True quantum-mechanical macroscopic effects in weak superconductivity

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CONTENTS

1. INTRODUCTION

There are at least three experiments[1-3] that were performed in 1981 with superconducting tunnel junctions, in which the quantum-mechanical behavior of one of the degrees of freedom of a macroscopic system has been reliably observed. The aim of this note is to review these experiments and discuss briefly further investigations and applications of the phenomena thus discovered.

The possible observation of quantum-mechanical behavior of macroscopic bodies has frequently been discussed since the very beginning of quantum-mechanics (see for example, Refs. 4 and 5). This discussion usually involves the dynamics of one of the enormous number of degrees of freedom of a macroscopic body. As an elementary example, which will turn out to be useful in our further discussion, let us consider a two-dimensional physical pendulum in the form of a solid body with a fixed axis in the gravitational field. Its mechanical motion as a whole can be characterized by a generalized coordinate φ—the angular deflection from the position of equilibrium, and the corresponding generalized momentum—the angular momentum M.

Although the motion of microparticles making up the given body can be essentially quantum-mechanical, and φ is a function of the coordinates in these particles, the variation of φ in time can be described with adequate precision by classical mechanics under normal conditions. Nevertheless, under certain conditions quantum-mechanical effects may have to be taken into account even in the dynamics of this degree of freedom. In fact, it follows from the commutation relation for the operators representing the conjugate variables φ and M

$$[\varphi, M] = i\hbar, \quad (1)$$

that these two quantities cannot be determined simultaneously with perfect precision. Because of this, the center of mass of the pendulum cannot be in a state of rest ($\varphi=0$, $M=0$) even at zero temperature T. In other words, in addition to thermal fluctuations, the center of mass must exhibit fluctuations of quantum-mechanical origin.[6,7]

The existence of such quantum-mechanical effects in macroscopic systems has long been seen by the majority of physicists as necessary,[4-10] despite the absence (as far as the present author is aware) of direct experimental evidence.[1-3] Nevertheless, the question of an acceptable description of such phenomena has given rise to considerable dispute. The essence of the discussion can be formulated as follows. Is it sufficient to perform the usual quantization of motion for a given degree of freedom without taking into account the presence in the macroscopic body of an enormous number of other (internal) modes? Thus, in the example of the pendulum, is it sufficient to use the usual Hamiltonian function.

$$H=\frac{M^2}{2J}+E_C(1-\cos\varphi) \quad (2)$$

where E_C is the amplitude of the potential energy and J is the moment of inertia, with φ and M being regarded as operators subject to (1)?

The source of doubt can be traced to the two fundamental differences between macrosystems and microsystems:

(1) The parameters of a macrosystem (for example, E_C and J) are averages over the motion of the component microparticles.

(2) An isolated degree of freedom may be coupled to internal modes, which may give rise, for example, to damping (friction).

Experiments[1-3] have given an affirmative answer to the above question. They have confirmed that, in the limit of negligible damping, an acceptable description of the quantum-dynamics of macroscopic bodies is given directly by quantizing the classical equations of motion of the given degree of freedom and, when damping is appreciable, it can be correctly described by the corresponding methods of quantum statistics.[6,7,11,12]

The experiments alluded to above have become possible as a result of the use of unique objects, namely, superconducting tunnel junctions. However, the exploitation of the phenomenon of superconductivity has given rise to certain difficultes in the interpretation of experi-

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ments, and even to a degree of confusion. Let us consider this question in more detail.

2. TWO TYPES OF QUANTUM-MECHANICAL MACROSCOPIC EFFECTS IN SUPERCONDUCTIVITY

The point is that, in addition to the "true quantum-mechanical" macroscopic effects that we have considered, it has been well-known since the early 1960's that there are other effects in superconductivity, which are usually referred to as either quantum-mechanical macroscopic effects or coherent effects (to avoid confusion, we shall use the latter term).

The best known coherent phenomena in superconductors are the quantization of magnetic flux and the Josephson effect (see, for example, Refs. 13 and 14). The essence of these effects is that certain macroscopic quantities characterizing superconducting systems are connected by relationships that follow directly from the fundamentals of quantum mechanics and explicitly contain Planck's constant. For example, the electric current I flowing across a weak contact between two superconductors (the Josephson junction) contains a specific component, namely, the supercurrent I_S, which is a periodic function

$$I_S = I_C \sin\varphi, \quad \varphi = \chi_1 - \chi_2, \tag{3a}$$

of the phase difference $\chi_{1,2}$ between the wave functions $\psi_{1,2}$ describing the state of the condensate of Cooper pairs in the contacting superconductors. The dependence of φ on the voltage V across the junction is then given by

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V, \tag{3b}$$

which follows directly[13,14] from the Schroedinger equation.

Real Josephson junctions exhibit a considerable deviation from (3a) (see, for example, the review given in Ref. 15). Moreover, the current I may contain a number of other components. More than that, the unusual relationship, given by (3), between the current and the electromagnetic field is responsible for the very complex and specific electrodynamics of Josephson junctions and structures containing them.[13-16] A total of about 4000 publications[17] has been devoted during the last two decades to the investigation of all these problems and to the practical application of weak superconductivity. Nevertheless, the above principle of describing coherent effects has been found to be completely satisfactory in practically all cases.

