Echo phenomena in plasmas

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Various types of echo oscillations in plasmas are discussed. Both unbounded plasmas (gaskinetic plasmas and plasmas in external fields) and bounded plasma systems are considered. Several practical applications of echo effects in radio wave propagation and plasma diagnostics are discussed. The second- and third-order echo oscillations in equilibrium isotropic and magnetized plasmas and in semiconductor plasmas are discussed. A charged-particle beam in a non-equilibrium plasma can lead to an amplification (over time) or a growth (over space) of echo oscillations. A ballistic theory is offered for echos in plasmas. This theory can describe echos in the case of large-amplitude external perturbations, under conditions such that the method of successive approximations cannot be used. The ballistic theory can also describe echo saturation. Echo phenomena in inhomogeneous plasmas are discussed. Section 2 reports a study of echos in bounded plasmas. In this case, additional echo phenomena result from the reflection of charged particles from the plasma boundaries and from the existence of surface waves. Research on echos in plasma slabs and metal films is reviewed. Echo effects can lead to an anomalous transmission of electromagnetic waves through a plasma slab which is opaque (in the linear approximation) to the fields of these waves. Section 3 reviews the basic experimental data on echos in the cyclotron, ion, and plasma zones and research on collisions and microscopic turbulence in plasmas by means of echos. The experimental data available are compared with the theoretical results.

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CONTENTS

INTRODUCTION

Even in a collisionless plasma, oscillations in the macroscopic properties decay over time (or space); the effect is called "Landau damping"[1] (and has been confirmed experimentally[2,3]). Landau damping is one of the most important phenomena in plasma physics. It plays a fundamental role in collective processes in collisionless plasmas. The reversible nature of Landau damping can be seen very clearly in the echo effect, predicted theoretically in 1967 by Gould, O'Neill, and Malmberg[4] and subsequently found experimentally.[5–9]

At the time, it was already known[1] that the damped oscillations of the macroscopic field in a collisionless plasma are accompanied by some undamped oscillations (microoscillations) of the distribution function at a frequency $\omega = \mathbf{k}\mathbf{v}$. These oscillations thus "remember" an external perturbation after the macroscopic field has disappeared (the microoscillations of the distribution function may be thought of as a set of modulated particle beams: Van Kampen waves[10]). Gould *et al.*[4] were the first to point out that the interference of the microoscillations of the distribution function modulated by the fields of two or several sources separated by a certain time interval (or a certain spatial distance) could lead to a phase focusing of the particles over time (or space) and that this focusing could be accompanied by the appearance of a macroscopic signal: a temporal (or spatial) echo. The immediate cause of the phase focusing of particles in a homogeneous plasma is a nonlinearity, so that in this setting the echo is an essentially nonlinear kinetic effect (in an inhomogeneous plasma, an echo effect may be linear in the field of the external perturbations).

Since the plasma echo is a coherent phenomenon associated with phase focusing of particles, it is similar in nature to the spin echo,[11] the cyclotron echo,[12,13] and the photon echo.[14] Kadomtsev published a penetrating physical analysis of the mechanism for the plasma echo (and also the spin

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and cyclotron echos) in a review[15] in 1968, at a time at which research on the plasma echo consisted exclusively of its prediction and discovery.

Subsequent years saw active research on echo phenomena in not only gaseous plasmas but also solid-state plasmas in semiconductors and metals.[28–30,50] Echo effects in solid-state plasmas may be useful for studying the band structure of semiconductors and the reflection of plasma particles from boundaries. Echo effects may also occur in plasmas in space; a cyclotron echo has been observed in the ionospheric plasma,[16] and numerical estimates suggest that a plasma echo can also occur there.[58]

In a bounded plasma there are some new possibilities for echo phenomena because of the reflection of particles from the plasma boundaries and the existence of surface waves.[71–74,76–79]

The spatial echo is nonlocal in nature, by which we mean that an echo signal can arise in a plasma region which cannot be reached by the original macroscopic waves. Information on external perturbations is carried by modulated particle beams through opaque regions, where these beams can regenerate a macroscopic signal under certain conditions. This possibility of plasma "brightening" through an echo effect was demonstrated in Refs. 18 and 19. This idea stimulated several theoretical and experimental studies of the brightening of inhomogeneous plasmas[20–23] and of the nonlinear transmission of electromagnetic waves through trans-critical plasma slabs[24–27] and metal films.[28–30] It can now be asserted confidently that the plasma echo is more than an elegant nonlinear phenomenon: It is also extremely useful for plasma diagnostics and for several practical applications in radio wave propagation. These applications are the motivation for this review.

We will adopt some simple examples to discuss the various types of echo oscillations in both unbounded plasmas (gaskinetic plasmas and plasmas in external fields) and bounded plasmas (semi-infinite plasmas, plasma slabs, and metal films). Two approaches are taken for a quantitative description of echos. One is based on a kinetic equation with a self-consistent field (this is the self-consistent approach), while the other uses the approximation of free-streaming electrons (the ballistic approach). We analyze the conditions for the applicability of these two approaches by working from theoretical and experimental data. We report some experimental data on echos, and we compare them with the theoretical results. We discuss the possibility of using echos to study collisions and microscopic turbulence in plasmas. Certain aspects of the plasma-echo phenomenon have been covered in some previous reviews.[31–33]

The review by Bachman, Sauer, and Wallis[31] focuses on research on the spin, cyclotron, and photon echos. They also set forth the ballistic theory for the temporal plasma echo and report experiments[9] revealing the temporal echo in a plasma.

Erokhin and Moiseev[32] reviewd the various types of wave conversions in inhomogeneous plasmas. One section of their review deals with research on echo effects (or related effects) in inhomogeneous plasmas.

Porkolab and Chang[33] have reviewed research on various nonlinear effects in plasmas (nonlinear Landau damping, decay instabilities, solitons, large-amplitude waves in plasmas, etc.). Their general approach was to compare theoretical and experimental results. Their review also contains, in particular, a brief review of results on echo phenomena in plasmas. They discuss primarily experimental data on echos of plasma oscillations,[5] ion acoustic oscillations,[6,7] and cyclotron oscillations[54,55] in homogeneous plasmas and on the use of echos to study collisions.[87]

1. ECHO PHENOMENA IN UNBOUNDED PLASMAS

a) Mechanism for the appearance of echos

Kadomtsev analyzed in detail the mechanism for the plasma echo and its relationship to Landau damping.[15] Kadomtsev was the first to suggest using the concept of Van Kampen waves to explain the echo effect. Let us review the basic thrust of this analysis. Ignoring collisions between particles, and restricting the discussion to longitudinal waves, we work from a nonlinear kinetic equation for the distribution function and an equation for the self-consistent electric field:

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}}+\frac{e}{m}\mathbf{E}\frac{\partial}{\partial \mathbf{v}}(f_{0\alpha}+f_\alpha)=0, \tag{1.1}$$

$$\operatorname{div}\mathbf{E}=4\pi e\int f_\alpha\,d\mathbf{v}, \tag{1.2}$$

where f_α is the deviation of the distribution function from the unperturbed function $f_{0\alpha}$, and the index α specifies the particle species ("e" or "i"). Assuming that the external perturbations are small, we can use the method of successive approximations to solve the system (1.1), (1.2). In other words, we seek the deviations of the distribution function f and of the field $\mathbf{E}$ in the form of the series

$$\begin{aligned} f&=f^{(1)}+f^{(2)}+\ldots,\\ \mathbf{E}&=\mathbf{E}^{(1)}+\mathbf{E}^{(2)}+\ldots, \end{aligned} \tag{1.3}$$

where $f^{(1)}$ and $f^{(2)}$ (and, respectively, $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$) are terms which are linear and quadratic in the external perturbations. We will take a more-detailed look at the linear solution in an electron plasma, ignoring the ion motion and setting $\alpha=\mathrm{e}$ (in the subsequent discussion the subscript "e" will be omitted). Landau showed[1] that for a correct solution of the small-oscillation problem the linearized version of Eq. (1.1) must be supplemented by specifying the initial (or boundary) perturbation of the distribution function $f^{(1)}$ at the time $t=0$. We choose this perturbation in the form of a plane wave $f^{(1)}(\mathbf{v},\mathbf{r},t=0)=g(\mathbf{v})e^{i\mathbf{kr}}$, where $g(\mathbf{v})$ is the velocity-dependent amplitude of the initial perturbation. The solution of system (1.1), (1.2) in the linear approximation is

$$f^{(1)}_{\mathbf{k}\omega}=-i\frac{e}{m}\frac{1}{\omega-\mathbf{kv}}\frac{\mathbf{k}}{k}\frac{\partial f_0}{\partial \mathbf{v}}E^{(1)}_{\mathbf{k}\omega}-i\frac{1}{\omega-\mathbf{kv}}g_{\mathbf{k}}(\mathbf{v}), \tag{1.4}$$

$$E^{(1)}_{\mathbf{k}\omega}=-\frac{4\pi i}{k}\frac{\rho^0_{\mathbf{k}\omega}}{\varepsilon(\omega,\mathbf{k})}, \tag{1.5}$$

where

$$\rho^0_{\mathbf{k}\omega}=e\int\frac{q_{\mathbf{k}}(\mathbf{v})\,d\mathbf{v}}{\omega-\mathbf{kv}},$$

and $\varepsilon(\omega, \mathbf{k})$ is the longitudinal dielectric permittivity of the plasma,

$$\varepsilon(\omega, \mathbf{k}) = 1 + \frac{4\pi e^2}{mk^2} \int \frac{\mathbf{k}(\partial f_0/\partial \mathbf{v})\,d\mathbf{v}}{\omega - \mathbf{kv}}. \tag{1.6}$$

For simplicity we restrict the discussion below to the one-dimensional case, with the waves propagating along the z axis; we adopt the notation $v_z = v$ and $k_z = k$.

Using the inverse Fourier transforms we can easily find the asymptotic behavior (in time) of $E^{(1)}$ and $f^{(1)}$. It follows from (1.5) that the asymptotic behavior of the field may contain, in addition to the natural oscillations whose complex frequencies are found from the equation $\varepsilon(\omega,\mathbf{k}) = 0$, some oscillations determined by the nature of the external perturbations, i.e., by the particular nature of the function $\rho^0_{\mathbf{k}\omega}$. In the evaluation of ε in (1.6), the integration is carried out along a Landau contour, with

$$\frac{1}{\omega - kv} \to \frac{1}{\omega - kv + i0} \to \frac{P}{\omega - kv} - i\pi\delta(\omega - kv),$$

where P means the principal value. We can then write the equation $\varepsilon(\mathbf{k},0) = 0$ in the form

$$\varepsilon(\omega, \mathbf{k}) \equiv \varepsilon_0(\omega, k) - \frac{4\pi e^2}{mk} \frac{\pi i}{|\mathbf{k}|} \frac{\partial f_0}{\partial v}\bigg|_{v=\omega/k} = 0, \tag{1.7}$$

where

$$\varepsilon_0(\omega, \mathbf{k}) = 1 + \frac{4\pi e^2}{mk^2} \int \frac{P(\partial f_0/\partial v)}{\omega - kv}\,dv = 0 \tag{1.8}$$

is the Vlasov dispersion relation,[15] and $f_0(v) = (m/2\pi T)\exp(-mv^2/2T)$ is a Maxwellian distribution function.

The solution of (1.7) determines the frequency and damping of the natural oscillations of an electron plasma, which for small values of $k \ll a^{-1}$ ($a = \sqrt{T_e/4\pi e^2 n_0}$ is the Debye length) are

$$\omega_k = \Omega\left(1 + \frac{3}{2}a^2k^2\right), \tag{1.9}$$

$$\gamma_k = \sqrt{\frac{\pi}{8}} \frac{\Omega}{(ak)^3} \exp\left(-\frac{1}{2a^2k^2}\right), \tag{1.10}$$

where $\Omega = \sqrt{4\pi e^2 n_0/m}$ is the plasma (Langmuir) frequency, and γ_k is the Landau damping rate.

If $g(v)$ is a smooth function (has no singularities), the asymptotic behavior of the field at large t is

$$E_k(t) \sim \exp(-i\omega_k t - \gamma_k t), \tag{1.11}$$

and the distribution function is

$$f_k^{(1)}(t) = f_k(v)\exp(-i\mathbf{kv}t) + f_k \exp(-i\omega_k t - \gamma_k t), \tag{1.12}$$

since the function $f^{(1)}_{k\omega}$ has the same singularities as the function $E^{(1)}_{k\omega}$, determined by the condition $\varepsilon(\omega,\mathbf{k}) = 0$, as well as another pole at the point $\omega = kv + i0$. Consequently, the plasma oscillations excited by some smooth initial perturbation $g(v)$ should be damped over time, while the function $f_k^{(1)}(t)$ will undergo undamped oscillations: The first term in (1.12) is undamped and is called the "ballistic" or "transit" term.

Van Kampen showed,[10] however, that undamped natural oscillations could also occur in a plasma. We find Landau's dispersion relation (1.7) by writing the distribution function $f^{(1)}_{k\omega}$ in the form

$$f^{(1)}_{k\omega} = \frac{P}{\omega - kv} \frac{e}{mk} \frac{\partial f_0}{\partial v} E^{(1)}_{k\omega} - i\pi\delta(\omega - kv) \frac{e}{mk} \frac{\partial f_0}{\partial v} E^{(1)}_{k\omega}. \tag{1.13}$$

If we introduce in the plasma, in addition to the initial perturbation $g(v)$, a modulated particle beam moving at a velocity equal to the phase velocity of the wave, then we can cancel the second term in (1.13) (if the density and phase of the beam are chosen appropriately). Substituting $f^{(1)}_{k\omega}$ into the field equation (1.2), we then find Vlasov's dispersion relation (1.8), which describes undamped plasma waves. Vlasov's solution thus describes a wave accompanied by a group of resonant particles.

Van Kampen showed[10] that Eqs. (1.1) and (1.2) describe a broader class of natural oscillations. To show this we need to supplement (1.13) with the solution of the homogeneous equation $(\omega - kv)f^{(1)}_{k\omega} = 0$ of the form $\lambda\delta(\omega - kv)\times E^{(1)}_{k\omega}$, where λ is some function of ω and k (it is obviously proportional to the density of the resonant particles). For $f^{(1)}_{k\omega}$ we can then write

$$f^{(1)}_{k\omega} = \frac{P}{\omega - kv} \frac{e}{m} \frac{1}{k} \frac{\partial f_0}{\partial v} E^{(1)}_{k\omega} + \lambda\delta(\omega - kv) E^{(1)}_{k\omega}, \tag{1.14}$$

and substituting this expression into field equation (1.2) we find the dispersion relation

$$\varepsilon_0(\omega, k) + \frac{4\pi e}{k^2|k|}\lambda = 0. \tag{1.15}$$

The second term in (1.14) describes a modulated beam of particles moving at the phase velocity of the wave: This is a Van Kampen wave. The contribution of Van Kampen waves to the dispersion is described by the second term in (1.15). It follows from (1.15) that, in contrast with the situation in (1.7), a given value of k can correspond to an arbitrary frequency ω; in other words, the spectrum of eigenvalues ω is continuous. It follows from (1.15) that for any frequency ω we can choose a value of λ (i.e., the density of resonant particles) such that solution (1.15) corresponds to an undamped wave: a Van Kampen wave with the given frequency ω. The second term in (1.15) is inconsequential if the perturbation frequency satisfies $\omega \sim \Omega$ [and thus $\varepsilon_0(\omega,k) \sim 0$], but it becomes significant if the frequency ω differs from the plasma frequency (under the condition $\omega \gg \Omega$, for example), and we have $\varepsilon_0(\omega,k) \neq 0$. It is in this case that solutions of (1.15) differ significantly from plasma waves, when the second term in (1.14) is comparable to or greater than the first term. In this case we can speak in terms of Van Kampen waves. This is also true if we deal with perturbations with $k \gg a^{-1}$, which are strongly damped, as Landau showed.[1] Perturbations of this sort may be considered a superposition of Van Kampen waves. There is no contradiction with Landau's solution, since Van Kampen showed that any initial perturbation $g(v)$ can be expanded in functions (1.14), so that the natural oscillations of an electron plasma are a superposition of Van Kampen waves and plasma waves. The ballistic term in expression (1.12) may be described as a set of Van Kampen waves. We find the same result for $f_k^{(1)}(t)$ from (1.14), where the second term contributes to the undamped oscillations.

