Electron bunches in nonlinear collective beam-plasma interaction

V. P. Kovalenko

Institute of Physics, Academy of Sciences of the Ukrainian SSR
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A number of facts established in numerous experiments on the nonlinear collective interaction of a monoenergetic electron beam with plasma and never previously satisfactorily explained became understandable after the concept of dense bunches, into which an electron beam separates, was developed in the 1970s. In this review, in addition to a description of these facts, the results of theoretical investigations of the formation and evolution of bunches under the conditions of stationary beam injection into a plasma with different parameters are presented. The role of higher-order harmonics of the electric field, dissipation, and plasma temperature is discussed. Special attention is given to experiments concerned with a direct study of the dynamics of electron bunches in plasma. Possible reasons for the observed rapid breakup of these bunches are enumerated. Results of experimental and theoretical investigations of the equilibrium state of bunches, created outside the plasma and then injected into it, are also examined. Under certain conditions, such bunches are conserved in a plasma much better than bunches that form spontaneously, while the periodic quasistationary waves created by them (Bernstein-Green-Kruskal waves) propagate much farther than evolving waves, which are excited when a continuous beam is injected into a plasma.

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CONTENTS

INTRODUCTION

Physical objects in which a collective interaction occurs between an electron beam and a plasma or in which at least the conditions for such an interaction are present are as widespread as they are varied. They include low-pressure arcs with a heated cathode, high-voltage glow discharges, polar auroras in the atmosphere, and plasma forming during electron-beam working of materials (welding and melting). Proposals and successful attempts are being made to use this phenomenon purposefully to achieve different aims: to create plasma in plasmochemical reactors, ion sources, in active experiments in space; to generate microwave radiation; and, to heat plasma. It is not at all surprising that the electron beam-plasma system is still attracting persistent interest after more than 50 years.

The interest in beam-plasma interaction, however, is equally due to the fact that these interactions reflect the fundamental properties of plasma. We should note especially the abundance of, in general, coupled instabilities and nonlinear oscillations that can occur in the beam-plasma system.

Let us first menion the easily excitable high-frequency electron branches of oscillations.[1] Different types of low-frequency excitations with the participation of ions can also be excited by beams.[2] For sufficiently intense electronic Langmuir oscillations, wave interaction

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processes (their decay, modulation instability, and collapse[3-5]) which also proceed with the participation of ions and which, in particular, lead to one of the channels for dissipation of high-frequency waves, could also be important. If to this we add that the appearance of collective fields in a weakly ionized medium is also accompanied by gas discharge effects,[6] then it becomes clear how difficult it is to create an all-encompassing unified description of an electron-beam plasma. As with many other complex systems, here, we must for the time being be satisfied primarily with the results that can be obtained by studying some particular process, chosen from those above, separately from the others.

The high-frequency electron motions in a plasma with a beam, examined in what follows, are in this sense not an exception. However, we can be sure that the information on the basic properties of nonlinear electron waves is necessary to understand correctly the entire complex of phenomena both in electron-beam plasma and in ion-beam systems.[7] On the other hand, many well-known experimental facts are explained by the motion of just the electron component by itself. It is not accidental that collective beam-plasma interaction was first manifested precisely in the excitation of high-frequency plasma oscillations.[8-11]

The mechanism of excitation of electron waves was understood after the discovery of the beam-plasma instability,[12,13] which stimulated extensive theoretical investigations of the beam-plasma interaction in the linear approximation, as well as corresponding experiments. The results obtained are contained in reviews and monographs (see, for example, Refs. 1, 14–16).

Further success in the development of the theory of relaxation of a beam in a plasma was achieved in the study of one of the types of beam-plasma systems, namely, a plasma containing a beam with a broad electron velocity distribution function. Based on a description of the excitation of a large number of waves with random phases in such a system, a quasilinear theory was constructed,[17-19] according to which the bump on the tail of the smoothed distribution function $f_0(v)$ must gradually spread, due to the interaction of small oscillations ($\bar{f}' \ll f_0$) toward small velocities until the final state, the state with a "plateau," is reached. In accordance with the evolution of the distribution function, the spectrum of excited waves must also become wider.

The basic results of the quasilinear theory were confirmed experimentally in Ref. 20, where special measures were taken so that the beam injected into the plasma consisted of electrons with a continuous velocity distribution over a wide range of values.

We should menion, however, that under the conditions of most of the other experiments the quasilinear theory is not applicable and it does not explain all the observed facts taken as a whole, although attempts have been made to interpret separate data based on this theory. The situation here was resolved only after the important role of phase focusing of the electrons in a fast monoenergetic beam into dense bunches was established by computer[21,22] and laboratory[23] experiments.

Subsequent theoretical and experimental investigations showed that many nonlinear properties of the beam-plasma system are determined by the dynamics of these charged bunches, called "macroparticles,"[24] in the field of the monochromatic wave excited by them.

A number of theoretical aspects of the dynamics of a beam in monochromatic plasma waves are clarified in Ref. 25. At the same time, the relationship between the results of numerous experimental and theoretical investigations has not yet been analyzed.

In this paper, we give a systematic description of the effects of beam-plasma interaction, related to excitation of regular longitudinal waves by bunches of electrons. Only nonrelativistic beams are considered, since unambiguous experimental data on bunching in a plasma have been obtained precisely for such beams. The experimental facts, which, having remained without a satisfactory explanation for a long time, stimulated the development of the theory of beam bunching in a plasma, are described in Sec. 2. Section 3 is concerned with Van Kampen waves.[26] The possibility of such waves in a plasma is related to modulated beams,[3] and for this reason it is impossible to circumvent the problem of their relation to the waves usually observed in the beam-plasma system. This section, together with Sec. 4, can be viewed as an introduction to the theory of bunching of a beam injected in a steady-state manner into the plasma (Sec. 5). Results of a direct experimental verification of this theory are presented in Sec. 6. The nonlinear periodic stationary Bernstein-Green-Kruskal waves (BGK waves),[27] in which bunches also play an important role, are discussed in Secs. 7 and 8. Special attention is devoted to conditions required for their excitation. It is precisely the clarification of these conditions that permitted realizing quasistationary BGK electron waves experimentally.[28,29]

2. CHARACTERISTICS OF ELECTRON-BEAM EXCITATION OF TRAVELING LONGITUDINAL WAVES IN A PLASMA

The variety of problems involving the interaction of electron beams with plasma naturally led to numerous experiments, distinguished by the parameters of the plasma and the beam, magnitude of external fields, diagnostic methods, etc. Nevertheless, the systems studied have a number of common characteristics. This fact is not always emphasized, but it is the common characteristics that, on the one hand, indicate the generality of the processes occurring under a wide range of experimental conditions and, on the other, form the first touch-stone in the theory of beam-plasma interaction. It is from this point of view that the discussion of the present empirical section, which is concerned with describing the "macroscopic" picture of beam excitation of traveling longitudinal waves, should be approached.

a) Experimental apparatus, conditions, and parameters

We shall list the most important conditions for the beam-plasma experiment (Fig. 1).

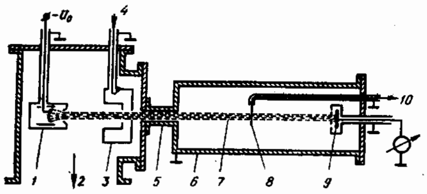


FIG. 1. Diagram of apparatus for a beam-plasma experiment. 1) electron gun, 2) to pump, 3) uhf resonator for modulating the beam, 4) uhf power input, 5) pressure drop channel, 6) plasma chamber, 7) beam in plasma, 8) probe, 9) beam collector, 10) to instruments analyzing oscillations.

1) The beam is introduced into the plasma from outside the plasma through the plasma boundary. In a number of arrangements, a small modulation is superposed on the beam entering the plasma by means of a uhf resonator, probe, or coil. With pulsed beam injection, the pulse duration greatly exceeds the time for an instability to develop, so that the conditions for beam-plasma interaction are, essentially, the same as for stationary beam injection.

2) The velocities of the electrons in the beam have a small thermal spread in the plane of injection, i.e., the relation $\Delta v/v_0 \ll 1$ is satisfied, where v_0 is the average velocity of electrons in the beam, and Δv is the interval over which the electron velocities are distributed.

3) The plasma is created in the interaction region either independently of the beam by some discharge or by the beam itself.

4) In many arrangements, an external magnetic field oriented along the beam axis is used. In spite of the fact that in a magnetic field the number of possible branches of oscillations increases, in many cases it was possible to separate out effects related to excitation of longitudinal waves traveling along the beam. As will be evident from the material presented in what follows, these effects do not differ essentially from those observed without a magnetic field.

5) The beam passing through the plasma falls on the collector, which limits the extent of the interaction region. The strong effect of the collector on the oscillations is well known.[30-32] This effect is related, in particular, to secondary electron emission.[32] However, in most of the experiments described below, this effect is apparently insignificant. Standard methods are used to analyze the longitudinal velocity distribution function for electrons in the beam in the plane of the collector.

The most characteristic data from a number of experiments, in which the "macroscopic" effects described below were observed, are displayed in Table I. In particular, this table presents information concerning the illustrations used in this review. The values of parameters which are of fundamental significance in comparing a particular experiment with theory ($\Delta f/f$, n_b/n_p, etc.) are also indicated.

b) Properties of excited oscillations

It is evident from Table I that, in accordance with the wide range of parameters of the beam-plasma system studied, in different experiments oscillations with, generally speaking, strongly differing frequencies in a wide region of the uhf range (10^2–10^4 MHz) were observed. In the absence of a magnetic field, the measured frequencies of the waves are close to the electron plasma frequency in complete agreement with the linear theory of beam-plasma interaction. The correspondence between the frequencies of traveling waves and the linear theory is also usually noted in experiments performed in an external magnetic field.

The most important characteristic of the excited waves is the spatial distribution of their intensity. For sufficiently sensitive detectors of the oscillations, it is possible to make measurements throughout the entire volume of the interaction chamber, and if the chamber consists of a dielectric material, then measurements can be made outside the chamber as well.[42,44,54] However, measurements with a probe show that the most intense oscillations are detected when a probe is introduced into the region penetrated by the beam. Outside the beam boundaries, the signal amplitude decreases by several orders of magnitude.[43]

Several examples of the axial distribution of the intensity of oscillations, taken from different papers, are presented in Fig. 2. Analogous dependences are also obtained in other experiments. In spite of the sometimes enormous difference in the experimental conditions (see Table I), these dependences are characterized by the presence of a distinct main peak: a region with strong oscillations. Over the preceding section, the amplitude usually increases exponentially with distance. Beyond the maximum, on the other hand, the nature of the damping of the waves depends strongly on the specific experimental conditions. In some cases,[34,39-41,50] the amplitude decreases rapidly and monotonically, as in Fig. 2b–d, and in others,[30,43,45,52] another peak is observed on the decreasing section (Fig. 2a, e, f), but with a much lower amplitude. Be that as it may, the results of numerous measurements (apparently, the only exception is Ref. 55) point to the existence of a localized region with the most intense oscillations at some distance away from the plane in which the beam enters the plasma. This distance depends on the parameters of the system, increasing with increasing beam energy and decreasing with increasing beam density.[11,34,42,51]

A very important property of the oscillations, which determines the behavior of the system at the nonlinear stage, is their high degree of regularity even in the absence of preliminary modulation of the beam. As is evident from the table, the half-width of the measured frequencies of the spectra Δf is usually much less than the fundamental frequency, especially in experiments with a weak beam and an independently formed plasma. As the beam density increases, when the oscillations begin to affect the average plasma parameters, the spectrum becomes somewhat wider. It has been established that in this case the oscillations can be generated in bursts, which correspond to different branches of the