Let us now note the fundamental difference between true quantum-mechanical and coherent effects. In the latter case, the formulas given by (3) show that all the quantities referring to a given degree of freedom (φ, V, I, $Q = \int I\, dt$, and so on) can have precise values at the same time. In other words, despite the quantum-mechanical nature of (3), the quantities involved in these relationships are, in fact, classical variables and, in that sense, coherent effects can be regarded as classical.

The latter conclusion becomes even more obvious if we rewrite (3) in terms of energy: the presence of the supercurrent I_S in the junction is equivalent[13-16] to the presence of the additional binding energy[1)]

$$U_i = E_C(1 - \cos\varphi) + \text{const}, \quad E_C = \frac{\hbar}{2e} I_C. \tag{4}$$

Coherent effects are thus seen to produce an additional "potential" energy U_i (φ) for the motion of the system along the coordinate φ, but the motion itself may exhibit completely classical behavior.

To summarize, we may say that coherent effects are none other than the coherent quantum-mechanical motion of microparticles (for example, Cooper pairs in a superconductor). At the same time, true quantum-mechanical effects consist of the quantum-mechanical motion of the macroscopic object as a whole.[2)]

We now have the natural question as to whether these two types of quantum-mechanical effect can coexist in a given system, for example, in a Josephson junction. Despite the fact that the possibility of small quantum-mechanical fluctations in such junctions has been considered for a relatively long time (for example, see Refs. 18–22, 52, and 55), it is only recent work that has given us a sufficiently clear understanding of the conditions necessary for the manifestation of true quantum-mechanical macroscopic effects in superconductivity.

As an example, let us consider a single Josephson function. If we take into account not only the "potential" energy given by (4) but also the "kinetic" energy of the electric field

$$T = \frac{Q^2}{2C} = \frac{M^2}{2J}, \quad J = \left(\frac{\hbar}{2e}\right)^2 C, \tag{5}$$

where C is the junction capacitance, we find that the Hamilton function again is given by (2), with the angular momentum M now given by

$$M = J\dot{\varphi} = \left(\frac{\hbar}{2e}\right)^2 C\dot{\varphi} = \frac{Q\hbar}{2e}, \tag{6}$$

which is proportional to the imbalance Q between the electric charges in the superconductors forming the Josephson junction.

According to the approach described in Section 1, to achieve the transition to the quantum-mechanical description, it is sufficient to use the Hamiltonian given by (2), with φ and M looked upon as noncommutating operators (1).[3)] The only "trace" of coherent effects is then the specific form of the function U_i (φ).

[1)] This can be easily verified by calculating ΔU_i as the sum of elementary amounts of work $I_S V dt$ when the phase varies slowly:

$$\Delta U_i = \int_1^2 I_S V\, dt = \frac{\hbar}{2e}\int_1^2 I_S\, d\varphi = -\frac{\hbar}{2e} I_C [\cos\varphi]_1^2.$$

[2)] Leggett[29] introduced a quantitative criterion to distinguish between these effects.

[3)] Another way of writing Eq. (1)

$$[\varphi, n] = i, \quad n = \frac{Q}{2e} = \hbar M,$$

demonstrates even more clearly its physical meaning: the modulus of the commutator of the imbalance n in the numbers of Cooper pairs in the electrodes and the phase φ is equal to unity.

It follows from the analogy between the properties of the above system and the well-known properties of the two-dimensional pendulum that its quantum-mechanical properties will clearly appear when the ground-state energy becomes comparable with the characteristic energy E_C and, at the same time, is greater than the masking thermal energy kT:

$$\hbar\omega \gg kT,\ aE_C, \tag{7}$$

where a is a coefficient of the order of unity, ω is the classical frequency of small oscillations of the system around the position of equilibrium $\varphi = \varphi_0$ and

$$\omega^2 = \frac{k}{m}, \quad k = \frac{\partial^2 U}{\partial\varphi^2}\bigg|_{\varphi=\varphi_0}, \quad \frac{\partial U}{\partial\varphi}\bigg|_{\varphi=\varphi_0} = 0 \tag{8}$$

(in the present case, $\varphi_0 = 0$). In the theory of the Josephson effect, the latter frequency is usually referred to as the plasma frequency.

The situation may, however, become complicated by the fact that Josephson junctions can exhibit appreciable damping. The basic mechanism responsible for this damping is that the current flowing through the junction has an essential quasiparticle component I_Q which is a function of the voltage V across the junction, in contrast to I_S given by (3). In the simplest case, this function may be regarded as linear and free of dispersion:

$$I_Q = \frac{V}{R}, \tag{9}$$

where R is usually not very different from the resistance of the junction in its normal state. It is clear from (3) and (9) that, in the classical limit, the inclusion of this component leads to the following equations for small oscillations:

$$\omega^{-2}\ddot{\varphi} + \omega_C^{-1}\dot{\varphi} + \varphi = 0, \tag{10}$$

where ω_C is usually called the characteristic junction frequency and is given by

$$\omega_C = \frac{k}{\eta}, \quad \eta = \left(\frac{\hbar}{2e}\right)^2 R^{-1}. \tag{11}$$

The ratio of the two frequencies ω and ω_C characterizes the rate of damping and is very dependent on the type of Josephson junction. For tunnel junctions,[14] the ratio ω/ω_C is usually in the range 0.1–0.001, the damping is small, and the condition for the transition to the quantum mechanical state is, as before, given by (7). Conversely, for junctions with indirect (non-tunneling) conductivity and in the form of point contacts or thin-film microbridges,[15] we have the reverse situation: damping is large and $\omega/\omega_C \gg 1$. There is as yet no rigorous quantum-mechanical theory of this case, but qualitative analysis[23,24] shows that, in most formulas, it is sufficient to replace ω by ω_C with a coefficient of the order of unity.

