Computer-aided tomography and physical experiment

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Computer-aided tomography (CAT) is a method of laminar reconstruction of the structure of an inhomogeneous and generally asymmetric three-dimensional object from a set of measured projections. Recently CAT has been widely used, not only in medical diagnostics, where most brilliant and impressive results have been achieved, but also in various areas in physics. Physical and technical applications of CAT are the topic of the present review. The basic principles of tomography research are described, CAT problems are classified from the point of view of integral geometry, and the main algorithms used in computational data processing are briefly commented upon. Particular attention is devoted to possible CAT applications to defectoscopy, microscopy, solid state physics, geophysics, Earth and planetary atmospheric physics, aero- and hydrodynamics, and plasma physics. The main developmental directions of theoretical and technical CAT in the near future are noted in conclusion.

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INTRODUCTION

The methods of tomographic, i.e., laminar (from the Greek tomos, meaning layer, cross section) study of the structure of inhomogeneous objects of diverse nature, have undergone substantial development in the last decade, thanks to progress in computational mathematics and contemporary equipment. Most of the recognition of computer-aided tomography (CAT) was achieved in biology and medicine. Particularly swift was growth in popularity of x-ray diagnostic CAT: the first sufficiently quantitative tomogram of a human brain was obtained in 1972, and in seven years, by 1979, tomographic instruments commercially manufactured by many companies despite their impressive cost, exceeding a million dollars for several models, operated in more than 2000 clinics throughout the world. At the same year the English engineer-researcher G. N. Hounsfield and the American mathematician A. M. Cormack were awarded the Nobel prize in medicine for prominent contributions to the development of x-ray CAT (their Nobel lectures were published in Refs. 1, 2).

At the same time the fast progress in medical tomography was accompanied by the onset and development of many other applications of this quite universal and informative method of introscopy (inner version). The CAT methods started penetrating more deeply into laboratory techniques of physical experiments, geophysics, cosmology, astronomy, analytic chemistry, and have introduced drastic changes into defectoscopy of industrial goods. At the same time it must be kept in mind that the basic ideas and principles of tomographic reconstruction of inhomogeneous objects were, generally speaking, well-known in physics, and moreover in various variants, long before they were adopted by biology and medicine. Thus, the first studies on reconstruction of the local characteristics of axially symmetric objects go back to the last century[3]; in the fifties Bracewell,[4] reconstructing patterns of solar UHF radiation from band sums, already used a purely tomographic computational technique in the contemporary sense of the word. Nevertehless, one cannot deny the fact that the substantial achievements of CAT diagnostics of recent years provided a powerful stimulus for developing applications in various areas of science and technology.

The review literature on CAT is already quite extensive. We first mention the monographs 5, 6, 179, and a collection of articles.[7] Popular expositions of the principles of CAT are contained in the lectures already mentioned,[1,2] as well as in the papers of Refs. 8–11. In reviews of more specific character, however, the accent was put either on medico-biological problems,[12–16] or on the purely computational aspect of the reconstruction problem of laminar images.[17–23] The physical aspects of tomography are usually touched upon only casually; in review papers by the present authors[24–26,180] the main topic is a special class of CAT problems, important for plasma physics and dynamics of a heated gas. Recent reviews[27,181,182] are devoted to NMR-introscopy, also using tomographic methods of study of objects.

In connection with these comments it is useful and timely to systematize the main results and important trends

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of CAT development, directly relevant to physics and technology. The review includes studies published up to mid-1982.

2. GENERAL PRINCIPLES OF TOMOGRAPHY

Contemporary tomography can be assumed, without exaggeration, to encompass all wave lengths. Depending on the nature of the specific problem of reconstructing the structure of an object, it can be based on registering beams of electrons, ions (including protons and alpha-particles), neutrons, photons over the whole electromagnetic spectral range, and sound waves. This also refers to exotic projects of neutrino tomography.[28]

The investigated object is in this procedure irradiated externally or is itself the emitter. The irradiating sources can be coherent (in a number of problems this is extremely important) or noncoherent. Both wide and narrow (approximately monochromatic) sections of emission, absorption, or scattering spectra. can be detected. Also measured are phase distortions of the wave front (Schlieren, shadow, and interferometric techniques), free induction signals (NMR-introscopy), rotational angles of polarization planes (CAT based on the Faraday effect), etc. Recently the electron-positron annihilation effect with use of a coincidence scheme has become more widely spread.

It might seem that with such diverse means and methods of obtaining data there cannot exist any even to some degree universal procedure allowing one to transform from measured quantities to elements of the internal structure of an object. However, the aforementioned complex of inverse problems contains a certain common core.

For definiteness we choose the case of a non-self-illuminating object, inaccessible to visual examination, which we can illuminate, for example, by x-ray beams.

The drawbacks of ordinary x-ray photographs, being two-dimensional shadow projections of a real, three-dimensional structure, are well known. These are, primarily, low contrast, low signal/noise ratio, unavoidable overlap of structural elements, and the impossibility of quantitative comparison of separate local fragments of the body according to density and other parameters. It is also clear that the set of such x-ray photographs, taken from various directions and available to the investigator, practically allows to extract little quantative information on the internal structure of the object if photographs are examined in succession in the standard fashion.

A method of substantial enhancement of information of x-ray photographs was suggested in the twenties by the French surgeon Bocage,[29] and his idea was practically realized soon after by the Italian engineer Vallebone, who named the instrument constructed by him- a tomograph. The essence of the idea is seen in Fig. 1. If during the taking of the photograph any two of the three participating components (x-ray tube, object, photographic film) are displaced in parallel planes, leaving the third one fixed, only one layer produces a sharp image on the film, while the elements of the remaining layers are smeared. Controlling the rates of displacement, one can isolate different layers and thereby successively investigate the structure of a three-dimensional object.

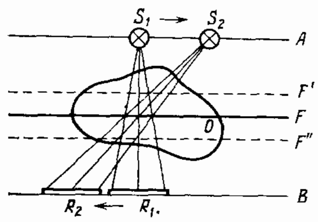

FIG. 1. Tomography scheme according to Bocage. The source S_1 is displaced to the position S_2 in the A plane, while the x-ray film R_1 moves to the position R_2 (the B plane). Sharply reproduced is the F plane, while the details of the object O corresponding to the planes F' and F'' are smeared out.

X-ray tomography according to Bocage (differently called focal plane tomography, planigraphy, biotomy) played an important role in medical radiology. The most practical applications were achieved in clinical pulmonary diseases (diagnostics of tubercular cavities, deeply located tubercular sites and infiltrators, chronic abscesses, and cancer of the lung). Tomography occupied a firm position in diagnostics of larynx diseases, since in ordinary x-ray photography the larynx is almost indistinguishable due to the superposition of shadows of the verterbrae on it.[30]

Nevertheless it is easily understood that the principle of successively extracting cross sections of the body by mutual displacements of components of the diagnostic instrument must not be assumed to be universal. Keeping in mind only medical radiology, even in this case a significant enhancement of the contrast of different organs cannot be achieved: this is due to unavoidable parasitic illumination of the other, out-of-focus cross sections of the body. Successive runs from one cross section to another are accompanied by large doses of radiation; as in ordinary x-ray diagnostics, only qualitative comparison of the densities of different portions is satisfactorily reached. As to tomography of self-luminous objects (plasma physics, high-temperature gas-dynamics), applications in electron microscopy, geophysics, atmospheric and cosmic studies, the principles described above are here quite ineffective.

Contemporary computer-aided tomography uses a totally different approach to extracting cross sections of interest to us of an inhomogeneous object. We explain it on an example belonging to x-ray diagnostics (Fig. 2). The radiation of the x-ray source may be well collimated: the beam is of the order of one or several millimeters in diameter, so that sufficient basis is provided for talking about probing rays. Each such ray, passing through the body, is characterized by its intensity, attenuated with distance. A detector at the exit establishes some integral quantity of this attenuation, a ray-sum, while the signal can be introduced into a computer memory in the form of a numerically digitized pulse by means of an analog-digital transformer. Further, displacing the rays according to a certain law in a selected plane (for example, parallel to itself, or fan-shaped with some angular resolution), we obtain a set of ray-sums determining the one-dimensional projection. For definiteness we consider parallel displacement of the probing rays; one projection then cor-

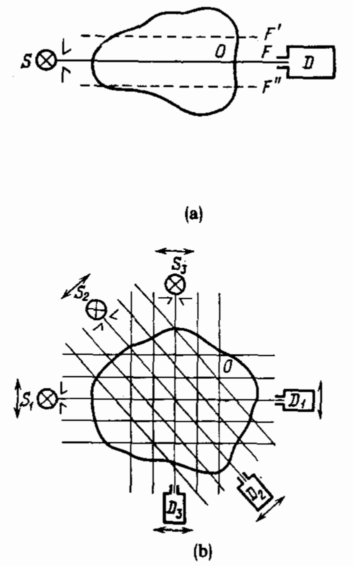

FIG. 2. Scheme of computer-aided tomography. a) A collimated beam traverses the F plane, is attenuated by the object O and is registered by a detector D; the effect of the planes F' and F'' is totally removed. b) The view on the F plane from above: the source and detector are displaced, forming a set of ray-sums and a projection. Rotation through a certain angle then takes place. The recording of three projections is shown.

responds to some angle relative to the reference system selected. Turning then the ray in the same plane by a small angle, we repeat the registration process of ray-sums and obtain a new projection, etc. The angular scanning process is repeated until the total angle of rotation becomes 180°. As a result the necessary information is accumulated in the computer memory for image reconstruction in an isolated plane (cross section). In this case there are no disturbances in the form of diffuse mapping elements corresponding to other planes, and theoretically one can expect quite high tomogram contrast.[6] The possibility of quanitative estimates of the density of structural elements by using computers is obvious. It is easily verified that the method is also well-suited to the study of emitting and scattering objects, combining well with techniques of holographic interferometry, NMR, EPR, polarization measurements, etc.

