

Trap trajectories and the cutoff of dissipative effects in semiconductors

I. B. Levinson

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In semiconductors, unlike gaseous plasmas, electrons scatter relatively often, and for this reason the dynamical properties of the carriers in motion along trajectories in crossed E and H fields usually have only a very weak effect on the kinetics. However, during markedly inelastic scattering by optical phonons, the topology of the trajectories in momentum space plays an important role, and a change in the topology will fundamentally alter the physical picture of the kinetics. A change in the topology can lead to a situation where the emission of optical phonons is impossible, so that dissipative effects are completely cut off. The topology of the trajectories also affects the nature of the nonequilibrium distribution function for hot electrons, forming regions of carrier accumulation in phase space.

1. TOPOLOGY OF TRAJECTORIES IN MOMENTUM SPACE

In pure semiconductors at low temperatures, the characteristic time τ for scattering by impurities and acoustical phonons is much longer than the time τ_0 for the emission of an optical phonon. There are thus two regions in p space in which the scattering has a completely different character; these regions are separated by a constant-energy surface $\varepsilon(p) = \hbar\omega_0$, where $\hbar\omega_0$ is the energy of a long-wavelength optical phonon. If $T \ll \hbar\omega_0$, then in the passive region $\varepsilon(p) < \hbar\omega_0$ the absorption of optical phonons is exponentially weak, and the emission of such phonons is impossible. In this passive region Ω , therefore, the scattering is due solely to impurities and acoustical phonons. Meanwhile, in the active region $\varepsilon(p) > \hbar\omega_0$ the dominant process is the spontaneous emission of optical phonons. In the simplest model of the spectrum, Ω is a sphere of radius $P_0 = \sqrt{2m\hbar\omega_0}$

There exists an interval of electric fields E such that

$$\tau \gg \tau_E \gg \tau_0, \quad (1)$$

where $\tau_E = p_0/eE$ is the time required for the acceleration of an electron from $\varepsilon = 0$ to $\varepsilon = \hbar\omega_0$. The left-hand inequality in (1) means that during its acceleration within region Ω there is insufficient time for the electron to scatter, while the right-hand inequality means that upon reaching the boundary of this region it instantly emits a phonon $\hbar\omega_0$.

After emitting the phonon the electron is situated at point $p = 0$, and then it moves under the influence of the field along a straight line parallel to E until it intersects the boundary of Ω at the point $p = p_0$ (this is the principal trajectory, shown by a heavy line in Fig. 1), whereupon it again emits a phonon, and so on.¹ On the average, in one out of every τ/τ_E acceleration cycles there is an elastic scattering event within Ω (dashed arrow 1). After this, however, the electron, in a time of order τ_E , accelerates along a collateral trajectory 2 to an energy $\varepsilon = \hbar\omega_0$ and emits a phonon (arrow 3), whereupon it returns to $p = 0$ and resumes its "reciprocating" motion along the principal trajectory. From this it is clear that the "lifetime" on the

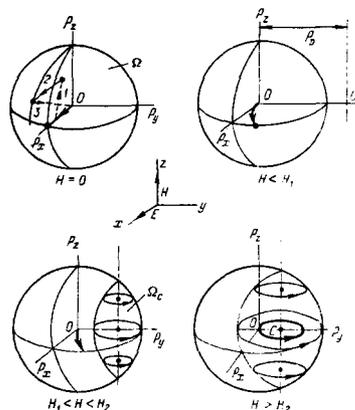


FIG. 1. Electron trajectories in momentum space in the case of crossed fields.

principal trajectory is of order τ , and the lifetime off it of order τ_E . The fraction of electrons that are off the principal trajectory is therefore small, of order τ_E/τ . The electrons of the principal trajectory form a sharply anisotropic needle-shaped distribution in momentum space. The average speed of the electrons is determined by the "center of gravity" of this trajectory and is equal to $(1/2)v_0$, where $v_0 = p_0/m$. The macroscopic manifestation of this circumstance is the saturation of the current j as a function of E .

