgs sector. On the other hand, the expected cross sections for the production of the H bosons are generally very small, and it is a rather complicated problem to identify the final state.

Since the constants for the interaction of Higgs bosons with quarks are of the order of $\sqrt{G_F\sqrt{2}}m_q$ in the standard model, the decays of heavy quarkonia $Q\bar{Q}$ are intense sources of H^0 bosons with a mass $m_H < 2m_Q$. In particular, for the vector quarkonium V_Q the probability for the decay $V_Q \to H^0+\gamma$ is determined by[157] (Fig. 31)

$$\frac{\Gamma(V_Q\to H^0\gamma)}{\Gamma(V_Q\to\mu^+\mu^-)}=\frac{Gm_q^2}{\sqrt{2}\,\pi\alpha}\left(1-\frac{m_H^2}{M_V^2}\right). \tag{4.17}$$

In the case of the Υ meson, for example, we find (with $m_H^2/M_V^2 \ll 1$) $BR(\Upsilon\to H^0\gamma)\approx 3\cdot 10^{-4}$ from this expression, so that there is the hope that this decay might be found from monochromatic γ rays. If $m_H \gtrsim 4$ GeV, its primary decay modes should be $H^0 \to \tau^+\tau^-, c\bar{c}$. We might note that in the standard model with a single Higgs doublet the requirement that the vacuum be stable and that perturbation theory apply gives rise to the restriction[158]

$$1\ \text{TeV} \gtrsim m_H \gtrsim 7\ \text{GeV}.$$

Models with several doublets can have even lighter scalar particles. The Crystal Ball group is presently analyzing data on the decay[41] $J/\psi \to \gamma\mu^+\mu^-$ to find a limitation on the cascade transition $J/\psi \to \gamma H^0 \to \gamma\mu^+\mu^-$ at 400 MeV $\lesssim m_H \lesssim 3$ GeV.

The ratio (4.17) increases sharply in the case of the t quark, and in view of the present restriction $m_t > 18.3$ GeV it would be difficult to obtain $[(\Gamma(V_t\to H^0\gamma))/(\Gamma(V_t\to\mu^+\mu^-))] > 0.12$ (provided that the mass m_t does not significantly exceed $M_Z/2$).

We turn now to the question of the charged Higgs bosons, $H^\pm$. If such an entity existed, with a mass $m_H < m_b - m_c$, the b quark would decay entirely by the semiweak transition $b \to H^- c$:

$$\frac{\Gamma(b\to H^-c)}{\Gamma(b\to c\bar{u}d')}\approx\frac{6\pi^2}{Gm_b^2}\sim 10^5.$$

The decay of H^- in the case $m_{H^-} > 2$ GeV would be dominated by the modes $H^- \to \tau^-\bar{\nu}_\tau, \bar{c}s$, which are extremely distinctive experimentally. As we saw in Subsection 1b2, the data on the decays of the B mesons rule out this possibility. Furthermore, analysis of the data on e^+e^- annihilation of $W \gtrsim 30$ GeV rules out the possible existence of $H^\pm$ with masses in the range 5–15 GeV and with the decay modes $H^- \to \tau^-\bar{\nu}_\tau, \bar{c}s$ (Refs. 20 and 21).

Consequently, the main hope for detecting $H^\pm$ bosons is now placed on the decay of the t quark: $t \to H^+ + b$.

The decays of heavy quarkonia yield some extremely important information on such a theoretical invention as the axion (a): a new, very light, pseudoscalar particle, which gives rise, in particular, to a naturalization of the *CP* invariance of strong interactions (see Ref. 147, for example). In recent years there has been an active search for this entity in experiments on proton and electron accelerators, in experiments on reactors, and in a study of the decays of excited nuclei.

In the standard theory for the axion[159,160] the constant of its interaction with u, c, and t quarks is of the order of $\sqrt{G\sqrt{2}}m_q x$, while that for the interaction with d, s, and b quarks is of the order of $\sqrt{G\sqrt{2}}m_q/x$; x is the standard parameter in the axion theory, equal to the ratio of the vacuum expectation values of two Higgs fields. As in the case of the decay $V_Q \to H^0+\gamma$, the ratio of the width of $V_Q \to a+\gamma$ to the leptonic width, for the J/ψ meson, for example, is [cf.(4.17)]

$$\frac{\Gamma(J/\psi\to a+\gamma)}{\Gamma(J/\psi\to\mu^+\mu^-)}=\frac{Gm_c^2x^2}{\sqrt{2}\,\pi\alpha} \tag{4.18}$$

(the corresponding expression for the b quark differs by the replacement $x^2 \to 1/x^2$).

The Crystal Ball group has carried out an experimental search[161] for the decay of the J/ψ meson to a γ ray (with $E_\gamma \approx m_{J/\psi}/2$) and a long-lived noninteracting entity with a small mass $m_a < 1$ GeV (an axion or some other exotic entity with similar properties). From a statistical base $\sim 2\cdot 10^6$ J/ψ events, the following upper limit was found on the relative probability for the decay $J/\psi \to a+\gamma$:

$$BR\,(J/\psi\to a+\gamma) < 1.4\cdot 10^{-5}\ (90\%\ \text{c.l.}). \tag{4.19}$$

Comparing (4.18) with (4.19) at the known value of $BR(J/\psi\to\mu^+\mu^-)$, we can easily find the limitation $x < 0.6$. We can thus immediately rule out the value $x = 3$, for example, deemed the preferred value in the experimental study by Faissner *et al.*,[162] which generated the claim that an axion had been observed.

