The mechanism of nuclear fission (Part II)

Ya. B. Zel'dovich and Yu. B. Khariton

Usp. Fiz. Nauk **139**, 501–527 (March 1983)

PACS numbers: 25.85. – w, 24.60.Dr

The content of Part II of the article (which was intended to be published in the next issue but has been delayed for over forty years: Ed. note) will be the analysis of experimental data on fission and the behavior of fragments in the light of the theory of fission.

FROM THE EDITOR OF SOV. PHYS. USPEKHI

Since the numeration of sections, formulas, figures, tables and references in Part II of the paper published in the March 1983 issue continues the numeration adopted in Part I of the article published in September 1941 which had not been previously translated into English and is diffucult to obtain in the Russian version, both parts have now been translated and are being published together in this issue.

PART II

§ 3. APPLICATION OF THE THEORY OF AN ACTIVATED COMPLEX TO NUCLEAR PROCESSES

A heavy nucleus is a system consisting of many particles characterized by many variables; therefore it is natural to apply *statistical* methods to the dynamics of the nucleus.

Applying the method of the activated complex which was developed initially for chemical reaction,[14,15] one usually starts by discussing the state of complete equilibrium. Let the reaction consist of a transition from one state of low energy A to another state—B (Fig. 9). In this diagram the energy as a function of the parameter describing the transition is plotted along the vertical axis.

In the state of complete equilibrium (in which the numbers of particles in A and B are in an equilibrium relationship) according to the principle of detailed balance the number of particles passing through any state, for example D, from left towards right and from right towards left is the same.

We assume that motion along the horizontal axis occurs according to the laws of classical mechanics without friction.

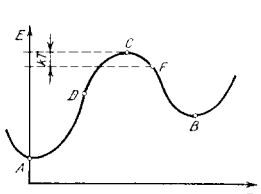

FIG. 9. Dependence of the minimal E-energy on the fission parameter and the concept of the activated complex C.

Particles moving through D from left to right, partially pass through to B, partially, as a result of forces acting in the opposite direction (the forces given by $-\partial E/\partial x$; cf., Fig. 9) return to A. From right to left move both those particles coming from B and also particles starting from A but reversing themselves along the segment DC.

Consequently of greatest interest is an investigation of the state C: counting the particles passing from left to right and from right to left we find the number of transitions from A to B and in the reverse direction; in equilibrium they are equal, and it is easy to find them by constructing an expression for the equilibrium number of particles in C and their average velocity u which is the same in both direction $u_+ = u_-$ (we speak everywhere of the rate of change of the parameter x).

Denoting by $n(C)$ the number of particles in the state C (more accurately—the density or the number of particles per unit length of the x axis) we obtain the number of transitions $A \to B$, which is equal to $\frac{1}{2}n(C)u_+$, and $B \to A$ correspondingly equal to $\frac{1}{2}n(C)u_-$.

However, the rate of reaction is of interest to us not when equilibrium between A and B exists, i.e., when the direct and inverse reactions balance each other. The final aim is the investigation of an irreversible reaction.

Let us assume that the concentration at B is equal to zero. The second assumption underlying the basis of the method of an activated complex[9)] consists of assuming that the number of transitions $A \to B$ is not changed by this, and particularly that it remains in the same relationship to the concentration in A which existed for the given concentration in A in the preceding case of detailed balancing, when an equilibrium number of particles existed in B and the inverse reaction took place.

To what extent are these assumptions mandatory? In his recent paper Kramers[16] has generalized the theory of the activated complex. He considers a system which during its motion along the x axis is subjected to random actions of external forces or of other degrees of freedom. As a result of this the motion takes on the character of a more or less random Brownian movement along the x axis. The random action externally manifests itself as viscosity: if the motion began with

[9)] The first assumption was the one concerning the classical nature of motion without friction.

 0038-5670/83/030279-16$01.80

a definite velocity, there exists a significant probability that this velocity will be reduced as a result of random interactions. Such a viscosity diminishes the probability per unit time of the passage through the state of the activated complex. The equilibrium number of particles passing in one direction or the other through the state C (Fig. 9) is unchanged in the state of total equilibrium. But now even in the absence of particles in B, i.e., when we are considering an irreversible process, it is not possible to neglect the reverse current at the point C: the particles that have passed C from left to right may in the interval CF due to the random interaction reverse themselves and pass through the point C from right to left. In order for this process to be to any extent probable it is necessary that the difference in the energies of C and F would be of order kT. Viscosity significantly reduces the probability of the transition when the "mean free path" of the system from one strong random interaction to another is smaller than the effective width of the transition state $2CF$; for the choice of the point F see above.

On the other hand, a certain small viscosity is necessary for the process: in its absence in the case of strictly inertial motion the system for which the energy of motion along the x coordinate is insufficient for a transition across the barrier will never be able to undergo a reaction, even though the total energy (part of which is distributed among the other degrees of freedom) were sufficiently great. The viscosity describing the interaction of the motion along the x coordinate and all the other degrees of freedom impedes the transition when the energy of motion along x is sufficiently great, but the same viscosity is necessary when the motion along x is weakly excited.

In his article Kramers notes that in the theory of nuclear fission we are apparently dealing specifically with the case of high viscosity. However at present there are no indications as to the order of magnitude of this viscosity so that in our subsequent discussion we shall present the treatment due to Bohr which does not take viscosity into account.

The mention here of the work of Kramers, in our opinion, is of interest, on the one hand, in connection with a discussion of assumptions underlying the concept of an activated complex; on the other hand, because in the very near future one can expect papers developing a theory of the viscous type (the type of Brownian motion or, as Kramers referred to it, the diffusion type) for the transition across the barrier.

Turning to the forthcoming calculations we should clearly visualize that their aim is not the exact calculation of the absolute value of the probability of fission, but first of all an elucidation of the dependence of the probability of fission on the energy of the nucleus. The effect of friction will reduce to a coefficient in the expression for the probability of fission which is almost independent of energy.

In the theory of the activated complex in chemical reactions one considers the statistical distribution of molecules over all values of energy and over all values of angular momentum. Physically this is associated with the fact that the reacting molecules experience a large number of collisions realizing all possible states.

In nuclear reactions the situation is significantly different. The reaction takes place always as a result of a single impact of some kind of a particle on the nucleus[10], after which the excited nucleus that has been formed is left to itself and, until the reaction has taken place, is characterized by a quite definite value of the energy and of the angular momentum.

Of all the possible (for a given excitation energy) values of the angular momentum in actual fact only those are realized which could be produced by the bombardment of the nucleus. Large values of the angular momentum turn out to be exceptional: in order to transfer a large angular momentum an impact is required which differs considerably from a central impact; the bombarding particle must move at a considerable distance from the center of the nucleus; in such a case the probability of it being captured is negligible.

It is still more important that the whole process occurs at constant energy.

Following Bohr we consider a microcanonical ensemble of nuclei having an excitation energy between E and $E+\mathrm{d}E$. We introduce the notation: $\rho(E)$ is the level density in the given energy interval; $\rho(E)\mathrm{d}E$ is the number of individual levels in the interval between E and $E+\mathrm{d}E$; $d=d(E)$ is the average distance between two adjacent levels, so that $\rho(E)=1/d(E)$ and, finally, Γ is the width of an individual level; the latter can be either less than d—in the case when we are dealing with discrete individual levels, or greater than d—in the case of overlapping levels[11].

The level width is extremely closely related to the lifetime of the state under consideration.

This relationship follows from Heisenberg's indeterminacy principle. In fact, only such a system can have an exactly determined value of the energy (level width equal to zero) which exists in the state under consideration for an infinitely long time. For a system whose time of occupying the given stage is finite, the indeterminacy principle

$$\Delta E\cdot\Delta t=\hbar \tag{34}$$

shows that any measurement of the energy will be associated with a lack of definition $\Delta E=\hbar/\Delta t$. The energy level turns out to be smeared out. Thus, for example, the theory of line width of spectral lines is well known and rests entirely on experimental data. This theory

[10] All the present methods of irradiation are too weak to be able to study processes associated with sequential action of several particles on a single nucleus. However, cascade processes are possible, for example a neutron capture which is followed by the re-emission of the neutron and only then by motion. Here prior to fission the angular momentum changes twice—both during capture, and during emission of the neutron.

[11] The broadening of levels, and even their overlapping, does not alter the invariant number of energy levels, although its calculation does become more complicated.

relates the finite width of these lines (for example, in a gas discharge) with the lack of definition of the energy of the excited level, arising as a result of the fact that collisions with other atoms or the process of radiation itself lead to a certain finite mean lifetime of the excited atom.

The total width of the level is related to the lifetime of the given state, i.e., to the probabilities of all the different possible processes.

The indeterminacy in the time in formula (34) should be interpreted as the lifetime of the nucleus, i.e., the quantity reciprocal to the total probability α of its going over to some other state:

$$t=\frac{1}{\alpha}=\frac{1}{\alpha_1+\alpha_2+\alpha_3+\dots}, \tag{35}$$

where $\alpha_1, \alpha_2, \dots$, etc., are the probabilities of the individual processes.

In accordance with (34) one can represent the total width

$$\Gamma=\frac{\hbar}{t}=\hbar\cdot\alpha=\hbar(\alpha_1+\alpha_2+\dots)=\Gamma_1+\Gamma_2+\dots \tag{36}$$

in the form of a sum of individual widths

$$\Gamma_1=\alpha_1\hbar,\quad \Gamma_2=\alpha_2\hbar,\ \dots \tag{37}$$

of the different processes. In future we shall often use this terminology and speak of the width of some process in order to characterize its probability.