The tomographic principle described above found one of the first applications in the early sixties by A. M. Cormack in model experiments, oriented toward medical x-ray diagnostics.[31] Kuhl and Edwards[32] practically simultaneously constructed an instrument for transverse scanning of a patient's body; a similar attempt was undertaken somewhat earlier by Oldendorf.[33] However, as already noted above, CAT acquired the status of a real diagnostic tool only in the early seventies, when an EMI-scanner was constructed under the guidance of G. N. Hounsfield, equipped with a dedicated computer for high contrast reconstruction of separate cross sections of a human brain.[34] Soon after Ledley developed an instrument, making it possible to scan any part of the human body or the whole body in the total volume. Practically at the same time started the process of intense use of different trechniques for CAT purposes, no longer related to absorption of x-ray beams, as well as the penetration of CAT ideas into various areas of science.

It is appropriate to stress that the scheme described above of obtaining tomograms must by no means be regarded as fixed or exclusive. Thus, computer application for image reconstruction from projections is, in principle, neither necessary nor inherent to tomography proper: the problem can often be solved (moreover, more cheaply and faster) by means of optical processors.[6] Recently the very term "tomography" has become all the more conditional, since a three-dimensional object, generally speaking, does not necessarily have to be investigated layer by layer. In particular, the set of two-dimensional projections, registered by means of several *camerae obscurae* (see Section 5) or other methods, makes it possible to reconstruct directly, without separation into layers, the real bulk structure of the body. Moreover, in designing any tomographic experiment quite an important role can be played by appropriately taking into account the symmetry properties of the object.

The exceptional flexibility of the CAT method gives rise, on the one hand, to the high diversity of physical, technical, and computational-mathematical solutions in obtaining tomograms, and, on the other hand, guarantees for this method an extremely wide field of applications.

3. CLASSIFICATION OF TOMOGRAPHIC PROBLEMS

The procedure of reconstruction the structure of an object from measurements of projections is an important part of any tomographic experiment. From a mathematical point of view this is a typical problem of integral geometry,[35] usually formulated as follows. Let $g(x)$ and $R(x, y)$ be sufficiently smooth functions, defined, respectively, in n-dimensional and $(n+k)$-dimensional spaces, let $x = (x_1,\dots,x_n)$ and $y = (y_1,\dots,y_k)$ be vectors, and let

$$\int_{M(x)} R(x, y)\, g(x)\, \mathrm{d}\sigma = f(y) \tag{1}$$

are assumed to be known as well as the weight functions R, with $\mathrm{d}\sigma$ being the measure element on $M(x)$. It is required to find $g(x)$. We note immediately that in most CAT problems $n = 2$ or 3, $k = 1$ or 2, $R = 1$, and the manifold M are points on a rectilinear or curvilinear trajectory. However, since more general statements of the problem are not excluded, it is useful to treat originally the multidimensional integral equation (1). In classifying CAT problems it is convenient to distinguish three large groups of these problems according to the degree of increasing complication of their solution.

a) Linear CAT Problems with strong *a priori* restrictions

In the given case the trajectories of the beams penetrating the object in a selected plane are assumed strictly rectilinear, and the *a priori* information concerning the unknown function $g(x)$ is usually of the following nature.

In the simplest case the functional form of $g(x)$ can be given, and then it remains only to determine several unknown parameters appearing in the problem. As is well-known, a preliminary analysis of the information base of this problem (calculation of the Fisher matrix) and parameter search (the maximum likelihood method) are well developed and widely used procedures. We note that this manner of formulating tomographic problems is often encountered in various applications of transport theory of γ-radiation[36] and neutrons,[37] in the physics of crystals,[38] in plasma spectroscopy,[39] and in gas dynamics.[40]

Further, the shape of the curves on which the unknown two-dimensional distribution $g(x)$ attains a constant value (isolines) can be assumed known. The simplest and most fully investigated case is that of isolines in the form of concentric circles, when the object is inhomogeneous only in the radial direction, and to reconstruct $g(x)$ (x is the radial avriable) only a single projection $f(y)$ is needed. Equation (1) with $R = 1$ degenerates in this case to the classical Abel equation

$$\int_0^y (y-x)^{-1/2} g(x)\, dx = f(y), \tag{2}$$

having for a continuously differentiable function $f(y)$ the single continuous solution

$$g(x) = \frac{1}{\pi} \frac{d}{dx} \int_0^x (x-y)^{-1/2} f(y)\, dy. \tag{3}$$

A large number of examples of physical problems leading to the Abel equation were considered in the review of Ref. 25. We note that if an axially symmetric object is self-illuminating, Eq. (2) is valid only in the limit, when reabsorption of radiation can be neglected. In the more general case[41,42] the solution of the problem evolves into a series of successive inversions of the Abel type (3).

It is important to recall that the existence of a simple inversion equation (3), seeming at first glance to solve the problem completely, in actual fact does not at all imply that the error in reconstructing $g(x)$ will be of the same order as the error in measuring the projection $f(y)$. In reality the problem belongs to a class of ill-posed (unstable) problems[43], and any algorithm of numerical reconstruction of $g(x)$ requires regularization, i.e., the use of additional restrictions on the function $g(x)$, consistent with random measurement errors in $f(y)$. Comparison of the effectiveness of several regularized algorithms of Abelian inversion was given in Ref. 44.

A number of more complicated situations was also investigated. Thus, the authors of Refs. 45–51, as well as of the paper of Ref. 52, described methods of extracting the local characteristics of inhomogeneous objects (mainly plasmons) from a minimal number of projections, one or two, when the isolines in the selected cross section are a system of nonconcentric circles, similar ellipses, as well as ellipses with regular displacements with respect to each other. In addition, the possibility was investigated of factorization of the unknown two-dimensional distribution function. Schemes were considered of illuminating the object by parallel beams, as well as fan-shaped projections; the optical density of the emitter of the radiation was taken into account.

The *a priori* restrictions on the function $g(x)$ can also be related to the use in the algorithm of the symmetry properties of the object, expressed not in terms of the given configuration of isolines, but directly in the language of group theory. In particular, many biomolecules, their complexes, as well as different types of viruses, bacteriophages, and albumin crystals possess spiral, dihedral, icosahedral, and other symmetries. Effective methods of three-dimensional reconstruction of this type of biological structures, investigated by means of electron microscopy, were developed in the Institute of Crystallography of the USSR Academy of Sciences under the guidance of B. K. Vaĭnshteĭn,[12,53] as well as by staff members of the Cambridge laboratory of molecular biology.[54,55]

We also note that a quite generally stated tomographic problem with isolines was investigated in Ref. 51, where the latter are described by a given system of arbitrary convex closed curves without self-intersections.

b) Linear CAT problems with weak *a priori* restrictions

Following in level of complications is the class of problems of two- and three-dimensional tomography in the direct sense of the word, when the object is asymmetric, parametrization or taking into account of the shape of isolines is not employed and the (weak) *a priori* restrictions introduced on $g(x)$ are related only to the regularization of the solutions. The beam trajectories are, as formerly, assumed to be given and rectilinear.

We turn again to the original Eq. (1), we put in it $R = 1$, and as $\{M(y)\}$ we consider the family of all possible hyperplanes in n-dimensional space. In this formulation the problem of seeking $g(x)$ was solved by one of the founders of integral geometry, the Austrian mathematician J. Radon, in 1917.[56] Figure 3 explains the results of Radon inversion in the two-dimensional case. Let L be a beam intersecting the object, with s the distance measured along it, O the origin of some coordinate system, φ the angle between the baseline OM in the selected plane and the perpendicular dropped from O onto L, p the shortest distance from O to L, and $\mathbf{n}$ the normal determined by the same angle φ. In this notation (1) must be replaced by

$$\int_L g(\mathbf{r})\, ds = f(p, \mathbf{n}), \tag{4}$$

where the two-dimensional vector $\mathbf{r}$, rotated with respect to OM by angle θ, characterizes the position on the plane of the point at which the distribution g is sought. As shown by Radon,

$$\begin{aligned} g(\mathbf{r}) &= \frac{\nabla^2}{2\pi^2} \int_0^\pi d\varphi \int_{-\infty}^{+\infty} f(p, \mathbf{n}) \ln |p - \mathbf{rn}|\, dp \\ &= \frac{1}{2\pi^2} \int_0^\pi d\varphi \int_{-\infty}^{+\infty} \frac{\partial^2 f(p, \mathbf{n})}{\partial p^2} \ln |p - \mathbf{rn}|\, dp \\ &= -\frac{1}{2\pi^2} \int_0^\pi d\varphi \int_{-\infty}^{+\infty} \frac{\partial f(p, \mathbf{n})}{\partial p} \frac{dp}{p - \mathbf{rn}} \\ &= -\frac{1}{2\pi^2} \int_0^\pi d\varphi \int_{-\infty}^{+\infty} \frac{f(p, \mathbf{n})\, dp}{(p - \mathbf{rn})^2}. \end{aligned} \tag{5}$$

In the calculations one can (of course, after an appropriate regularization, since the problem remains ill-posed) use any of expressions (5); the inner integrals are understood in the sense of Cauchy principal value.