Bearing this in mind, we can readily write out the final condition for the appearance of true quantum-mechanical macroscopic effects:

$$\min\,[\hbar\omega,\ \hbar\omega_C] \gg kT,\ aE_C. \tag{12}$$

This condition clearly shows why true quantum-mechanical macroscopic effects have been successfully observed in Josephson tunnel junctions. It is precisely these junctions that retain the strong nonlinearity at helium temperatures ($kT \simeq 10^{-15}$ erg), i.e., they have a small E_C (thus, for a junction with $I_C \approx 30\ \mu$A, Eq. (4) shows that $E_C \approx 10^{-14}$ erg). Moreover, the frequencies involved in (12) can be relatively high. Thus, with present tunnel junction technology (cf. for example, the review given in Ref. 25), the critical current densities can be up to about 10^5 A/cm^2 or more, which corresponds to plasma frequencies $\omega \gtrsim 10^{13}$ s^{-1}. At the same time, the characteristic frequency ω_C remains close to the theoretical limit,[14] $\pi\Delta(0)/\hbar$, where $\Delta(T)$ is the energy gap, which for typical superconductors (Pb, Nb) is about 10^{13} s^{-1}. It is thus clear that the value of $\hbar\omega_C$ for modern tunnel junctions can be taken up to about 10^{-14} erg, so that both conditions in (12) are satisfied.

3. MACROSCOPIC QUANTUM-MECHANICAL TUNNELING

We now proceed to the analysis of specific experiments. We note at once that, even when the conditions given by (12) are satisfied, the detection of true quantum-mechanical macroscopic effects is not a trivial problem. In fact, direct measurement of quantum-mechanical fluctuations encounters a fundamental difficulty because any measuring device is also subject to such fluctuations (zero-point fluctuations) (see, for example, Refs. 26 and 27). We can escape from this dilemma by exploiting the internal nonlinearity of the system under investigation so as to "amplify" the quantum-mechanical fluctuations in one way or another, i.e., use them to stimulate some process that occurs with relatively high intensity. When this intensity reaches the "classical" level, the process can be examined with conventional devices without taking into account their quantum-mechanical properties.

The first such possibility was implemented in the experiments reported in Refs. 1 and 2, and was specifically put forward in 1978 by Leggett[22] (see also Ref. 29), although the basic possibility of quantum-mechanical measurements of this type was frequently considered by Blokhintsev (see, for example, Lecture 12 in Ref. 30). Suppose that a Josephson junction is connected to a simple external circuit in the form of a source of direct current I, just below the critical current for the junction (Fig. 1a). The Hamiltonian for the system must then also include the term U_e which is equal to the product of the generalized coordinate φ and the corresponding generalized force

$$F_\varphi = \frac{\partial U}{\partial\varphi} = \frac{\hbar}{2e} I, \tag{13}$$

so that the resultant potential $U = U_1 + U_e$ has the "washboard" form (Fig. 1a):

$$U = E_C\left(1 - \cos\varphi - \frac{I}{I_C}\varphi\right), \tag{14}$$

where the local minima occur at the points

$$\varphi_0 = \arcsin\frac{I}{I_C} + 2\pi k, \quad k = 0,\ \pm 1,\ \pm 2,\ \ldots, \tag{15}$$

and correspond to "classically" stable states of the phase difference, i.e., the usual superconducting state of the Josephson junction.[13-16]

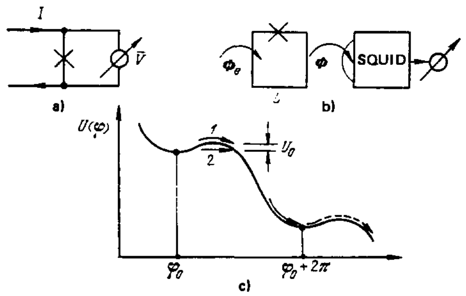

FIG. 1. Principles of experiments on macroscopic quantum-mechanical tunneling in Josephson junctions with small (a) and large (b) damping, and dependence of potential energy U on the Josephson phase difference φ in these experiments (c). Arrows on the last figure indicate schematically the mechanism of the decay of the metastable state $\varphi \simeq \varphi_0$: thermal activation over the barrier U_0 (1) and quantum-mechanical tunneling under the barrier (2).