A substantially more complex and detailed picture of the electron motion arises when a field H is turned on perpendicular to E .^{2,3} The trajectories in p space become circles lying in planes perpendicular to H , with their centers on a straight line parallel to H and displaced from $p=0$ in the Hall direction by a distance $p_D = my_D$, where $v_D = cE/H$ is the Hall drift speed (see Fig. 1). The electrons revolve along these circles at the cyclotron frequency $\omega_c = eH/mc$.

As long as the line of centers lies outside the sphere Ω , i. e., for $v_D > v_0$, all of the trajectories inside Ω are open, (i. e., not closed), and the motion of the electrons is the same as for the case $H=0$ except that all the trajectories, both the principal and collateral, are curved. Such a picture obtains for $H < H_1 \equiv eE/v_0$ or, equivalently, for $\omega_c \tau_E < 1$. In a field $H = H_1$ the topology of the trajectories changes: For $H > H_1$, closed collateral trajectories appear in Ω , forming a spindle-shaped region Ω_c (the accumulation region). Trajectories in Ω_c are traps for electrons—an electron falling onto a closed trajectory can return to the principal trajectory only after scattering elastically out of Ω_c onto an open collateral trajectory. In other words, the lifetime in Ω_c is of the same order as the lifetime on the principal trajectory, and therefore the fraction of electrons in Ω_c is of the order of the ratio of phase volumes $\Omega_c / (\Omega - \Omega_c)$. At $H = H_2 = 2H_1$ there is a second topological change: The principal trajectory closes. For $H > H_2$, almost all the electrons are found within the spindle; the fraction of electrons on open trajectories outside the spindle is τ_E/τ .

2. CUTOFF OF DISSIPATIVE EFFECTS

The degree of "magnetization" electrons in the spindle is different from that of the electrons on an open principal trajectory: For the first, it is governed by the quantity $\omega_c \tau \gg 1$, and for the second by $\omega_c \tau_E \approx 1$. Strongly magnetized electrons first appear in a field $H = H_1$, and at $H = H_2$ the weakly magnetized electrons vanish. It is this circumstance that accounts for the peculiar effect of the magnetic field on the kinetic phenomena during inelastic scattering.

The spindle electrons contribute only to the Hall current j_y ; the dissipative current j_x is entirely due to the electrons on the principal trajectory. Therefore, at $H = H_2$, when the emission of optical phonons is cut off, the dissipative current should drop abruptly.^{2,3}

When the spindle appears in a field $H = H_1$, a cyclotron-resonance signal should arise in an alternating test field E ,⁴ at $H = H_2$ this signal should grow abruptly.

If E and H are not perpendicular, the circular trajectories go over into helices, and an electron on any trajectory can be accelerated to $\varepsilon = \hbar\omega_0$. However, it is easy to see that in this case the length of any trajectory is an oscillatory function of H , since an increase in H adds new turns to the helix in region Ω . This may account for the interesting oscillatory behavior of j as a function of H .⁵

3. SMEARING OF THE CRITICAL FIELDS H_1 AND H_2

In reality, the optical phonon is not emitted instantly, and so the electron penetrates into the active region to a finite depth $\Delta\varepsilon/v_0$, with the result that it does not return precisely to $p=0$, but rather to a small spherical region of radius $\delta p \approx \sqrt{2m\Delta\varepsilon}$. This means that the principal trajectory acquires a finite "thickness" δp , which is responsible for a smearing of the features in the kinetic coefficients that appear as a result of the closing of the principal trajectory. The penetration depth can be estimated from the condition that the probability for the emission of an optical phonon during the time of acceleration from $\varepsilon = \hbar\omega_0$ to $\varepsilon = \hbar\omega_0 + \Delta\varepsilon$ be of order unity, i. e.,

$$\int_{\hbar\omega_0}^{\hbar\omega_0 + \Delta\varepsilon} \frac{dt}{\tau(\varepsilon)} = 1, \quad dt = \frac{dp}{eE} = \frac{d\varepsilon}{eEv_0}, \quad (2)$$

where the characteristic time for the emission of a phonon near the threshold is given by

$$\frac{1}{\tau(\varepsilon)} = \frac{1}{\tau_0} \left(\frac{\varepsilon}{\hbar\omega_0} - 1 \right)^{1/2}. \quad (3)$$