The Radon solution is in the case $n = 3$

$$g(\mathbf{r}) = -\frac{\nabla^2}{8\pi^2} \int_{S_0} f(\mathbf{rn}, \mathbf{n})\, d\Omega, \tag{6}$$

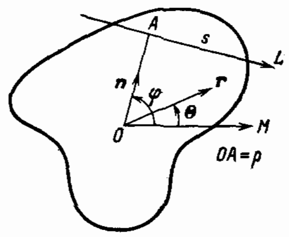

FIG. 3. Definition of the variables used in Eqs. (4) and (5). Explanations are given in the text.

where S_0 is the surface of the unit sphere, $d\Omega$ is the solid angle element, determined by the normal $\mathbf{n}$, and the function $f(\mathbf{rn},\mathbf{n})$ is the surface integral of the function g, evaluated in the plane passing through the point $\mathbf{r}$ and normal to $\mathbf{n}$.

The case of reconstructing the two-dimensional distribution of the emission coefficient of a self-illuminated optically dense object was considered in Ref. 50; in this case $R \neq 1$ in (1), and is a function found from an auxiliary experiment.

c) Tomographic problems for curvilinear trajectories

The transition to curvilinear beam trajectories, which can be primarily caused by taking into account refraction of the medium under consideration, gives rise to new, more complicated, and quite diverse classes of CAT problems. In a number of geophysical applications the form of the curvilinear trajectories is known ahead of time (gravimetry, the inverse kinematic problem of seismology, investigated already early in this century by G. Herglotz, E. Wiechert, and others). Many important analytic results were obtained in this area, and effective numerical algorithms were created (see, for example, Ref. 57).

Axially symmetric optical inhomogeneities, as well as inhomogeneities in a planar layer due to strong beam refraction were investigated by a number of authors.[58–62] The eikonal approximation is usually used in these studies. Curiously, despite the beam bending under conditions of axial symmetry, one again obtains the Abel integral equation, though the variables occurring in it have a totally different meaning than in the case of rectilinear trajectories.

When the shape of the curvilinear trajectories is not known ahead of time, and the object is asymmetric, even in the simplest approximation of geometric optics we encounter a nonlinear tomographic problem. Many attempts were undertaken of solving this problem by means of iteration schemes.[63–67] Definite success was achieved on this path,[65,66] but as a whole the problem is far from completed, since convergence problems of the iteration process have not been investigated. The Radon equation is repeatedly solved in this process, and at each step the refractive index field and the trajectory are "corrected" to the measured projections. Besides, due to the effect of virtual beam intersection,[61,68] as well as effects of the type of Maxwell's "fish eye,"[67] the danger exists here of loss of existence and uniqueness of the solution of the problem.

Though within the eikonal approximation one often succeeds in avoiding several crude artifacts of refraction nature, the resolving power of the reconstructed image can remain inadmissably low. As shown in Ref. 20, further progress in this direction is related to taking into account diffraction effects, i.e., to solutions of the full wave equation or the Helmholtz equation. These solutions are usually found by using the Born approximation or by transforming the wave equation into the Riccati equation (the Rytov approximation).[69] The theory of the corresponding CAT problems is only starting to be developed.

Finally, we mention several mathematical treatments of methods of solving nonlinear CAT problems, obtained by M. M. Lavrent'ev's school and containing important general results concerning the uniqueness and stability of the solutions.[70,71] It must be said that in specific computational algorithms and programs, this still remains to be settled.

In conclusion it must be noted that the classification suggested above must by no means be considered as encompassing all conceivable types of applied CAT problems. For example, the extension of tomography principles to laser scattering methods could make it possible to reconstruct directly the distribution function of gas or plasma particles in 6-dimensional phase space. A wide variety of CAT problems is generated as strong coherent fields are employed under conditions of occurrence of nonlinear optical effects such as self-focusing, self-diffraction, the Fredericks effect in liquid crystals, etc. Since, however, the formulations of such inverse problems are not as yet supported by any solid experimental material, they must for now be considered speculative only, and the corresponding classification—premature.

4. BRIEF CHARACTERISTICS OF ALGORITHMS OF RECONSTRUCTION OF TOMOGRAPHIC IMAGES

The CAT literature is abundant with descriptions of numerous algorithms used in reconstructing images. This is related both to the exceptional diversity of the formulations of tomographic problems and methods of their experimental realization, and to the continual efforts of specialists to develop such algorithms, which would exceed the available ones by one or two practically important parameters of the problem: quick operation, requirements of computer memory, resolving power, contrast, amount of required projections, etc. In the present review there is no possibility (or necessity) of detailed description and illustration by examples, and comparison of the various algorithms with each other. Therefore we only enumerate the important ones, and provide brief commentaries of qualitative character. Moreover, we restrict ourselves only to algorithms of problem solutions, which by the classification of the preceding section were placed in the second group (asymmetric objects, rectilinear trajectories) and are most frequently applied in practice.

Consider first an elementary example of reconstructing a two-dimensional object, when the object itself is extremely simple, the algorithmic "mechanism" is quite transparent, and computer implentation is generally not required.

As such an object we choose the square $ABCD$ (Fig. 4), divided into 9 equal boxes (cells). The numbers 1 to 9, dispersed over the boxes, correspond to the density or any other

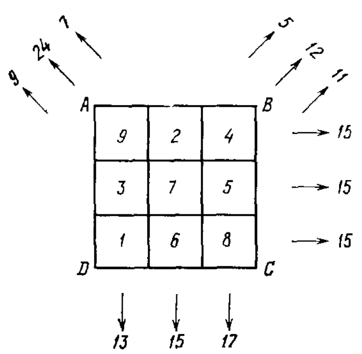

FIG. 4. A test object of nine elements. The arrows show the 12 ray-sums used in four projections.

characteristic, found tomographically. Let 4 projections be known, determined by the directions of the sides AB and AD of the square and its diagonals AC and BD. If in each projection one takes 3 ray-sums, then in the first two cases contributions are provided by all boxes, and in the other two only 7 out of the 9 boxes. Thus, we start from 12 values of ray-sums, and seek 9 structural elements of the object, i.e., we are solving an overdetermined problem.

We select for the reconstruction an algorithm based on the scheme of "balancing of projections," which is a special case of the quite universal iteration method of solving linear inverse problems.[72]

We start with the projection formed by the beams parallel to the side AB. We divide each value of the ray-sum by the number of intersected boxes, and we ascribe to these boxes the obtained value (5 in the present case). We adopt the obtained result as the first iteration (Fig. 5a). As we see, in our case and for the projection selected the object appears to be completely homogeneous; if the reconstruction error is estimated by the equation

$$\sigma_i = \Big(\sum_{k=1}^{9} (g_k^{(i)} - g_k^{(0)})^2 \Big/ \sum_{k=1}^{9} k^2\Big)^{1/2} \cdot 100\%, \qquad (7)$$

where the subscript k enumerates the box, i denotes the iteration number, and $g_k^{(0)}$ are the starting values on Fig. 4, then $\sigma_1 = 45.8\%$.

We turn now to the next projection (the beams propagate along AD). For each beam one must now make a correction to the sum of numbers obtained after the first iteration using the known ray-sum of the given projection. Thus, the sum in the first column (15) must, obviously, be decreased by 2 and 2/3 must be subtracted from each number; in the third column one must, on the other hand, add 2/3 to each box; there is no change in the second column. It is seen that in our example the second iteration is practically noninformative: the preceding uniform distribution is only slightly deformed, creating a small gradient along AB and not at all affecting the complex structure of the object (Fig. 5b). The reconstruction error estimated from (7) even increases somewhat in comparison with the first iteration: $\sigma_2 = 48.7\%$.

The third iteration (Fig. 5c, the beams are parallel to the diagonal AC) already alters the situation sharply, since the inhomogeneity of the object is distinctly manifested in the ray-sum values. The balancing principle remains in effect, only that the corresponding differences must be distributed uniformly over two or three boxes, depending on which ray-sum is taken in the calculatin; $\sigma_3 = 28.7\%$. A similar situation occurs in the fourth iteration (Fig. 5d), completing the first cycle of the procedure; now the object already recalls the original $\sigma_4 = 14.6\%$.

Further one can again invoke the first procedure, and thus start the second cycle of balancing. Figures 5d, e show the results obtained after the sixth ($\sigma_6 = 9.94\%$) and the eighth ($\sigma_8 = 4.25\%$) iterations. The process could of course, be extended even further (according to Ref. 72 its convergence is guaranteed), but it is already clear from the calculations performed that 7–8 iterations make it possible to obtain a fairly good reconstruction result.