We will now use the concept of Van Kampen waves to explain the mechanism for the temporal echo. If perturbations of the electric field of the form $\exp(ik_1z)$ are specified in the plasma at the time $t=0$, these oscillations will decay over time with the decay constant $\gamma_{k_1}^{-1}$ determined by (1.10). However, the field oscillations leave a "wake" in the plasma in the form of microoscillations of the distribution function, i.e., Van Kampen waves. Their phase evolution is described by the expression $f_1^{(1)} \sim f_1(v)\exp(i(k_1z - k_1vt))$ (the duration of this phase memory is evidently determined by the particular processes which destroy it, e.g., collisions). There are no macroscopic manifestations of these oscillations at $t \gg \gamma_{k_1}^{-1}$; i.e., we have $\int \exp(-ik_1vt)dv = 0$, since the integrand oscillates at a progressively higher frequency. This phenomenon is called "phase mixing" (or "randomization"). For the particles to transfer energy back to the wave (for the particles to excite a macroscopic signal), we must reverse the evolution of the phases of the microoscillations of the distribution function; i.e., we must arrange phase focusing of the modulated particle beams. This reversal of the phase evolution can be arranged by applying a second perturbation. Specifically, if, at the time τ ($\tau \gg \gamma_{k_1}^{-1}$), we again excite oscillations of the type $\exp(-ik_2z)$ of the electric field in the plasma, then these oscillations will give rise to undamped oscillations of the distribution function of the type $f_2^{(1)} \sim f_2(v) \exp\{i[-k_2z + k_2v(t-\tau)]\}$. If we ignore nonlinear effects, then these oscillations would have to exist independently of the oscillations excited previously, and at $t-\tau \gg \gamma_{k_2}^{-1}$ they would have no macroscopic manifestations. However, because of the nonlinear interaction of the beams the second perturbation also gives rise to secondary Van Kampen waves of the form $f^{(2)}(z,v,t) \sim f_1(v)f_2(v)\exp[-i(k_2-k_1)z - i(-k_1+k_2)vt + ik_2v\tau]$. At the time $t = \tau' = k_2\tau/(k_2-k_1)(k_1<k_2)$, the phase in $f^{(2)}$ will obviously be independent of the velocity, so that at the time $t=\tau'$ macroscopic oscillations of the field will reappear in the plasma: These oscillations are the temporal echo, which is thus an essentially nonlinear effect.

If the wave numbers k_1 and k_2 of the external perturbations satisfy the strong-damping condition, i.e., if $ak_{1,2} \gg 1$, we may speak of these perturbations as being Van Kampen waves proper at an arbitrary time, and in this case it is not necessary to satisfy the conditions $t \gg \gamma_{k_1}^{-1}$ and $t-\tau \gg \gamma_{k_2}^{-1}$.

To draw a qualitative picture of the spatial echo, we place in the $z=0$ plane in a plasma a grid to which we apply a monochromatic signal of frequency ω_1 satisfying $\omega_1 \gg \Omega$. Under these conditions, Van Kampen waves proper will propagate away from the grid [in this case, solution (1.15) differs greatly from plasma waves]. The longitudinal field of the external perturbation of frequency ω_1 modulates the particle beams, and this modulation constitutes Van Kampen waves of first order, $f_1^{(1)} \sim f_1(v) \times \exp(-i\omega_1 t + i^{\omega_1 z}/v)$. We now assume that at a distance l from the first grid there is a second grid, to which we apply an alternating potential at a frequency $\omega \gg \Omega$. Van Kampen waves will then propagate away from this grid also:

$$f_2^{(1)} \sim f_2(v)\exp\left[-i\omega_1 t + i\frac{\omega_2(z-l)}{v}\right].$$

In addition, however, this grid will modulate the f_1 density wave. The beams begin to be deformed; i.e., particles are transferred from some beams to others, and secondary beams are produced. In particular, a Van Kampen wave is excited at the difference frequency $\omega_2-\omega_1$:

$$f^{(2)} \sim f_1(v) f_2(v)\exp\left[-i\omega_1\left(t-\frac{z}{l}\right) + i\omega_2\left(t - \frac{z-l}{v}\right)\right].$$

At $z^* = \omega_2 l/(\omega_2-\omega_1)$ the phase in the exponential function becomes independent of the velocity (at this point, the secondary particle beams become bunched in phase), and at this point macroscopic oscillations appear at the frequency $\omega = \omega_2 - \omega_1$: an echo. Most experiments on plasma echos have been concerned with the spatial echo, which is easier to observe. We might note that under the condition $\omega_{1,2} \sim \Omega$ the picture becomes slightly more complicated, since both plasma waves and Van Kampen waves are excited in this case. The plasma waves, however, are then damped (they are "bled" by Van Kampen waves because of the phase spreading), and at distances $|z| \gg \Gamma_{1,2}^{-1}$ ($\Gamma_{1,2}^{-1}$ is the resonant-damping length for oscillations at the frequency $\omega_{1,2}$) from the sources of the original waves we are left with only Van Kampen waves. The position and frequency of the echo are not changed, but the amplitude and dispersion of the echo signal are changed.

A novel demonstration of the echo phenomenon (with optical screens) was proposed by A. A. Vedenov and A. M. Dykhne (see Ref. 34).

The examples discussed above correspond to second-order non-linearities. In a plasma, however, there can be echo effects of higher order.[34,35] For example, spatial echo oscillations of order $(m+n)$ arise at the frequency $\omega_{mn} = m\omega_2 - n\omega_1$ at a distance $l_{mn} = m\omega_2 l/(m\omega_2 - n\omega_1)$ from the first source. The temporal echo of order $(m+n)$ reaches a maximum intensity at the time $t_{mn} = mk_2\tau/(mk_2 - nk_1)$.

The echo effect thus proves the reversible nature of Landau damping: In the course of collisionless damping, the energy of a wave is transferred to resonant particles, which "remember" the external perturbation after it disappears. When the velocities of the particles reverse, the entire process would unfold in the opposite direction; i.e., the particles would transfer energy back to the wave, and the effect would be seen as an echo. However, is all the energy of the original oscillations "put back?" Let us consider the picture of the spatial echo. In moving away from the sources of the original oscillations, all the energy of the oscillations is transferred to modulated particle beams, as we have seen. As these beams move along the z axis they become bunched in places and create macroscopic signals; i.e., memory "knots"—the echos of second, third, etc., orders—are "tied" at the points $z' = \omega_2 l/(\omega_2-\omega_1)$, $z'' = 2\omega_2 l/(2\omega_2 - \omega_1)$, etc., at the frequencies $\omega_2-\omega_1$, $2\omega_2-\omega_1$, etc. It is quite clear that these signals excite only certain groups of particles, so that only a fraction of the original energy is returned to the echos of second, third, and other orders; just which fraction can be determined by direct calculations (or experimentally). In an experiment carried out by Ikezi and Takagashi,[7] for example, the maximum power of an echo of ion acoustic waves (this power is proportional to the square of the amplitude of the echo at the point of maximum intensity) was of the order of 1% of the power of the ion waves which were excited.

b) Collective echo phenomena of second and third orders in an isotropic equilibrium plasma

Two approaches are taken to a quantitative description of echo phenomena. The first uses a nonlinear equation for the distribution function with a self-consistent field, i.e., the system of equations (1.1), (1.2). This approach must evidently be taken when collective properties of the plasma are important and when it is necessary to consider perturbations of not only resonant particles but also all other particles. Quantitatively, the amplitude and shape of the collective echo in such cases depend strongly on both the form of the external perturbations and the dielectric properties of the plasma [the second-order response function $f^{(2)}$ contains $\varepsilon(\omega_1,k)$, $\varepsilon(\omega_2,k)$, and $\varepsilon(\omega_2-\omega_1,\mathbf{k})$, the dielectric permittivities of the plasma]. We call such echos "collective." It is perfectly clear that the dielectric properties of the plasma are important when the frequencies or wave numbers (in the cases of the spatial and temporal echos, respectively) of the oscillations correspond to the transparency region. Langmuir waves, for example, are weakly damped if $\omega\sim\Omega$ and $k\ll a^{-1}$.

If the waves of interest are instead strongly damped, or if their frequencies lie in the opaque region, then we may ignore the dielectric properties of the plasma and approach the problem in the free-streaming approximation. In an electron plasma, for example, we have seen that at frequencies $\omega\gg\Omega$ (or under the condition $ak\gg1$) the modulated particle beams play a governing role in the excitation of the echo. This simplified approach has been termed the "ballistic theory of echos."

To study the collective echo we start from system (1.1), (1.2), which we will solve by successive approximations, assuming that the amplitudes of the external perturbations are small. We consider the temporal and spatial echos which arise in a plasma under the influence of external charges,

$$\rho^0(\mathbf{r},\ t)=\rho_1\exp(i\mathbf{k}_1\mathbf{r})\,\delta(\omega_0 t)+\rho_2\exp(i\mathbf{k}_2\mathbf{r})\,\delta(\omega_0(t-\tau)),\tag{1.16}$$

$$\rho^0(z,\ t)=\rho_1\exp(i\omega_1 t)\,\delta(k_0 z)+\rho_2\exp(-i\omega_2 t)\,\delta(k_0(z-l))\tag{1.17}$$

respectively. In other words, we assume that the external perturbations are applied to the plasma at the times $t=0$ and $t=\tau$ or at the points $z=0$ and $z=l$ (ω_0 and k_0 are arbitrary quantities having the dimensions of a frequency and a reciprocal length, respectively). The oscillations which are produced in the plasma by the external charges are also described by Eqs. (1.4) and (1.5), provided that we simply discard the second term in (1.4). Perturbations (1.16) and (1.17) give rise to only resonant oscillations.

From kinetic equation (1.1) we find the nonlinear correction to the distribution function, $f^{(2)}$; substituting this correction into the equation for the field, (1.2), we find the second-approximation correction $E^{(2)}$ to the field:

$$E^{(2)}_{\mathbf{k}\omega}=-\frac{1}{\varepsilon(\mathbf{k},\ \omega)}\sum_{\omega',\ \mathbf{k}'}\varkappa^{(2)}(\omega-\omega',\ \mathbf{k}-\mathbf{k}';\ \omega',\ \mathbf{k}')\times E^{(1)}_{\mathbf{k}-\mathbf{k}',\ \omega-\omega'}E^{(1)}_{\mathbf{k}'\omega'},\tag{1.18}$$

where

$$\sum_{\omega',\ \mathbf{k}'}\ldots=\int\frac{d\omega}{2\pi}\int\frac{d\mathbf{k}}{(2\pi)^3}\ \ldots,$$

and $\varkappa^{(2)}(\omega_1,\ \mathbf{k}_1;\ \omega_2,\ \mathbf{k}_2)$ is the nonlinear susceptibility of the plasma, given by

$$\begin{aligned}\varkappa^2(\omega_1,\ \mathbf{k}_1;\ \omega_2,\ \mathbf{k}_2)&=\sum\frac{(-i)}{2}\,\frac{4\pi e^2}{m}\,\frac{e}{m}\,\frac{1}{k_1k_2\,(\mathbf{k}_1+\mathbf{k}_2)}\\&\times\int d\mathbf{v}\,\frac{1}{\omega_1+\omega_2-(\mathbf{k}_1+\mathbf{k}_2)+i0}\\&\times\Big[\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\Big(\frac{1}{\omega_2-\mathbf{k}_2\mathbf{v}+i0}\,\mathbf{k}_2\frac{\partial}{\partial\mathbf{v}}\Big)\\&+\mathbf{k}_2\frac{\partial}{\partial\mathbf{v}}\Big(\frac{1}{\omega_1-\mathbf{k}_1\mathbf{v}+i0}\,\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\Big)\Big]f_0(\mathbf{v}).\end{aligned}\tag{1.19}$$

We find explicit expressions for the fields in the linear approximation, $E^{(1)}$, from (1.5), where ρ_0 is given by (1.16) for the case of the temporal echo. Using these linear solutions and expression (1.19) for the nonlinear susceptibility $\varkappa^{(2)}$, we then find the following expression for the second-order field:

$$\begin{aligned}\mathbf{E}^{(2)}_{\mathbf{k}}(t)&=8(2\pi)^4\,ik\,\frac{e^3\rho_1\rho_2}{m\omega_0^2k_1^2k_2^2k^2}\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\\&\times\int d\mathbf{v}\int_{-\infty}^{\infty}d\omega\,\frac{e^{-i\omega t}}{(\omega-\mathbf{k}\mathbf{v}+i0)^2\,\varepsilon(\omega,\ \mathbf{k})}\int_{-\infty}^{\infty}d\omega'\\&\times\Big\{\frac{\mathbf{k}\mathbf{k}_1\exp(i\omega'\tau)}{(\omega'-\mathbf{k}_2\mathbf{v}+i0)\,\varepsilon(\omega-\omega',\ k_1)\,\varepsilon(\omega',\ k_2)}\,\mathbf{k}_2\frac{\partial f_0}{\partial\mathbf{v}}\\&-\frac{\mathbf{k}\mathbf{k}_2\exp[i(\omega-\omega')\tau]}{(\omega'-\mathbf{k}_1\mathbf{v}+i0)\,\varepsilon(\omega-\omega',\ \mathbf{k}_2)\,\varepsilon(\omega',\ \mathbf{k}_1)}\,\mathbf{k}_1\frac{\partial f_0}{\partial\mathbf{v}}\Big\}.\end{aligned}\tag{1.20}$$

The integration over ω' and ω can be carried out by the Cauchy theorem, with the integration contour closed on the corresponding complex plane by a semicircle of infinite radius, in the upper or lower half-plane, depending on the sign of the coefficient of ω' or ω in the exponential function. If the quantity τ and the time interval between the instant at which the echo appears and the second perturbation are large in comparison with γ^{-1} (γ is the Landau damping rate), then we need consider only the pole at the point $\omega'=\mathbf{k}_1\mathbf{v}$ in the integral over ω'. The contributions from the poles at the points where the dielectric permittivity vanishes can be ignored by virtue of the condition $\gamma\tau\gg1$. Carrying out the integration over ω in a similar way, and considering only the second-order pole at $\omega=\mathbf{k}\mathbf{v}$, we find

$$\begin{aligned}E^{(2)}_{\mathbf{k}}(t)&=-8(2\pi)^6\,\mathbf{k}\,\frac{e^3\rho_1\rho_2}{m^2\omega_0^2k_1^2k_2^2k^2}\,(\mathbf{k}\mathbf{k}_2)\,(t-\tau)\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\\&\times\int d\mathbf{v}\,\frac{\exp[-i(\mathbf{k}\mathbf{v}t-\mathbf{k}_2\mathbf{v}\tau)]}{\varepsilon(\mathbf{k}\mathbf{v},\ \mathbf{k})\,\varepsilon(\mathbf{k}_1\mathbf{v},\ \mathbf{k}_1)\,\varepsilon(\mathbf{k}_2\mathbf{v},\ \mathbf{k}_2)}\,\mathbf{k}_2\frac{\partial f_0}{\partial\mathbf{v}}.\end{aligned}\tag{1.21}$$

The exponential factor in the integrand here vanishes at the time

$$t\equiv\tau'=\frac{\mathbf{k}_2\mathbf{v}}{\mathbf{k}\mathbf{v}}\,\tau,\tag{1.22}$$

which determines the time at which the echo appears. A necessary condition for the appearance of an echo is that τ' exceed τ. Direct calculation shows[18] that an echo can appear only when the vector $\mathbf{k}_2$ is directed opposite to $\mathbf{k}_1$ and, furthermore, $|\mathbf{k}_2|>|\mathbf{k}_1|$. When there is an echo, the angle between the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ may differ from π by only a small quantity $(\tau ks)^{-1}$. The factors $\varepsilon^{-1}(\mathbf{k}\mathbf{v},\mathbf{k})$, $\varepsilon^{-1}(\mathbf{k}_1\mathbf{v},\ \mathbf{k}_1)$ and $\varepsilon^{-1}(\mathbf{k}_2\mathbf{v},\ \mathbf{k}_2)$ in (1.21) describe the influence of the dielectric properties of the plasma on the external perturbations and on the echo field.