It is clear that the thermally activated decay of this metastable state will have a finite probability at a finite temperature T. Classcial theory[31,32] (see also Chapter 8 in Ref. 16) gives the following expression for the rate of this decay:[4)]

$$\tau_{cl}^{-1} = \frac{\omega_A}{2\pi}\exp\left(-\frac{U_0}{kT}\right), \tag{16}$$

where U_0 is the height of the energy barrier (Fig. 1) and the "attempt frequency" ω_A is approximately (exactly, in the limit of small and large damping) equal to $\min|\omega, \omega_C|$. The frequencies ω and ω_C are again given by (8) and (11), but now with allowance for the contribution of the current I to the potential U in (14). In the most important case $I \to I_C$, we have

$$\omega = \left(\frac{2eI}{\hbar c}\right)^{1/2}(2\varepsilon)^{1/4}, \quad \omega_C = \left(\frac{2eI_C R}{\hbar}\right)(2\varepsilon)^{1/2}, \tag{17}$$

$$U_0 = \frac{2}{3}E_C(2\varepsilon)^{3/2}, \quad \varepsilon \equiv \left(\frac{I}{I_C}\right) - 1 \ll 1, \tag{18}$$

so that, as $I \to I_C$, the height of the barrier decreases and the rate of decay increases.

The possibility of detection of this type of decay is very dependent on damping in the junction.[16] When this damping is small enough, so that

$$\frac{\omega}{\omega_C} \leqslant (2\varepsilon)^{-1/2}, \tag{19}$$

finite inertial will ensure that the system will not stay at the next minimum, but will slide down the "washboard" and gradually acquire a velocity $\dot{\varphi}$ until the average velocity $\bar{\dot{\varphi}}$ reaches a constant value because of increasing dissipation. From the point of view of the observer, this process corresponds to a jump from the superconducting state of the junction ($\bar{\dot{\varphi}} = \bar{V} = 0$) to the "resistive" state with sufficiently large potential difference $\bar{V} = (\hbar/2e)\,\bar{\dot{\varphi}}$, usually of the order of a few millivolts. A potential difference of this magnitude can easily be determined by a classical device because it substantially exceeds the level of not only quantum-mechanical but also thermal fluctuations in practical voltmeters. It is precisely in this way that the thermally activated decay of the metastable states of Josephson junctions was first measured[13] in 1974, and good agreement was achieved between the decay rate τ^{-1} and (16) (for $\omega/\omega_C \ll 1$).

On the other hand, when the condition opposite to that given by (19) is satisfied, damping will confine the system to the nearest local minimum $\varphi_1 = \varphi_0 + 2p$, which again corresponds to the superconducting state. The resulting short ($\sim\omega_C^{-1} \sim 10^{-12}$ s) and weak ($\int U dt \approx h/2e \sim 10^{-15}$ V.s) voltage pulse is very difficult to measure. A different path is more successful: the Josephson junction is included as part of a superconducting ring with high enough inductance $L \gtrsim (\hbar/2e)I_C^{-1}$ (Fig. 1b). A current I can then be established by applying an external magnetic field B_e to the system ("single-contact interferometer"[14,16]):

$$I \approx \frac{\Phi_e}{L}, \quad \Phi_e = \int B_e\, dS. \tag{20}$$

The jump of the system to the neighboring metastable state ($\Delta\varphi \approx 2\pi$) then leads to small jumps in the magnetic flux and the current:

$$\Delta I \approx \frac{\Delta\Phi}{L}, \quad \Delta\Phi \approx \Phi_0 = \frac{h}{2e}, \tag{21}$$

which are readily detectable, for example,[37] by a superconducting quantum-mechanical interference device ("squid") using, by the way, a similar Josephson interferometer as the sensor.[5)] These measurements, also performed[34] in 1974 have demonstrated good agreement with (16) for junctions with high damping ($\omega/\omega_C \gg 1$).

If we now reduce the temperature and thus increase τ_{cl}, the true quantum-mechanical effects will ensure that the lifetime of the metastable state will not increase without limit, but will reach a certain constant τ_{qm} as $T \to 0$. In fact, quantum-mechanics shows that there should be a finite probability that the "particle" (in the present case, the macroscopic system) will succeed in passing under the energy barrier U_0. Simple calculations based on the principles indicated above show that the rate of this "macroscopic quantum-mechanical tunneling" is given by the following expression[35] at $T = 0$:

$$\tau_{qm}^{-1} = \frac{\omega}{2\pi}\left(864\pi\frac{U_0}{\hbar\omega}\right)^{1/2}\exp\left(-\frac{36}{5}\frac{U_0}{\hbar\omega}\right), \tag{22}$$

which is valid for systems with small damping ($\omega/\omega_C \ll 1$). We note that the use of the WKB approximation to solve this problem[2,36,52] leads to an incorrect pre-exponential factor; this coefficient was erroneously omitted from formula (8) in Ref. 35.

The transition from the classical limit (16) to the quantum-mechanical limit (22) can also be easily performed. To do this, it is sufficient to sum the decay intensities due to all the thermally excited energy states of the system in the region of the metastable state. The result of this type of calculation[35] is illustrated in Fig. 2, from which it is clear that the transi-

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4) This formula, as well as all the subsequent formulas for the lifetime τ, is valid under the usual condition $\omega_A\tau \gg 1$, in the absence of which the very meaning of the concept of lifetime is ambiguous.

5) Another, even a more convenient, method consists of measuring such spontaneous sudden changes in the magnetic flux in the interferometer of the squid itself utilizing the fine structure of the plateau in its r.f. characteristic (cf. Ref. 16, Ch. 14, and also Refs. 38, 51).

