
V. A. Mikheev. *Quantum diffusion and localization of He^3 atoms in solid He^4 .* One of the interesting characteristics of quantum diffusion of atoms in crystals is that according to the theoretical predictions of Kagan, practically complete localization of diffusing atoms in an ideal lattice can occur at low temperatures. It is demonstrated in the theory constructed¹ that in quantum crystals, particles of one type dif-

fusing in a matrix consisting of another type of particle appear as a kind of defects, interacting elastically with one another. This leads to random displacement of energy levels at neighboring nodes and if the magnitude of the displacement $\delta\varepsilon$ exceeds the width of the energy band Δ of the impurity quasiparticles (impuritons²), then in the limit $T \rightarrow 0$, spatial localization of particles must occur. As the temperature

ture increases, the localization gradually disappears due to fluctuations of levels as a result of interactions with phonons and, in the case $T \gg \delta\epsilon$, when two-phonon processes are effective, the diffusion coefficient of particles D is described by the expression

$$D = \frac{za^2 J^2 \Omega_p(T)}{3\hbar (\delta\epsilon)^2 + \Omega_p^2(T)}, \quad (1)$$

where $zJ = \Delta$, J is the tunneling amplitude, z is the coordination number, $\delta\epsilon = \alpha U_0 x^{4/3}$, U_0 is the elastic interaction constant, x is the particle concentration, $\Omega_p(T) = B(T/\Theta)^9 \Theta$, Θ is the Debye temperature, and B and α are numerical coefficients. It is interesting to note that for $\delta\epsilon = 0$, Eq. (1) goes over into the well-known Andreev-Lifshitz equation,² predicted and confirmed for quantum diffusion in a gas of impurities interacting with phonons in the lattice, when D increases sharply with decreasing temperature according to the law $D \sim T^{-9}$. For $\delta\epsilon > \Delta$, the dependence D has an entirely opposite form and $D \sim T^9$, i.e., the diffusion coefficient drops just as sharply with decreasing temperature as for $T = 0$ and $D = 0$, which should not occur in the case of band type quantum diffusion.

The purpose of the present work was to investigate experimentally the diffusion of He^3 in the hexagonal closely packed phase of the solid solutions $\text{He}^3\text{-He}^4$ over a wide range of He^3 concentrations and densities and to observe the strong localization of He^3 atoms. The $\text{He}^3\text{-He}^4$ system is apparently one of the most suitable systems for studying the phenomena predicted in Ref. 1. The extremely narrow energy band ($\Delta \sim 10^{-4}$ K) permits having a disordered system ($T \gg \Delta$), which does not decay ($T > T_s$, T_s is the separation temperature of the solution) down to quite low temperatures. Due to the inequality $T \gg U_0$, where $U_0 = 10^{-2}$ K, all configurations of He^3 atoms are statistically equivalent in the range of temperatures investigated (≥ 0.4 K), but the system is strongly interacting ($U_0 \gg \Delta$).

To perform the investigations, a special low-temperature pulsed magnetic-field gradient technique was developed. This technique permitted measuring record low values of the diffusion coefficient 10^{-12} cm^2/s in solid helium using a pulsed nuclear magnetic resonance apparatus.

Figure 1 shows the results of the experimental investigations of the diffusion coefficient of He^3 over a wide range of He^3 concentrations and temperatures, where practically all currently known mechanisms of diffusion in solid helium are clearly evident. At the lowest He^3 concentrations, coherent band-type quantum diffusion occurs, due to scattering of impurities by phonons and by one another. As the concentration increases, the trend in the temperature dependence of D changes and for maximum concentrations of $\sim 5\%$ He^3 , a regime with strong localization with $D \sim T^9$, predicted by Kagan and recently discovered experimentally,³ becomes clearly evident. At high temperatures ($T > 1.5$ K), He^3 diffusion through vacancies, which makes a significant contribution only for concentrated solutions, is added.