Substituting (2) into (3), we find $\Delta\varepsilon/\hbar\omega_0 \approx (\tau_0/\tau_E)^{2/3}$, which permits us to estimate the degree of smearing of the critical field as

$$\frac{\delta H_2}{H_2} \approx \frac{\delta p}{p_0} \approx \left(\frac{\tau_0}{\tau_E} \right)^{1/3}. \quad (4)$$

The size of the jump in the current j_x that should be observed at a field H_2 is determined by the ratio of the number of electrons on open collateral trajectories to the number of electrons on the principal trajectory, i. e.,

$$\frac{\delta j_x}{j_x} \approx \frac{\tau_E}{\tau}. \quad (5)$$

In other words, to observe the features at H_2 one must have

$$\tau > \tau_E \gg \tau_0. \quad (6)$$

The features that occur at a field H_1 are due to the appearance of magnetized electrons. These features therefore appear only if $\omega_c \tau \gg 1$. The field H_1 is itself smeared because there is a thin layer in the active region around Ω in which the phonon emission probability (3) has not yet exceeded the elastic scattering probability $1/\tau$. After estimating the thickness Δp of this layer we find

$$\frac{\delta H_1}{H_1} \approx \frac{\Delta p}{p_0} \approx \left(\frac{\tau_0}{\tau} \right)^2. \quad (7)$$

This means that the features occurring at H_1 will be observed if

$$\tau \gg \tau_E, \quad \tau_0; \quad (8)$$

the relationship between τ_E and τ_0 is not important.⁶

4. GAUSS-AMPERE CHARACTERISTICS (EXPERIMENT)

The first attempts to detect a cutoff of the dissipative current at a field H_2 were made in p -type Ge,⁷⁻⁹ where the distribution of hot holes in strong E fields at $H=0$ was known to be markedly anisotropic.¹⁰ In the best of the samples, τ was equal to 12 ps, so that the left-hand inequality in (6) would require $E > 0.3$ kV/cm. In such a field, however, we find from (4) with the known value $\tau_0 = 0.4$ ps, that $\delta H_2/H_2 \approx 0.3$. It is not surprising, therefore, that sharp features were not detected in the experiments of Refs. 7-9.

Success was achieved in experiments on AgBr and AgCl crystals of high purity.¹¹⁻¹³ For electrons in these crystals the elastic scattering time τ is around 10 ps, approximately the same as for holes in Ge, but because of the highly ionic character of alkali halide crystals the interaction with optical phonons is much stronger: $\tau_0 \approx 0.01$ ps. This gives a ratio $\tau/\tau_0 \approx 1000$, compared to $\tau/\tau_0 \approx 30$ for p -Ge. Figure 2 shows the results of measurements of the Hall angle Θ ; the points are for various samples of AgCl in various fields from 1 to 3 kV/cm.¹³ The solid curve is theoretical²:

$$\operatorname{tg} \theta = \frac{1}{h} \left(\frac{\arcsin h}{h} - \sqrt{1-h^2} \right), \quad h \equiv \frac{H}{H_2} = \frac{H}{2H_1}. \quad (9)$$

This expression was calculated under the assumption that all the electrons are found on the principal trajectory; in this case the current is determined simply by the center of gravity of the trajectory. It is seen in Fig. 2 that for $H < H_1$, when almost all the electrons actually do move along the principal trajectory, the dependence of Θ on H/H_1 is universal and very close to the theoretical. For $H > H_1$, some of the electrons go over to the closed collateral trajectories in the spindle, where they have a substantially larger effective $\omega_c \tau$, and the experimental curves therefore go above the theoretical curve, which corresponds to the principal trajectory. The fraction of the electrons which are in Ω_c is determined by an elastic-scattering balance³