Since $BR(\Upsilon\to a\gamma)\sim M_\Upsilon^2$, and since the product $BR(\Upsilon\to a\gamma)\times BR(J/\psi\to a\gamma)$ does not depend on x and is predicted unambiguously for the standard Weinberg-Wilczek axion, a joint analysis of the reactions $J/\psi \to a\gamma$ and $\Upsilon \to a\gamma$ has attracted much interest. Data obtained on monochromatic γ rays in the reactions $\Upsilon \to a\gamma$(LENA) and $\Upsilon'' \to a\gamma$(CUSB) rule out the existence of an axion with the standard properties.[41] We wish to emphasize that in the searches for the axion the decays of heavy quarks have proved much more effective than other methods of study. In the future, the decays of heavy quarks will apparently become the most severe test of new axion models.

5. HEAVY QUARKS AND PERTURBATION THEORY IN QUANTUM CHROMODYNAMICS

Since the physics of heavy quarks involves small distances, where asymptotic freedom holds, many problems appear much simpler for heavy quarks than for light quarks. The large mass of the quark means that the standard perturbation-theory methods can be used

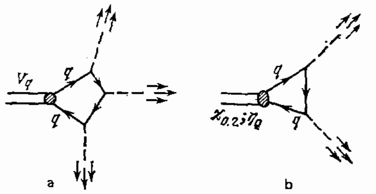

FIG. 32. Diagrams describing the decay of vector (a) and C-even (b) quarkonia in quantum chromodynamics.

(e.g., the summation of leading logarithms; see Refs. 11, 163, and 164, for example) to describe the final state in various hard processes, primarily for the fragmentation function of the heavy quark. Combining the perturbation-theory results with the parton arguments, we find a closed picture for the production and hadronization of heavy quarks, in which the bremsstrahlung and confinement effects are intimately related.

Furthermore, the decays of heavy quarkonia are in principle the purest source of gluon jets. The vector quarkonia can decay into three gluons, while the pseudoscalar and P-wave states with $J^{PC} = 2^{++}, 0^{++}$ can undergo transitions to two gluons also (Fig. 32). A study of the decays of heavy quarkonia will thus yield important information about the properties of the jet produced by a gluon, in particular, the specifics of the hadronization of gluons. Also related to a study of gluon jets is a determination of the quantum numbers of the gluon (its spin, parity, etc.). It is pertinent to recall here that perturbation theory is not a self-consistent approximation, and it is always necessary to be prepared for effects not controlled by the perturbation theory. Experiments of this type can reveal the conditions under which confinement does not prevent colored partons from exhibiting the characteristics of distinct individual entities with completely definite properties. In this manner, we accumulate information on just how "soft" the decoloring mechanism is.

Finally, we note that since the intensity at which soft gluons are emitted by a gluon in quantum chromodynamics is 9/4 times the intensity emitted by a quark, there is the possibility in principle of comparing the yields of particles of different species in the decays of quarkonia and in e^+e^- annihilation (see Refs. 11 and 164, for example). In this manner we can test the consequences of the picture of the cascade multiplication of gluons and their subsequent hadronization.

We will now briefly discuss the spectrum of heavy hadrons in e^+e^- annihilation, the study of the spin and parity of the gluon, and the quantum-chromodynamics prediction regarding the yields of hadrons of various species in the decays of quarkonia.

a) Distributions of heavy hadrons in jets

We will first discuss the specific features which would be caused in gluon bremsstrahlung and hadronization by the large mass of the Q quark, M_Q (Refs. 67 and 164). We assume $M_Q R \gg 1$, where R is a characteristic decoloring radius, determined by a nonperturbative interaction of bremsstrahlung gluons with $k_\perp \sim R^{-1}$ $[\alpha_s(k_\perp^2 \sim R^{-2}) \sim 1]$ and a vacuum condensate of light quarks, $R^{-1} \approx 250$–300 MeV. Since Q has a large mass, the formation time for bremsstrahlung with an energy ω and a transverse momentum $k_\perp$,

$$t_{rad} \approx \frac{E_Q z(1-z)}{k_\perp^2 + M_Q^2 z^2},$$

is, at $z \sim 1$, always parametrically shorter than the gluon hadronization time $t_{\text{hadron}} \approx \omega R^2$, when the interaction becomes strong. For fast gluons we would typically have $k_\perp \gtrsim M_Q$, and there would be an intense emission of only parametrically soft gluons with $z \leq (M_Q R^{-1}) \ll 1$, $k_\perp \sim R$. We can thus draw the important conclusion that the energy lost by a heavy quark on emission is slight and is controlled by perturbation theory.[67,155,164] Taking this approach, we can find an explanation for the clearly defined heavy-quark (and hadron) leader effect,[165] which is kinematically reminiscent of the baryon leader effect in the pp interaction.

If the parameter $M_Q R$ is large enough, the spectra of H_Q hadrons containing heavy Q quarks should be essentially the same as the spectrum of the quark Q: $|x_Q - x_{H_Q}| \leq (M_Q R)^{-1}$ at $1 - x_{Q,H_Q} \gg (1/M_Q R)(x_{Q,H_Q} = 2E_{Q,H_Q}/W)$. When a sum is taken over all types of hadrons with the quark Q, the agreement should become even better. At $|1 - x_{H_Q}| < 1/M_Q R$, the H_Q spectra fall off in proportion to a power of $(1 - x_{H_Q})$.

Since the distances which are important in the hard emission of gluons by quarks Q are determined by the quantity $M_Q^{-1} \ll R$, at $(1 - x_Q) > 1/M_Q R$ the Q spectrum is described completely by perturbation theory and is an infrared-stable quantity.