TABLE I. Parameters of beam–plasma experiments.*

Parameters	1939 r.[31]	1959 r.[33]	1959 r.[34]	1960 r.[35]	1960 r.[36]	1963 r.[37], 1965 r.[38]	1963 r.[30]	1965 r.[39]	1967 r.[40]	1967 r.[41]	1969 r.[23]
Energy of beam electrons, eV	20	20.6	40—120	$8\cdot10^{4}$	$<3\cdot10^{3}$	$15\cdot10^{3}$	100	20—40	$(1-1.5)\cdot10^{3}$	$4\cdot10^{2}-10^{3}$	100—200
Beam current, A	0.02—0.1	0.034	$10^{-3}-5\cdot10^{-2}$	1	$(1-2)\cdot10^{-3}$	5—8	0.13	0.03—0.25	$(5-25)\cdot10^{-3}$	0.02—0.1	$(3-30)\cdot10^{-3}$
Beam diameter, cm	0.5		0.3—0.5	0.5	0.2—0.4	1	0.5		0.3	0.9	≈ 0.5
Type of gas, pressure, mm Hg	Hg $5\cdot10^{-3}$	Hg $1.2\cdot10^{-3}$	Hg $1\cdot10^{-3}$	$9\cdot10^{-2}$	Ar, H_2, N_2 $10^{-2}-10^{-3}$	Air $10^{-4}-10^{-3}$	Hg $(1-2)\cdot10^{-3}$	Hg $(0.2-1)\cdot10^{-3}$	H_2 $2.5\cdot10^{-2}$	He $6\cdot10^{-4}$	Ar $10^{-3}-10^{-2}$
Frequency of excited oscillations f, GHz	1.5	0.7	0.75—1.5	2.7	2—3	0.83	2	0.1—0.5	1.35	0.3—0.9	0.5—1.5
Plasma frequency, GHz	$\geqslant f$	≈ 0.65	$\approx f$	2.4	$\approx f$	1.26		$\approx f$	$\approx f$	0.6—1.5	$\leqslant f$
Electron cyclotron frequency, GHz	0	0	0	0	0	2.6	0	0	0.5	1.1	0
Relarive half-width of spectra of oscillations, $\Delta f/f$		$2\cdot10^{-2}$		0.14	0.2	0.06—0.1 (unmodulared)		0.15—0.2	≈ 0.08	0.15—0.2	
Temperature of plasma electrons, eV		2—8				40		3		10—25	
Ratio of beam density to plasma density, n_b/n_p	0.1—0.4		0.02—0.2	0.03		0.1	0.15		0.07	0.04	0.01—0.1
Origin of plasma**	a	a	b	b	b	c	a	a	c	d	c
Pulse duration, μs	Stationary	Stationary	Stationary	2	Stationary	3.6	Stationary	Stationary	Stationary	3	Stationary
Method of external modulation	Absent	Absent	Absent	1) Absent 2) Resonator	Absent	1) Absent 2) Coil	Absent	Absent	Absent	Probe	1) Absent 2) Resonator

*The range of the parameters or their average value is presented.

**a) Near-cathode region of the arc with heated cathode; b) auxiliary discharge, created along the beam path; c) beam-plasma discharge; d) plasma flowing out of the auxiliary discharge into the vacuum.

Parameters	1969 r.[42]	1969 r.[43]	1971 r.[44]	1972 r.[45]	1973 r.[46, 47]	1973 r.[48] 1976 r.[49]	1973 r.[50]	1973 r.[51]	1975 r.[52]	1979 r.[53]
Energy of beam electrons, eV	$(1-2)\cdot10^{4}$	100—200	300—500	385	100—500	100—1000	10^{3}	100	$(2-10)\cdot10^{3}$	$14\cdot10^{3}$
Beam current, A	0.1—1	$(4-250)\cdot10^{-6}$	$10^{-5}-10^{-4}$	$2\cdot10^{-3}$	$(0.5-8)\cdot10^{-3}$	$\leqslant3\cdot10^{-3}$	0.02—0.1	$10^{-5}-10^{-3}$	$(2-60)\cdot10^{-3}$	$(2-20)\cdot10^{-3}$
Beam diameter, cm	1	0.6		0.6		0.6		0.5	0.5	
Type of gas, pressure, mm Hg	Ne $4\cdot10^{-2}$	H_2 $1.5\cdot10^{-5}$	Hg 10^{-3}	Ar $5\cdot10^{-4}$	H_2 10^{-5}	He $(2-8)\cdot10^{-4}$	He $5\cdot10^{-3}-5\cdot10^{-2}$	Ar $10^{-4}-10^{-3}$	Ar $7\cdot10^{-5}-5\cdot10^{-4}$	He, Ne $(3-8)\cdot10^{-2}$
Frequency of excited oscillations f, GHz	27—39	0.145	4	0.31	0.1	0.71	0.2—1	1	1.5	8.8
Plasma frequency, GHz	$\approx f$	0.19	$\approx f$	0.28		0.6	0.6—1	$\approx f$	1.3	$1.03f$
Electron cyclotron frequency, GHz	0	0.78	0.28	0.195	2.6	0.5	0	0.56	0.94	0
Relative half-width of spectra of oscillations, $\Delta f/f$		0.1		$1.5\cdot10^{-2}$ (unmodulated) $3\cdot10^{-4}$ (mod.)		$7\cdot10^{-2}$	0.3	0.1	$\leqslant2\cdot10^{-3}$	$1.5\cdot10^{-3}$
Temperature of plasma electrons, eV	1	11	4	8	14	1—8		3—4	4—6	≈ 0.7
Ratio of beam density to plasma density, n_b/n_p	$10^{-5}-10^{-4}$	$3\cdot10^{-3}-0.1$	$5\cdot10^{-5}$	$3.7\cdot10^{-2}$	$5\cdot10^{-4}-10^{-2}$	$10^{-3}-10^{-2}$	10^{-2}	$10^{-4}-10^{-3}$	$10^{-3}-2\cdot10^{-2}$	
Origin of plasma**	b	d	b	c	d	d	c	b	d	b
Pulse duration, μs	1.5—10	Stationary	Stationary	Stationary	Stationary	Stationary	Stationary	Stationary	3	1.5—10
Method of external modulation	Absent	Probe	1) Absent 2) Coil	Probe	Absent	Probe	1) Absent 2) Resonator	Absent	Resonator	not indicated

instability, i.e., the uhf spectrum changes with the time.[58,59] Oscillograms of the shapes of the high-frequency signal under these conditions, together with a subsequent correlation analysis of realizations, showed that the oscillations can be represented by alternating segments of quasiharmonic functions with different frequencies and amplitudes.[60] The lengths of some quasiharmonic trains attain many tens of periods.[47,61]

Even more regular oscillations are excited in a plasma when the beam is weakly modulated by a monochromatic signal. If the amplitude of the modulation is sufficiently high, then only oscillations with the imposed frequency remain in the system.[62-64] Oscillations at other frequencies are suppressed.

Some idea of the frequency spectra in a beam-plasma system and their evolution with distance is given by the examples in Fig. 3.

Finally, we should note the nonlinearity of excited regular oscillations. The presence of higher-order harmonics in their frequency spectrum was already noticed in Ref. 65. Higher order harmonics were then observed also in other experiments (see, for example, Refs. 23, 38, 44, 46, 47). A detailed investigation of the behavior of the harmonics was made in Refs. 41, 48, 51, 66. From the dependences presented in Fig. 2, it is evident that all harmonics have a maximum situated at the same location as the maximum of the first harmonic. The higher the order of the harmonic, the farther

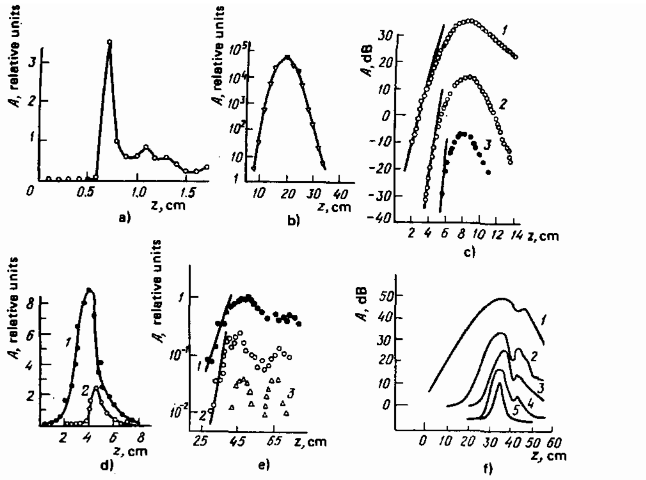


FIG. 2. Intensities of the fundamental oscillation and the harmonics (noted by numbers) as a function of the distance to the plane of beam entry into the plasma, obtained in Ref. 33 (a), 42 (b), 51 (c), 23 (d), 45 (e), and 48 (f).

its growth begins from the entry point. This indicates that the oscillations of harmonics do not arise at the point of beam entry, but are formed at some distance during the beam-plasma interaction.

c) Evolution of the distribution function of electrons in the beam

Another very characteristic manifestation of excitation of longitudinal waves by a beam in a plasma is the strong spreading of the velocity distribution function of electrons in the beam under conditions when pair collisions are so rare that they should not affect the beam significantly. For the time being, we are talking about a distribution function averaged over a time interval that is not only longer than the period of the high-frequency oscillations, but also longer than the duration of separate quasiharmonic trains, which we mentioned above. At the early stages of beam-plasma studies, in a series of experiments, data were obtained only for this relatively easily measured distribution function.[11,34,37,40,42]

As an illustration of the nature of the change in the averaged distribution function with distance, we pre-

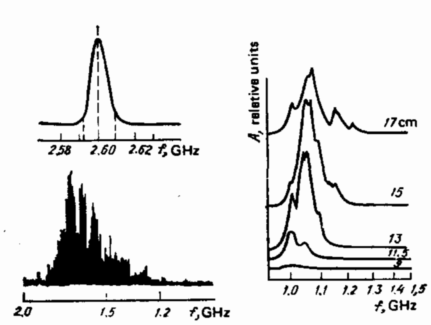


FIG. 3. Examples of narrow and wide spectra from Ref. 59 and evolution of a spectrum with distance.[51] Unmodulated beam.

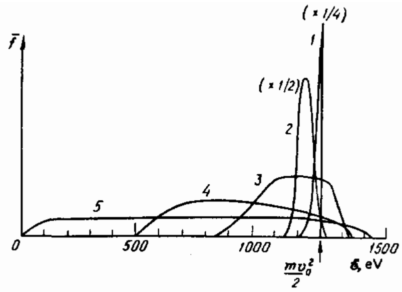


FIG. 4. Normalized averaged energy distribution functions of electrons in the beam. Measured at a distance of 26 cm from the gun for different values of the beam current, I_b (mA): 2 (1), 10 (2), 12 (3), 16.5 (4), and 22 (5).

sent Fig. 4 from Ref. 40. As noted in Ref. 40, the increase in the beam current is equivalent (in the sense of its effect on the distribution function) to a displacement of the probe-analyzer from the point of beam entry into the plasma.

It is evident that the beam ceases to be monoenergetic as it propagates into the bulk of the plasma. The spread of electrons over longitudinal velocities increases especially rapidly near the maximum of the axial dependence of the amplitude of the oscillations. Beyond this region, the width of the distribution function can become comparable to the unperturbed velocity of the beam. An important point here is the acceleration of part of the electrons in the beam. Electrons with energies 1 to 1.5 times greater than the initial energy were also observed in other experiments, in particular, experiments performed with the use of the optical technique,[67,68] which definitely does not introduce any perturbations into the system studied.

An equally strong broadening of the averaged distribution function accompanying the appearance of accelerated electrons is also observed when a beam that is weakly modulated prior to being injected into the plasma interacts with the plasma.[23]

d) Beam-plasma discharge

When the plasma is not highly ionized and the beam is quite dense, the excited oscillations are manifested in the glow of plasma. The oscillations affect the external form of the beam-plasma system in two ways. First, the high-frequency field distorts the trajectories of the electrons in the beam and, in the absence of a magnetic field, thereby alters the form of the more or less clearly delineated glowing trace of the beam, arising due to excitation of atoms by collisions with fast electrons in the beam. On the other hand, since atoms excited by impacts of relatively slow plasma electrons, whose average kinetic energy depends on the amplitude of the collective oscillations, also make an important contribution to the visible radiation, the brightness and color of different regions of the system correlate with the spatial distribution of the intensity of high-frequency fields.