It is a simple matter to carry out the integration over the velocity components perpendicular to the vector **k** in (1.21). The remaining integration over the velocity v can be carried out by going to the complex v plane and closing the integration contour with a semicircle of infinite radius in either the upper half-plane (if $t<\tau'$)or the lower half-plane (if $t>\tau'$). (The singularity at $v=i\infty$ in $\partial f_0/\partial v$ in the numerator is cancelled by the same singularity in the imaginary part of $\varepsilon(\mathbf{k}v, \mathbf{k})$ in the denominator.) The time evolution of the echo is determined by the zeros of the functions $\varepsilon(\ldots)$ in (1.21). The growth of the echo is exponential, $\exp[-(k_3/k_1)\gamma_1(t-\tau')]$, while the damping is $\exp[-(k_3/k_2)\gamma_2(t-\tau')]$ and $\exp[-\gamma(t-\tau')]$, where γ_1, γ_2, and γ are the Landau damping rates for the waves with the wave vectors $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}$. We see that the echo signal is asymmetric in time.

As an example, we give the explicit expression for the field of the echo signal in the case in which all three oscillations correspond to plasma frequencies and the conditions $ak\ll 1$ and $k_2=2k$ hold[4,18]:

$$E^{(2)}(\mathbf{r},\ t)=-32\pi^2 i\frac{e}{m}\frac{\rho_1\rho_2}{\omega_0^2k^2}\Omega\tau\sin\varphi\mathbf{k}\exp[i\mathbf{kr}-\gamma(t-\tau')]\times\cos[\Omega(t-\tau')+\varphi], \qquad (1.22')$$

where $\mathrm{tg}\varphi=[2k/(k-k_1)]\gamma/\Omega$. In this particular case the echo is symmetric.

To analyze the spatial echo, we choose the external perturbations in the form (1.17), and, assuming $\Gamma_i l\gg 1$ [Γ_i^{-1} is the resonant-damping length for waves of frequency ω_i ($i=1,2$)], we find the following expression for the echo field from (1.18) (Ref. 19):

$$E^{(2)}(z,\ t)=16\pi^{3/2}\frac{e}{m}\frac{\rho_1\rho_2}{\omega_1\omega_2}\frac{z-l}{a^2k_0^2}e^{-i\omega t}\times\int\limits_0^\infty\frac{d\xi\exp[-\xi^{-2}+i(\omega/s)\xi(z-l')]}{\xi^2\varepsilon[(\omega/s)\xi,\ \omega]\,\varepsilon[-(\omega_1/s)\xi,\ -\omega_1]\,\varepsilon[(\omega_2/s)\xi,\ \omega_2]}, \qquad (1.23)$$

where $l'=\omega_2 l/(\omega_2-\omega_1)$ is the point at which the echo reaches its maximum intensity. The integration over ξ in (1.23) can be carried out by the method of steepest descent $|((\omega/s)|z-l'|\gg 1)|$. It follows from (1.23) that the spatial evolution of the echo oscillations is determined by the zeros of the functions $\varepsilon(\ldots)$; the growth of the echo signal is determined by the exponential function $\exp[(\omega_3/\omega_1)\Gamma_1(z-l')]$, while the damping is determined by $\exp[-(\omega/\omega_2)\Gamma_2(z-l')]$ and $\exp[-(\omega/\omega_1)\Gamma_1(z-l')]$. The echo oscillations are therefore asymmetric in space. Let us consider the case in which ω_1 and $\omega=\omega_2-\omega_1$ correspond to plasma frequencies, and the condition $\omega_2=2\omega$ holds. In this case the echo oscillations are symmetric in space:

$$E^{(2)}(z,\ t)=32\pi^2\frac{e}{m}\frac{\rho_1\rho_2}{\omega_1\omega_2k_0^2}(z-l)\,k\Gamma\exp(-i\omega t)\times\frac{\exp[-\Gamma(z-l')]\exp[ik(z-l')]}{[1-(\Omega^2/\omega_1^2)][1-(\omega_1^2/\omega^2)+12ik\Gamma a^2]}, \qquad (1.24)$$

where

$$k\equiv k(\omega)=\frac{\omega}{s}\sqrt{\frac{2}{3}}\sqrt{\frac{\omega^2-\Omega^2}{\omega^2}},\quad \Gamma\equiv\Gamma(\omega)=\frac{1}{6}\sqrt{\frac{\pi}{2}}\frac{1}{a^5k^4}\exp\left(-\frac{m\omega^2}{2Tk^2}\right). \qquad (1.25)$$

The temporal and spatial echo oscillations associated with a wave conversion in a plasma were studied in Refs. 19 and 36. Sitenko *et al.*[19] studied the low-frequency acoustic echo oscillations which result from the superposition of high-frequency plasma oscillations. These echos are asymmetric in space: The growth is $\sim\exp[(\omega/\Omega)\Gamma_1(z-l')]$, while the damping $\sim\exp[(\omega/\omega_1)\Gamma_2(z-l')]$ and $\exp(\Gamma_s(z-l'))$ where Γ_s is the damping rate of the acoustic oscillations,

$$\Gamma_s=\sqrt{\frac{\pi}{2}}\frac{\omega_s}{\Omega}\exp\left(-\frac{m\omega_s^2}{2Tk^2}\right), \qquad (1.26)$$

$\omega\approx kv_s$, $v_s=\sqrt{T/m_i}$ is the velocity of nonisothermal sound, and the values of $\Gamma_{1,2}=\Gamma(\omega_{1,2})$ are given by (1.25). Vodyanitskiĭ and Repalov[41] also studied the spatial echo in an isotropic plasma.

External perturbations (1.16) and (1.17) have been chosen to be δ-shaped (point perturbations) for simplicity, but they could also be chosen as wave packets over frequency (or over wave vector). The effect of the shape of the exciting fields on the nature of the echo oscillations was taken into account in Refs. 42–44.

As an example of a higher-order echo effect we consider the three-pulse echo in a plasma which results from the superposition of three successive perturbations separated by time intervals longer than the characteristic decay times of the corresponding oscillations.[36] We choose the perturbations in the form of plane waves, and we assume that the wave vectors of the perturbations do not satisfy the collinearity conditions. We can thus rule out the possibility of a second-order echo, but if the wave vectors of all three perturbations lie in a common plane there is the possibility of a third-order echo. In this case the echo oscillations have not only a longitudinal component but also a transverse component; i.e., the three-pulse echo is accompanied by wave conversion in the plasma. The time of appearance and the shape of the signal are very dependent on the time intervals between the successive perturbations.

We choose the external charge density to be

$$\rho^0(\mathbf{r},\ t)=\rho_1\exp(i\mathbf{k}_1\mathbf{r})\,\delta(\omega_0 t)+\rho_2\exp(i\mathbf{k}_2\mathbf{r})\,\delta(\omega_0(t-\tau))+\rho_3\exp(i\mathbf{k}_3\mathbf{r})\,\delta(\omega_0[t-(1+\lambda)\tau]); \qquad (1.27)$$

i.e., we assume that the external perturbations are applied to the plasma at the times $t=0$, $t=\tau$, and $t=(1+\lambda)\tau$, where $\tau\gg\gamma^{-1}$ and $\lambda\gtrsim 1$. Because of the longitudinal nature of the perturbations, there are no transverse oscillations of the electric field in the linear approximation.

The solution of Eqs. (1.1) and (1.2) was used in Ref. 36 to derive an expression for the field of the third-order echo oscillations; that expression contains both longitudinal and transverse components: $E^{(3)}=(\mathbf{k}/k)E^l+\boldsymbol{\eta}E^t$, where $\boldsymbol{\eta}$ is a unit vector perpendicular to $\mathbf{k}$.

The oscillations of the longitudinal field reach a maximum amplitude at the time $t=\tau'$, for which the coefficient of v in the exponential function in the nonlinear response $f^{(3)}$ vanishes. It was found in Ref. 36 that τ' is determined by

$$\mathbf{k}_1\tau'+\mathbf{k}_2(\tau'-\tau)+\mathbf{k}_3(\tau'-(1+\lambda)\tau)=0. \qquad (1.28)$$

Equation (1.28) has a solution only if the vectors $\mathbf{k}_1$, $\mathbf{k}_2$, and

$\mathbf{k}_3$ lie in a common plane, i.e., are coplanar. Projecting vector equation (1.28) onto the direction perpendicular to $\mathbf{k}$, we find

$$\mathbf{k}_{2\perp}+(1+\lambda)\,\mathbf{k}_{3\perp}=0 \tag{1.29}$$

and, furthermore,

$$\mathbf{k}_{1\perp}+\mathbf{k}_{2\perp}+\mathbf{k}_{3\perp}=0. \tag{1.30}$$

Equations (1.29) and (1.30) determine the conditions under which the three-pulse echo can arise. These conditions can be satisfied by setting

$$k_{1\perp}=\lambda k_{3\perp},\quad k_{2\perp}=-(1+\lambda)\,k_{3\perp}. \tag{1.31}$$

Since the echo can arise only after the third perturbation, i.e., since $\tau'>(1+\lambda)\tau$, yet another condition must be satisfied for an echo to occur:

$$k_{2\parallel}+(1+\lambda)\,k_{3\parallel}>(1+\lambda)\,k. \tag{1.32}$$

Assuming that conditions (1.31) and (1.32) hold, Sitenko *et al.*[36] derived an explicit expression for the amplitudes of the longitudinal and transverse echos, E^l and E^t.

As in the case of the two-pulse echo, it can be shown that a violation of the coplanarity of the wave vectors $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ results in the disappearance of the three-pulse echo. Consequently, the angle θ between one of the wave vectors and the plane containing the two others must satisfy

$$\theta\ll(\tau ks)^{-1}. \tag{1.33}$$

In this manner it was shown in Refs. 18 and 36 that there can be a nonlinear wave conversion in a plasma (a conversion of high-frequency waves into low-frequency waves and of longitudinal waves into transverse waves) because of an echo effect.

We call the spatial echo which results from the interaction of oscillations with frequencies ω_1 and ω_2 at the points $z=0$ and $z=l$ the "primary" echo. In addition to this primary echo, there is also a secondary echo in the plasma, which results from an interaction of one of the primary oscillations with the echo oscillation. The frequency ω_{mn} and the position of the secondary echo oscillations are the same as for the primary echo of order $(m+n)$, but the shape of the secondary echo of order $(m+n)$ differs from that of the primary echo of the same order, in agreement with experiment.[8]

Since the echo is an essentially nonlinear effect (in a homogeneous plasma), we wish to call attention to the following circumstance to show the position occupied by echo phenomena among other nonlinear processes in plasmas. Research on nonlinear processes in plasmas (the decay and coalescence of waves, the induced scattering of waves by particles, nonlinear Landau damping, etc.) has made extensive use of a nonlinear equation for the field in which the coefficients are the dielectric permittivity and the nonlinear susceptibilities of the plasma.[37–40] The nonlinear susceptibilities were introduced in Refs. 37–39. The nonlinear equations for the field can be used to study echos. For example, expression (1.18) for $E^{(2)}$ which we used to calculate the second-order echo follows immediately from the nonlinear equation for the field if the frequency and wave-vector resonance conditions $\omega_{\mathbf{k}}=\omega_{\mathbf{k}_1}+\omega_{\mathbf{k}_2}$ and $\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2$ on the waves involved in the interaction are not satisfied (or if they are not satisfied simultaneously). In a similar way we can find the field correction $E^{(3)}$ and use it to analyze the third-order echo, etc. (The nonlinear susceptibilities $\varkappa^{(2)}$ and $\varkappa^{(3)}$, in terms of which $E^{(2)}$ and $E^{(3)}$ are expressed, can be found in Refs. 37–40.) There is thus no need to carry out special calculations to find the response functions $f^{(2)}, f^{(3)}, \ldots$ and to use them to calculate the fields $E^{(2)}, E^{(3)}, \ldots$, as has usually been the approach in the theory for the collective echo in a homogeneous plasma (see, for example, Refs. 4, 6, 18, and 19; see also the monographs in Refs. 45 and 46, where a detailed theory of the echo is set forth).

Echo effects are seen under conditions such that the ordinary nonlinear interactions of waves (both resonant and nonresonant interactions) are unimportant. Some special conditions are imposed on the fields of the primary signals: They must be separated in time (or space) by intervals of time (or distance) significantly larger than the characteristic times (or lengths) for their linear damping; i.e., $\tau\gg\gamma_{k_i}^{-1}$ $(l\gg\Gamma^{-1}(\omega_i))$. Consequently, the macroscopic fields of these oscillations cannot interact directly through a nonlinear coupling. Under these conditions there can be only a nonlinear interaction of beams of particles which are modulated by the fields of the primary oscillations, and it is this interaction which is responsible for the appearance of the macroscopic echo signal. The echo effect is thus one example, although an extremely unusual one, of a nonresonant and nonlinear wave interaction.

c) Echo oscillations whose dispersion is determined by the nature of the perturbations of the distribution function

It follows directly from the system of equations (1.4), (1.5) that if the initial perturbation $g(v)$ is not a smooth function but instead has singularities then the asymptotic behavior of the field $E_k^{(1)}(t)$ is also influenced by certain other oscillations, whose dispersion is determined by the nature of the singularities of $g(v)$. Can the interaction of these oscillations give rise to a nonlinear echo signal? It was shown in Refs. 18 and 19 that in this case some additional echo oscillations do in fact arise, with a dispersion determined by the nature of the external perturbations (not by the plasma properties). In particular, a study was made of the second-order temporal echo in an electron plasma.[18] For this purpose, system (1.1), (1.2) should be supplemented by the specification of the initial perturbation of the distribution function (a discontinuity of the distribution function) at the time $t=0$ and the specification of a repeated perturbation at the time $t=\tau$. The perturbations of the distribution function are chosen to be plane waves:

$$f(\mathbf{r},\ \mathbf{v},\ t)|_{t=0}=g_1(\mathbf{v})\exp(i\mathbf{k}_1\mathbf{r}),$$
$$f(\mathbf{r},\ \mathbf{v},\ t)|_{t=\tau+0}-f(\mathbf{r},\ \mathbf{v},\ t)|_{t=\tau-0}=g_2(\mathbf{v})\exp(i\mathbf{k}_2\mathbf{r}), \tag{1.34}$$

where $g(\mathbf{v})$ and $g_2(\mathbf{v})$ are the velocity-dependent amplitudes of the initial and repeated perturbations. As an example we consider perturbations with the amplitudes

$$g_i(v)=g_i\frac{v_i'}{(v-v_i)^2+v_i'^2},\qquad i=1,2. \tag{1.35}$$

It follows from (1.5) that their contribution to the asymptotic behavior of the field in the limit $t\to\infty$ is

$$E_{\mathbf{k}}^{(1)}(t)\sim g_i\exp(-kv_i't-ikv_it). \tag{1.36}$$

The frequency and damping of the oscillations which are excited in the plasma by perturbations (1.35) are thus kv_i and kv_i', respectively. Perturbations of this type can be arranged by injecting a modulated particle beam into the plasma (v_i is the beam velocity, and v_i' is the beam width). It was shown in Ref. 18 that the nonlinear interaction of such oscillations can give rise to second-order echo oscillations whose dispersion is determined by the nature of the external perturbations if the following inequalities hold:

$$\tau^{-1}\ll k_1v_i',\ k_2v_i',\ kv_i'\ll\gamma_1,\ \gamma_2,\ \gamma. \tag{1.37}$$

Assuming for simplicity $v_i\gg v_i'$, restricting the discussion to the particular case $v_1+iv_1'=v_2+iv_2'=v_0+iv_0'$, and using (1.5) and (1.35), we find from (1.18) the following expression for the echo-oscillation field, which in this case is symmetric:

$$E^{(2)}(\mathbf{r},\ t)\propto g_1g_2(t-\tau)\,\mathbf{k}\exp(i\mathbf{kr})\times\frac{\exp[-ik_0v_0(t-\tau')]}{\varepsilon(kv_0,\ k)\,\varepsilon(k_2v_0,\ k_2)}\exp[-kv_0'(t-\tau')]. \tag{1.38}$$

The field amplitude determined by (1.38) is higher by a factor of v_i/v_i' than the amplitude of the same field in the case $v_1+iv_2'\neq v_2+iv_2'$. A resonant increase in the amplitude also occurs under the conditions $\varepsilon(k_iv_1,k_i)=0$ and $\varepsilon(k_iv_2,k_i)=0$, i.e., when the velocities of the particles of the modulated beam coincide with the phase velocities of the corresponding waves in the plasma. In Ref. 19 it was shown that there can also be spatial echo oscillations with a dispersion determined by the nature of the external perturbations.

