The large-scale structure of the universe

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Usp. Fiz. Nauk **139**, 83–134 (January 1983)

A survey is given of theories for the origin of large-scale structure in the universe: clusters and superclusters of galaxies, and vast black regions practically devoid of galaxies. Special attention is paid to the theory of a neutrino-dominated universe—a cosmology in which electron neutrinos with a rest mass of a few tens of electron volts would contribute the bulk of the mean density. The evolution of small perturbations is discussed, and estimates are made for the temperature anisotropy of the microwave background radiation on various angular scales. The nonlinear stage in the evolution of smooth irrotational perturbations in a low-pressure medium is described in detail. Numerical experiments simulating large-scale structure formation processes are discussed, as well as their interpretation in the context of catastrophe theory.

PACS numbers: 98.50.Kg, 98.50.Tq, 98.80.Bp

CONTENTS

INTRODUCTION

There has been much talk of late regarding the large-scale structure of the universe. Just in the past six years the subject has been reviewed three times; two Soviet national conferences have been devoted to it, as well as two international symposia. Everyone seems to be thinking about it in light of the massive-neutrino problem. When regions devoid of galaxies were discovered recently, there was a new upsurge of interest in these matters, with discussions not merely in scientific journals but in popular magazines and even in the daily press. And two books have been published, both bearing the same title as our present review: one a monograph by Peebles,[1] in 1980, and the other the proceedings of an International Astronomical Union symposium[2] that was held at Tallin in September 1977.

a) The status of cosmology today

Over the past 10 or 15 years cosmology has developed into a branch of physics and astronomy in its own right, concerned with the evolution of the universe as a whole, with the questions surrounding the early stages of the universe—theories of the processes that occurred close to the singularity, and finally, with problems of the origin and evolution of global structure in the universe and structure on the scale of galaxies and clusters of galaxies.

In coping with these problems cosmologists rely both on major advances made in observational extragalactic astronomy, and on progress in fundamental physical theory. At one time, up to the early 1960s, cosmology was dominated by applications of the general theory of relativity, which had been founded not so very long before—theories of spacetime and gravitation. A fundamental principle was laid down during that period: the universe was established to be globally homogeneous, and was discovered to be expanding isotropically. Influenced by the pioneering work of Alexander Friedmann and Edwin Hubble, cosmologists occupied themselves mainly with studying various global models of the universe. In the cosmological context, general relativity for the first time came to the fore not simply as a theory of gravitation offering minor adjustments to Newtonian theory, but as a road toward the radically new concept of a finite, closed (if unbounded) world. General relativity provides a logically faultless capability for analyzing the dynamics of an infinite universe.

Once Penzias and Wilson had made their 1965 discovery[3] of the microwave background radiation, which confirmed George Gamow's idea[4] of a big bang model uni-

 0038-5670/83/010046-31$01.80

verse and essentially marked the birth of physical cosmology, success began to be achieved, on the one hand, in the theory of processes near the singularity, describing the very early steps in the evolution of the "standard" model universe, and on the other, in the evolutionary theory for the large-scale structure of the universe.

At the moment, the new ideas being developed in cosmology stem chiefly from the rapid strides in elementary-particle and quantized-field theory (the two are synonymous!). For many years it has been felt that the task facing physics is to describe nature on the basis of specified properties inherent in some minimal family of particles. Protons and electrons had been regarded as making up that family; then quantum theory led to the concept of photons as particles, and nuclear physics discovered the neutron. Studies of β decay enriched physics with some new particles: the neutrinos. The theory of nuclear forces gave rise to the idea of mesons as quanta of the corresponding field. Finally, antiparticle theory nearly doubled the family of particles, but in doing so it restored the symmetry between positive and negative electric charge.

The inspiring goal of describing nature with maximum economy by minimizing the number of particles has been remarkably fruitful. The whole of chemistry, biology, solid state physics, plasma physics, and many other branches of natural science are firmly grounded on this principle. Yet high-energy physics, cosmic-ray research, and especially experiments with powerful accelerators have soundly refuted the "economic minimum" approach as a principle underlying all science. As early as the 1940s, unnecessary extra particles were being discovered—the muon and the strange particles (the Δ, Σ, Ξ hyperons, the K meson). The number of particles began to climb at a menacing tempo. "Let's discover a new elementary particle this fall!" So went the slogan from the wall newspaper in the film "Nine days of one year," and you might think it sounds ironic; but if you remember that in 30 years some 300 particles have been discovered, why, you can say that the plan was overfulfilled!

Under the impact of experimental accomplishments such as this, the theorists had to change their tune too. Theories sprang up that organized the particles into subfamilies; there were theories of particle symmetry and supersymmetry. Renormalization conditions led to the idea of spontaneous symmetry breaking, and new types of fields, the Higgs fields, were predicted.