In this case the inclusive distribution $\bar{D}_Q(x)$ of the Q quark in the reaction $e^+e^- \to Q(x) + \ldots$ is given in the leading logarithmic approximation (LLA) by [164,16]

$$\bar{D}_Q^{LLA}(x, W) = \int \frac{dj}{2\pi i} x^{-j} \exp\left\{\Delta\xi \cdot C_2\left[3 + \frac{2}{j(j+1)} - 4\psi(j+1) - 4\gamma_E\right]\right\}, \tag{5.1}$$

where ψ is a known function of Γ'/Γ, $\gamma_E = 0.577\ldots$ is the Euler constant,

$$C_2 = \frac{N^2-1}{2N}, \quad \Delta\xi = \xi(W^2) - \xi(M_Q^2) = \frac{1}{b}\ln\frac{\alpha_s(M_Q^2)}{\alpha_s(W^2)}, \tag{5.2}$$
$$b = \frac{11}{3}N - \frac{2}{3}n_f = 9 \quad \text{(u, d, s-quarks)}$$

N is the number of colors ($N = 3$), and n_f is the number of "thawed" quarks. The integration in (5.1) is along a contour running parallel to the imaginary axis, to the right of all the singularities, i.e., at $\mathrm{Re}\,j > 0$. Distribution (5.1) increases with increasing x. At arbitrary values of $\Delta\xi$ and x, the quantity $D_Q^{LLA}(x)$ can be found numerically. At realistic energies, however, $\Delta\xi$ is small, for the b quark, for example, and we can use simple interpolations, for example,

$$\bar{D}_Q(x) \approx N(\Delta\xi)\frac{1+x^2}{2}(1-x)^{-1+4C_2\Delta\xi}, \tag{5.3}$$

where $N(\Delta\xi)$ is determined by the normalization conditions. In principle, more-accurate calculations could reveal the logarithmic corrections to the LLA expression, (5.1), in the form of a series in α_s.

In the LLA, we find the following expression for the average fraction of the energy carried off by a heavy quark:

$$\langle x_Q \rangle^{\mathrm{LLA}} = e^{-8C_2\Delta_5^2/3} = \left(\frac{\alpha_s (M_Q^2)}{\alpha_s (W^2)} \right)^{-32/81}. \tag{5.4}$$

This result corresponds to the results for valence quarks in a hadron in deep inelastic scattering.[167] For the c quark this result means that at the energies presently attainable ($W \approx 30$–40 GeV) this quark would lose only about a fourth of its energy on radiation, $\langle x_c \rangle \approx 0.75$. The b quark would be even stingier with its energy, $\langle x_b \rangle \approx 0.85$. An experimental test of these predictions of perturbation theory would be of major interest. The first data on the production of charm, recently reported, confirm the theoretical expectations: $\langle x_c \rangle_{\mathrm{expt}} \approx 0.7$.

The properties of the mets produced by heavy quarks are being studied experimentally in the "trigger" decays of the daughter heavy hadrons, which carry off nearly all the momentum of the heavy quark. For example, one could detect the transition $c \to D^* + \ldots$ or hard leptons from the semileptonic decays of Q. Analysis of the lepton spectrum will make it possible not only to distinguish events involving heavy quarks but also to distinguish between the contributions of different heavy quarks. Furthermore, it will become possible to distinguish between the jets of a quark and a gluon and those of a quark and an antiquark. This circumstance opens up a path for studying not only strong-interaction problems (the properties of the gluon,[155,176] for example) but also the manifestations of the weak interactions of heavy quarks (Subsection 4b). Events involving b and c quarks can be separated by making use of the transverse momentum (transverse with respect to the axis of the hadron jet) of the leptons, $p_\perp^l$; the condition $p_\perp^l > 1.2$–1.5 GeV corresponds primarily to events involving b quarks.[67] Other characteristic manifestations of events involving heavy quarks in e^+e^- annihilation could also be cited. For example, along with a lepton one should observe in such events a substantial energy loss (carried off by neutrinos) and an apparent imbalance of transverse momenta.

b) Test of the spin and parity of the gluon

Several perturbation-theory methods have now been worked out for determining the spin and parity of the gluon, and they have already been tested experimentally.[11,16,168] Taken together, the results show a preference for the hypothesis $J_g^{PC} = 1^{--}$. However, an interpretation of any experiment on the properties of quarks and gluons requires additional assumptions, related, in particular, to the hadronization of the quarks and the gluons. It is therefore particularly important to measure the quantum number of the partons in various experiments, in order to check the various levels in the hierarchy of hypotheses regarding hadronization.[11] In this case, comparison of the results would represent a test of both the predictions of perturbation-theory quantum chromodynamics and the ideas regarding the nature of the binding of partons and hadrons, i.e., confinement.

In efforts to determine how sensitive a theory is to some prediction or other of quantum chromodynamics it is customary to compare it with the predictions of other models, in particular, models in which the quantum numbers of the gluon are changed. It should of course be kept in mind here that such models are only auxiliary models, since they do not represent versions of a systematic theory. For example, the asymptotic freedom indicated by, in particular, the observed properties of the quarkonia can prevail only in gauge theories. In this regard, chromodynamics has no competition.

