The mechanism of nuclear fission (Part I) [*** from Usp. Fiz. Nauk 25(4), 381–405 (September 1941)]

Ya. B. Zel'dovich and Yu. B. Khariton

Usp. Fiz. Nauk **25**, 381–405 (September 1941)

PACS numbers: 25.85.Ec, 27.90. + b, 01.65. + g

The picture is presented as developed in the early forties of nuclear fission, both neutron induced and spontaneous, and of the fission products decay and of neutron emission accompanying fission. The paper was written when the authors were working in the Institute of Chemical Physics in close contact with I. V. Kurchatov, N. N. Semenov and I. I. Gurevich.

FROM THE EDITORIAL BOARD OF USP. FIZ. NAUK

From the moment of the discovery of the fission of uranium nuclei by Hahn and Strassmann (January 1939), Igor' Vasil'evich Kurchatov participated very actively in projects along this new direction of research having understood its tremendous significance and, possibly, having a presentiment of the role which, he was fated to play in the solution in our country of the "uranium problem". He immediately initiated relevant research in his laboratory in the Leningrad Physicotechnical Institute and followed with unflagging attention the theoretical work on the physics of fission being carried out in Leningrad in the Physicotechnical Institute and in the Institute of Chemical Physics.

Therefore in connection with the eightieth anniversary of the birth of I. V. Kurchatov, the editorial board of Usp. Fiz. Nauk considers it appropriate to publish the article by Ya. B. Zel'dovich and Yu. B. Khariton which was written in Leningrad in 1941 and is closely related to the scientific interests of I. V. Kurchatov.

We note that the first part of this article was published in 1941 in the September issue of Usp. Fiz. Nauk, i.e., already after the beginning of the war (this issue was sent to press on July 15, 1941). It turned out that the next issue of "Uspekhi" appeared only after a large gap—in 1944, when the situation with the uranium problem had undergone a change and the corresponding publications were for obvious reasons suspended in the whole world.

The article by Ya. B. Zel'dovich and Yu. B. Khariton which is being published now is of definite historical interest. It is published using the typewritten text prepared for typesetting and retaining the markings of the technical editor of the Journal and the approval of the then editor-in-chief—É. V. Shpol'skiĭ.

The editorial board takes this opportunity to congratulate one of the authors—Yu. B. Khariton on the award to him of the highest distinction of the Academy of Sciences of the USSR—the Lomonosov medal for 1983.

PART I

In our earlier article[1] we have presented in detail the history of the discovery of a new type of radioactive processes—the fission of a uranium nucleus irradiated by neutrons into two fragments of approximately equal mass. We have also described there the principal experimental fact established at the present time as a result of research carried out in a number of laboratories and published in a number of articles which followed rapidly one after another throughout 1939 and the first half of 1940.

In the present article we shall treat in greater detail the problem of the theoretical description of this completely new type of radioactive processes. The principal articles in this field are by Niels Bohr and his collaborator Wheeler.[2] Simultaneously and independently the theory has been developed by Ya. I. Frenkel' (Physicotechnical Institute, Leningrad).[3]

In our presentation we shall follow the more detailed paper of Bohr except for those parts of it which at the present time are subject to doubt.

At the present time all authors are in agreement with the qualitative treatment of the new type of phenomenon which was advanced by L. Meitner and Frisch.[4] Developing the general scheme of nuclear reactions proposed by Bohr,[5] in which the nucleus is compared to a liquid drop, Meitner and Frisch note that for a sufficiently large nucleus surface tension can no longer oppose the action of the forces of electrostatic repulsion. With a constant ratio of the electric charge to mass (as is approximately the case in the periodic system of the elements) the long-range forces of Coulomb repulsion increase more rapidly as the nuclear mass and charge increase simultaneously than the short-range forces of "surface tension" of the nuclear liquid. Therefore for a large nucleus one can expect a process analogous to the fission of a large charged drop into smaller droplets.

Below we shall start with the problem of the energy balance in fission; later in the second section we shall discuss the problem of the critical size of a nucleus and the current state of the problem of the critical shape through which the nuclei pass on fission; in Part II of this article we shall discuss in greater detail the problems of the kinetics of nuclear fission and the probability of other processes competing with fission, i.e., problems determining the probability of nuclear fission

 0038-5670/83/030266-13$01.80

upon neutron capture; we shall there also deal with the state of the fission fragments and with the theory of the processes of the emission of neutrons that follow fission; and, finally, with the problem of nuclear fission under the action of various particles other than neutrons.

§1. ENERGY BALANCE IN THE REACTION OF NUCLEAR FISSION

We must first of all investigate the problem of the change in the energy of the nucleus when it undergoes fission into two parts approximately equal in charge and mass. The principal difficulty in the case of interest to us of fission of heavy nuclei is the fact that the ratio of mass to charge in the periodic system is not quite constant. It increases slowly as the charge and mass increase. A direct examination of the Mendeleev table, the atomic weights and the atomic numbers of the elements convinces us that in fission, for example, of the uranium nucleus into two equal parts retaining the total mass and the total charge we obtain two palladium nuclei with a mass of approximately 119–120 units of atomic weight, while the atomic weight of ordinary palladium is considerably smaller and amounts to only 106.7. Thus, as a result of fission of a heavy nucleus we obtain two nuclei with an unusual charge to mass ratio. On the one hand this unusual charge to mass ratio is the reason for the instability of such a fragment nucleus leading to a series of further radioactive transformations of the fragments. It is the investigation of these radioactive transformations that led to the discovery of the fission processes. On the other hand the unusual charge to mass ratio in fragment nuclei impedes us from directly utilizing the fairly extensive information on mass defects accumulated by the present time, i.e., information concerning internal energies and stability of ordinary isotopes of the elements.

It will now be necessary for us to find a method for estimating the energy of nuclei with a very unusual charge to mass ratio.

We recall that relativity theory establishes a relationship between the mass and energy of a body which for our purposes can be written in the form $E - E_0 = (M - M_0)c^2$, where E and M are the energy and mass in one state, and E_0 and M_0 are the same quantities in the other state, and c is the velocity of light. One unit of atomic weight is equal to the energy of $(3\cdot 10^{10})^2/6\cdot 10^{23} = 1.5\times 10^{-3}$ ergs per atom $= (3\cdot 10^{10})^2/96\,500\cdot 10^7 = 933\,000\,000$ eV $= 933$ MeV, the electron mass is equal to the energy of 0.51 MeV. In future we shall refer to mass defects expressed in millions of electron-volts.

According to present concepts a nucleus consists of neutrons and protons. We shall express the nuclear charge in numbers of elementary charges; Z coincides with the ordinal number of the element in Mendeleev's table. Z is equal to the number of protons in the nucleus and is an integer. The mass defects of elements with respect to oxygen are less than 500 MeV, therefore rounding off the atomic weights of the isotopes of the elements to the integer A we shall directly obtain the number of heavy particles of which the nucleus consists, i.e., the sum of the number of protons and the number of neutrons.

We also note that when we speak concisely of the mass defect of a nucleus, we in fact mean the mass defect of the neutral atom, i.e., of a nucleus $+Z$ electrons. If we compare the mass defects of two nuclei of the same atomic weight (isobars), but with a charge differing by unity, then we obtain the energy of the process of transformation of one atom into the other one. Physically the transition from Z to $Z+1$ is the emission of an electron or β-activity; the inverse process is the capture by the nucleus of an electron from a shell (K-electrons nearest to the nucleus are captured). The existence of different nuclei of a given atomic weight (isobars) in nature is associated with the stability of a nucleus with respect to spontaneous processes of β-transformation and electron capture which do not require external irradiation by some sort of particles, do not require overcoming energy barriers and therefore proceed relatively easily. If the ordinary stable elements correspond for a given atomic weight to a minimum of the total energy of the nucleus, then for elements in which for a given atomic weight the nuclear charge deviates from this value, we can expect that the energy will be expressed by the following formula:

$$M(Z, A) - M(Z_A, A) = \frac{1}{2} B_A (Z - Z_A)^2. \qquad (1)^{1)}$$

In this formula Z is the charge of the nucleus under consideration. Z_A is the charge mentioned above at which the energy of the nucleus is minimal for a given A. Generally speaking, Z_A need not be an integer. The isobars occurring naturally are grouped around the value of Z_A. The values of Z_A are given in Fig. 1 by a solid line.