We thus see that if perturbations of microscopic quantities, e.g., the electron distribution function, are specified then echo oscillations appear in a plasma under certain conditions with a dispersion determined by the nature of the external perturbations (not by the plasma properties). We thus have an opportunity to control the nature of the nonlinear oscillations by means of external perturbations.

d) Echo oscillations in a nonequilibrium plasma

The simplest example of a nonequilibrium plasma is a beam-plasma system, i.e., an equilibrium plasma through which passes a neutralized beam of charged particles. At certain beam velocities $u>u_c$, where u_c is some critical velocity, an instability of natural oscillations of the plasma occurs (the values of u_c differ for different wave types). To determine the nature of the nonlinear echo oscillations in a nonequilibrium beam-plasma system and to determine whether an instability of oscillations of this type can occur, we adopt the unperturbed electron distribution function

$$f_0(\mathbf{v})=n_0\left(\frac{m}{2\pi T}\right)^{3/2}\exp\left(-\frac{mv^2}{2T}\right)+n'\left(\frac{m}{2\pi T'}\right)^{3/2}\exp\left(-\frac{m(\mathbf{v}-\mathbf{u})^2}{2T'}\right), \tag{1.39}$$

$$\frac{n'}{n_0}\ll 1. \tag{1.40}$$

We consider the temporal echo in a plasma of this type.[18] We assume that the perturbations are caused by external charges (1.16) and that the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are directed along the beam velocity $\mathbf{u}$. We choose the wave numbers k_1 and k_2 to satisfy the conditions $ak_1,\ ak_2\gg 1$, while their difference, $k=k_2-k_1$, in contrast, satisfies the condition $ak\ll 1$. In the linear approximation, as we have seen, the nature of the field oscillations is determined by the poles corresponding to the equations $\varepsilon(\omega,k)=0$, $\varepsilon(\omega,k_1)=0$, and $\varepsilon(\omega,k_2)=0$. At short wavelengths ($ak_1,\ ak_2\gg 1$) the solutions of these equations are known to describe strongly damped aperiodic oscillations ($|\mathrm{Im}\,\omega|\gg|\mathrm{Re}\,\omega|$). The effect of the beam on the damping, on the other hand, is important only for long-wave oscillations ($ak\ll 1$), since these waves are weakly damped. The solution of the equation $\varepsilon(\omega,\mathbf{k})=0$ is thus of the form $\omega=\pm\Omega-i\gamma$, where γ is determined by

$$\gamma=\sqrt{\frac{\pi}{8}}\frac{\Omega}{(ak)^3}\left[\exp\left(-\frac{m\Omega^2}{2Tk^2}\right)+\frac{n'}{n_0}\frac{\Omega-\mathbf{ku}}{\Omega}\left(\frac{T}{T'}\right)^{3/2}\exp\left(-\frac{m(\Omega-\mathbf{ku})^2}{2T'k^2}\right)\right]. \tag{1.41}$$

The field of the echo oscillations can be determined from (1.20), where $f_0(\mathbf{v})$ is taken from (1.39). In the integration over ω and ω' in (1.20) we need to take into account the pole in the function $\varepsilon^{-1}(\omega,\mathbf{k})$, since the condition $\gamma\tau\gg 1$ does not hold, because the damping rate γ can become arbitrarily small or even negative. In the instability region ($\gamma\lesssim 0$) the echo field is given by

$$E^{(2)}(\mathbf{r},\ t)=32\pi^2 i\,\frac{e\rho_1\rho_2}{m\omega_0^2k_1k_2}\gamma\mathbf{k}\times\exp(i\mathbf{kr}+|\gamma|(t-\tau')-i\Omega(t-\tau'))\times\begin{cases}\dfrac{\tau'-t}{k_1}, & \tau<t<\tau',\\[2mm] \dfrac{\tau}{k}, & t>\tau'.\end{cases} \tag{1.42}$$

It can be seen from (1.42) that under the condition $\gamma<0$ the echo oscillations can grow with time. We thus see that, in the first approximation, the field oscillations which arise in a nonequilibrium plasma are rapidly damped as a result of the superposition of short-wave perturbations. Nevertheless, the echo oscillations may grow in the second approximation.

The echo signal in (1.42) is affected by resonant electrons of both the plasma and the beam, since $E^{(2)}\sim\gamma\exp\times(\gamma'|t-\tau'|)$. The role played by the beam is to disrupt the stability of the echo oscillations in the plasma when the beam velocity $\mathbf{u}$ exceeds a certain critical velocity, which for plasma oscillations is of the order of the thermal velocity of the plasma electrons (the beam "pumps" the echo oscillations, since they are weakly damped, while the external perturbations are strongly damped and are stable under the same conditions). Hinton and Kawabe[49] have studied a second-order spatial echo which arises simply from a modulation of the beam electrons.

It was shown in Ref. 19 that spatial echo oscillations could be amplified by plasma oscillations, and it was shown in Ref. 48 that they can be amplified by ion acoustic waves in a beam-plasma system. It also follows from Refs. 19 and 48 that, although the primary oscillations with frequencies

$\omega_{1,2}$ ($\omega_{1,2} \gg \Omega$) are strongly damped, a nonlinear echo signal appears at a distance $l' = \omega_2 l/(\omega_2 - \omega_1)$, from the source of the oscillations with the frequency ω_1. This nonlinear echo signal has a frequency $\omega = \omega_2 - \omega_1$, which corresponds to the transparency region, $\omega \gtrsim \Omega$ (i.e., there is a "brightening" of the nonequilibrium plasma).

e) Nonlinear echo oscillations in the relativistic case. Plasma echos in semiconductors

In this subsection we examine the echo in an electron plasma when relativistic effects are significant, and we also consider the echo in a plasma in an $A^{III}B^{V}$ semiconductor.[50,52] In a relativistic plasma the electron velocity is bounded from above; it cannot exceed the speed of light c. In type $A^{III}B^{V}$ semiconductors, the relationship between the momentum $\mathbf{p}$ of the conduction electrons and their energy $\mathscr{E}$ is (according to Kane[51])

$$p^2 = 2m\mathscr{E}\left(1 + \frac{\mathscr{E}}{\mathscr{E}_g}\right), \tag{1.43}$$

where m is the mass at the bottom of the band, and $\mathscr{E}_g$ is the width of the energy gap. For a band of this shape the electron velocity,

$$v = \frac{d\mathscr{E}}{dp} = \left[\frac{2\mathscr{E}}{m}\left(1 + \frac{\mathscr{E}}{\mathscr{E}_g}\right)\right]^{-1}\left(1 + \frac{2\mathscr{E}}{\mathscr{E}_g}\right)^{-1},$$

is again bounded from above as the electron energy increases; it cannot exceed a certain limiting velocity $v_n = \sqrt{\mathscr{E}_g/2m}$.

The upper bound on the electron velocity should have some important consequences for processes in which the tail of the Maxwellian distribution function plays an important role. One such process is the Landau damping of plasma waves. The appearance of a limiting velocity for the conduction electrons is reflected directly in the Landau damping rate and thus in the shape of the echo oscillations.

For a quantitative description of the collective temporal echo in a relativistic plasma we can work from expression (1.20), where f_0 must be understood to be the momentum distribution function of the particles of a relativistic ideal gas. We take the external perturbations in the form in (1.16) and (1.17).

We choose the wave numbers to satisfy $\Omega/k_1, \Omega/k_2 \ll c$, while their difference $k = k_2 = k_1$ satisfies $\Omega/k \lesssim c$. In this case the poles corresponding to the solutions $\varepsilon(\omega, -k_1) = 0$ and $\varepsilon(\omega, k_2) = 0$ describe strongly damped oscillations, while the pole corresponding to the solution $\varepsilon(\omega, k) = 0$ describes weakly damped oscillations and contributes to the echo field.

Using the method of steepest descent ($\Omega|t - \tau'| \gg 1$) we find the following expression for the field of the echo oscillations[50] (for $t > \tau'$):

$$E^{(2)}(\mathbf{r}, t) = -i32\pi^2 \frac{e\rho_1\rho_2}{m\omega_0^2 k_1 k_2}\Omega(t-\tau)\tilde{\gamma}\left(1 - \frac{\Omega^2}{k^2c^2}\right)^{3/2} \times \mathbf{k}\exp(i\mathbf{kr})\exp(-\tilde{\gamma}(t-\tau'))\cos(\Omega(t-\tau')), \tag{1.44}$$

where

$$\tilde{\gamma} = \sqrt{\frac{\pi}{8}}\frac{\Omega}{(ak)^3}\left(1 - \frac{\omega^2 l}{k^2c^2}\right)^{-1} \times \exp\left\{-\frac{mc^2}{T}\left(\left(1 - \frac{\omega^2}{k^2c^2}\right)^{-1/2} - 1\right)\right\}. \tag{1.45}$$

In the limit $\Omega/k \to 0$ we have $\tilde{\gamma} \to 0$, and the echo oscillations decay slowly over time.

The temporal echo in the plasma of an $A^{III}B^{V}$ semiconductor was studied in Ref. 52. It was shown that as the wave phase velocity tends toward a limit (set by the band width) the echo oscillations decay with time. The shape of the echo oscillations is thus sensitive to the band shape.

Echo oscillations can arise only if the resonant damping exceeds the collisional damping, i.e., only if $\gamma > \nu$. This is a restrictive condition for a semiconductor plasma. For the semiconductor n-InSb with a carrier density $n = 4 \cdot 10^{13}$ cm^{-3} at $T = 10$ K we find $\nu = 3 \cdot 10^9$ s^{-1} (electron-phonon scattering). Taking $ak = 0.25$, we find $\gamma/\Omega = 10^{-2}$ and $\gamma \sim 10^{10}$ s^{-1}. We thus see that conditions for the appearance of echo oscillations can be arranged in n-InSb with these parameters. We might note that Landau damping has been observed experimentally in an $A^{III}B^{V}$ semiconductor—by Tell and Martin[53] in n-GaAs.

f) Echos in magnetized plasmas

In a magnetized plasma we would expect to find some nonlinear effects not seen in an isotropic plasma, because the external magnetic field determines the way in which the charged particles move in the plasma and thus alters the dielectric properties of the plasma (the spectrum of natural oscillations becomes significantly broader, and another damping mechanism, cyclotron damping, comes into play along with Landau damping). The external magnetic field thus has an important effect on the nature of the Van Kampen waves. The solution of the kinetic equation for the function $f^{(1)}$ is (for the simplest case, in which the waves are propagating along the external magnetic field $\mathbf{B}_0 \| Oz$).

$$f^{(1)}_{\alpha k\omega} = -i\frac{e_\alpha}{m_\alpha}\left(\frac{1}{2}\frac{E^{(1)}_{1k\omega}\,\partial f_{0\alpha}/\partial v_\perp e^{i\theta}}{\omega - kv_z + \omega_{B\alpha} + i0} + \frac{1}{2}\frac{E^{(1)}_{2k\omega}\,\partial f_{0\alpha}/\partial v_\perp e^{i\theta}}{\omega - kv_z - \omega_{B\alpha} + i0} + \frac{E^{(1)}_{zk\omega}\,\partial f_{0\alpha}/\partial v_z}{\omega - kv_z + i0}\right), \tag{1.46}$$

where θ is the azimuthal angle in velocity space, $E_{1,2} = E_x \mp iE_y$, and $\omega_{B\alpha} = e_\alpha B_0/m_\alpha c$. It can be seen from (1.46) that in a magnetized plasma we have, in addition to the undamped oscillations of the distribution function at the frequency $\omega = kv_z$, some oscillations of the distribution function at the frequencies $\omega = kv_z \pm \omega_{B\alpha}$. Accordingly, in addition to the longitudinal echo oscillations at the point $z = l' = \omega_2 l/(\omega_2 - \omega_1)$ caused by the interaction of the longitudinal primary oscillations there is also an echo at the points $z = l'_\alpha(\omega_2 + \omega_{B\alpha})/(\omega_2 \pm \omega_1)$ due to the interaction of the transverse oscillations, and there is a transverse echo at the points

$$z = l_{\alpha 1,2} = \pm\frac{\omega_2 \pm \omega_{B\alpha}}{\omega_3' \pm \omega_{B\alpha}}l \quad (\omega_3' = \mp\omega_1 \pm \omega_2)$$

in the interaction of a transverse oscillation with a longitudinal oscillation (at the points $z = 0$ and $z = l$, there are sources of perturbations with the frequencies ω_1 and ω_2, respectively). In a magnetized plasma, therefore, there can also be some nonlinear echo oscillations associated with wave conversion.