1) *Decays of heavy quarkonia*

The most reliable confirmation of the spin and parity of the gluon would come from a comparison of the decay widths of heavy quarkonia, since this comparison would be based exclusively on the minimum assumption that the parton states are complete.[168,169] In particular, for the ratio of the widths of direct hadron decays (Γ^{dir}) of the pseudoscalar ($^1S_0$) and vector ($^3S_1$) states,

$$r^S = \frac{\Gamma^{\mathrm{dir}}(^1S_0)}{\Gamma^{\mathrm{dir}}(^3S_1)}, \tag{5.5}$$

quantum chromodynamics predicts $r_{\mathrm{QCD}}^S \sim 6(\alpha_s/\pi)^{-1}(1 + O(\alpha_s))$ (Fig. 32). In the case of J/ψ and η_c, for example, we would have $r_{\mathrm{QCD}}^S \sim 10^2$ (Section 2). A similar value would be expected for the case of Υ, η_b.

If, on the other hand, the gluons are pseudoscalar particles ($J_g^P = 0^-$), the $^1S_0$ state could not decay into gg, and both states, $^1S_0$ and $^3S_1$, would convert into three gluons, despite the difference in *C* parity. The difference between widths becomes much smaller than in the case $J_g^P = 1^-$. In the case of the J/ψ and η_c states, for example, we would expect[168] $r_p^S \approx 10$. At $J_g^P = 1^+$, the theoretically expected widths $\Gamma^{\mathrm{dir}}(^1S_0)$ is much smaller than in the standard case, with $J_g^P = 1^-$, and perhaps even smaller than $\Gamma^{\mathrm{dir}}(^3S_1)$.

if $J_g^P = 0^+$, then $^3S_1$ can again decay into three gluons, but in a direct decay of $^1S_0$ at least four gluons would form. In this case we could expect[11] $\Gamma_{\eta_c}^{\mathrm{tot}} < \Gamma_{J/\psi}^{\mathrm{tot}}$.

Consequently, the ratio $r_{\mathrm{expt}}^S \approx (3 \pm 1) \cdot 10^2$ observed experimantally for J/ψ and η_c (Table XIII) is in excellent agreement with the standard gluon of quantum chromodynamics, and it rules out the other possibilities which have been discussed. An even more reliable test of J_g^P will come from measurements of the S-wave states of $b\bar{b}$.

A comparison[169] of the decay widths of the $^3P_0$ and $^3P_2$ states also requires nothing more than the completeness of the parton states.

2) $\Upsilon \to 3g$

The use of the three-jet processes $e^+e^- \to q\bar{q}g$ and $\Upsilon \to 3g$ (Refs. 170–172, 63, 17) to study the spin of the gluon requires more-serious hypotheses, primarily the hypothesis that the hadrons "remember" the emission direction and the properties of the partons which generated them (this is the so-called hypothesis of soft decoloring[163]).

In experiments at the Υ and Υ' resonances, the dis-

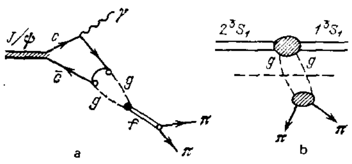

FIG. 33. Diagram representation of the decays $J/\psi\to\gamma f_{\lfloor\to\pi\pi}$ (a) and $(2\,^3S_1)\to 1\,^3S_1)\pi\pi$ (b).

tributions of the elongation axis T of events[12)] with respect to the direction of the initial e^+e^- beams,[62] $\sim 1 + \alpha_T\cos^2\theta[(\alpha_T)_{\rm expt}=0.35\pm0.11]$, and with respect to the normal to the gluon emission plane,[62] $\sim 1 + \alpha_N\cos^2\beta[(\alpha_N)_{\rm expt}=-(0.29\pm0.06)]$, have been measured. The results agree well with the expectations of quantum chromodynamics[171] $[(\alpha_T)_{\rm QCD}=0.39, (\alpha_N)^T_{\rm QCD}=-0.33]$ and completely refute the hypotheses $J^P_g=0^-$ and 0^+.

3) $J/\psi\to\gamma f\to\gamma\pi\pi$ (*Ref.* 173) *and* $2^3S_1\to\pi^+\pi^-\ (1^3S_1)$ (*Refs.* 116 *and* 118).

These decays are shown in Fig. 33 in the approximation of two-gluon exchange. A study of these decays will represent the next steps in the "hierarchy" of hypotheses.

In particular, the assumption of masslessness of intermediate gluons has been used for the decay $J/\psi\to\gamma f$, and the calculations of Ref. 173 included even further assumptions. For this reason, it is not surprising that we do not find a good agreement between the predictions of Ref. 173 and experiment in measurements of the polarization state of the f meson in the radiative decay[42] (cf. the note with Table II with $x_{\rm theo}=0.76$ and $y_{\rm theo}\approx 0.54$).

It was shown in Refs. 116 and 118 through the use of a multipole expansion of the gluon fields that the ratio of the widths of the cascade transitions of ψ' and Υ',

$$k=\frac{\Gamma(\Upsilon'\to\Upsilon\pi\pi)}{\Gamma(\psi'\to J/\psi\pi\pi)},\tag{5.6}$$

should depend strongly on the spin of the gluon: $k_{\rm QCD}\approx 0.1$, $k_{0^+}\approx 1$. In contrast with the scalar case, the quantum-chromodynamics amplitude for the process is proportional to $\langle r^2\rangle$ [Subsection 2c and Eq. (2.52)].

The experimental result,[62] $k_{\rm expt}=(8.5\pm6)\cdot10^{-2}$, indicates a clear preference for the quantum-chromodynamics expectation.

