Superfluid ^{3}He: introduction to the subject

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An account is given in this review of the basic experimental results and theoretical concepts concerning the superfluid A and B phases of ^{3}He. The A and B phases of ^{3}He possess the most complicated order parameters of all presently known ordered materials, resulting in a rich spectrum of properties of these phases: magnetic, acoustic, superfluid, and others. The theoretical description has been carried out on a phenomenological level, establishing the structure of the superfluid phases of ^{3}He on the basis of general concepts of Cooper pairing with nonzero angular momentum and the set of experimental data. A separate section is devoted to linear and nonlinear NMR in superfluid ^{3}He. NMR experiments and theory have played an important role in understanding the structure of the superfluid phases of ^{3}He.

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1. INTRODUCTION

The helium isotope with atomic weight 3, as well as helium-4, forms a liquid which does not solidify at normal pressure all the way down to absolute zero temperature. The nuclear spin of ^{3}He is 1/2; therefore, unlike the spinless ^{4}He, liquid ^{3}He is a Fermi liquid with a degeneracy temperature near 1 K. It is well known that at a sufficiently low temperature and in the presence of arbitrarily weak interparticle attraction a Fermi liquid is unstable to Cooper pairing of particles, resulting in a transition to a superfluid state. More accurately, the transition occurs even if the attraction takes place at some one value of L, the orbital angular momentum of relative motion of paired particles. Thus, in superconductors Cooper pairing of electrons occurs in the s-state. The search for superfluidity in ^{3}He started in 1959, when it was shown theoretically by L. Pitaevskiĭ[1a] that due to the strong repulsion of ^{3}He atoms at short distances atomic pairing, caused by Van der Waals attraction, must occur in a state with nonvanishing angular momentum L (for an estimate of the transition temperature see Ref. 1b). Models for superfluidity of ^{3}He were developed in following years in the spirit of the BCS theory of superconductivity, among which two models of superfluid Fermi liquids with Cooper pairing in the p state stand out, the so-called Anderson-Morel[2] and Balian-Werthamer[3] models.

Meanwhile, the development of cryogenic techniques made it possible to work in the temperature region of the order of 10^{-3} K. There exist three methods by which one can reach such low temperatures; solution of ^{3}He in ^{4}He, adiabatic magnetization, and the Pomeranchuk effect. The latter consists of the fact that at temperatures below 0.3 K the entropy of liquid ^{3}He $S_l = \gamma T$ is lower than the paramagnetic entropy of solid ^{3}He $S_s = \ln 2$. Therefore, compressing ^{3}He adiabatically till the melting curve, we transform part of helium-3 from the liquid to the solid state, thus lowering the temperature of the mixture. Cooling by the Pomeranchuk effect was first achieved by Anufriev[4] in 1965. It is by this method that Osheroff, Richardson, and Lee[5] discovered in 1972 a small break, and then, after an interval, a jump on the curve of the time dependence of the pressure of a liquid and solid ^{3}He mixture in the millikelvin temperature region.

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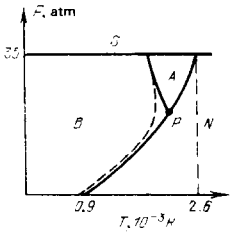

FIG. 1.

It was shown that the break was explained by a phase transition ($T_c = 2.6 \cdot 10^{-3}$ K) of liquid helium from the normal state to a superfluid phase, called the A phase, and the jump—by a phase transition ($T_{AB} = 2.07 \cdot 10^{-3}$ K) of the A phase to another superfluid phase, called the B phase (see the phase diagram in Fig. 1). As a result of intense experimental and theoretical study of the new phases it was established that the A phase corresponds to the model of p-pairing, theoretically considered by Anderson and Morel,[2] and the B phase—to the Balian and Werthamer[3] model. The properties of the new phases are surprisingly diverse, and recall in a number of cases the properties of other ordered materials: superconductors, magnets, and liquid crystals. Experimental work in the millikelvin region is carried out in several laboratories in the world, and now, after a lapse of 10 years following the discovery of the small singular features on the melting curve of ^{3}He, a whole new area in the physics of superlow temperatures was generated, the physics of superfluid ^{3}He.

The literature on superfluid ^{3}He, numbering hundreds of publications, contains several experimental[6-11] and theoretical[12-16] review articles. Two collections of papers on superfluid ^{3}He[17,18] were translated into Russian. The superfluid phases of ^{3}He are such rich and interesting objects, that each group of properties, magnetic, acoustic, superfluid, etc., merits writing a separate review. And yet there exists hardly any review literature in Russian on superfluid ^{3}He. The present paper contains a brief description of the basic experimental results and theoretical concepts, making it possible to establish the structure of the superfluid phases of ^{3}He. A separate section is devoted to nuclear magnetic resonance, playing a major role in identifying the phases of ^{3}He.

The present article is subtitled "introduction to the subject", and, naturally, contains many gaps which, the author hopes, will be filled in subsequent more specialized reviews. One of these gaps is partially compensated by a popular soft-cover book,[19] devoted to the superfluid properties of ^{3}He-A, possibly the most interesting and unusual aspect of what the discovery of the new phases of liquid ^{3}He brought to physics.

2. BASIC EXPERIMENTAL PROPERTIES OF THE SUPERFLUID PHASES OF ^{3}He

a) Phase diagram

The phase diagram of the new phases of ^{3}He is shown schematically in Fig. 1. Here S denotes the region of solid ^{3}He, starting above 35 atm, N denotes the region of normal liquid ^{3}He, and A and B, respectively, the regions of superfluid ^{3}He-A and ^{3}He-B. The phase transition between the normal and superfluid phases is a second order transition. The transition between the A and B phases is of first order, with a latent heat near the melting line close to 20 erg/mol. Two essential differences are introduced by the switching on of a magnetic field. First, the direct transition from the N to the B phase disappears in the region below 20 atm, i.e., as shown by the dashed line in Fig. 1, in the presence of a magnetic field the B phase is separated from the N phase by a narrow band of the A phase. Secondly, in a magnetic field the T_c line splits into two transitions (see Fig. 2) with temperatures T_{c1} and T_{c2} (the so-called A_1 and A_2 transitions),

$$T_{c1} - T_{c2} = 6 \cdot 10^{-9} \left(\text{K/Gauss}\right) H,$$

between which ^{3}He is in the state of the A_1 phase.[20]

b) Specific heat

The temperature dependence of the specific heat is similar to the behavior of the specific heat in superconductors. The specific heat ratio C_S/C_N at T_c, equal to 2.43 in the case of superconductors, is for He near 2.8 at high pressures, this being a manifestation of strong coupling effects (see Ref. 21), while for vanishing pressure C_S/C_N approaches 2.43.

c) Susceptibility

Magnetic susceptibility measurements of the A-phase show that it coincides, within a few tenths of a percent, with the magnetic susceptibility of ^{3}He in the normal state. This implies that if the transition mechanism is Cooper pairing, the transition occurs in states with pair spin $S = 1$ and two equally probable spin projections on the direction of the field $S_z = \pm 1$. It is precisely for this pairing that pairs react to a magnetic field as atoms with spin projections $S_z = \pm\frac{1}{2}$. The small change in the susceptibility is due to renormalization of the Fermi liquid parameters (of the order of $\Delta/\varepsilon_F \sim 10^{-3}$) during the transition.

Following a sharp drop in the susceptibility with the transition from the A to the B phase, a drop which disappears together with the A phase with decreasing pressure (see Fig. 3), in the B phase the magnetic susceptibility starts decreasing smoothly with temperature, reaching a value $\sim 0.3\chi_N$ at $T = 0$. This implies the appearance in the transition to the B phase of states with $S_z = 0$, along with the paired states with spin projections

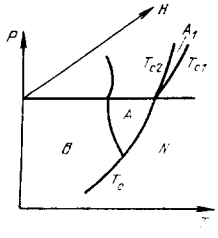

FIG. 2.

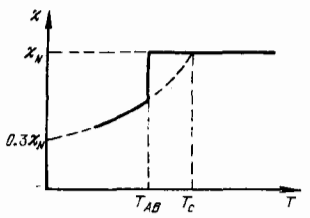

FIG. 3.

$S_z = \pm 1$ along the field direction which are characteristic of the A phase. The susceptibility of a Fermi gas, paired in the $S = 1$ state with three equally probable projections $S_z = 0, \pm 1$, should tend to $2\chi_N/3$ when the temperature tends to zero, but due to Fermi liquid corrections[7,22,23] the limiting value of $\chi^B(T = 0)$ is near $0.3\chi_N$ (see Ref. 7). The dependence of the susceptibility of the B-phase on T/T_c, where T_c is the transition temperature at a given pressure, is nearly universal. This implies that the transition temperature T_c is common for the A and B phases. This is also indicated by the absence of a break in the T_c curve at the termination point of the first order transition line (the point P on Fig. 1).

The fact that the susceptibility of the B phase is always smaller than that of the A phase leads to an important thermodynamic consequence. Specifically, the A phase becomes energetically more favorable than the B phase in the presence of a magnetic field in a narrow region near T_c. Therefore, as already noted, the region of existence of the B phase in a magnetic field is always separated from the normal liquid by an A phase band, which is illustrated on Fig. 1 by dashed lines.

We also note here that when the temperature is lowered in a magnetic field the phase transition to the superfluid state occurs initially at T_{c1} for particles with spins directed along the field, and then at T_{c2} for particles with spins against the field. This follows[20] from the dependence of the transition temperatures on the density of states at the Fermi surface, being different for particles with spins along and against the field:

$$T_{c.1,2} = \omega \exp\left(-\frac{1}{VN_0(\varepsilon_F \pm \mu H)}\right).$$

For this reason the A_1 phase (Fig. 2) is a superfluid liquid containing pairs with $S_z = +1$ only. The total spontaneous magnetization of the A_1 phase, however, is insignificant, of the order of $\sim(\Delta/\varepsilon_F)^2 \simeq 10^{-6}$, for the magnetic moment of the superfluid component is compensated with just this accuracy by the magnetic moment of the normal component, having the opposite spin orientation.

d) Nuclear magnetic resonance

In normal ^{3}He, placed in a constant magnetic field H_0 and a varying field perpendicular to it, absorption occurs at the Larmor frequency of spin precession $\omega_0 = \gamma H_0$. In ^{3}He-A this frequency is shifted,

$$\omega_{A_\perp}^2 = \omega_0^2 + \Omega_A^2(T).$$

The quantity Ω_A depends on temperature, vanishing at $T = T_c$, and far from T_c it corresponds to a field of approximately 50 Gauss. Moreover, in the A phase there occurs also resonance absorption in a varying field parallel to the constant field H_0, at a frequency equal to $\omega_{A_\parallel} = \Omega_A(T)$. Absorption similarly occurs in the B phase at frequency $\omega_{B_\parallel} = \Omega_B(T)$, of the same order of magnitude as $\Omega_A(T)$. There is no frequency shift of the transverse resonance in the B phase.