The quantity B_A, which characterizes the sharpness of the maximum, i.e., which characterizes the change in energy on deviation from the charge which minimizes the energy, cannot be obtained from the existing experimental material on mass defects and energy of stable nuclei, since in stable nuclei the deviations of the value of Z from Z_A are too small. However, we can approach the calculation of the value of B_A theoretically with the aid of the following considerations.

We shall attempt to establish what determines the usual ratio between the charge and the mass of a nucleus, i.e., the ratio between the number of neutrons and the number of protons in a nucleus corresponding to minimal energy. In any theory of nuclear forces acting between neutrons and protons one might expect that these forces reach saturation and yield a minimum of energy for equal numbers of neutrons and protons in the nucleus. And indeed, for light elements the mass to charge ratio is quite close to 2, which exactly corresponds to $Z - A = A$, the equality of the numbers of neutrons and protons. What determines the deviation of

1) Near the minimum in the expansion in terms of $(Z - Z_A)$ the first order term is absent and we neglect terms of order higher than the second.

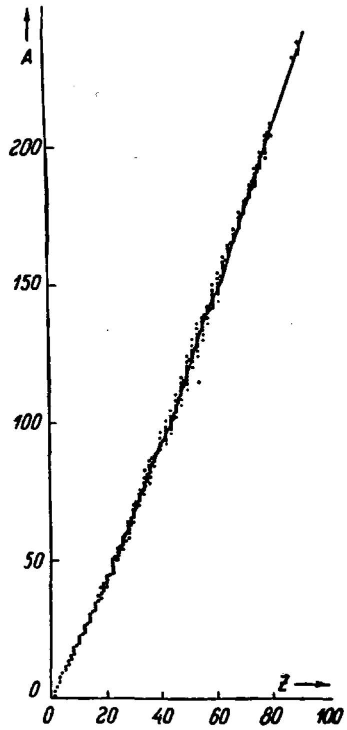

FIG. 1.

TABLE I.

A	Z_A	B_A MeV	A	Z_A	B_A MeV
50	23,0	3,5	150	62.5	1.2
60	27,5	3,3	160	65.4	1,1
70	31,2	2,5	170	69.1	1,1
80	35,0	2,2	180	72.9	1,0
90	39,4	2,0	190	76.4	1,0
100	44,0	2,0	200	80,0	0,95
110	47,7	1,7	210	83.5	0,92
120	50,8	1,5	220	87.0	0.88
130	53,9	1,3	230	90.6	0.86
140	58,0	1,2	240	93,9	0.83

this ratio of A/Z from 2, i.e., the deviation of the minimum of the energy from conditions in which the number of neutrons and the number of protons are equal? On the one hand we should take into account that the self-energy of protons and neutrons is slightly different; on the other hand—and this is more significant, —in addition to the short range "chemical" nuclear forces holding the nucleus together it is also necessary to take into account the forces of electrostatic repulsion which are the greater, the larger is the nuclear charge. It is these forces that introduce asymmetry between protons and neutrons and determine the fact that the minimum energy corresponds to a number of protons smaller than the number of neutrons. If the chemical energy of the nucleus is a minimum when the two numbers are equal, then the existence of the electrostatic energy will give rise to a displacement of the minimum in the direction of nuclei for which the number of neutrons is greater than the number of protons. But it is not difficult to evaluate the magnitude of the electrostatic energy of the nucleus. In principle, as well as the electrostatic forces the difference between the neutron and proton masses is also significant; this difference determines the energy that might be released by the transformation of a neutron into a hydrogen atom outside the nucleus. This energy is known and amounts to only 0.78 MeV.

We write the expression for the mass of a nucleus in the following form:

$$M(Z, A)=C_A+\frac{1}{2}B'_A\left(Z-\frac{1}{2}A\right)^2+\left(Z-\frac{1}{2}A\right)(M_p-M_n)+\frac{3Z^2e^2}{5r_0A^{\frac{1}{3}}}, \qquad (2)$$

where the value of C_A does not depend on Z, $(Z-\frac{1}{2}A)$ is one-half of the difference between the number of protons and the number of neutrons in the nucleus $[Z-(A-Z)]/2$. In this formula the first two terms describe the chemical energy of the nucleus; the form in which they are written corresponds to the fact that the chemical energy is a minimum for $Z=A-Z$; the third term represents the difference between the neutron and proton masses; the fourth term represents the energy of the electrostatic interaction. The electrostatic energy was obtained by considering a sphere with a constant density of volume charge and making the usual assumption that the nuclear radius is equal to $r_0A^{1/3}$, where r_0 is the radius referred to a single particle. The value of r_0 has been known for a long time from the theory of α-decay in which the probability of decay depends on the value of the energy barrier near the surface of the nucleus, i.e., depends in an essential manner on the nuclear radius. The most probable value is $r_0=1.48\cdot 10^{-13}$ cm.

Differentiating formula (2) with respect to Z and setting the resultant expression equal to zero we obtain the value of Z_A which makes the energy a minimum, and which depends on B'_A. Comparing this with the known data on the average, most probable, value of Z_A for stable elements, we can obtain the value of B'_A in formula (2) which characterizes the minimum of the chemical energy, and from this with the aid of elementary calculations obtain the value of B_A in formula (1).

Table I gives the values of Z_A and B_A in formula (1) as functions of atomic weight. We shall not here go into greater detail of Bohr's calculations that take into account small fluctuations of energy depending on whether the numbers of neutrons and protons in the nucleus are odd or even.

With the aid of the estimate obtained above of the energy of the nucleus as a function of its atomic weight and the number of charges over a wide range of variation of both quantities we can now answer the question concerning the energy liberated in the breakup of a heavy nucleus.

Table II gives the results of such calculations for several typical nuclei.

In the third column of Table II we have given the energy liberated in the fission of the initial nucleus (first column) into two products shown in the second column; in fission the sum of the charges and the sum of the atomic weights are conserved. However, the fission products obtained undergo further transformations as a result of the unusual ratio of the number of protons to

TABLE II.

Initial nucleus	Fission products	Energy liberated, in MeV	
		in fission	subsequently
Ni^{61}_{28}	$Si^{30,31}_{14}$	—11	2
Sn^{117}_{50}	$Mn^{58,59}_{25}$	10	12
Er^{167}_{68}	$Se^{83,84}_{34}$	94	13
Pb^{206}_{82}	$Nb^{103,103}_{41}$	120	32
U^{239}_{92}	$Pd^{119,120}_{46}$	200	31

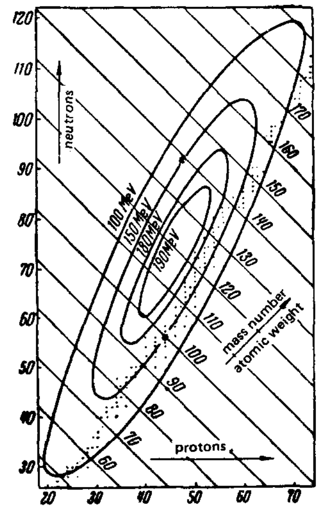

FIG. 2.

the number of neutrons; thus, a product of uranium fission, palladium, of atomic weight 120 must transform into the stable tin isotope Sn^{120}_{50} by emitting four electrons (four β-particles). The additional energy liberated in this process is given in the last column of Table II; for greater details concerning the β-activity of the fragments see Part II.