And there was the notion of subparticles—the quarks. Particle physicists introduced 15 or 18 species of quarks, with just as many antiparticles, and an enormous variety of strongly interacting particles—the hadrons. The baryons—the proton and neutron, no less—turned out to be composite, made up of three quarks; the antiparticles corresponding to them consisted of three antiquarks; and the mesons combined a quark with an antiquark. The qualitatively new thing about this whole situation was that, as a matter of principle, it was not possible to detach and observe a single individual quark.

It was against this background that a major rethinking of the prevailing views and concepts in cosmology began to happen five or seven years ago. The theory of the hot, big bang universe, which has now gained general acceptance as the standard model, had been developed chiefly on "economic" precepts.[5,6] Only particles whose existence had been definitely established were considered. For simplicity, and for want of contrary evidence, the neutrino mass was taken to be zero. All particles (except perhaps for gravitons) were thought to have been in thermodynamic equilibrium at some early epoch. As the universe cooled, the massive exotic particles would have faded away without appreciably violating the equilibrium. By the time the temperature had dropped to about 10 MeV, the state of the universe would have become *independent* of its prior history—and that idea served as a firm basis for subsequent analysis.

But now these starting premises have to be reconsidered, to bring them into accord with the latest tenets of high-energy physics. Furthermore, in order to acquire information on the particles themselves, effective use has to be made of cosmology. To the scientist interested in elementary-particle physics, the early universe is a paradise, where he can explore processes taking place at energies that his accelerators cannot match.

Today the following problems are the ones being worked on most intensively in conjunction with cosmology.

1. The question of whether matter predominates over antimatter in the universe, that is, whether the universe is asymmetric with respect to substitution of antiparticles for particles. The story behind this question and its current status have recently been reviewed in these pages[7] and elsewhere,[6,8] and need not be repeated here. Processes in which the baryon charge is not conserved and symmetry is broken will have determined such important parameters of the universe as the ratio of the photon and baryon densities, $\overline{n}_\gamma/\overline{n}_b \approx 10^9-10^8$, a measure of the specific entropy of the universe.

2. The existence of a finite rest mass for the neutrino[9,10] or any of the other particles (such as the photino, gravitino, majoron, goldstino) which would interact with one another and with plasma only gravitationally. Estimates for the age of the universe place definite constraints on the neutrino mass[11]: $m_\nu < 200$ eV, while the cosmic helium abundance restricts the number of species of low-mass particles[12] to a level $n \lesssim 5-7$. These limits on the muon- and τ-type neutrino masses are in fact much sharper than all other estimates to date.

If the neutrino mass, averaged over the three species $(\nu_e, \nu_\mu, \nu_\tau)$, should exceed 20 eV, the density of the universe would be supercritical. Even if the neutrino mass were just 0.1 eV, the mean neutrino density $\overline{\rho}_\nu$ would exceed the density $\overline{\rho}_b$ of visible matter (stars, galaxies, and so on). We shall call a world in which $\overline{\rho}_\nu > \overline{\rho}_b$ a "neutrino universe."

3. Topics in the theory of processes near the singularity—very early phase transitions associated with Higgs fields, and the like.[13,14] In attempting to develop

a theory for the singularity and a theory of large-scale structure, "total" cosmological theories have been devised, wherein the very same factors would underlie elementary-particle physics and the laws governing the evolution of the universe as a whole, as well as the perturbations which we perceive today as large-scale structure in the universe.

In principle, there could have been two types of perturbations: a) unified fluctuations in the metric and the total density, as discussed many years ago by Lifshits and Khalatnikov[15,16]; b) a feature peculiar to the big bang model—fluctuations in the photon/baryon density ratio n_γ/n_b (or, in general, in the entropy) not initially accompanied by metric and total-density fluctuations.[17] To determine from general principles the large-scale asymptotic behavior of the fluctuations, their amplitude, and their spectrum is a most urgent task. It probably can be attacked successfully in terms of de Sitter cosmology, in which the expansion would occur exponentially.

At this point we come up against the question of what role very massive particles, particularly magnetic monopoles, might play in cosmology. Whether they actually exist, what their abundance would have been in the early universe, how actively they would have participated in the evolution process—all these questions have yet to be squarely faced.

4. To supplement this inventory we can add the eternal dilemma about the cosmological constant Λ. Introduced in 1917 by Albert Einstein to provide for a closed, steady-state model universe, the cosmological constant became superfluous following the work of Friedmann and Hubble. At one stage, not so long ago, it seemed that certain observations of quasars agreed better with models including the Λ-term.[18,19] In the context of quantum theory, the dilemma lies on a different plane: one has to explain why $\Lambda = 0$, or at any rate, why the Λ-term is so small.[20,21] It is just lately that the way toward an answer seems to have been marked out.