The frequency shift in the A phase implies the presence of a preferred direction in the liquid, to which the spin system is locked. The theoretical treatment of NMR, as well as comparison with experimental data, will be carried out in the third section of this review.

e) Ultrasound attenuation

Measurements of ultrasound absorption in ^{3}He show a clearly expressed peak in transitions from the normal phase to both the A phase and the B phase of superfluid ^{3}He,[25,26] explained by a destruction mechanism of Cooper pairs.[15] In magnetic fields above 4 kOe a splitting of the absorption peak into two peaks is observed for the transition to the A phase,[25] from which it can be concluded that the phase transition is split into two transitions with temperatures T_{c1} and T_{c2} (see sub section 2a).

The existence of a strong dependence of the speed and attenuation of sound in the A phase on the mutual orientations of the sound propagation direction and the direction of the applied magnetic field was demonstrated experimentally,[27,28] and this along with NMR experiments implies anisotropy of the A phase.

Besides ultrasound absorption at frequency $\omega = 2\Delta/\hbar$ we have specific absorption in the superfluid phases of ^{3}He at lower frequencies, related to oscillations of the absolute value of the order parameter, i.e., structural oscillations of Cooper pairs (see reviews 15, 16). For propagating waves of this type in the B phase of ^{3}He a behavior was recently discovered characteristic of waves in nonlinear optical systems.[29,30]

f) Superfluid properties.

1) *Viscous damping of an oscillating wire.*[31] The first proof of superfluidity in ^{3}He was provided by experiments with an oscillating wire, in which the amplitude and resonance frequency of a tight wire in a chamber with liquid ^{3}He were measured. The oscillation amplitude in normal ^{3}He $a \sim (\rho\eta)^{-1/2}$ decreases with temperature, which corresponds to an increase in the viscosity of a normal Fermi liquid: $\eta \sim T^{-2}$. At the A transition the amplitude starts increasing, at the B transition it increases by a jump, and then increases more quickly than could be expected from the growth generated by the reduced viscosity, which in a superfluid Fermi liquid must behave near T_c as

$$\eta(T) = \eta(T_c)\left[1 - A\left(1 - \frac{T}{T_c}\right)^{1/2}\right]$$

(see Ref. 32).

The splitting of the A transition in strong magnetic fields was also verified by this method.[31c]

2) *Thermal fluxes.*[33,7,11] As is well known from the example of ^{4}He, the heat transfer mechanism in a

superfluid liquid is mostly due to the motion of the normal component, so that the total current satisfies $j = \rho_n v_n + \rho_s v_s = 0$. Measurements of heat resistance, i.e., the ratio of the temperature difference ΔT at the edges of a capillary to the heat flow ΔQ show that a hydrodynamic mechanism of heat transfer is also generated in liquid ^{3}He below the transition temperature T_c. A sharp increase is also observed experimentally in the heat resistance for a given thermal flux in superfluid ^{3}He at temperatures near T_c. This effect is explained within the two-fluid model by the fact that the thermal flux is proportional to the momentum of the normal component $\Delta Q \sim \rho_n v_n \sim -\rho_s v_s$, and conservation of ΔQ for $T \to T_c$, i.e., for $\rho_s \to 0$, is possible only for increasing v_s. A disruption of the laminar superfluid flow occurs at some temperature $T < T_c$ when a critical velocity value v_s^c is reached. Critical velocity values thus measured near T_c (see Ref. 33, as well as Refs. 7, 11) are near 0.5 cm/sec in ^{3}He-B, and near 0.1 cm/sec in ^{3}He-A.

3) *Fourth sound.*[34,35,7] A characteristic property of a superfluid liquid is the fourth sound, propagating in capillaries and pores filled by helium, whose sizes are smaller than the viscous penetration depth of the wave $\lambda = (\eta/\omega\rho_n)^{1/2}$. The speed of fourth sound is proportional to the square root of the density of the superfluid component $c_4 \sim \rho_s^{1/2}$. Measurements of c_4 near T_c are in good agreement with the theoretical behavior $\rho_s/\rho \sim \{1 - (T/T_c)\}$.

4) *Critical velocity and critical current.* Measurements of the period and damping of the oscillatory motion of superfluid ^{3}He through a narrow orifice inside a toroidal channel gave critical velocity values in both phases of ^{3}He of the order of several millimeters a second.[36] Similar values of v_c were observed by measurements of transverse NMR frequency shifts in channels with moving ^{3}He-A.[37] The critical velocities of superfluid flow in a capillary, thus found, differ strongly from the Landau critical velocity, at which excitations start being created in the superfluid liquid. The Landau critical velocity can be measured by the motion of charged ions in ^{3}He. More precisely, the velocity of moving ions is proportional to the applied field $v = \mu eE$. For low fields the mobility coefficient μ is determined by the ion deceleration due to collisions with excitations in the superfluid liquid. In high fields the dominant contribution to the slowdown is provided by the mechanism of breakdown of Cooper pairs. A change in slope in the dependence of velocity on the applied field occurs at speeds of the order of the Landau critical velocity $v_c(T) = \Delta(T)/p_F$ ($v_c(0) = 10$ cm/sec), which is satisfied experimentally quite well.[38,39]

The temperature dependence of the critical current through thin capillaries in the B phase of ^{3}He was also found experimentally,[40] being in complete agreement with the law $j_c \sim \{1 - (T/T_c)\}^{3/2}$, following from the Ginzburg-Landau theory.

5) *"Andronikashvili" type experiments.*[41-45] The viscosity and density of the normal component of both the A and B phase were measured[41-45] by the torsional oscillation method, first used by Andronikashvili in superfluid ^{4}He. The results obtained are in fairly good agreement with theoretical predictions (a discussion and references to numerous theoretical papers can be found in a review of Ref. 16). In the following section we will address the problem of the temperature dependence of $\rho_n(T)$ in the A and B phases of ^{3}He. We note here only that in the B phase $\rho_n(T)$ is a scalar function, as in normal superfluid ^{4}He, while in the A phase the quantity $\rho_{ij}^n(T)$ is a tensor, i.e., it depends on the directions of the anisotropy axes of the liquid.

6) *Second sound.* As is well known, the propagation of temperature waves, second sound, being density oscillations of the normal and superfluid components originating in the opposite phase, is possible in superfluid liquids. So far it has not been possible to observe the second sound signal in the A and B phases due to the large value of damping α ($c_2^2 = \rho_s T S^2/\rho_n C \simeq 1$ cm^2/sec^2),

$$\alpha = \frac{\omega^2}{2\rho c_2^3}\left[\frac{\rho_s}{\rho_n}\left(\frac{4}{3}\eta + \zeta\right) + \frac{k}{C}\right] \approx 10^5 \text{ cm}^{-1}$$

at frequency $\omega \simeq 1$ kHz. In the A_1 phase, however, the superfluid component of which consists of particles with spin oriented along the external field, and whose normal component has oppositely oriented spins, second sound is simply oscillations of magnetization, propagating with velocity $c_2^2 = (\rho_\perp^s \rho/\rho_\perp^n \chi)(\gamma\hbar/2m)^2 \approx 1$ m^2/c$^2 \cdot 10^2 [1 - T/T_{c1}]$ (see Refs. 46, 47), where χ is the magnetic susceptibility, and γ is the gyromagnetic ratio. The measurements of this quantity[48] are in excellent agreement with the dependence given. The sound signal disappears upon transition to the A phase ($T = T_{c_2}$). It was established from the polarization of the transient signal in this experiment that the superfluid component of the A_1 phase indeed has a magnetization along the external field, though the opposite orientation of pair spins is also theoretically possible, since the density of states at the Fermi surface could also decrease with increasing Fermi level in a real Fermi liquid (see subsection 2b).

With this we conclude the brief list of basic experimental results for the superfluid phases of ^{3}He. Several experiments will be described after the introduction of the necessary theoretical concepts. To these belong: linear and nonlinear NMR in the A and B phases, experiments related to the anisotropic properties of the A phase, the measurement of the normal density tensor, and orbital magnetism.

For more detail on experimental results we refer to reviews of Refs. 6–11, papers in collections of Refs. 17, 18, as well as the proceedings of the last two low-temperature conferences LT-15 and LT-16.[49,50]

For experiments related to the dynamics of orbital degrees of freedom in ^{3}He-A see Volovik's review of Ref. 19.

3. STRUCTURE OF THE SUPERFLUID PHASES OF ^{3}He

a) The order parameter in the A and B phases of ^{3}He

Magnetic susceptibility measurements (see Section 1) justify the hypothesis that Cooper pairing of ^{3}He atoms occurs in states with $S = 1$. According to the general principles of quantum mechanics the angular momentum of Cooper pairs with $S = 1$ can acquire only odd values $L = 1, 3, 5, \ldots$ Cooper pairing with $L = 1$ is realized in superfluid ^{3}He, because any other angular momentum value would make the existence of an isotropic B phase impossible.

Indeed, the wave function ψ of a pair with spin 1 can be expanded in components of vector spin wave functions $\chi = (\chi_1, \chi_0, \chi_{-1})$, where

$$\left.\begin{aligned} \chi_1 &= |\uparrow\uparrow\rangle = \frac{\nu_\xi + i\nu_\eta}{\sqrt{2}}, \quad S_\zeta = 1, \\ \chi_0 &= |\uparrow\downarrow + \downarrow\uparrow\rangle = \nu_\zeta, \quad S_\zeta = 0, \\ \chi_{-1} &= |\downarrow\downarrow\rangle = \frac{\nu_\xi - i\nu_\eta}{\sqrt{2}}, \quad S_\zeta = -1 \end{aligned}\right\} \tag{1}$$

are the eigenfunctions of the projection of the spin operator $\hat{S}_\zeta$ on some preferred direction ζ, and ν is a unit vector in spin space with coordinate axes (ξ, η, ζ). The expansion of ψ, linear in the components of ν, can be written in the form

$$\psi = \sqrt{3}\, \mathbf{d}\nu. \tag{2}$$

Here the expansion coefficients d_i are linear combinations of the eigenfunctions $Y_{LM}(\theta, \phi)$ of the projection operator of orbital angular momentum L_z. The average over the spin variable

$$\int \frac{d\Omega_\nu}{4\pi} \psi^*\psi = \mathbf{d}^*\mathbf{d}$$

is the probability density of finding a particle at the point with angular coordinates θ and ϕ. In an isotropic state, however, $\mathbf{dd}^*$ is independent of angle, and this can happen only if the three components of the vector $\mathbf{d}$ form an irreducible representation. The rotational group has only one three-dimensional irreducible representation, whose basis are the eigenfunctions of the angular momentum projection operator L_z with $L = 1$, i.e., the spherical harmonic functions $Y_{LM}(\theta, \phi)$:

$$\left.\begin{aligned} \frac{\sin\theta e^{i\varphi}}{\sqrt{2}} &= \frac{n_x + in_y}{\sqrt{2}}, \quad M = 1 \\ \cos\theta &= n_z, \quad M = 0, \\ \frac{\sin\theta e^{-i\varphi}}{\sqrt{2}} &= \frac{n_x - in_y}{\sqrt{2}}, = \quad M = -1. \end{aligned}\right\} \tag{3}$$

Here $\mathbf{n} = (n_x, n_y, n_z)$ is a unit vector, directed from one particle in the Cooper pair to the other.[1)]

Thus, as components of the spin vector d one can choose the functions $Y_{1M}(\theta, \phi)$, or simply the components of the unit vector $\mathbf{n}$, i.e., in the B phase $\mathbf{d} = \Delta(T)\mathbf{n}$, where $\Delta(T)$ is a temperature dependent factor. In the general case [retaining the requirement $\mathbf{d}^*\mathbf{d} = \Delta^2(T)$] the spin space can be rotated, so that for the B phase

$$d_i^B = \Delta(T) R_{ik} n_k e^{i\Phi}, \tag{4}$$

where R_{ik} is an arbitrary three-dimensional rotation matrix (in the absence of spin-orbit interaction), and $e^{i\Phi}$ is a phase factor. The matrix

$$A_{ik}^B = \Delta(T) R_{ik} e^{i\Phi}, \tag{5}$$

common for all Cooper pairs, is an order parameter in the B phase of ^{3}He.