We see that up to atomic weight equal to 100 the nuclei are energetically quite stable with respect to fission[2)]. Above this limit the transformation of the nucleus into two fragments of equal mass separated by a great distance becomes energetically favorable.

It is interesting that the fission of a nucleus into three equal parts becomes favorable beginning with the atomic weight of 110. Uranium upon fission into three equal parts would liberate even somewhat more energy than upon fission into two parts. Fission of uranium into approximately ten equal parts would take place without liberation and without expenditure of energy.

We are particularly interested in energy relationships in the case of uranium fission. Just as in Table II we consider the fission of a nucleus obtained by the capture of one neutron by the nucleus of the main uranium isotope U^{238}_{92} giving rise to U^{239}_{92}.

Figure 2 shows the amount of energy liberated in the fission of uranium brought about by different methods. The number of protons in the nucleus of the fragment formed is plotted along the horizontal axis and the number of neutrons in the same nucleus along the vertical axis. The atomic weight of the fragment equal to the sum of these two numbers is constant along straight lines forming an equilateral right-angle triangle with apex at the origin. Stable nuclei are shown by points in the diagram. Finally, the ellipses shown in the figure are lines along which the liberation of energy in uranium fission is constant and is equal to the quantity marked on the ellipse (expressed in MeV). Specifying the charge and atomic weight of one of the fragments we naturally completely determine the charge and atomic weight of the other fragment.

Thus, if one of the fragments is a normal nucleus of ruthenium (charge 44, atomic weight 100—lower asterisk), then the second nucleus must be cadmium (charge 48, atomic weight 139, number of neutrons 91—upper asterisk). In the fission of a uranium nucleus into two such fragments 150 MeV of energy must be liberated.

It is easily seen that points representing the two fragments in Fig. 2 must always lie on a straight line passing through the center of the ellipses on opposite sides of the point corresponding to the energetically most favorable direction of nuclear fission. Therefore for the energetically most favorable direction of the fission process the nuclei formed lie at a considerable distance from the Milky Way representing the stable isotopes. Transformation of the fragments into stable isotopes is associated with the emission from three to six β-particles.

The estimate of the energy of such unusual nuclei developed above will enable us later to elucidate completely the question of the subsequent radioactive transformations of the fragments.

The elementary calculations carried out above combining the most general experimental data on atomic weights of different nuclei and simple theoretical considerations concerning the nature of nuclear forces show in complete agreement with experiment that fission of a heavy nucleus is a process liberating the greatest amount of energy. The tremendous energy of the fragments and their great ionizing power are widely used by experimenters who can unambiguously distinguish fission from other radioactive processes.

§2. STABILITY OF A HEAVY NUCLEUS

The energy stability of a nucleus with respect to fission into two fragments separated by a large distance does not guarantee directly the possibility of such a process because in actual fact fission must pass

2) At an atomic weight near 100 the fission of a nucleus stable with respect to a change of Z (cf., Fig. 1 and Table I) leads to the formation of two nuclei which differ little from nuclei stable with respect to a change in Z or corresponding atomic weight. Therefore an estimate for the limit of energy stability can be given directly on the basis of Aston's mass defect curve.

through a state when the two fragments are close to one another or even incompletely separated. As a result of electrostatic repulsion of the fragments the energy of such a state will be much larger than the energy of the state considered in the preceding section, in which the fragments have been separated by a large distance and their interaction can be neglected. In order to elucidate the problem of the probability of nuclear fission and of the conditions existing in the course of fission it is necessary to examine more closely the mechanism itself of the process and to determine those intermediate phases through which the process of nuclear fission proceeds.

We shall preface the presentation of the mathematical theory by a curious calculation due to I. I. Gurevich.

Representing the nucleus as a sphere consisting of individual small spheres—protons and neutrons, we find the fraction of them lying on the surface; it turns out that with the densest packing a nucleus consisting of 238 particles has on its surface approximately 130 particles—more than half of the total number.

Naturally, under such conditions all calculations in which the energy is divided into a volume and a surface part or calculations made of the change in energy accompanying a change of nuclear shape, cannot lay claim to accuracy. The calculations are of an illustrative nature, their results must be checked by experiment as much as possible. A sober evaluation of the degree of approximation will enable us to omit a number of calculations.

With the above qualifications we now begin the discussion of the energy of different nuclear shapes.

Above in formula (2) we investigated the dependence of the energy on the charge for a nucleus of constant shape and dimensions and a constant total number of particles. In this approach we have included the surface energy together with other terms in the constant C_A. Now, in contrast, we consider the changes in the shape of the nucleus for constant mass and charge and a constant ratio of the numbers of neutrons and protons. The terms in the expression for the energy that depend on this ratio may be omitted.

We represent the total energy of a nucleus at rest as a sum of the electrostatic and surface energies

$$E = W + Q. \tag{3}$$

Following Frenkel', Bohr, and Wheeler we calculate the electrostatic energy for a body of a given shape, whose volume is equal to $4\pi r_0^3 A/3$ [cf., formula (2) of §1] with a constant volume charge density, with the total charge being equal to eZ. We represent the surface energy as a product of the surface of the body by the (constant) value of the surface tension q.

According to the estimate of Feenberg[6] the best value is

$$4\pi r_0^2 q = 14 \text{ MeV}. \tag{4}$$

For curiosity sake we point out that from (4) it follows that $q = 10^{20}$ dyne/cm² (in the case of water $q = 80$, in the case of liquid mercury $q = 500$).

Formula (3) with a constant q, as Ya. I. Frenkel' justifiably observes, is closer to the truth than the calculation of Frisch and Meitner based on the influence of the charge on surface tension.

We now seek the dependence of the energy on the shape of the body. The electrostatic energy is a maximum for a sphere; in contrast, the surface energy attains a minimum in the case of a sphere. The spherical shape will always give an extremum of the total energy. Let us investigate the energy of shapes close to a sphere.

It is obvious that the surface energy will always increase for any deviation from the spherical shape of the nucleus, while the electrostatic energy of the nucleus, conversely, attains a maximum for a spherical shape, in which the individual elementary charges are situated closest to each other. Any perturbation of the spherical surface diminishes the electrostatic energy and increases the surface energy. If the former is sufficiently great with respect to the latter the gain in electrostatic energy in the deformation of the nucleus may exceed the energy required to increase the surface, i.e., the work done against the surface tension forces.

Frenkel' considers ellipsoids of revolution. Denoting the semiaxes by a and b, restricting ourselves to small deviations from the spherical shape, $a - b \ll a$, and taking into account the constancy of the volume $ab^2 = \text{const}$, we write his result in the following form:

$$\left.\begin{aligned} W &= W_0\left[1 - \frac{4}{45}\left(\frac{a-b}{a}\right)^2\right], \\ Q &= Q_0\left[1 + \frac{8}{45}\left(\frac{a-b}{a}\right)^2\right], \end{aligned}\right\} \tag{5}$$

where W_0 and Q_0 refer to a sphere. Formulas (5) are equally valid for prolate ($a > b$) and oblate ($a < b$) ellipsoids of revolution. The total energy is given by

$$E = E_0 + \frac{4}{45}\left(\frac{a-b}{a}\right)^2 (2Q_0 - W_0). \tag{6}$$

Thus, the stability of the spherical shape of a charged drop depends on whether the electrostatic energy is greater than or less than twice the surface energy. The sphere is stable when

$$W_0 < 2Q_0. \tag{7}$$

The same result was obtained more rigorously by Bohr and Wheeler. In general terms they describe a small perturbation of the spherical surface by a sum of spherical harmonics and expand the total energy in a series in terms of the coefficients of the harmonics. For $W_0 < 2Q_0$ the minimum corresponds to the vanishing of all the coefficients, i.e., to an unperturbed sphere. When $W_0 > 2Q_0$ deformation of the sphere becomes energetically favorable.