The fact that these different methods agree with each other in terms of the gluon spin is extremely important, since it justifies our arguments regarding the properties of confinement. In particular, we find further experimental support for the hypothesis of soft decoloring.

c) Hadron multiplicities in the decays of quarkonia

The experimental data on the multiple production of hadrons in e^+e^- annihilation at high energies[23,174] furnish qualitatively good support for the clear predictions of the picture of a cascade multiplication of gluons (see Refs. 11 and 164, for example)—predictions which are specific to quantum-chromodynamics perturbation theory: a sharp acceleration of the growth of the average charged-particle multiplicity $\langle n_{\rm ch}\rangle$ with increasing W, the characteristic plateau structure, etc. Also in agreement with this picture is the observed increase in the yield of baryons ($p, \bar p$ and $\Lambda, \bar\Lambda$) in hadron jets.[164] The latter fact also indicates that in the fragmentation of gluons into hadrons the formation of light glueballs does not play a governing role (saturation of the final state by glueballs with $m_g<2.5$ GeV would have reduced the relative yield of baryons).

In the doubly logarithmic approximation of quantum-chromodynamics perturbation theory there is a general functional technique for calculating the asymptotic behavior of various characteristics of the production of hadrons in jets: the multiplicities, spectra, correlation functions, etc.[175] When the known gluon distributions in the decays of quarkonia are used, this technique can also be used to calculate the characteristics of the multiple production of particles in these decays in the case in which the relative gluon emission angles are quite large. With the more or less symmetric configurations in the decay $V_Q\to 3g$, however, we can find some simple relations between the characteristics measured directly in the direct decays of quarkonia with the corresponding quantities in e^+e^- annihilation at $W=(2/3)M_{V_Q}$ (Ref. 11). Here the theoretical uncertainties, e.g., those surrounding the corrections to the doubly logarithmic formulas of the perturbation theory, become less important. It must be stipulated, however, that the estimates found below may be somewhat crude, particularly since the actual three-jet symmetric configurations of gluons in the V_Q decay are not absolutely dominant.

Here we will discuss only the emission of hadrons of various species h in the direct decays of quarkonium V_Q. V_Q. The average multiplicities $\langle n_h(V_Q)\rangle$ can be related to the contribution of bremsstrahlung gluons to the corresponding multiplicities in e^+e^- annihilation, $\langle n_h(e^+e^-)\rangle$ by means of[11,164]

$$\langle n_h(V_Q)\rangle=\frac{3}{2}\cdot\frac{9}{4}\,\Delta\left\langle n_h\left(e^+e^-,\ W=\frac{2}{3}M_{V_Q}\right)\right\rangle+\langle n_h(J/\psi)\rangle,\tag{5.7}$$

where $\Delta\langle n_h\rangle$ is the increase in the multiplicity in e^+e^- annihilation as the energy is increased from ~ 2 GeV to W. The factor of 3/2 results from the transition from two jets to three, and the factor of 9/4 results from the higher probability for the emission of soft gluons by a gluon than by a quark (see Ref. 176, for example).

For the Υ meson, expression (5.7) gives a reasonable estimate of the charged-particle multiplicity $[n_{\rm ch}(\Upsilon)\approx 8-10]$. For t quarkonium (toponium) in the mass interval 40–50 GeV the estimated value of $\langle n_{\rm ch}(T)\rangle$ is 35–40.

Since the baryon yields are low at low e^+e^--annihilation energies, $W\lesssim 3$ GeV [at $W\approx 3$ GeV, we have $n_p(e^+e^-)=(2\pm1)\cdot10^{-2}$], essentially the entire observed multiplicity of nonleading baryons can be attributed to a cascade breeding of perturbation-theory gluons, by

12) The T axis lies near the direction of the highest-energy gluon jet.[11]

replacing $\Delta\langle n_{\bar{p}}(e^+e^-)\rangle$ in (5.7) by $\langle n_{\bar{p}}(e^+e^-)\rangle$. Hence we immediately find the prediction that the multiplicity of baryons in direct decays of quarkonium should be significantly higher than in the background.[164] Interestingly, this situation holds even in the case of the J/ψ meson. For example, we have[177] $n_{\bar{p}}(J/\psi)\approx(6.3\pm1.8)\cdot10^{-2}$, which is 2–3 times higher than away from the resonance. This result may be interpreted as an argument for a three-gluon nature of the direct decays of J/ψ. When we work from data on the p, $\bar{p}$ and $\Lambda, \bar{\Lambda}$ yields in e^+e^- collisions (see Ref. 23, for example), Eq. (5.8) leads to values in the case of the Υ resonance which are in good agreement with experiment (Table VI): $\langle n_{p+\bar{p}}(\Upsilon)\rangle = 0.74\pm0.08$ and $\langle n_{\Lambda+\bar{\Lambda}}(\Upsilon)\rangle = 0.22\pm0.04$.

We wish to emphasize that in this picture of events the increases in the multiplicities for the hadrons of the various species associated with the emission of perturbation-theory gluons are similar in nature. In the case of t quarkonium, where the preasymptotic effects associated with the size of the substrate are small, we would expect to find an approximately identical ratio of the baryon and meson yields at resonance and away from it.

6. TOPONIUM AND WHAT IT MIGHT BE

Superheavy quarkonium may be thought of as an amazing world in which the weak electromagnetic and strong interactions play roles opposite those with which we have become familiar. The weak-interaction effects, which are negligible in charmonium and bottonium, may prove important or even dominant for $t\bar{t}$. From the theoretical standpoint, everything is transparent here: The Coulomb description holds within the limits determined by the theory, but the decay properties are very peculiar. Let us briefly look at the expected properties of toponium, T, the 1^3S_1 state of $t\bar{t}$ (see Refs. 86, 87, and 141, for example, for more details on these questions). As we have already mentioned, hopes for developing a gluon factory are pinned on toponium. This factory would permit a detailed study of the properties of gluons and their hadronization. Some unique possibilities for studying the properties of weak interactions are also presented by the decays of T (Section 4).