It is easy to establish the numerical relationship between these quantities: since $\hbar=10^{-27}$ erg·s and 1 eV $=4.77\cdot10^{-10}/300=1.59\cdot10^{-12}$ erg, then the lifetime $t=1$ s and the probability of decay 1 s^{-1} correspond to the width

$$\Gamma=\frac{10^{-27}\ \text{erg}\cdot\text{s}}{1\text{c}}=10^{-27}\ \text{erg}=6\cdot10^{-16}\ \text{eV}. \tag{38}$$

Conversely, a line width of the order of 1 eV corresponds to a lifetime equal to $6\cdot10^{-16}$ s and to the probability of the decay process of $1.6\cdot10^{15}$ s^{-1}.

Thus, below we shall speak of neutron widths associated with the probability of emitting a neutron, of the width for fission, associated, according to (37), with the probability of fission, etc.

So let us consider a microcanonical ensemble of nuclei having an excitation energy between E and $E+dE$, and choose the number of nuclei under consideration exactly equal to the number of levels. Thus, in this set of nuclei on the average each possible state corresponds to a single nucleus.

In accordance with the definition of partial widths the number of nuclei undergoing fission per unit of time in our microcanonical ensemble will be equal to

$$\rho(E)\,dE\frac{\Gamma_f}{\hbar}, \tag{39}$$

where the first factor represents the number of nuclei under consideration in the microcanonical ensemble, and the second factor represents the probability of fission per unit time (the subscript f refers to fission).

In our ensemble in which the average number of nuclei per level is equal to unity the number of nuclei undergoing fission per unit time must be equal to the number of nuclei in the transition state which penetrates the fission barrier per unit time.

Let us determine the number of states for nuclei per unit length of the barrier at its apex. The motion of a nucleus through the barrier in the direction of fission is equivalent to the inertial motion of a particle on which no forces act, since at the apex of the barrier the energy is a maximum. The derivative of the energy with respect to the coordinate taken along the barrier is equal to zero. Thus, in evaluating the number of states we shall, for the degree of freedom corresponding to the motion of the particle through the barrier, have to evaluate the partition function in the same way as we do for the motion of a free particle in one dimension.

The dimension of a cell in phase space is equal, as is well known, to $h^n=(2\pi\hbar)^n$, where n is the number of dimensions of the space. In the case under consideration the translational motion takes place only along one coordinate—along the coordinate describing fission. Thus, one level corresponds in p_x, x phase space to a cell of area $2\pi\hbar$. The number of levels per unit length in the interval of variation of the momentum between p and $p+dp$ is equal to $dp/2\pi\hbar$.

We introduce the quantity ρ^*—the density of levels of the nucleus under consideration in the transition state, taking into account in the calculation of ρ^* all the degrees of freedom which are excited in the nucleus with the exception of that degree of freedom motion along which describes fission. Then the final number of states of the microcanonical ensemble with an energy between E and $E+dE$, with a momentum between p_x and p_x+dp_x per unit length of the x coordinate in the transition state is given by the following expression:

$$dE\rho^*(E-E_f-K)\frac{dp}{2\pi\hbar}. \tag{40}$$

Expression (40) has the dimensionality of cm^{-1} if the x coordinate (motion along which describes fission) has the dimensionality of length. The number of states (40) depends on the energy associated with all the degrees of freedom included in ρ^*, i.e., all the degrees of freedom with the exception of motion in the direction of fission. This latter energy is equal to the total energy of the nucleus E less the potential energy of fission equal to the barrier height E_f and the kinetic energy K of the motion of the system in the direction of fission.

A perfectly general expression for the differential of the kinetic energy establishes the connection between the velocity of motion and the direction of fission, the momentum p and the kinetic energy K:

$$dK=v\,dp. \tag{41}$$

In the initial state we have a single nucleus in each individual quantum level. In accordance with this equilibrium the levels of the transition state are also filled with the same average density. We shall find the number of fission events occurring per unit time by constructing an expression for the total flux of particles across the barrier, i.e., by multiplying the density of particles in each elementary momentum interval by the velocity with which these particles move in the direction of fission. Thus we shall obtain for the number of

fission events by integrating over all the values of the momentum the formula

$$dE \int \rho^*(E-E_f-K)\frac{dp}{2\pi\hbar}v=\frac{dE}{2\pi\hbar}\int \rho^* dK=\frac{dE\cdot N}{2\pi\hbar}, \qquad (42)$$

in which we have denoted by N the total number of levels in the transition state, accessible for a given excitation energy. The dimensionality of v is cm/s, the dimensionality of expression (40) is cm^{-1}, the number of fissions (42) has the dimensionality s^{-1}.

We recall that dE is the total magnitude of the energy interval in the microcanonical ensemble under consideration, so that the total number of nuclei under consideration is proportional to dE. In (42) the integration is carried out over values of the energy from $K=0$ to $K=E-E_f$ which makes the argument of ρ^* equal to zero. Comparing this expression (42) with the definition of the fission "width" Γ_f (39), we finally obtain the following expression:

$$\Gamma_f=\frac{N^*}{2\pi\rho(E)}=\frac{d}{2\pi}N^* \qquad (43)$$

for the fission width expressed in terms of the level density per unit energy interval or in terms of the quantity d—the average distance between energy levels in the nucleus under consideration. With other conditions being equal, the greater is the density of levels ρ in the nucleus, the smaller is the probability of the nucleus entering the transition state.

The derivation which we have just utilized is correct only in the case if N^* (the number of levels in the transition state) is sufficiently great compared to unity. Only in that case can one introduce the density ρ^* and integrate (42). In accordance with formula (43) this condition coincides with the condition that the fission width should considerably exceed the average distance between levels. The physical meaning ot the latter condition is quite clear: if the width of each individual level is considerably greater than the average distance between the levels, the latter strongly overlap, quantization becomes unnecessary; in accordance with the correspondence principle we can use classical mechanics. On the other hand, if the excitation E either exceeds the critical energy by only a small amount or is even less than E_f, the number of levels becomes small or might even turn out to be equal to zero. This means that fission will occur due to specific quantum mechanical effects of the type of a tunnelling transition under the barrier. Experimental data, as well as exploratory calculations (cf., the preceding section) which take into account the large mass of the fragments, show that the probability of such a transition under the barrier in the case of fission of heavy nuclei falls off very rapidly with decreasing excitation energy. In principle, in an exact discussion we must obtain a gradual transition from the formula of the theory of an activated complex to the quantum mechanical expression for barrier penetrability. However, physically the tunnelling fission of uranium is not very important.

We can in the same manner by utilizing the theory of the activated complex approach the problem of the probability of the re-emission of a neutron. It is specifically the existence of re-emission of neutrons that limits the fission yield at a high excitation energy. Calculations of neutron evaporation were repeatedly given in the literature; these questions were dealt with in particularly great detail by L. D. Landau[17] and V. F. Weisskopf.[18]

Let us consider once again the microcanonical ensemble introduced above. Only small changes will be needed compared with the preceding material. Now the transition state is represented by a nucleus in which the neutron being emitted is situated in the thin spherical shell just at the surface of the nucleus, whose area is equal to $4\pi R^2$, where R is the nuclear radius.

The critical energy of the process correspondingly coincides with the neutron binding energy E_n. The density of the levels of excitation in the transition state is determined by the density ρ^{**} of the spectrum of the excited nucleus remaining after evaporation.

We write down the number of quantum states in the microcanonical ensemble with a given momentum of the neutron being evaporated between p and $p+dp$ and with a given direction of emission of the neutron. We characterize the latter by the solid angle Ω and seek the number of levels per unit thickness of the spherical layer for the state in which the momentum of the neutron being evaporated lies between p and $p+dp$, while the direction of evaporation lies within the element of the solid angle $d\Omega$.

The volume within which the neutron is situated in the transition state referred per unit length along the direction of emission of the neutron from the nucleus is numerically equal to the magnitude of the nuclear surface $4\pi R^2$. Finally the total number of levels in the transition state referred per unit length along the coordinate R describing neutron evaporation is given by the following expression:

$$\frac{(4\pi R^2 p^2\,dp\,d\Omega)}{(2\pi\hbar)^3}\rho^{**}(E-E_n-K)\,dE, \qquad (44)$$

in which the first bracketed factor represents the number of levels of the neutron per unit length, and the second factor represents the number of levels of the remaining nucleus (dimensionless), whose excitation energy is equal to $E-E_n-K$.

In order to obtain the number of neutrons emitted per unit time we must multiply the density of neutrons in the spherical layer (per unit thickness) by the velocity of their motion away from the nucleus, which is equal to $v\cos\theta$, where θ is the angle between the direction of motion of the neutron and the position vector drawn from the center of the nucleus.

Substituting $p^2=2mK$, $v\,dp=dK$ and integrating over the whole hemisphere, we obtain the following expression for the number of events of neutron emission occurring per unit time:

$$dE\left(\frac{4\pi R^2\cdot 2\pi m}{(2\pi\hbar)^3}\right)\int \rho^{**}(E-E_n-K)K\,dK. \qquad (45)$$

We must identify this expression with the expression for the neutron width which is found in a completely analogous manner to the expression for the fission width. Expressing in this manner the probability for neutron emission in energy units (as the level width)

we obtain the number of events of neutron emission in the form

$$\Gamma_n = \frac{1}{2\pi\rho(E)} \cdot \frac{2mR^2}{\hbar^2} \int \rho^{**}(E - E_n - K)\, K\, dK. \quad (46)$$

From formulas (45) and (46) it can be seen that the probability for the remaining nucleus to land on some one or another excited level is not the same. The probability of each level is directly proportional to the kinetic energy of the evaporating neutron. But from the law of conservation of energy it follows that the greater is this kinetic energy, the lower is the excitation energy of the nucleus after emission. Thus, for the nucleus remaining after emission of the neutron the probability referred to each individual level turns out to be lower for levels with high energy. Evaporation of a neutron with a high kinetic energy is relatively the more probable. However, this algebraic factor K in the integrand in (45) and (46) is more than compensated by the fact that the number of levels in the nucleus remaining after evaporation increases rapidly with increasing excitation energy.