The temporal and spatial echos in a magnetized plasma were studied in Refs. 54–59. In particular, the longitudinal and transverse temporal echos of second order were studied in Refs. 56 and 57, respectively. Echo oscillations of ion-acoustic and cyclotron waves were studied in Ref. 58, and it was shown that in a nonequilibrium magnetized plasma with a low-density charged-particle beam there can be a growth of a nonlocal, nonlinear echo signal at the sum and difference frequencies $\omega = \omega_2 \pm \omega_1$. A self-consistent approach was taken to study the echo in Ref. 58. The dispersion relation for the natural oscillations for propagation along the magnetic field breaks up into three independent equations[47]: equations for the longitudinal wave (which is of the same form as in an isotropic plasma) and for the left-hand and right-hand polarized transverse waves,

$$\varepsilon_{1,2}(\omega, k) = k^2c^2 - \omega^2 - i\sqrt{\pi}\sum_{\alpha}\frac{\omega\Omega_\alpha^2}{ks_\alpha}w\left(\frac{\omega\pm\omega_{B\alpha}}{ks_\alpha}\right) = 0. \tag{1.47}$$

Revenchuk and the present author[58] studied the second-order longitudinal echo at the ion acoustic frequency which results from the interaction of transverse electron cyclotron waves for the following selection of frequencies of the external perturbations:

$$\left|\frac{\omega_{1,2}-(\omega_{Be})}{\omega_{1,2}}\right| \leqslant \left(\frac{s\Omega}{c\omega_{Be}}\right)^{2/3},$$

and

$$\omega_3 = \omega_2 - \omega_1 \approx \omega_s$$

(for this selection of frequencies, the oscillations with ω_1 and ω_2 experience a strong cyclotron absorption, while the echo oscillations at ω_3 are only weakly damped). The expression derived there for the second-order longitudinal echo is

$$E^{(2)}(z, t) = \frac{8\pi^{5/2}i\omega_1 a^2 (z-l)\,k_s^5}{\omega_s^3 k_0'^2}\, j_1 j_2 \exp(-i\omega_3 t)$$
$$\times \sum_\alpha \frac{e_\alpha \Omega_\alpha^2(\omega_2\pm\omega_{B\alpha})}{m_\alpha s_\alpha}$$
$$\times \frac{\exp[-(\omega_s/k_s s_\alpha)^2]\exp[i(k_s+i\Gamma_s)(z-l'_\alpha)]}{\varepsilon_1\left(\frac{\omega_s+\omega_{B\alpha}}{\omega_s}k_s, \omega_2\right)\varepsilon_2\left(-\frac{\omega_2+\omega_{B\alpha}}{\omega_s}k_s, -\omega_1\right)}, \tag{1.48}$$

where k_s and Γ_s are the wave number and damping rate of the ion acoustic waves, and j_1 and j_2 are the amplitudes of the external current. It thus follows from (1.48) that although the primary waves at the frequencies ω_1 and ω_2 are strongly damped there is a reradiation of a wave at the frequency $\omega_3 = \omega_2 - \omega_1$ in the transparency region by virtue of echo effects (i.e., there is the possibility of "brightening" opaque regions of a magnetized plasma).

Porkolab and Sinnis[54] studied the second-order spatial echo oscillations which result from an interaction of electrostatic oscillations at frequencies which are multiples of the cyclotron frequency when they propagate at an angle with respect to the external magnetic field ($k_\perp \neq 0$); Bernstein modes). Their analysis was carried out by the method of Ref. 59; they found that the echo at the frequency ω_{Be} reached a maximum intensity at the point

$$z = z^* = \frac{\omega_2 - n\omega_{Be}}{\omega_2 - \omega_1 - p\omega_{Be}}\, l$$

(n and p are integers). The experimental results of Ref. 54 are discussed in Section 3 of the present review.

It can be concluded from the echo studies in Refs. 54–59 that the echo in a magnetized plasma has the following distinguishing features: 1) Cyclotron damping, rather than Landau damping, plays a governing role in the formation of the echo; 2) the echo arises at not only the difference frequency but also the sum frequency, $\omega = \omega_2 \pm \omega_1$; 3) the amplitude of the echo signal depends strongly on the magnetic field.

Numerical estimates show that conditions for the occurrence of echos can occur in the ionospheric plasma (according to data reported in Ref. 60, the resonant-damping lengths for electron cyclotron waves, ion cyclotron waves, and plasma waves for the ionospheric F layer are $\Gamma_1^{-1} \sim 10^{-2}$ m, $\Gamma_2^{-1} \sim 1$ m, and $\Gamma_0^{-1} \sim 10^{-2}$ m, respectively, and the mean free path of the particles is $\lambda \gtrsim 10^3$ m; i.e., $\lambda \gg \Gamma_i^{-1}$). A cyclotron echo has been observed in the ionospheric plasma during topside sounding of the ionosphere.[17] The echo in the plasma of the solar corona was studied in Ref. 61.

g) Ballistic theory of echos

We have been discussing the echos in plasmas by working from a kinetic equation with a self-consistent field. We turn now to a simpler theory, the ballistic theory of echos, which was derived in Refs. 63–65. This theory is based on the free-streaming approximation, in which only the field of the external perturbations is incorporated in the kinetic equation (the field of the external perturbations induces modulated particle beams or Van Kampen waves in the plasma, but we are ignoring the effect of the field of the external perturbations on the dielectric properties of the plasma).

We work from the Boltzmann kinetic equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} + a(z, t)\frac{\partial f}{\partial v} = 0, \tag{1.49}$$

where $f(z,v,t)$ is the distribution function, and $a(z,t)$ are the external perturbations. Equation (1.49) is solved by the method of characteristics:

$$\dot{z} = v, \quad \dot{v} = a, \quad \dot{f} = 0, \tag{1.50}$$

and t is the parameter of these equations. The solution of Eqs. (1.50) is

$$z = z(z_0, v_0, t), \quad v = v(z_0, v_0, t),$$
$$f = f(z_0, v_0, t). \tag{1.51}$$

Returning to the old variables, we write

$$z_0 = z_0(z, v, t), \quad v_0 = v_0(z, v, t),$$
$$f = f(z_0(z, v, t), v_0(z, v, t), 0). \tag{1.52}$$

Knowing f, we can find macroscopic properties: the density, the electric field, and so forth. For example, the spatial Fourier component of the charge density can be found from the following expression:

$$\rho_k(t) = e\int dz_0\, dv_0 \exp(-ik(v_0, z_0, t))\, f(v). \tag{1.53}$$

To examine the temporal echo we choose the external perturbations to be

$$a(z, t) = a_1(z)\,\delta(t) + a_2(z)\,\delta(t-\tau), \tag{1.54}$$

where $a_i = C_i \exp(ik_i z)$, and $C_i = \dfrac{eE_i}{m\Omega}$. We then find the following expression[6] for the charge density of order $(p + n)$:

$$\rho(z, t) = \sum_{p,\, n}^{\infty} \rho_{pn}(t) \exp(ik_{pn}\tau),$$

$$\begin{aligned}\rho_{pn}(t) = (-i)^{p+n} eJ_p \{\theta(t)\, C_1 k_{pn}(t - \tau_{pn})\} \\ \times J_n \{\theta(t-\tau)\, C_2 k_{pn}(t-\tau_{pn})\} \\ \times \int dv \exp(-ik_{pn}(t-\tau_{pn}))\, f(v),\end{aligned} \qquad (1.55)$$

where $k_{pn} = nk_2 - pk_1$, and $\tau_{pn} = nk_2\tau/k_{pn}$. At the time $t = \tau_{pn}$ the phase factor in the exponential function in (1.55) becomes independent of the velocity, and the macroscopic charge density becomes nonzero: This is a temporal echo. The spatial echo was studied by the ballistic theory in Refs. 64 and 65, and its amplitude of order $(p + n)$ was found to be

$$\begin{aligned}E^{(p+n)} \sim J_n\left(n\frac{e}{m}\frac{k_1^2}{\omega_1^2}E_1 l\right) J_p\left(p\frac{e}{m}\frac{k_2^2}{\omega_2^2}E_2 l\right) \\ \times \exp\left[-\frac{3}{2}\left(\frac{\omega}{s}|z-l'|\right)^{2/3}\right],\end{aligned} \qquad (1.56)$$

where $l_{pn} = p_1\omega_2/(p\omega_2 - n\omega_1)$ is the point at which the echo reaches its maximum intensity, and $J_p(x)$ is the Bessel function of index p. We might note that expressions (1.55) and (1.56) for the macroscopic echo signal were derived without assuming that the external perturbations are small, in contrast with the cases discussed earlier, where a method of successive approximations in the fields of the external perturbations, E_i was used.

h) Comparison of the results on collective and ballistic echos

This discussion has shown that both the self-consistent approach and the ballistic approach are required for a quantitative description of the effects, depending on the formulation of the problem. Specifically, (1) if the frequencies and the wave vectors of the oscillations under consideration correspond to the plasma transparency region (e.g., at a frequency $\omega \sim \Omega$ and wave numbers $k \ll a^{-1}$ for an electron plasma), the self-consistent approach must be taken to study the echo, while (2) in the case of strongly damped waves ($\omega \gg \Omega$ and $k \gg a^{-1}$ for an electron plasma) the ballistic approach is sufficient.

The collective echo has some characteristic features.

1. The shape of the echo signal is asymmetric in time or in space. For example, the spatial evolution of the echo signal in the case of the interaction of plasma waves can be described as follows: a growth of an echo signal $\sim \exp[(\omega_2/\omega_1)\Gamma_1(z - l')]$ and a decay $\exp[-(\omega/\omega_2)\Gamma_2(z - l')]$ and $\exp[-(\omega/\omega_1)\Gamma(z - l')]$.

2. The amplitudes of the echo signal of order $(m + n)$ are

$$E^{(m+n)} \sim E_1^m E_2^n \tau^n, \qquad (1.57)$$

$$E^{(m+n)} \sim E_1^m E_2^n l^n, \qquad (1.58)$$

for the temporal and spatial echos, respectively, since the method of successive approximations was used for the echo calculations. From the conditions for the applicability of this method ($E^{(2)} \ll E^{(1)}$, $E^{(3)} \ll E^{(2)}$, etc.) we find some restrictions on the fields E_i $(i = 1,2)$ of the primary perturbations $(E_i \sim \rho_i/k)$, working from expressions (1.22) and (1.24) for the fields of the second-order echo oscillations, $E^{(2)}$:

$$\frac{eE_i k\tau}{m\gamma} \ll 1, \qquad (1.59)$$

$$\frac{eE_i k^3 l}{\omega^2 m\Gamma} \ll 1. \qquad (1.60)$$

[Under conditions (1.59) and (1.60) we can use the linear theory of Landau damping for the primary waves.] The amplitudes of the temporal and spatial echos of second order are determined by (1.55) and (1.56) with $p = 1$ and $n = 1$.

3. If the echo is a collective response to the bunching and debunching of electrons, then its wavelength is determined by the frequency of the echo, that of one of the external perturbations, and the plasma dispersion relation.

Two aspects of the ballistic echo are worthy of note.

1. The ballistic echo is symmetric in time and space. For the second-order spatial echo, for example, the amplitude can be described by $\sim \exp\{-(3/2)[(\omega/s)|z - l'|]^{2/3}\}$, i.e., is symmetric for an arbitrary relationship between the frequencies of the external perturbations. The amplitude is furthermore proportional to

$$J_1\left(\frac{e}{m}\frac{k_1^2}{\omega_1}E_1 l\right) J_1\left(\frac{e}{m}\frac{k_2^2}{\omega_2}E_2 l\right).$$

2. If the echo is due exclusively to the bunching and debunching of free-streaming electrons, then its wavelength is a function of only the echo frequency and the characteristic electron velocity.

For small amplitudes of the primary fields, for which conditions (1.59) and (1.60) hold, we can expand the Bessel functions in series in their small argument in (1.55) and (1.56), and the amplitudes of the temporal and spatial echos are again given by (1.57) and (1.58) with $m = 1$ and $n = 1$.

Setting the dielectric permittivities equal to unity, $\varepsilon = 1$, in expressions (1.22) and (1.24) for the amplitudes of the collective echo, we find the same expressions for the echo field as are predicted by the ballistic theory in (1.55) and (1.56) for small field amplitudes E_i satisfying conditions (1.59) and (1.60). As the amplitudes of the external perturbations increase, and conditions (1.59) and (1.60) no longer hold, we must abandon the method of successive approximations, using (1.55) and (1.56) to describe the echo amplitudes. The ballistic theory thus predicts a saturation of the echo amplitude with increasing amplitude of the external perturbations. Experimental data confirm this interpretation of the collective and ballistic echos.

Ikezi and Takagashi[7] and Wong and Baker[88] have used the theory of a collective echo to study ion-wave echos. Their experimental results showed that the echo is asymmetric in that the echo signal rises and falls in different ways; this asymmetry is in agreement with the theory. Goforth and Gentle[44] carried out experiments on the structure of the third-order plasma-wave echo, determining the growth rate and decay rate of the echo signal with respect to the point of maximum intensity and also determining the echo wavelength. Those experimental results agree well with the theory of the collective echo. Malmberg *et al.*[5] showed experimentally that the echo is determined primarily by collective effects (the frequencies of the external perturbations and of the echo were $\omega_{1,2}$ and $\omega_3 \sim \Omega$).

Ripin and Pechacek[65] experimentally demonstrated the ballistic nature of the second-order spatial echo at $\omega \gg \Omega$: The wavelength and the frequency are related by $\omega/k = v_c$, where v_c is approximately equal to the velocity at which the electron distribution function has its maximum slope. This distribution is symmetric. Moeller[67] studied the echo saturation at large amplitudes of the external perturbations and showed that the echo amplitude can be described well by the Bessel function J_1, in accordance with the ballistic theory.

If the amplitudes of the external perturbations are large enough to satisfy $eE_i/m\omega s \gg 1$, trapped particles must be taken into account.[68–70] Echo phenomena involving trapped particles were studied theoretically and experimentally in Refs. 68–70, and the possibility of exploiting them to study the turbulence in fusion-research devices was discussed.

i) Echos in inhomogeneous plasmas

We have been discussing the echo in a homogeneous plasma, which is a nonlinear echo. In an inhomogeneous plasma in which a wave interacts resonantly with only a certain group of particles in each region of the plasma, the echo effects may be linear. Studies have been carried out[20–22] on nonlocal echo effects, which can occur only in an inhomogeneous plasma, in particular, the linear echo which arises during the propagation of a wave along a nonuniform external magnetic field, the longitudinal echo from two transverse sources in an opaque region of an inhomogeneous plasma, and the second-order echo at the sum frequency from two longitudinal sources in an inhomogeneous, isotropic plasma. The role played by echo effects in the brightening of the opaque regions of an inhomogeneous plasma was examined in those studies. The transmission of electromagnetic waves through an opaque region in an inhomogeneous magnetized plasma was studied experimentally in Ref. 23.

Let us examine, for example, the propagation of an extraordinary wave of frequency ω along a nonuniform magnetic field. The current induced in the plasma is

$$j_\omega(z) = -\frac{\Omega^2}{4\pi}\int_0^\infty \frac{dv}{v} f_0(v)\Big[\int_z^\infty dz' E_\omega(z')\,e^{-i\psi} - \int_\infty^z dz' E_\omega(z')\,e^{i\psi}\Big], \qquad (1.61)$$

where the phase factor is

$$\psi = \int_{z'}^{z} \frac{dy}{y}\,(\omega - \omega_B(y)).$$

A solution for the electric field is sought in the WKB approximation:

$$E_\omega(z) = A_\omega(z)\exp\Big(i\int_{z'}^{z} k(z')\,dz'\Big). \qquad (1.62)$$

The phase factor in the exponential function in (1.62) becomes

$$\theta = \int_{z'}^{z} a_\omega(z'')\,dz'', \qquad (1.63)$$

where $a_\omega(z'') = k - (\omega_B(z'')/v)$.

In the integration over the velocity v in a homogeneous plasma, the phase factor θ causes a thermal dissipation of the perturbation in accordance with $\sim \exp[-(3/4)(\theta z/s)^{2/3}]$. In an inhomogeneous plasma the z dependence may cause the phase θ to have an extremum along the v scale, so that a macroscopic signal—an echo—arises as a result of the integration over velocity.