A photograph of the cathode region of an arc discharge in mercury is presented in Fig. 5a.[69] The fol-

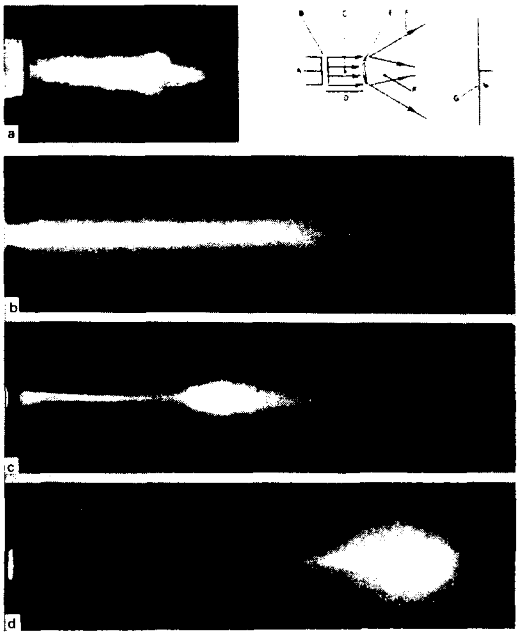


FIG. 5. External view of beam-plasma discharge without a magnetic field. a) Meniscus discharge and diagram illustrating it; b, c) modulated beam injected into gas, $U_0 \sim 100$ V; d) unmodulated beam, $U_0 = 1.35$ kV.

lowing regions are clearly distinguished in the plasma penetrated by the electron beam: the cathode layer B; almost parallel beam of electrons C; clearly delineeated, from the cathode side, zone with high brightness E; and, deflected electron rays F. The sharp maximum in the axial distribution of the amplitude (see Fig. 2a) is observed precisely in region E, which was given a special name, the meniscus. In some cases, radial focusing of the beam was noted before the vigorous scattering in the meniscus.[32,70]

The deflected electron rays and the glowing scattering region are also observed when a premodulated beam is injected into a gas,[23] and in addition the details of the light effects depend on the modulation frequency (Fig. 5b and c). As the accelerating voltage increases, both in modulated and unmodulated beams, the boundaries of the plasma formation, where the most intense oscillations are localized, become more diffuse (Fig. 5d).[71]

The characteristics of the plasma luminescence suggest the obvious idea that the beam-excited longitudinal high-frequency field can have a large effect on the charged particle balance in the plasma. And indeed, the measured values of the plasma density, created by injecting a dense beam into a gas, exceed by orders of magnitude the values which would be expected when only pair collisions of beam electrons and atoms are included. Moreover, the variation of the plasma density with pressure has the character of gas breakdown: when some critical pressure is attained, the plasma density increases discontinuously with simultaneous appearance of intense uhf oscillations.

These and several other facts, established initially in a magnetic field[72-76] and later without it,[23,50,71,77] served as the basis for the description of a new form of gas discharge: a beam-plasma discharge.[6,78,79] In this type of discharge, ionization is realized primarily by plasma electrons, which are accelerated in the high-frequency field of collective oscillations excited by the beam. Many of the data presented in this review were obtained under conditions when the plasma is created by the beam itself by means of a beam-plasma discharge.

3. FOR PLASMA OSCILLATIONS

Before proceeding to an explanation of the characteristics of regular electronic oscillations noted above (this will be given in Secs. 5 and 6), it is useful to consider the difference between the waves excited in the plasma by a given beam and the waves that can appear in the plasma in the absence of a beam.

We shall examine one-dimensional longitudinal waves in a plasma with a stationary ionic background. At first, we shall neglect the thermal motion of electrons. In the general case, a beam of charged particles with current density j_b with a constant component j_0 can pass through the plasma. In the case of small perturbations of the plasma, using a linear approximation for the plasma (but not for the beam), it is easy to obtain an equation for the irrotational electric field E:

$$\frac{\partial^2 E}{\partial t^2} + \omega_p^2 E = -4\pi \frac{\partial j_b}{\partial t}, \tag{3.1}$$

where $\omega_p = \sqrt{4\pi e^2 n_p/m}$ is the plasma frequency and n_p is the plasma density. Equation (3.1) is the equation of an oscillator, oscillating under the action of the force created by the modulated beam.

Assume that at time $t=0$ a beam with harmonically modulated density moving with velocity v_0 is introduced into the plasma. Let us further consider the hypothetical situation in which the fields arising in the plasma do not affect the motion of the particles in the beam, so that for $t>0$ we have $j_b = j_1 \sin(\omega t - k_0 z)$, where $k_0 = \omega/v_0$. In this case, the smallest dissipation [not reflected in Eq. (3.1)] leads to the fact that after some transient process, a state with forced oscillations, described by the following particular solution of Eq. (3.1), is established in the system:

$$E = -\frac{4\pi\omega}{\omega_p^2 - \omega^2} j_1 \cos(\omega t - k_0 z), \quad \omega \neq \omega_p. \tag{3.2}$$

Thus, if without the beam in a cold plasma oscillations are possible only with $\omega = \omega_p$, then in the presence of the beam waves with any frequency ω can exist: there is no definite dispersion relation for these waves.

Is it possible to extend this result to a plasma with nonzero temperature? For this purpose, restricting the analysis to the case of small perturbations, we shall use the linearized Vlasov equation for the plasma electrons and the Poisson equation, in which we include the presence of a modulated beam and, in addition, we shall again assume that the beam is "rigid", i.e., the density is not changed by modulation.

Let us write:

$$\frac{\partial f'}{\partial t} + v \frac{\partial f'}{\partial z} - \frac{e}{m} E \frac{\partial f_0}{\partial v} = 0, \tag{3.3}$$

$$\frac{\partial E}{\partial z} = -4\pi e \int f' \, dv + 4\pi \frac{j_1}{v_0} \sin(\omega t - k_0 z), \tag{3.4}$$

where f_0 is the equilibrium velocity distribution function for plasma electrons and f' is the deviation from equilibrium. The system of equations (3.3), (3.4) easily reduces to an inhomogeneous equation for f':

$$\frac{\partial^2 f'}{\partial t\,\partial z}+v\,\frac{\partial^2 f'}{\partial z^2}+\frac{4\pi e^2}{m}\,\frac{\partial f_0}{\partial v}\int f'\,\mathrm{d}v=\frac{4\pi e j_1}{m v_0}\,\frac{\partial f_0}{\partial v}\sin(\omega t-k_0 z). \quad (3.5)$$

Since the solution of the homogeneous equation, corresponding to (3.5), in the case $\partial f_0/\partial v<0$ describes a wave process with collisionless damping, while in the real plasma collisional damping also exists, it is natural to expect that oscillations excited by the given beam, after the transient process terminates, will go over into the steady-state regime. Only a stationary wave, synchronous with the beam, will remain in the system, as in the case of a cold plasma.

The stationary wave is described by the particular solution of Eq. (3.5)

$$f'(v,\,z,\,t)=f_1(v)\sin(\omega t-k_0 z).$$

Substituting this solution into (3.5) and integrating with respect to the velocity, we obtain an equation for the amplitude of plasma density oscillations $n_1=\int f_1\,\mathrm{d}v$:

$$n_1+n_1\frac{4\pi e^2}{m k_0}\int\frac{\partial f_0'/\partial v}{\omega-k_0 v}\,\mathrm{d}v=\frac{4\pi e j_1}{m k_0 v_0}\int\frac{\partial f_0/\partial v}{\omega-k_0 v}\mathrm{d}v. \quad (3.6)$$

This equation corresponds to a dispersion relation, which includes an arbitrary constant j_1, which does not depend on k_0 and ω, i.e., any dependence between the real ω and k_0 is possible.

Such a dispersion relation was first obtained by Van Kampen.[26] Van Kampen waves can be interpreted as polarization waves, accompanying the modulated beam.[3] We would like to draw attention to the conditions for the existence of linear stationary Van Kampen waves, which are as follows:

1) The beam should not deform during motion through the plasma. This condition is approximately satisfied if a density modulated electron beam, so weak that the excited electric field is negligibly small, is injected into the plasma. A heavy ion beam is even better.

2) After injection of the given undeformed beam into an initially unexcited plasma begins, some time should pass in order for the transient regime in the system to disappear.

We note that the experiments described in Sec. 2 clearly indicate the nonstationariness of regular waves in the usual beam-plasma systems, in spite of the fact that the conditions for the steady state (the beam is continuously injected) exist.

Based on the analysis presented above it is natural to relate this circumstance to the spatial variations in the variable density of the beam. To describe better the behavior of the latter in a plasma, we recall the basic properties of beam dynamics in a vacuum, which have been studied in detail in connection with problems in uhf electronics.[80,81]

4. BUNCHING OF AN ELECTRON BEAM IN A VACUUM

Let a one-dimensional monoenergetic electron stream pass through a stationary ionic background, compensating the static electric field. We shall modulate the beam velocity v harmonically in the initial section ($z=0$):

$$v|_{z=0}=v_0-v_1\sin\omega t,\quad j_b|_{z=0}=j_0.$$

The perturbations introduced into the stream by the modulations move together with the stream in the direction $z>0$. For small perturbations (the criterion for smallness will be presented below), the behavior of the system can be described in terms of traveling (fast and slow) linear waves of volume charge. On the other hand, for arbitrary amplitudes, another, also well-known approach involving the study of the evolution of moving electron bunches, turns out to be more effective. Here we shall present only a qualitative picture of this process. The method for describing it quantitatively will be briefly mentioned in connection with the analysis of beam dynamics in a plasma.

Let us suppose that the stream at times $t=2n\pi/\omega$ (n is an integer) is separated at the coordinate $z=0$ into parts by imaginary planes oriented perpendicular to the stream and moving along the z axis with constant velocity, equal to the unperturbed velocity of electrons v_0. Since the behavior of all such parts, having an extent $\lambda=2\pi v_0/\omega$, will be identical due to the periodicity of the process in time, we shall follow the electron layers filling one of them. For this purpose, we shall "mark" the layers by introducing the parameter t_0, the time at which the layer passes by the coordinate z_0, and we shall represent some of them, chosen with identical intervals Δt_0, by points in the phase plane $[(v-v_0)/v_1,\,\omega t]$ in a coordinate system moving with velocity v_0 (Fig. 6). For each coordinate $z\geq 0$ the time, measured from the moment when the first of two imaginary planes mentioned above arrives at this coordinate, is marked along the abscissa axis. The change in the position of the chosen electron layers in the phase plane as a function of z illustrates well the dynamics of the beam.

For small z the motion of points in the phase plane depends only on the initial state of the system. The points in the positive half-period in Fig. 6a will move to the left with increasing z and the points in the lower half-period will move to the right. As a result the electron density begins to increase in the vicinity of the phase π. This bunching process will further greatly depend on the ratio of the kinetic energy of modulation to the possible potential energy, related to charge separation in the system.

If the magnitude of this ratio, which we shall denote by η, is less than unity, then the electric field arising with bunching and opposing it (Fig. 6b), turns out to be sufficient first to stop the points in the phase plane and then impart to them a reverse motion. In this case, as shown in Fig. 6a–d, a periodic bunching and debunching of electrons along z will occur. In other words, these are nonlinear Langmuir oscillations, localized between two chosen moving planes. For $\eta\ll 1$, the degree of

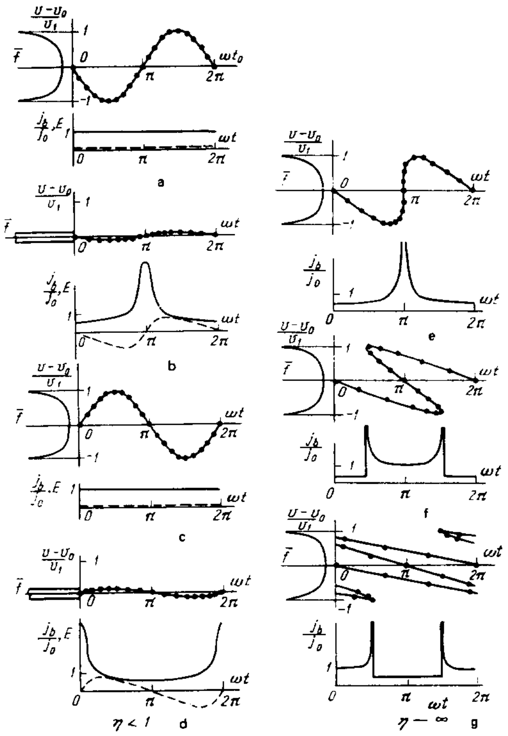


FIG. 6. Dynamics of electron layers in the phase plane, evolution of time-averaged distribution function $\bar{f}(v)$ (left), beam current density j_b (continuous lines), and electric field E (dashed lines). $\eta = v_1\omega/v_0\omega_b$, $\omega_b = \sqrt{4\pi e j_0/m v_{0B}}$—characteristic frequency of the beam; a) $z=0$, b) $z \lesssim (\pi/2)v_0/\omega_b$, c) $z=\pi v_0/\omega_b$, d) $z \lesssim (3/2)\pi v_0/\omega_b$, e) $z = S_k \equiv v_0^2/v_1\omega$, f) $z=3S_k$, g) $z=6S_k$.

periodicity of beam bunching is low: the oscillations are linear. In this case, the entire process can be described as resulting from superposition of fast and slow waves of volume charge propagating with constant amplitude.