According to the general principles of statistical mechanics we can relate the density of levels to the entropy of the excited nucleus. The entropy is the logarithm of the total number of levels with an energy lower than the given energy[12]:

$$S = \lg N = \lg \int^{E} \rho(E)\, dE, \quad \int^{E} \rho(E)\, dE = e^{S(E)}, \quad (47)$$

$$\rho(E) = \frac{de^S}{dE} = e^S \frac{dS}{dE} = \frac{1}{T} e^S. \quad (48)$$

We shall determine what energy of the neutrons being emitted corresponds to the maximum of the integrand in formulas (45) and (46), i.e., what is the most probable energy of the neutrons being emitted:

$$\rho(E - E_n - K)\, K = \max. \quad (49)$$

We find the logarithmic derivative

$$\frac{d}{dK} \lg[\rho(E - E_n - K)\, K] = \frac{d}{dK} \lg \rho(E - E_n - K) + \frac{1}{K} = 0. \quad (50)$$

Differentiating expression (48) for the density of levels we neglect the changes in the factor preceding the exponential:

$$\frac{d \lg[\rho(E - E_n - K)]}{dK} + \frac{1}{K} = \frac{dS(E - E_n - K)}{dK} + \frac{1}{K} = -\frac{1}{T(E - E_n - K)} + \frac{1}{K} = 0,$$

$$K = T(E - E_n - K). \quad (51)$$

In equations (48) and (51) we have utilized the general thermodynamic relationship[13].

$$dE = T\, dS, \quad \frac{dS}{dE} = \frac{1}{T}.$$

As calculations have shown the average energy is

$$\overline{K} = 2T(E - E_n - K), \quad (52)$$

while for free particles in thermal equilibrium we would evidently have $\overline{K} = 3T/2$.

[12] Everywhere below we measure the temperature in units of energy (in ergs or in electron-volts). In such a system the entropy is dimensionless, and the Boltzmann constant is equal to unity.

[13] In formulas (51) and (52) $T(E - E_n - K)$ is the temperature of the nucleus at an energy $E - E_n - K$.

It is of particular importance that the energy of the neutrons being emitted and the probability itself of neutron emission depend on the temperature of the nucleus remaining after neutron emission, and not on the initial temperature of the nucleus [the formulas contain $\rho^{**}(E - E_n - K)$, and not $\rho(E)$].

Bohr transforms equation (46) by introducing zero kinetic energy of the particle situated in the nucleus: if each particle corresponds to a volume in the form of a cube of edge x, then the de Broglie wavelength can not be greater than x and this corresponds to a momentum of the order of

$$p \approx \frac{\hbar}{x}$$

(in accordance with Heisenberg's indeterminacy principle) and to an energy

$$K' \approx \frac{p^2}{2m} \approx \frac{\hbar^2}{2mx^2}.$$

In a nucleus with total radius R Bohr substitutes $x = R/A^{1/3}$;

$$K' = \frac{A^{2/3}\hbar^2}{2mR^2}. \quad (53)$$

As a result formula (46) is brought to the form

$$\Gamma_n = \frac{d}{2\pi} \left(\frac{A^{2/3}}{K'}\right) \sum_i K_i. \quad (54)$$

The summation is taken over all the values of the kinetic energy corresponding to different energy levels of the remaining nucleus. Bohr emphasizes the analogy of expression (54) and the expression (43) for the probability of fission: the total number of levels N^* appearing in (43) can be written as $\sum_i 1$ over all the N^* levels so that

$$\Gamma_f = \frac{d}{2\pi} \sum_i 1. \quad (43a)$$

Bohr estimates the numerical value of Γ_n in equation (54) as 9.3 eV using the data on nuclear radii. The following transformation seems to us to be no less instructive. We introduce the kinetic energy K of a neutron for which the de Broglie wavelength is of the order of the dimensions of the whole nucleus (and not of a segment associated with a single particle in the nucleus, as was the case previously):

$$K'' \approx K' A^{-2/3}, \quad \Gamma_n = \frac{d}{2\pi} \frac{1}{K''} \sum_i K_i. \quad (55)$$

In (55) we can clearly see the connection between both conditions of applying the theory of the activated complex: it is necessary that the kinetic energy of the emitted neutrons should exceed K in order that it should be possible to speak of their *position* and of the *direction* of the neutron at the moment of emission.

But if at least some of the K_i would exceed K'', then Γ_n would be greater and the levels would overlap.

According to the principle of detailed balance the probabilities of neutron capture and neutron re-emission are interrelated. Formulas (45), (46), (54), (55) correspond to the capture cross-section for fast neutrons which is equal to the geometric cross-section of the nucleus of order

$$\sigma_c = \pi R^2 = \pi (1.48 \cdot 10^{-13})^2 A^{2/3} \approx 3 \cdot 10^{-24}.$$

For slow neutrons quantum mechanics leads to a gradual increase in the capture cross-section as the energy decreases proportional to $1/V$, i.e., proportional to $1/K^{1/2}$ (in the case of overlapping levels).

The re-emitted neutron has an energy which is exactly equal to the energy of the incident neutron (this follows from the distribution of levels in the spectrum of the remaining nucleus; cf., below). The probability of emission of a slow neutron is

$$\Gamma_n \approx 10^{-3}\sqrt{K}, \tag{56}$$

if Γ and K are expressed in eV.

Summation is not required here, since the neutron can be emitted only in the case when the nucleus is left in the ground state: the energy is insufficient to excite the nucleus.

We shall briefly consider the probability of other processes. The binding energy of protons and neutrons in nuclei is practically the same; we have already noted that a nucleus with a high proton binding energy is β-active, while a nucleus with a large neutron binding energy diminishes its charge by the capture of a K-electron or—in the case of sufficient energy—by emitting a positron[14)].

It would appear that if the binding energy of a neutron and a proton were the same the emission of a proton should be just as probable as the emission of a neutron. In actual fact, as a result of the electrostatic repulsion by the nucleus a proton whose energy is sufficient to be removed from the nucleus to infinity cannot be situated near the nucleus. In the case of a low energy of the emitted proton, i.e., in the case when the energy of the initial nucleus exceeds the binding energy of the proton by only a small amount, the emission of a proton is possible only as a result of the quantum mechanism of tunnelling barrier penetration. The probability of such a process is very low. But also in the case when the energy stored in the nucleus is sufficient for a classical transition of a proton with a high energy above the barrier such a process is of low probability.

We saw that a neutron is emitted from the nucleus with an energy of the order of T or $2T$, where T is the temperature of the nucleus.

The same considerations apply to a proton near a nucleus. But if the kinetic energy of the proton near a nucleus is of order T, then in going away from the nucleus the proton will be accelerated in the field of the nucleus, and at a large distance its kinetic energy will be of the order of $2T + 9$ MeV (the estimate of the electrostatic energy has been made for the case of uranium). Carrying away a larger amount of energy, the proton after emission leaves the nucleus in a state which is less excited compared to the nucleus remaining after neutron emission. The less is the nucleus excited the smaller is the number of individual levels, i.e., the lower is the probability of the process.

In heavy nuclei ($Z > 30$) one can neglect proton emission for any excitation energy, since it is always by a factor of millions weaker than the emission of neutrons.

The same considerations also apply to the emission of α-particles. The barrier in this case is twice as high (15–20 MeV). But then in heavy nuclei the binding energy of the α-particle is negative as is indicated by natural α-radioactivity. These two considerations act in opposite directions, so that from general considerations it is difficult to establish the relationship of the probabilities of emission of an α-particle and a neutron by an excited nucleus, whose energy is sufficient for neutron evaporation.

In the cases of interest to us experiments show that the probability of α-decay is sufficiently small, and there is no need to take it into account.

Nuclear processes associated with the emission of light charged particles—electrons and positrons—are completely inaccessible to a classical description. For us the knowledge is sufficient that their probability is negligibly small in comparison with the probability of other processes[15)] and moreover depends comparatively weakly on the energy of the nucleus. We shall examine these processes below in connection with the question of the fate of the fission fragments.

The last in this ordering, but not the least in significance, is the process consisting of the emission of γ-quanta. Again the statistical theory in examining the equilibrium between the excited nuclei and the equilibrium (black-body) radiation at a temperature of several MeV (~10^{10} K), establishes the relationship between the cross-section for the capture of γ-rays and the probability of their emission by the excited nucleus.

The capture cross-section of the order of 10^{-26} cm^2 corresponds in this case to the time for the γ-transformation of the order of 10^{-14} s, i.e., to the probability of emission of 10^{14} s^{-1} or ~0.1 V in energy units.

Our information concerning the probability of the different processes are summarized in Fig. 10.

Along the horizontal axis we have plotted the excitation energy of the nucleus under consideration in MeV, and along the vertical axis we have plotted the probabilities of the different kinds of transformation (indicated

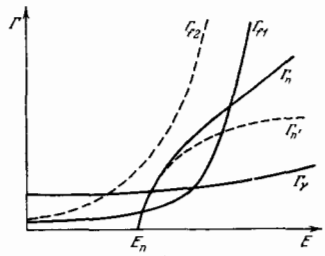

FIG. 10. Schematic dependence of the probability (width) of different processes on the energy of the nucleus.

[14)] More accurately, the neutron binding energy must be greater than the binding energy of the proton by 0.7 MeV—the difference between the masses of a neutron and a hydrogen atom expressed in energy units.

[15)] Except for the tunnelling processes the probability of which can be very small.

for each curve).

In order to exhibit in a single diagram such processes, as spontaneous fission (with a half-life of ~10^{20} years) and neutron evaporation from a highly excited nucleus (10^{-19} s), one has to utilize a highly deformed scale.

The left hand scale shows probabilities expressed in energy units, —level widths.