It is easily verified that in the isotropic B phase the mean values of the projection operator of the pair spin and angular momentum on any axis vanish. Indeed, the pair spin operator $\hat{S}$ is a generator of rotations in spin space:

$$\hat{S}_i d_j = -ie_{ijk} d_k. \tag{6}$$

Consequently, $(d_j^B)^* \hat{S}_i d_j^B = 0$. We also have for the angular momentum operator of the pair

$$\hat{L} = \left[\frac{\mathbf{n}}{i}\frac{\partial}{\partial \mathbf{n}}\right], \quad \int (d_j^B)^* \hat{L}_p d_j^B \, d\Omega = 0. \tag{7}$$

On the other hand, it is easily verified that the wave function (2), (4) is an eigenfunction of the operators $\hat{\mathbf{S}}^2$ and $\hat{\mathbf{L}}^2$ with eigenvalues $S(S+1) = L(L+1) = 2$.

We turn now to the A phase of ^{3}He. Magnetic susceptibility measurements (see Section 1) show that there exists in the A phase Cooper pairing with $S = 1$ and two equally probable spin projections $S_\zeta = \pm 1$ in the external field direction. The wave function of this state must be of the form $\psi^A \sim (\chi_1 + e^{i\Phi}\chi_{-1})$. Due to the equal probability of the ± 1 spin projections, the coefficients of χ_1 and χ_{-1} can differ from each other only by an arbitrary phase factor $e^{i\Phi}$. Using expression (1), we obtain for the pair wave function (2) of the A phase

$$\psi^A = \sqrt{3} f(\mathbf{n}) \mathbf{V}\nu, \quad \mathbf{d}^A = f(\mathbf{n})\mathbf{V}, \tag{8}$$

where V is a unit spin vector, perpendicular to the spin quantization axis ζ ($V_\xi = \cos(\Phi/2)$, $V_\eta = \sin(\Phi/2)$), and $f(\mathbf{n})$ is the coordinate wave function of an A phase pair, whose absolute value squared $|f(\mathbf{n})|^2$ must contain information on the ansiotropy of the A phase. As we already know (see subsection 2c), the A and B phases have an identical critical temperature T_c; consequently, Cooper pairing in the A phase, as well as in the B phase, occurs in the p-state ($L = 1$). The experimental data (primarily NMR) are explained very well by assuming that $f(\mathbf{n})$ is proportional to the spherical harmonic $Y_{11}(\theta, \phi)$, the eigenfunction of the angular momentum projection operator $\hat{L}_z$ with eigenvalue $M = 1$ (see Eq. 3).

In the general case of arbitrary orientation of the coordinate axes the coordinate wave function of a pair in the A phase of ^{3}He is

$$f(\mathbf{n}) = \frac{1}{\sqrt{2}} \Delta(T) (\Delta' + i\Delta'', \mathbf{n}), \tag{9}$$

where Δ' and Δ'' are two orthogonal unit vectors, and $[\Delta'\Delta''] = \mathbf{l}$ specifies the direction of the quantization axis of angular momentum or the anisotropy axis of the A phase. The square of the absolute value of the spatial wave function of the A phase is $|f(\mathbf{n})|^2 = (1/2)\Delta^2(T)[\mathbf{nl}]^2$, so that Cooper pairs in the A phase are flattened in the direction of the anisotropy axis $\mathbf{l}$.

The matrix

[1)] By $\mathbf{n}$ we can also denote the common direction of opposite momenta of particles in a Cooper pair.

$$A_{ik}^{A}=\frac{1}{\sqrt{2}}\Delta(T)V_i(\Delta_k'+i\Delta_k''), \tag{10}$$

common for all Cooper pairs, is the order parameter in the A phase of ^{3}He. We note that in the A_1 phase, where pairing occurs only between particles and field spins (see Section 2), the wave function of the pair is $\psi\sim\chi_1$. Therefore, taking into account (1) and (10), we obtain the order parameter in the A_1 phase

$$A_{ik}^{A_1}=\frac{1}{2}\Delta(T)(V_i'-iV_i'')(\Delta_k'-i\Delta_k''),$$

where $\mathbf{V}'$ and $\mathbf{V}''$ are unit orthogonal vectors, perpendicular to the spin quantization axis ζ.

Usually, instead of expanding in terms of the vector wave functions (1), (2) in spin space, one uses an expansion in components of a symmetric spinor, i.e., a basis of symmetric, unitary, two-row matrices

$$\hat{\psi}=i(\hat{\sigma}\mathbf{d})\sigma_y, \tag{11}$$

where the vector

$$d_l=A_{ik}n_k=-\frac{i}{2}\mathrm{Sp}(\hat{\sigma}_y\sigma_l)_{\alpha\beta}\psi_{\alpha\beta}, \tag{12}$$

is already familiar, $\hat{\sigma}=(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z)$ are the Pauli matrices, and $\hat{\psi}$ is an abbreviated notation for the spinor $\psi_{\alpha\beta}$. It is useful to reduce the expressions for the order parameters of the A and B phases of ^{3}He to the notation of Eq. (11). From (4) we have (for $R_{ik}=\delta_{ik}$)

$$\hat{\psi}^{B}=\Delta(T)e^{i\Phi}\begin{pmatrix}-n_x+in_y & n_z\\ n_z & n_x+in_y\end{pmatrix}, \tag{13}$$

and from (8) [for $V=(0,0,1)$]

$$\hat{\psi}^{A}=f(\mathbf{n})\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \tag{14}$$

where $f(\mathbf{n})$ is given by expression (9).

The notation (11), (12) is convenient in many respects; thus, for example, it follows from (6) and (11) that

$$\hat{\mathbf{S}}\hat{\psi}=[\sigma\mathbf{d}]\sigma_y, \tag{6a}$$

whence it is seen that in either phase $\mathbf{d}$ is orthogonal to the pair spin $\hat{\mathbf{S}}$.

b) The normal density and susceptibility of the A and B phases of ^{3}He

The energy of single-particle excitations in superfluid Fermi liquids, where the order parameter $\hat{\psi}$ of (11) is a unitary matrix (to which, as is easily seen, the A and B phases belong), is determined by the equation

$$E=\sqrt{\xi^2+\frac{1}{2}\mathrm{Sp}\,\hat{\psi}\hat{\psi}^{+}}=\sqrt{\xi^2+\mathbf{dd}^*}, \tag{15}$$

where ξ, as usual, is the energy of normal Fermi quasiparticles, measured from the Fermi level. Using (13) and (14), we obtain in the B and A phases, respectively, $E^{B}=\sqrt{\xi^2+\Delta^2(T)}$ and $E^{A}=\sqrt{\xi^2+(1/2)\Delta^2(T)[\mathbf{nl}]^2}$, whence it is clear that the A phase possesses anisotropic, and the B phase—isotropic physical properties.

Thus, for example, the normal component density, defined as

$$\rho_{ij}^{n}v_j^{n}=\sum_{\mathbf{k},\sigma}k_i f(E-\mathbf{kv}^{n})|_{v^{n}\to 0}=-\sum_{\mathbf{k},\sigma}k_ik_j\frac{\partial f}{\partial E}v_j^{n},$$

where f is the Fermi quasiparticle distribution function, is isotropic in the B phase

$$\rho_{ij}^{nB}=\rho Y(T)\delta_{ij}, \tag{16}$$

and is a tensor in the A phase:

$$\rho_{ij}^{nA}=\rho[Y^{\parallel}(T)l_il_j+Y^{\perp}(T)(\delta_{ij}-l_il_j)]; \tag{17}$$

where $Y(T)=\int d\Omega Y(\mathbf{n},T)/4\pi$ is the Yoshida function, equal to 0 for $T=0$, and 1 for $T=T_c$:

$$Y(\mathbf{n},T)=\frac{1}{4T}\int_{-\infty}^{+\infty}d\xi\,\mathrm{sech}^2\frac{E}{2T},$$
$$Y''(T)=3\int\frac{d\Omega}{4\pi}(\mathbf{nl})^2Y(\mathbf{n},T),\quad Y^{\perp}(T)=Y(T)-Y^{\parallel}(T). \tag{18}$$

Fermi liquid corrections to the normal densities in the A and B phases were not taken into account in Eqs. (16), (17). The corresponding expressions can be found in the reviews of Refs. 13, 16.

Near the walls of the ^{3}He-A container the direction $\mathbf{l}$ coincides with the normal to the wall (see subsection 3g). Far from the walls the direction of $\mathbf{l}$ can vary with the magnetic field (see subsection 3e). Measurements of the superfluid component density of the A phase, $\rho_{ij}^{s}=\rho\delta_{ij}-\rho_{ij}^{n}$, carried out by the Andronikashvili method,[42,43] have confirmed the tensor nature of this quantity.

We now find expressions for the magnetic susceptibility of the A and B phases. The calculation is conveniently performed in the general case (15).

We choose the direction of the pair spin quantization along the z-axis; the vectors $\mathbf{d}(\mathbf{n})$ are then located in a plane perpendicular to this axis (see the end of the preceding subsection). If the external magnetic field $\mathbf{H}$ is also perpendicular to the z-axis, for example, $\mathbf{H}\parallel\mathbf{d}(\mathbf{n})$, Cooper pairs will not react to it, and a contribution to the susceptibility will be provided only by Bogolyubov excitations with energy

$$E^{\perp}=\sqrt{\xi^2+\mathbf{dd}^*}-\sigma\mu H,$$

where μ is the nuclear magneton, and $\sigma=\pm1$. We note that the vector $\mathbf{d}$ depends on the momentum direction $\mathbf{n}$ on the Fermi surface, therefore the susceptibility must be calculated separately for each direction $\mathbf{n}$. Thus, the susceptibility in directions perpendicular to the direction of the spin quantization axis is defined as

$$\chi_n^{\perp}=\frac{\mu N_0}{2}\sum_{\sigma}\int d\xi\left(\frac{\partial}{\partial H}f(E^{\perp})\right)_{H=0}, \tag{19}$$

where N_0 is the density of states at the Fermi surface, and f is the Fermi distribution function of Bogolyubov quasiparticles. Expression (19) can be rewritten in the form

$$\chi_n^{\perp}=\chi_N Y(\mathbf{n},T), \tag{20}$$

where $\chi_N=\mu^2N_0/2$ is the susceptibility of the normal Fermi liquid, and the quantity $Y(\mathbf{n},T)$ was defined in (18).