2. ECHO PHENOMENA IN BOUNDED PLASMAS

A spatially bounded plasma can exhibit some qualitatively new echo phenomena, which are not seen in an unbounded plasma. These new effects stem from the reflection of charged particles from the plasma boundaries and from the existence of surface waves. Surface echo waves arise in addition to volume echo waves. Furthermore, echo phenomena can give rise to an anomalous transmission of electromagnetic waves through dense plasma slabs[24,27] and metal films.[28–30]

Since the kinetic approach is required for describing echo phenomena, an important point here is to select the reflection conditions for the plasma particles at the boundary. In the case of a sharp boundary, we specify

$$f^+(x, y, z=0; v_x, v_y, v_z) = \rho f^-(x, y, z=0; v_x, v_y, -v_z), \qquad (2.1)$$

where f^- and f^+ are the deviations from the equilibrium distribution function for particles moving toward and away from the boundary, respectively (the plasma occupies the $z \geqslant 0$ half-space). Condition (2.1) means that a fraction ρ of the total number of particles incident on the boundary are reflected without a loss of directed velocity, i.e., in a specular fashion, while a fraction $1-\rho$ are reflected in a diffuse fashion.

One of the earliest experimental studies[5] carried out to detect echos in plasmas dealt with not only echo phenomena in an unbounded plasma but also the echo which results from the specular reflection of electrons from a metal plate bounding a plasma.

As in the case of an unbounded plasma, it is convenient to use the nonlinear equation for the field derived in Ref. 71 for a quantitative description of echo phenomena in a semi-infinite plasma with the model of specular reflection ($\rho = 1$). For mixed specular-diffuse reflection, $0 < \rho < 1$ (in both semi-infinite plasmas, on the one hand, and plasma slabs and metal films, on the other), the echo is described by the simpler ballistic approach.

a) Echo phenomena in a semi-infinite plasma with a sharp boundary for the model of specular reflection

Let us examine the echos in a semi-infinite plasma with a sharp boundary under the assumption of specular reflection of the electrons at the boundary (this situation can arise, for example, if a plasma is confined by an insulator). The boundary causes (first) additional Van Kampen waves (because of the specular reflection of particles) and (second) a substantially different dispersion of the waves propagating through the plasma. These two circumstances give rise to some qualitatively new echo effects, as we will now see.

The second-order collective echo in a semi-infinite electron plasma was studied in Refs. 72–74 under the assumption of specular reflection of electrons from the boundary (a spatially homogeneous, steady-state plasma fills the $z\geqslant 0$ half-space, while the $z<0$ half-space is filled with an insulator with a dielectric permittivity ε_0). The external perturbations are assumed to be external charge densities in the planes $z=\pm\alpha x\pm\sqrt{1+\alpha^2}l$, tilted at an angle θ with respect to the plasma surface ($\tan\theta=\alpha$):

$$\rho^0(x, z, t) = \sum_{i=1,2}\sum_{\pm}\rho_i\exp(i\omega_i t)\,\delta(k_0'(\sin\theta x\pm\cos\theta(z-l))). \tag{2.2}$$

In other words, we are actually dealing with a two-dimensional case in which all quantities are functions of x and z. The field of the echo oscillations at the frequency $\omega=\omega_2-\omega_1$ consists of a volume field and a surface field:

$$\mathbf{E}^{(2)}_{\mathbf{k}\omega}=\mathbf{E}^{(2)v}_{\mathbf{k}\omega}+\mathbf{E}^{(2)s}_{\mathbf{k}\omega}. \tag{2.3}$$

The field of the volume-oscillation echo at the frequency $\omega=\omega_2-\omega_1$ is

$$E^{(2)v}_x(x, z, t)=16\pi^{3/2}\frac{\alpha}{1+\alpha^2}\frac{e}{m}\frac{\rho_1\rho_2(z+\alpha x-\sqrt{1+\alpha^2}\,l)}{\omega_1\omega_2 a^2 k_0'^2}$$
$$\times\exp(-i\omega t)\int d\xi\,\frac{\exp[(i\omega/s\xi)(\sin\theta x+\cos\theta z-l')-\xi^{-2}]}{\xi^2\varepsilon(-\omega_1, \omega_1/s\xi)\,\varepsilon(\omega_2, \omega_2/s\xi)\,\varepsilon(\omega, \omega/s\xi)},$$
$$E^{(2)v}_z(x, z, t)=\frac{1}{\alpha}E^{(2)v}_x(x, z, t), \tag{2.4}$$

where $l'=(\omega_2+\omega_1)l/(\omega_2-\omega_1)$. It can be seen from (2.4) that the maximum of the echo signal at the frequency $\omega=\omega_2-\omega_1$ lies in the plane $z=-\alpha x+\sqrt{1+\alpha^2}l$. If $\theta=0°$ we have $\alpha=0$ (meaning that the planes containing the charge grids with ω_1 and ω_2 coincide and run parallel to the plasma boundary, lying at a distance l from it). The density waves (Van Kampen waves) propagating from the grid with the frequency ω_1 toward the plasma boundary are reflected from it and are then modulated again by the field at the frequency ω_2 from the same grid. The result is the excitation of secondary Van Kampen waves at the frequency $\omega_2-\omega_1$, which produce a macroscopic echo signal at a distance $z=l'$ from the plasma boundary.

If the frequencies $\omega_{1,2}$ of the external perturbations and the frequency $\omega=\omega_2-\omega_1$ of the nonlinear signal satisfy the conditions $\omega_{1,2}>\Omega$, $\Omega/\sqrt{1+\varepsilon_0}<\omega<\Omega$, then the echo field is determined primarily by the surface term in (2.3), i.e., $E^{(2)s}$. In this case the nature of the echo signal is determined by the dispersion of the surface waves, and it reaches a maximum intensity at the point $x=(\omega_2+\omega_1)l/(\omega_2-\omega_1)$ on the plasma boundary. The z dependence of the amplitude of this signal is described by

$$E^{(2)s}\sim\begin{cases}\exp\left[-\frac{3}{2}\left(\frac{z\omega}{2s}\right)^{2/3}\right], & \frac{z\omega}{s}\ll\left(\frac{\omega}{k_1 s}\right)^3,\\ \exp(-k_1 z), & \frac{z\omega}{s}\gg\left(\frac{\omega}{k_1 s}\right)^3,\end{cases} \tag{2.5}$$

where k_1 is the wave number of the surface wave at the frequency ω, given by[79]

$$k_1=\left(1+\frac{\varepsilon_0}{\varepsilon}\right)\frac{\omega}{s}\,\mathrm{Re}\,\beta^{-1}\left(\frac{\omega}{\Omega}\right), \tag{2.6}$$

with

$$\varepsilon=1-\frac{\Omega^2}{\omega^2},\quad \varepsilon>-\varepsilon_0\quad s<\frac{\omega}{k}<\frac{c}{\sqrt{\varepsilon_0}},$$
$$\beta\left(\frac{\omega}{\Omega}\right)\approx 2.5-i0.3$$

and $\omega=\Omega/\sqrt{2}$.

It follows from (2.5) that the echo signal is concentrated near the surface bounding the plasma and is exponentially small only a very short distance from this surface, at a few times the Debye length. In other words, the echo signal is of the nature of a nonlinear surface wave.

The dispersion of the collective echo in a semi-infinite electron plasma is thus determined by the dispersion of both volume and surface waves.

The temporal echo in a semi-infinite plasma was studied in Ref. 76, and the spatial echo in a semi-infinite magnetized plasma was studied in Ref. 77 (in each case, the model of a sharp boundary with specular reflection of particles from the boundary was used).

b) Echos in a semi-infinite plasma with a sharp boundary for the model of specular-diffuse reflection

If the plasma is bounded by an insulator then the reflection of particles from this boundary may be mixed: Some particles may settle on the insulator and lose their directed velocity, while others may be reflected in a specular manner as a result of Coulomb repulsion [in this case we would have $0<\rho<1$ in (2.1)]. The echo signal should depend strongly on the nature of the particle reflection at the boundary, since only the specularly reflected particles contribute to the echo. Studies[78,79] of the spatial ballistic echo in a semi-infinite electron plasma with $0<\rho<1$ show that this is indeed the case.

To study the echos we work from kinetic equation (1.1) for the distribution function, specifying the electric field $E^{(1)}$ to be (we ignore the self-consistent field)

$$E^{(1)}_x=\sum_{i=1,2,3'}E_{0i}[\exp(i\omega_i t)+\exp(-i\omega_i t)]$$
$$\times\exp(-\varkappa_i|z-l|), \tag{2.7}$$

where $\varkappa_i^{-1}$ is the skin depth,

$$\delta_i\equiv\varkappa_i^{-1}=\frac{c}{\omega_i}\sqrt{\frac{\Omega^2}{\omega_i^2}-1}. \tag{2.8}$$

[Since the frequencies of the external perturbations satisfy $\omega_i<\Omega$, expression (2.7) means that the field penetrates a distance equal to the skin thickness δ_i on each side of the source at $z=l$.] The effect of the field (2.7) reduces to one of modulating the particle fluxes, which we assume to be directed normal to the boundary. The distance l from the boundary is chosen to satisfy $\varkappa_i l\gg 1$. Assuming that the external perturbations are small ($eE_{0i}/m\omega_i s\ll 1$), we can solve the kinetic equation by the method of successive approximations. A longitudinal echo can occur in the second approximation. It arises in the plasma volume at a distance $l'=(\omega_2+\omega_1)l/(\omega_2-\omega_1)$ from the boundary. In the third approximation in the field, there can be a transverse echo at the frequency $\omega_e=\omega_3+\omega_2-\omega_1(\omega_e<\Omega)$:

$$E_x^{(3)}(z,\ t)=\frac{4\sqrt{3}}{\pi^2}\,\rho\,\frac{e^2\Omega^2 s^2 l^2}{m^2c^2}\,\frac{\varkappa_1\varkappa_2\varkappa_3\,(\varkappa_2\,(\omega_3+2\omega_2)+\varkappa_3\omega_2)}{\omega_0\omega_2^2\omega_3\,(\omega_2+\omega_3)^2}$$
$$\times E_{01}E_{02}E_{03}\exp(-i\omega_e t)\exp\left(-\frac{3}{2}\frac{\omega_2}{2s}\,|z-l_1|\right)^{2/3}$$
$$\times\,[1-i\sqrt{3}\,\mathrm{sign}\,(z-l_1)],\qquad(2.9)$$

where $z = l_1 = (\omega_3 + \omega_2 + \omega_1)l / \omega_e$ is the point at which the echo reaches it maximum intensity. We see that the amplitude of echo (2.9) is proportional to ρ, i.e., to the fraction of electrons reflected specularly from the boundary, and we see that the shape of this echo is symmetric in space with respect to the position of the maximum, l_1. A similar approach was taken in Ref. 80 to study the echo of electromagnetic waves in an unbounded plasma.

The longitudinal spatial echo in a semi-infinite plasma was also analyzed by the ballistic theory in Ref. 78. Monochromatic signals with frequencies ω_1 and ω_2 satisfying $\omega_{1,2} \gg \Omega$ (Van Kampen waves are propagating normal to the boundary) are specified on a grid positioned a distance l from the boundary. A general expression is derived for the longitudinal echo field of order $(p + q)$ (without the assumption that the external perturbations are of small amplitude, so that the saturation of the echo can be described). The maximum intensity of this field occurs at the point

$$z_{pq}=\frac{q\omega_2+p\omega_1}{q\omega_2-p\omega_1}\,l.\qquad(2.10)$$

The amplitude of the echo of order $(p + q)$ is also proportional to ρ, the fraction of electrons reflected specularly. By measuring the amplitudes of the echo signal it is thus possible to carry out a direct experimental study of the nature of the electron reflection from the plasma boundary.

c) Spatial echo in a plasma with a diffuse boundary in the approximation of a given field

The boundary of a plasma is frequently not sharp and is instead characterized by some distance over which the density changes near the plasma surface (we will call this the "plasma-vacuum transition layer"). A boundary of this type may form as a result of some particular potential profile, for example. It was shown in Ref. 81 that the nature of the longitudinal spatial echo in this case depends strongly on the potential profile. As in the case of a plasma with a sharp boundary, sources of external perturbations with frequencies ω_1 and ω_2 are positioned at a distance l from the boundary $(\omega_{1,2} > \Omega)$. We supplement the kinetic equation for the electron distribution function with the condition for specular reflection of the electrons by the potential profile:

$$f^+(z=z_\varepsilon,\ v)=f^-(z=z_\varepsilon,\ -v),\qquad(2.11)$$

where the electron reflection point $z = z_\varepsilon$ is determined by the condition $\frac{e}{m}U(z) = \mathscr{E}$, where $\mathscr{E} = \frac{e}{m}U(z) + v^2/2$ is the total electron energy (for simplicity we are dealing with the one-dimensional case). A detailed analysis was made of the case of a parabolic potential profile,

$$U(z)=\begin{cases}-U_0(z/a-1)^2 & z\leqslant a,\quad U_0>0,\\ 0, & z>a,\end{cases}\qquad(2.12)$$

and a linear potential profile,

$$U(z)=\begin{cases}-U_0(z/a-1), & z\leqslant a,\\ 0, & z>a\end{cases}\qquad(2.13)$$

(the parameter a determines the width of the plasma-vacuum transition layer and is of the order of the Debye length). Figure 1 shows the shape of the real part of the echo field, $E^{(2)}_{\omega_3}(\omega_3 = \omega_2 - \omega_1)$ as a function of z for the following parameter values: $kT = 100$ eV, $\Omega = 1.78\cdot10^9\ \mathrm{s}^{-1}$, $\omega_1 = 1.2\Omega$, $\omega_2 = 2.8\Omega$, $a = 3.4$ cm, $l = 11.75$ cm, $l' = 25.845$ cm, and $kT/eU_0 = 0.1$. We see that the echo amplitude has different shapes, depending on the potential profile; this situation raises the possibility of experimentally studying the potential profile in the plasma-vacuum transition layer by measuring spatial-echo effects.

d) Echos in plasma slabs and metal films

The modulated particle beams which undergo repeated specular reflection from the boundary of a plasma slab may be focused in phase and may produce a macroscopic echo signal (some examples of such boundaries are insulating plates and the surfaces of metal films). The most interesting situation here is evidently that in which the slab is opaque to the fields of the external electromagnetic waves which are incident on the slab, e.g., under the condition $\omega_i < \Omega$ (ω_i is the frequency of the electromagnetic wave, and Ω is the plasma frequency of the slab), in which case the fields penetrate a distance equal to the skin thickness δ_i into the plasma, while the slab thickness is $d \gg \delta_i$. Van Kampen waves will propagate away from the slab boundary. These waves will be reflected from the boundaries of the slab and repeatedly modulated by the fields of the external perturbations (over the skin thickness), and they may become focused in phase near the opposite side of the slab and excite a nonlinear echo signal.

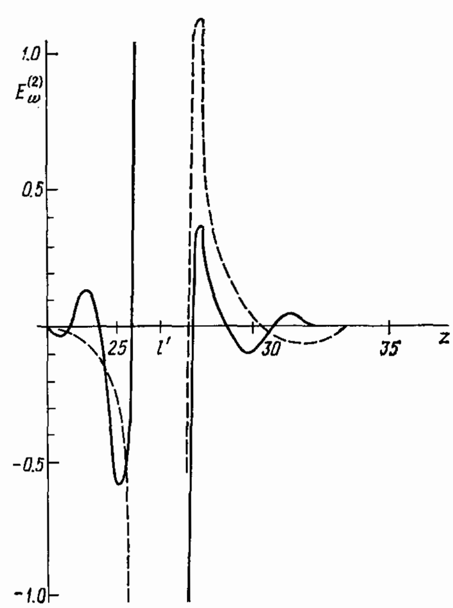

FIG. 1. Profile of the asymptotic shape of the echo field $E^{(2)}_{\omega}$ along z. Dashed curve—linear potential; solid curve—parabolic potential.

This signal may be radiated into vacuum, i.e., there may be a nonlinear "brightening" of the plasma slab.