The reader desiring to get a better feeling of the role of the various factors affecting the quality of reconstruction of a tomographic image is advised to equip himself with the simplest calculator and independently practice on a square divided into 16 boxes, for example. In this case the calculations do not become too complicated, but the possibilities of a computational experiment increase substantially. Thus, one can vary the number of projections and the number of ray-sums in them, solve both overdetermined and underdetermined problems, select various initial element distributions and change the "contrast" in these distributions, and to simulate a real experiment by introducing into the ray-sum values random and systematic errors, and so on. Possibilities also open up for trying various iteration schemes for reconstruction.

What, briefly, is the situation with algorithms used in solving real CAT problems? They can be divided conventionally into two categories. To the first group belong those which are directly based on general, widely known methods

5	5	5
5	5	5
5	5	5

(a)

$4\frac{1}{3}$	5	$5\frac{2}{3}$
$4\frac{1}{3}$	5	$5\frac{2}{3}$
$4\frac{1}{3}$	5	$5\frac{2}{3}$

(b)

$7\frac{1}{3}$	$3\frac{1}{6}$	$5\frac{2}{3}$
$4\frac{1}{6}$	8	$3\frac{5}{6}$
$4\frac{1}{3}$	$4\frac{5}{6}$	$8\frac{2}{3}$

(c)

$7\frac{1}{3}$	2	$3\frac{2}{3}$
3	6	5
$2\frac{1}{3}$	6	$8\frac{2}{3}$

(d)

8	$2\frac{8}{9}$	$4\frac{1}{9}$
$3\frac{1}{3}$	$6\frac{5}{9}$	$5\frac{1}{9}$
$1\frac{2}{3}$	$5\frac{5}{9}$	$7\frac{7}{9}$

(e)

$8\frac{5}{9}$	2	$3\frac{22}{27}$
3	$6\frac{22}{27}$	5
$1\frac{10}{27}$	6	$8\frac{1}{3}$

(f)

FIG. 5. The various states (a–f) of reconstructing the test object shown in Fig. 4 by the iteration method of "balancing the projections." Detailed explanations are given in the text.

of computational mathematics. The second group consists of special type algorithms, created especially for CAT problems.

Speaking of algorithms belonging to the first category, one must first mention classical methods of direct matrix inversion (see, for example, Ref. 73), the iteration algorithms ART (algebraic reconstruction technique)[74] and SIRT (simultaneous iterative reconstruction technique),[75] involving among them the techniques of the finite element method,[76] the maximum entropy method (MENT),[77–79] the Monte-Carlo method,[80,81] and methods of reducing CAT problems to deconvolution problems with subsequent application of A. N. Tikhonov's regularization method.[82,83]

The procedure of direct matrix inversion, used quite often at the early stages of CAT development, is presently considered almost unanimously as nonconstructive, and is practically not used any more. The remaining algorithms mentioned, having their merits and drawbacks, find numerous applications and continue being developed on a theoretical level. Particularly often is used the ART iteration algorithm, which was successfully used in the first medical tomography of Hounsfield, and later in electron microtomography. A distinguishing feature of the MENT algorithm is the possibility of using it in three-dimensional tomography, and of requiring in this case quite a small number of projections, while other algorithms give rise to undesirable artifacts. Many specialists associate with the Monte-Carlo method hopes for a reliable and successful solutions of CAT problems of high dimensionality, and even nonlinear problems; moreover, it is assumed that statistical modelling will allow better economy than, for example, the use of iteration methods, in the distribution of data in a computer memory. The algorithm based on Tikhonov regularization forms the foundation for the system of mathematical provision for the first Soviet x-ray tomograph SRT-100, designated for studying the human brain.[82] The already accumulated experience in exploiting this device confirms the high quality of reconstructing fine details of a two-dimensional image in a selected cross section.

Among the numerous specialized CAT algorithms, of particular interest are schemes of layered splitting,[26,46,51,84] the technique of polynomial expansion,[31,45,85] Fourier synthesis algorithms,[6,86] filtered back projection (FBP),[6,19,87] as well as direct use of the Radon transform (5).[23,88–91]

Layered splitting, often called "onion peeling," can also be considered in some sense as a successful tomographic version of the finite element method. As a result the possibility arises of solving, instead of the original two-dimensional problem, a series of one-dimensional integral equations (Volterra equations of first kind), which gives a number of significant computational advantages. Moreover, the method can be generalized to complicated physical situations: the presence of a nontransparent body, overlapping a part of the beams, an optically dense emitter, etc.

The use of polynomial expansions has been for a long time popular among experimental physicists. Among the expansions used particularly often by Cormack and Maldonado,[31,85] the Chevyshev, Laguerre, Hermite and Zernike polynomials are prominent, while Cormack reduced the problem to a relatively compact system of linear algebraic equations, and the authors of Ref. 45 developed a regularized version of the Maldonado algorithm with a restricted number of series terms on the basis of discrepancy criteria.

The merit of the Fourier synthesis algorithm is primarily due to the possibility of using the widely well-known Cooley-Tukey scheme (the fast Fourier transform). Besides, the algorithm is easily realized in an analog form[6] and makes it possible to take into account effectively the *a priori* information concerning the object symmetry. The fast Fourier transform is also successfully applied to NMR-introscopy, when the gradient of the external magnetic field during the decay of the free induction signal (following the application to the sample of a pulsed resonance RF field) is in turn oriented along each of the three mutually orthogonal coordinate axes.[183] In this case reconstruction is immediately obtained of the entire three-dimensional structure of the object,[182] though it is necessary to take into account the fact that fast switching of field gradients can lead to the appearance of undesirable induction currents.

Filtered back projection is used in most contemporary x-ray diagnostic tomographs and, thus, plays a particularly important role from the purely utilitarian point of view. Arranging a set of filters, one can stress or attenuate any details or aspects of the image. Besides, the algorithm admits the so-called "conveyer" handling of measurements, when in the calculation process only a part of the projection data is used at a certain stage, while the angular scanning is still being carried out.

Application of the Radon inversion equations, though usually accompanied with enhanced expense in computer time, nevertheless makes it possible to achieve good qualitative reconstruction fo the structure using a restricted set of discrete values of the scanning angle. In this case, as shown by the authors of Ref. 91, an important role can be played by the use of smoothing splines, as well as the use of a deformed grid.

Not touched upon the above short list of CAT algorithms is a variety of combinations of image reconstruction, methods of interpretation of data obtained with a fan-shaped scanning beam, special methods of multiplex tomography,[6] etc. Nevertheless the general conclusion which can be drawn from everything that was said reduced to the fact that practically each of the algorithms mentioned, except those based on direct matrix inversion, has its own positive qualities (and, naturally, weak points). Therefore it would be improper to give unconditional preference to any one or two of them, stating that the others are ineffective and not deserving further development. As already emphasized, the large diversity of experimental formulations of tomographic problems also gives prominence in some cases to different algorithm requirements.

An important aspect of the activity of specialists, developing and perfecting CAT algorithms, is the thorough verification of the suggested new versions using standard, unified test objects (heterogeneous phantoms). The Cormack[92] and Cho[94] phantoms have become widely known lately as applied to problems of medical radiology. Figure 6a shows the cross section of the simplest Cormack phantom; in the given

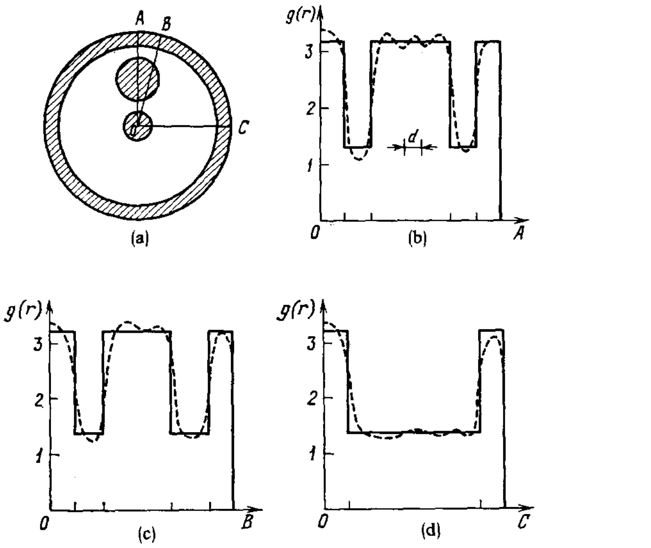

FIG. 6. The Cormack phantom[92] and typical results of its reconstruction in three cross sections: along the lines *OA*, *OB*, and *OC*. The density values are given in rel. un.; the scales along the abscissa axis in Figs. b)–d) are double that in Fig. a). *d* is the width of the illuminating beam.

given case the phantom components are prepared from aluminum (the shaded areas) and lucite (the unshaded inner circle). The ratio of absorption coefficients of aluminum to lucite for illuminating γ-rays, emitted by the ^{60}Ni or ^{60}Co isotopes, equals approximately 3, which simulates the situation occurring in emission x-ray diagnosis of human brain tumors (the aluminum corresponds to the skull bone and the affected areas, and lucite to the healthy brain). Figures 6b–d show by solid lines the absorption coefficient as a function of the radial coordinate, measured in the directions of *OA*, *OB*, and *OC*. The beam width *d* is assumed finite, and the beam cross section is assumed rectangular.