The curve showing the probability of neutron emission is vertical at an excitation energy equal to the binding energy of the neutron since there are no tunnelling transitions of any kind, no barrier. When the energy of the nucleus is slightly lower than the binding energy of the neutron emission of a neutron is completely impossible; in the case of a small excess of energy emission of a low velocity neutron is very probable.

Figure 10 shows the total probability of emission of a neutron Γ_n, corresponding to formulas (54) or (55) in which the summation is carried out over all the states of the remainder nucleus and correspondingly over all the possible (for a given excitation energy) values of the kinetic energy of the neutron being emitted.

Along with this the curve of $\Gamma_{n'}$ is given which corresponds to the first term in the sum in expressions (54) and (55); $\Gamma_{n'}$ is the probability of a neutron being emitted from the nucleus carrying away all the energy and leaving the nucleus unexcited in its ground state. If the energy of excitation of the nucleus exceeds the binding energy of the neutron only by an amount which is less than the excitation energy of the remainder nucleus, the neutron must necessarily carry away all the energy, since it cannot be retained by the nucleus. Therefore for low energies both curves for Γ_n and $\Gamma_{n'}$ coincide. In contrast, at a high energy the probability is very low that the neutron would carry away all the energy, since the number of degrees of freedom of the nucleus is considerably greater than the number of degrees of freedom of the neutron; as shown in Fig. 10 in this case $\Gamma_n \ll \Gamma_{n'}$.

The probability of emission of an α-particle and the probability of fission make a smooth transition from the classical values to the curve of the (quantum mechanical) penetrability of the barrier by particles of insufficient energy.

In the system of coordinates with a highly deformed scale the straight line in Fig. 10 indicates a strong exponential dependence of the fission probability on the energy of the nucleus. The critical energy of fission determines that value of the excitation energy of the nucleus at which the curve of fission probability passes through the value $\Gamma_f \sim 0.1$ V, which separates the classical and the quantum mechanical parts of the curve.

The curve for the fission probability is given in the diagram in two variants: for a critical energy of fission exceeding the neutron binding energy by 1 MeV (solid curve) and being below it by 1 MeV (the dashed curve); the latter curve is displaced with respect to the former by 2 MeV in the direction of lower energy.

The probability of emission, as shown in the diagram, depends weakly on the energy and is represented by an almost horizontal line.

Before discussing the experimental data we briefly summarize the data on the distribution of levels in the spectra of heavy nuclei. It has been possible to establish from the study of γ-spectra that the lower levels are very sharp and that the distance between them is of the order of 50,000–100,000 V. As the excitation energy increases the number of levels increases rapidly. The statistical treatment of the concepts of temperature and entropy of the nucleus is applicable only when the number of levels become sufficiently great.

General theoretical arguments force is to expect that up to energies of hundreds of MeV the free energy of the nucleus will depends on its temperature according to

$$F = -\frac{\alpha}{2} T^2. \tag{57}$$

We apply the simple thermodynamic transformations

$$S = -\frac{\partial F}{\partial T} = \alpha T, \quad E = F + TS = \frac{\alpha T^2}{2}, \quad S = \sqrt{2\alpha E}. \tag{58}$$

Expression (57) follows from the requirement of the Nernst theorem that the entropy should vanish[16] at zero temperature. Formulas (57) and (58) lead to a specific heat proportional to the absolute temperature. Proportionality of the specific heat to the temperature was observed experimentally by Keesom in metals at low temperature. At the same time the specific heat of the lattice depends on the temperature according to T^3 (the Debye law), so that the specific heat observed by Keesom referred to the electrons in the metal.

Although the analogy between electrons in a metal and matter in a nucleus is not rigorous, nevertheless in any case expressions (57) and (58) are a sensible approximation which agrees not badly with the experimental data on the levels of heavy nuclei. Formula (58) indicates a rapid increase in the number of levels with increasing energy proportional to $\exp(2\alpha E)^{1/2}$. Bohr's article[5] describing the properties of level distribution corresponding to formula (58) has already been published in "Uspekhi". For nuclei close to uranium one can expect that the average distance between levels, which is of the order of 50,000–100,000 V for the first few levels, decreases to 20 V at an excitation energy of 6 MeV and to 0.2 V at an excitation energy of 8.5 MeV.

We shall require these data below in order to describe experimental data.

§4. DISCUSSION OF EXPERIMENTAL DATA ON THE PROCESS OF FISSION INDUCED BY NEUTRONS

The work of the last year and a half has contributed greatly to the elucidation of the basic facts. A number of conclusions which Bohr established by means of a subtle comparison of different possibilities, have now received direct experimental confirmation; in particular this refers to the question of the role played by the

[16] Entropy equal to zero means that the system is in a single definite level, in a completely defined state. In our discussion we follow L. D. Landau.[17]

different isotopes of uranium (cf., the end of §2).

In our article we shall present the concepts accepted at the present time which are based on the theory discussed in §1–3, omitting proofs of their uniqueness.

Mass-spectrographic investigations have established the existence of three uranium isotopes: with atomic weights 238 (99.3%), 235 (0.7%) and 234 (0.006%). In §2 we have established that the critical energy of fission which depends on the ratio Z^2/A must be the lowest for the lightest, and correspondingly the least abundant, isotope with $A = 234$, is greater for U^{235} and greater still for U^{238}.

The concentration of the lightest isotope is very small and at present there are no direct data on its behavior, nor on any features of the behavior of the natural mixture of uranium isotopes the explanation of which would require involving U^{234}. Therefore we shall omit it from further discussion.

Uranium of atomic weight 235 has remarkable properties.[17] This isotope gives rise to the actinium series. Just as U^{238}, U^{235} exhibits natural α-radioactivity, but with a somewhat shorter half-life. Thus, several hundred million years ago the isotopic composition of uranium was more favorable. Bringing about a chain reaction of uranium in that far distant era would have been a much more easy problem. The light isotope is of interest for us because it undergoes fission under the action of slow neutrons.

The latest calculations of I. I. Gurevich and the present authors show that in order to realize a chain reaction in uranium with the liberation of tremendous quantities of energy approximately ten kilograms of the pure uranium-235 isotope would be sufficient.

Such properties are explained, on the one hand, by the fact that the nucleus which is formed on capture of a neutron by uranium-235, —the nucleus of uranium-236—has a relatively low critical energy for fission, of only 5 MeV; on the other hand the binding energy of a neutron in the nucleus of uranium-236 consisting of an even number of protons and of neutrons is great. According to Bohr's estimate the binding energy of the neutron is of the order of 6.2 MeV. Thus, in Fig. 10 we are dealing with the case which corresponds to the dashed fission curve. At all energies the probability of fission is by a large factor (at least by a factor of 100) greater than the probability of neutron re-emission. The energy of excitation of the nucleus formed on neutron capture is not less than the binding energy of the neutron. In this case, as can be seen from Fig. 10, the probability of fission considerably exceeds also the probability of loss of energy—emission of a γ-quantum. Thus, when uranium-235 is bombarded by neutrons the capture of a neutron with a probability that does not differ from unity, leads to fission.

In order for the neutron to be captured a resonance (equality of energies) is required between the nucleus-neutron system and the compound nucleus that is formed (U^{236} in the case under discussion).

The nucleus being irradiated is in the ground state with a strictly defined energy. The U^{236} nucleus obtained upon neutron capture has an energy of excitation of approximately 6 MeV (upon capturing a thermal neutron). With such an excitation energy the average distance between the individual energy levels of the compound nucleus is of the order of 10–20 eV. But the width of the levels, which depends on the probability of fission of the excited nucleus, exceeds the mean distance between them; the energy spectrum of the compound nucleus should be regarded as being continuous (consisting of overlapping levels).

Experiment establishes the effective fission cross-section under the action of thermal neutrons at room temperature to be of the order of $2.5 \cdot 10^{-24}$ cm^2 for the natural mixture of uranium isotopes. The cross-section referred to uranium-235 turns out to be equal to $350 \cdot 10^{-24}$ cm^2. As the neutron energy increases the cross-section diminishes in inverse proportion to the velocity, i.e., it falls off as $K^{-1/2}$ (K—kinetic energy of the neutron).

On the other hand, when the nucleus is bombarded by very fast neutrons its cross-section does not differ from the geometric cross-section, i.e., it is of the order of $2.4 \cdot 10^{-24}$ cm^2. Connecting the two regions in the simplest manner we obtain the cross-sections for fission:

$$\sigma_f = \frac{350\sqrt{0.025}}{\sqrt{K}} \cdot 10^{-24} = \frac{55}{\sqrt{K}} \cdot 10^{-24}\ \text{cm}^2, \quad K < 10^3\ \text{V},$$
$$\sigma_f = 2.4 \cdot 10^{-24}\ \text{cm}^2, \quad K > 10^3\ \text{V}. \tag{59}$$

In actual fact it is very difficult to predict the behavior of the cross-section as a function of the energy in the range of K lying between 10^3 and 10^6 V. Expression (59) can be used in this range only in the absence of other data (cf., below).

Unfortunately, the principal uranium isotope behaves differently. The large mass for the given charge diminishes the electrostatic energy; the critical energy of fission of the compound nucleus uranium-239, which is formed when the principal isotope captures a neutron, is of the order of 5.5 MeV. At the same time the formation of a nucleus with an odd atomic weight from an even nucleus with an even charge and weight is accompanied upon capturing a neutron by a smaller amount of energy liberated—approximately 5 MeV. We see the interrelationship of the probabilities of the different processes in Fig. 10 taking into consideration the (solid) curve for fission.

For a low energy of the incident neutron the energy of the compound nucleus being formed is close to the neutron binding energy. For an energy lower than the critical, fission is possible only as a result of a tunnel transition with the probability of such a process being quite small.