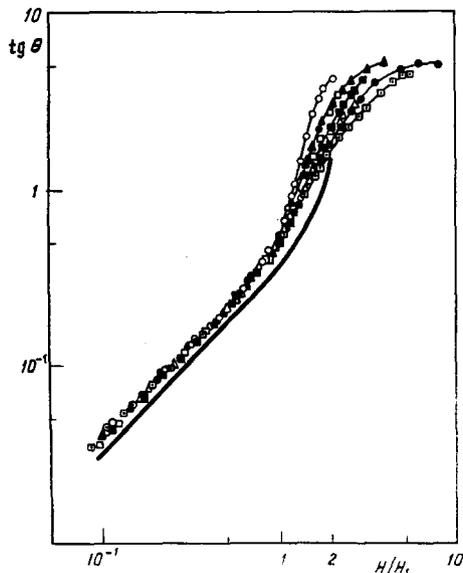


FIG. 2. Hall angle in AgCl at $T=4.2$ K, $H_1=14.5$ kOe for $E=2$ kV/cm; $m=0.43$.

within Ω , the details of which depend on the number of impurities. It is for this reason that the functions $\Theta(H)$ in the region $H > H_1$ are different for the different samples.

Figure 3 shows the measured current j_x arising when a weak alternating field E_x is applied along H .^{12,13} The curves are labeled by the values of the strong fields E_x . The acceleration in the z direction of the electrons on the principal trajectory is disrupted by scattering by optical phonons, which occurs in a time τ_E ; in other words, the differential mobility of these electrons along z is of order $(e/m)\tau_E$. Meanwhile, the spindle electrons, moving along a helix, are accelerated in the z direction over a time τ (it is assumed that the field E_x is so weak that elastic scattering occurs more frequently than the helix "sticks into" the boundary of the region Ω). The mobility of these electrons is therefore of order $(e/m)\tau$, somewhat larger than in the former case. In other words, when the spindle appears the average mobility should begin to grow with the spindle volume Ω_c .¹² This circumstance is beautifully confirmed by experiment (the fields H_1 are indicated by arrows in Fig. 3).

5. NONEQUILIBRIUM NATURE OF THE DISTRIBUTION IN CROSSED FIELDS

The distribution of electrons having a pronounced dynamical motion along trajectories, which was studied in Refs. 2 and 3, has in some sense a more nonequilibrium character than the usual, almost isotropic distribution of hot electrons which arises in the case of quasielastic energy relaxation.¹⁴ For example, in the approximation $\tau = \infty$ and $\tau_0 = 0$, the differential conductivity $\sigma(\omega)$ at $H=0$ is zero,¹⁵ whereas the intensity of the current noise $S(\omega)$ is nonzero, having delta-function-like peaks at frequencies which are multiples of $2\pi/\tau_E$.^{15,16} We have the maximum possible violation of the fluctuation dissipation theorem. For finite τ and τ_0 one obtains $\sigma(\omega) \neq 0$, but then, in a number of cases, one has $\sigma(\omega) < 0$ in the frequency region $\omega \approx 2\pi/\tau_E$.¹⁷⁻¹⁹

When a spindle-shaped region Ω_c is present, the specific properties of the distribution can be interpreted in terms of a population inversion.²⁰ The advent of the spindle Ω_c implies the appearance of a group of electrons with energies close to $\hbar\omega_0$; a peak arises on the energy distribution function $f(\epsilon)$, i. e., the distribution becomes inverted. Quasielastic scattering, on the other hand, always gives a noninverted

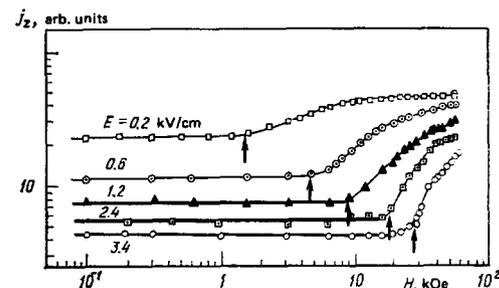


FIG. 3. Gauss-ampere characteristic for longitudinal current in AgCl at $T=4.2$ K and $E_x=15$ V/cm.

monotonically decreasing distribution $f(\varepsilon)$.¹⁴ An example of an inverted distribution $f(\varepsilon)$ for heavy holes in Ge, which was calculated by the Monte Carlo method, is shown in Fig. 4.

Another nonequilibrium aspect of the distribution in crossed fields is the two-pronged character of the distribution $f(p_y)$ of the electrons on a closed principal trajectory over the Hall component p_y of their momentum. It is easy to see from geometrical considerations that $f(p_y)$ goes to infinity at $p_y = 0$ and $p_y = 2p_D$, i.e., at the points where the principal trajectory intersects the p_y axis.