Let us examine the results which have been found on echos in plasma slabs and metal films. Revenchuk and the present author[24,25] predicted a nonlinear transmission of electromagnetic waves through a dense slab of an electron plasma occupying the interval $0 \leqslant z \leqslant d$ with sharp boundaries (boundary 1 at $z = 0$ and boundary 2 at $z = d$). This nonlinear transmission was to be a consequence of echo effects. Let us examine the case in which electromagnetic waves of frequency $\omega_i \ll \Omega$ ($i = 1,2,3$) are incident normally on boundary 1 from vacuum. These waves penetrate a distance equal to the skin thickness, δ_i, into the plasma. Since the layer thickness d satisfies

$$\delta_i \ll d \ll \lambda \tag{2.14}$$

(λ is the electron mean free path), the linear transmission coefficient for the waves (the ratio of the amplitudes of the incident and transmitted waves) is exponentially small, $k_{\text{lin}} \sim \exp(-d/\delta)$. If particles are reflected specularly from boundaries 1 and 2 (at boundary 1 the particles are modulated by the fields of the incident waves), and if the frequencies ω_i are chosen correctly, then a transverse third-order macroscopic echo signal at the sum and difference frequencies $\omega_3 - \omega_1 + \omega_2$ will arise near boundary 2. These echo signals may be radiated into vacuum. The nonlinear wave transmission coefficient in this case is significantly higher than the linear coefficient. For a plasma slab with $\Omega = 10^{10}$ s^{-1}, $s = 0.5 \cdot 10^6$ m/s, $d = 20\delta$ ($\delta \approx c/\Omega = 3 \cdot 10^{-2}$ m), and $\omega_i = 0.1$ we find $k_{\text{non}}/k_{\text{lin}} \gtrsim 10^3$ when the amplitudes of the incident waves satisfy $eE_0/m\omega_i s \approx 10^{-2}$. Echo phenomena in a plasma slab were studied in Ref. 27 for the cases in which s- and p-polarized waves are incident from vacuum.

In a nonequilibrium plasma slab along whose boundary an electron beam is moving, a second-order nonlinear echo signal may be generated as a result of modulation of the beam particles.[82]

Dryakhlushin and Romanov[26] studied the spatial echo in a plasma slab under the condition of specular reflection of the plasma particles at the slab boundaries. They dealt with electrostatic waves, and they put the sources of the "external" fields at the same point inside the plasma slab. They derived a general expression for all points for which a plasma echo at the frequency $\omega_2 - \omega_1$ is possible (ω_1 and ω_2 are the frequencies of the external fields). Under the condition $\alpha\omega_1 = \beta\omega_2$ (α and β are integers), a large number of echo oscillations may be superimposed at a single point, and an echo resonance of a sort may arise [the amplitude of the echo-oscillation field is proportional to $(\lambda/d)^2$, where λ is the mean free path of the particles, and d is the slab thickness].

Plasma echos in metal films were studied in Refs. 28–30 under the assumption that the mean free paths of the charged particles were much larger than the film thickness and that a certain fraction of the particles undergo specular reflection from the film boundaries.

It was shown in Ref. 29 that in an anisotropic crystal a plasma echo arises even in second order, because of electrons which are reflected specularly from the metal boundary. It was assumed there that electromagnetic waves with frequencies $\omega_{1,2} < \Omega$ are incident on both surfaces of the metal film (boundary 1 at $z = 0$ and boundary 2 at $z = d$) and decay over the skin thickness. An echo signal at the frequency $\omega_2 - \omega_1$ arises near boundary 1 in the second approximation in the field. This signal has a sharp maximum at $\omega_2 = 2\omega_1$. A corresponding effect can be seen at $\omega_2 = \omega_1/2$ near surface 2. These results are valid under the rather stringent conditions

$$\frac{v_0}{\Omega} = a \ll \frac{v_0}{\omega_i} \ll \delta_i \ll d \ll \lambda, \tag{2.15}$$

where $v_0 = [h(3\pi^2 n)^{1/3}]/m$ is the Fermi velocity, m is the effective electron mass, $\delta_i \approx c/\omega_i$ is the skin thickness, and $\lambda = v_0/v$ is the mean free path in the metal. Some numerical estimates were also carried out in Ref. 29 for a bismuth single crystal, in which inequalities (2.15) can be satisfied at $\omega \sim 10^{10}$–10^{11} cm^{-1}, so that the conditions required for a plasma echo can be satisfied.

Leviev and Potapenko[30] observed the plasma echo experimentally in a bismuth plate in a magnetic field directed perpendicular to the surfaces of the plate. These measurements involved determining the dependence of the power level of the nonlinear echo signal on the magnetic field.

In summary, an anomalous transmission of electromagetic waves at sum and difference frequencies through plasma slabs with sharp boundaries and through metal films was predicted in Refs. 24, 25, and 28–30. The effect was understood to result from a nonlinear phase focusing of particles reflected in a specular manner by the boundaries of the slab or film.

3. EXPERIMENTAL RESULTS ON PLASMA ECHOS. USE OF ECHOS FOR PLASMA DIAGNOSTICS

a) Experimental research on echos of plasma waves, ion-acoustic waves, and cyclotron waves

Spatial echo phenomena at the plasma frequency were first discovered in Ref. 5. In those experiments, a plasma column 180 cm long and 5 cm in diameter with a central density of $1.5 \cdot 10^8$ cm^{-3} was placed in a magnetic field of 305 G. The background pressure was $\sim 1.5 \cdot 10^{-5}$ torr for H_2; at this pressure the mean free path for collisions of electrons with neutrals was 10 m; in other words, the plasma was collisionless. The plasma temperature was 9.4 eV, and its Debye length was 2 mm. The plasma was surrounded by a cylinder 5.2 cm in radius which served as a waveguide for the waves propagating through the plasma. The plasma column was bounded in the longitudinal direction by a negatively charged plate whose electric field reflected electrons. Oscillations were excited and studied with an array of probes which could be moved along the radius and along the axis. A signal of adjustable amplitude and frequency from an external generator was applied to one of the probes in order to arrange dispersion and damping of the plasma waves. Malmberg *et al.*[5] concluded from the experimental dispersion curve [$\omega = f(k)$] that the dispersion of the oscillations in the region $k < 1$ cm^{-1} depends on the radius of the system, while at $k > 1$ cm^{-1} the dispersion is dominated by finite-temperature effects. For the frequencies studied the condition $k_i > 2$ cm^{-1} ($i = 1,2,3$) holds; in other words, the plasma can be assumed unbounded in the radial direction. For observation of echos, a signal of frequency $f_1 = 120$ MHz was applied to one of the probes (at $z = 0$ cm), while a signal at $f_2 = 130$ MHz was applied to another probe, at a distance $z = l = 40$ cm from the first. In agreement with the theory, a third-order echo was observed at the frequency $f_3 = 2f_2 - f_1 = 140$ MHz at a distance $l^* = 2f_2 l/(2f_2 - f_1) \approx 75$ cm from the first probe. The signal was amplified by 20 dB. The position of the echo maximum was measured as a function of the distance between the generators [i.e., $l^* = f(l)$] for various frequencies of the primary waves, f_1 and f_2. The dependence was found to be linear in all cases, as expected (Fig. 2). The peak echo power P^* was also measured as a function of the power of the primary waves, P_1 and P_2; more precisely, P_3 was measured as a function of P_1 and P_2^2, where $P^* \sim |E^{(3)}_{\omega_3}(z = l')|^2$, $P_1 \sim E_1^2$, $P_2 \sim E_2^2$ [E_1 and E_2 are the amplitudes of the external perturbations, and $E^{(3)}_{\omega_3}(z = l')$ is the amplitude of the echo at the point of the maximum , l'] (see Fig. 3). By virtue of the relationship $P^* \sim P_1 P_2^2$, which holds for small signals, linear dependences were found. With increasing amplitudes of the external perturbations, the echo amplitude was found to reach saturation (and this was again a predictable result, since higher-order effects come into play in the interaction as the wave intensities increase). The absolute power levels from Ref. 5 (see Fig. 3 of the present paper) are extremely crude values, because the plasma-probe interaction is not known accurately. For this reason, the results in Fig. 3 and elsewhere corresponding to those experiments cannot be used for a detailed comparison with theoretical results (regarding points for which absolute values of the signals are required).

Let us assume that, as follows from (1.56), the perturbation theory breaks down at fields

$$\frac{eE_1k^3l}{m\omega^3\Gamma} \sim 1, \tag{3.1}$$

in accordance with the estimate in Ref. 5. According to Malmberg *et al.*,[5] these fields correspond approximately to

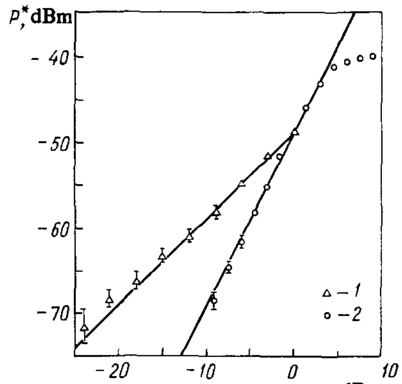

FIG. 3. Peak power of the third-order echo, $P_3 = |E^{(3)}_{\omega}\times(z = l')|^2$, as a function of the power of the primary waves.[3] 1—P_1 is varied, $P_2 = -29$ dBm; 2—P_2 is varied, $P_1 = -26$ dBm. The absolute values of the power are only approximate.

those primary-wave fields at which the saturation becomes clear. The perturbation theory does not predict saturation of the echo amplitudes. The saturation was also studied experimentally in Refs. 57 and 58, where it was shown that the behavior of the echo amplitude at fields exceeding those in (3.1) is described well by Eq. (1.52) with $m = n = 1$—i.e., by the ballistic theory. In addition, a second-order echo was observed at the frequency $f_3 = f_2 - f_1 = 160$ MHz ($f_1 = 120$ MHz, $f_2 = 280$ MHz), and the maximum of this echo occurred at the point $l' = f_2 l/(f_2 - f_1)$, again in agreement with the theory. Also observed there was a third-order echo, with a single generator; this echo resulted from the reflections of electrons from a negatively charged plate. If a generator is positioned at a distance l from the boundary, the electrons reflected from the plate pass twice through the grid to which the signal ω_1 is applied (this situation corresponds to the model of a semi-infinite plasma with specular reflection of electrons[73,78]), and the echo arises at a distance

$$z' = \frac{2\omega_2 + \omega_1}{2\omega_2 - \omega_1} l = 3l \qquad (\omega_2 = \omega_1), \tag{3.2}$$

according to Eq. (2.10), as was observed experimentally (Fig. 4). The generator was positioned at a point $l = 30$ cm from

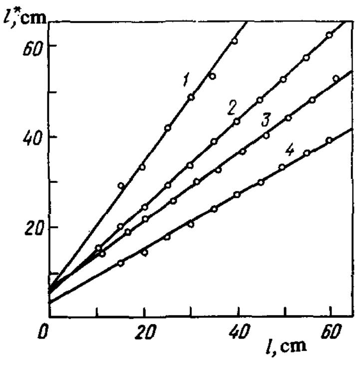

FIG. 2. Position of the maximum of the third-order echo as a function of the distance between the generators[3] [$l^* = 2f_2 l/(2f_2 - f_1)$]. Solid curves—theoretical; circles—experimental data for various values of f_1, f_2, and $f_3 = 2f_2 - f_1$(MHz). 1) 150, 130, 110; 2) 140, 145, 150; 3) 110, 130, 150; 4) 110, 150, 190.

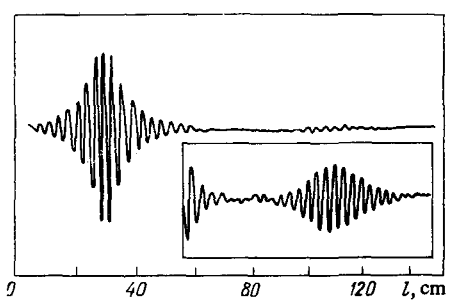

FIG. 4. Third-order echo due to reflections ("sheath echo").[5] This is a plot of the output signal from the interferometer (arbitrary units) as a function of the distance from the reflecting plate. The amplification is 30 dB; $f_1 = f_2 = f_3 = 120$ MHz.

the plate, and a frequency of 120 MHz was applied to it. A third-order echo was observed at the same frequency; its maximum intensity occurred at the point $z' = 90$ cm, as would be expected from (3.2). The dependence of the peak power P^* of this echo on the power P_1 of the primary wave was the same as in the preceding case: At small values of E_1 there was a linear dependence ($P^* \sim P_1^3$), and as the generator power was increased further the echo amplitude reached saturation (Fig. 5). As in the preceding case, the absolute values of the power are extremely approximate.

The echos of ion-acoustic waves were studied by Baker *et al.*[6] Their experiments were carried out in the highly ionized cesium plasma of a Q machine. The plasma was bounded in the radial direction by an axial magnetic field of 4 kG. The plasma density was $5\cdot10^9$ cm$^{-3} < n < 2\cdot10^{11}$ cm^{-3}, and the temperatures were $T_e = T_i = 0.2$ eV. Under these conditions we have $\lambda/lk \gg 1$, where λ is the ion mean free path, and l is the distance between the grids to which the signals at frequencies f_1 and f_2 were applied. These frequencies were varied over the interval 35–215 KHz, and the distance l between the generators was varied over the interval 1.5–15 cm. Figure 6 shows the measured dependence of the echo amplitude on the distance between the grids. We see that the echo amplitude falls off as this distance is increased; Baker *et al.*[6] attributed this effect to an influence of collisions on the amplitude of the echo signal, as in the case of electron waves. According to the theory of Ikezi and Takagashi,[8] the echo should reach its maximum intensity at the point $l' = f_2 l/(f_2 - f_1)$, and this result was in fact observed experimentally. The echo appeared only under the condition $f_2 > f_1$. The dependence $l' = f(l)$ was measured for $f_2/(f_2 - f_1) = 3.33, 2.33$, and 1.75. The dependence was found to be linear in all three cases. The nature of the echo was also found to be asymmetric with respect to the point of the maximum. Ikezi and Takagashi[7] also observed a second-order spatial echo of ion waves. A more detailed study of the ion-wave spatial echo was carried out in Ref. 8, where the effect of collisions on the shape of the echo was also studied. The parameters of the plasma there were approximately the same as those in Ref. 6. Higher-order echos and the secondary echo were also studied in Ref. 8, and good agreement between theory and experiment was found. Heymann and Sauer[83] observed second-

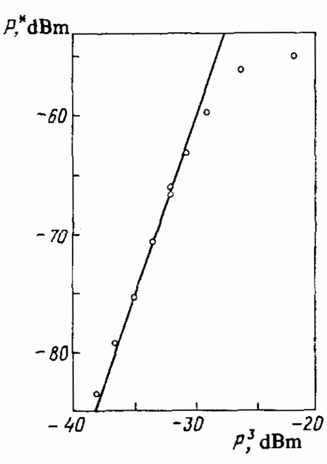

FIG. 5. Power of the echo due to reflections as a function of the generator power.[3] The circles are experimental data (the absolute values of the power are only approximate). With increasing generator power, the power of the echo signal reaches saturation.