The principal competing processes after capture of a low-energy neutron are the re-emission of the neutron and the emission of a γ-quantum. The emission of a γ-quantum, after which the excitation energy of the nu-

[17] The so-called actinouranium.

cleus becomes in any case smaller than the binding energy of the neutron, leads to the formation of a still heavier isotope—uranium-239[18].

The probability of re-emission of a neutron at a low energy is of the order of $\Gamma_n = 10^{-3} K^{1/2}$, where K and the probability Γ_n are expressed in volts.

The probability of emitting a γ-quantum is of the order of 0.1. Up to an energy of the incident neutrons of 1,000–10,000 V the capture of a neutron leads to the formation of a nucleus of uranium-239 in an unexcited state after the binding energy has been radiated away. But a nucleus of uranium-239 contains an excess of neutrons. Emission of β-rays takes place and uranium-239 transforms into element 93—a transuranium element—with the same atomic weight. We note that specifically the production of transuranium elements with an atomic number higher than 92 was the original aim of investigators who subjected uranium to the action of neutrons. The β-transformation of uranium-239 into a transuranic element occurs with a half-life of approximately 20 min. The quantity of element 93 that is formed is very small so that little is known of its subsequent fate and properties.

MacMillan and Abelson[19] established that element 93-239 in its turn undergoes a further β-decay with a half-life of 2.3 days, producing the element 94-239. Investigation of the latter did not exhibit any kind of activity. The sensitivity of the method enables one to assert that the lifetime for spontaneous fission or for spontaneous α-decay of element 94-239 is not less than a million years[19]. This means that neutrons captured by the principal isotope uranium-239 with the formation of the transuranium element 93 should be regarded as lost for the chain reaction process. It is specifically this that makes it impossible to realize a chain reaction in metallic uranium without isotope separation.

When the energy of the incident neutrons exceeds the difference between the critical energy for fission and the neutron binding energy in the compound nucleus the excitation energy of the compound nucleus exceeds the critical energy for fission; fission according to the classical mechanism becomes possible. The calculation of the probability for this process was carried out in the preceding section by the method of the activated complex. The probability of fission increases as the excitation energy increases corresponding to the fact that the number of levels of the nucleus in the critical state through which the transition is possible increases. Soon after that the probability of fission becomes considerably greater than the probability for the emission of γ-quanta. Now at high energies of incident neutrons two competing processes remain—fission and the re-emission of a neutron. The ratio of the probabilities for the two processes for uranium-239 and thorium is shown in Fig. 11, adapted from Bohr and Wheeler.[2]

Bohr assumes that for uranium-239 the difference between the critical energy for fission and the neutron binding energy is equal to 0.75 MeV, and for thorium-1.75 MeV.

The upper part of the diagram shows the ratios of the probability of re-emission of a neutron and the probability of fission to the average distance d between the levels in the nucleus. The kinetic energy of the neutrons is plotted along the horizontal axis. The energy of excitation of the nucleus formed upon neutron capture is equal to the sum of the neutron binding energy and its kinetic energy.

For a high neutron energy the total capture cross-section is determined by the geometrical dimensions of the nucleus

$$\sigma = \pi R^2. \tag{60}$$

After capture the nucleus has a definite probability for fission Γ_f and the probability of neutron re-emission Γ_n. Thus, with reference to one of the processes, for example to fission, the cross-section is given by the formula

$$\sigma_f = \sigma \frac{\Gamma_f}{\Gamma_f + \Gamma_n} = \pi R^2 \frac{\Gamma_f}{\Gamma_f + \Gamma_n}. \tag{61}$$

The additional quantity to make up the total cross-section πR^2 is the cross-section for neutron scattering (capture with a subsequent re-emission)

$$\sigma_n = \sigma \frac{\Gamma_n}{\Gamma_f + \Gamma_n} = \pi R^2 \frac{\Gamma_n}{\Gamma_f + \Gamma_n}, \quad \sigma_f + \sigma_n = \sigma = \pi R^2. \tag{62}$$

In the roughest approximation, neglecting all the factors preceding the exponential, we will find the ratio of Γ_f and Γ_n by noting that each of these quantities is proportional to the number of levels of the transition state for the corresponding process. By assuming for the "specific heat" (the connection between the energy, temperature and entropy) of the transition state the

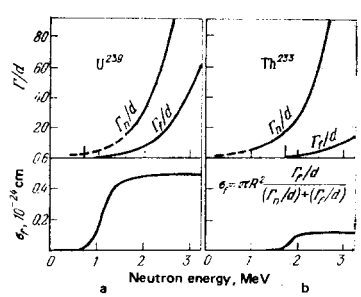

FIG. 11. Dependence of the probability of different processes Γ for U-239 (a) and Th-233 (b) and of the fission cross sections of U-238 and Th-232 on the neutron energy (adapted from Bohr and Wheeler).

[18] According to a remark of L. D. Landau one can expect on the basis of the laws of black body radiation that the most probable energy of a γ-quantum is of the order of the temperature of the nucleus expressed in energy units. The temperature of the nucleus is lower than its energy so that the nucleus excited by the capture of a neutron very likely gives up its energy in several steps in the form of several γ-quanta.

[19] According to the very latest data[20] (February 1941) the behavior of thorium irradiated by neutrons is completely analogous to the behavior of uranium with the following processes taking place: $(n\gamma)$-capture, β-emission with a half-life of 20 min with the production of Pa_{91}^{233} and a further β-emission (half-life of 27 days) with the production of a long-lived new uranium isotope U_{92}^{233}.

laws formulated above we can write

$$\Gamma_f \approx \exp \sqrt{2\alpha(E-E_f)}, \quad \Gamma_n \approx \exp \sqrt{2\alpha(E-E_n)}. \tag{63}$$

Each of the probabilities depends on the excess of energy over the minimum amount required for the process, i.e., on the energy of excitation of the transition state.

The lower part of the diagram shows the curves for the fission cross-section of uranium-238 and of thorium drawn tentatively by Bohr and Wheeler in accordance with formulas (62), (63) and experimental data which they had in the summer of 1939.

In a brief abstract of a report by four American physicists published in the middle of 1940 there is an assertion that the threshold for fission in uranium is lower than Bohr and Wheeler had assumed and amounts to 0.35–0.1 MeV.[21] However, the brief abstract without an indication of the method used and of the specific results of the experiment does not permit us to judge the degree to which this conclusion is justified.

On the other hand Italian scientists[22] observed fission of uranium with a cross-section of $0.1 \cdot 10^{-24}$ cm^2 at a neutron energy of approximately 0.2 MeV. However, in interpreting their results these scientists write that there are no reasons for reexamining the threshold which was assumed by Bohr to be approximately 0.7 MeV; fission with the cross-section of $0.1 \cdot 10^{-24}$ cm^2 should be ascribed to the isotope U^{235}, for which the cross-section in such a case turns out to be equal to $137 \cdot 0.1 \cdot 10^{-24} = 14 \cdot 10^{-24}$ cm^2. This cross-section is greater than the geometrical dimensions of the nucleus [cf., formula (59)], but does not noticeably exceed $\lambda^2/\pi = 13 \cdot 10^{-24}$ cm^2—the limit established for the cross-section by wave mechanics (λ is the de Broglie wavelength for a neutron with a given energy equal to 0.2 MeV).

Of considerable interest is protactinium Pa_{91}^{231}, which according to its properties occupies an intermediate position between the two isotopes of uranium, since based on the value of Z^2/A the critical energy for fission of the the Pa_{91}^{232} nucleus being formed is lower than in the case of U_{92}^{239} (although it does exceed the energy of U_{92}^{236}).

Indeed, experiment[23] has shown that the cross-section for the fission of protactinium is large—approximately $2.3 \cdot 10^{-24}$ cm^2—for fission by fast neutrons. The threshold energy of neutrons inducing fission is comparatively low (<0.25 MeV).

In a letter complementing the basic article, Bohr and Wheeler in correcting the earlier inaccuracy show that these data agree with their theory.[24] In a later article[25] Bohr gives an estimate of the quantity of greatest significance:

$$\Delta E = E_f - E_n = -0{,}27(238-A) + 1.32(92-Z) + \begin{cases} -0.6 & (A-Z) \text{ even}, \\ +0.4 & (A-Z) \text{ odd}. \end{cases} \tag{64}$$

All the quantities, including A, refer to the nucleus obtained *after* neutron capture.

Formula (64) gave a satisfactory description of the experimental date (until the appearance of the American paper[21], which alledgedly reduced ΔE for the principal uranium isotope). We demonstrate its application on the examples in Table IV in which the nuclei are arranged in order of increasing difficulty of fission *under neutron bombardment*, whereas earlier we arranged them according to the critical energy for fission without paying attention as to the source of this energy.

Experimental data show in agreement with theory that as ΔE increases not only is there an increase in the neutron energy necessary to induce fission, but there *is a decrease in the fission cross-section for neutrons* having sufficient energy. It is expedient to make the comparison only for a fairly high neutron energy when the wavelength does not exceed the dimensions of the nucleus and there are no specific ($1/v$) effects.

Thus we can make the following comparison: σ is equal to $3 \cdot 10^{-24}$ cm^2 for Pa; $0.5 \cdot 10^{-24}$ cm^2 for U^{238} and $0.1 \cdot 10^{-24}$ cm^2 for Th.

For other elements (radium and one of the mercury isotopes have been arbitrarily chosen as examples in Table IV) one should expect very small fission cross-sections. Consequently the difficulty for their undergoing fission is associated not only with the necessity to provide the nucleus with a high excitation energy, but also with the fact that for an excitation energy exceeding the binding energy of a neutron in the nucleus the probability is overwhelmingly great that the excitation energy will be expended on neutron evaporation and not on fission.