Investigation of small perturbations gave us the criterion for stability. What will occur in the case of a strong perturbation corresponding to a significant deformation of the sphere? Will the nucleus for $W_0 > 2Q_0$ find a stable shape differing from a sphere? Are there limits of stability for $W_0 < 2Q_0$?

There are several theoretical articles in the literature in which the authors, taking the shape of the nucleus to be in the form of an ellipsoid of revolution, seek the dependence of the energy of the nucleus for given values of charge and mass on the ratio of the axes of the ellipsoid without restricting themselves to small deformations. However, in considering an ellipsoid of revolution we inevitably arrive at the result that even when the spherical shape becomes unstable there exists a certain finite ratio of the axes of the ellipsoid corresponding to a minimum energy, i.e., seemingly a stable one; we reach the conclusion that a heavy nucleus can exist in a shape significantly different from a sphere. Here we should note at once that such a method of investigation is incorrect in principle, since it does not follow from any considerations, and in particular in the case of finite large deformations of the nucleus, that it must at all times remain an ellipsoid of revolution. It is directly obvious that the appearance of a minimum in the energy in the case of a finite deformation is associated with our artificial assumption that the nucleus must be an ellipsoid, since under this assumption we cannot achieve a transition from a single nucleus to two separate fragments.

Thus, the method of considering an ellipsoid of revolution which is mathematically complex must moreover be applied with great care to the problem of nuclear fission and has sense only for small perturbations.

The available information on surface tension leads in the case of uranium to the relationship $W_0 = 1.71\ Q_0$. Apparently the uranium nucleus (and *a fortiori* all other nuclei) is stable in the shape of a sphere. A small deformation requires an expenditure of energy. And yet, as we have determined in §1, the formation from a uranium nucleus of two nuclei separated by a great distance is accompanied by a tremendous liberation of energy. Taking the spherical state for one limiting point and the separated fragments for the other limiting point of a line, motion along which describes fission, we determine that for uranium and for other nuclei the energy attains a maximum somewhere in the middle of this line.

Bohr points out justifiably that estimates of stability based on values of surface tension obtained by indirect methods are not reliable, and the information of interest to us must be obtained from experimental data on fission by comparing them with theory. We continue the theoretical discussion of the problem.

Let us find the energy of the fragments. The volume of a fragment is equal to $\frac{1}{2} = 2^{-1}$ while the surface of a fragment is equal to $2^{-2/3}$ of the surface of the initial nucleus so that for two equal fragments we have

$$Q_1 = Q_2 = 2^{-\frac{2}{3}} Q_0, \tag{8a}$$

where Q_0 refers, as before, to the initial nucleus prior to fission.

The charge of the fragment is equal to half of the initial charge, the radius has been decreased by a factor of $2^{1/3}$ so that we have

$$W_1 = W_2 = \mathrm{const}\,\frac{Z^2}{R} = \mathrm{const}\,\frac{\left(\frac{Z_0}{2}\right)^2}{R_0 \cdot 2^{-\frac{1}{3}}} = 2^{-\frac{5}{3}} W_0. \tag{8b}$$

We compare the energy of the system after fission E' with the initial value E_0;

$$E' = W_1 + W_2 + Q_1 + Q_2 = 2^{-\frac{2}{3}} W_0 + 2^{\frac{1}{3}} Q_0, \tag{9}$$

$$E' - E_0 = (2^{\frac{1}{3}} - 1)\, Q_0 - (1 - 2^{-\frac{2}{3}})\, W_0. \tag{10}$$

Fission becomes energetically favorable beginning with the value

$$W_0 > \frac{2^{\frac{1}{3}} - 1}{1 - 2^{-\frac{2}{3}}} Q_0; \quad W_0 > 0.7 Q_0. \tag{11}$$

The result obtained above (Table II) for uranium and other nuclei according to which the energetic possibility of fission is attained considerably earlier than the limit of stability of the spherical shape, is thus of a general nature and follows from a comparison of (11) and (7).

We obtain significant information by calculating the energy at the moment when the nucleus has already separated into two parts, but the fragments still have not had time to become separated and are in contact with each other. Obviously such a state is a necessary link in the chain no matter through what shapes the process of fission proceeds. Since the fragments carry charge of the same sign the energy of the system at the time of contact E'' is certainly greater than the energy E' in the state when the fragments have become separated.

Frenkel' has evaluated E'' for two spherical fragments in contact with each other. The electrostatic and surface energy of each fragment retains its values (8a) and (8b). But to them there is added the electrostatic energy of the interaction between the fragments

$$W_{12} = \frac{\left(\frac{eZ}{2}\right)^2}{2R_0 \cdot 2^{-\frac{1}{3}}}, \tag{12}$$

where Z_0 and R_0 are the charge and radius of the initial nucleus, and $Z/2$ and $R_0 \cdot 2^{1/3}$ are the same quantities for the fragments. Comparing this with the expression for W_0, we obtain

$$W_{12} = \frac{5}{24} \cdot 2^{\frac{1}{3}} W_0;\ E'' = E' + W_{12} = 2^{-\frac{2}{3}} W_0 + 2^{\frac{1}{3}} Q_0 + \frac{5}{24}\, 2^{\frac{1}{3}} W_0$$

$$= \frac{17}{24} \cdot 2^{\frac{1}{3}} W_0 + 2^{\frac{1}{3}} Q_0; \tag{13}$$

$$E'' - E = (2^{\frac{1}{3}} - 1)\, Q_0 - (1 - \frac{17}{24} \cdot 2^{\frac{1}{3}})\, W_0. \tag{14}$$

For the possibility of fission from the condition

$$E'' - E \leqslant 0$$

we obtain

$$W_0 \geqslant \frac{(2^{\frac{1}{3}} - 1)}{1 - \frac{17}{24}\, 2^{\frac{1}{3}}} Q_0 = 2.4\ Q_0{}^{3)}. \tag{15}$$

---

3) The value 2.17 in Frenkel's paper is either a misprint or an error.

The calculation leads to an unexpected result! An impression is created that for W_0/Q_0 exceeding 2 by not very much, i.e., for

$$2<\frac{W_0}{Q_0}<2.4, \tag{16}$$

the heavy nucleus can no longer exist in the shape of a sphere, but also it cannot fall apart for it lacks the energy to go through the shape of two fragments in contact. Frenkel' concluded from these calculations that already for uranium the condition (16) holds and the uranium nucleus in this case exists in a shape significantly different from a sphere. This question was discussed recently by Yu. A. Zysin and one of the authors of this article.[7]

It turns out that Frenkel's results are essentially associated with the arbitrary assumption that fission proceeds through the shape of two spheres in contact. It is evident that at the last moment of fission when the fragments are in contact with each other at only one point the energetically most favorable shape will be that of two elongated pears. An exact determination of this shape encounters very great mathematical difficulties, but the extremum nature of the problem makes such an exact calculation unnecessary. Considering ellipsoids of revolution elongated along the axis along which the contact occurs Zel'dovich and Zysin have shown that for the most favorable choice of the ratio of the axes of the ellipsoid the total energy of the system turns out to be less than the energy of the initial heavy nucleus not only for $W/Q_0=2.4$, but also for $W_0/Q_0=2$ and beyond right down to $W_0/Q_0=1.64$.

Consideration of shapes still closer approaching the minimal ones, and in particular of pearlike asymmetric shapes can lead only to a further diminution of the value 1.64 obtained, and this is of no great physical interest. Already from the result obtained above we can draw the physical conclusion essential for us that in the case when near $W_0/Q_0=2$ the spherical shape of the nucleus becomes unstable, the fission of the nucleus passing through the shape of two elongated ellipsoids in contact with each other is quite possible and is not energetically forbidden. This overthrows the arguments of Ya. I. Frenkel', and the assumption proposed by him of the possibility of existence of nonspherical nuclei becomes unfounded.