For $\eta > 1$, bunches evolve in a completely different manner. Now, the electric field can no longer establish electron layers converging to the center of the period. They overtake one another and the motion becomes aperiodic. The extreme case, corresponding to $\eta \to \infty$ is illustrated in Fig. 6a, e, f, and g. All points in the phase plane move with increasing z parallel to the abscissa axis with constant velocities, fixed at $z=0$. An electron bunch, whose density at the center approaches infinity forms at $S_k = v_0^2/v_1\omega$: so-called phase focusing occurs. Beyond the coordinate of the phase focus, due to the fact that some electrons overtake others or, in other words, due to the toppling of the wave, the bunch consists of electrons that are distributed between three streams, moving with different velocities (Fig. 6f). The total width of the bunch increases, while its average density decreases, although narrow density peaks are observed at the boundaries. Still farther away from S_k the electrons move beyond the boundaries of the interval examined (Fig. 6g). As z increases, the number of combined streams into which the beam separates increases in an unbounded manner, and the electron density equalizes everywhere, with the exception of density spikes which become increasingly more narrow with their number remaining constant.[82]

5. THEORY OF SPATIAL EVOLUTION OF ELECTRON BUNCHES IN PLASMA

It is natural to expect that phase focusing will also occur in a plasma when conditions arise for modulation of the beam velocity. The corresponding analysis, more intuitive than well-justified, was made already in the first investigations, but it appeared that it could not be accommodated in any way to the rigorous linear theory of beam-plasma interaction. In addition, the modulating fields observed at the plasma boundary were much smaller than required for satisfactory correspondence of the quantity S_k with the experimental data. The problem remained unsolved until it was shown ambiguously in numerical[21,22] and laboratory[23] experiments that the mechanism of the nonlinearity of electron waves, excited by the beam in the plasma, is precisely phase focusing, leading to the formation of dense bunches; in addition, the linear stage of beam instability turned out to be included in a natural in this process, describing its initial stage. The nonlinear dynamics of a beam in a plasma under different conditions was then analyzed in a number of papers both as a temporal problem[83-89] and as a spatial problem.[90-96] Here we shall consider primarily the latter, as satisfying the conditions for real experiments with stationary beam injection into a plasma.

a) Computational method

Assuming that the beam entering the plasma is somehow modulated at frequency ω, we introduce the variable t_0, the time that an electron layer of the beam enters the plasma, and we shall use the equation of conservation of charge in the beam in the form

$$j_b(0, t_0)\,dt_0 = j_b(z, t)\,dt. \tag{5.1}$$

In the established periodic (in time), the time t at which the chosen layer passes through the coordinate z is related to the time t_0 that this layer enters the plasma by the obvious relation

$$\omega t = \omega t_0 + \frac{\omega z}{v_0} + g(z, t_0). \tag{5.2}$$

The function $g(z, t_0)$ represents the phase delay of the electron layer compared to the case of its unperturbed motion with velocity v_0. This function exhaustively describes the spatial evolution of the beam. In particular, the quantity $\omega^{-1}\partial g/\partial t_0$, as is evident from Eq. (5.1), characterizes the degree of deformation of the beam that occurs in the spatial interval $[0, z]$ under consideration. Phase focusing corresponds to oscillations of the function $\omega^{-1}\partial g/\partial t_0$ with amplitude equal to unity. If the current density in the beam is represented as a Fourier series of traveling waves of the form

$$j_b - j_0 = \sum_{n=1}^{\infty} \left\{A_n(z)\sin\left[n\omega\left(t - \frac{z}{v_0}\right)\right] + B_n(z)\cos\left[n\omega\left(t-\frac{z}{v_0}\right)\right]\right\}, \tag{5.3}$$

then the coefficients in this series

$$A_n(z) = \frac{\omega}{\pi}\int_0^{2\pi/\omega} j_b(z, t)\sin\left[n\omega\left(t-\frac{z}{v_0}\right)\right]dt,$$
$$B_n(z) = \frac{\omega}{\pi}\int_0^{2\pi/\omega} j_b(z, t)\cos\left[n\omega\left(t-\frac{z}{v_0}\right)\right]dt \tag{5.4}$$

are also expressed by means of relations (5.1) and (5.2) in terms of the function $g(z, t_0)$.

To find $g(z, t_0)$, it is necessary to integrate the equations of motion of the electron layers in the electric field $E(z, t_0)$, taken along their trajectories, including the initial modulation of the beam $\tilde{v}\,|_{z=0} = v_1 \sin\omega t_0$. In the simplest case of small perturbations of the velocity $\tilde{v}(z, t_0) \ll v_0$ the relation between g and E has the form

$$g(z,\ t_0) = \frac{e\omega}{mv_0^3}\int_0^z dz \int_0^z E(z,\ t_0)\,dz + \frac{v_1 z\omega}{v_0^2}\sin\omega t_0. \qquad (5.5)$$

The system of equations is closed if the electric field is expressed in terms of the harmonics of the beam current (5.4). For a cold plasma this can be done by means of Eq. (3.1). In the steady state regime, we obtain

$$E = 4\pi\sum_{n=1}^{\infty}\frac{n\omega}{\omega_p^2 - n^2\omega^2}\left\{B_n(z)\sin\left[n\omega\left(t - \frac{z}{v_0}\right)\right] - A_n\cos\left[n\omega\left(t - \frac{z}{v_0}\right)\right]\right\},$$
$$n\omega \neq \omega_p. \qquad (5.6)$$

b) Role of plasma dispersion and higher-order harmonics

It is expression (5.6) that reveals the fundamental difference in the structure of electric fields, accompanying the modulated beam in plasma and in vacuum.

In vacuum (this case is also described by Eqs. (5.1)–(5.6), if in (5.6) we set $\omega_p = 0$), the phase difference between the harmonics of the electric field and the charge density is such that the field always strives to stretch a bunch and to eliminate deformation of the beam (see Fig. 6). In plasma, on the other hand, the beam, whose density is harmonically modulated at a frequency $\omega < \omega_p$, excites a field which is phase shifted by π compared to the vacuum case (the dielectric permittivity is negative). This occurs because the plasma electron density oscillates with opposite phase relative to charge oscillations in the beam so that inside the electron bunch there is an excess charge of positive ions. Thus the electric field in the plasma at frequencies $\omega < \omega_p$ compresses the condensations appearing in the beam, moreover, all the more strongly the higher the amplitude of the first harmonic of the beam density. As a result, the bunching process has an exponential character not only at the initial stage, described by the linear theory, but it is nearly exponential at later times as well. Phase focusing, in contrast to the vacuum case, occurs with the smallest amplitudes of initial velocity modulation. The coordinate of the phase focus, taking into account the exponential increase in velocity modulation, was approximately calculated in Refs. 23 and 91. In particular, for ω close to ω_p, it is determined by the equation:

$$S_v \approx \frac{1}{\gamma}\ln 2\gamma S_k, \qquad (5.7)$$

where γ is the spatial increment for growth of oscillations in the linear theory, and S_k is the kinematic coordinate of the phase focus (see Sec. 4). It is now clear that due to the logarithmic dependence of S_v on v_1, the exact value of the latter in all estimates does not play that prominent role, which caused the initial concept of phase focusing to be criticized. Moreover, phase focusing can also occur in plasma in the absence of initial modulation of the beam velocity. It is enough to modulate the beam density a little. The expression for the coordinate of the phase focus in this case has the form[97]

$$S_j \approx \frac{1}{\gamma}\ln\frac{2j_0}{j_1}, \qquad (5.8)$$

where j_1 is the amplitude of the harmonic modulation at $z = 0$.

Meanwhile, it follows from expression (5.6) that due to the distortions resulting from phase focusing of the initially harmonic profile of the beam density, higher-order harmonics should appear in the total electric field. Under the condition that $\omega_p/2 < \omega < \omega_p$, these harmonics are phased relative to the harmonics of the beam density just as in a vacuum and, therefore, they will inhibit compression of bunches. As a result, the electric field wave must acquire a specific profile,[91] presented in Fig. 7, while the structure of electron bunches becomes more complicated even before the trajectories intersect. As expected, the repulsive action of the higher-order harmonics increases with increasing ω_p/ω.

A decrease of the frequency of the initial harmonic modulation to values $\omega < \omega_p/2$ must lead to a new effect, arising due to the fact that the second harmonic, created with the distortion of the wave profile, falls into the region of amplification of oscillations. Since in this case the spatial increment of the wave with frequency 2ω is much larger than the increment of the initial wave, beginning with some distance the second harmonic dominates.[90,91,98] Actually, the beam turns out to be modulated at frequency 2ω and two bunches form within a period of the initial modulation. As ω/ω_p decreases further, a higher order harmonic can appear.

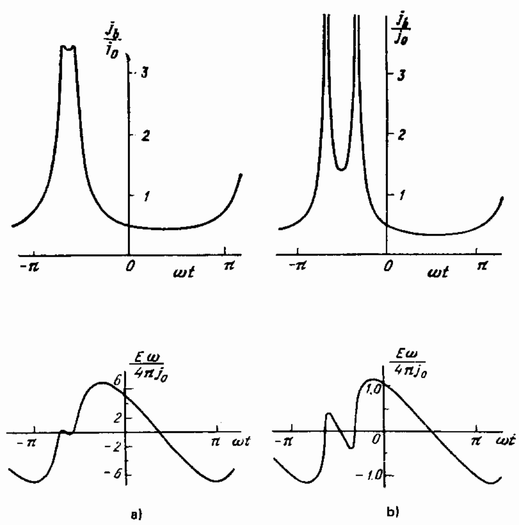


FIG. 7. Computed wave structure at the coordinate of the phase focus S_V. The time origin is arbitrary, but identical for the curves of j_b and E. a) $\omega = 0.91\omega_p$; b) $\omega = 0.6\omega_p$.

c) Beam behavior beyond the phase focus

The dynamics of bunches in a cold plasma at distances exceeding S_v has been studied by numerically calculating the trajectories of electron layers.[99] It follows from this calculation, first, that in spite of the counteraction of the higher order harmonics of the field to compression of bunches, in phase focusing some electron layers overtake others. Observation of bunches in a system of coordinates moving with velocity v_0 reveals that they consist of electrons oscillating at the bottom of potential wells of the first harmonic of the wave. These electrons are called "trapped," in contrast to "free flight" electrons in the beam, which overcome the potential barriers between the wells. The first phase focusing in the coordinate S_v indicates that trapped electrons, initially uniformly distributed in some interval of phases of the wave, having completed 1/4 of the complete oscillation in a well whose depth increases exponentially with time, collect at the bottom of the well. In the next quarter period, the extent of the bunches increases. For this reason, the amplitude of the wave in the corresponding spatial interval decreases. Subsequently, alternating bunching and debunching of trapped electrons lead to spatial oscillations of the wave amplitude.

As is evident from Fig. 8, if ω is sufficiently far from ω_p then with the first phase focusing a significant part of the electrons is distributed between two distinct groups. These groups of electrons ("macroparticles") are preserved for some further time as well, oscillating with opposite phase in the wells of the wave. Spatial changes in the amplitude in this case are nearly periodic.

If, on the other hand, ω is close to ω_p, then the amplitude of the wave after the first distinct maximum changes with distance irregularly. The nonperiodicity of oscillations in both cases is related to the fact that due to the nonparabolicity of the potential wells the oscillation frequencies of different trapped electrons do not coincide, and neither do their amplitudes. Moreover, these parameters do not remain constant even for any one electron. For this reason, the trajectories in the phase plane represent complicated, generally speaking, open curves and, in addition, the particles can go from a trapped state to a free transit state. The complexity of the trajectories is a result of both the effect of higher-order harmonics and the fact that the depth of the wells, in which the electrons oscillate, changes with a characteristic time close to the period of oscillations of the electrons themselves.