The expected properties of T depend most strongly on its mass, M_T. Unfortunately, the existing theoretical predictions of this mass stretch over a broad range, from values just slightly above the level which has been attained experimentally, $\gtrsim 37.6$ GeV, up to $M_T\approx150$ GeV (see Ref. 178, for example) or even higher.

The basic decays of superheavy vector quarkonium, $Q\bar{Q}$, can be described by the diagrams in Fig. 34. Furthermore, transitions involving Higgs bosons, e.g., $Q\bar{Q}\to Z^0H,\ldots$ (Fig. 31), are possible in principle.

We are particularly interested in transitions described by the diagrams in Figs. 34h and 34i, which correspond to weak decays of one of the quarks in heavy quarkonium. In the case of the t quark, the diagram in Fig. 34h corresponds to the transition

$$t\bar{t}\to t+\bar{b}+\text{leptons or hadrons}. \tag{6.1}$$

The t and $\bar{b}$ quarks can convert into a new hadron $t\bar{q}$ and a B meson, or they can form a superheavy hadron $t\bar{b}$. We thus see the real possibility of studying new elements of the quark mixing matrix U [in particular, U_{tb}; see (4.4) and Subsection 4a]. The contribution of decays (6.1) to the total width of T becomes appreciable ($\gtrsim5\%$) at $m_t\gtrsim25$ GeV and increases with increasing mass of the t quark; at $M_Z\lesssim M_T\lesssim 2M_W$ it becomes dominant (more on this below). Experimentally, these decays can be detected reliably from, for example, the hard leptons from the transition $W^-\to e\bar{\nu}_e$ or from the semileptonic decay of the t quark. The diagram in Fig. 34i works at $m_t>M_W$.

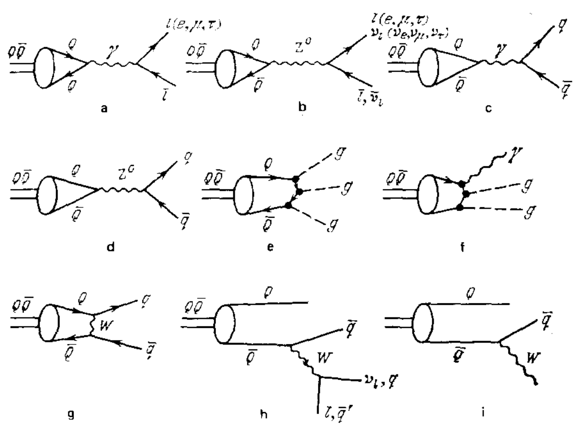

FIG. 34. Basic diagrams corresponding to the decays of superheavy vector quarkonium $Q\bar{Q}$.

We turn now to the diagrams in Figs. 34(a–34g). All correspond to short-range $Q\bar{Q}$ annihilation (and are thus called annihilation diagrams) and are proportional to the square of the quark wave function at the origin. Taking γ and Z^0 exchanges into account [Figs. 34(a) and 34(b)], we can describe the leptonic width $\Gamma(T\to e^+e^-)$ by the following expression [cf. (2.17), Subsection 4b, and (4.11)][13]:

$$\Gamma(T\to e^+e^-)=\Gamma_T^e\left[1-\frac{2v_e v_t R_Z}{Q_t}+\frac{(v_e^2+a_e^2)v_t^2R_Z^2}{Q_t^2}\right], \tag{6.2}$$

where

$$R_Z=\frac{G}{8\pi\alpha\sqrt{2}}\frac{M_T^2M_Z^2}{M_T^2-M_Z^2}\approx5.5\cdot10^{-2}\left(\frac{M_T}{35\ \text{GeV}}\right)^2\frac{1}{(M_T^2/M_Z^2)-1}, \tag{6.3}$$

and $\Gamma_T^e\equiv\Gamma(T\to e^+e^-)_{QED}$ is the purely electromagnetic contribution to the leptonic width of T. The axial constant a_t and the vector constant v_t for the t quark are determined in the standard model[9,154] by Eqs. (4.12), while those for the electron are given by

$$a_e=-1,\quad v_e=(4\sin^2\theta_W-1), \tag{6.4}$$

i.e., $v_e\ll1$, since $\sin^2\theta_W\approx0.23$. We recall (Subsection 2a3) that Γ_T^e depends comparatively weakly on the quark mass, and the energy dependence of the leptonic width is determined essentially completely by the last term in square brackets in (6.2). In particular, $\Gamma(T\to e^+e^-)$ increases sharply at values of M_T near M_Z.

Since we have $v_\nu=a_\nu=1$ for neutrinos, the width of the

[13]Expression (6.2) changes near the Z^0 peak, in particular, because of the modification of R_Z by radiative effects and the incorporation of Γ_Z (Refs. 179 and 180).