All these problems, however, have already received some attention in the pages of *Uspekhi*.[7,22] In this review we shall deal only with comparatively late phases in the evolution of the universe; in terms of classical macroscopic physics we shall inquire into how structure has developed in the universe from small random perturbations. Questions of this kind are currently under study in very diversified branches of physics. Interestingly enough, in cosmology these problems have turned out to be intimately connected with catastrophe theory[23] and percolation theory.[24,25] As a consequence one can lay down some sweeping rules which do not rely on any particular (approximate) method of analysis.

b) The observations

In the realm of optical and radio astronomy, the most outstanding advance of the past few years has come from *en masse* redshift measurements of individual galaxies, furnishing estimates of their distance. As a result, astronomers have been able to pass from analyzing the apparent distribution of galaxies in the plane of the sky to studying the true three-dimensional structure of that distribution. Such analyses have been performed for the close neighborhood of our Galaxy, within a radius of about 100 Mpc (1 megaparsec $\approx 3 \times 10^{24}$ cm). Furthermore, in some fields of the celestial sphere velocities have been determined for all galaxies brighter than a definite apparent magnitude.

The space distribution suggests that huge regions exist, up to 100 Mpc across, in which no galaxies are observed. Groups and clusters of galaxies tend to collect near comparatively thin surfaces or elongated filaments bordering these empty cells.[2,26-40] We shall give only a very brief account (in Sec. 8) of the current evidence on large-scale structure and clusters and superclusters of galaxies. A special review of the subject has been prepared by Oort.[41]

These observational findings have been gathered concurrently with the development of a nonlinear theory of gravitational instability to describe the process whereby cell structure has originated in the universe. As a first step, one of the authors established[42] that as large-scale perturbations evolve (with small-scale ones suppressed), flat, pancake-shaped structures should form, strongly compressed in one direction and confined by shock waves.[42] Later it was shown, largely by numerical calculations, that the growth of the perturbations should cause the pancakes to intersect, forming a lattice cell structure.[43-45]

Another approach, closely tied in with analyses of galaxy catalogs, was worked out during the 1970s.[1,46-48] By applying universal correlation-analysis methods (and similar techniques) it was shown that the distribution of visible matter in the universe has certain regular features: galaxies are distributed on one definite scale,[1,46] and clusters of galaxies on another.[48-51]

The mission confronting both theoretical and observational astronomy right now is to take the next step, and to try to describe more fully (and explain!) the picture observed.

It is pertinent to recall here that any theory for the origin of structure is bound to be statistical in character, and can do no more than explain the statistical properties of the observed structure. Unfortunately, extragalactic observations in many respects are limited to data on specific objects; statistical parameters describing the present large-scale structure are still few and far between.

A most important role in the growth of our understanding of the structure of the universe has been played by analysis of the fluctuations in the temperature of the microwave background radiation. Shortly after this primordial radiation had been discovered, several authors pointed out[52-54] that its temperature could not be perfectly constant over the celestial sphere. The density and velocity irregularities whose buildup has given rise to the pattern seen in the universe today should have left their traces in the background radiation as well. The temperature of the microwave background ought to depend on direction, with the amplitude of the temperature variations being intimately related to the amplitude of the irregularities in the density and velocity of matter in the early universe.

From 15 years of radio observations, in which the steadfast efforts of Pariĭskiĭ[2,55] and Partridge[56] have played an especially valuable part, it has been shown that the amplitude of those perturbations must have been exceedingly small! Thus far only upper limits have been set at all scales, except for the dipole component, yielding a value (about[57-59] 600 km/sec) for the velocity of the Galaxy with respect to the cosmic background radiation. Estimates of the background fluctuations enter as a necessary ingredient into all theoretical scenarios for the origin of structure in the universe, and set very serious constraints on them.

In 1980 there were two claims[58,59] that quadrupole anisotropy had been observed in the background radiation at a level $(\delta T/T)_Q \approx (1-3)\times 10^{-4}$, although the results have not been confirmed. If instead the amplitude $\delta T/T$ of the quadrupole mode of the cosmological perturbations should prove to be significantly smaller than this value, then analysis of the angular dependence of $\delta T/T$ [or of the correlation function $f(\theta) = \langle \delta T/T(\varphi)\cdot \delta T/T(\varphi+\theta)\rangle$] for large angles would furnish invaluable information on the large-scale asymptotic behavior of the initial perturbation spectrum.

c) Development of structure in the universe

In very broad outline, the structure found in the universe today is thought to have taken form in the following way.

1. When the temperature had dropped below 100 MeV, with all phase transitions having ended and only the presently known particles having come into play, the universe presumably proceeded to evolve according to the "standard" model: hot, homogeneous plasma in a state of thermodynamic equilibrium experienced isotropic Hubble expansion.[5,6] However, small disturbances of the density, the velocity, the metric, and perhaps the entropy are believed to have been present. Since these perturbations would have been weak, we are justified in neglecting their influence upon the nucleosynthesis, general expansion, and other global properties of and processes in the universe.

Gravitational-instability theory describes how the perturbations would have evolved in the linear approximation,[15,16] and as in any linear theory, the spectral approach plays the leading role. The initial properties of the perturbations should have been determined by processes that had taken place in the "very early" universe, as disclosed by theories of processes operating close to the singularity. For now we shall as a rule limit attention to adopting some premise or other with regard to the initial perturbation spectrum (assuming random phases).