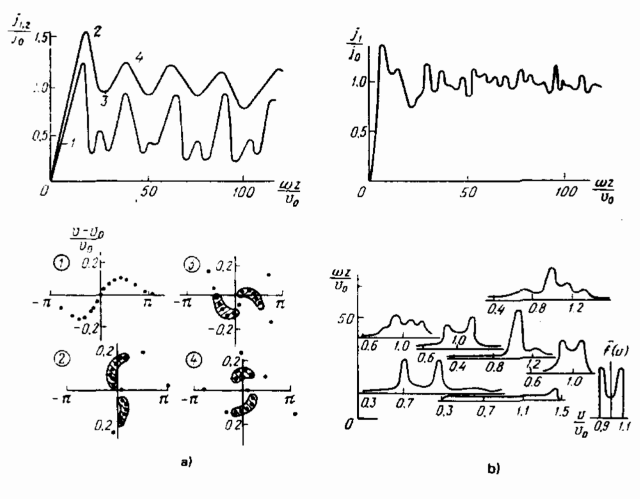


FIG. 8. Amplitudes of harmonics of the beam current density j_1 and j_2, distribtuion of electron layers in the phase plane (they are presented for the coordinates noted) and the averaged velocity distribution function of electrons (b, bottom) as a function of distance $v_1/v_0 = 0.1$, $\omega_b^2/\omega^2 = 0.01$, a) $\omega_p^2/\omega^2 = 1.6$; b) $\omega_p^2/\omega^2 = 1.1$.

We should note the form of the velocity distribution function for the electrons in the beam averaged over a period of the high-frequency oscillations. A calculation shows that it characteristically has several pronounced maxima. Under the conditions, corresponding to Fig. 8, the electric field of the wave is so high that already at the beginning of the nonlinear stage of the interaction the velocity of the monoenergetic beam oscillates with an amplitude of the order of the unperturbed velocity of the beam. Naturally, the averaged distribution function also has the same width, although the true distribution function is not smeared out.

Using a description in terms of the separation of the beam into bunches, it is possible to determine the maximum amplitude of the growing wave. For this, it is necessary to substitute into expression (5.6) the value of the amplitude of the fundamental harmonic of the current density, attained with the first phase focusing, namely, a quantity of the order of the unperturbed density of the beam.[1)]

d) Resonance conditions

The above analysis of the formation and evolution of bunches, as noted already, concerns nonresonant interaction conditions. The steady state regime of oscillations in the resonant case, when $\omega = \omega_p$, can be analyzed only if dissipative mechanisms, limiting the amplitude, are included. This is done in Refs. 95 and 96 by introducing some effective collision frequency for plasma electrons ν, which is justified, at least, for Coulomb interactions of particles, when collisions between fast electrons in the beam and plasma particles can be neglected, and only the collisions between slow plasma electrons and ions need be included.[86]

Introducing ν into the dielectric permittivity of the plasma eliminates the divergence of the amplitude and changes the phase relations between the bunches and the wave that they excite. The latter results in a monotonic decrease in the energy of the beam with distance (Fig. 9) in disagreement with the fact that under nonresonant dissipationless conditions the beam in the steady-state regime passes through the plasma without energy losses, in spite of the spatial amplification of the wave.

[1)] For a beam with an average, over the period, current density j_0, the maximum possible value of the amplitude of the first harmonic equals $2j_0$. Such an amplitude would occur if all electrons in each period were collected in infinitely thin bunches.

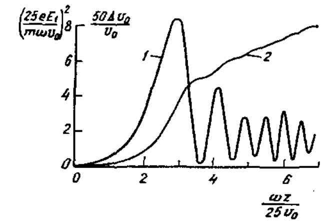


FIG. 9. Spatial dependences of the square of the amplitude of the critical field E_1 (1) and deceleration of the beam Δv_0 (2), calculated including dissipation with $\omega=\omega_p(\nu=625\cdot\omega_b^2/2\omega_b)$.

We note that higher-order harmonics of the field are neglected in this numerical calculation and for this reason the details of the true behavior of the wave beyond the first maximum can differ from that presented in Fig. 9. This conjecture is based in part on a comparison of the temporal evolution of the beam in the plasma when higher-order harmonics are and are not included, as was done in Ref. 100.

e) Oscillations of bunches in a plasma with $T_e \neq 0$

Another factor, limiting the amplitude of the wave at $\omega=\omega_p$ is the thermal motion of electrons in the plasma, leading to spatial dispersion. In this case, the weak beam amplifies waves propagating somewhat slower than itself. The nonlinear dynamics of a beam in such a wave were studied in Refs. 92 and 93 neglecting higher-order harmonics of the field. As follows from the universal solution obtained, the process is to a certain extent similar to the behavior of a beam in a traveling wave tube. Dense bunches form in the decelerating phase of the wave and then, with only small deformations, they oscillate as a whole in potential wells, periodically going from accelerating phases into decelerating phases. As a result of the periodic exchange of energy between bunches and the wave, its amplitude also changes periodically with distance and, in addition, these spatial oscillations are more regular (Fig. 10) than in the case of a cold plasma, when they are caused by the sequential bunching and debunching of the beam.

Further numerical investigations[94,101] showed that regular oscillations of bunches in the presence of weak dissipation in the plasma must break down. Bunches already breakup at $\nu/\omega_p=2\cdot10^{-3}$ in the second period.[101] This is related to the considerable decrease in amplitude of the wave in the first minimum compared to the dissipationless case. Due to the smallness of the amplitude, a large part of the trapped electrons spills out of the potential well here, and later the bunching of the beam no longer attains a high degree. Breakup of bunches is especially clearly demonstrated in the computer experiment in Ref. 102 (Fig. 11).

As noted in Ref. 92, the analysis of the wave dynamics taking into account T_e concerns only systems with a very weak beam, whose density satisfies the inequality

$$\left(\frac{n_b}{n_p}\right)^{1/3} \ll 3\frac{v_T^2}{v_0^2}. \tag{5.9}$$

If the density of the beam is such that this inequality is not satisfied, then for a kinetic description of the plasma it is necessary to take into account its nonlinearity. However, it is evident that for plasma oscillations with large amplitudes, when $v_T/\tilde{v}<1$, but $n_1/n_p \ll 1$ the linearized hydrodynamic description of the plasma and the corresponding results obtained by neglecting the thermal velocities of electrons are still applicable.

f) Trapping of plasma electrons

Neither an analytic theory nor a universal numerical calculation of electron waves in a beam-plasma system taking into account the nonlinearity of the plasma component exists as yet. Information on the dynamics of the beam and plasma in this case is obtained from computer experiments concerning spatial-temporal[21] and temporal problems.[103] As also in a linear plasma, bunches forming in the beam are not conserved, and this leads to spatial localization of oscillations.[21] In the region of maximum amplitude, the plasma electron density wave is no longer harmonic, but has a sharply pointed crest (so-called cnoidal wave[5]). The ratio n_1/n_p reaches values much greater than unity. In this case, the order of electron layers can change even in an initially cold plasma. Part of the electrons on a crest breaks away from the main mass of particles and moves in the direction of the beam: trapping of plasma electrons occurs. The role of this process increases with decreasing beam energy and increasing plasma temperature.[103] The nonlinearity of the plasma together with dissipation and spatial dispersion is an additional factor limiting the amplitude of the electric field.

In concluding this section, we present Table II, which shows the interrelation of different theoretical investi-

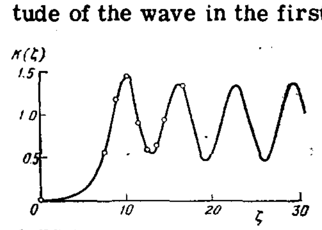


FIG. 10. Amplitude of the electric field wave as a function of distance for $T_e \neq 0$.

$$\zeta=\frac{\omega_p z}{v_0}\left(\frac{n_b v_0^2}{3n_p v_T^2}\right)^{1/3},$$

$$K=\frac{E_1}{\sqrt{4\pi n_b m v_0^2\left(\frac{n_b}{n_p}\right)^{1/3}\left(\frac{v_0}{3v_T}\right)^{8/3}}};$$

n_b is the unperturbed beam density, v_T is the average thermal velocity of electrons in the plasma.

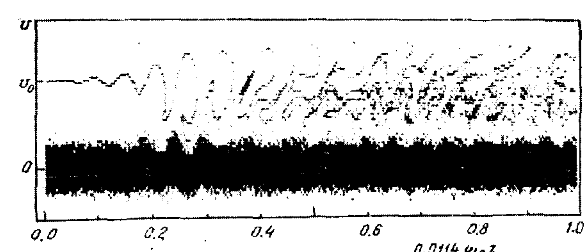
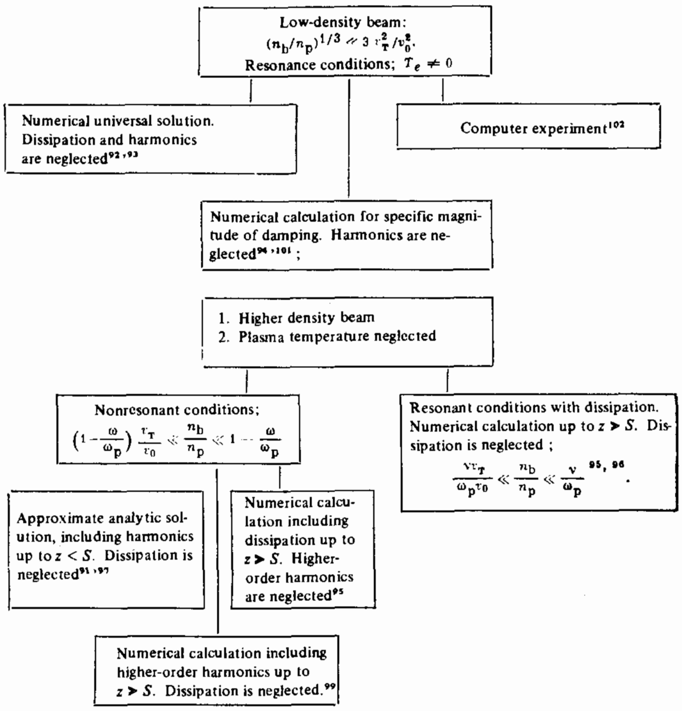


FIG. 11. Distribution of electrons in the plasma and in the beam in the phase plane $[v;\ 0.0114\omega_p z/v_0]$ at some time during the steady-state regime in a numerical experiment. $n_b/n_p=10^{-3}$, $v_0/v_T=9.9$.

TABLE II. Theoretically studied cases of spatial evolution of electron waves in a beam–plasma system.



gations of the spatial evolution of bunches in a plasma and, in particular, the criteria for applicability of particular calculations are indicated.

6. EXPERIMENTAL OBSERVATIONS OF ELECTRON BUNCHES IN PLASMA

a) Data on phase focusing

The reader has probably already thought of the fact that the theoretical conclusions to a certain extent agree with the facts presented in Sec. 2. This agreement is also manifested in the presence of harmonics, in the axial dependence of their amplitudes up to the first minimum, and in the spatial evolution of the smoothed distribution function. But, direct evidence also exists for bunching of the beam in a plasma. The first such evidence[23] was obtained even before the rigorous theoretical investigations, described in the preceeding section, and was based on a comparison of the location of the glowing region of intense nonlinear oscillations (see Fig. 5) with the coordinate of the phase focus, calculated from the approximate equation (5.7). It turned out that the first coincides with the second for all possible variations in the parameters. The curves shown in Fig. 12 are examples. The results of Ref. 23 simultaneously confirm the validity of relation (5.7). Equation (5.8) also satisfactorily agrees with experiment.[97]

b) Wave "toppling"

Experimental information on the distribution of particles in the phase plane should apparently be viewed as the most convincing illustration of phase focusing of the

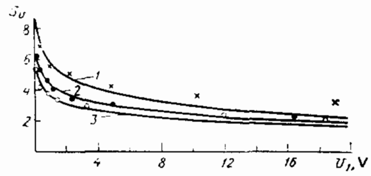


FIG. 12. Experimental dependences of the distances to the regions of intense oscillations on the amplitude of the modulating uhf voltage U_l under different conditions (different marks). Continuous lines: corresponding computed curves of S_v, 1—$\nu = 0.82$ cm^{-1}, 2—$\nu = 1.15$ cm^{-1}, 3—$\nu = 1.5$ cm^{-1}

beam and subsequent toppling of the wave (Fig. 13). The first such data for a beam in a plasma were obtained by means of a specially developed electron-beam analyzer, measuring the particle distribution function "instantaneously" over a time much shorter than the period of the oscillations.[104] With somewhat worse resolution, analogous results were also obtained with an energy analyzer with a decelerating field, which is switched on only at certain phases of the wave.[47,48]

