M. A. Leontovich's researches in electrodynamics

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A brief review is given of the principal results achieved by M. A. Leontovich in electrodynamics. Among them are approximate boundary conditions on the surface of a good conductor, the parabolic equation method for solving diffraction problems, the theory of thin antennas, the inertial theory of constriction of a powerful pulsed discharge, conditions for suppressing the kink instability of a current-carrying conductor, and other results.

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M. A. Leontovich's researches in electrodynamics can be divided into two periods. The first comprises the series of papers on the theory of propagation of electromagnetic waves, and was largely completed in the 1940s. The second was concerned with the confinement of high-temperature plasmas in a magnetic field.

There is no doubt that M. A. Leontovich's derivation of the boundary conditions on the surface of a good conductor ranks among his most significant achievements in electrodynamics. These conditions are now referred to as the impedance, or the Leontovich, conditions. Their significance flows from the fact that they reduce the solution of the wave problem for two media, one of which is a good conductor, to the problem of only the field outside the conductor. The electric and magnetic components of this external field are related on the surface of the conductor by a universal expression that involves the surface impedance of the conductor, which in turn depends on the skin-layer depth at the particular frequency.

For nearly a decade, these boundary conditions were frequently employed in high-frequency electrodynamics with references to M. A. Leontovich as the author of these conditions but without citing the specific literature reference because, having pointed out these conditions in 1938–1939, M. A. Leontovich did not actually publish them until 1948.[1] Even Leontovich himself wrote about his boundary conditions with references to papers published by others, where these conditions were acknowledged as having been originally proposed by him. Since then the range of boundary-value problems in electrodynamics involving the presence of good conductors has continued to increase, and Leontovich's boundary conditions have become commonplace in high-frequency physics and engineering.

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Another fundamental paper by M. A. Leontovich appeared in 1944 and gave an approximate solution for the propagation of electromagnetic waves over the Earth's surface (Sommerfeld's problem). It was in this paper that Leontovich put forward the method of the parabolic equation.[2] He introduced the longitudinal and transverse scales of variation of the so-called attenuation function (a slowly-varying factor in the expression for the field) and showed that—as follows from his impedance boundary conditions—this function satisfies a parabolic differential equation that can readily be solved. In particular, Sommerfeld's solution for a vertical dipole above a homogenous flat Earth (Weyl–van der Pol formula) appears at the end of only the fourth page of that paper, whereas the derivation given by Frank and Mises in their book occupies 20 large pages. However, the subject matter of that paper does not merely reduce to this special case although it is considered (and rightly so) to constitute one of the major achievements of twentieth century electrodynamics. M. A. Leontovich himself points out in the introduction to his paper that the method of the parabolic equation can also be used in more complicated cases involving the continuously inhomogeneous Earth (this was in fact demonstrated in his paper where the problem is reduced to the solution of the Volterra integral equation) and the propagation of electromagnetic waves around a spherical Earth. The latter problem was solved in a joint paper by M. A. Leontovich and V. A. Fock[3] in 1946. By using this method, Fock was also able to take into account refraction in the radially inhomogeneous atmosphere surrounding the Earth. However, the importance of the method extends far beyond the range of problems of this kind.

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It has been applied, and increasingly continues to be applied, to the quantitative description of a wide range of wave processes, including wave propagation in nonlinear and randomly-inhomogeneous media.

During this period, M. A. Leontovich also was engaged in research in a completely different area. This was the theory of thin wire antennas. In a paper written jointly with M. L. Levin,[4] he used a characteristic small parameter to obtain an approximate solution of the integrodifferential equation for the current in a thin conductor. The small parameter was the reciprocal of the logarithm of the ratio of the length of the conductor to its width. The resulting equation was of the quasitelegraph form, and this enabled him to give a single solution for both resonance and nonresonance modes of excitation of the antenna, and to consider both receiving and transmitting antennas. In contrast to previous solutions obtained for wire antennas, this approach enabled him to cover the case of bent conductors and systems of conductors of arbitrary cross section.[5] An analogous approach was subsequently used by Ya. N. Fel'd as the basis of his theory of slotted antennas.

In these three topics initiated by him, L. I. Leontovich adopted an essentially physical approach—so typical of him—in which small parameters were used at an early stage in the formulation of a problem rather than in the course of a search for approximate expressions that follow from exact statements that are physically difficult to interpret and are ineffective in practice. This does not, of course, mean that M. A. Leontovich was not interested in exact solutions. On the contrary, he derived exact solutions for many electrodynamic problems formulated by him.

For example, in the posthumously published paper entitled "Radiation emitted through an aperture in a resonator" he generalized the Rayleigh method to the case of the electromagnetic wave field and obtained an exact formulation of the Babinet principle for arbitrary apertures in an infinitesimally thin, plane, perfectly conducting screen.[6] In particular, he used this theorem together with the results reported in Ref. 5 to derive the effective transmission cross section of a narrow straight slot when the magnetic field in the incident wave is parallel to the edges of the slot. We note that a rigorous solution for the case where the electric field is parallel to the slot was given and discussed by Leontovich as far back as 1933. It is included as an addendum to Lecture 5 of Mandel'shtam's course.[7]

Next, we note the paper written jointly with S. M. Rytov[8] in which they discussed electrical fluctuations of thermal origin. In particular, M. A. Leontovich was responsible for the transformation (within the framework of quasistationarity) from the Nyquist integral emf to the extraneous fluctuational electric field responsible for it. Such extraneous fields—both electric and magnetic—were subsequently used by him to develop a general fluctuational electrodynamics free from the condition of quasistationarity.