Above we have collected all information on the energy relationships in the case of fission which could be obtained by simple calculation. It is summarized in Figs. 3 and 4. In both diagrams the energy of the system is plotted along the vertical axis and the parameter Φ describing the process of fission is plotted along the horizontal axis; this parameter is so chosen that it is equal to 0 for a spherical initial nucleus and takes on the value equal to 1 for the case of two fragments in contact, and 2 for fragments separated by a large distance. Solid lines represent reliable results of calculations; dotted lines connect them in the simplest manner, i.e., they represent the thinking of a person with minimal fantasy who holds to established facts as close as possible.

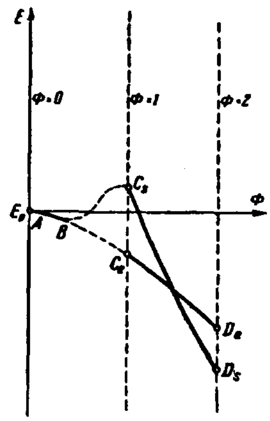

FIG. 3.

Figure 3 refers to the case $2<W_0/Q_0<2.4$. A small deviation from the spherical shape near $\Phi=0$ leads to a decrease in the energy (the line AB), in accordance with the instability of a sphere for $W_0/Q_0>2$. For $\Phi=1$ the point C_s for spherical fragments (Frenkel') lies above E_0; for $\Phi=2$ for the same fragments the point D_s lies considerably below E_0. The curve ABC_sD_s must inevitably have a minimum for $0<\Phi<1$ at some point.

However, calculations for fragments in the shape of ellipsoids (the points C_e and D_e) enable us to draw a monotonic curve ABC_eD_e. Strictly speaking it is not possible to assert that we have excluded the possibility of a minimum on the energy curve in the course of fission, but in any case our calculations have made the existence of such a minimum very improbable.

Figure 4 has been drawn for $2>W_0/Q_0>1.64$. The segment AB describing the effect of small deformations shows an increase in energy. The position of C_s and D_s can be seen from the diagram; for us it is more important that, as before, the points C_e and D_e lie below E_0[4].

There must necessarily be a maximum in the energy at some point M along the line ABC_eD_e. The height of the maximum determines the energy of excitation required for fission.

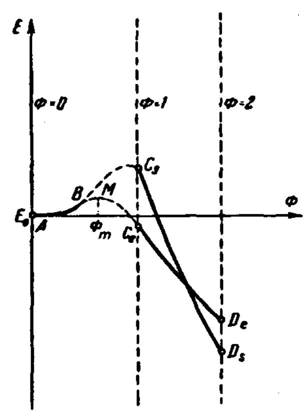

FIG. 4.

[4]Making the calculation more precise we can most likely lower the value of W_0/Q_0, for which the relationship $E''<E_0$ still holds and C_e lies below E_0. The point D_e always lies below C_e since the fragments repel each other no matter what their shape.

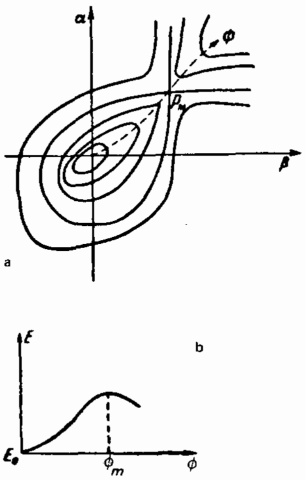

FIG. 5.

While small deformations of the nucleus require an expenditure of energy, a sufficiently strong deformation leads to a state in which the nucleus will be unstable (this will occur after M, for $\Phi > \Phi_M$ in Fig. 4).

The existence of a maximum, the existence of a definite critical energy are extremely important for the understanding of the process of fission.

However, Fig. 4 is clearly insufficient: we are attempting to describe the change of shape by a single parameter; without a more precise specification such a description is ambiguous. We shall improve the situation by going over to the following diagram.

Figure 5a shows lines of constant energy as a function of two variables α, β characterizing the shape of the nucleus. In actual fact one should imagine a similar picture in a space of a very large number of dimensions, but even in Fig. 5a we can determine the principal qualitative features of the process.

The origin represents the spherical shape of the nucleus. Small deformations of it, i.e., small displacements from the origin of coordinates in any direction are associated with an increase in energy. However, for a certain finite deformation we arrive at a saddle point, and with subsequent deformation the energy falls again.

The dotted line shows a path leading through the saddle point. The height of the pass above the valley surrounding the origin of coordinates represents that minimum energy which must be supplied to the nucleus in order that it could pass through the saddle point and fall apart. Figure 5b shows the variation of the energy as the point moves along the dotted line of Fig. 5a passing through the saddle point.

In order that in Fig. 4 the height of the energy maximum should correspond to the critical energy it is necessary to choose the parameter Φ in such a manner that as Φ is varied the nucleus would move along the dotted line through the saddle point of the energy surface.

The calculation of the nuclear shape at the saddle point needed for the determination of the critical energy presents great mathematical difficulties which up to the present time have not been overcome by anybody. A rigorous formulation of the problem starts with the circumstance that in the desired state the energy is an extremum: it is a maximum along one coordinate (P) and a minimum along all the other coordinates.

Physically the extremum of the energy means that mechanical equilibrium has been attained.

In a liquid with a free surface and a surface tension q, the value of the pressure P at the surface of the liquid depends on the curvature of the surface:

$$P = q \cdot g; \quad g = \frac{1}{r_1} + \frac{1}{r_2}, \tag{17}$$

where g is the average curvature, the sum of the reciprocal radii of curvature in two perpendicular directions.

In a charged liquid each element of volume experiences volume electrostatic forces together with forces of pressure; denoting the intensity of the field by V, the potential by φ and the charge density by d we obtain the conditions for equilibrium:

$$\left.\begin{array}{c} \operatorname{grad} P = -Vd = -d \operatorname{grad} \varphi, \\ P + \varphi \cdot d = \text{const.} \end{array}\right\} \tag{18}$$

Finally the equation for equilibrium yields at the surface of the drop

$$q \cdot g + \varphi \cdot d = \text{const.} \tag{19}$$

In (19) q and d are constants characteristic of the nuclear liquid; g depends on the shape of the surface at a given point, the potential φ at the given point of the surface can be calculated in the usual manner for a given spatially distributed charge.

Equation (19) yields as a result of this a most complicated integrodifferential equation for the surface bounding the nucleus.

Important results can be obtained utilizing the methods of similarity theory.

We shall below interpret Φ as the totality of parameters required for a complete description of the shape of the nucleus. The total energy of the system is given by

$$E(\Phi) = W(\Phi) + Q(\Phi). \tag{20}$$

The sign of the function in (20), just as the variable Φ, is of symbolic nature. E, W, Q are functionals of the shape of the surface.

We seek the "value" Φ_e, for which $E(\Phi_e)$ is an extremum (a saddle point) and we are particularly interested in the value of the critical energy $E(\Phi_e) - E_0$, where E_0 refers to the unperturbed nucleus (sphere). The functionals E, W, Q depend not only on the shape, but also, naturally, on the charge, the magnitude of surface tension and the nuclear radius. For different nuclei the critical shape and the critical energy are different.

From dimensional considerations alone, even without writing out the expressions for W and Q, it may be easily seen that these quantities depend on the charge and the surface tension in the same manner as W_0 and Q_0. Therefore they can be represented in the following manner:

$$W = W_0 \cdot a(\Phi); \quad Q = Q_0 \cdot b(\Phi), \tag{21}$$

where a and b are dimensionless functionals of the shape, common for all nuclei; thus, b is the ratio of the surface of a body of shape Φ to the surface of a sphere of equal volume. From this we obtain

$$E = W_0 \cdot a(\Phi) + Q_0 \cdot b(\Phi) = Q_0 \left[\frac{W_0}{Q_0} a(\Phi) + b(\Phi) \right] = Q_0 \cdot c\left(\frac{W_0}{Q_0}, \Phi \right). \tag{22}$$

In the expression (22) c is a dimensionless functional which depends in addition to the shape Φ only on the dimensionless ratio W_0/Q_0.