The absence of broadening of the instantaneous distribution function under conditions when the average energy spread of electrons in the beam is very large led to the conclusion that the previous interpretation of the experimental results based on the quasilinear theory is

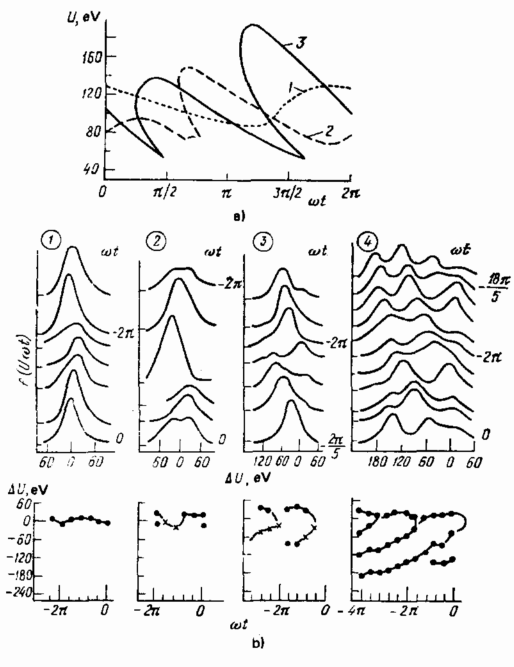


FIG. 13. Measured electron distributions in the beam in the phase plane [ev, ωt]; eU is the kinetic energy of the electrons. a) Data from Ref. 104: 1) profile of beam velocity wave for $z < S_v$ (see (5.7)). 2, 3) formation of three-velocity stream for $z > S_v$, 3) toppling of wave twice within a period; b) instantaneous distribution functions $f(U)$ at different ωt and corresponding wave profiles of the beam velocity[48] in front of the phase focus (1), near the phase focus (2), beyond the phase focus (3), and in the region of strong damping (see Fig. 2, f) (4), $eU_0 = 390$ eV, $\Delta U = U - U_0$.

not justified and it is necessary to use the concept of bunches of particles.[71]

c) Similarity of the dynamics of modulated and unmodulated beams in plasma

If we do not appeal to direct, but complex "instantaneous" measurements, then information on the nature of the beam–plasma interaction can be obtained by measuring the electron velocity distribution function, averaged over a time greater than the period of the uhf oscillations, but shorter than the characteristic time of low-frequency oscillations in the system. According to calculations, such a distribution function for the single-mode regime must be not monotonic, but must have a multihump structure (Fig. 8). It turned out that this is in fact observed in experiments with both modulated and unmodulated beams (Fig. 14).[105] The nonmonotonic form of the distribution function again confirms the universality of the description in terms of bunches in a monoenergetic beam-plasma system.

We note that the fact that prior modulation of the beam is not necessary to excite a longitudinal, nearly monochromatic wave appears to be natural, if we keep in mind the following unavoidable factors, which narrow the wave spectrum in the system:

1) Strong frequency nonuniformity of the gain, increasing with distance[25,106];

2) Feedback between the region of intense oscillations and the point of beam entry into the plasma[107].

d) Profile of electric field wave

As follows from a calculation of the functions $E(t)$ (Fig. 7), waves excited by a strongly bunched beam, must be nonharmonic waves. The presence of higher-order harmonics in the spectrum of the oscillations (Fig. 2) can be viewed as an experimental confirmation of this fact. However, because it is not clear which particular physical quantity is associated with the oscillations detected by the probe and there is no information on the phase relations between the harmonics, it is impossible to reconstruct the profile of the electric field wave from the spectrum.

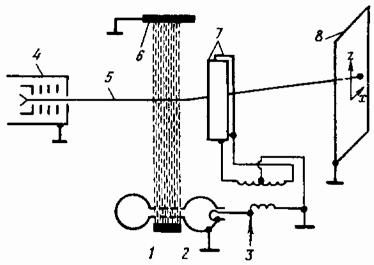


FIG. 15. Diagram of electron beam probe of beam-plasma system. 1) Cathode, 2) modulating resonator, 3) uhf power input, 4) auxiliary electron gun, 5) low intensity electron beam, 6) collector.

The wave profile $E(t)$ in a beam–plasma system was determined directly by electron-beam probing.[108] A weak-current electron beam 5 (Fig. 15) was passed perpendicularly to the main beam through its axis, without exciting the system under study. Deflecting in the field of the plasma oscillations along the z axis, and then in the synchronous harmonic field between the plates 7 along the x axis, the beam traced a figure on a luminescent screen 8, from which the amplitude of the electric field wave and its profile were found.

Data on the spatial evolution of the wave were obtained by probing the system at different distances z from the point of entry of the beam into the plasma.[109] The characteristic series of figures on the luminescent screen and the corresponding curves of $E(t)$ are shown in Fig. 16 by the continuous lines. It is evident that the

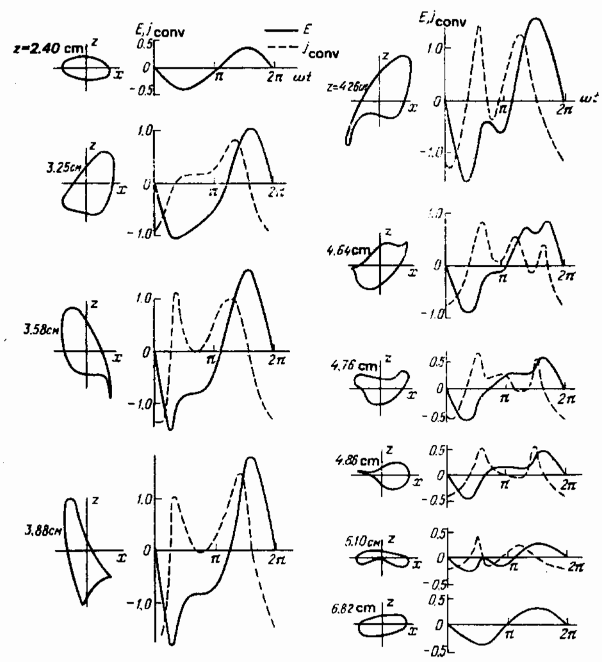


FIG. 16. Spatial evolution of temporal profiles of electric field wave E (continuous curves) and convective current densities in the plasma j_{conv} (dashed curves). The corresponding figures appearing on a luminescent screen are presented on the left.

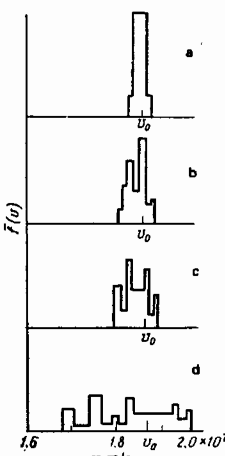


FIG. 14. Averaged velocity distribution functions of beam electrons, measured over a short time, at different stages of beam–plasma interaction in a beam–plasma discharge. p (mm Hg) $5\cdot10^{-6}$ (a), $1.3\cdot10^{-3}$ (b), $2\cdot10^{-3}$ (c) and $2.7\cdot10^{-3}$ (d).

wave profile with strong beam bunching (Fig. 16e) is close to that predicted theoretically. The difference lies only in the fact that the sharp deviation from harmonicity, arising with the passage of the bunch, is observed not at the zero phase of the fundamental harmonic, as in Fig. 7, but somewhat displaced into the decelerating half-period. This phase displacement of the bunch can be explained by the presence of dissipation and nonzero electron temperature, factors which are not taken into account in the theoretical calculation of the profile.

e) Dynamics of bunches

Using the dependences $E(t)$, it is possible to establish the fine structure of electron bunches.[109] To do so, it is necessary to keep in mind the fact that the derivative $\partial E/\partial t$ (dashed lines in Fig. 16) determines the convective current j_{con} in the plasma, while the latter represents the superposition of an almost sinusoidal current of the fundamental harmonic (this current is created by plasma electrons) and the current in the bunch, which forms the well in the positive half-period of the function $j_{con}(t)$. Therefore, the relief of the well is determined by the profile of the bunches, while the spatial evolution of the well is a good illustration of their dynamics, in which we can identify the following most important points.

1) The electron bunches formed are not conserved in further motion: their size increases and they are observed to break up into two parts ($z = 4.64$ cm). The bunches later again coalesce. At the next stage, the debunching becomes so strong that the wave again becomes practically sinusoidal ($z = 6.82$ cm).

2) The electron bunches break up more rapidly than their phase changes. From Fig. 16 it is evident that the center of the bunches is always displaced into the decelerating half-period of the field. Only when the bunches break up into two parts can one of the parts go over into the accelerating phase of the wave ($z = 4.76$ cm).

These facts, concerning a comparatively dense beam, agree with the theory of its evolution in a cold plasma (Sec. 5c). The same conclusion can be arrived at concerning the results obtained by probing the system at different ω/ω_p. As this ratio decreases, the contribution of higher order harmonics increases and, in particular, almost 100% conversion of the initial wave into the wave with frequency 2ω or 3ω was observed.

The dynamics of a less dense, initially unmodulated beam in a plasma situated in a magnetic field was already investigated by the decelerating field method, mentioned above, with the analyzer switched on at certain phases of the wave.[46,47] As is evident from Fig. 17, in this case a transition is observed from deceleration of a bunch to its acceleration, which agrees with the theoretical description of the oscillations of bunches (Sec. 5e).

Thus direct experimental data on the evolution of bunches in the vicinity of a phase focus exist (in the interval $0 < z \lesssim 3S_v$). Beyond the phase focus, depending on the conditions, both of the theoretically predicted wave damping factors—debunching and removal of energy from the field by bunches—are observed. However, compared to the first theoretical results, this spatial damping is deeper and, apparently, irreversible, which indicates the complete breakup of bunches. We can identify the following reasons for their rapid breakup:

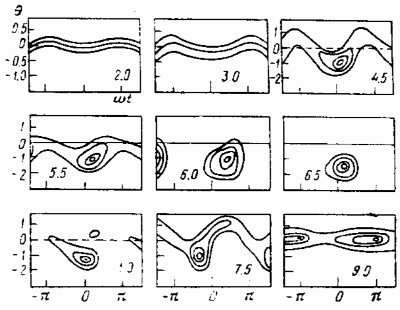


FIG. 17. Electron density distribution in the beam in the phase plane as a function of distance from the origin of the system. $\theta = (v - v_0)/v_0(n_b/2n_p)^{1/3}$. Lines of maximum density and half-width of the instantaneous distribution function are indicated in the coordinates 2.0 and 3.0. The width is close to the instrumental width. For large z the inner contour corresponds to maximum density, normalized to 1. For the other contours: 0.8, 0.5, and 0.2.

1. Angular divergence of the beam under the action of the high frequency field in experiments without a magnetic field.[23,109] This divergence can, in particular, explain the fact that the amplitude of the wave does not increase significantly with repeated merging of bunches (Fig. 16).

2. Dissipation in the plasma together with the large velocity spread in a bunch. It is known that the attainable degree of beam bunching with a discrete collection of velocities is much smaller than for a single-velocity beam.[110] For this reason, if electrons leave the trapping region on the section with decreasing amplitude, then their further strong bunching is no longer possible (Fig. 11). The critical effect of damping on the preservation of bunches was also observed in an experiment with a traveling wave tube.[111]

3. Parametric instability of a monochromatic wave, leading to the growth of side-band waves. This will be discussed in greater detail in Sec. 7d. Here we refer only to the above-mentioned experiment with traveling wave tubes, in which break-up of bunches was observed accompanying excitation of a growing wave with frequency close to the fundamental frequency.

4. Dependence of plasma parameters in a beam-plasma discharge on the amplitude of the oscillations. The decrease in plasma density due to partial debunching of the beam and the corresponding decrease in the wave amplitude in the region beyond the phase focus can facilitate total break-up of bunches.

7. STATIONARY PERIODIC BERNSTEIN-GREEN-KRUSKAL WAVES

a) Possibility of equilibrium distributions of trapped electrons

Bunches, forming in the usual beam–plasma experiments when an unmodulated or weakly modulated beam is injected into the plasma and moving synchronously with the wave are thus nonequilibrium formations. The exponential process of electron bunching is replaced by a quite rapid stage in which bunches break up. As a result, the amplitude of the waves excited by such beams changes strongly (by several orders of magnitude) along the direction of motion of the beam, so that the oscillations are practically localized in a small region at some distance from the point of beam entry: the waves are not stationary.