2. As the universe evolved, it would have passed through critical eras when one regime gave way to another, causing characteristic scales to emerge in the perturbation spectrum. One of the most important critical periods was the era of hydrogen recombination, when the temperature of the universe was $T \approx 4000\,^\circ$K. The plasma would have become neutral during this era; matter and radiation would have ceased to interact, and the perturbations could no longer have been dissipated. Several scales of interest would have become established at this time,[53,60-64] including[1)] the horizon scale R_H dissipative scale $R_c \approx 20-50$ Mpc.

Another critical period, and a highly important one in a neutrino universe, would have been the era when the neutrinos became nonrelativistic, that is, when the universe had cooled to the point where $3kT_\nu \approx m_\nu c^2$. That epoch will depend on the mass of the neutrino, so the horizon scale at the time similarly will depend on the neutrino rest mass[65-70]:

$$R_\nu \approx 4.8\left(\frac{m_\nu}{30\,\mathrm{eV}}\right)^{-1}\mathrm{Mpc}.$$

The perturbation spectrum should exhibit breaks at the corresponding scales. Some of these breaks should develop later, during the nonlinear stage, representing a characteristic scale, a fundamental length in terms of which the statistical parameters of the large-scale structure are expressed; other breaks should also manifest themselves in the angular pattern of the microwave background fluctuations.

3. The structure now observed in the universe would have originated in the nonlinear stage, after the perturbations had managed to grow.

Several different theories have been worked out thus far to explain how large-scale structure formed in the universe. They differ in the assumptions they make regarding the properties of the primordial perturbations, and hence the routes by which the observed structure developed. The fullest theory at present is the adiabatic theory for the growth of structure (the A-theory), which rests on the premise that the initial perturbations were adiabatic. For a neutrino-dominated universe, the A-theory would fit the observations quite well, and it is the case of the A-theory applied to a neutrino universe that we shall discuss below.

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resolve certain observational paradoxes.[1,71-73] Lately, however, much preparatory work has been carried through, both in seeking to recognize what influence such particles might have and in devising techniques for describing the processes whereby structure has formed and evolved in a neutrino universe. The A-theory for the development of structure can be generalized to the neutrino-universe model in a very natural way, and accordingly that is the model which has received the most detailed and concentrated attention.[65-70,74-83] Several authors have recently discussed the cosmological role of other particles of finite mass—the photino, gravitino, majoron, goldstino.[84-88]

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Massive neutrinos could play the decisive role in solving certain problems in cosmology. The first problem that we shall describe briefly is the disparity in the estimates for the mean density of the universe.

1. One of the first methods proposed for evaluating the mean density of the universe[89] was to estimate its luminosity per unit volume, and then to convert that quantity to the mean density by using the mass–luminosity relation for galaxies. Estimates of this kind have been carried through time and again over the past two decades, and they indicate that the universe has a density[90,91] $\rho_b \approx (2-4)\times 10^{-31}$ g/cm^3. This value for the mean density is reasonably consistent with current estimates of the deuterium abundance in the interstellar gas,[92] $[D/H]\approx 1.5\times 10^{-5}$, from which on the basis of cosmological nucleosynthesis theory one can set a limit $\bar{\rho}_b < 7\times 10^{-31}$ g/cm^3. When the mean density of the universe is estimated from the deuterium abundance in the interstellar gas, one can only place an upper bound on $\bar{\rho}_b$, because deuterium will easily be consumed as stellar fuel and its abundance may tend to diminish. On the other hand, the deuterium contained in the interstellar gas in galaxies might be replenished through accretion of intergalactic gas, for example.

The available data on the relative abundance of primordial helium by mass (0.21–0.25) similarly yield mean densities $\bar{\rho}_b \approx (1-3)\times(T_\gamma/3\,°\mathrm{K})^3\times 10^{-31}$ g/cm^3, independently of the value of the Hubble constant.

2. But the values just quoted for the mean density of the universe do not fit in with certain other observations. Even in the 1930s it was realized[93] that if the mass of clusters of galaxies is estimated from the velocities of their member galaxies, the result will much exceed the mass of the same clusters evaluated from their luminosity. One is faced with the celebrated "virial paradox"—the missing-mass problem. If clusters of galaxies are in a steady state, as suggested both by the regular shape that many of them have and by the way in which their galaxies are distributed, then the velocity dispersion of the member galaxies should be related to the mass and size of the cluster by the virial theorem. Current estimates for the ratio of the dynamical mass M_v of clusters to their mass M_L based on the luminosity of their galaxies[94-100] range from 10 to 60. Measurements of the velocities of the galaxies in clusters are in good accord with the temperature of the hot intergalactic gas, as determined from the x-ray spectra of clusters.[100,101]

Furthermore, evidence has emerged in the past few years indicating that certain individual galaxies not visible optically are surrounded by massive coronae, or halos.[2)] When the halos are included, the mass of the galaxies turns out to be 5–10 times their estimated visible mass.[103-109] With this hidden mass, the universe could have a mean density 10–50 times as high as the values given above. Indeed, if the unseen mass were not confined merely to clusters and superclusters of galaxies, one could even have densities $\Omega = \bar{\rho}/\rho_{cr} \geq 1$.