§5. EMISSION OF NEUTRONS IN FISSION AND CHAIN REACTION BASED ON FISSION

The interest in the fission of uranium is to a considerable extent associated with the possibility in principle of realizing a chain process due to the fact that in fission along with the fission fragments neutrons are also *produced in numbers in excess of one.*

The phenomenon of the production of neutrons in fission has been investigated in detail under the action of slow neutrons on a natural mixture of uranium isotopes. In this case fission leads to the production of between 2 and 3.5 neutrons (according to the measurements of different authors the most probable value is 2.4).

As we know, the process in this case is due to the interaction between neutrons with the U^{235} isotope.

Up until the present time there are no definite data on the yield of neutrons in fission of the principal U^{238} isotope under the action of fast neutrons. The problem is

TABLE IV.

Nucleus	Z	A	A_{comp}	$\Delta E = E_f - E_n$
U	92	235	236	$-0.27 \cdot 2 + 1.32 \cdot 0 - 0.6 = -1.14$
U	92	234	235	$-0.27 \cdot 3 + 1.32 \cdot 0 + 0.4 = -0.41$
Pa	91	231	232	$-0.27 \cdot 6 + 1.32 \cdot 1 + 0.4 = +0.14$
U	92	238	239	$-0.27 \cdot 1 + 1.32 \cdot 0 + 0.4 = +0.67$
Th	90	232	233	$-0.27 \cdot 5 + 1.32 \cdot 2 + 0.4 = +1.69$
Ra	88	226	227	$-0.27 \cdot 12 + 1.32 \cdot 4 + 0.4 = +2.46$
Hg	80	200	201	$-0.27 \cdot 38 + 1.32 \cdot 12 + 0.4 = +5.98$

being investigated at present by Kurchatov and Flerov in the Leningrad Physicotechnical Institute. Data for other cases (Th, Pa) are even more difficult to obtain.

What is the mechanism for the emission of neutrons in fission? Bohr and Wheeler advance two hypotheses. On the one hand they point out that in the fission of a liquid drop there is usually observed the formation of several droplets at the point where the constriction is being formed. Neutrons might be identified with such droplets. However such a hypothesis appears to us to be very artificial. If the term "drop" is justified as applied to nuclei containing 100–200 particles, then the term "droplet" might be applied to some kind of small nuclei, but certainly not to neutrons. Here the analogy between a nucleus and a liquid drop has been carried too far.

The opposite assumption that the neutrons are emitted by the fragments excited in the process of fission is much more likely to be correct. This assumption is organically related to considerations which establish the necessity of fission through the deformed states of the fragments (§2). At the instant when the fragments are still in contact the energy of the system is considerably lower if the fragments are shaped like a pear and are in contact at their elongated ends. The state in which two spherical fragments are in contact would require an energy higher than the critical energy by 60–100 MeV.

Fission is known to go through the stage of deformed fragments in contact with each other. As long as they are in contact the deformation is needed in order to reduce the energy of the system. But as soon as the fragments are separated by a considerable distance (so that the energy of their electrostatic interaction would be low) it will turn out that the minimal energy corresponds just to such a spherical shape. If we "freeze" fragments in the shape which they had at the instant of fission (the moment of contact) and separate them by a considerable distance then it will turn out that the fragments are "excited"—have a considerable potential energy of deformation each of the order of 30–40 MeV (Zel'dovich and Zysin; cf., Part I).

When the fragments separated in their "frozen" state are "thawed out" potential energy of deformation will go over into vibrational energy, will be diffused between the different degrees of freedom of the nucleus and could be utilized for evaporating a neutron.

In contrast, if the fragments are on purpose separated slowly, they will pass through a sequence of states corresponding to minimum energy, and in such a case will acquire at a sufficiently great separation a spherical shape and will yield unexcited nuclei.

Thus, the question concerning the excitation of the fragment nuclei and of the evaporation of neutrons from them turns out to be associated with the details of the motion of the fragments after the critical state and the condition of the fragments in contact have been passed. Problems of this type are too complicated; we know too little concerning the dynamics of a nuclear liquid. One can only assert that the considerations developed above do not contradict experimental data, but all the considerations available at the present time are insufficient in order to predict any kind of dependences, for example the dependence of the neutron yield on the energy of the incident neutron or on the kind of nucleus which is being bombarded.

At first sight it might appear that both the energy of the neutrons inducing fission, and the critical energy of fission are so small compared to the total liberation of energy in fission (up to 200 MeV)[20] that a change in them can in no way affect the neutron yield. Actually we have seen that the fraction of the energy utilized to excite the fragments depends in an essential manner on the rate of passage through the state of fragments being in contact in which the energy of fission has not yet been liberated. Therefore even the restricted assertion concerning the constancy of the yield of neutrons in different cases of fission should be regarded as not having been proved.

The energy of the neutrons being emitted is determined by the excitation energy of the fragment nuclei. At the same time it is necessary to have in mind that the neutrons are emitted by an excited nucleus 10^{-14}–10^{-16} sec after it has been excited. (We recall that the direct experiments mentioned in our article prove that the time for fission and the time for the emission of neutrons is less than $5\cdot10^{-3}$ sec).

The kinetic energy of the fragments is 100 MeV, and this corresponds to a velocity of the order of 10^9 cm/s. During the time of evaporation of a neutron that follows from theoretical estimates the fragments will be separated by a distance of 10^{-5}–10^{-7} cm. This distance is sufficiently great for the electrostatic energy of interaction of the fragments to have become converted into the kinetic energy of separation of the fragments; this justifies *post factum* the assumed value of the velocity of separation of 10^9 cm/s.

At the same time a distance of 10^{-5}–10^{-7} cm is considerably smaller than the distance over which a heavy fragment is slowed down even in a dense substance ($\sim10^{-3}$ cm). Thus, a neutron is evaporated from a fragment moving with a high velocity.

To the energy obtained by a neutron from an excited fragment (depending on the temperature of the nucleus remaining after evaporation) there is added the average kinetic energy of motion per particle in the fragment which, as a whole, is moving with an energy of 100 MeV. Thus, the average energy of neutrons formed in fission exceeds 1 MeV.

This energy is sufficient to induce fission of the principal isotope uranium-238. But the cross-section for uranium fission which is equal to $0.5\cdot10^{-24}$ cm^2, amounts to no more than 1/6 of the geometrical cross-section of the nucleus. Consequently, in five collisions out of six the neutron will be captured, but will be reemitted without causing fission.

In such a process in the majority of cases a neutron

[20] M. Henderson has determined the energy liberated in fission by a dirct calorimetric experiment.[26] His result is 180 ±5 MeV.

in leaving the nucleus will carry away only a part of the kinetic energy with which it entered, leaving the nucleus in an excited state. For a neutron energy of 2–3 MeV the energy of excitation of the nucleus which re-emitted the neutron is insufficient for fission[21)]; this energy will be emitted in the form of γ-quanta. At the same time we obtain a decelerated neutron (on the whole the process is called inelastic scattering). In order that inelastic scattering should take place with the extraction from the neutron of a definite fraction of the kinetic energy it is necessary that the scattering nucleus should have a corresponding energy level. Since the first few excited levels are separated from the ground level by 0.1 MeV, inelastic energy becomes less probable beginning with an energy for incident neutrons of 0.1–0.2 MeV. As can be seen from Fig. 10 at this energy the total neutron width Γ_n which characterizes the total probability for the emission of a neutron by the compound nucleus does not differ from the partial width Γ_n' for the probability of the emission of a neutron without loss of energy.

Thus, for the possibility of a chain reaction fission of the principal isotope of uranium the following considerations are essential: the number of neutrons formed in the act of fission, the ratio between the probability of fission and the probability of inelastic scattering, the ratio between the critical energy of a neutron required for fission and the limiting energy up to which slowing down by inelastic scattering is effective.

As the authors of the present article have shown in their first paper,[27] it is essential that loss of energy by elastic scattering of neutrons by light elements be excluded. But even if metallic uranium is used an estimate made of the different factors leads to pessimistic conclusions that a chain reaction is not realizable. However, our knowledge of all these factors enumerated above is so poor that a final decision can be provided only by a direct experiment.

On the other hand, a chain reaction process is very probable if one uses protactinium. In this case the fission cross-section is close to the cross-section for inelastic scattering; moreover, even after several events of inelastic scattering a neutron is still capable of inducing fission of protactinium. The frequency of occurrence of protactinium in nature is equal to the frequency of occurrence of radium. The ratio of their frequencies of occurrence depends on the ratio of the frequencies of occurrence of the isotopes giving rise to the uranium series and the actinium series—uranium-238 and its isotope—uranium-235—and on the ratio of the decay periods.

[21)]See §7 concerning the action of very fast 6–7 MeV neutrons. However, in the case of fission the number of neutrons with such an energy is negligible and in discussing chain reaction fission we can neglect effects arising in such a case. For the same reason, since the average energy of neutrons produced in fission does not exceed 2–3 MeV (although individual neutrons with energies up to 10 MeV have been observed) it is not possible to assist the chain reaction process significantly by means of the $n \to 2n$ reaction of knocking out neutrons from a nucleus by fast neutrons.

Chemical separation of protactinium is considerably more complicated than chemical separation of radium, so that at the present time protactinium is considerably less available for experimenters than radium, and accumulation of a critical mass of the order of ten kilograms is a very difficult and expensive enterprise.

For the question of chain fission of uranium-235 acted upon by slow neutrons the capture of neutrons by the principal uranium isotope, which is also present, is of the greatest significance. The mechanism of the process was discussed above, in §4. Emission of γ-quanta after neutron capture leads to the formation of U^{239}, which as a result of β-decay transforms into the transuranium element Ekarhenium $EkaRe_{93}^{239}$. For neutron capture a resonance between the incident neutron and the excited nucleus being formed is essential.