Meanwhile, it was already predicted in Refs. 13 and 27 that stationary nonlinear waves with nondeforming bunches of trapped particles can exist in a plasma. The theory of such stationary waves (usually called BGK waves) is based on the assumption that a stationary plane wave already exists in the plasma. Then, in a moving system of coordinates in which the wave is stationary, all quantities are independent of time and it is possible to use the general solution of the stationary nonlinear Vlasov equation for the electron distribution function

$$f=f\left(\frac{mv^2}{2}-e\varphi\right), \qquad (7.1)$$

where φ is the electric potential of the wave and v is the velocity of the electrons. To simplify the problem, we shall neglect the perturbation of the background ion density n_i of the wave. We shall separate the electrons into two groups: electrons that are trapped by the wave and electrons that are not trapped. In this case, substituting (7.1) into the Poisson equation

$$\frac{\partial^2\varphi}{\partial z^2}=4\pi e\left(\int f\,dv-n_i\right), \qquad (7.2)$$

we obtain an integral equation for the normalized energy distribution function of trapped electrons $f_{trap}(\mathscr{E})$:

$$4\pi e n_{trap}\int_{-e\varphi}^{-e\varphi_{min}}\frac{2f_{trap}(\mathscr{E})\,d\mathscr{E}}{\sqrt{2m(\mathscr{E}+e\varphi)}}=\frac{\partial^2\varphi}{\partial z^2}+4\pi e n_i-4\pi e n_{transit}\int_{-e\varphi_{min}}^{\infty}\frac{2f_{transit}(\mathscr{E})\,d\mathscr{E}}{\sqrt{2m(\mathscr{E}+e\varphi)}}, \qquad (7.3)$$

where n_{trap} and $n_{transit}$ are the average densities of trapped and transit electrons, respectively, $\mathscr{E}=(mv^2/2)\sim e\varphi$. It follows from Eq. (7.3) that specifying arbitrarily the profile of the wave $\varphi(z)$ and the energy distribution of untrapped electrons $f_{transit}$, it is always possible to select a matching equilibrium distribution function of electrons trapped in the potential wells of the wave. It is because the distribution of trapped particles in the BGK wave is an equilibrium distribution that these waves differ from Van Kampen waves, where the reverse effect of the field on the resonant particles is neglected (as a result of which, Van Kampen waves are not realizable in practice).

Specific examples of BGK waves, corresponding to different distribution functions of trapped electrons, were examined in Refs. 13 and 112. Since electron bunches in neighboring wells are separated by a potential minimum, their distributions can be independent of one another and aperiodic solutions are also possible in addition to periodic solutions, for example, in the form of separate pulses of the potential.[13]

b) Excitation of stationary waves by electron bunches and role of plasma dispersion

In the usual analysis of stationary waves in a system of coordinates moving with the wave, the conditions for their excitation[27,113] and the physical reason for the equilibrium of bunches remain unclear. There exist only qualitative considerations,[3,85] confirmed by numerical modeling,[114] of the fact that a type of stationary wave can be established as a result of the temporal development of the beam-plasma instability.

Another possibility, predicted in formulating the problem of the behavior of preformed electron bunches periodically injected into the plasma, is much more realistic.[115] Stationary waves with such bunches are also described by solutions of Eq. (7.3). But, together with this, the analysis of these waves within the framework of the dynamic equations (5.1)–(5.6) also reveals the conditions for their excitation. If the current pulses of the beam are given a rectilinear form in the injection plane (Fig. 18), then for a definite pulse duration a quasistationary wave is established at finite z. The electric field of the wave vanishes at values of the phase at which the electrons in the beam are concentrated. For this reason, in the absence of initial velocity modulation, cold bunches propagating synchronously with the excited wave are not deformed. Such an equilibrium is a result of dispersion leading to competition between the harmonics of the electric field of the wave. Part of them strive to compress the bunches, while the remaining harmonics strive to prevent compression (Sec. 5b).

The physical meaning of the stationary state is especially easily revealed in the case of very thin bunches ($\tau/2\pi\ll 1$), injected into the plasma at a frequency ω somewhat less than the plasma frequency. Substituting into expression (5.6) the amplitude of the first harmonic of the current density $A_1\approx 0, B_1\approx 2j_0$, we find the field created by the plasma charge inside a bunch and compressing it:

$$E_p\approx 4\pi\frac{2j_0\omega}{\omega_p^2-\omega^2}\sin\left[\omega\left(t-\frac{z}{v_0}\right)\right]\approx 8\pi\frac{j_0\omega^2}{\omega_p^2-\omega^2}\left(t-\frac{z}{v_0}\right). \qquad (7.4)$$

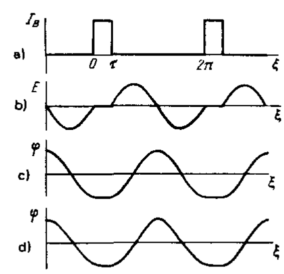


FIG. 18. Computed (a–c) and experimentally measured (d) structure of a stationary wave excited by cold electron bunches ($\xi=\omega t-\omega z/v_0$).

The intrinsic field of a bunch, on the other hand, which pushes it apart, can be determined from Gauss's theorem:

$$E_b = -4\pi j_0 \cdot \frac{2\pi}{\tau}\left(t - \frac{z}{v_0}\right). \tag{7.5}$$

Equating to zero the resulting field $E = E_p + E_b$, we find the equilibrium duration of the bunches:

$$\frac{\tau}{2\pi} \approx \frac{\omega_p - \omega}{\omega}. \tag{7.6}$$

A more rigorous analysis[28,116] shows that cold bunches are in equilibrium for any $\omega < \omega_p$ and $n\omega \neq \omega_p$ if the condition

$$\frac{\tau}{2\pi} = \frac{\omega_p - s\omega}{\omega_p}, \tag{7.7}$$

where s is an arbitrary integer, is satisfied.

If the temperature of the bunches T_b differs from zero, then compression of bunches is likewise prevented by the gas kinetic pressure of the electrons. As ω approaches ω_p and T_b increases, the influence of higher-order harmonics of the field on the shape of the bunches can be neglected compared to the thermal motion. Assuming that the beam electrons have a Boltzmann distribution in the harmonic stationary wave, it is possible to determine the dependence of the wave amplitude on T_b (Fig. 19) and the equilibrium profile of the bunches.[117] For $T_b > \beta$, bunches cannot be in equilibrium.

We emphasize again that, in any case, the equilibrium arises only for $\omega < \omega_p$, when the dielectric permittivity of the plasma is negative and the bunches are located in potential wells. In this case, the approximation of linearity of plasma oscillations used for simplicity is not of fundamental significance. In particular, the parameters of the stationary wave in a nonlinear plasma with trapped cold bunches are determined in Ref. 118.

c) Quasistationary waves in a dissipative plasma

Since the idea of equilibrium of bunches concerns a steady-state oscillatory regime, while the existence of such a regime presumes the presence of some dissipation mechanism in the plasma, it is necessary to clarify the effect of the plasma on the waves under study. In Ref. 119, damping was included, as usual, by introducing some collision frequency ν into the equation of motion of plasma electrons, as a result of which the term $\nu\partial E/\partial t$ appears on the left side of Eq. (3.1).

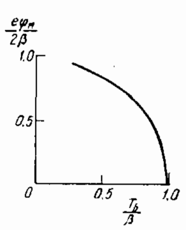


FIG. 19. Amplitude of the potential φ_M of the stationary wave as a function of the temperature of the bunches T_b. $\beta = \frac{n_b \omega_p m v_0^2}{2 n_p (\omega_p - \omega)}$.

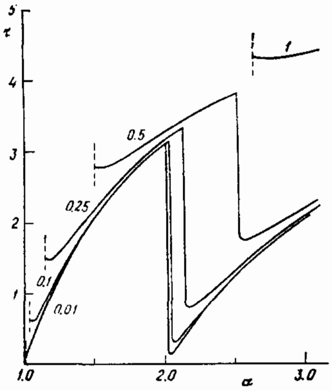


FIG. 20. Equilibrium duration of cold bunches as a function of $a = (\omega_p/\omega)$ for different magnitudes of dissipation ν/ω. Equilibrium is impossible to the left of the dashed lines.

It turned out that waves with constant profile and amplitude can also propagate in such a plasma. Damping leads to a definite shift in the phase of the electric field with respect to the dissipationless case, so that cold bunches are displaced from the bottom of moving potential wells onto their decelerating slopes. As a result, the equilibrium shape and duration of bunches do not change significantly. More significant is the fact that bunches, moving into the bulk of the plasma, decelerate monotonically and, correspondingly, the quasistationary wave slows down: its length decreases. In addition, the range of frequencies of these waves (Fig. 20) is shortened: the stronger the dissipation, the farther the boundary of existence of equilibrium moves away from ω_p.

d) Instability of bunches in quasistationary waves

Of course, any real bunch former cannot provide the exact parameters of bunches injected into the plasma required for equilibrium. On the other hand, plasma fluctuations have a perturbing action on the system. For this reason, there arises the question of the stability of the equilibrium of bunches.

The behavior of perturbations in stationary waves was examined in a number of papers.[120-125,116,117] Different mechanisms for their instability were predicted and analyzed. Since this is a very broad problem, which has not been completely solved and is closely related to the more general problem of wave interaction in a plasma, a complete discussion of the problem would take us far outside the scope of this review. Here, we shall consider only two aspects directly related to excitation of quasistationary waves by cold bunches.

First, we note that for stationary injection of bunches into a plasma the possibility of realizing BGK waves depends both on the spatial increment of perturbations and on the initial magnitude of these perturbations. Taking into account the possibility of stabilizing the frequency of the bunch former, we can expect that the strongest initial perturbations will be identical, for each bunch, deviations from the equilibrium profile. As the bunches move into the bulk of the plasma, these perturbations, not leading to the appearance of new frequencies in the spectrum, but changing the relation of the amplitudes of the harmonics of the nonlinear wave, evolve as $\exp(i\varkappa z)$,[116] and, in addition, $\varkappa$ must satisfy the dispersion relation

$$\frac{\sqrt{\varkappa^2-h^2}}{\varkappa}\sin\frac{\varkappa[1-(s\omega/\omega_p)]\pi}{(\omega/\omega_p)\sqrt{\varkappa^2-h^2}}=0, \qquad (7.8)$$

where $h^2=4\pi e j_c/mv_0^3$ and j_c is the current density in a bunch.

It follows from Eq. (7.8) that the perturbations of the equilibrium shape of a bunch do not grow in space for $\omega/\omega_p>\tau/2\pi$ or, which is the same thing, for $S>(\omega_p-\omega)/\omega$. In particular, perturbations are not amplified in waves for which $\omega_p/2<\omega<\omega_p$. On the other hand, for lower frequencies, narrow bunches are conserved, while the wide bunches must breakup. It is interesting that the instability being examined includes the instability of an unmodulated cold beam in a plasma as a special case, corresponding to $\tau/2\pi=1$.

We shall now consider perturbations of a different type: perturbations that change the mutual positioning of bunches in a wave and, correspondingly, giving rise to new frequencies in the spectrum. In Refs. 117 and 120, the behavior of such perturbations was analyzed for stationary waves with a frequency close to the plasma frequency, which permitted neglecting the higher-order harmonics. In addition, trapped bunches were treated as undeformed bunches. The following method was used to analyze the stability of the system.

Let us suppose that perturbation waves arise in the plasma together with a high-amplitude stationary wave; these are pilot waves, displacing bunches out of the equilibrium position in the potential wells. Using the equation of motion, the displacement of each bunch can be expressed in terms of the electric field of both the fundamental and pilot waves, the perturbed density of the beam can then be determined, and taking into account the dielectric permittivity of the plasma, the electric field created by this density perturbation can be found from Poisson's equation. A closed system of equations is obtained if the field found is identified with the field of the perturbation waves. In this case, a dispersion relation was obtained for the frequencies Ω and wave numbers $\varkappa$ of the pilot waves, which for a cold plasma assumes the form:

$$\frac{\omega_b^2}{(\Omega-\varkappa v_0)^2-\omega_0^2}\left[\frac{1}{\varepsilon(\Omega)}+\frac{1}{\varepsilon(\Omega-2\omega)}\right]=1, \qquad (7.9)$$

where ω_b is the plasma frequency of the beam, corresponding to its average density, ω_0 is the frequency of

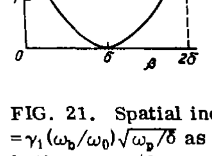


FIG. 21. Spatial increment of side-band instability $\mathrm{Im}\,\varkappa=\gamma_1(\omega_b/\omega_0)\sqrt{\omega_p/\delta}$ as a function of the frequency of the perturbation wave ($\delta=\omega_p-\omega$, $\beta=\omega_p-\Omega$).