The ratio W_0/Q_0 is the only determining criterion for the problem, the only quantity varying from one nucleus to another when we seek Φ_e. We finally obtain

$$\begin{aligned} &\Phi_e = \Phi_e\left(\frac{W_0}{Q_0}\right); \quad E(\Phi_e) = Q_0 c'\left(\frac{W_0}{Q_0}\right), \\ &E(\Phi_e) - E_0 = Q_0 \cdot c'\left(\frac{W_0}{Q_0}\right) - W_0 - Q_0 \\ &= Q_0 \left[c'\left(\frac{W_0}{Q_0}\right) - \frac{W_0}{Q_0} - 1 \right] = Q_0 f\left(\frac{W_0}{Q_0}\right), \end{aligned} \tag{23}$$

where c' and f are no longer functionals, but ordinary functions of the variable W_0/Q_0.

The nature of the dependence of f on W_0/Q_0 is shown in Fig. 6. In constructing f we proceed by considering limiting cases.

For the value of the parameter $W_0/Q_0 = 2$ the critical energy is equal to 0. For the value of the same parameter equal to 0, i.e., in the case of fission of an uncharged drop, fission must proceed entirely as a result of external forces overcoming the total difference between the surface energy of the initial drop and the two fragment droplets; in the latter case the required energy expressed in terms of Q_0 is equal to $2 \cdot 2^{-2/3} - 1 = 0.261$, while in the case of a charged nucleus a considerable part of the increase in the surface energy is compensated by the decrease in the electrostatic energy[5].

The middle portion of the curve for f in Fig. 6 between 0 and 2 has been interpolated by Bohr with the aid of approximate methods of calculation developed by him. However, this interpolation, in particular in the region of greatest interest from 1.2 to 2, is apparently not very reliable, although up till the present we have not seen anything better in the literature.

Bohr describes the deformation of the nucleus by spherical harmonics. The second spherical harmonic $P_2(\cos\theta)$ (where θ is the width of the point being considered on the surface of the sphere) describes the elongation of the sphere.

Figure 7a shows the shape corresponding to

$$R(\theta) = R_0[1 + \alpha_2 P_2 \times (\cos\theta)]; \quad \alpha_2 = 0. \tag{25}$$

[5]Bohr obtains by more complicated arguments also the first term in the expansion of f near zero:

$$f = f_1 = 0.261 - 0.108\frac{W_0}{Q_0}. \tag{24}$$

f_1 characterizes the energy of spheres in contact: $f_1 = 0$ for $W_0/Q_0 = 2.4$, cf. formula (15) and the dashed line in Fig. 6.

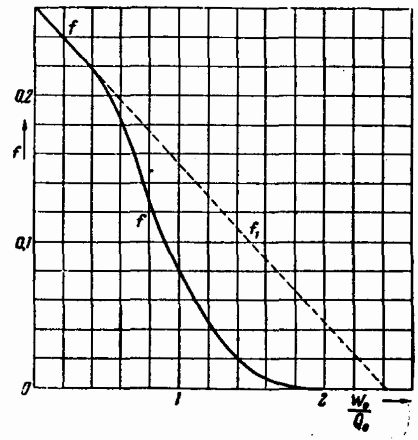

FIG. 6.

Figures 7b and 7c show shapes corresponding to perturbations taken in the form of $P_3(\cos\theta)$ and $P_4(\cos\theta)$ respectively with positive coefficients α_3 and α_4.

Bohr seeks the critical shape in the following manner: having assumed a definite value of α_2, i.e., a definite elongation of the nucleus, he obtains the value of α_4, which makes the energy a minimum for a given value of α_2. It turns out that, $\alpha_4 = \alpha_4(\alpha_2) < 0$; comparing Fig. 7a and Fig. 7b we see that the conditions $\alpha_2 > 0$, $\alpha_4 < 0$ correspond to the shape of Fig. 7d in which a constriction is formed at the middle of the nucleus.

If α_4 is expressed as a function of α_2 from the condition that the energy be a minimum, one can, on neglecting all the other terms, obtain the energy as a function of α_2 for a sequence of shapes sensibly describing the deformation leading to fission. A direct calculation yields the value of α_2 which maximizes $E[\alpha_2, \alpha_4(\alpha_2)]$ and the critical value of E itself.

The dimensionless parameter W_0/Q_0 introduced above enters into all the calculations. It is evident that the calculation is good only as long as the critical deformation is small, which occurs near $W_0/Q_0 = 2$ (near the stability limit). Bohr's result

$$\frac{E_{cr} - E_0}{Q_0} = f = \frac{49}{540}\left(2 - \frac{W_0}{Q_0}\right)^3 - \frac{1421}{68\,850}\left(2 - \frac{W_0}{Q_0}\right)^4 \tag{26}$$

is given in terms of the powers of the deviation from the limiting value.

Figure 6 has been interpolated by Bohr more or less arbitrarily between the limiting expressions (24) and (26).

In the region of practical interest Bohr in fact utilizes expression (26). Formula (26) is very sensitive to W_0/Q_0, and therefore an inverse determination of W_0/Q_0 from experimental data on the critical energy of fission is expedient. As we shall see the energy of fission of uranium of atomic weight 239 having captured a neutron is close to 6 MeV. The surface energy of heavy nuclei Q_0 is of order of 530 MeV. Comparing these values with formula (26) Bohr found $W_0/Q_0 = 1.48$ for uranium 239. This value agrees quite sensibly with the available information on nuclear radii; nevertheless, too great a significance should not be attached to it.

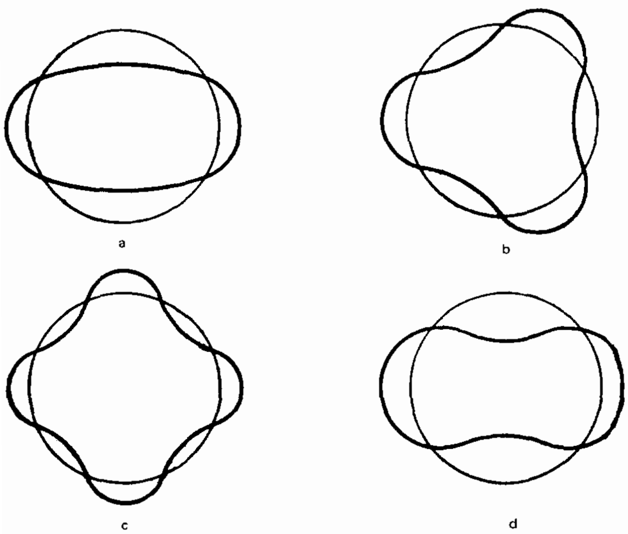

FIG. 7.

There exist indications of the inapplicability of Bohr's approximate calculation for $W_0/Q_0 < 1.8$; at the same time very recently the value of the critical energy for the fission of uranium 239 has itself been subjected to reexamination[6)].

However, even if we become reconciled to the fact that we do not know the exact shape of the curve for $f(W_0/Q_0)$ the very concept of the existence of a smooth curve will enable us to order the nuclei of different elements in terms of the increasing difficulty for fission, and will allow us to encompass the set of experimental facts.

We write the expression for W_0 and Q_0:

$$W_0 = \frac{3(Ze)^2}{5r_0 A^{\frac{1}{3}}}; \qquad (27)$$

$$Q_0 = q\cdot 4\pi (r_0 A^{\frac{1}{3}})^2. \qquad (28)$$

We recall that r_0 is the radius ascribed to a single elementary particle, $r_0 A^{1/3}$ is the radius of a nucleus of atomic weight A containing A elementary particles

$$\frac{W_0}{Q_0} = \frac{3e^2}{20\pi r_0^3 q}\cdot\frac{Z^2}{A}. \qquad (29)$$

The factor $3e^2/20\pi r_0^3 q$ consists of constants common to all nuclei.