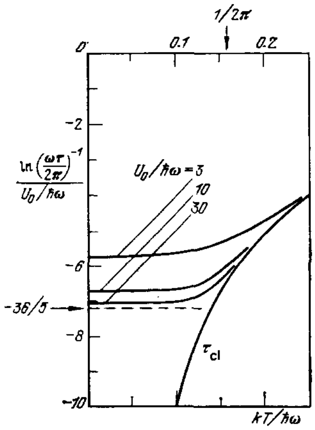

FIG. 2. Lifetime τ of the metastable "superconducting" state $\varphi \simeq \varphi_0$ as a function of the temperature T for junctions with small damping and different values of the ratio of the barrier height U_0 to the energy $\hbar\omega/2$.

tion from thermal activation to macroscopic quantum-mechanical tunneling is continuous but relatively rapid, and occurs at temperature T_0 given by

$$kT_0 \approx \frac{\hbar\omega}{2\pi}. \tag{23}$$

The situtation in the case of appreciable damping is much less clear. Caldeira and Leggett have shown[23] that a small ($\omega/\omega_C \ll 1$) linear dispersion-free damping introduces an additional exponential into the expression given by (22), namely,

$$\exp\left(-2\pi\alpha\frac{U_0}{\hbar\omega}\right), \quad \alpha = \frac{81}{\pi^4}\zeta(3) \approx 1. \tag{24}$$

Qualitative analysis shows[23,24] that, in the case of arbitrary damping, the lifetime τ_{qm} is given by the same formula:

$$\ln\left(\frac{\omega_A \tau_{qm}}{2\pi}\right)^{-1} \sim \exp\left(-2\pi\frac{U_0}{\hbar\omega}\right)\exp\left(-2\pi\frac{U_0}{\hbar\omega_C}\right), \tag{25}$$

but the question as to what is the preexponential factor and what are the precise coefficients in the arguments of the exponentials (25) remains open.[6)]

All the above expressions were obtained back in 1980, but the validity of the simple idea on which they are based (see Section 1) has frequently been disputed. Some authors consider that, when macroscopic quantum-mechanical effects are calculated, it is necessary to take into account, in one way or another, the effect of internal degrees of freedom (which is different from damping). In other words, it may be that the contribution of coherent effects to the true quantum) mechanical effects do not simply reduce to the appearance of the effective potential $U_i(\varphi)$ of the form given by (3), and that the entire quantum-mechanical calculation must be performed "from first principles," i.e., at the microscopic level. The early experiments in this field[37,38] did not make the situation any clearer, since they did not lead to any definite conclusions.

These doubts were, however, removed by two striking experiments performed at the Bell[1] and IBM[2] laboratories. In both experiments, Josephson tunnel junctions with relatively small damping were connected to a source of current I, and the current was gradually increased until the voltmeter connected across the junction recorded the spontaneous jump to the resistive state. The corresponding current was recorded and then reduced to zero (to take the Josephson junction back to the superconducting state), and the cycle was repeated. This experiment was repeated many times, and the result was the distribution of the jump probability density w along the current axis, which took the form of a peak lying a little below I_C. From this distribution using the formula[7)]

$$w(I) \equiv \frac{dp}{dI} = \left(\frac{dI}{dt}\right)^{-1}\tau^{-1}(I)(1-p) \tag{26}$$

one can readily establish the lifetime of the metastable superconducting state τ as a function of the current I, and then compare the result with the above formulas taken together with (18).

Although the experiments reported in Refs. 1 and 2 are basically similar, the conditions given by (12) were satisfied in different ways. The IBM group used Nb-Nb tunnel junctions with relatively low current density (about 10^1–10^2 A/cm^2) and relatively high specific capacitance (about 10^{-5} F/cm^2). The plasma frequencies of these junctions were relatively low, of the order of 3×10^{11}–10^{12} s^{-1}, so that they had to be cooled to temperatures of the order of 30–100 mK to reach the quantum threshold (23). This was done by placing the junction directly in the mixing chamber of a helium refrigerator, which meant that the measurements could be performed at temperatures down to 3 mK (!). The technical level for this experiment is also indicated by the following figure: the width of the peak in $w(I)$,[7)] which is given by

$$\sigma = \langle (I - \langle I \rangle)^2 \rangle^{1/2} \tag{27}$$

reached 3 nA at low temperatures, and it was shown that external pick-up had no appreciable effect even under these conditions (anyone working with Josephson junctions will know how difficult it is to reduce the pick-up level even down to a few hundred nanoamperes).

Figure 3a illustrates the basic result of this experiment, namely, the dependence of σ on temperature for a junction with $I_C \simeq 1.6\ \mu A$, $R \simeq 530\ \Omega$, $C = 100$ fF. It is clear that, for $T \lesssim 1000$ mK, σ (and, therefore, the lifetime τ) is a constant. However, this constant agrees with (22) only for junctions with relatively low current density, i.e. small damping ($\omega/\omega_C \sim 0.01$), whereas for junctions with $\omega/\omega_C \simeq 0.2$, the argument in the exponential has to be increased by a factor of two to achieve comparable agreement with this formula. This difference could be explained by the influence of damping if the coefficient α in (24) were of the order of five rather than unity. The origin of this discrepancy is

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6) Recently Golub and Iordatii[53] obtained estimates in agreement with formula (25).