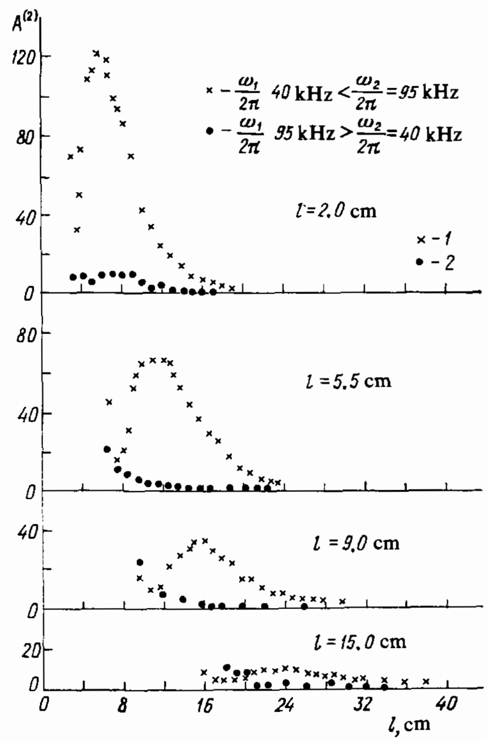

FIG. 6. Amplitude of the ion-acoustic echo (arbitrary units) as a function of the distance (l) to the first generator, for four values of l (Ref. 4) 1—$\omega_2 > \omega_1$; 2—$\omega_2 < \omega_1$.

and third-order spatial echos of azimuthally symmetric surface waves propagating in a column of an unmagnetized plasma (these waves were called surface waves since all the electric field components reached their maxima at the plasma boundary and decayed with distance from the boundary[84]). Porkolab and Sinnis[54] carried out experimental studies of the echos of cyclotron waves in magnetized plasmas with $k_\perp \neq 0$, i.e., under conditions allowing resonances at the higher harmonics $\omega = n\omega_{Be}$ ($n = 2,3$). These experiments were carried out in a column of an argon plasma with properties similar to those of the plasma studied in Ref. 5; the condition $\Omega > \omega_{Be}$ held. Cyclotron waves were excited at an angle from the magnetic field ($k_\perp \neq 0$). It follows from the results that the echo amplitude depends on the magnetic field and that the echo exists at frequencies above $2\omega_{Be}$, in accordance with the theory. Figure 7 shows the position of the echo as a function of the distance (l) between the generators, described by $z^* = (\omega_2 - n\omega_{Be})/(\omega_2 - \omega_1 - p\omega_{Be})$. Three curves are shown here for various values of n and p [(2,1), (3,1), (3,2) with $s = z^*/l = 2.3$, 2, and 1.54, respectively]. As expected, these plots coincide with straight lines.

The experimental results thus show that echo oscillations of second and higher orders can propagate in the plasma. In all cases the theoretical results on the shape and position of the echo oscillations are in accordance with the experimental data.

The temporal echo in a plasma was observed experimentally by Droanh and Rohne.[9] Two pulses were applied to an argon plasma at a time separation $\tau = 10$–60 μs; these pulses excited standing ion waves. A third-order echo was

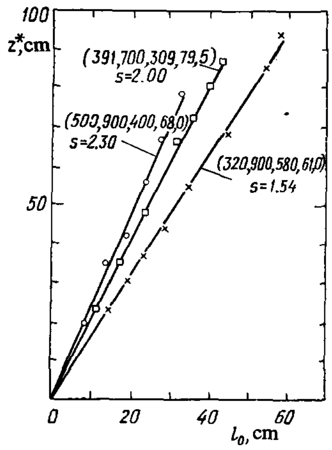

FIG. 7. Position of the maximum intensity of the electron cyclotron echo, z^* as a function of the distance between generators, l_0 (Ref. 55). $z^* = (f_2 - n f_c)l_0/(f_2 - f_1 - pf_c)$. For the lines with $s = 2.3, 2.00$, and 1.54 ($s = z^*/l_0$) the values of n, p are, respectively, (2, 1), (3, 1), and (3, 2).

observed at a time $\tau' = 2\tau$ after the first pulse with a wave number $k_3 = k_2 = k_1 (k_3 = 2k_2 - k_1)$.

b) Possible applications of echos in plasma diagnostics

The echo effect has been used to obtain important results on collisions and microscopic turbulence in plasmas.

The effect of collisions on echos has been the subject of many theoretical and experimental studies.[8,67,68,85–93]

Let us write the collision integral in the form of a Fokker-Planck equation in which the effect of collisions reduces to dynamic friction and diffusion of particles in velocity space. The kinetic equation for the distribution function is then

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} + \frac{e}{m}E\frac{\partial f}{\partial v} = -\frac{\partial}{\partial v}(D_1(v)f) + \frac{\partial^2}{\partial v^2}(D_2(v)f), \tag{3.3}$$

where $D_1(v)$ is the dynamic friction coefficient, and $D_2(v)$ is the diffusion coefficient in velocity space. We assume that two grids in the electron plasma at $z = 0$ and $z = l$ excite self-consistent waves with Fourier components $E(k,\omega_1)$ and $E(k,\omega_2)$. Since the deviation from an equilibrium distribution function is $\sim \exp(i(\omega_1/v)z)$, the collision integral is dominated under the condition $\frac{\omega_1 z}{l} \gg 1$ by the term with $D_2(v)$, where we differentiate the rapidly oscillating function $\exp(i(\omega_1/v)z)$ twice with respect to v. Solving (3.3) and substituting the resulting solution into the Poisson equation for the field, we find the following expression for the echo amplitude (the echo maximum is at the point $l' = \omega_2/(\omega_2 - \omega_1)l$):

$$E^{(2)}(k,\ \omega_3) = \frac{\pi}{2}\left(\frac{e}{m}\right)^2 \frac{e\omega_1 l E^*(\omega_1/v,\ \omega_1)\,E(\omega_2/v,\omega_2)}{k^2v^3\,\varepsilon(\omega_3/v,\ \omega_3)}\frac{\partial f_0}{\partial v} \times \exp\left(-\frac{\omega_1^2\omega_2 D_2(v)}{3\omega_3 v^5}l^3\right)\Big|_{v=\omega_3/k} \tag{3.4}$$

It can be seen from (3.4) that measurements of the dependence of $\lg E^{(2)}_{k\omega}/l$ on l^3 can reveal the damping factor $b(v) = \omega_1^2\omega_2 D_2(v)/3\omega_3 v^5$, and thus the diffusion coefficient $D_2(v)$.

Moeller[67] has measured the echo decay which results from diffusion in velocity space and has found the velocity dependence of the diffusion coefficient. That experimental apparatus used a plasma column 60 cm long in a magnetic field of 1 kG. The electron density was $n = 1.1\cdot10^8$ cm^{-3} and the electron temperature $T_e = 0.65$ eV. The echo was excited by two grids to which frequencies ω_1 and ω_2 were applied. Here we will discuss the results obtained at $f_1 = 350$ MHz, $f_2 = 600$ MHz, and $f_3 = 250$ MHz. The Fourier components of the echo amplitude were plotted for various distances l. Here the value $k = 10$ cm^{-1} corresponds to a velocity of $1.5\cdot10^8$ cm/s. The shape of this curve corresponds to Eq. (3.4) with corrections for the grid form-factor. Actually, collisions of electrons with neutrals also play a role. For the given number k the amplitude of the Fourier component in this case is proportional to

$$la\left(\frac{\omega_3}{k}\right)\exp\left[-b\left(\frac{\omega_3}{k}\right)l^3 - \frac{l'}{\lambda(\omega_3/k)}\right],\quad l' = \frac{\omega_2}{\omega_2-\omega_1}l. \tag{3.5}$$

Knowing the pressure of the neutrals we can determine $\lambda(v)$. Figure 8 shows results corrected for collisions with neutrals for $v = 8.5\cdot10^7$ cm/s and $k = 18.5$ cm^{-1}. From these results, the damping factor $b(\omega_3/k)$ can be determined [$b(\omega_3/k)$ is determined by the slope of the line]. From the series of these results for various wave numbers, the dependence $b = f(v)$ can be found. Figure 9 shows both the velocity dependence of the diffusion coefficient according to these experimental results and the velocity dependence of the diffusion coefficient for various temperatures according to the theory of Ref. 94:

$$D_2(v) \approx \frac{e^2}{m}\frac{\Omega^2}{v_{0\parallel}}\frac{A}{\pi}\ln(ak_0), \tag{3.6}$$

where $v_{0\parallel}$ is the electron velocity along the magnetic field, $A = \mathrm{Im}\,\omega'(v_{0\parallel}/s)$, $k_0 = ms^2/e^2$, and we have $\ln(ak_0) \approx 13$ for this experiment. Figure 9 demonstrates the correspondence between the theoretical and experimental results.

Moeller[67] also noted that expression (3.4), which was used for the experimental determination of $b(v)$, is valid only for small amplitudes of the external perturbations, under condition (1.60).

Jensen *et al.*[87] studied the effect of microscopic turbulence on the third-order echo in the presence of boundaries (the experiments were carried out in the same apparatus as was used in Ref. 5). When diffusion is occurring, the echo amplitude is reduced by a factor of e^{-k*}, where

$$k^* = \int_0^l D\left(\frac{\omega}{k},\ z\right)\frac{k^5z^2}{\omega^3}dz + \int_l^{2l} D\left(\frac{\omega}{k},\ z\right)\frac{k^5(2l-z)^2}{\omega^3}dz. \tag{3.7}$$

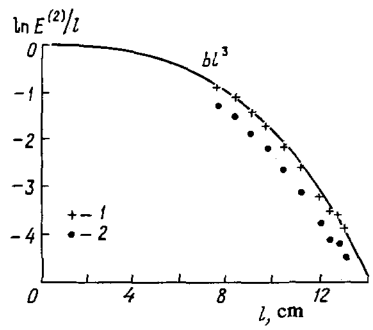

FIG. 8. The damping effect with a correction for collisions of electrons with neutrals.[91] 1—With correction for neutrals; 2—without this correction.

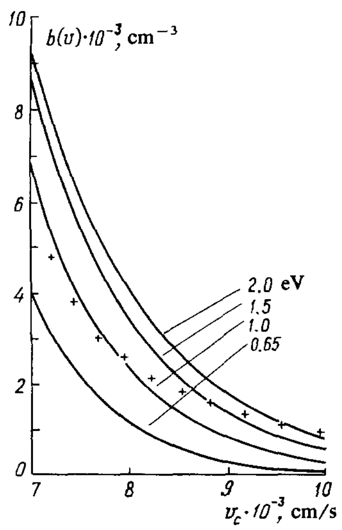

FIG. 9. Curves—theoretical curves of the damping factor for a Maxwellian plasma at various temperatures; points—experimental data.[91]

At the point $z = 2l - l'$ there was a source of noise which increased the diffusion coefficient in velocity space, since diffusion in velocity space is related to the spectral density of the microturbulence according to the quasilinear theory:

$$D_2(v, z) = 2\pi \left(\frac{e}{m}\right)^2 \frac{\Delta|E(\omega, k_0(\omega))|^2}{\Delta\omega} \exp[-2k_i(z-2l+l_1)], \qquad (3.8)$$

where $\Delta\, |E(\omega,k)|^2/\Delta\omega$ is the spectral density of the noise at the lowest mode, and k_i is the Landau damping factor for this mode. The noise source, which produced a noise band 20 MHz wide around the generator frequency of 160 MHz, was a probe near the transmitter. It follows from (3.7) and (3.8) that the damping factor depends on both the noise power and the position of the noise-generating probe, in agreement with the experimental data in Fig. 10; this figure shows the logarithm of the echo amplitude A'_E (at the maximum) as a function of both the noise power (at a fixed l_1) and the position of the noise source (at a fixed noise power). Moeller also measured the dependence of $\ln(A_E/l^2)$ on l^3 (A_E is the echo amplitude when the diffusion is caused by the background

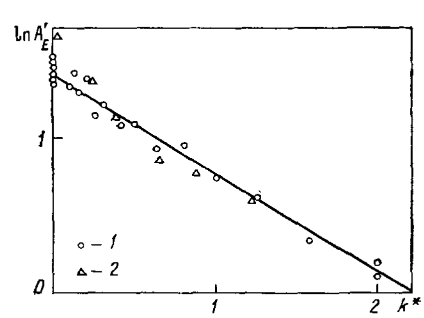

FIG. 10. Logarithmic k^* dependence of the amplitude of the second-order echo, A'_E ($A'_E = A_E/l$) (Ref. 87). Line—theoretical; 1—noise power is varied, $l_1 = 40$ cm; 2—l_1 is varied from 10 to 40 cm, and the noise power is held constant.

noise in the plasma). Figure 11 shows the results of these measurements. From the slope of this line, the absolute value of the diffusion coefficient $D_2(v)$ was found; the result was found to agree approximately with (3.6).

In deriving (3.4), Moeller[67] and O'Neil[85] assumed that the echo was determined primarily by the poles of $v = \omega_3/k$, i.e., by resonant electrons. O'Neil[85] evaluated the echo amplitude near the point of maximum intensity by the method of steepest descent, while at the periphery he used the pole approximation. As a result, the echo should become sharper with increasing l. In the experiment of Ref. 8, however, the echo amplitude decreased with increasing l, and the echo became flatter (rather than sharper). Nichikawa and Gould[91] carried out some corresponding numerical calculations, which yielded results in agreement with the experimental data of Ref. 8. It was also found that the echo maximum varies $\sim\exp(-l^{1/4})$, and this result was confirmed experimentally in Ref. 93. Further experiments on the effect of collisions on the spatial echo were carried out in Ref. 66.

Agreement has thus been found between the theoretical and experimental results on the effect of collisions on the echo shape and on the velocity dependence and absolute value of the diffusion coefficient.

Dryakhlushin and Romanov[95] have proposed an interesting method for determining the distribution function of the plasma particles: by working from the spatial structure of the echo field in an unbounded plasma and the frequency dependence of the amplitude of a transmitted wave in a homogeneous plasma slab.

Éĭdman[96] has studied the echo in a plasma with moving sources. Nemtsov and Éĭdman[97] have pointed out that Van Kampen waves can produce a localized echo field pulse which moves at a velocity $v = l/\tau$, where l and τ are respectively the spatial and temporal separations of the external perturbations.

CONCLUSION

We have attempted to demonstrate not only the variety of forms taken by plasma echos in various plasma media—laboratory plasmas, plasmas in space, metals, and semiconductors—but also the clear benefit to be gained from studying echos: for reaching a better understanding of collective processes in plasmas and for several practical applications in radio propagation and plasma diagnostics.

The foremost point to be made here is that the echo

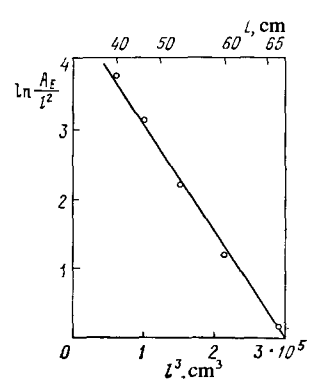

FIG. 11. Dependence of $\ln A_E/l^2$ on l^3. Here A_E is the echo amplitude, and l is the distance between the generator and the reflecting plate.[87]

phenomenon confirms the reversible nature of Landau damping and confirms that information about a perturbation is preserved in a plasma (in the form of nondecaying oscillations of the distribution function) even after the disappearance of the macroscopic field.[5–9]

Important results have been obtained through the use of echos for plasma diagnostics. Specifically, the velocity dependence of the diffusion coefficient has been measured, and microscopic turbulence in a plasma has been studied.[67–70,86]

The echo effect has stimulated the suggestion of some new methods for wave conversion in plasmas[19,48,58] and for the "brightening" of transcritical plasma slabs and opaque regions in plasmas.[24–27,20–23]

Some extremely promising suggestions for new experiments are to use echos to study the mechanisms for the interaction of charged particles with surfaces bounding plasmas (insulators and magnetic walls),[72–78,81] to study the reflection of electrons from a metal boundary[28–30] (since measurements of the surface impedance are relatively insensitive to the nature of the reflection of the particles—specular or diffuse—the echo method is unique here, since the echo amplitude is extremely sensitive to the nature of the particle reflection), and to study the band structure of semiconductors and metals.[29,52]

Some extremely interesting and promising suggestions have been made regarding the use of echos to study distribution functions[95] and the use of trapped-particle echos to study turbulence in fusion-research devices.[68–70]

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