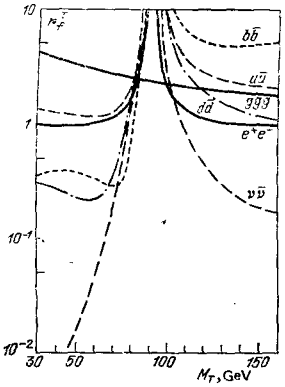

FIG. 35. Dependence of the ratios $r_f^T = \frac{\Gamma(T \to f\bar{f})}{\Gamma(T \to e^+e^-)_{QED}}$ and $r_{ggg}^T = \frac{\Gamma(T \to ggg)}{\Gamma(T \to e^+e^-)_{QED}}$ [87].

transition $T \to \nu\bar{\nu}$ [Fig. 34(b)] is given by

$$\Gamma(T \to \nu\bar{\nu}) = 2\Gamma_T^e \frac{v_t^2}{Q_t^2} R_Z^2. \tag{6.5}$$

At $M_T \sim M_Z$ the transition $T \to \sum_\nu \nu\bar{\nu}$ is a significant fraction (0.1–0.2) of the total T width, and it would be very interesting to observe this transition, by making use of, for example, the radiative tail[179]: $e^+e^- \to T + \gamma$. Such experiments would be particularly interesting in connection with the effort to obtain information on the number of lepton generations. On the other hand, the decay $T \to \nu\bar{\nu}$ might be observed in a study of the cascade[14)]

$$2\,{}^3S_1 \to \underbrace{1\,{}^3S_1}_{\to\, \nu\bar{\nu}} + \pi^+ + \pi^-. \tag{6.6}$$

The width of the decay $T \to q\bar{q}$ corresponding to the diagrams in Figs. 34(c) and 34(d) is [cf. (6.2)]

$$\Gamma(T \to q\bar{q}) = 3\Gamma_T^e \left[Q_q^2 - \frac{2v_q v_t Q_q}{Q_t} R_Z + \frac{(v_q^2 + a_q^2) R_Z^2}{Q_t^2} \right]. \tag{6.7}$$

We might note that in the case of the transition $T \to b\bar{b}$ we must add the contribution of the t-channel exchange of a W boson [Fig. 34(g)] to the amplitude described by Fig. 34(d). The W-exchange itself generates a contribution to the total width given at $U_{cb} = 1$ by[181]

$$\Gamma_W^T = \frac{1}{2} \frac{M_T^4}{[M_W^2 + (1/4) M_T^2]^2} \left(\frac{\Gamma_t^e}{Q_t^2} \right). \tag{6.8}$$

Figure 35 shows the dependence on the T mass of the ratio

$$r_f^T = \frac{\Gamma(T \to f\bar{f})}{\Gamma_T^e}, \quad f = e,\ \nu,\ q, \tag{6.9}$$

of the widths $\Gamma(T \to f\bar{f})$ to the electromagnetic width Γ_T^e. Shown for comparison here is the ratio $r_{3g}^T = \Gamma(T \to 3g)/\Gamma_T^e$ calculated from an expression like (2.41). We see from this figure (see also the discussion below) that the contribution of three-gluon annihilation decreases with increasing M_T, not exceeding 30% at $M_T \gtrsim 60$ GeV.

The difference between the $d\bar{d}$ and $b\bar{b}$ contributions results primarily from the constructive interference of the diagrams in Fig. 34(c) and 34(d) with the diagram in Fig. 34(g). Near Z^0 ($M_T \approx 80$–90 GeV), the behavior of the $T \to f\bar{f}$ channels is determined completely by the pole term, modified[13)] in accordance with Refs. 179 and 180. A further increase in the quark mass leads to an obvious "interlacing" of the various forces. Near the Z pole, for example, the leptonic widths of the excited states lying closer to Z must be larger than the leptonic width of the ground level.[182]

Experimentally, the $T \to q\bar{q}$ transitions may give rise to two-jet configurations; the $T \to b\bar{b}$ mode can be singled out without any particular difficulty by making use of the distinctive features of the decay of b quarks (see Subsections 1b and 4a).

At $M_T \gtrsim 100$ GeV, the modes $T \to H^0\gamma$ [see Eq. (4.17)], $T \to Z^0\gamma$, and $T \to H^0Z^0$ become comparable to the annihilation transitons discussed above. As we have already emphasized, however, the decays of the free t quark become dominant at these energies [Figs. 34(h) and 34(i)]. In contrast with the annihilation diagrams (where the M_T dependence is quite weak, except for the effects of the Z^0 pole), the corresponding contribution to the total T width in this case is proportional to m_t^5.

At $m_t < M_W$, ignoring all the fermion masses other than m_t and m_b, and setting $|U_{tb}| = 1$, we can write the following expression for $\Gamma_t^T \to W + \ldots$:

$$\Gamma_{t \to W + \ldots}^T = 2 \cdot 9 \frac{G^2 m_t^5}{192\pi^3} f\left(\frac{m_t^2}{M_W^2},\ \frac{m_b^2}{M_W^2} \right). \tag{6.10}$$

The factor of 9 reflects the presence of nine allowed decay channels. The function f reflects effects associated with the phase volume and the W propagator (Fig. 36). Figure 37, taken from Ref. 87, compares

$$\Gamma(T \to 3g),\ \Gamma_f^T = \sum_{f = l,\, \nu,\, q} \Gamma(T \to f\bar{f}) \ \text{and}\ 18\Gamma(t \to b\,\mu\,\nu)$$

for various values of the mass M_T. We see from this figure that the decay of the free quark becomes significant $([(\Gamma_{t \to W + \ldots}^T)/\Gamma_T] \gtrsim 6\%)$ at $M_T \gtrsim 50$ GeV; at $M_T \gtrsim 70$ GeV, its probability is roughly twice $\Gamma(T \to 3g)$ and amounts to ~1/9 of the total T width Γ_T.