The closest level of the excited nucleus lies, as experiment demonstrates, several volts above the energy of the system U_{92}^{238} + a neutron at rest. The width of the level is determined by the probability of having γ-emission (which here is greater than the probability of neutron re-emission); for γ-radiation at an excitation energy of the order of 5 MeV one can expect widths of the order of 0.1 eV. We note that the average distance between levels in the compound nucleus at the given energy of excitation is of the order of 20–50 eV. Thus, we are dealing with isolated, non-overlapping levels; this conclusion agrees with the experimental data according to which in uranium resonance capture of neutrons of definite energy takes place. At the same time (the following conclusion is difficult to check experimentally) it should be expected that along with the resonance level that has been investigated there also exist others corresponding to higher energy and separated from each other by 20–50 V. Apparently it is necessary to take them into account in order to describe the absorption of neutrons which are being slowed down in the mixture of uranium and hydrogen (cf., the article by the present authors[28]).

Resonance absorption of neutrons has also been observed in the case of thorium.[20]

§6. BEHAVIOR OF FRAGMENTS

In §1 we have determined the energy liberated in the fission of uranium or its nearest neighbors into two nuclei with an anomalous ratio between neutrons and protons (with an excess of neutrons). There is reason to think (§2, 5) that the fragments at the instant of formation also have a considerable energy of excitation.

Part of it is expended on evaporation (1–1.5 neutrons for each fragment), part is taken away by γ-quanta.

After a time shorter than 10^{-11} sec these processes come to an end. We are then dealing with fragments that are unexcited, but which have retained their excess neutrons almost entirely. By comparison with stable nuclei of the same charge the excess neutrons amount to approximately 15–20 neutrons for both fragments.

Does this mean that such a number of neutrons will be emitted? Certainly not. When we speak of an excess

of neutrons this does not at all mean that the binding energy of a neutron is zero. The excess of neutrons means that the binding energy of a neutron is less than the binding energy of a proton although it is positive[22] and points to the possibility of β-transformation.

For several series of consecutive β-transformations the data on the liberation of energy are summarized in Fig. 12 adapted from Bohr and Wheeler.[2] The atomic weight of the nucleus is plotted along the horizontal axis, the charge of the nucleus is conserved along oblique lines rising from left to right; the use of such a coordinate system makes the diagram very compact. A β-transformation which increases the charge with a constant atomic weight is shown by a vertical arrow directed downward. The energy liberated in MeV is shown beside each arrow for the corresponding transformation. In order to determine the possible primary fragments one can write a multitude of variants of the fission reaction satisfying the conservation laws. For example, we have

$$U_{92}^{238} = Pd_{46}^{119} + Pd_{46}^{118} + 2n,$$
$$U_{92}^{235} = Ru_{44}^{109} + Cd_{48}^{114} + 2n.$$

Concerning experimental data with respect to the mass and the charge of the fragments see §8.

In Fig. 12 the stable isotopes of the elements are shown by solid circles, the dashed line shows the most favorable charge for a given atomic weight (cf., Fig. 7).

The amounts of energy indicated on the diagram refer to β-transformations in which the initial and final nuclei are both in their ground states. As is well known in such a transition the actual energy distribution of the emitted electrons is such that the energy of the reaction represents only the upper boundary of the distribution. The average energy is considerably lower; the difference is ascribed to the energy of a neutrino, a light uncharged particle that has not yet been observed.

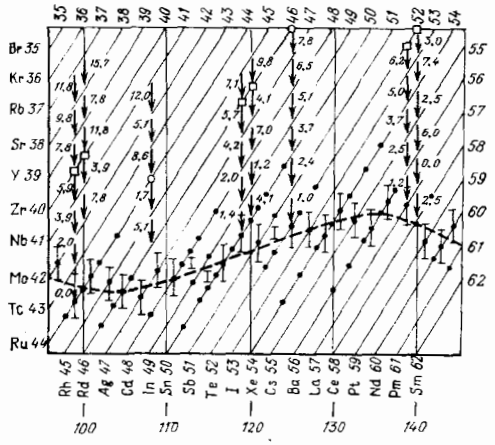

FIG. 12. β-decay of fission products. Numbers at the arrows are energies of decay in MeV (adapted from Bohr and Wheeler).

[22] To the binding energy of a proton one must also add the difference between the self-energies of a free neutron and a free proton (cf., §1).

In this case we must also take into account the fact that for a given unexcited state of the initial nucleus transitions are possible not only to the ground state but also to excited states of the nucleus being formed.

On the one hand this leads to the fact that the energy spectrum of electrons turns out to be a superposition of many elementary spectra (Fig. 13). On the other hand the excited nucleus that is formed is capable of further reactions and first of all to the emission of γ-quanta (Fig. 14). The probability of electron emission is proportional, according to the Fermi theory, to the fifth power of the electron energy[23].

However, the greater is the electron energy the lower is the energy of excitation $E(T)$ of the nucleus being formed, and consequently the lower is the number of levels of the nucleus being formed per unit energy interval.

Statistical considerations show that the most probable amount of energy liberated (the sum of the energy of the electron and the neutrino) is equal to $5T$ where T is the temperature of the remaining nucleus. Thus, the state of the excited nucleus in the case of the most probable process is determined by the equation

$$E_{\text{transf}} = E(T) + 5T, \tag{65}$$

here E_{transf} is the energy of transformation of the initial nucleus in the ground state to the final nucleus in the ground state (Table V).

The entire curve for the distribution of the nuclei being formed with respect to the excitation energy is shown in Fig. 14. The curve in Fig. 13 is not a complement to Fig. 14 since part of the energy is carried away by neutrinos.

Apparently the emission of neutrons with a half-life of approximately 10 sec, observed after irradiating uranium by neutrons is associated specifically with excitation of a nucleus as a result of β-transformation. Such an assumption explains the considerable delay in the emission of a neutron which would have been inexplicable for direct evaporation, since in such a case either the neutron leaves during a time shorter than 10^{-12}–10^{-13} sec, or the excitation energy leaves in the form of γ-quanta and the emission of a neutron becomes impossible. But our assumption that 10 sec is nothing other than the half-life of β-decay is quite a natural one agreeing in order of magnitude with the half-lives of other β-decays. After the emission of a β-particle the

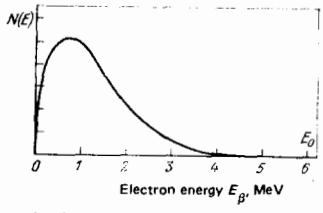

FIG. 13. Electron spectrum in β-decay of fission fragments (adapted from Bohr and Wheeler).

[23] More accurately the sum of the electron and neutrino energies.

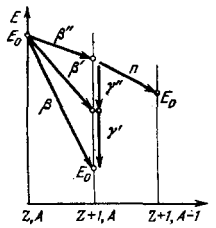

FIG. 14. Scheme of β-decay to ground and excited levels of the nucleus with subsequent emission of γ rays or delayed neutrons.

emission of a neutron (if there is enough energy for this) occurs practically instantaneously.

An estimate of the energy of β-transformation and of the neutron binding energy (cf., Table V) allows such a process. Moreover the observed number of delayed (with respect to time of emission) neutrons is not great—on the average approximately one neutron per 120 fragments.

As the present authors have shown,[29] near the critical conditions for the development of a chain reaction, when the system is very sensitive, even this small number of neutrons appreciably modifies such properties as the relaxation time of the system.

§7. FISSION INDUCED BY DIFFERENT PARTICLES

Investigation of fission under the action on the nucleus of other particles is of considerable interest for adding support to all the arguments in which the binding energy of the neutron played an important role along with the critical energy for fission.

Thus, during the time elapsed since the publication of the article by Bohr and Wheeler fission of uranium under the action of γ-quanta has been observed.[30] In this case the fission cross-section amounted to $(3.6 \pm 0.1) \cdot 10^{-27}$ cm^2 for a quantum energy of 6.3 MeV. This has directly established the critical energy for fission (but this was for U^{238}, while in the preceding discussion we were interested primarily in the compound nucleus U^{239}). The photofission cross-section for thorium amounted to $(1.7 \pm 0.5) \cdot 10^{-27}$ cm^2 for the same γ-quanta.

Comparing the values of Z^2/A for uranium and thorium nuclei we can estimate the critical energy for fission; the critical energy is always somewhat lower than for the same nucleus which has captured a neutron and increased its A by unity. We have taken this information, actually obtained as a result of a theoretical treatment of data on fission induced by neutrons, from Table III of Part I: E_f is equal to 5.8 MeV for uranium-238 and 6.6 MeV for thorium. Thus, for uranium the process is quite possible; for thorium it takes place with a somewhat diminished probability, requires a tunnelling transition, but with a very small barrier height above the energy level of the nucleus (or entirely above the barrier if all the values of the critical energies are somewhat overestimated; cf., Ref. 21).

TABLE V.

Transformation		Energy liberated, MeV	Binding energy of neutron in final nucleus, MeV	Transformation		Energy liberated, MeV	Binding energy of neutron in final nucleus, MeV
Initial nucleus	Final nucleus			Initial nucleus	Final nucleus		
Zr^{99}_{40}	Nb^{99}_{41}	6.3	8.2	In^{130}_{49}	In^{130}_{50}	7.6	7.1
Nb^{100}_{41}	Mo^{100}_{42}	7.8	8.6	Te^{140}_{52}	J^{140}_{53}	5.0	3.5
Pd^{125}_{46}	Ag^{125}_{47}	7.8	6.7	J^{140}_{53}	Xe^{140}_{54}	7.4	5.9
Ag^{125}_{47}	Cd^{125}_{48}	6.5	5.0				

In discussing the problem of fission under neutron bombardment we saw that the most important process competing with fission is emission of neutrons by the excited nucleus. In the general scheme of nuclear reactions due to Bohr this circumstance is entirely unconnected with the method of exciting the nucleus: neutron emission remains in competition with the fission process no matter by what method the nucleus is excited.