If the field $\mathbf{H}$ is directed along the spin quantization axis z, all particles (i.e., both pairs and excitations) contribute:

$$\chi_n^{\parallel}=\frac{\mu N_0}{2}\sum_{\sigma}\int d\xi\left(\frac{\partial}{\partial H}n(E^{\parallel})\right)_{H=0}=\chi_N; \tag{21}$$

where

$$n(E)=\frac{1}{2}\left[1-\frac{\xi}{E}(1-2f(E))\right]$$

is the particle distribution function, and

$$E^{\mathrm{II}}=\sqrt{(\xi-\sigma\mu H)^2+\mathbf{dd}^*}.$$

Thus, χ_n^{II} coincides with the susceptibility of the normal Fermi liquid.

The full expression for the susceptibility tensor is obtained by integrating the combination of (20) and (21) over all directions **n**:

$$\chi_{ij}=\int\frac{d\Omega}{4\pi}\left[\left(\delta_{ij}-\frac{d_i^*d_j}{\mathbf{d}^*\mathbf{d}}\right)\chi_n^{\mathrm{II}}+\frac{d_i^*d_j}{\mathbf{d}^*\mathbf{d}}\chi_n^{\perp}\right], \tag{22}$$

whence, using expressions (4) and (8), we obtain for the susceptibilities in the A and B phases:

$$\chi_{ij}^{\mathrm{A}}=\chi_N[\delta_{ij}+V_iV_j(Y(T)-1)], \tag{23}$$

$$\chi_{ij}^{\mathrm{B}}=\chi_N\delta_{ij}\left[\frac{2}{3}+\frac{1}{3}Y(T)\right]; \tag{24}$$

where $Y(T)=\int d\Omega Y(\mathbf{n},T)/4\pi$ is the Yoshida function. As could be expected, the susceptibility of the A phase depends on direction. We note, however, that since χ^{A} is maximum in directions perpendicular to **V**, the equilibrium position of **V** is perpendicular to the external field, while the experimentally measured χ in the free geometry always coincides with χ^{N}. More accurate expressions for the susceptibilities in the A and B phases, including Fermi liquid corrections, are (see Refs. 13, 16):

$$\chi_{ij}^{\mathrm{A}}=\chi_N\left\{\delta_{ij}+V_iV_j\frac{(Y(T)-1)[1-(Z_0/4)]}{1+(Z_0Y(T)/4)}\right\}, \tag{25}$$

$$\chi_{ij}^{\mathrm{B}}=\chi_N\delta_{ij}\frac{[(2/3)+(Y(T)/3)][1+(Z_0/4)]}{1+(Z_0/4)[(2/3)+(Y(T)/3)]}; \tag{26}$$

where Z_0 is the Fermi liquid constant.

We note that the energy difference of the A and B phases near T_c is of the order of the condensation energy, i.e., $(F^{\mathrm{A}}-F^{\mathrm{B}})\sim\{1-(T/T_c)\}^2$ (see Ref. 13), so that the difference of magnetic energies of the A and B phases in a magnetic field is

$$F_m^{\mathrm{A}}-F_m^{\mathrm{B}}=-\frac{H^2}{2}(\chi^{\mathrm{A}}-\chi^{\mathrm{B}})\sim-\frac{\Delta^2(T)}{T_c^2}H^2\chi_N\sim-\left(1-\frac{T}{T_c}\right)H^2\chi_N.$$

Therefore, as already noted in Section 1, in the presence of a magnetic field the A phase is always energetically more favorable than the B phase in a sufficiently close neighborhood of the transition temperature.

c) General problem of the phases of p-pairing in ^{3}He

As has already been mentioned, the order parameter in a superfluid Fermi liquid with p-pairing is a complex 3×3 matrix A_{ik} (see Eq. (12), depending in the general case on 18 real parameters. Expressions for A_{ik} in the B and A phases are given by Eqs. (5) and (10). The matrix A_{ik} transforms as a vector in the first subscript for rotations in spin space, and as a vector in the second subscript for rotations in coordinate space. The specific shape of A_{ik} for a given phase is determined by minimizing the A_{ik}-dependent energy. In the absence of spin-orbit interaction, which, as we will see below, is extremely small, the free energy of ^{3}He is real and independent of the mutual orientation of the quantization axes in the spin and coordinate spaces. Consequently, it should not change under transformations of the order parameter of the type

$$A_{ik}\to A_{pq}=R_{pi}^sR_{qk}^ce^{i\Phi}A_{ik}; \tag{27}$$

where R_{pi}^s and R_{qk}^c are, respectively, the matrices of three-dimensional rotations of spin and coordinate space, and $e^{i\Phi}$ is a gradient transformation. Near the phase transition, where A_{ik} is small, the Ginzburg-Landau expansion in powers of the order parameter can be used for the free energy:

$$F_{\mathrm{cond}}=-\alpha A_{ik}A_{ik}^*+\beta_1|A_{ik}A_{lk}|^2+\beta_2(A_{ik}A_{ik}^*)^2+\beta_3A_{ik}A_{il}^*A_{mk}A_{ml}^*+\beta_4A_{ik}A_{il}^*A_{mk}^*A_{ml}+\beta_5A_{ik}A_{il}A_{mk}^*A_{ml}^*. \tag{28}$$

The formulated invariance requirements are satisfied here because the first subscript of A_{ik} in (28) is contracted only with the first, the second—with the second, and F_{cond} is real. The three-dimensional rotations form the group SO_3, and the gradient transformations form the group U(1), equivalent to the group of one-dimensional rotations, or the circle S^1. Thus, the full group G, with respect to which the free energy of superfluid ^{3}He is invariant, is the direct product $SO_3\times SO_3\times U(1)$.

The functional (8) has several extrema, determined by the equations $\delta F_{\mathrm{cond}}/\delta A_{ik}=0$. Which of the extrema are minima, and which of the minima is absolute, depends on the relations between the coefficients β_i, which, in turn, depend on temperature and pressure. The problems of listing all minima, and then choosing from among them the absolute minimum, are complicated and have been solved only partially.[51-55]

The calculation of the coefficients β_i from the microscopic theory is possible only in the "weak coupling" approximation (see Ref. 51a) and is not particularly useful, since in this case the absolute minimum in the whole temperature and pressure region is realized for the order parameter (5), corresponding to the B phase.[3,13] In calculations within the "weak coupling" approximation, $\Delta\ll\varepsilon_F$, the interaction between Fermi quasiparticles is assumed to be independent of the type of pairing. However, as was first shown by Anderson and Brinkman[56] (see also Refs. 13, 21, 57–60), the mechanism of Cooper pairing in ^{3}He is exchange of spin density fluctuations, i.e., paramagnons. The spin susceptibility, as well as the effective pair interaction in the A phase is larger than in the B phase, which makes the A phase energetically more favorable than the B phase in the region of high temperatures and pressures.

d) Degeneracy spaces of the A and B phases

Many, primarily superfluid,[19] properties of superfluid liquids result from the structure of the region of variation of the order parameters, having in the A and B phases the form [see Eq. (10) and (5)]:

$$A_{ik}^{\mathrm{A}}=\sqrt{\frac{1}{2}}\Delta(T)V_i(\Delta_k'-i\Delta_k''),$$

$$A_{ik}^{\mathrm{B}}=\Delta(T)R_{ik}e^{i\Phi}.$$

Among all transformations (27), which keep the free energy of the system invariant, only a part varies the

order parameter. Thus, in the A phase these are the three-dimensional rotations of coordinate space, which alter the positions of the triplet of vectors $(\Delta', \Delta'', \mathbf{l})$ and form the SO_3 group. Among the spin rotations only those are important for the A phase, which vary the orientation of the vector $\mathbf{V}$, spanning the two-dimensional sphere S^2, and rotations around the direction of $\mathbf{V}$ are unimportant. The gradient transformations of (27) are equivalent for the A phase to the rotations Δ' and Δ'' around the direction of $\mathbf{l} = [\Delta'\Delta'']$, therefore they have already been taken into account. Moreover, A^{A}_{ik} does not change under discrete transformations of the form $(\mathbf{V}, \Delta', \Delta'') \to (-\mathbf{V}, -\Delta', -\Delta'')$, and consequently, the points of the region of variation of the order parameter of the A phase, going over into each other under such transformations, are equivalent. The operation of identification ("gluing together") of points of a set according to this rule is called factorization. In the given case we have a factorization of the direct product $S^2 \times SO_3$ in terms of the set of two points Z_2, denoted by $(S^2 \times SO_3)/Z_2$. Thus, the complete region of variation of the order parameter of the A phase is $R^A = (S^2 \times SO_3)/Z_2$. At all points of this region the functional (28) assumes the same value which is the absolute minimum in a certain region of the phase diagram. Thus, as is usually the case in second order phase transitions, the minimum energy is realized in the whole region of variation of the order parameter, which it is naturally to call the degeneracy space. It is easily seen that for the A phase the degeneracy space R^A is five-dimensional, and the 5 variables parametrizing R^A are the Goldstone variables. We recall that in a superconductor, or in the normally superfluid ^{4}He, where the order parameter is a complex function $\psi = |\psi| e^{i\Phi}$, the minimum of the condensation energy is realized on the circle S^1, the region of variation of the phase factor $e^{i\Phi}$. It is well known that the stable singularities, quantized vortices in ^{4}He, correspond to closed contours, traversing the circle S^1, the degeneracy space of ^{4}He, an integral number of times. It can be shown similarly that the stable singularities in ^{3}He-A correspond to closed contours in the five-dimensional space $(S^2 \times SO_3)/Z_2$, which cannot be contracted into a point.[61-64]

In the B phase the region of variation of the factor $e^{i\Phi}$ is the circle S^1, while the region of variation of the real orthogonal matrix R_{ik} is the group of three-dimensional rotations SO_3. Thus, the complete degeneracy space of the B phase is $R_B = S^1 \times SO_3$.[61-64]

e) Spin-orbit interaction

1) *Dipole energy*. The spin-orbit interaction in electrically neutral ^{3}He results from the magnetic dipole interaction of the nuclear spins of particles:

$$H_D = \frac{\gamma^2\hbar^2}{2} \int d^3r_1 d^3r_2 \left[\frac{\sigma(\mathbf{r}_1)\sigma(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - \frac{3(\sigma(\mathbf{r}_1)(\mathbf{r}_1 - \mathbf{r}_2))(\sigma(\mathbf{r}_2)(\mathbf{r}_1 - \mathbf{r}_2))}{|\mathbf{r}_1 - \mathbf{r}_2|^5} \right]; \quad (29)$$

where γ is the gyromagnetic ratio. The dipole energy F_D is obtained by averaging Eq. (29) using the apparatus of second quantization (see Ref. 13). However, to find the form of the energy of the mutual orientation of the spin and coordinate axes, it is sufficient to solve the simpler quantum-mechanical problem for two particles with spin $\frac{1}{2}$ and wave function (11) $\hat{\psi} = i(\hat{\sigma}\mathbf{d})\hat{\sigma}_y$, interacting according to

$$\hat{V} = 2g_D(\mathbf{S}^1\mathbf{S}^2 - 3(\mathbf{S}^1\mathbf{n})(\mathbf{S}^2\mathbf{n})); \quad (30)$$

where $\mathbf{n}$ is the direction of the vector from the particle with spin $\mathbf{S}^1$ to the particle with spin $\mathbf{S}^2$. The dipole energy is, obviously,

$$F_D = \frac{1}{2} \mathrm{Sp} \int \frac{d\Omega}{4\pi} \hat{\psi}^+ \hat{V} \hat{\psi}; \quad (31)$$

where $\hat{\psi}$ is determined by expression (11).