Both real and purely computational experiments can be carried out using phantoms. The results of a real experiment, carried out in 1963 by Cormack and Hennage, in which 19 ray-sums were measured in 25 projections with an error not exceeding 1%, were later handled by an algorithm based on polynomial expansions (the Cormack variant). The dashed lines in Fig. 6 illustrate typical reconstruction results. In a mathematical simulation of the same problem over a closed cycle (calculation of ray-sums and projections, introduction of random errors into the data, solution of the inverse problem) results are obtained, practically indistinguishable from those given in Fig. 6; taking the beam width into account also plays a secondary role up to a certian limit. In this case one can, of course, use various reconstruction algorithms and compare them with each other for various characteristics. Calculations show that substantial improvement of the quality of reconstruction can be achieved by adding *a priori* information on the phantom structure (sign definiteness of functions, existence of discontinuous derivatives,[94] etc.), examples of the so-called descriptive regularization. This kind of regularization is most simply realized within the SIRT, MENT, and Monte-Carlo methods.

5. COMPUTER-AIDED TOMOGRAPHY IN PHYSICS AND TECHNOLOGY

In the present section we are not concerned with applications belonging to medical diagnostics, but center our attention on CAT applications to physical and technological problems. Here, as in medicine, many important and striking results were obtained, though the amount of penetration of ideas and methods of CAT into the different areas of physics varies. In several cases tangible fruitful results have already been achieved, and the investigation front has advanced quickly; in others—only future experiments have been planned, and only model calculations have been carried out so far.

a) Defectoscopy

The role of CAT as an effective and quite universal method of nondestructive control of industrial products, all kinds of materials, and even food products, becomes more significant with every passing year. Practically any radiation (translucent) method of nontomographic type encounters the same difficulties as in classical medical x-ray diagnostics: the super position of structural detail in shadow imaging, low contrast, insufficient resolving power, background interferences, absence of a valid qualitative measure of the configuration, depth of occurrence and properties of defects, etc. The application of CAT changes the situation radically, though one must not forget that tomographic defectoscopy, in addition to the increased cost of the required equipment and the unfamiliarity of the personnel carrying out radiation control, is also accompanied with the necessity of overcoming difficulties of fundamental nature. The first is correction of measurement data for Rayleigh and Compton scattering, as well as for the "beam hardening" effect,[19,25] due to the filtering action of many investigated media on low energy photons.

In Ref. 95 one finds discussions of the features of γ-ray control with translucent emission energy from 310 to 662 KeV of wood and plastic products, as well as welds, voids and cracks in concrete, and radio-technological blocks. In particular, a tomogram of a silver fir shaft of a diameter of the order of 40 cm gives a distinct pattern of annular growth, worm holes, and internal dense pockets.

In a number of papers the objects of study are poorly accessible and at the same time most vulnerable such as construction elements of a nuclear reactor: fuel elements, coolers, and so forth.[96–99] Figure 7 shows tomography results of a hexagonal lattice of 37 fuel elements, in which three elements are missing, and four have voids.[97] The reconstruction was carried out by the MENT algorithm using 3, 6, and 18 projections. The conclusion was drawn that even six projections seemed to be sufficient for the reconstruction solution of the defectoscopic problem, while three sufficed for identification of a fuel element absent or unfit for utilization, if it is known *a priori* that the defect occurs only at one lattice site. We note that a similar problem is solved in Ref. 98 with utilization of a phantom with irradiation of the latter by a neutron beam of energy of 24 KeV.

Also tomographic control has been achievd of sites of

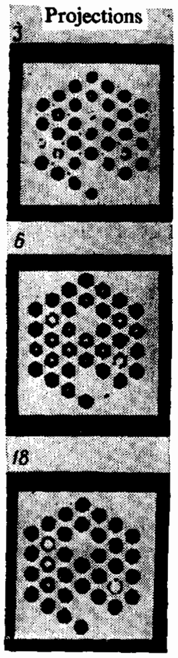

FIG. 7. An example of defectoscopy of a hexagonal lattice of fuel elements.[97] Three elements are missing, and four require replacement. The reconstruction was carried out from 3, 6, and 18 projections.

jet engines and turbines,[99] nonmetallic construction elements of helicopters,[100] and capacitors designed for high currents and voltages.[101] Some very interesting results were obtained in the Scientific Research Institute for Introscopy (Moscow).[102] Thus, in a dielectric layer of a product of diameter 240 mm one easily observes the presence of a copper wire of diameter 240 mm one easily observes the presence of a copper wire of diameter 15 μm; a quite sharp picture is obtained of the transverse cross section of a tube, which, in addition to a thin external coating, contains three layers of density packed metallic tubes, and in the internal region has a spiral construction, involving six layers of a metal with dielectric interlayers.

A method of a kind of preventive defectoscopy was suggested by V. D. Zimin and P. G. Frik,[103] who showed the possibility of reconstructing a complex temperature field over a fragment of a radioelectronic circuit (a montage plate). Hardly any method other than tomography could be used to identify in advance the elements of the circuit, which are in the first instance liable to be damaged by overheating.

Finally, one must mention the wide-ranging diagnostic possibilities of CAT, based on NMR-spectroscopy and making it possible to obtain the 2- and 3-dimensional fields of the nuclear spin relaxation time.[27] Thus, one can realize noninvasive local control of the quality of butter, cheese, margarine, and other food products directly in commercial form, which was earlier done only for a large bulk of the product "on the average."[104,105]

b) Microscopy

The importance of using CAT methods in many microscopic studies is mostly determined by the fact that often focusing of the selected transverse cross section of small thickness is not achievable as a result of which superposition of a large number of two-dimensional images of one or even several objects occurs and provides little information. In optical microscopy such a situation occurs usually in the analysis of almost transparent samples, when the phase map has to be somehow transformed into an amplitude map.[20,106] This concerns both the simplest methods of realization of a phase contrast (the dark field method, the Schlieren method), and the more universal methods, using the quarter-wavelength Zernike plate, anoptral and interference schemes.[107] We note that Berry and Gibbs,[106] developing the methodology of interpretation of three-dimensional phase objects, anticipated many results in the development of CAT algorithms (Fourier synthesis, the use of the Radon inversion with and without taking into account the symmetry properties of the body, the calculation of statistical structural characteristics), which then often appeared in later publications without mentioning Ref. 106.

The significance of CAT as applied to electron microscopy was already discussed in Section 3. Here, perhaps, one must only add that alternative methods of investigating objects such as biomolecules and their associations (viruses, riboscomes, albumin crystals, etc.), for example, by means of stereopairs or neutron-diffraction cameras, are in any case at the present time clearly inferior in their properties to tomographic electron microscopy. Indeed, stereomicroscopy does not make it possible to obtain quantitative estimates of the depth, and does not eliminate the mutual blocking of the details of the object;[20] as to the inverse structural problem of scattering in neutron diffraction, its solution is not unique in the general case.[108]

We mention several interesting results which appeared after the publication of B. K. Vaĭnshteĭn's famous review[12] on this subject. A group of Japanese authors using phantoms and real associates (viruses) has investigated in detail the reconstruction possibilities of micro-objects employing a restricted range of scanning angles ($2\pi/5$ instead of π) and a small number of projections.[109] TGhe ART and MENT algorithms were used, and preference was given to the latter on the basis of the quality of reconstruction. Figure 8 reproduces the results of tomographic studies of bacteriophage No. 5 Bacillus Megaterium 337 from data of a Soviet study;[53] in this case a variant of the FBP algorithms was used, and 47×47 nodes of a square grid were selected with the size of the dicretization step being 6Å.

Speaking of the role of CAT methods in electron microscopy, one must mention the prominent achievements of the Cambridge group of investigators, led by A. Klug, in the analysis of the structure of the most important nucleo-protein complexes. The successful combination of the technique of optical diffractometry with tomography enabled these investigators to develop a quite universal, precision method for determining the detailed structure of transport RNA, nucleosomes, complex histones occurring in them, etc. As is

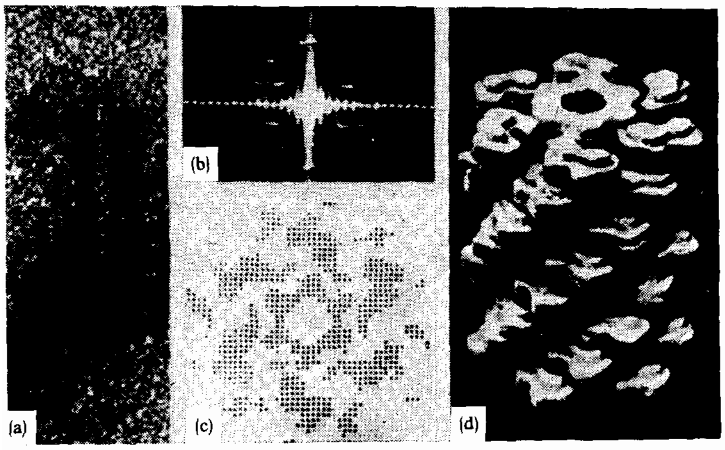

FIG. 8. Microphotography of a bacteriophage (a), its optical diffractogram (b), a tomogram of the central cross section of an elementary disk (c), and a spatial model of the bacteriophage (d).[53]

well-known, on the basis of work A. Klug was awarded the Nobel prize in chemistry in 1982 (see Ref. 184).

c) Solid state physics

The results of real and even model experiments in solid state physics utilizing CAT methods are so far quite modest and rare, but the prospects in this area of research are quite promising. With some degree of arbitrariness the corresponding studies can be considered as tomographic defectoscopy of a higher level of complexity, since here we are dealing with identification of fine "defects" of the type of fatigue microcracks, extrusions and intrusions on the free surface of residual microstresses, weak variations of density or electrical resistance due to changes in the number of dislocations, etc.