7) In order to obtain it, it suffices to take into account the fact that the probability p of the jump having occurred increases with time according to $\dot{p} = \tau^{-1}(I)(1-p)$.

8) The angle brackets denote statistical averaging

$$\langle \ldots \rangle = \int (\ldots)\, w(I)\, dI.$$

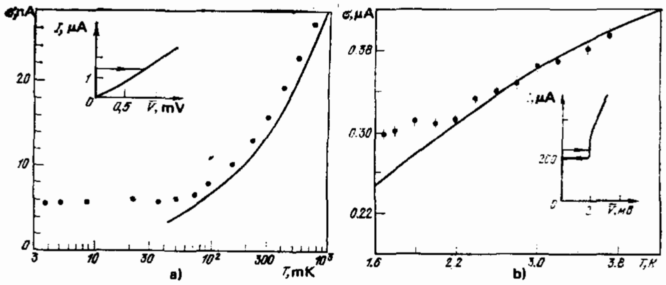

FIG. 3. Width σ of the probability distribution of jumps to the resistive state as a function of temperature in the experiments performed by the IBM group[2] (a) and the Bell group[1] (b). Solid lines are calculated from classical theory [see Eq. (16)]. Inserts show the current-voltage characteristics of the Josephson junctions employed.

still not clear, but there is no doubt that macroscopic quantum) mechanical tunneling has now been detected (following the heated discussions during the Sixteenth International Conference on Low Temperature Physics in August 1981).

In contrast to the experiment just described, the Bell group used Pb(In)–Pb tunnel junctions with much higher current densities (up to 5×10^5 A/cm²) and somewhat lower specific capacitance (~0.6×10^{-5} F/cm²). Such junctions have very high plasma frequencies (~10^{13} s^{-1}), so that the experiments could be performed at "ordinary" helium temperatures, i.e., above 1 K. Figure 3b shows the function $\sigma(T)$ for a junction with $I_C\simeq 300\ \mu A$, $R\simeq 5\ \Omega$, and $C\simeq 10$ fF. It is clear that, below 2.2 K, the measured dependence deviates from the classical behavior. However, in this case, σ cannot be directly compared with the corresponding theoretical predictions for $T\to 0$. The point is that, in contrast to the niobium junctions used in Ref. 2, lead junctions[1] have highly nonlinear current-voltage characteristics (see the insert in Fig. 3), which shows that there is strong dispersion of damping in such junctions. Comparison with quantum-mechanical calculations performed in the dispersion-free model can therefore be only approximate under these conditions. The authors of Ref. 1 found that agreement with (22) and (24) could be achieved by assuming an effective resistance R between 15 and 35 Ω (normal resistance 5 Ω), which is not inconsistent with existing data on the impedance of tunnel junctions of this kind.

The conclusions that can be drawn from these two experiments[1,2] are thus seen to be very similar: the existence of macroscopic quantum-mechanical tunneling can be regarded as demonstrated, with high degree of confidence, but it is too early to conclude that there is quantative agreement with the theory.[9)] This quantitative comparison can be approached in two ways:

(1) The Caldeira-Leggett theory[23] can be generalized to the case of dispersion-free damping of arbitrary strength ($\omega/\omega_C\sim 1$), and the results can be compared with the data[2] on Nb–Nb junctions.

[9)] Very recently a communication[51] appeared concerning the observation of this effect in contacts with high damping (using a squid).

(2) The theory of macroscopic quantum-mechanical tunneling can be developed on the basis of the microscopic theory of the Josephson effect,[39,40] and the result can be compared with data on junctions made of metals other than transition metals, for example, Pb(In)–Pb.[10)]

Finally, we note that macroscopic quantum-mechanical tunneling can also be observed in superconducting structures of a different type. It can now be regarded as demonstrated that the low-temperature tail of the $R(T)$ curve of a resistive phase transition between sufficiently thin superconducting films is the consequence of the thermal activation of pairs of antipolar Abrikosov vortices (Refs. 41–43).[11)] The width of the transition can be estimated from the following expression which follows from the theory of Aslamazov and Larkin[44] for the high-temperature tail of the transition:

$$\Delta T\approx T_C\frac{R_\square}{R_{qm}}, \tag{28}$$

where $R\ =(\sigma_N d)^{-1}$ is the normal resistance "per square" of the film and $R_{qm}=4\hbar/e^2\simeq 16.5$ kΩ is the quantum-mechanical unit of resistance. It is natural to assume that the resistance of still thinner films $R\gtrsim R_{qm}$) should remain finite even at $T\simeq 0$ because pairs of vortices will be generated by quantum-mechancial fluctuations in the superconducting condensate. The relevant experiments have already begun[45] and, in all probability, an acceptable theory of this phenomenon will soon emerge. Distributed Josephson junctions and granular superconducting structures exhibit similar properties.