TABLE III.

	A	Z	$\frac{Z^2}{A}$	$E_{cr} - E_0$ MeV
Least abundant (0.006%) uranium isotope	234	92	36.2	4.8
Rare (0.7%) uranium isotope	235	92	36.0	5.1
Protactinium	231	91	35.9	5.2
Principal (99.3%) uranium isotope	238	92	35.5	5.8
Thorium	232	90	35.0	6.6
Radium	226	88	34.4	7.6
Mercury	196—204	80	32.7—31.3	11—15
Tin	112—124	50	22.5—20.2	40—50

Thus, comparing the expression for Z^2/A for different nuclei we establish the order in which they must be placed according to increasing difficulty of fission[7)] as shown in Table III.

Nothing is known concerning the fission of the last three substances, and they have been included in Table III only for purposes of comparison.

In Table III we, following Bohr, have also attempted to estimate the critical energy of fission for different nuclei.

Our estimate is based on an analysis of experimental data on fission accompanying neutron bombardment; it should be regarded as being more reliable than the formula (26) itself with the aid of which the extrapolation has been carried out.

Before going on to a detailed examination of the most important process of fission following neutron capture we consider the problem of the probability of spontaneous fission. As we have seen there is every reason for thinking that for all existing atoms, in particular, for the heaviest uranium nucleus, the value of W_0/Q_0 is considerably smaller than two, is smaller than the critical value; from this follows the existence of an energy barrier. The critical energy required for fission (the difference between the energy at the saddle point and the energy of the unexcited nucleus) is supplied as a result of neutron bombardment in the form of the binding energy ("condensation energy") of the neutron and in the form of kinetic energy of the neutron. However, along with this, quantum mechanics establishes the possibility of the passage of a nucleus through a state forbidden by classical theory, the so called tunnelling transition below the barrier. Together with establishing the correct point of view concerning the mechanism of fission Frisch and L. Meitner noted that spontaneous fission of a nucleus by means of the tunnelling transition is not very probable, since the nuclear mass is very high.

The critical energy needed for uranium fission has by now been established quite well. However, a calculation of the probability of the barrier transition requires the knowledge not only of the critical energy (the height of the barrier), but also an estimate of the width of the barrier, i.e., the length of the tunnel. Moreover the

[6)] In a preliminary communication Knipp and Present[8] point out that from their calculations there follows the possibility of an asymmetric critical shape of the nucleus which in this case describes the experimental fact that the fragments obtained in fission, as a rule, differ appreciably from each other in mass. Thus,[1] fission is observed with the ratio A_1: $A_2 = 0.37$: 0.63. We might rather have expected that in the case of a symmetric shape it would be more likely that the mass and charge of the fragments would be closer to each other. However, the principal result—specifically the critical ratio of the electrostatic energy to the surface energy, equal to 2, and qualitatively the picture of Fig. 6, remains unaltered under these conditions. It is not altered also by the considerations of Berestetskiĭ and Migdal.[9]

[7)] The critical energy is equal to the product $Q_0 \cdot f(W_0/Q_0)$. We assume that in going from one nucleus to another f varies much less sharply than Q_0 and arrange the nuclei according to the decreasing values of the argument of f.

width of the barrier appears in the expression for the probability of the process in the exponential term. It is exceedingly difficult to make an estimate of it.

The calculation presents a natural generalization of the theory of α-decay. The expression for the barrier penetrability is given by the exponential

$$\exp\left(-\sqrt{2A\,(E_{cr}-E_0)}\cdot\frac{\alpha}{\hbar}\right), \tag{30}$$

where A is the mass of the nucleus, and α is the effective width of the barrier.

For the term preceding the exponential Bohr chose the frequency of oscillation of the nucleus about its spherical shape.

The reciprocal probability of spontaneous fission (equal to the mean lifetime in the absence of other radioactive processes) is expressed by the formula

$$t = 10^{-21}\exp\left(\sqrt{2A\,(E_{cr}-E_0)}\cdot\frac{\alpha}{\hbar}\right)\ \text{sec.} \tag{31}$$

If the lifetime is expressed in years, A in units of atomic weight, the energy in MeV, and the width of the barrier in units of r_0 (cf., §1), then the formula can be brought to the following form:

$$t = 10^{-29+0{,}145\alpha\sqrt{A_{\alpha}(E_{cr}-E_0)}}\ \text{years.} \tag{32}$$

For uranium, substituting $E_{cr}-E_0 \cong 6$ MeV, $A = 238$ and assuming $\alpha = 3(238)^{1/3}/2$—a value intermediate between the diameter and the radius of a nucleus, Bohr found

$$t = 10^{-29+51} = 10^{22}\ \text{years} = 10^{30}\ \text{sec.} \tag{33}$$

For comparison we note that the time for fission of a nucleus possessing sufficient energy (not requiring a tunnelling transition) is of the order of 10^{-15} sec.

Bohr's estimate leads to a lifetime tremendous even by comparison with the lifetime for the α-decay of uranium ($4\cdot 10^9$ years). The time for spontaneous fission of 10^{22} years would correspond to the formation of a single pair of fragments per day in a mass of 1 kg of uranium; a process with such a probability could hardly be observed, particularly taking into account the existence of atmospheric (cosmic) neutrons and the difficulty of complete shielding from them.

The experiments of the Sovient physicists Petrzhak and Flerov[10] noted in our previous article show that in actual fact uranium fission proceeds with a considerably greater probability with a half-transformation time of the order of 10^{16} years, which makes it accessible to observation using contemporary experimental techniques.

The time of 10^{16} years is obtained if we ascribe the observed number of fissions to the principal uranium isotope. Ascribing it to the 235 or 234 isotopes we obtain respectively 10^{14} and 10^{12} years.

It follows from Bohr's estimate that the critical energy of fission of uranium isotopes is lower, of the order of 5.1 MeV for U^{235} and 4.8 MeV for U^{234}.

Retaining $\alpha = 1.5A^{1/3}$, we obtain respectively 10^{18} and 10^{16} years for the isotopes.

Bohr's estimate describes the experimental data of Petrzhak and Flerov if one assumes that the observed spontaneous fission is due to the light isotopes, and one chooses for α (barrier width) the value 1.38 $A^{1/3}$ instead of 1.5 $A^{1/3}$.

One should admire the tact with which Bohr chose prior to the experiments of Petrzhak and Flerov a sensible value for the width of the barrier; some discrepancies between the measured probability and the probability predicted by Bohr for spontaneous fission are associated with the extreme sensitivity of expressions (31),(32) to factors contained in the exponential.

In popular press one often meets the assertion that the discovery of Petrzhak and Flerov explains why the periodic system of the elements does not extend further, and is terminated at the element No. 92-uranium[8)]. Formally this is not quite so; for the existence of the last element—uranium α-decay is much more important than spontaneous fission; it is quite probable that for the nearest transuranium elements this relationship remains in force, and we do not observe them not because they undergo fission too fast, but because of the excessively rapid α-decay. The discovery of spontaneous fission does not establish an exact boundary for the periodic system. But in a wider sense it is indubitable that there is a connection between spontaneous fission and the boundaries of the periodic system.

With a significant increase in Z (by several units) the increase in Z^2/A will lead to a sharp decrease in the critical energy, and then very rapidly as a result of the exponential dependence of expressions (31),(32), the probability of spontaneous fission will grow catastrophically.