In an already mentioned study Cagnasso *et al.*[100] investigated by tomography a component of a helicopter rotor (composition of glass and of an epoxy resin), enabling them to determine clearly the position and sizes of fatigue microcracks, as well as density gradients due to the polymerization process. An analysis of the residual stresses can be carried out, using methods of both ultrasonic CAT[21] and holographic tomointerferometry.[68] The original tomographic variants of shadow and interference methods for reconstruction of temperature fields and thermoelastic stresses were developed in the polymer physics department of the Ukr. Sci. Center of the USSR Academy of Sciences.[111-113]

For solid state physics problems the combination of CAT with the contemporary EPR and NMR techniques is quite promising. Using EPR introscopy (the Lauterbuhr variant, the ART algorithm) Hoch and Day[114] measured the spatial impurity distribution of singly substituted nitrogen is a crystal lattice of diamond, showing in this case that spatial resolution of several microns can be achieved (Fig. 9). Mansfield and Grannell[115] suggested an effective method, making it possible to suppress the dipole NMR line width in the solid, and to retain in this case a sufficiently high gradient of the magnetic field. The method was verified in an experiment with solid camphor. As mentioned in Ref. 115, if the field gradient were to be enhanced by 4–5 orders in comparison with that already achieved, a unique possibility would be uncovered of visualizing the crystal lattice, showing vacancies, fine structure of dislocations, and other details. Advances by experimenters in this direction is continuing.

d) Geophysics

As in the case of solid state physics, the number of tempting ideas and projects, related to the use of CAT, that have been put forward so far exceeds noticeably the number of real studies performed. Speaking of geotomography, a number of specific features must be mentioned. These are, primarily, the large sizes of the investigated areas and the distances d from the source to the detector, usually tens or hundreds of meters; the use for transillumination by radiowaves with a wave length λ of the order of several meters (more rarely—decimeters and centimeters), and the spatial resolution corresponding to these λ values. Also possible is the application of seismic probing. This means the use of bore holes, usually several dozen meters in depth, in which sources and detectors are located. Often there are only two such holes, and they may be inclined with respect to each other. Thus, the angular scanning is strongly restricted, and even in the geometric optics approximation there may not be a distinguished plane in which the image is reconstructed (Fig. 10). Finally, to verify the assumption of rectilinear wave propagation it is necessary, according to Ref. 116, that: a) $\pi d \gg \lambda$, b) $\pi\delta \gg \lambda$ (δ is the thickness of the skin-layer in the medium), and c) the refractive index of the medium should vary quite smoothly.

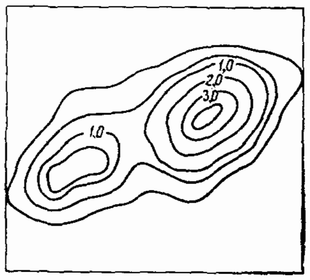

FIG. 9. Isolines of nitrogen impurity distribution in the crystal lattice of two diamonds[114]; tomographic EPR-introscopy. The distances between the diamond centers and their diameters are ~2.5 mm, and the spatial resolution is ~300 μm.

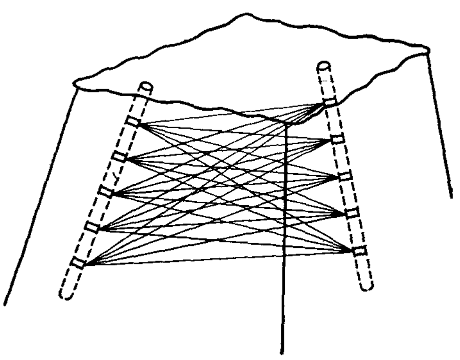

FIG. 10. Geotomography scheme with two inclined holes. One of the holes contains sources, and the other—detectors.

We provide several examples of geotomographic studies. A number of test objects was investigated in Ref. 117, a reconstruction algorithm (an ART variant), was being developed, and then the results obtained were used for interpreting an experiment in which the nature of variation of rocks was investigated along a section of a subway line being planned. The holes were inclined and had a depth of 46 to 79 m; the frequency of probing radio waves was 50 MHz. The *same* operating frequency was used[118] to investigate the distribution of the electrical conductivity in the process of underground coal gasification, where the combustion front could be controlled. In handling the data preference was given to the ART and SIRT algorithms. Among other possible applications discussed in the literature we note Refs. 117–120: discovery and estimation of petroleum reserves with secondary exploitation of beds, contouring of salt deposits, mine mapping during the search for tapered and lost deposits, inspection of reservoirs of depleted nuclear fuel, and so forth. It can also be expected that CAT methods will change drastically the traditional practice of archeological excavation.

In discussing geotomography problems their conceptual similarity to the inverse kinematic problems of blast seismology must be noted.[121] The general formulation of the latter is the following: on some observation surface x, y a system of hodographs of different types of seismic waves (reflected, refracted, diffracted, exchanged, etc.) from a series of known sources is given; the velocity distribution law of the corresponding waves is sought in the x, y, z space. A number of studies during recent years shows that methods of deep seismic probing and CAT methods are similar in many respects and supplement each other successfully in their development.[57]

e) Earth and planetary atmospheric physics

If studies of the Earth atmosphere are separated from the general group of geophysical problems, then the present stage of penetration into the arsenal of these studies of CAT ideas and methods must, apparently, be regarded as only initial. Nevertheless, a number of tomographic problems of atmospheric physics has already become quite specific.

The reconstruction problem of concentration fields of contaminating impurities of industrial origin is discussed in Refs. 122–124 for atmospheric volumes of characteristic size ~ 10–20 km. Compared with Lidar and other remote sensing methods of contamination analysis,[125] the CAT technique allows to construct directly the laminar distribution maps of components such as NO_2, NO, CO, O_3, SO_2, C_2H_4, etc. In this case the requirements on laser power are substantially reduced, since for atmospheric tomography the signal/noise ratio is much higher than for a Lidar.

It must be said that the authors of the references mentioned have so far restricted themselves to mathematical simulation problems only, but theoretically the problem was investigated quite thoroughly: the signal/noise ratio was varied, as were the laser power (continuous and pulse regimes), sizes and geometry of objects, methods of recording projections and their number, etc. From the algorithmic point of view preference has been given to inverse projection with the Shepp-Logan filter. The experimental scheme suggested in Ref. 124 appears in all respects more constructive than the one in Ref. 122: it employs one laser, a system of mirrors producing fan-shaped beams, and a set of detectors: the mirrors and detectors are located on a circle in the selected plane at a required height, and the laser itself can be situated, generally speaking, on the surface of the earth (Fig. 11). An FBP variant with application of fan-shaped projections was developed by Herman and Naparstek.[126]

An interesting variety of CAT problems is generated in ionospheric physics in the study of propagation of short waves around the Earth.[127] These problems arise due to the substantial progress in the techniques of inclined and retrograde-inclined probing of the ionosphere.[128] The results of Ref. 127 can be considered as an extension of the Radon inversion to the case in which an unknown even continuous function is given on a sphere, and its integrals over the great

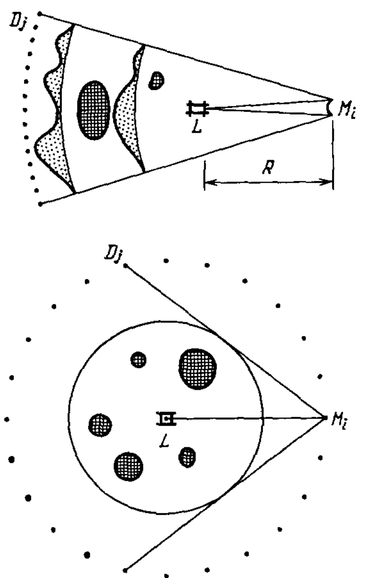

FIG. 11. Possible variant of a CAT of the atmosphere with illumination of the investigated region by a fan-shaped beam from a single laser L.[124] The system of mirrors M_i and detectors D_j are located along a circle. The regions of atmospheric contamination are shaded. Also given are two intensity distributions of the laser beam.

circles are measured. Also proposed was a regularization method of the corresponding ill-posed inverse problem. The actual algorithm consists of using a truncated expansion of the right hand side in eigenfunctions of the operator of the problem (these are Laplace functions).[129]

The authors of Ref. 130 studied the structure of temperature inhomogeneities of the Earth atmosphere from observations of light refraction by means of the orbiting station "Salyut-6." In this case a variant of the defocused diaphragm method[131] was used, i.e., Schlieren measurements were essentially carried out, allowing a purely tomographic data analysis.