To calculate the trace it is sufficient to transform the operator $\hat{V}$ to the form

$$\left.\begin{aligned} \hat{V} &= g_D^{ij}\left(\hat{S}_i\hat{S}_j - \tfrac{1}{2}\delta_{ij}\right), \\ g_D^{ij} &= 2g_D(\delta_{ij} - 3n_in_j), \\ \hat{\mathbf{S}} &= \hat{\mathbf{S}}^1 + \hat{\mathbf{S}}^2. \end{aligned}\right\} \quad (32)$$

By means of relation (6a) the trace of (31) is now easily calculated:

$$F_D = g_D \int \frac{d\Omega}{4\pi} [3(\mathbf{dn})(\mathbf{d}^*\mathbf{n}) - \mathbf{dd}^*]. \quad (33)$$

Hence we obtain from (12) the expression

$$F_D = \frac{g_D}{5}\left(A^*_{ii}A_{jj} + A^*_{ij}A_{ji} - \frac{2}{3}A^*_{ij}A_{ij}\right), \quad (34)$$

in which the contractions of the space and spin subscripts appear directly. For A_{ij} in (34) we have in mind the normalized matrix $A_{ik}A^*_{ik} = 1$, whose absolute value has already been taken into account in the dipole interaction constant g_D.

The magnitude of the dipole-dipole interaction constant g_D is defined[7] as the product of the number of spins per unit volume N by the dipole interaction energy of two spins $\gamma^2\hbar^2/a^3$ and by the fraction of orderd particles $(\Delta/\varepsilon_F)^2$. Taking into account that $Na^3 \simeq 1$ and that $N/\varepsilon_F = 2N_0$, where N_0 is the density of states at the Fermi surface, we obtain $g_D = (2\gamma\hbar N_0\Delta)^2$. An exact calculation[13] gives

$$g_D(T) = \frac{\pi}{2}\left(\frac{\gamma\hbar}{2} N_0\Delta \ln\frac{\varepsilon_F}{\Delta}\right)^2. \quad (35)$$

Near the critical temperature[13]

$$g_D(T) = 10^{-3}\left(1 - \frac{T}{T_c}\right) (\mathrm{erg/cm^3}), \quad (36)$$

while the condensation energy (28) is

$$F_{cond} \approx \left(1 - \frac{T}{T_c}\right)^2 (\mathrm{erg/cm^3}). \quad (37)$$

Therefore the spin-orbit interaction energy (34) is everywhere low in comparison with the condensation energy (37), except for a narrow region $\Delta T \simeq 10^{-6}$ K near the transition temperature, where, as shown in Ref. 65, the so-called vector phase of ^{3}He must exist. The presence of spin-orbit interaction must also lead to the appearance of nuclear ferromagnetism, though extremely small, of the A phase of ^{3}He with magnetic moment density[65]

$$M \approx 10^{-14}\left(1 - \frac{T}{T_c}\right)^{-1} (\mathrm{Gauss/cm^3})$$

(compare with electron ferromagnetism of the A phase, considered in the following subsection).

2) *The* A *phase.* Using the expression for the order parameter of the A phase (10), we obtain from (33)

$$F_D^A = \text{const} - \frac{3}{5} g_D(T) (\mathbf{Vl})^2. \quad (38)$$

Thus, in the A phase the dipole energy is minimum when $\mathbf{V} = \pm\mathbf{l}$. In the presence of a spin-orbit interaction the order parameter of the A phase (10) is

$$\tilde{A}_{ik}^A = \sqrt{\frac{1}{2}} \Delta(T) [\Delta'\Delta'']_i (\Delta_k' + i\Delta_k''). \quad (39)$$

The region of variation of the order parameter (39), the degeneracy space of the A phase in the presence of the dipole interaction $\tilde{R}_A$ coincides with the region of variation of the reference expression $(\Delta', \Delta'', \mathbf{l})$ the three-dimensional rotation group SO_3.

3) *The* B *phase.* Using for the real orthogonal matrix R_{ik}, i.e., the matrix of three-dimensional rotations of (5), a parametrization by means of the rotational angle θ around the direction $\hat{\omega}$:

$$R_{ik} = \delta_{ik} + (\hat{\omega}_i\hat{\omega}_k - \delta_{ik})(1 - \cos\theta) - e_{ikl}\hat{\omega}_l \sin\theta, \quad (40)$$

we obtain in the B phase from (34)

$$F_D^B = \text{const} + \frac{2}{5} g_D(T) \left(\frac{1}{2} + 2\cos\theta\right)^2. \quad (41)$$

The dipole energy of the B phase is minimum when the angle of rotation θ of the spin space relative to the coordinate space around an arbitrary axis (!) $\theta_0 = \arccos(-\frac{1}{4}) \simeq 104°$. The order parameter of the B phase in the presence of spin-orbit interaction is

$$\tilde{A}_{ik}^B = \Delta(T) e^{i\Phi} R_{ik}(\hat{\omega}, \theta_0). \quad (42)$$

To elucidate the geometric structure of the degeneracy space of the B phase in the presence of spin-orbit interaction we recall that the group of three-dimensional rotations SO_3, parametrized by the angle θ and the axis $\hat{\omega}$, is a sphere of radius π, whose points are the ends of the vectors $\theta\hat{\omega}$ of length θ, directed along $\hat{\omega}$, and drawn from the center of the sphere. Diametrically opposed points on the surface of this sphere correspond to the same rotation, since rotations by an angle π around oppositely directed axes coincide. The spin-orbit interaction fixes the length of the vectors $\theta\hat{\omega} = \theta_0\hat{\omega}$. The degeneracy space of the B phase in the presence of spin-orbit interaction is $\tilde{R}^B = S^1 \times S^2$,[63,64] where the circle S^1 is the region of variation of the phase factor $e^{i\Phi}$ in (42), and the sphere S^2 is the region of variation of the vector $\theta_0\hat{\omega}$ (see Fig. 4).

f) The superfluid phases in a magnetic field

1) *The* A *phase.* As follows from the expression for the magnetic susceptibility (25), the magnetic energy density of the A phase can be written in the form

$$F_{magn}^A = -\frac{1}{2}\chi_{ij}^A H_i H_j = \text{const} + \frac{\chi_a}{2}(\mathbf{VH})^2,$$
$$\chi_a = \chi_N \frac{(1 - Y(T))|1 + (Z_0/4)|}{1 + (Z_0/4) Y(T)}. \quad (43)$$

The energy (43) is minimum when the vector $\mathbf{V}$ is perpendicular to the field $\mathbf{H}$. Recalling that the spin-orbit interaction aligns the vectors $\mathbf{l}$ parallel or antiparallel to $\mathbf{V}$, we obtain that in the presence of a constant magnetic field ^{3}He-A acquires the so-called Leggett configuration: $\mathbf{l} \parallel \mathbf{V} \perp \mathbf{H}$. The degeneracy in this case is still not fully removed. Indeed, the vector $\mathbf{V}$ can rotate freely around the direction of $\mathbf{H}$ in the plane perpendicular to $\mathbf{H}$, and, moreover, the vectors Δ' and Δ'' can rotate around the direction $\mathbf{l} \parallel \mathbf{V}$. Consequently, in the presence of a spin-orbit interaction and a magnetic field the degeneracy space of the A phase is $\tilde{\tilde{R}}^A = S^1 \times S^1$.

Near T_c (see Ref. 13)

$$F_{magn}^A = 5 \cdot 10^{-7} \left(1 - \frac{T}{T_c}\right) H^2 \left(\frac{\text{erg}}{\text{cm}^3\ \text{Gauss}^2}\right). \quad (44)$$

Comparison of this quantity with the dipole energy (36) shows that near T_c the magnetic energy exceeds the dipole energy in fields above 30 Gauss.

2) *Orbital magnetism of the* A *phase.* It was shown by Leggett[66] that the presence of orbital angular momentum in each Cooper pair must lead to a shift in the electron terms of atoms, i.e., to the appearance of electron rotation around atoms (electron-rotation interaction), a consequence of which is a finite magnetic moment of each pair, directed for each pair in the common $\mathbf{l}$ direction. Thus, ^{3}He-A is a liquid orbital ferromagnet, and its Hamiltonian in a magnetic field must contain a term linear in the field, besides the quadratic one:

$$F_{magn}^A = \chi_a (\mathbf{lH})^2 - \lambda_m \mathbf{lH}.$$

An estimate gives[66]

$$\lambda_m \approx 10^{-2}\ (\text{Gauss}) \left(1 - \frac{T}{T_c}\right) \chi_N,$$

which exceeds by several orders the nuclear magnetic moment of the A phase, considered in the preceding section ($\chi_N \sim 10^{-8}$). We know (see subsection 2e) that the sound attenuation in the A phase depends strongly on the mutual orientation of the vector $\mathbf{l}$ and the direction of sound propagation.[15,16,27,28] The dependence of sound attenuation on the sign of the applied field was established precisely by this method,[67] and the quantity λ_m found was in fair agreement with Leggett's estimate.[66] We also note here that similarly to the magnetic moment in the A phase Leggett has theoretically predicted an electric dipole moment in the B phase, generated by the parity-violating weak electron-nuclear interaction.

3) *The* B *phase.* We turn now to the B phase in a magnetic field. The spin-orbit interaction in the B phase fixes the angle of rotation $\theta_0 = \arccos(-\frac{1}{4})$, but retains the degeneracy in directions of the rotation axis $\hat{\omega}$. In this case we have for the rotation matrix [see Eqs. (40), (42)]:

$$R_{ij} = \frac{1}{4}(-\delta_{ij} + 5\hat{\omega}_i\hat{\omega}_j - \sqrt{15}\, e_{ijk}\hat{\omega}_k). \quad (45)$$

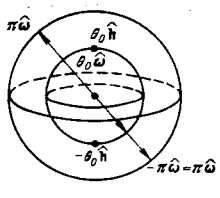

FIG. 4.

The susceptibility in the B phase (26) is isotropic, therefore the magnetic field does not exert a direct orienting action on the axis direction $\hat{\omega}$. An effect of orientation is generated only through corrections to the order parameter of the B phase due to the presence of a magnetic field, since any further interaction "deforms" the original phase (compare with the "unpairing" effect of a magnetic field in superconductors). Thus, to find the order parameter in the presence of a magnetic field, it is necessary to minimize the sum

$$F_{\text{cond}} + F_{\text{magn}}, \tag{46}$$

where the condensation energy F_{cond} is given by expression (28), and the magnetic energy F_{magn} can be written as

$$F_{\text{magn}} = \frac{\delta\chi}{2} A_{ik} A^*_{jk} H_i H_j; \tag{47}$$

where $\delta\chi = \chi_N - \chi^B$ [see Eqs. (22), (26)], and the matrix A_{ik} is normalized to unity, $A_{ik}A^*_{ik} = 1$. A minimum of (46) will be realized (see Refs. 69, 14) on the deformed B phase

$$\begin{aligned} A^B_{ik} &= \frac{e^{i\Phi}}{\sqrt{3}} (R_{ik} + \delta R_{ik}), \\ \delta R_{ik} &= \alpha H^2 R_{ik} - \beta H_i H_j R_{jk}, \end{aligned} \tag{48}$$

where α and β are constants of the order of $\delta\chi/F_{\text{cond}}$. The mutual orientation of H and $\hat{\omega}$ can be found by substituting (48) into expression (34) for the dipole energy:

$$F_{\text{magn}} = \text{const} - \frac{2}{5}\beta g_D (H_i R_{ij} H_j R_{kk} + H_i R_{ij} R_{jk} H_k). \tag{49}$$

Taking into account (45), we obtain from (49) (see Ref. 69):

$$F_{\text{magn}} = -g_D \beta (\hat{\omega}\mathbf{H})^2. \tag{50}$$

Calculations[69] give in the weak coupling limit the quantity $\beta g_D \simeq 4 \cdot 10^{-12}$ erg/cm^3 Gauss^2, independent of temperature.