Thus some observations imply a low mean baryon density while others indicate that the universe has a high mean density.

3. Low values for the mean density also are incompatible with the observed fluctuations in the temperature of the cosmic background radiation.

For example, if $\Omega_t = \Omega_b = 0.01$, then during the time elapsed since the era of hydrogen recombination, when the fluctuations $\delta T/T$ originated, perturbations would have grown by less than a factor of 10 (see Fig. 4 in Sec. 4). Hence in order for the perturbations to have achieved their nonlinear growth phase ($\delta\rho/\rho \geq 1$) by the present epoch, they would have had to have possessed too large an amplitude during the cosmological recombination era—unacceptably large, from the standpoint of the fluctuations in the background radiation. With present experimental limits on the background temperature fluctuations, the observed structure of the universe could not have managed to develop in any model having $m_\nu = 0$.

This contradiction can easily and naturally be resolved by introducing finite-mass neutrinos into cosmology (or any other species of massive collisionless particles). Neither measurements of luminosity nor observations of the deuterium abundance can furnish estimates for anything other than the baryon component of the mean density; but virial estimates and arguments based on the temperature fluctuations of the background radiation pertain to the total density, including the neutrino as well as the baryon component.[3)] One would merely have to explain why these two components do not always have similar density distributions. This is another question that we shall explore more fully below.

A final advantage of a neutrino-dominated universe is that it can yield near-minimum fluctuations in the background radiation for some specified large-scale structure of the universe, because the neutrinos will interact with the radiation only gravitationally. Evidently it would not be possible to avoid all gravitational influence of cosmological irregularities upon the radiation, but in models with $m_\nu = 0$ the perturbations of the baryon component, interacting with the radiation directly, would

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[2)] Karachentsev[102] has cast new doubt on the existence of halos around galaxies.

[3)] Incidentally, efforts to attribute the missing mass to stars or any other form of ordinary matter would not solve the deuterium and helium problem.

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The available data on the relative abundance of primordial helium by mass (0.21–0.25) similarly yield mean densities $\bar{\rho}_b \approx (1-3)\times (T_\gamma/3\,^\circ\mathrm{K})^3 \times 10^{-31}$ g/cm^3, independently of the value of the Hubble constant.

2. But the values just quoted for the mean density of the universe do not fit in with certain other observations. Even in the 1930s it was realized[93] that if the mass of clusters of galaxies is estimated from the velocities of their member galaxies, the result will much exceed the mass of the same clusters evaluated from their luminosity. One is faced with the celebrated "virial paradox"—the missing-mass problem. If clusters of galaxies are in a steady state, as suggested both by the regular shape that many of them have and by the way in which their galaxies are distributed, then the velocity dispersion of the member galaxies should be related to the mass and size of the cluster by the virial theorem. Current estimates for the ratio of the dynamical mass M_v of clusters to their mass M_L based on the luminosity of their galaxies[94-100] range from 10 to 60. Measurements of the velocities of the galaxies in clusters are in good accord with the temperature of the hot intergalactic gas, as determined from the x-ray spectra of clusters.[100,101]

Furthermore, evidence has emerged in the past few years indicating that certain individual galaxies not visible optically are surrounded by massive coronae, or halos.[2)] When the halos are included, the mass of the galaxies turns out to be 5–10 times their estimated visible mass.[103-109] With this hidden mass, the universe could have a mean density 10–50 times as high as the values given above. Indeed, if the unseen mass were not confined merely to clusters and superclusters of galaxies, one could even have densities $\Omega = \bar{\rho}/\rho_{cr} \geq 1$.

Thus some observations imply a low mean baryon density while others indicate that the universe has a high mean density.

3. Low values for the mean density also are incompatible with the observed fluctuations in the temperature of the cosmic background radiation.

For example, if $\Omega_t = \Omega_b = 0.01$, then during the time elapsed since the era of hydrogen recombination, when the fluctuations $\delta T/T$ originated, perturbations would have grown by less than a factor of 10 (see Fig. 4 in Sec. 4). Hence in order for the perturbations to have achieved their nonlinear growth phase ($\delta\rho/\rho \geq 1$) by the present epoch, they would have had to have possessed too large an amplitude during the cosmological recombination era—unacceptably large, from the standpoint of the fluctuations in the background radiation. With present experimental limits on the background temperature fluctuations, the observed structure of the universe could not have managed to develop in any model having $m_\nu = 0$.

This contradiction can easily and naturally be resolved by introducing finite-mass neutrinos into cosmology (or any other species of massive collisionless particles). Neither measurements of luminosity nor observations of the deuterium abundance can furnish estimates for anything other than the baryon component of the mean density; but virial estimates and arguments based on the temperature fluctuations of the background radiation pertain to the total density, including the neutrino as well as the baryon component.[3)] One would merely have to explain why these two components do not always have similar density distributions. This is another question that we shall explore more fully below.