4. OBSERVATION OF QUANTUM-MECHANICAL CURRENT FLUCTUATIONS

Practically simultaneously with the experiments on the macroscopic quantum-mechanical tunneling, true quantum-mechanical macroscopic effects were observed under somewhat different conditions at the University of California (Berkeley). Let us again consider a Josephson junction with a given constant current I, but let us now suppose that $I>I_C$. We then have Josephson generation[13–16] in the junction with frequency ω_V, which, according to (3b), is proportional to the constant voltage across the junction.

$$\omega_V\equiv\bar{\dot\varphi}=\frac{2e}{\hbar}V. \tag{29}$$

Fluctuations in the phase φ, including quantum-mechanical fluctuations, contribute to the low-frequency fluctuations in the voltage across the junction, and these can be measured relatively simply.

The theory of such fluctuations in the voltage V has been developed in a series of papers[18–22,46] for different special cases. Let us consider the simple case of the Josephson junction with strong dispersion-free damping,

[10)] First steps in this direction were taken in Ref. 54.

[11)] Here it is necessary to note that even small inhomogeneties of the film can lead to the pinning of these vortices with the thermal activation of such pinning centers producing a considerable masking effect. Apparently it is just for this reason that quantitative agreement with the simple theory[41,42] was successfully achieved very recently utilizing very perfect films.[43]

so that the dynamics of the phase φ in the absence of fluctuations is described by (3) and (9):

$$I = I_C \sin\varphi + \frac{\hbar}{2eR}\dot{\varphi}. \tag{30}$$

Analysis of this "resistive" model shows[21] that the spectral density $S_V(\omega)$ for small fluctuations in the voltage at low frequencies $\omega \ll \omega_V$ is given by the simple expression

$$S_V(\omega) = R_d^2 \left\{ S_I(\omega) + \left(\frac{I_C}{2I}\right)^2 [S_I(\omega+\omega_V) + S_I(\omega-\omega_V)] \right\}, \tag{31}$$

where $R_d(I)$ is the differential resistance at the working point and $S_I(\omega)$ is the spectral density of the Langevin force (in this case, of the current I_f), which must be inserted into the right-hand side of (30) to achieve adequate description of the fluctuations. Generally speaking, this source, $I_f(t)$, is a complicated nonlinear combination of fluctuations of thermal, quantum-mechanical, and shot origin.[22,46] However, when the dissipative subsystem is in thermodynamic equilibrium, the fluction-dissipation theorem[6,7] is valid for I_f:

$$S_I(\omega) = \frac{2}{\pi R}\theta(\omega, T),$$
$$\theta = \frac{\hbar\omega}{2}\operatorname{cth}\frac{\hbar\omega}{2kT} = \frac{\hbar\omega}{2} + \hbar\omega\left[\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right]^{-1}. \tag{32}$$

In addition to the expected first term in braces, the formula given in (31) contains two further terms. In the language of radio engineering, their appearance is explained by the downward conversion of the fluctuation frequency, which occurs when they are mixed with intrinsic Josephson generation in the contact. The fundamental point is that the corresponding conversion coefficient $(I_C/2I)^2$ can be of the order of unity even for $\omega \ll \omega_V$, and this means that the conversion of high-frequency current fluctuations to low-frequency voltage fluctuations occurs with a very high ($\sim \omega_{V^2}/\omega^2$) quantum-mechanical gain. We note that this amplification is forbidden by the Manley-Rowe relations[47] and becomes possible only because of the specific nature of the nonlinearity of the Josephson junction.[14]

Let us now suppose that the temperature is low enough ($kT \lesssim \hbar\omega_V$) although, of course, $kT \gg \hbar\omega$. The formula given by (32) then provides us with a simple description of the quantum-mechanical fluctuations in the current $\theta(\omega_V, T) \approx \hbar\omega_V/2$, and (31) shows that these fluctuations may provide an essential contribution to the low-frequency fluctuations in the voltage. We note that, owing to the high quantum-mechanical gain mentioned above, we again need not consider quantum-mechanical fluctuations in the measuring instrument because their contribution at the frequency of the voltage measurements is proportional to $\hbar\omega/2$, which is much smaller than $\hbar\omega_V/2$.

Such measurements of low-frequency fluctuations in the voltage across the tunnel junction have, in fact, been performed.[3] To produce a system as close as possible to the resistive model (30), the Pb(In)–Pb tunnel junction with critical current $I_C \simeq 1.5$ mA and capacitance $C \simeq 0.7$ pF was shunted by an external resistor with $R \simeq 0.1\ \Omega$, which was much smaller than the internal junction resistance. The resistor was in the form of a thin (40–100 nm) Cu(Al) film, 10 μm wide and 5 μm long. The length L was such that[15,16]

$$L \gg \frac{e\bar{V}}{kT}\Lambda_Q, \tag{33}$$

which is a necessary condition for the equilibrium of the electron subsystem and, consequently, for the validity of the fluctuation-dissipation theorem (32). In the above expression, $\Lambda_Q \approx (D\tau_E)^{1/2}$ is the electron diffusion length corresponding to the energy relaxation time τ_E (in typical cases, $\Lambda_Q \lesssim 1\ \mu$m).

The experimental results are illustrated in Fig. 4, and it is clear that they are in good agreement with (31) and (32) when the experimental values of R_d (solid curve) are used. On the other hand, if we adopt the frequently expressed but poorly justified view that zero-point fluctuations are "fundamentally unobservable," and exclude the corresponding term $\hbar\omega/2$ from (32), the calculations (broken curve) turn out to be in clear conflict with the experimental results (full points).