Thus, in the B phase the equilibrium direction of the vector $\hat{\omega}$ is parallel or antiparallel to the external magnetic field. In the presence of a field the region of variation of the vector $\hat{\omega}$, the sphere S^2 (Fig. 4), is compressed to the two points Z_2, lying at the ends of the diameter of this sphere, directed parallel to the external field direction H. The degeneracy space of the B phase in a magnetic field is $\tilde{R}^B = S^1 \times Z_2$.[70]

g) The superfluid phases in an inhomogeneous state

1) *Gradient energy.* In the presence of inhomogeneities in superfluid ^{3}He the condensation (28), dipole (34), and magnetic (43), (50) energies must be supplemented by a gradient energy, whose density in the lowest quadratic order in the gradient has for $T \to T_c$ the general form:

$$F_{\text{grad}} = \frac{K_1}{2} \nabla_i A^*_{mj} \nabla_i A_{mj} + \frac{K_2}{2} \nabla_i A^*_{mj} \nabla_j A_{mi} + \frac{K_3}{2} \nabla_i A^*_{mi} \nabla_j A_{mj}. \tag{51}$$

Calculations in the weak coupling approximation[13] give for $T \to T_c$:

$$K_1 = K_2 = K_3 = \frac{21\zeta(3)}{80} \frac{\hbar^2 N}{m^* T_c^2}.$$

2) *Characteristic lengths.* Naturally, a restriction to corrections quadratic in the gradient is possible only in the case of sufficiently slow spatial variations of the order parameter, the characteristic scale of which is smaller than the coherence length $\xi(T)$. An expression for $\xi(T)$ is obtained, as usual, by comparing the condensation energy (28) with the gradient energy (51): $\xi(T) = \sqrt{K/\alpha} \sim \xi_0/\sqrt{1-(T/T_c)}$, where K is the common value of the coefficients K_i. Numerically ξ_0 can be found from the BCS estimate $\xi_0 = \hbar v_F/\Delta_0$. Since both the Fermi velocity v_F and the gap Δ_0 in ^{3}He are smaller by approximately 3 orders of magnitude than in superconductors, the coherence length in ^{3}He is of the same order of magnitude as in superconductors, $\xi_0 \simeq 10^{-5}$–10^{-6} cm.

Similarly comparing the gradient (51) and dipole (34) energies, we obtain the dipole length $\xi_D \simeq (K\Delta^2/g_D)^{1/2} \simeq (10^2$–$10^3)\xi_0$. Similarly comparing the gradient and magnetic energies (43) and (50), we find the magnetic length $\xi_m \simeq (K\Delta^2/F_m)^{1/2}$. In the A phase $\xi_m < \xi_D$ in fields stronger than 30 Gauss, and for the B phase $\xi_m > \xi_D$ for practically all fields and temperatures not too close to T_c.

3) *Currents.* An expression for the superfluid current is obtained from Eq. (51) by a Galilean transformation of the order parameter

$$A_{ij} \to \exp\left(-i\frac{2mru}{\hbar}\right) A_{ij}.$$

In the approximation linear in u we have

$$F_{\text{grad}} \to F_{\text{grad}} - \mathbf{j}^s \mathbf{u},$$

where

$$j^s_i = \frac{2m}{\hbar} \operatorname{Im} \{K_1 A^*_{mj} \nabla_i A_{mj} + K_2 A^*_{mj} \nabla_j A_{mi} + K_3 A^*_{mi} \nabla_j A_{mj}\}. \tag{52}$$

Substituting the expression for the order parameter in the A phase (10) into (52), we obtain[71]

$$\begin{aligned} \mathbf{j}^{sA} = \frac{2m}{\hbar} \Delta^2 \Big\{ \frac{2m}{\hbar} \Big[\Big(K_1 + \frac{K_{23}}{2}\Big) \mathbf{v}_s - \frac{1}{2} K_{23} \mathbf{l} (\mathbf{l}\mathbf{v}_s) \Big] \\ + \frac{K_3}{2} \operatorname{rot} \mathbf{l} - \frac{K_{23}}{2} \mathbf{l} (\mathbf{l} \operatorname{rot} \mathbf{l}) \Big\}, \end{aligned} \tag{53}$$

where

$$K_{23} = K_2 + K_3, \quad \mathbf{v}_s = \frac{\hbar}{2m} \Delta_i^* \nabla \Delta_i^*. \tag{54}$$

Similarly, in the B phase we have, using (52) and (5)

$$\mathbf{j}^{sB} = \frac{2m}{\hbar} \Delta^2 (3K_1 + K_{23}) \nabla\Phi. \tag{55}$$

It is hence seen that the superfluid current in the B phase is proportional to the superfluid velocity potential $\mathbf{v}_s = (\hbar/m)\nabla\Phi$, implying that the superfluid properties of the B phase do not differ from the superfluid properties of ordinary ^{4}He. As a consequence of the more complicated structures of the order parameter of the A phase we have a more complicated expression for the superfluid current (53). Volovik's review[19] is devoted to the unusual properties of the A phase.

Similarly one also finds the spin current:

$$j^\sigma_{ij} = -\frac{2}{\hbar} \frac{\partial F_{\text{grad}}}{\partial \nabla_i \theta_j}, \tag{56}$$

where θ_j is the angle of rotation of the vector $\mathbf{d}$ in spin space (for the corresponding expressions see Refs. 72, 14).

h) Interaction with the walls

1) *The* A *phase.* The boundary conditions for the order parameter of the A phase at the surface of the container of ^{3}He were obtained in Ref. 73, where it was shown that the longitudinal component A_{iz} of the order parameter A_{ik} (the z-axis is directed along the normal ν to the wall) changes significantly more strongly (it vanishes) than the transverse A_{ix}, A_{iy}. This implies that the surface energy of the A phase will be minimum if the order parameter $A^{A}_{ik} = \sqrt{1/2}\Delta(T) \times V_i(\Delta'_k + i\Delta''_k)$ has no longitudinal components at all, i.e., $\mathbf{l} = \boldsymbol{\Delta}' \times \boldsymbol{\Delta}'' = \pm\boldsymbol{\nu}$. The vector **V** is arbitrarily directed.

2) *The* B *phase.* In contrast to the anisotropic A phase, the presence of a preferred direction ν must alter the nature of the spherically symmetric B phase. The result of a treatment similar to that we carried out for the B phase in a magnetic field (see preceding subsection) gives[72,74,14] the boundary condition $\hat{\omega} = \pm\nu$.

Thus, in the presence of walls or external fields (as well as superfluid currents) the order parameter distribution in a container with superfluid ^{3}He can have a quite complicated structure. These distributions, which have been designated as textures, have been investigated intensely (see Refs. 14, 16, 19, 63, 64, 70, 72).

4. NMR IN SUPERFLUID ^{3}He

Studies of magnetic properties of ^{3}He have played the principal role in solving the problem of which model of Cooper pairing can be used to describe the observed A and B phases of superfluid ^{3}He. The principal contribution is due to A. Leggett,[75] who constructed the theory of spin dynamics of ^{3}He, on the basis of which the problem of phase identification was solved. Various NMR experiments and their interpretation within the Leggett theory are still the source of rich information on diverse properties of ^{3}He, such as relaxation of magnetization, spin waves, solitons and textures, second sound in the A_1 phase, and others. The two nonlinear Leggett equations for the vectors **S** and **d** (see below), having many qualitatively different solutions, are interesting also from a theoretical point of view, and their study is ongoing. In this section we derive the Leggett equations, and elaborate several experimental consequences of these equations.

a) The Legett equations and their simplest consequences in the A and B phases.

1) *The Leggett equations.* The dynamics of a spin system in a constant magnetic field $\mathbf{H}_0$ is described by the equation

$$\dot{\mathbf{S}} = \gamma [\mathbf{S}\mathbf{H}_0] \tag{57}$$

(here γ is the gyromagnetic ratio), from which it follows that the magnetization $\gamma\mathbf{S}$ precesses around the field direction with angular frequency $\omega_0 = \gamma H_0$. Therefore, if the system is placed in an external varying field $\mathbf{H}(t)$, perpendicular to $\mathbf{H}_0$, resonance absorption will be observed at the frequency ω_0.

As already noted (see subsection 2d) transverse field absorption is observed in superfluid ^{3}He at the frequency $\omega = (\omega_0^2 + \Omega_A^2(T))^{1/2}$, and, moreover, resonance absorption of longitudinal oscillations $\mathbf{H}(t) \| \mathbf{H}_0$ occurs at frequency $\Omega_A(T)$.[24] This implies that the spin system does not precess freely, but clings to a preferred direction inside the liquid. This direction, as we know, is the direction of the orbital angular momentum **l**. The engagement of the spin system occurs due to the spin-orbit interaction

$$F_D^A = -\frac{\chi\Omega_A^2}{2\gamma^2}(\mathbf{lV})^2, \tag{58}$$

which tends to align the vector spin wave functions $\mathbf{d} = (1/2^{1/2})\mathbf{V}(\boldsymbol{\Delta}' + i\boldsymbol{\Delta}'', \mathbf{n})$ in the direction parallel or antiparallel to **l**. Unlike (38), in (58) we used the notation $\chi\Omega_A^2/2\gamma^2 = (3/5)g_D$, the convenience of which becomes clear below. Thus, the equations of spin dynamics consist of equations of motion for the vectors **S** and **d**.

We note, to avoid confusion, that the notation $\gamma\mathbf{S}$ is used here for the magnetization generated under the action of an external magnetic field in the A and B phases of ^{3}He which do not have a spontaneous magnetic moment (we do not take into account the negligibly small orbital ferromagnetism of the A phase; see subsection 2e). Unlike the total spin of the Cooper pair, for which we also used in Section 3 the notation **S**, the total spin of the liquid per unit volume does not have to be perpendicular to the vector spin wave function **d**. The components of the total spin **S** are interrelated and related with the components of the vector **d** by the usual quantum-mechanical commutation relations:

$$[S_i, S_j] = ie_{ijk}S_k, \tag{59}$$

$$[S_i, d_j] = ie_{ijk}d_k. \tag{60}$$

Writing down the Hamiltonian of the system in an external field taking the spin-orbit interaction into account

$$\hat{H} = \frac{\gamma\mathbf{S}^2}{2\chi} - \gamma\mathbf{S}\mathbf{H} + F_D, \tag{61}$$

where in the general case (see Eq. 33)

$$F_D = 3g_D(T)\int\frac{d\Omega}{4\pi}|\mathbf{n}\,\mathbf{d}(\mathbf{n})|^2, \tag{62}$$

and χ is the magnetic susceptibility, we obtain with the aid of (59)–(62) the Leggett equations[75]:

$$\begin{aligned}\dot{\mathbf{S}} &= \gamma[\mathbf{S}\mathbf{H}] + \mathbf{R}_D,\\ \dot{\mathbf{d}}(\mathbf{n}) &= \gamma\left[\mathbf{d}(\mathbf{n})\left(\mathbf{H} - \frac{\gamma\mathbf{S}}{\chi}\right)\right];\end{aligned} \tag{63}$$

where

$$\mathbf{R}_D = -\int\frac{d\Omega}{4\pi}\left[\mathbf{d}(\mathbf{n})\left(\frac{\delta F_D}{\delta\,\mathbf{d}(\mathbf{n})} + \text{c.c.}\right)\right] \tag{64}$$

is the moment of dipole forces.