As examples of investigating the structure of planetary atmospheres of the solar system we mention Refs. 132, 133. In both cases the toipic is the Jovian atmosphere, and while the authors of Ref. 132 essentially used the assumption of central symmetry of the problem, a quite general method was developed in Ref. 132 for data processing of radio transillumination of the atmosphere, making it possible to take into account refraction, horizontal gradients of the index of refraction, and the "oblate" shape of the planet. Mathematically the inverse problem reduces to solving a system of nonlinear equations of integral geometry.

f) Aero- and hydrodynamics

'he special feature of applying CAT methods to prob- ·f dynamics of a liquid, gas, and plasma, discussed in the present and the following sections, is that quite often one succeeds in avoiding the general two-dimensional or, even more so, the three-dimensional formulation. In the applications discussed above the possibilities of simple parametrization of the unknown function or direct use of the symmetry properties of the object were, as a rule, very restricted. On the other hand, in describing flows of liquids and gases, as well as many plasma systems, the introduction of phenomenological parameters and various dimensionless quantities, characterizing the self-preserving nature of the problem and other similarity properties, is traditional. In this connection the Abel inversion, its generalization to the case of elliptic isolines, coordinate decoupling, taking into account the symmetry planes of the reconstructed field, etc.,[24–26] become quite important.

Nevertheless, also in this wide area of studies the significance of general CAT methods, applied to asymmetric field parameters (density, temperaeture, flow velocity, etc.) becomes predominant as the complexity of the investigated objects increases. Examples of this nature are discussed below.

The first successful studies of spatial flows of complex shape were carried out by Soviet authors.[134,135] Although the algorithms used in these early studies (they are described in detail in the monograph of Ref. 136) should hardly be recommended to experimenters from the point of view of the present level of development of computational CAT methods, on the whole they made it possible to solve specific problems, basically related to the flow around axially symmetric bodies under angles of attack.

The large amount of information provided by CAT methods in aerodynamics was demonstrated by Collins *et al.*[137–139] The experiments were performed using methods of pulsed holographic interferometry; the characteristics of an inclined air jet and of substantially asymmetric density fields near the cone were investigated for varying angles of attack, as well as transonic flow near the wing of an aircraft model. A similar problem was posed in Ref. 140 (a cone under an angle of attack in a supersonic aerodynamic tube). Different CAT algorithms were used in these cases: while Collins' group used Maldonado polynomial expansions, Fourier synthesis was used in Ref. 140.

CAT methods were successfully used by a number of authors in studying structures of convective flows of liquids and gases. Vest *et al.*,[141,142] using holographic interferometry with a phase lattice, reconstructed temperature fields of complex configuration in water over heated bodies with an error not exceeding ± 1 °C. V. D. Zimin and P. G. Frik[103] investigated by Schlieren methods the optical inhomogeneity of a liquid, produced by the temperature field of a convective turbulent jet flowing from a nozzle. Fitzgerald and Hörster[143] used holographic interferometry with compensation of phase distortions to study the correlation of the temperature distribution and pressure in a lamp filled with argon. Later a group of Japanese authors carried out an experiment with a similar object (an automobile bulb) with a holographic device with a viewing angle near 180°.[144] The reconstruction of the refractive index field was carried out by the ART algorithm, with more than 30 projections; also investigated was a convective air flow over a heated tungsten wire. Other examples related to the study of heat- and mass-transfer are given in the monograph of Ref. 68. Speaking of these experiments it is appropriate to stress their significance for verifying conclusions and further development of the contemporary theory of stochastic self-oscillations, involving the concept of a strange attractor.[145,146]

In problems related to dynamics of liquids and gases CAT methods can, of course be combined not only with interferometric measurement techniques. Thus, in French studies on combustion physics[147] the emission coefficient fields of a turbulent hydrogen-oxygen flame were reconstructed on the basis of purely emission data. A method was suggested and experimentally tried[148] of determining the local values of the emitting capability of a medium irradiated by a deutron beam (with energy ~ 35 MeV) from a linear accelerator. This method, based on the MENT algorithm, is important both from the point of view of defectoscopy problems of a number of materials, and for solving many diagnostic problems of dilute gas dynamics, where the electron beam technique of exciting an isolated portion of a flow[149] is widely used.

One study[150] is devoted to applying tomographic methods to laser absorption diagnostics of turbulent jets. Figure 12a shows the scheme of illuminating the object by radiation of a single laser, making it possible to obtain 5 projections with pulse repetition frequencies up to 20 kHz. This is used to reconstruct the space-time pattern characterizing the flow structure; an example of one of the "slices" of such a pattern, corresponding to the outflow of a mixture of 10% CH_4–90% Ar, is given in Fig. 12b. In processing the data over a small

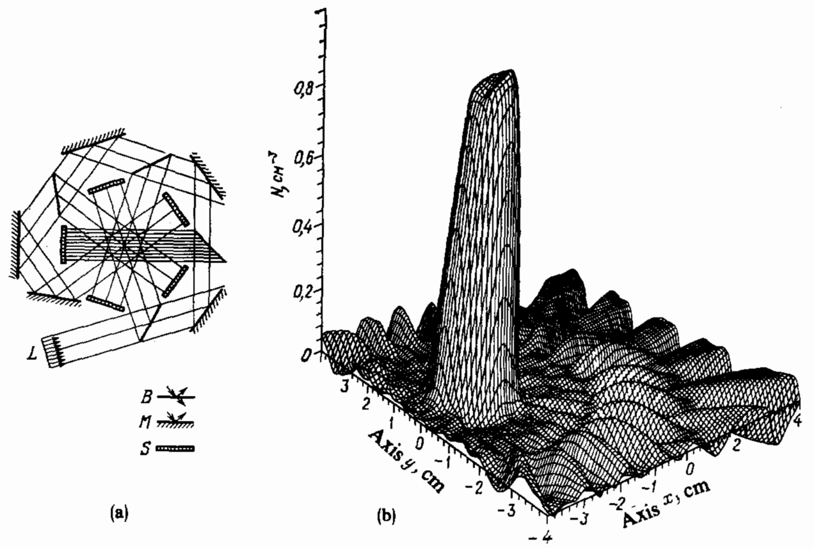

FIG. 12. a) CAT absorption scheme for studying nonstationary jet flows.[150] A single laser is used with a high pulse repetition frequency (B are semitransparent mirrors, M are mirrors, and S are detectors). b) Reconstruction results of the concentration field of the absorbing component in one of the cross sections.

number of projections (about 100 ray-sums in each) the authors of Ref. 150, after careful preliminary modeling of the problem, gave preference to the modified algorithm of inverse projecting with a Shepp-Logan filter.

We do not dwell in detail on the possibilities of combining techniques of NMR-spectroscopy and CAT as applied to reconstruction of the velocity field of a liquid flow in a channel; several examples of this dind are given in the review of Ref. 27. We mention here only the promising studies of Lauterbur and coauthors, in which "slices" were reconstructed of pulsed flow of a liquid inside a glass tube, placed in a stationary liquid of the same kind,[151] and also the reconstruction problem using projections of vector (velocity) fields was investigated.[152]

g) Plasma physics

In many problems of plasma diagnostics the procedure of obtaining local characteristics must be combined with good temporal resolution, taking non-equilibrium into account, reabsorption of radiation, presence inside the investigated volume of structurally complicated flows of material and heat, presence of electric and magnetic fields, etc. Angular scanning and the measurement of a set of projections are often extremely difficult (particularly for closed systems, used in thermonuclear studies, objects investigated in astropysics, etc.). All these factors are serious obstacles for the wide use of CAT methods in plasma physics; thus is explained the strong adherence of a number of specialists in plasma diagnostics to simple models, assuming parametrization of the unknown distributions, Abelian inversion, or its simplest generalizations.

Nevertheless, the number of examples convincingly illustrating the possibilities of tomographic study of an asymmetric plasma is already quite large at the present time. We discuss several of them.

In several studies relating to the physics of a *low-temperature plasma* discharge arcs were investigated, whose axial symmetry was substantially destroyed by the application of an external magnetic field or the interaction with the flow of the incoming gas. Apparently, the first in this direction was a study,[153] where in two cross sections from seven distributions a reconstruction was made of the two-dimensional distribution of the emission coefficient in the Ar line $\lambda = 696.5$ nm for arc discharge in a magnetic field at a pressure of 1.1 atm. Sebald[154] not only imposed a magnetic field on the arc, but also placed it in a nitrogen, low-turbulence aerodynamic tube. The local-equilibrium temperature field was found, as well as the flow velocity field of the heated gas, penetrating to the core of the arc. The plasma possessed a symmetry plane, and the error of reconstructing the emission coefficient from the same seven projections as in Ref. 153 was estimated to be within 3%. Using a magnetic field Plessl[155] caused the arc itself to rotate, which under the assumption of stationary discharge in a rotating reference system made it possible to gather quite simply the required number of projections. The two-dimensional temperature field of the gas was determined from the H_β-line with the deviation of the plasma state from local thermodynamic equilibrium being taken into account; in this case a complex structure of the flow was made evident, including several vortices and stagnation points. We note that a reconstruction algorithm based on Maldonado polynomial expansions was used in all the studies mentioned.