In the case the fission cross-section amounted to that for the U^{238}_{92} and Th^{232}_{90} nuclei all the quantities Z, A, $A - Z$ are even, i.e., the binding energy is great. Calculating in accordance with formula (64) the expression $E_f - E_n = \Delta E$ we obtain: for uranium-238 $\Delta E = -0.6$ MeV and for thorium $\Delta E = +0.4$ MeV.

Thus, for uranium there is every reason to expect that each absorption of a γ-quantum must lead to fission.

The cross-section of $3.5 \cdot 10^{-27}$ cm^2 corresponds to the available information on the cross-sections for the interaction of heavy nuclei with γ-quanta. In the case of thorium, apparently, photofission proceeds with the probability of approximately 1/2 per absorbed γ-quantum.

When uranium is bombarded by charged heavy particles first of all it is necessary that the particle should overcome the forces of electrostatic repulsion. These forces create around the nucleus an energy barrier whose effective height (for a particle with charge $+e$) is of the order of 10 MeV.

Protons and neutrons with such energy interact with the heavy nucleus with a cross-section of the order of the geometrical dimensions of the nucleus. The resulting excitation is quite great; moreover an increase of the charge by unity reduces the critical energy for fission.

The result of the process after capture again is determined by the competition between fission and neutron emission.

Applications of Bohr's formula (64) shows that for a nucleus formed as a result of the capture of a neutron by the principal isotope, the fission probability is close to unity, since the energy of excitation of the nucleus is higher than 15 MeV, while for the element with $Z = 93$, $A = 240$, $E_f - E_n = \Delta E = -0.4$ fission is more probable than neutron emission.

For a lower energy of the bombarding particles the cross-section will be determined by the probability of their passage under the barrier.

According to the estimates of Bohr and Wheeler at an energy of 6 MeV one can expect for protons cross-sec-

tions of 10^{-28} cm^2 and for deuterons of 10^{-29} cm^2.

It is of interest to compare these results with the experimental data of Jacobsen and Lassen[31], obtained using the cyclotron of the Institute of Theoretical Physics in Copenhagen. Fission of uranium and thorium irradiated with deuterons was observed. Fission was observed at 7.5 MeV and continued to increase so that extrapolation gave 10^{-24} cm^2 for 11 MeV (the experimental curve has been traced only up to 9.5 MeV).

The cross-section of theoreium over the whole extent of the energies investigated amounted to 0.75 of the uranium cross-section.

Of interest are Bohr's considerations[25] concerning the action of very fast neutrons on uranium-238. No matter how high would be the energy in the resulting excited nucleus of uranium-239, the probability of fission is lower than the probability of neutron re-emission; the fission cross-section varies but little. But, beginning with a neutron energy of approximately 6 MeV, a new factor enters; the nucleus remaining after neutron emission may turn out to be sufficiently excited to undergo fission. Then fission becomes possible in the case of *inelastic scattering of the neutron*, in contrast to fission accompanying neutron *capture* which we have discussed in detail in §4.

In its nature fission accompanying inelastic scattering is closer to fission under the action of a γ-quantum, as a result of which we have placed discussion of it in the present section.

In the case of uranium the circumstance is particularly favorable that after inelastic scattering neither the charge nor the atomic weight are altered. In formula (64) we expect to have $\Delta E = -0.4$ MeV. Consequently, emission of a neutron from the excited U^{238} is less probable than fission. The new possibility of fission of an excited nucleus must lead to a significant increase in the fission cross-section for uranium when the neutron energy exceeds 6 MeV. An increase in the cross-section for very fast neutrons has been actually observed by Italian investigators.[32]

One should again note that for the problem of a fission chain reaction this increase in the cross-section at high energies is not significant, since the energy of the overwhelming majority of the neutrons is not sufficient for such a cascade process with an increased cross-section.

§8. THE MASS OF THE FRAGMENTS

Energy considerations developed in Part I of this article establish that fission into two parts which are equal both in charge and mass is the most favorable mode. However this does not at all establish the necessary direction of the process, since even in the case of very unsymmetric fission (and also, for example, in fission into three parts) the process remains exothermic.

After the system has surmounted the critical state, its kinetic energy increases rapidly. The direction of the velocity vector in the multidimensional space of parameters describing fission can be quite varied, particularly in the case of "viscous motion" of the nuclear liquid.

Even in the case when the critical shape is symmetric, as assumed by Bohr and Wheeler, production of a considerable variety of fragments is possible in fission. The high energy of the particles inducing fission increases the excess of the energy of the system above the critical energy; in this case the passage of the particle at a great distance to the side of the most favorable saddle point becomes possible. One might expect, according to a direct indication by Bohr and Wheeler, that at high energy the asymmetry of fission will only be increased. But particularly in this problem until now it has not been possible to bring the available experimental material into agreement with the theory.

The most detailed work of Petrzhak[33] shows that in fission the kinetic energy of the fragments is not equal. The fragments are divided into two quite sharp groups with energies of 60 and 85 MeV.

Assuming the total kinetic energy of nuclei formed in fission to be the same, Petrzhak concludes that the nucleus falls apart into two unequal parts[23)]: the kinetic energy of each part is inversely proportional to its mass since according to the law of conservation of momentum $m_1|u_1| = m_2|u_2|$. The ratio of the energies corresponds to the fission of the nucleus of total weight approximately 140 and 100. The chemical facts confirm this asymmetry.

American authors have published[34] a communication concerning an unfinished quantitative investigation of different series of β-transformations of the fragments.

In the case of asymmetric fission the heavy fragments are identified better than the light ones. It is possible to identify only the fragments which undergo subsequent transformations.

We reproduce data on the distribution of the number of fragments as a percentage of the number of fissions as a function of their atomic weight (in the process of subsequent β-transformations A remains constant, only Z increases):

A:	127	129	131	133	135	139	140
%:	0.18	0.34	1.6	7.6	9	6.4	8.4

A fraction of the series (not shown here) has not been identified, a fraction has not been observed, the total balance does not agree with the number of fissions.

At the same time there are indications of a more symmetric fission when the energy of the bombarding neutrons is greater.[35]

At the present time it is still difficult to form a judgment on the validity of the facts and their interpretation given above. It is difficult to form a judgment whether these facts will require some improvement of

[24)] By analyzing more accurately the energy distribution of the fragments Petrzhak arrives at the conclusion that together with asymmetric fission the process with the production of fragments of equal energy and equal mass also takes place.

the theory taking into account individual properties of the nuclei or a radical restructuring of the theory. In any event we hope that the value of the present theoretical concepts concerning the mechanism of fission is sufficiently clearly evident from all the preceding discussion of the problems of spontaneous fission, the role played by the different isotopes and the behavior of fragments.

Within the framework of the present article we do not have the possibility of discussing the very interesting problems of the behavior of fragments with a charge of approximately 40, mass of approximately 100 and energy up to 100 MeV in their passage through a gas, or through a photoemulsion. The problem of their charge, i.e., of the number of electrons which the fragment drags along with itself, of their range, of the rate of loss of energy, etc. is of interest. Here we can only refer the reader to the literature.[36,37]

In conclusion the authors consider it their pleasant duty to express their sincere gratitude to I. I. Gurevich for examining the manuscript and for a number of valuable suggestions.

[14]H. Eyring, J. Chem. Phys. **3**, 107 (1935).
[15]E. Wigner, Trans. Faraday Soc. **34**, 29 (1938).
[16]H. A. Kramers, Physica **7**, 284 (1940).
[17]L. D. Landau, Zh. Eksp. Teor. Fiz. **7**, 819 (1937).
[18]V. F. Weisskopf, Phys. Rev. **52**, 295 (1937).
[19]E. MacMillan and P. H. Abelson, Phys. Rev. **57**, 1185 (1940).
[20]A. V. Grosse, E. T. Booth, and J. R. Dunning, Phys. Rev. **59**, 322 (1941).
[21]R. O. Haxby, W. E. Shoupp, W. E. Stephens, and W. M. Wells, Phys. Rev. **58**, 199 (1940).
[22]M. Ageno, E. Amaldi, D. Bocciarelli, B. N. Cacciapuotti, and G. C. Trabacchi, Ric. Sci. **11**, 302 (1940).
[23]A. V. Grosse, E. T. Booth, and J. E. Dunning, Phys. Rev. **56**, 382 (1939).
[24]N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 1065 (1939).
[25]N. Bohr, Phys. Rev. **58**, 864 (1940).
[26]M. C. Henderson, Phys. Rev. **58**, 200 (1940).
[27]Ya. B. Zel'dovich and Yu. B. Khariton, Zh. Eksp. Teor. Fiz. **4**, 1425 (1939).
[28]Ya. B. Zel'dovich and Yu. B. Khariton, Zh. Eksp. Teor. Fiz. **10**, 29 (1940).
[29]Ya. B. Zel'dovich and Yu. B. Khariton, Zh. Eksp. Teor. Fiz. **10**, 477 (1940).
[30]R. O. Haxby, W. E. Shoupp, W. E. Stephens, and W. H. Wells, Phys. Rev. **59**, 57 (1941).
[31]I. C. Jacobsen and N. O. Lassen, Phys. Rev. 58, 867 (1940).
[32]M. Ageno, E. Amaldi, D. Bocciarelli, and G. Trabacchi, Ric. Sci. **11**, 413 (1940).
[33]K. A. Petrzhak, Dokl. Akad. Nauk SSSR **27**, 208 (1940).
[34]H. L. Anderson, E. Fermi, and A. V. Grosse, Phys. Rev. **59**, 52 (1941).
[35]Y. Nishina, T. Yasaki, K. Kimura, and M. Ikawa, Phys. Rev. **59**, 323 (1941).
[36]A. B. Migdal, Zh. Eksp. Teor. Fiz. **9**, 1163 (1939).
[37]N. Bohr, Phys. Rev. **59**, 270 (1941).

Translated by G. M. Volkoff