The averaged values of D (see Fig. 1) in the region of the low-temperature plateau are shown in Fig. 2, where analogous results for other densities are also shown. The behavior of D , as is evident in Fig. 2, clearly deviates from band diffu-

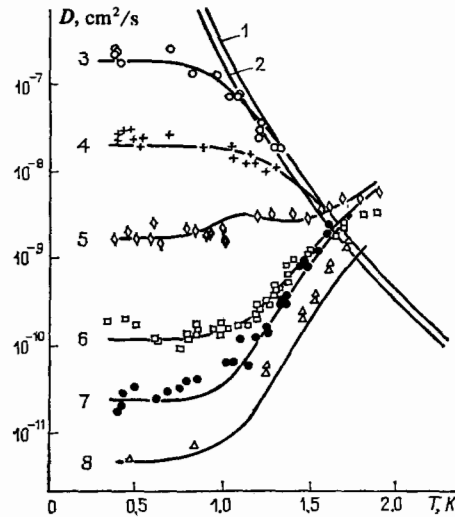


FIG. 1. Temperature dependence of the diffusion coefficient of He^3 in the hcp phase of solid solutions. 1, 2) curves for $\nu = 21.0$ and $20.5 \text{ cm}^3/\text{mole}$, respectively, characterizing quantum diffusion for $\delta\epsilon = 0$; 3) 0.006%; 4) 0.05% He^3 , $\nu = 21.0 \text{ cm}^3/\text{mole}$; 5) 0.25%; 6) 2.17%; 7) 4.0% He^3 , $\nu = 20.7 \text{ cm}^3/\text{mole}$; 8) 4.98% He^3 , $\nu = 20.5 \text{ cm}^3/\text{mole}$.

sion (dashed line) and has a tendency to go over into the state of localization with concentration equal to some critical value. As Kagan and Maksimov recently showed,⁴ in this case, the percolation approach to diffusion of He^3 in solid He^4 is valid; they confirmed the interpolation equation, describing the well-known mechanisms of quantum diffusion in the entire x - T plane. In the region of the low-temperature plateau, as the critical concentration is approached, the critical behavior of D , related to a decrease in the number of neighboring nodes with a displacement less than the width of the band and entering into the formation of an infinite cluster, must occur. The corresponding critical dependence has the form

$$D = \frac{\text{const}}{x} \left(\frac{x_c - x}{x_c} \right)^t. \quad (2)$$

The continuous curves in Fig. 2 correspond to critical depen-

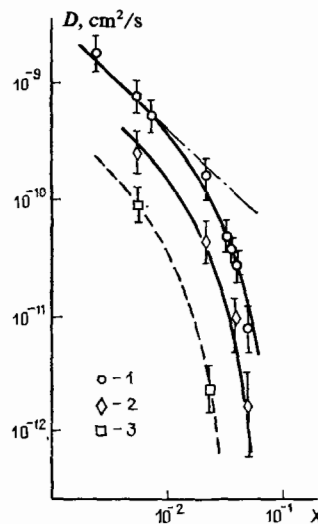


FIG. 2. Concentration dependence of the coefficient of quantum diffusion in the region of the low temperature plateau for different densities: ν (cm^3/mole) = 20.7 (1), 20.5 (2), and 19.9 (3).

dences with $t = 1.7$. The value of the critical concentration for $\nu = 20.7 \text{ cm}^3/\text{mole}$ lies in the range $6 \lesssim x_c \lesssim 8\% \text{ He}^3$. We note that the index t found is close to the critical index of electrical conduction with a metal-insulator phase transition.⁵ Using the interpolation equation in Ref. 4 with well-known quantum diffusion parameters and the found values of x_c and t , it was possible to calculate the theoretical curves of D for the entire range of concentrations and densities studied. The continuous curves in Fig. 1 are the result of this calculation. As is evident in Fig. 1, theory and experiment agree quite well.

Thus, in this work, we investigated the diffusion of He^3 in solid $\text{He}^3\text{-He}^4$ solutions up to concentrations $\sim 5\% \text{ He}^3$ and we observed the phenomenon of strong localization of He^3 atoms and phonon delocalization in complete agreement with the predictions of Kagan's theory. In the region of

strong localization, a critical behavior of the diffusion coefficient of He^3 was found. The theory of localization developed was compared with the experimental data over a wide range of parameters and their agreement was demonstrated.

¹Yu. Kagan in: *Defects in Insulating Crystals: Proc. Intern. Conference, Riga, May 1981, Springer-Verlag, Berlin (1981)*, p. 17.

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⁴Yu. Kagan and L. A. Maksimov, *Zh. Eksp. Teor. Fiz.* **84**, 92 (1983) [*sic*].

⁵B. I. Shklovskii and A. L. Éfros, *Élektronnye svoïstva legirovannykh poluprovodnikov (Electronic Properties of Doped Semiconductors)*, Nauka, Moscow (1979).