In a series of papers T. S. Mel'nikova and the present authors[48,51,156] studied a nonstationary turbulent plasma, and a large diversity of shapes of the plasma filament in a longitudinal magnetic field was recorded. Figure 13a shows the scheme used in the case of a 6-channel tomograph, in which short-focus lenses L_1–L_6 were used to focus the radiation from a certain cross section of the plasma on the surfaces of the light-pipes P_1–P_6. The opposite edges of the light-pipes were assembled in a block, and the pattern from it at a wavelength selected by an interference light filter (F) is registered by a fast movie camera (K). Figure 3b shows an example of an axonometric projection of the field of reconstructed temperatures in a particular cross section of the filament. A regularization algorithm of the Radon inversion was used in the reconstruction.

Despite the significant technical difficulties of obtaining short time intervals of several angular projections, CAT

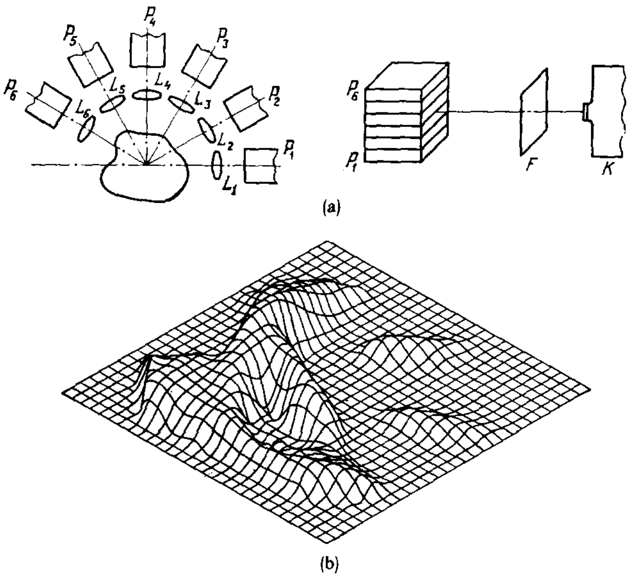

FIG. 13. a) Scheme of a 6-channel tomograph for diagnostics of a nonstationary low-temperature plasma from emission spectra[156]; b) example of reconstructing the temperature field of a turbulent plasma of an arc discharge $T_{max} = 10500$ K).

methods have also spread to the region of *high-temperature plasma* physics. Thus, a reconstruction algorithm was developed[157] for purposes of local phase measurements of the plasma density in the toroidal instrument "Tokamak-3," similar to that suggested earlier by A. M. Cormack[31] for medical x-ray diagnostics. A successful realization of fast polychromatic registration of four projections of a selected cross section was achieved in the Lawrence (Berkeley) laboratory on the "Tormak-4" plasma device by means of lightpipes, and then, using the ART algorithm, the local distributions of spectral line profiles of hydrogen and helium were established, allowing further reconstruction of the density and temperature fields of the plasma.[158] Similarly to the already mentioned experiment of Plessl,[155] the authors of Ref. 159 obtained on the "Alcator-A" Tokamak up to 18 fan-shaped projections of x-ray emission from a plasma rotating with a period $T = 0.456$ msec.

Diagnostic experiments on the "Pulsator"[160] and "Tokamak-10"[161] devices can be regarded though with some reservations, as tomographic studies. Since it was not possible to obtain more than two projections in these studies, the authors were forced to resort to a definite parametrization of the problem. The latter corresponded in one case[160] to the imposition of two systems of nonconcentric circular isolines, and in the other[161]—to oval type isolines.

The possibilities of tomographic diagnostics were recently demonstrated not only on closed magnetic systems of the tokamak or stellarator type, but also on devices with laser heated targets. Thus, using the well-known "Cyclops" device (Livermore laboratories, USA) in studying two-sided laser compression of a glass microsphere of diameter ~ 100 μm, filled with a deuterium-tritium mixture, the multiplex tomography method with a zone plate was utilized.[162] The x-ray emission generated during the target heating was registered, and a spatial resolution of the order of 8 μm was achieved. A three-dimensional tomographic reconstruction of the source function from four two-dimensional projections, fixed by a system of *camerae obscurae*, was realized in Ref. 163. A special (three-dimensional) version of the MENT algorithm, carefully checked in advance on test objects,[164] was used in analyzing the experiment. It was shown that if the initial diameter of the spherical target was 200 μm, then following two-sided laser compression an oblate ellipsoid of revolution with a mean diameter ~ 60 μm was obtained. A number of interesting details of the implosion process was also observed, particularly the presence of an intensely emitting "satellite" in the x-ray spectrum, arising from the residual fragment of the glass shell.

Williamson and Clarke[165] proposed to use the equations of three-dimensional Radon inversion to reconstruct the asymmetric distribution function of plasma electrons with respect to the velocity vector. It is suggested in this case to register the projections consisting of spectra of Thomson scattering. Relativistic corrections are included in finding the distribution function, and the foundation is laid for the application of the method for purposes of plasma diagnostics in problems of controlled thermonuclear fusion. Model calculations show the possibility of entirely satisfactory reconstruction of the distribution function from four projections.

As to the use of CAT methods in problems of *space plasma* physics, in most cases investigators were forced to resort to quite rigid *a priori* restrictions, and only isolated attempts are known of reconstructing the characteristics of truly asymmetric objects. For example, in studying the solar corona plasma the standard assumptions made are of central or axial symmetry,[166] the possibility of describing the electron density distribution function by a small number of parameters,[167] the correctness of calculating the three-dimensional magnetic field distribution from two-dimensional (photosphere) data by using the Laplace equation,[168] etc. Recently Altschuler and coauthors attempted to realize a tomographic experiment on reconstructing the electron density field of the corona in a quite general form.[77] In this case, however, it was necessary to process projections obtained successively during half the period of the Sun rotation around its axis, i.e., during 14 days. Thus, a quite serious assumption was made concerning the constancy of plasma parameters during the time mentioned.

The studies already mentioned of Bracewell (their detailed review is contained in Ref. 7), which contain investigations of UHF emission of solar plasma, fixed by the method of band sums are purely tomographic in their mathematical approach. To the same class also belong studies on lunar radar ranging,[169] and on the interpretation of data obtained on a radioheliograph.[170]

Somewhat specific, but undoubtedly promising, is a method of studying the three-dimensional spatially inhomogeneous structure of planetary nebulae from the planar pattern of its isophots.[171] By comparing with earlier calculations for the planetary nebula NGC 7662, in which the latter was assumed homogeneous in density, substantially improved results were achieved within the variable density model.

6. CONCLUSION

The development of CAT continues in many directions, and embraces all new areas of physics and technology. A tendency is distinctly traced of enhancing the amount of information in each of the ray-sums and each of the projections. This is primarily achieved by the fact that the recorded values in the same direction of observation are becoming not simply numbers, but functions of frequency, polarization characteristics, relaxation parameters, etc. Due to this fact two- and three-dimensional fields of various quantities, describing the properties of an inhomogeneous object, can be reconstructed. One example is so-called tomochemistry,[172] opening up the possibility of determining the composition of the material at each point of an object. Quite promising in this respect is also the combination of CAT principles with scattering spectroscopy.[173] A large amount of information is also characteristic of new variants of tomographic NMR-introscopy.[174]

The problem of registering the number of projections necessary for good reconstruction of the structure of an object remains urgent. As seen from several examples above, significant effect here is provided by the use of holography methods (phase lattices, conic reflectors with a viewing angle near 180°, etc.), as well as multiaspect laser schemes with a high pulse repetition frequency. Apparently, employment of the phenomenon of wave front reversal can also be useful. In x-ray CAT ever more complex devices with a large number of sources and detectors (dynamic core tomography) continue to be constructed.[175] The purely mathematical aspects of the problem of image reconstruction from a restricted number of projections are also being investigated.

Quite significant future possibilities are contained in optical methods of realizing the process of reconstruction of the tomographic image. In particular, very attractive is the variant of multiple image processing, carried out by analog methods in which high resolution is achieved initially, followed by high contrast (high signal/noise ratio).[6] New interesting possibilities are offered by coded aperture methods. A purely physical approach to significant enhancement of image contrast, proton tomography, was suggested by Cormack and Koehler.[176] This direction of research is particularly useful in medico-biological applications, since it makes it possible to decrease substantially the total dosage of object irradiation.

Among the new promising ideas and directions of CAT development it is appropriate to mention tomosynthesis with filtration of two-dimensional projections of an object in high and low frequencies (by transition to Fourier transforms), the so-called ektomography.[177] Sclieferstein and Chain[178] recently started developing an important general method of analysis, making it possible to assess with a certain degree of accuracy to what extent the reconstructed structure repeats the original for a given algorithm. A number of interesting results have already been achieved by this method, and the fundamental role of statistical simulation algorithms has been demonstrated.

The improvement of techniques of forming wide ribbon beams from lasers operating both in the pulsed and continuous regimes opened new possibilities for the development of resonance fluorescence tomography with high temporal resolution,[185] as well as interference holographic versions, making it possible to reconstruct two-dimensional fields of a liquid velocity vector, for example, in convective (Bénard) flows.[186]

A new promising direction of research, which is, admittedly, in its early stage, is the so-called inversion tomography.[187,188] Its aim is the multiplex synthesis in real time of a three-dimensional image of an object from a finite set of tomographic "slices."

The fast and continuous progress in both techniques and computational CAT methods leaves no doubt that tomographic study of various physical objects will lead time and time again to new experimental results of a fundamental nature.

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