One of the amplification mechanisms associated with the principal trajectory is negative absorption at the cyclotron resonance, which was discovered by Al'ber *et al.*²¹ Let us consider the situation which arises when the principal trajectory has just barely closed, i.e., H is slightly larger than H_2 . Let the sample be placed in an alternating electric field \vec{E} of frequency $\omega = \omega_c$, polarized in the x, y plane. The effect of E on an electron of the principal trajectory depends on the phase relationship between \vec{E} and the revolution of the electron. Electrons which increase their energy under the action of \vec{E} move over to trajectories of larger radius, reach the boundary of Ω , emit a phonon, and return to $p = 0$. This will occur until such time as the electron, upon arriving at $p = 0$, finds itself in such a phase that its energy will decrease in the field \vec{E} . This results in a bunching of the electrons in phase, after which all the electrons will give up their energy to the alternating field, amplifying it. Numerical calculations show²² that a necessary condition for this amplification to occur is $\tau/\tau_0 \geq 200$, a value achievable at currently attainable purity levels only in perfect alkali halide crystals.

6. HOLE INVERSION IN GERMANIUM

It would appear that a population inversion is more easily achievable when there are several groups of carriers with different cyclotron masses and the emission of a phonon $\hbar\omega_0$ can lead to mutual conversion of the carriers between the different groups. The distribution function of the carriers in such a situation has been studied for electrons in germanium and silicon, which have valleys that are oriented nonequivalently with respect to the magnetic field,²³ and also for a hole band in germanium with light and heavy holes.⁴ It is this latter case that relates to all the experimental attempts that have been mounted to data in an effect to create an inversion and use it to generate radiation.

Because of the difference in the masses of the light and heavy holes (in germanium, $m_l/m_h = 0.12$), there is

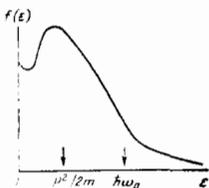


FIG. 4. Inverted distribution of heavy holes in germanium at $T = 21.5$ K, $E = 2.7$ kV/cm, and $H = 20$ kOe.

an interval of fields H in which a spindle-shaped accumulation region Ω_c^l exists in the passive region Ω^l of the light holes but no such region exists in the passive region Ω^h of the heavy holes. In this field interval, as was shown by Vosilyus,⁴ the conversion of light holes into heavy holes through emission of a phonon $\hbar\omega_0$ is ruled out, and the ratio of the number of light holes to the number of heavy holes, which in equilibrium is $(m_l/m_h)^{3/2} = 4\%$, increases by a factor of τ/τ_E^h (τ_E^h is the acceleration time for a heavy hole; the scattering times τ are the same for holes of both types). The growth in the density of light holes can lead to an inversion in some region of p space, where the inequality $f_l(p) > f_h(p)$ will hold. Then laser action will arise at the transitions between the light and heavy hole bands,²⁴ with the emission of photons $h\nu = \varepsilon_l(p) - \varepsilon_h(p)$ (Fig. 5).

Inversion of the distribution of light holes relative to that of heavy holes in p -Ge has been confirmed by absorption measurements²⁵ at the transitions from the light and heavy hole bands l and h to the empty band s , which is split off by the spin-orbit interaction (see Fig. 5); knowing the spectral variation of the absorption, one can evaluate f_l and f_h . In a field $H = 23$ kOe and at rather high values $E \approx 4$ kV/cm, an inversion $f_l/f_h \approx 2-4$ is reached in a certain region of momentum p , but no features are observed in the inversion when the field E is varied at the points where the topological changes occur in the trajectories. Therefore, although the range of fields E and H in which the inversion is observed agrees with the predictions of the theory, the experiment of Ref. 25 does not fully settle the question of the inversion mechanism.