At the very beginning of work on the problem of controlled thermonuclear fusion, M. A. Leontovich carried out a number of important researches into the electrodynamics of current-carrying plasma.

Foremost among them is the analysis of the effect of a metal enclosure on the equilibrium of toroidal plasma. In the first paper devoted to this topic,[9] he calculated the forces that arise when a current carrying conductor is displaced relative to an enclosure with a finite conductivity. The second paper[10] was devoted to the effect of ports and cuts in the perfectly conducting enclosure that are necessary for introducing the induced electric field into the discharge chamber. He showed that, in the case of a thin enclosure, the presence of even a narrow cut produces a reduction in the confining force that corresponds to the exclusion of a segment of the enclosure of length roughly equal to the minor diameter. The essential point is that Leontovich reduced the problem of cuts to a purely internal boundary-value problem with boundary conditions representing the presence of cuts. This method and the boundary conditions obtained by Leontovich eventually led to the solution of many other important problems in the theory of plasma equilibrium, including problems involving the effect of cuts on the equilibrium displacement of the plasma ring in the tokamak and the penetration of the confining field through the joints.

One of the key pieces of research into controlled thermonuclear fusion was carried out in 1952 by M. A. Leontovich (jointly with V. D. Shafranov). It was concerned with the stabilization of the kink instability of a high-current discharge by an external longitudinal magnetic field.[11] M. A. Leontovich proposed a method for calculating the forces acting on a conductor on the assumption that its cross section was constant and there was no magnetic field in the interior of the conductor. This subsequently led to the demonstration of the fact that current driven plasma instabilities could be stabilized. This paper was the starting point for extensive studies of the behavior of a current-carrying plasma ring in a longitudinal magnetic field, and has stimulated experimental work on the stabilization of instabilities and was fundamental to the theory of the tokamak.

Another paper due to Mikhail Aleksandrovich, published[12] during this period (jointly with S. M. Osovets), laid down the foundations for the physics of the self-constricting discharge, i.e., the pinch. Early experiments on powerful pulsed discharges appeared to lead to the natural conclusion that the pinching of the discharge occured adiabatically, so that the gas-kinetic and electrodynamic compressive forces were in balance at each instant of time. However, experimental data did not fit this. In particular, the sharp valley on the rising part of the current oscillogram presented a puzzle. The peak current, which was termed the singular current, was lower than the maximum current, and the plasma temperature was found to be lower than expected. The observed facts received a natural explanation within the dynamic inertial theory of discharge pinching put forward by Leontovich and Osovets.

This was the first theoretical study of controlled fusion the results of which were in complete agreement with experiment. It constituted an important psychological breakthrough for those concerned with high-temperature plasma (this fickle object became knowable!). This work on inertial compression is now a classic in the physics of powerful pulsed discharges.

In addition to the immediate specific problems that arose in the course of research into controlled fusion, Mikhail Aleksandrovich was, as always, interested in fundamental general problems in physics.

The necessity for taking the thermal motion of charges into account in the theory of propagation of waves through plasma led Leontovich to the examination of the general propagation properties of electromagnetic waves in media with spatial dispersion. He generalized[13] the well-known Kramers-Kronig formulas to the case of media with spatially nonlocal constitutive equations (spatial dispersion), and showed that the principle of causality, taken together with the demands of the theory of relativity, finds its most general expression in this case, and leads not only to the relationship between the real and imaginary parts of the permittivity, but also to a definite limitation on each of them.

It seemed, following the advent of the special theory of relativity, that the concept of magnetic and electric lines of force became totally archaic and its utility would be confined to illustrative purposes in teaching. In fact, the magnetic lines of force in plasmas underwent a "materialization" in the form of electrons revolving around them, which immediately led to the question of a relativistically invariant definition of a line of force. This was precisely the way in which this problem was formulated by Leontovich[14] who showed that, when the electric and magnetic field vectors are perpendicular, it is possible to construct moving lines of force that are independent of the choice of the inertial frame of reference. This paper provided a clear solution of the problem, and is one of the most outstanding papers in the literature of electrodynamics.

Finally, let us pause to consider another, somewhat later, paper by Mikhail Aleksandrovich. The concept of "shear," which is a measure of the difference between the pitch of magnetic lines of force on different magnetic surfaces of a given toroidal system, occupies an important position in the theory of plasma stability in a magnetic field with helical lines of force. Leontovich considered the relation between shear and taper—a local geometric characteristic of the behavior of a set of lines of force—and used the theory of congruence of curves to show that these two quantities did not reduce to one another. This small piece of research illustrates the characteristic style of M. A. Leontovich's work: fundamental approach, wide erudition, brevity, and classic clarity of exposition.

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