If we base ourselves (in the absence of a better one) on the estimate of the critical energy (26) and on the value of $W_0/Q_0 = 1.48$ for uranium following from it, the absolute limit of stability ($W_0/Q_0 = 2$) will be attained for $Z_2/A = 48$, i.e., for Z of the order of 125 (we assume roughly that A increases proportionally to Z). Such a nucleus will live no longer than for 10^{-20} sec. We now consider spontaneous fission occurring according to the mechanism of the tunnel transition discovered by Petrzhak and Flerov and we shall seek a nucleus with a lifetime of the order of 1000 years. From formulas (30), (31), and (32) it follows that the corresponding probability of fission will be attained at a critical energy of fission of the order of 2.5–3 MeV; for such a decrease in the critical energy we require Z^2/A to be of order of 40–40.5, which corresponds to Z of the order of 100–102. It is not possible to construct a stable nucleus with a larger value of Z by increasing A for reasons indicated in §1: in such a nucleus processes of β-transformation will take place, Z will increase for a constant A, and a nucleus with a larger value of Z and Z^2/A will undergo spontaneous fission.

From the experimentally observed probability of spontaneous fission one can make important qualitative conclusions for our further discussion: accordance to

[8)]The amount of uranium, which is of the order of 10^{-6} of the total mass of the earth, is very great.

a remark of I. I. Gurevich the effective width of the barrier of the order of a nuclear radius, itself indicates that the critical deformation leading to fission can by no means be regarded as small and that all the calculations of the critical shape made on the assumption of the smallness of the deformation may at best be only of an illustrative nature.

On the other hand, the exceedingly small probability of the quantum mechanical tunnelling mechanism definitely indicates the classical nature (with very small corrections for quantum mechanics) of energetic fission induced by neutron or some other bombardment.

We return to the question of the fission of nuclei under neutron bombardment. Present day views developed by Bohr[5] are based on the fact that a heavy nucleus is a system consisting of many particles with a large number of degrees of freedom. Every process occurring as a result of some method of exciting the nucleus begins from the formation of an excited (heated) compound nucleus with a comparatively long lifetime. The lifetime of an excited nucleus of the order of 10^{-15} sec is much longer than the time during which a neutron of energy of several MeV traverses a distance of the order of a nuclear radius, $10^{-12}/10^{9}=10^{-21}$, and is considerably longer than the period of nuclear oscillation, 10^{-20} sec.

Such a relationship enables one to speak in the case of neutron bombardment of the formation of a compound nucleus for which the value of Z does not differ from the value of Z of the initial nucleus, while as a result of neutron capture A has been increased by unity.

Figure 8 shows values of the critical energy of fission (according to Bohr's semi-empirical estimate) for different compound nuclei obtained as a result of neutron capture; the nuclei are placed in a series according to the values of the parameter Z^2/A. The reader will note that in place of U_{92}^{238}, U_{92}^{235}, Pa_{91}^{231} the figure shows U_{92}^{239}, U_{92}^{236}, Pa_{91}^{232}.

The numerical value of the energy taken off this curve, and particularly the value of the slope of the curve, possibly, differ considerably from the true values. However, the order in which Bohr has placed the elements in the figure must indubitably be preserved. As can be seen from the diagram, the nuclei that should undergo fission most easily are the rare light isotopes of uranium. Protactinium occupies a position intermediate between the main and light isotopes of uranium.

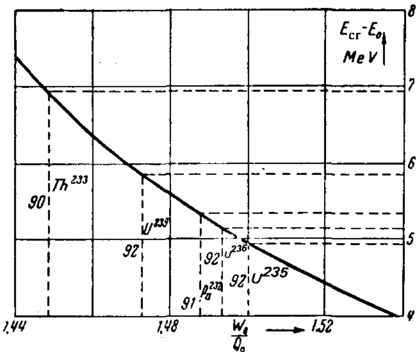

FIG. 8.

Finally, the fission of thorium must require a considerably greater expenditure of energy and correspondingly is more difficult and occurs with a smaller probability. At the same time the capture of a neutron not only changes the atomic weight of the nucleus, but simultaneously, and this is the most important feature, is also the principal source of energy for the process of fission. Even in the case when fission occurs under the action of slow, thermal neutrons, whose kinetic energy is negligible, the process of fission differs significantly from spontaneous fission. Indeed, even the slowest thermal neutron on fusing with a nucleus liberates a considerable amount of energy, of the order of 5–6 MeV—the condensation energy of the neutron. The compound nucleus that captures the neutron becomes energetically highly excited. Fission represents one of the possible reactions for such an excited system. At the same time re-emission of a neutron is also possible. Capture which is followed by re-emission of a neutron is nothing other than neutron scattering. If the nucleus captures a neutron of considerable kinetic energy then on being re-emitted it is not very probable that the neutron would carry away all the kinetic energy with which it approached the nucleus thus leaving the latter in a changed state. It is much more probable that the evaporating neutron will carry away only a part of the initial kinetic energy leaving the nucleus excited. However, at best the energy of excitation does not exceed the kinetic energy of the neutron, while in the case of neutron capture the energy of condensation (binding energy) of the neutron is added to the kinetic energy. In the case of neutrons with energy not exceeding 4–5 MeV inelastic scattering does not lead to fission.

Finally, the last possibility for the excited compound nucleus still containing both the kinetic energy of the absorbed neutron and its heat of condensation, is the emission of energy in the form of a γ-quantum. As a result of the loss of energy the total energy of the nucleus will turn out to be lower than the heat of evaporation of a neutron, the nucleus can no longer reevaporate the neutron, and also cannot undergo fission. We thus obtain a relatively stable nucleus of atomic weight increased by unity.

The question of what exactly happens under neutron bombardment of a given element is first of all a question of competition, of the ratio of the probabilities of the three most important processes indicated above—fission, neutron scattering, i.e., capture with a subsequent reevaporation of the neutron, and, finally, the capture of a neutron with the emission of energy in the form of a γ-quantum. In the next section we shall examine in greater detail the probability of each of the three processes enumerated above, utilizing for this purpose the method of the activated complex. The general indefiniteness which still exists in the theory of fission will not allow us to draw quantitative conclusions; however, the use of the theory of the activated complex will enable us to establish on the basis of general theoretical considerations the character of the principal dependences and above all—the dependence of the predominating direction of the process on the excitation energy.

The best proof of the power of theoretical analysis is the explanation given by Bohr[11] of the involved situation of the action on uranium of neutrons of different energy. This explanation (cf., §4), ascribing different effects to different uranium isotopes guided experimenters and at the present time has led to direct confirmation in experiments with isotopes separated with the aid of mass-spectrographs.[12,13]

[1]Ya. B. Zel'dovich and Yu. B. Khariton, Usp. Fiz. Nauk **23**, 329 (1940).

[2]N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).

[3]Ya. I. Frenkel', Zh. Eksp. Teor. Fiz. **9**, 641 (1939).

[4]L. Meitner and O. R. Frisch, Nature **143**, 239 (1939).

[5]N. Bohr, (Russ. Transl. Usp. Fiz. Nauk **16**, 425 (1936)).

[6]E. Feenberg (cf. remark on fission), Phys. Rev. **55**, 504 (1939).

[7]Ya. B. Zel'dovich and Yu. A. Zysin, Zh. Eksp. Teor. Fiz. **10**, 831 (1940).

[8]R. D. Present and J. K. Knipp, Phys. Rev. **57**, 751 (1940); *ibid*. p. 1188.

[9]V. Berestetskiĭ and A. Migdal, Dokl. Akad. Nauk SSSR **30**, 701 (1941).

[10]K. A. Petrzhak and P. G. Flerov, Zh. Eksp. Teor. Fiz. **10**, 1013 (1940); Usp. Fiz. Nauk **25**, 171 (1941).

[11]N. Bohr, Phys. Rev. **55**, 418 (1939).

[12]A. O. Nier, E. T. Booth, J. R. Dunning, and A. V. Grosse, Phys. Rev. **57**, 546, 748 (1940).

[13]K. H. Kingdon, H. C. Pollock, E. T. Booth, and J. R. Dunning, Phys. Rev. **57**, 749 (1940).

Translated by G. M. Volkoff