Equations (63) are, naturally, valid in the frequency region $\omega \ll \Delta/\hbar$, for which the absolute value of the order parameter remains constant, and only the angle-dependent spin part of the order parameter varies with time.

2) *The* A *phase.* In the case of the A phase $\mathbf{d}(\mathbf{n}) = \mathbf{V}(\boldsymbol{\Delta}' + i\boldsymbol{\Delta}'', \mathbf{n})/2^{1/2}$, and Eqs. (63) acquire the form

$$\dot{\mathbf{S}} = \gamma[\mathbf{SH}] + \frac{\chi\Omega_A^2}{\gamma^2}[\mathbf{VI}](\mathbf{VI}),$$
$$\dot{\mathbf{V}} = \gamma\left[\mathbf{V}\left(\mathbf{H} - \frac{\gamma\mathbf{S}}{\chi}\right)\right]. \quad (65)$$

In Eqs. (65) the motion of spin variables is considered for frozen orbitals $\mathbf{l}$ = const. (the adiabatic hypothesis), since τ, the relaxation time of $\mathbf{l}$, is very long (see the review of Ref. 19) in comparison with the reciprocal of the magnetization precession frequencies: $\omega\tau > 10^3$.

The spin precession around the field direction $\mathbf{H}_0$, parallel to the z-axis, is described, for small transverse spin deviations from the equilibrium value $S_z = S_0 = \chi H_0/\gamma$ and small deviations of $\mathbf{V}$ from the equilibrium direction $\mathbf{V} = \mathbf{l}$, parallel to the x-axis, by the equations

$$\dot{S}_x = \gamma H_0 S_y,$$
$$\dot{S}_y = -\gamma H_0 S_x + \frac{\chi\Omega_A^2}{\gamma^2} V_z, \quad (66)$$
$$\dot{V}_z = -\frac{\gamma^2}{\chi} S_y,$$

which are obtained from (65) by linearization. Hence

$$\ddot{S}_y = -[(\gamma H_0)^2 + \Omega_A^2] S_y,$$

and, consequently, the transverse resonance frequency is $\omega_\perp = (\omega_0^2 + \Omega_A^2)^{1/2}$.[75]

Similarly we have from (65) for the spin longitudinal oscillations, i.e., oscillations of the z-component of the spin about the equilibrium value S_0,

$$\dot{S}_z = -\frac{\chi\Omega_A^2}{2\gamma^2}\sin 2\Phi,$$
$$\dot{\Phi} = \frac{\gamma^2}{\chi}(S_z - S_0), \quad (67)$$

where Φ is the angle of rotation of $\mathbf{V}$ around the direction of $\mathbf{H}_0 \parallel \hat{\mathbf{Z}}$, measured from the direction of $\mathbf{l} \perp \mathbf{H}_0$. As seen from these equations, small oscillations take place about the equilibrium positions $S_z = S_0$, $\Phi = 0$ with frequency $\Omega_A(T)$.

The oscillations of the longitudinal magnetization in the A phase are found in complete analogy with the nonstationary Josephson effect.[13,75] As we know, the superfluid part of ^{3}He-A can be treated as a mixture of two superfluid components with spin projections $S_z = \pm 1$ on the direction of the magnetic field. The difference in the number of particles of these components is proportional to the equilibrium magnetization: $S_0 = \chi H_0/\gamma$. The deviation from its equilibrium value of the difference in particle numbers corresponds to an appearance of a "chemical potential" difference $\delta\mu = (\gamma^2/\chi)(S_z - S_0)$, equal to the rate of change of the "phase difference" Φ of the wave functions of the two components. Oscillations of the number of particles of each component arise as a result of the oscillations of the phase difference Φ. The maximum oscillation amplitude corresponds to a rotation of the vector $\mathbf{V}$ by the angle π to the equivalent position $-\mathbf{V}$. This regime is similar to "phase slipping" in superconductors and in ^{4}He. In the given case the Josephson effect takes place in the whole volume of the liquid, and the spin-orbit interaction plays the role of the tunnel junction.

It follows from the form (58) of the dipole energy in the A phase that there exist two equilibrium directions $\mathbf{V} = \pm\mathbf{l}$. Therefore, in a container with ^{3}He-A it is possible to have domains, regions with opposite mutual orientations of $\mathbf{V}$ and $\mathbf{l}$, separated by thin transition layers—the domain walls. These domain walls have a thickness of the order of the dipole length $\xi_D = (K\Delta^2/g_D)^{1/2}$, and are topologically stable structures[70]—planar solitons. The frequency shift of longitudinal and transverse NMR in ^{3}He-A, related to local oscillations of the vectors $\mathbf{S}$ and $\mathbf{V}$ near domain boundaries, was observed experimentally[76]; theoretically, this problem was treated in Ref. 77.

3) *The* B *phase.* In the B phase there is no shift of the transverse resonance; however, as in the A phase, longitudinal resonance does take place. The spin dynamics equations of the B phase are obtained from the general Leggett equations (63)–(64). Recalling that in the B phase $d_i(\mathbf{n}) = R_{ik}n_k e^{i\Phi}$, where R_{ik} is the matrix of three-dimensional rotations (40) around the direction $\hat{\boldsymbol{\omega}}$ by the angle θ, we have (see Ref. 78):

$$\dot{\mathbf{S}} = \gamma[\mathbf{SH}] + \frac{4}{15}\frac{\chi\Omega_B^2}{\gamma^2}\hat{\boldsymbol{\omega}}\sin\theta(1 + 4\cos\theta),$$
$$\dot{\theta} = \gamma\hat{\boldsymbol{\omega}}\left(\frac{\gamma\mathbf{S}}{\chi} - \mathbf{H}\right), \quad (68)$$
$$\dot{\hat{\boldsymbol{\omega}}} = -\frac{\gamma}{2}\left[\hat{\boldsymbol{\omega}}\left(\frac{\gamma\mathbf{S}}{\chi} - \mathbf{H}\right)\right] + \operatorname{ctg}\frac{\theta}{2}\left[\hat{\boldsymbol{\omega}}\left[\hat{\boldsymbol{\omega}}\left(\frac{\gamma\mathbf{S}}{\chi} - \mathbf{H}\right)\right]\right],$$

where the notation $g_D = \chi\Omega_B^2/3\gamma^2$ has been used. At equilibrium with $H = H_0$ the magnetization is $S_0 = \chi H_0/\gamma$ and is directed opposite to the field ($\gamma < 0$), $\hat{\boldsymbol{\omega}}$ is in the field direction, and $\theta = \theta_0 = \arccos(-\frac{1}{4})$. For longitudinal small magnetization oscillations we have from (67):

$$\dot{S}_z = -\frac{\Omega_B^2\chi}{\gamma^2}(\theta - \theta_0),$$
$$\dot{\theta} = \frac{\gamma^2}{\chi}(S_z - S_0), \quad (68a)$$

whence follows the presence of longitudinal resonance with frequency $\Omega_B(T)$. It is also seen from Eq. (68) that no frequency shifts occur for small transverse oscillations of the spin S. Indeed, in this case the first of Eqs. (68) becomes $\dot{\mathbf{S}}_\perp = \gamma[\mathbf{S}_\perp\mathbf{H}]$.

b) Nonlinear NMR in the A and B phases

A remarkable confirmation of the correctness of Leggett's equations was provided by NMR pulse experiments, in which the precession of magnetization S was studied for large angles of deviation from the direction of a constant field.[79] The most impressive discovery was the appearance of a frequency shift of transverse NMR in the B phase at deviation angles of the magnetization from the external field larger than 104°. Solutions of the Leggett equations for nonlinear NMR in strong fields $\gamma H \gg g_D$ ($\omega_0 \gg \Omega_{A,B}$) were first obtained by Brinkman and Smith.[80] In the present discussion we follow the general method of finding solutions, well-suited for the case $\omega_0 \gg \Omega_{A,B}$, suggested by Fomin.[81]

1) NMR *equations in strong fields.* For strong magnetic fields the theory contains the parameter $(\Omega_{A,B}/\omega_0)^2$, small in the whole temperature interval for fields $H > 30$ Gauss in the A phase, and fields $H > 100$ Gauss in the B phase; $\Omega_B^2/\Omega_A^2 = 5/2$. To understand better the simplifications occurring in strong fields, we transform the Leggett equations (63) to a coordinate system rotat-

ing with the Larmor frequency $\omega_0 = \gamma H_0$

$$\dot{\mathbf{S}} = \mathbf{R}_D(\mathbf{d}),$$
$$\dot{\mathbf{d}}(\mathbf{n}) = \frac{\gamma^2}{\chi}[\mathbf{S}\,\mathbf{d}(\mathbf{n})]. \tag{69}$$

As follows from (69), in this system the vectors d, retaining their mutual orientation, precess quickly with angular velocity $\gamma^2 S/\chi \simeq \omega_0$ around the direction of S, and both the absolute value and direction of the vector S vary slowly during a time $\sim\Omega_{A,B}^{-1}$. It is easily shown that under these conditions the projection of the direction of S on any of the vectors $\mathbf{d}(\mathbf{n})$ is conserved. Indeed, taking the scalar product of the second of Eqs. (69) by $s = S/S$ and integrating over time from 0 to $t \gg 1/\omega_0$, we obtain

$$\mathbf{s}(t)\,\mathbf{d}(t) = \mathbf{s}(0)\,\mathbf{d}(0) + \int_0^t \mathbf{d}(t)\,\dot{\mathbf{s}}(t)\,dt.$$

The integral in the right hand side of this expression obviously vanishes, as does the integral of the product of a quickly oscillating function by a slowly varying (almost constant) function. The conservation of projection of the direction of S on the direction of any d, i.e., the conservation of the orientation of S with respect to the system of vectors $\mathbf{d}(\mathbf{n})$ makes it possible to reduce the number of variables in the original system of equations (63) from six to four.