At $M_T > 2M_W$, the decay of the t quark accompanied by the formation of a real W boson becomes predominant [see the diagram in Fig. 34(i)]. The corresponding width of the decay $T \to W + \ldots$ is ($U_{tb} \approx 1$)

$$\Gamma(T \to W + \ldots) = 2 \frac{G}{8\pi\sqrt{2}} m_t^3 \left(1 + 2\frac{M_W^2}{m_t^2} \right) \left(1 - \frac{M_W^2}{m_t^2} \right)^2. \tag{6.11}$$

We wish to emphasize that the various hadronic decay modes of T can be distinquished experimentally by mak-

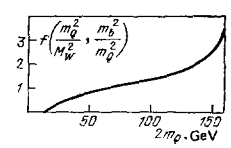

FIG. 36. The function $f(m_Q^2/M_W^2,\ m_b^2/m_Q^2)$ at $m_b \approx 5$ GeV and $M_W = 80$ GeV.

[14)]The possibility of finding limits on the decay $1\,{}^3S_1 \to \nu\bar{\nu}$ from cascade (6.6) became the subject of an active discussion immediately after the discovery of the J/ψ and ψ' mesons (see Ref. 97, for example).

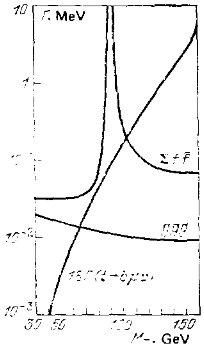

FIG. 37. Comparison of the widths of the decay[87]

ing use of the distinctive features of the corresponding events. For example, transitions of the type $T \to q\bar{q}$ correspond to primarily two-jet events, $T \to 3g$ transition correspond to a planar topology, while the decays of the free t quark give rise to events which are, on the average, isotropic. Furthermore, as we have mentioned earlier, hard direct leptons correlated with K mesons (from the decays of the b quark) should be observed in the latter case.

At the T-resonance peak, the cross section for the e^+e^- annihilation corresponding to the decay of the free t quark is[179]

$$\sigma(e^+e^- \to t\bar{t})_{\text{free quark decay}} = R_{\text{free quark decay}}\,\sigma(e^+e^- \to \mu^+\mu^-), \tag{6.12}$$

$$R_{\text{free quark decay}} = \frac{9}{2\alpha^2}\sqrt{\frac{\pi}{2}}\left(\frac{\Gamma(T \to e^+e^-)}{\Gamma_T}\right)\left(\frac{\Gamma^T_{t\to W^+\ldots}}{\sigma}\right)\left(\frac{2\sqrt{2}\sigma}{M_T}\right)^{\beta}, \tag{6.13}$$

where

$$\beta = \frac{4\alpha}{\pi}\left[\ln\left(\frac{M_T}{m_e}\right) - \frac{1}{2}\right],$$

and σ is the energy spread in the e^+e^- beams ($\sigma \sim W^2$). At $M_T \lesssim 70$ GeV, we note that $\Gamma(T \to e^+e^-)/\Gamma_T$ and thus the ratio $R_{\text{free quark decay}}$ depend only slightly on the quark wave function.

The cross sections corresponding to other T decay modes, $\Gamma(T \to f')$, can be found from (6.12) through the replacement $\Gamma^T_{t\to W^+\ldots} \to \Gamma(T \to f')$.

We have seen that the possibilities for experimentally observing T and studying it in detail depend strongly on σ (see Ref. 59, for example). For toponium with a mass $M_T \approx 40$–50 GeV at σ (in MeV) $= 22 \cdot 10^{-3} M_T^2$ (in GeV^2), for example (the figures corresponding to the PETRA), we find that the maximum cross section exceeds the background by a factor of 2–3 in the hadronic mode and by a factor of 1.5–2.5 in the muonic mode. There should thus be no difficulty in measuring the leptonic widths and reconstructing the total. At $M_T > M_Z$, we might note, the chances for studying toponium become slimmer because of the large radiative tail of Z^0; an attempt should be made to "tune away" from this tail.[155,179]

7. OUTLOOK

Although we know a lot about heavy quarks, it appears that there is still more to learn. We can expect more striking and important discoveries. In particular, the observation of the t quark and measurement of its mass could strongly affect the theory of quarks and leptons. The t quarks may prove a unique factory for generating exotic entities such as Higgs particles. Furthermore, we do not rule out the possibility that a fourth generation of quarks (if it exists in nature) will be observed.

Many problems remain unresolved in the fields of charmed and beauty quarks. The information which can be obtained here deals primarily with the properties of the strong interaction, in particular, gluon dynamics. As always, the most attractive areas are the frontier areas which have not yet been reached by the theory. From this standpoint the $c\bar{q}$ and $b\bar{q}$ systems—charmed and beauty particles—are surprisingly rich. Here it is necessary to study the exclusive weak decays, to learn how to calculate the transition form factors of the type $f_+^{D\to K}$, and to resolve finally the problem of the lifetimes of all the charmed hadrons. An analysis of transitions of the type $D^* \to D\gamma$ and $D^* \to D\pi$ would be interesting. There are of course many unmapped areas in the family of charmonium and, especially, bottonium. It is believed that some P levels remain to be discovered. The radial excitations of J/ψ above 4 GeV may hold some surprises. Important information can be extracted from hadronic transitions between the levels of charmonium (or bottonium) and in the radiative decays of J/ψ and Υ.

In accumulating data on the various decays of heavy hadrons we are obtaining information on the elements of the quark mixing matrix. Information on the transitions of the b quark to c and u quarks would be particularly interesting at this point.

Progress in the physics of heavy quarks has been surprisingly rapid. There have been at least 10–15 major theoretical and experimental findings in the past few years; more importantly, the increasing effort in this field will definitely be rewarded.

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