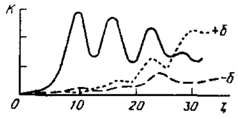


FIG. 22. Computed axial dependences of the amplitudes of the fundamental (resonant) wave with frequency ω_0 (see Fig. 10) and pilot waves with frequencies $\omega_0\pm\delta$, $\delta=0.5(\omega-\omega_0)/\omega_0(n_b 3v_T/n_p v_0)^{4/3}$. The amplitudes of the pilot waves at $z=0$ are fixed an order of magnitude smaller than the amplitude of the fundamental wave.

oscillations of a bunch in the potential well of the wave, and ε is the dielectric permittivity of the plasma.

If we are interested in the spatial amplification of the perturbations, then assuming that in (7.9) Ω is a real quantity, it is easy to determine the frequency dependence of the spatial increment.[29] This dependence is shown in Fig. 21. It is evident that the side-band perturbations, whose frequencies are shifted approximately by an amount $\pm(\omega_p-\omega)$ from the frequency of the fundamental wave, must be maximally amplified. The tendency of the increment to become infinite is related to the fact that the thermal motion of the plasma and dissipation of oscillations were not included in the calculation. These effects must limit the increment at resonant points just as in the problem of amplification of waves in the case of the usual beam-plasma interaction.

Of course, the linear theory of instability does not predict the future fate of the bunches. For this reason, the results of the numerical calculation of the effect of the side-band instability on the fundamental wave are very important,[126] even though they were obtained not for stationary waves. In Fig. 22, it is evident how, as a result of the breakup of bunches under the action of the side-band waves, the fundamental wave damps out with time.

8. EXPERIMENTAL INVESTIGATION OF EXCITATION OF QUASISTATIONARY WAVES BY ELECTRON BUNCHES

The behavior of electron bunches, created independently of the beam-plasma interaction and later injected into the plasma, was studied in Refs. 28, 29, and 127, which differ fundamentally in the formulation of the problems and experimental conditions.

In the experiment described in Ref. 127, a wave, which also propagates in the absence of an electron beam, was excited from an external oscillator in a magnetized plasma. Electron bunches, with duration $\tau\ll 2\pi$ and velocity equal to the velocity of the wave, were injected synchronously with the wave into the plasma along the same direction. The density of the bunches was so low that they did not affect the dispersion of the wave. At the same time, their oscillations in the potential wells led to periodic exchange of energy between the bunches and the wave and corresponding spatial oscillations of the latter. The amplitude and phase of these spatial oscillations were regulated by the change in the initial phase of the bunches in the wave in complete correspondence with the theoretical predictions (Sec. 5e).

In contrast to these experiments, in Ref. 28, preformed bunches excited in an unmagnetized plasma a wave that did not exist without the beam. The characteristics of the wave were determined by the parameters of the injected bunches and, with a careful choice of these parameters, approached the properties of stationary waves. In Fig. 23, the axial dependences of the amplitudes of such waves are compared with the corresponding dependence for the usual exponentially growing waves, which are excited when a weakly modulated beam is injected into the plasma. It is evident that the amplitude of the quasistationary wave from the very beginning is close to the maximum possible amplitude for the given beam, and what is especially important, is conserved at distances at which initially the growing wave is already almost completely damped. The measured equilibrium profile of the potential wave corresponds to the theoretical profile (Fig. 18) and also does not change over the entire path of the quasistationary wave.[29] If the parameters of the bunches formed are fixed, then they are in equilibrium only for a definite value of the ratio ω/ω_p, which corresponds qualitatively to expression (7.6). We note that in the experiment radially restricted bunches are formed and the existence of a quasistationary wave indicates not only their longitudinal, but also their transverse equilibrium. The appearance of forces, which prevent radial expansion of the bunches in the absence of a magnetic field, is related to the negative value of the dielectric permittivity of the plasma for $\omega < \omega_p$.[128]

The properties of quasistationary waves are clearly manifested in the configuration of the beam-plasma discharge, maintained by the wave in a much larger volume than in the case of the unmodulated or weakly modulated beam (Fig. 24). Thus the experimental facts confirm that preformed bunches can be in equilibrium in the field of the wave excited by them. Nevertheless, at some distance the amplitude of the wave is observed to decrease. In the region with damping, the synchronization between the plasma oscillations and the modulating voltage breaks down. With the help of spectral analyzers, it has been established[129] that in this case the probe detects together with the oscillations at the fundamental frequency ($f \approx 0.4$ GHz) satellite waves with frequencies 400 ± 30 MHz and 30 MHz that are spontaneously excited and that grow along the beam (Fig. 25).

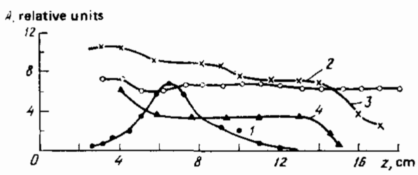


FIG. 23. Axial dependences of amplitudes of waves excited in one case (curve 1) by a small modulation of the beam, and in other cases (2–4) by injection of bunches with different parameters into the plasma.

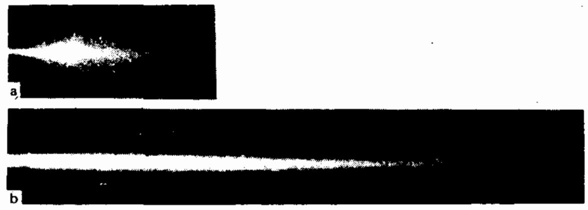


FIG. 24. Beam-plasma discharge with weak modulation of beam (a) and in the case of a quasistationary wave (b).

Comparing the observed experimental evolution of the spectra with the computed behavior (see Fig. 21), their qualitative agreement is evident, although the experimental values of the side-band frequencies do not correspond to the computed values. One of the likely reasons for this disagreement could be the fact that the theory constructed for small $\omega_p - \omega$ becomes too inaccurate for the values of $\omega_p - \omega$ realized in the experiment.

The fact that the theory does not predict amplification of oscillations in the low-frequency region of the spectrum at frequencies near 30 MHz is also apparently related to the use of simplifying assumptions. Meanwhile, these oscillations play an important role in the dynamics of the system, as demonstrated by the following experiment. A small voltage with frequency 30 MHz was applied to the grid modulating the beam together with a uhf voltage exciting the quasistationary wave. In so doing, the spectra of all side-band waves appreciably narrowed, while the region of damping of the fundamental wave approached the point of entry of the beam. Figure 26 presents the spatial dependences of the amplitudes of the fundamental and low-frequency waves. The amplitudes of waves with the combination frequencies (400 ± 30 MHz) vary along the beam analogously to the curve illustrated by the dashed line. The data presented indicate that the reason for the damping of the

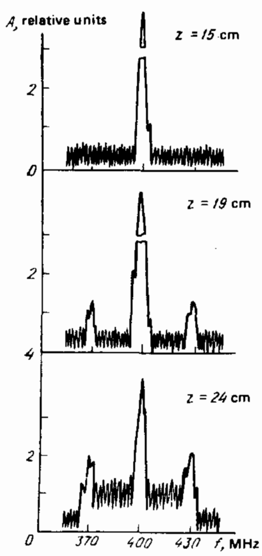


FIG. 25. Spatial evolution of the frequency spectrum of a quasistationary wave.

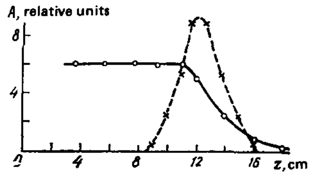


FIG. 26. Spatial distribution of the amplitudes of the fundamental (continuous curve) and low-frequency (dashed curve) waves. The scales of the curves along the vertical axis are different.

fundamental wave is the growth of the satellite waves. These waves remove bunches from the equilibrium state and the bunches then break up.

It should be noted that in the experiment, together with the breakdown of equilibrium of bunches, two other effects arise, which give rise to additional damping of the fundamental wave. This involves, first of all, angular scattering of electrons in the beam, observed visually. Second, it involves a strong change in the beam-plasma discharge regime and, therefore, in the plasma parameters in the region with decreasing amplitude. The measured axial variations of the radial fluxes of electrons and ions have a decreasing character in this region. For this reason, the direct influence of side-band waves on the fundamental wave is manifested distinctly only at the initial stage of damping of the latter, and later it is masked by the effects indicated above.

Breakup of electron bunches naturally leads to damping of the side-band waves as well.

9. CONCLUSIONS

Thus, for stationary injection of an electron beam into a plasma, regular nonlinear waves, which represent a periodic sequence of electron bunches accompanied by a polarization cloud of charges in the plasma, are excited under a wide range of conditions. The spatial evolution of these waves is a result of the corresponding change in the parameters of the bunches due to the initial modulation of the beam velocity and the action of the electric field corresponding to them. This conclusion is based both on the results of a theoretical analysis of the interaction of a monoenergetic beam and the plasma under different conditions and the available convincing experimental proofs of the existence of bunches, including direct observation of their dynamics.

The entire complex of phenomena described in Sec. 2 is easily explained based on the concept of electron bunches. In so doing, we can conventionally single out, in the wide range of experimental systems studied, the following stages of the beam-plasma system that succeed one another in the axial direction.

1. Linear stage. Here the quasimonochromatic wave and its exponential growth in space become separated from the noise. The degree of bunching of the beam and the amplitude of other variable quantities is very small.

2. Region of bunching. In this region, modulation of the beam velocity continues. Simultaneously, due to this modulation, the variable density of the plasma, increasing in magnitude, strongly deviates from a harmonic variation. The wave is enriched with harmonics. This stage terminates with the formation of bunches. The characteristic length of the bunch forming process is given by Eqs. (5.7) and (5.8).

3. Region of interaction of the bunches formed and the plasma. Here most of the electrons in the beam are concentrated in bunches, and this, depending on the conditions, leads to more or less appreciable deviation of the profile of the electric field wave from the harmonic profile. The bunches are not rigid formation, but have a fine internal structure, which changes in a complicated manner in the course of their translational motion. Although the degree of these changes depends on the specific conditions, generally speaking, they involve debunching and repeated bunching of electrons, which must lead to corresponding spatial changes in the amplitudes of the oscillations. In cases when bunches interact with a wave, which also propagates in the plasma without the presence of bunches, the spatial oscillations in the amplitude are caused primarily by energy exchange between the wave and the bunches.

The electrons forming the bunches have a relatively high velocity relative to one another. This predetermines the tendency of bunches to break up, since a decrease in the amplitude of the wave for any reason involves spilling of electrons in the bunches out of the potential wells in which they are confined. Breakup of bunches leads to damping of the wave.

According to data obtained from numerous measurements, the extent of the region in which strong bunching of the beam remains is of the order of (1–2) S, while the total length of the system studied experimentally, including the initial stage, equals approximately $3S$. At this distance the beam consists of a small number of mutually interpenetrating velocity-modulated streams, i.e., the beam is far from having relaxed. There is no experimental information on the subsequent dynamics of the beam and the fundamental problem of its relaxation channels remains open.

The mechanisms affecting the flow of energy out of the Langmuir oscillations,[129] in particular, the modulation instability and wave collapse in a collisionless plasma, play a very important role in the theoretical analysis of this process. However, considerations of the asymptotic behavior of the beam,[25,130] based on an analysis of the electron oscillations only, the subject of the present review, are of definite interest as well.

According to these models, in the case of a low-density beam, in spite of the continuous increase in the number of combined streams as a function of distance, the magnitude of their variable velocity remains insignificant. As a result, the distribution function of the electrons in the beam spreads out, and this under the conditions of a wide spectrum of small-amplitude oscillations can lead the system into a quasilinear relaxation regime.

Apparently, another situation, characteristic for high beam densities, when in the bunching process the variable velocity of the beam is of the same order of magnitude as v_0, can also arise. In this case, in the multivelocity stream formed, the amplitudes of the variable quantities are such that the linear description of the beam is inapplicable, and the beam is in a strongly turbulent regime, relaxing through mixing and scale division.

Be that as it may, collective fields strongly decrease in the region of multistream motion arising with the breakup of bunches. The maximum level of oscillations can be maintained over a large interval if preformed bunches, exciting a quasistationary wave, are injected into the plasma.

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