A final advantage of a neutrino-dominated universe is that it can yield near-minimum fluctuations in the background radiation for some specified large-scale structure of the universe, because the neutrinos will interact with the radiation only gravitationally. Evidently it would not be possible to avoid all gravitational influence of cosmological irregularities upon the radiation, but in models with $m_\nu = 0$ the perturbations of the baryon component, interacting with the radiation directly, would

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[2)] Karachentsev[102] has cast new doubt on the existence of halos around galaxies.

[3)] Incidentally, efforts to attribute the missing mass to stars or any other form of ordinary matter would not solve the deuterium and helium problem.

induce far stronger fluctuations in the background. If the baryon density is low enough ($\Omega_b \approx 0.01$), then perturbations of the baryon component in a neutrino universe during the era when the background fluctuations were taking shape would have been much smaller than the perturbations of the neutrino component, and their influence upon the cosmic background radiation would have been weakened.

e) Theories for formation of structure in the universe

Let us now outline briefly the main features distinguishing the several current theories for the development of large-scale structure in the universe. As intimated above, the linear theory of gravitational instability can describe the evolution of all types of disturbances. During the nonlinear stage, however, the different models diverge fundamentally. We begin with the adiabatic theory (the A-theory).

According to the A-theory, objects of maximum scale would have formed at the outset—"pancakes" of gas and neutrinos, identifiable with superclusters of galaxies.[42,66,78] A gaseous pancake would have been confined by strong shock waves in which the gas flowing toward the pancake would have been compressed and heated. In the central regionsof the pancake the gas would have cooled off rapidly through intensive radiation of its thermal energy, and its density would accordingly have risen.[110] Inside the pancake the gas flow would have possessed a substantial vorticity, resulting from the compression of shocked matter at the surface of the pancake[111] (Thomson's theorem regarding the conservation of vorticity would thereby be violated). The vortical flow within the pancake would have been unstable against turbulization, and that property, together with the thermal instability of the compressed gas, would have caused the gas to fragment into clouds of mass $M \approx 10^6\ M_\odot$, which would have represented the basic elements in subsequent processes of star and galaxy formation.[112,113] Enlarging in size, the pancakes in due course would have merged, producing the observed large-scale lattice structure of the universe.[43-45] Galaxy- and star-formation processes would have operated intensively in the dense pancakes, but no galaxies would have formed in the rarefied places in between the pancakes; that would explain the origin of the vast dark regions bordered by bright superclusters of galaxies.[114,115]

After some delay, the outlying parts of a pancake would have gone through the same evolutionary process as the central zone: radiative cooling of the gas, accompanied by a breakup into separate clouds in which the first stars would have formed, with the clouds simultaneously collecting into galaxies and the galaxies into groups and clusters. However, in the different parts of the pancake these processes would have operated at different relative rates; that could have affected the entire process as a whole, with diverse results in the various parts of the pancake. Perhaps this is why we observe such a wide variety of structures.

The process whereby large-scale structure would develop in the A-theory is also of interest as an example of a nonlinear problem. In formal terms, when a highly compressed pancake is created and shock waves and caustics are generated, short wavelengths of substantial amplitude will appear in the Fourier integral of the density.

These amplitudes will not, however, correspond to the breakup of large masses into small ones: the amplitude of the short waves will here represent the small thickness of the pancake and the rapid change in density across the pancake. There will be no intuitive relationships between the wavelength and the mass of the separate structures, because certain phase relations will hold here among the short waves; one cannot apply to them the random-phase concept which ordinarily is taken for granted when speaking of the spectrum of irregularities.[116] A strong correlation will be maintained among perturbations of differing scale; it will manifest itself as a quasiorganized distribution of matter and of empty cells—as a formation of lattice structure. By analogy with conventional and acoustic turbulence, this phenomenon may be termed "gravitational turbulence."

This state constitutes an interesting example of intermediate asymptotic behavior[117-119]: a well-developed structure will persist for a limited time. To begin with, new types of instability will arise in the material compressed into a pancake: kinetic instability in the neutrinos, and thermal and hydrodynamic instability in the gas. Turbulent flows will set in. At the same time, as numerical calculations have shown, the lattice structure will break up into separate clouds, which will progressively amalgamate into ever bigger formations.[44,45,120]

Some alternative scenarios for the development of large-scale structure in the universe have been worked out in the context of the entropic theory for the formation of structure[1,2,5,17,121-129] (the E-theory). Several different schemes have been discussed in terms of the E-theory. All these versions of the E-theory presuppose the existence of entropic perturbations. Entropic perturbations are disturbances in the baryon-photon density ratio n_b/n_γ which would probably have arisen during phase transitions of one type or another in the early universe. In such transitions the total energy would have been conserved, so the total density of the universe would have remained constant (in space).