Another source of information on the distribution function is provided by experimental measurements of the far-infrared emission at the $l \rightarrow h$ transitions in p -Ge in crossed fields.²⁶⁻²⁸ Figure 6 shows the H dependence of the integrated emission intensity from p -Ge in the 50-120 μm range (the sensitivity region of a Ge/Ga photodetector). Arrows A and B indicate the fields H_l^l and H_l^h at which an accumulation region arises in the light and heavy hole band, respectively. The rise of the emission at $H = H_l^l$ is also confirmed by the experiments of Kromiyama,²⁸ which were done in the same range of E and H . In those experiments²⁸ the drop in intensity at $H = H_l^h$ was observed only in strong fields $E \approx 1$ kV/cm; in weaker E fields the emission intensity reached a plateau. The presence of emission does not signify an inversion, but the experiments did show that

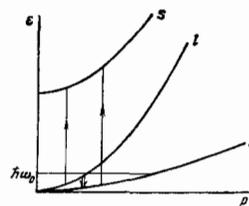


FIG. 5. Hole band in germanium. h) Heavy holes; l) light holes; s) spin-split-off holes. The short arrows are transitions $l \rightarrow h$, which give radiation in the far infrared; the long arrows are transitions $h \rightarrow s$ and $l \rightarrow s$, by which the hole distribution is measured.

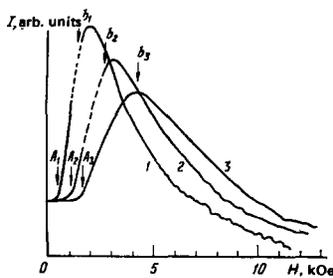


FIG. 6. Integrated emission intensity as a function of H for various fields E : 1) 0.36 kV/cm; 2) 0.67 kV/cm; 3) 1.1 kV/cm (after Ivanov²⁶).

the intense emission arises concomitantly with a region of accumulation of light holes. This conclusion is also supported by the spectral measurements made in that study.²⁸

Extremely weighty evidence for inversion was obtained in the experiment of Vorob'ev *et al.*,²⁹ where it was shown that if a sample of p -Ge is placed in a resonator which is selective for oscillations with $\lambda \approx 100 \mu\text{m}$, the intensity of the infrared emission from the sample increases by one and a half to two orders of magnitude. It is seen in Fig. 7 that the threshold E field at which appreciable emission appears is extremely close to the field $E_1 = 4.3 \text{ kV/cm}$ at which the spindle-shaped accumulation region for the heavy holes vanishes, thereby creating favorable conditions for inversion of the light holes, whose accumulation region exists for fields up to $E_2 = 12 \text{ kV/cm}$.

7. ALTERNATING FIELDS

There are also a number of effects which occur in the case when the strong field E is an alternating field (of frequency ω). In the field E is circularly polarized in a plane perpendicular to H , the problem in the isotropic model reduces to the static case³⁰; one need only transform to a coordinate system rotating with the field and change H in such a way that the cyclotron frequency in the new field is $\omega'_c = \omega_c - \omega$. It follows that the cyclotron-resonance line in the strong field E has an unusual shape³⁰: The line terminates abruptly at a distance $\Delta H = 2(c/v_0)E$ from the resonant field $H_0 = mc\omega/e$. In the case of linear polarization the "half-width" ΔH is the same, although the cutoff is not so sharp.³¹ This dependence of ΔH on E is well confirmed by an experiment on AgBr.³²

In an alternating field E there are also accumulation

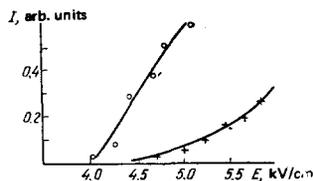


FIG. 7. Intensity of infrared emission at $\lambda \approx 100 \mu\text{m}$ from a p -Ge sample placed in a resonator, as a function of the electric field. $H = 22.5 \text{ kOe}$, $T = 10 \text{ K}$ (upper curve) and $T = 80 \text{ K}$ (lower curve).

regions which oscillate inside Ω with the period of the field.³³

The experiments described above indicate that in pure alkali halide crystals and in p -type germanium the kinetic phenomena in strong fields take place against the background of a pronounced carrier dynamics.^{2,3} From a practical standpoint the most interesting circumstance here, I believe, is that the state of the carriers under these conditions differs more strongly from the equilibrium state than in the case of ordinary heating. This raises the hope of a larger spectrum of instabilities and the possibilities that that would entail.

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