Experiments on NMR pulses are set up as follows. An equilibrium magnetization $S_0 = \chi H_0/\gamma$ is initially created in some direction. Then a magnetic field H_1, perpendicular to H_0, is switched on for a time τ, and the equilibrium magnetization precesses around the direction of H_1, inclined to the original direction by the angle $\beta_0 = \gamma H_1 \tau$. Following that, the field H_1 is switched off, and the magnetization starts precessing around the direction of the external field H_0, while at the same time the system of vectors d precesses around the direction of magnetization. With the purpose of describing this motion we introduce, in addition to the fixed coordinate system x, y, z ($\hat{z}$ is parallel to H_0), a moving system ξ, η, ζ, rigidly bound with the vectors d, and we assume that initially S is directed along ζ and has the equilibrium value $\chi H_0/\gamma$. As can be proved, the direction of S always coincides with the direction of ζ, implying that the variables describing the motion will be the three Euler angles α, β, γ, specifying the orientation of the moving coordinate system with respect to the fixed one, and the magnitude of the spin S. We recall that α and β are, respectively, the azimuthal and polar angles of the fixed ζ-axis, and γ is the angle of rotation of the ξ and η axes around the ζ-direction (Fig. 5). Instead of the variable β it is convenient to consider the projection of S on the z-direction: $S_z = S\cos\beta$. In these variables the Hamiltonian (61) can be rewritten as

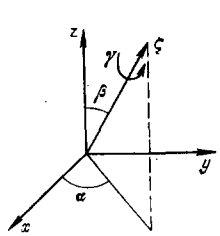

FIG. 5.

$$\hat{H} = \omega_0\left[\frac{S^2}{2} - S_z + \tilde{U}(\alpha, \beta, \gamma)\right], \tag{70}$$

where the measurement unit of S is taken to be $S_0 = \chi H_0/\gamma$, the magnetic field unit is H_0, and $\tilde{U} = U_D/\chi H^2$.

The variables S_z and S are naturally considered as independent canonical momenta, while the angles α and γ are their conjugate coordinates. The Hamilton equations following from (70) are

$$\dot{S} = -\omega_0\frac{\partial\tilde{U}}{\partial\gamma}, \quad \dot{\gamma} = \omega_0\left(S + \frac{\partial\tilde{U}}{\partial S}\right),$$
$$\dot{S}_z = -\omega_0\frac{\partial\tilde{U}}{\partial\alpha}, \quad \dot{\alpha} = \omega_0\left(-1 + \frac{\partial U}{\partial S_z}\right). \tag{71}$$

As is seen from the equations, the angular velocities $\dot{\alpha}$ and $\dot{\gamma}$ can be in resonance $\dot{\gamma} = -\dot{\alpha}$, near which it is convenient to transform from the two fast variables α and γ to one fast variable and one slow variable $\Phi = \alpha + \gamma$. The new momenta, canonically conjugate to these variables, are, respectively, $P = S_z - S$ and $S - 1$. The generating function of this canonical transformation is

$$\Pi = \alpha(P + S) + \gamma S.$$

In the new variables the Hamiltonian (70) appears as:

$$H = \omega_0\left[\frac{(S-1)^2}{2} - P + \tilde{U}\left(\alpha, \Phi, \frac{P}{S}\right)\right]. \tag{72}$$

Averaging the Hamiltonian (70) over the fast variable α, we obtain a new Hamiltonian

$$\overline{H} = \omega_0\left[\frac{(S-1)^2}{2} - P + V\left(\Phi, \frac{P}{S}\right)\right], \tag{73}$$

where

$$V\left(\Phi, \frac{P}{S}\right) = \overline{\tilde{U}\left(\alpha, \Phi, \frac{P}{S}\right)}. \tag{74}$$

The new Hamilton equations are

$$\dot{S} = -\omega_0\frac{\partial V}{\partial\Phi}, \quad \dot{\Phi} = \omega_0\left(S - 1 + \frac{\partial V}{\partial S}\right),$$
$$\dot{P} = 0, \quad \dot{\alpha} = \omega_0\left(-1 + \frac{\partial V}{\partial P}\right). \tag{75}$$

As seen from these equations, the momentum $P = S_z - S$ is conserved. The invariance of P is, naturally, adiabatic, i.e., P is conserved accurately up to terms neglected in the averaging. The quantity P is determined by its initial value $P = \cos\beta_0 - 1$. The system (75) has stationary solutions $\Phi = \Phi_0$ and $S = S_0$, satisfying the equations

$$\frac{\partial V}{\partial\Phi} = 0,$$
$$S_0 = 1 - \frac{\partial V}{\partial S}. \tag{76}$$

When the stability condition of these equations, $\partial^2 V/\partial\Phi^2 > 0$ is satisfied, the magnetization S performs a precession around the direction of the field H at a frequency (see Eq. 75)

$$\omega_\perp = \omega_0\left(-1 + \frac{\partial V(\Phi_0, P)}{\partial P}\right), \tag{77}$$

where, as can be seen, the frequency shift depends on the initial angle of inclination of the magnetization, β_0. We recall that the gyromagnetic ratio γ is negative for ^{3}He, therefore formally $\omega_0 = \gamma H < 0$.

Small deviations of S and Φ from the stationary values S_0 and Φ_0 lead to oscillations with frequency (see Eq. 75)

$$\omega_{\parallel}^2 = \omega_0^2 \frac{\partial^2 V(\Phi, P)}{\partial \Phi^2}\Big|_{\Phi=\Phi_0}, \tag{78}$$

transforming at $\beta_0 = 0$ to the ordinary longitudinal magnetization oscillations.

To find explicit expressions for $\omega_\perp$ and $\omega_\parallel$ in the A and B phases of ^{3}He we need expression (74) for the average dipole energies in the A and B phases.

2) *The* A *phase.* Substituting into (62) the order parameter of the A phase, expressed in terms of the Euler angles:

$$d_i = \frac{1}{\sqrt{2}} R_{ij}(\alpha, \beta, \gamma) V_j (\Delta' + i\Delta'', \mathbf{n}),$$

where

$$\mathbf{V} \perp \mathbf{H}_0, \quad \mathbf{V} \| \mathbf{l} = \Delta' \times \Delta'', \quad R_{ij}(\alpha, \beta, \gamma)$$

is the matrix of three-dimensional rotations, parametrized in terms of the Euler angles (see Ref. 81), we obtain

$$\tilde{U}_A = -\frac{\Omega_A^2}{2\omega_0^2}\left[\left(\frac{P}{S}+2\right)\cos\Phi + \frac{P}{S}\cos(2\alpha - \Phi)\right]^2. \tag{79}$$

The averaging of $\tilde{U}_A$ over the fast variable α gives

$$V_A = \text{const} - \frac{\Omega_A^2}{2\omega_0^2}\left[\left(1+\frac{P}{S}\right)^2 + \frac{1}{2}\left(2+\frac{P}{S}\right)^2 \cos 2\Phi\right]. \tag{80}$$

The potential energy (80) for any deviation angles β of the magnetization, i.e., for any P varying in the interval from 0 to -2, has qualitatively the same structure characterized by two stable minima $\Phi = 0, \pi$, as is the case for $P = 0$ (see Eq. 58). By means of (80) we obtain from (77) an expression for the magnetization precession frequency (see Ref. 80a):

$$\omega_{\perp A} = -\omega_0 - \frac{1}{8}\frac{\Omega_A^2}{\omega_0}(4+3P) = -\omega_0 - \frac{1}{8}\frac{\Omega_A^2}{\omega_0}(1+3\cos\beta), \tag{81}$$

going over as $\beta \to 0$ to the result of the linear theory for the frequency shift in the A phase (see subsection 4a2).

The frequency of small longitudinal magnetization oscillations (78) is given by the expression[81a]

$$\omega_{\parallel A}^2 = \left(\frac{\Omega_A}{2}\right)^2 (1+\cos\beta)^2. \tag{82}$$

3) *The* B *phase.* Similarly for the B phase, substituting into (62)

$$d_i = \frac{R_{ij}(\alpha, \beta, \gamma)}{4}(-\delta_{jk} + 5\hat{\omega}_j\hat{\omega}_k - \sqrt{15}\, e_{jkm}\hat{\omega}_m) n_k,$$

where $\hat{\omega} \| H_0$, and integrating over the solid angle, we have:

$$\tilde{U}_B = \frac{2}{15}\frac{\Omega_B^2}{\omega_0^2}\left[\frac{1}{2} + \frac{P}{S} + \left(2+\frac{P}{S}\right)\cos\Phi\right]^2, \tag{83}$$

implying $\tilde{U}_B = V$. The angle Φ in (83) is measured from $\theta_0 = \arccos(-\frac{1}{4})$, i.e., $\Phi = \alpha + \gamma + \theta_0$. Expression (83) naturally transforms for $P = 0$ to expression (41) for the dipole energy in the B phase. The two minima of this potential, determined by the equation

$$\cos\Phi_0 = -\frac{P+1/2}{P+2},$$

start approaching each other as P decreases from zero to $P = -5/4$. In this interval ($0 < \beta < 104°$) we have $V(P, \Phi_0) = 0$, and, consequently, there is no precession frequency shift. At the point $P = -5/4$ ($\beta = 104°$) bifurcation occurs, the minima coalesce, so that in the interval from $P = -5/4$ to $P = -2$ ($104° < \beta < 180°$) there is one minimum $\Phi_0 = 0$:

$$V(P, 0) = \frac{8\Omega_B^2}{15}\left(P + \frac{5}{4}\right)^2,$$

and a precession frequency shift[80b] takes place

$$\omega_{\perp B} = -\omega_0 + \frac{16}{15}\frac{\Omega_B^2}{\omega_0}\left(P+\frac{5}{4}\right) = -\omega_0 + \frac{16}{15}\frac{\Omega_B^2}{\omega_0}\left(\frac{1}{4}+\cos\beta\right). \tag{84}$$

The frequency of small longitudinal oscillations for P varying from 0 to $-5/4$ equals

$$\omega_{\parallel B}^2 = \Omega_B^2\left(1+\frac{4}{5}P\right), \tag{85}$$

and in the interval $(-5/4, 2)$:

$$\omega_{\parallel B}^2 = -\frac{8}{15}\Omega_B^2 (P+2)\left(P+\frac{5}{4}\right). \tag{86}$$

The agreement of expressions (81), (84) with experimental results,[79,82] among which we want to point out the appearance of a frequency shift of the transverse resonance in the B phase for deviation angles of the magnetization from the magnetic field direction higher than 104°, is a convincing confirmation of the theoretical concepts concerning the structure of the A and B phases of superfluid ^{3}He.

The expressions for the NMR frequencies in the A and B phases (81) and (84) were obtained by an approach valid in strong magnetic fields, developed by I. A. Fomin.[81] A remarkable fact, noted by S. P. Novikov,[83] is that for the B phase the Leggett equations admit a periodic solution in magnetic fields of arbitrary strength. Indeed, expression (83) for the dipole energy of the B phase depends only on the angle β and on the combination $\alpha + \gamma$. Thus, the variable α in the exact equations, containing not 4 but 6 variables, will be cyclic. This implies that one can seek a solution stationary in all variables except α. The corresponding calculations were carried out by I. A. Fomin *et al.*[84] It has been shown that in a magnetic field of arbitrary strength there is no frequency shift of the transverse resonance in the B phase for deviation angles of the magnetization from the magnetic field direction less than 104°. For angles larger than 104° the frequency shift is determined by the equation

$$\omega_\perp(\omega_\perp + \omega_0) = -\frac{16}{15}\Omega_B^2\left(\cos\beta + \frac{1}{4}\right), \tag{87}$$

whose solution transforms into (84) for strong fields $\omega_0 \gg \Omega_B$, and to the frequency of the periodic solution in a vanishing field ($\omega_0 = 0$), found in Ref. 85.

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