Occasionally the term entropic perturbations is applied to a nonuniform distribution of baryons against a uniform radiation background, and for that reason another name is used: "isothermal perturbations." Actually this intuitive picture is somewhat imprecise: since baryons carry weight, the condition that the mean density was constant at the time the perturbations developed implies that some (small) perturbation must have existed in the radiation density (and temperature), and would not have disappeared subsequently. If this fact were neglected,[64] the entropic and adiabatic perturbation modes would blend together. However, for short waves, the most typical ones in the E-theory, the perturbations in the radiation temperature should indeed have been small.

Very likely it will prove feasible to express the n_b/n_γ ratio in terms of the parameters of grand-unification theory. It then should automatically turn out that the fluctuations in this ratio (the entropic mode!) are notably small. Rough estimates suggest that not even the phase transitions would have produced significant fluctuations in the n_b/n_γ ratio. We therefore shall give preference to developing a pure A-theory. There are, in addition, reasons to believe that the large-scale perturbations of interest for cosmology originated during the era of exponential expansion[130] and probably did not contain the entropic mode.

The versions of the E-theory under discussion at the present time essentially rely on a mixture of adiabatic and entropic perturbations. But these E-theories may reflect certain interesting features of the structure formation processes, and they do warrant discussion quite apart from the assumptions made regarding the initial inhomogeneities. Finally, even in a neutrino universe variant theories are possible which bear some resemblance to the classical E-theories.[88] We therefore believe it worthwhile to give a concise account of the E-theory model.

Entropic perturbations would early on have evolved into objects of mass $M \approx 10^6\ M_\odot$. All versions of the E-theory agree in this respect. But the models differ in the subsequent course of evolution.

Probably the best-known E-theory scenario is the concept of successive clustering[1,121,122]—the "escalation" theory. According to this approach, the first objects to form would be gaseous and stellar bodies of comparatively small scale, such as globular clusters and dwarf galaxies of mass $M \approx 10^6 - 10^7\ M_\odot$. Later these bodies would undergo gravitational clustering into objects of increasingly large mass, up to the scale of galaxies ($M \approx 10^{10}\ M_\odot$) and clusters of galaxies ($M \approx 10^{14}\ M_\odot$). The theory has not been adequately worked out mathematically, but numerical experiments indicate that this scheme probably would be capable of explaining certain significant features of the observed structure of the universe.[2,131-140]

Lately the question has again been raised of what impact the explosion of numerous supernovae (millions or billions) might have had on the large-scale structure of the universe.[129] In 1967 we had looked into such a mode of galaxy and cluster formation[17] (the large-scale structure was not discovered until later), but subsequently we came to feel that a version of the A-theory was more promising. We still hold this view today, without wishing to deny, however, that supernova outbursts of differing scale may have played a major role in enriching the intergalactic medium with heavy elements and in generating internal structure in clusters and superclusters of galaxies, and even, perhaps, in producing the rotation of galaxies.

One distinctive version of the E-theory has been developed by Rees[125] and his colleagues.[141-147] Assuming an early formation of hypothetical third-generation stars (the first type would be ordinary stars resembling the sun; the second generation would consist of galactic-halo-type stars, deficient in heavy elements), they seek to explain not only the observed structure but also the observed entropy of the universe. The early stars are assumed to have accounted for (90–95)% of the observed radiation background.

This theory relies heavily on measurements of departures of the cosmic background spectrum from a Planck spectrum in the millimeter wavelength range.[148,149] Whether those departures are real and what their amplitude is remains an open question.[150]

A word is also in order here concerning the vortex theory of galaxy formation (the V-theory), which was being widely discussed in the early 1970s.[2,128,151-153] Once it had been shown that the problem of the rotation of galaxies can successfully be resolved on the basis of the A-theory,[5,111-113,154,155] the V-theory lost most of its appeal, its adherents having accepted that it faces many intrinsic difficulties[156,157] (in particular, a non-Friedmann singularity had to be accommodated[158]). Very little has been said about the V-theory lately. Nonetheless it did play a positive role in the development of cosmology.

f) Mathematical aspects of the adiabatic theory for the formation of structure

We proceed now to examine in greater detail some mathematical points encountered in the A-theory for the formation of structure in the universe. The A-theory is based upon an approximate nonlinear theory of gravitational instability. In Sec. 5 we shall discuss the fundamental properties and accuracy of the A-theory as well as the results that have been obtained from it. First we wish to set forth here certain arguments which, independently of how accurate the approximate theory may be quantitatively, persuade us that it undoubtedly is correct in a qualitative sense. Early developments will be found in 1970 papers by one of the authors.[42,43] A most important contribution to the mathematical elaboration of the A-theory for the formation of structure has been made by the Soviet mathematician Vladimir I. Arnol'd.[159-161]

Following the recombination era, the Jeans mass would have been small compared with the characteristic scales of the adiabatic perturbations; hence at a certain stage one may neglect the influence of pressure (adopting the cool-medium approximation) and regard the evolution of the disturbances as controlled entirely by gravitational forces. In the problem at hand the gravitational potentials and characteristic velocities will not reach relativistic values, and one may use Newtonian gravitation theory.