Thus, the experiment described in Ref. 3 was the first to measure directly the quantum-mechanical fluctuations in an isolated degree of freedom of a macrosopic physical system. Detailed analysis of such measurements shows that they are possible (or, more precisely, that the influence of fluctuations in the measuring instrument can be neglected) only when certain definite conditions are satisfied. In particular, it clearly follows from the derivation of (31) by the method of slowly-varying phases (see Ref. 48, or section 6.2 of the monograph in Ref. 14) that $S_V(\omega)$ contains the contribution not of the entire Langevin force I_f but of only one of its quadrature components, which is in phase with Josephson generation by the contact. However, measurement of one of the quadrature components is, in fact, one of a class of "quantum nondemolition measurements"[8-10] for which the effect of fluctuations in the measuring instrument can, in principle, be made arbitrarily small. The experiments described in Ref. 3 are thus interesting from another standpoint as well, namely, in all probability, this is the first realization of quantum nondemolition measurements.[12)]

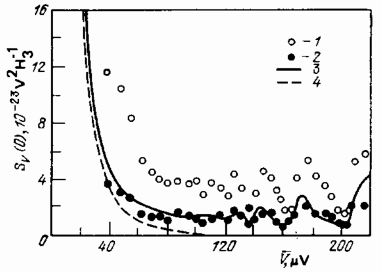

FIG. 4. Spectral density of low-frequency fluctuations in the voltage across a shunted tunnel junction as a function of the average voltage across the junction: 1—total density of fluctuations measured in the experiment described in Ref. 3; 2—measured fluctuations with the component $R_d^2 S_I(\omega)$ subtracted; 3—calculated values of the latter component based on (31) and (32); 4—the same calculation but with $\hbar\omega/2$ omitted from (32).

12) This remark is due to R. F. Giffard.

5. CONCLUSION

Thus, by exploiting the specifically nonlinear properties of the Josephson junctions, it has been possible[1-3] to demonstrate experimentally the presence in weak superconductivity of not only previously well-known quantum-mechancial macroscopic or "coherent" effects, but also "true quantum-mechanical" effects. In the more general context, these measurements constitute the first experimental demonstration of the quantum-mechanical behavior of an isolated degree of freedom of a macroscopic physical system.

It is possible that the existence of such effects is intuitively obvious to some people and does not, therefore, require experimental verification.

However, it seems to the present author that the observation of true macrocopic quantum-mechanical effects in superconductivity is a significant achievement, at least for the following reasons:

(1) The experiments described above lead us to the important realization that "all is well" with quantum mechanics even at the macroscopic level, i.e., the transition from classical to quantum-mechanical behavior in macro-objects occurs in precisely the same way as in micro-objects. This gives us confidence in planning actual quantum-mechanical experiments and (no less important) in considering thought experiments with macro-objects.

(2) Attempts to obtain quantitative agreement between experiment and theory have shown that there are certain basic gaps in our knowledge of even the simplest quantum-mechanical systems. Thus, as was noted in Section 3, the problem of quantum-mechanical tunneling in highly-damped systems has not as yet been solved. Generalizing a little, we may say that, in all probability, we do not know the simple equations describing the quantum-mechanical evolution of a one-dimensional Brownian particle without inertia in a potential $U(\varphi)$. In other words, it turns out that we do not know how to generalize the Fokker-Planck equation to the quantum-mechanical case.

(3) Finally, experiments[1-3] have shown that the Josephson tunnel junctions fabricated by modern technology (i.e., having high current densities) are the most suitable objects for experiments with macro-objects at the quantum-mechanical level. There are at least two groups of such experiments that are of interest at present.

First, we face the real possibility of being able to realize the recently proposed[9,10] schemes for the optimum detection of extremely weak signals and, as a consequence, we may well be able to overcome the so-called "quantum-mechanical limit" of the sensitivity[26,49] of detecting systems in a broad frequency range, extending from a few tens of kilohertz to hundres of gigahertz.

Secondly, it is now possible to formulate experiments designed to verify the fundamental assumptions of the quantum theory of measurement. The point is that this theory was developed in the early days of quantum mechanics[4,5] and was essentially designed for measurements of the parameters of micro-objects. The quantum-mechanical analysis of measuring devices is then unrealistic because of their complexity, and this has been the reason for the sharp differentiation between the object and the measuring device in the theory of measurement.

The situation is radically altered by the advent of macro-objects with quantum-mechanical behavior. For example, experiments on the macroscopic quantum-mechanical tunneling constitute an obvious realization of the famous "Schroedinger cat" paradox[50] (see also Ref. 4, Chapter 11). However, in this experiment, the "measuring device" is, in fact, not the voltmeter but the Josephson junction itself. Its nonlinearity leads to an increase in the signal to a level such that the realization of one of the quantum-mechanical alternatives (whether or not tunneling has taken place up to a given instant of time) can be recorded by a very crude external device which is definitely known to have no influence on the dynamics of the processes involved. It thus becomes possible to achieve a complete quantum-mechanical description of the system consisting of the object plus the measuring device.

It may well be that actual or thought experiments performed with such systems will throw new light on the fundamentals of quantum mechanics.

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