In the initial state, immediately after hydrogen recombination, we will be dealing with a near-uniform density distribution and nearly Hubble velocities. The perturbation amplitudes will be of order 10^{-3}. Since small-scale perturbations will be suppressed, the velocity field will be a smooth vector field. An important feature is that gravitational instability will single out potential (irrotational) flows associated with density perturbations which are growing through the action of the potential gravitational field and which cause the corresponding velocities to rise as well. Particle motion of this type can formally be treated as a single-parameter family of differentiable mappings: after time t the map will take the starting point into its final position. For sufficiently short times t the mappings will be one-to-one, but as time passes the particles will begin to overtake one another and the maps will develop singularities.

If at initial time the particles are distributed over space at nearly constant density, then after a certain interval the density will become infinite at those points which represent critical values of the mapping after elapsed time t. The trajectories of infinitesimally separated particles will intersect there.

The set of singular points will form a caustic curve. Caustics themselves may have singularities, but excessively intricate singularities will be unstable: they can be avoided by stirring things up slightly—that is to say, by making, for example, minor changes in the initial perturbations. The remaining singularities will be structurally stable: they will be insensitive to small deformations in the Lagrangian of the manifold in phase space that characterizes the state of the system, provided that the manifold does remain Lagrangian.

The imprecision in the approximate solution may itself be treated as just such a deformation in the Lagrangian of the manifold of the system, resulting from regular evolution of the system. The approximate solution represents a Lagrangian mapping, just as the exact solution which is unknown to us; and it is this common property which preserves all the qualitative aspects of the exact solution in the context of an approximate description.

In view of this property of the approximate solution, one is able to rely on the extremely general character of the results that have been obtained in the theory of Lagrangian singularities,[23,159-164] or properly speaking, in one particular case: the theory of gradient-mapping singularities. These results are applicable for arbitrary smooth potential fields for the initial velocities of a general state, assuming that the force field is irrotational (for example, in a medium consisting of noninteracting particles moving by inertia, or in a medium whose particles are moving in a gravitational field). There is a valid analogy in optics: the structure of the caustic curves formed at a certain distance from a "random" lens resembles the structure of the density singularities in the mechanical systems described above.[165]

It is highly important to understand that the Lagrangian-mapping theory investigates the structure of singularities in a general state that will develop in the absence of any special symmetries inherent in the initial data. At present we have no evidence that the perturbations during the early evolution of the universe were organized in any fashion, apart from the strong suppression of small-scale adiabatic disturbances. It therefore is natural to expect that the objects which are produced would take the form of very simple, nonremovable singularities. The best-known type of such singularities, the ones which would have marked the beginning of structural development in the adiabatic scenario, are the pancakes—highly flattened clouds with an enhanced particle density, separated from the ambient medium by a caustic, a surface on which the density is infinite.

Some structurally stable gradient-mapping singularities have now been established in space of one, two, and three dimensions; their normal forms have been obtained, and detailed geometrical descriptions and drawings have been given for them.[23,159-161,166]

That caustics might develop in matter with $p=0$ in the context of general relativity was pointed out two decades ago, in discussing problems of singularity formation in a synchronous reference frame.[16,167] Grishchuk[168] demonstrated that the corresponding solution possesses the maximum possible arbitrariness. More recently, Novikov[169] has carried over the basic early results[42] to general relativity theory. Emphasis has here been placed on the fact that if allowance were made for the pressure, the caustics would disappear. While the pressure is indeed finite in the problems of interest to us, it will be small. Its influence may be assessed by constructing a small dimensionless parameter from the pressure and the perturbation parameters. Thus although there will not be any caustic-type singularities in a literal sense, the theory describing the emergence of such singularities proves to be quite meaningful. By applying that theory one not only can solve the problem of how structure developed, but can also obtain a good description of the manner in which regions of high (but not infinite!) density must have evolved.

These matters as well are discussed again briefly in Sec. 5g.

1. MEAN DENSITY AND AGE OF THE UNIVERSE

Let us begin by considering how a neutrino universe would evolve as a whole. First we shall estimate the age of the universe and establish a relation between its age and mean density.

According to Hubble's law, distant galaxies are receding from us at a velocity $\mathbf{v}$ proportional to their distance $\mathbf{r}$:

$$\mathbf{v} = H_0\mathbf{r}. \tag{1.1}$$

Recent measurements of the Hubble "constant" H_0 yield values[170,171]

$$H_0 = 100h \text{ km sec}^{-1}\text{ Mpc}^{-1} \approx 0.3\, h\cdot 10^{-17}\text{ sec}^{-1}, \quad 0.5 \leqslant h \leqslant 1. \tag{1.2}$$

As the simplest possible estimate for the age of the universe (the elapsed time from the start of the expansion—from the singularity), one may take

$$t_U = H_0^{-1} \approx 10\, h^{-1}\cdot 10^9 \text{ yr}. \tag{1.3}$$

This estimate will be accurate if the present mean density $\bar{\rho}$ of the universe is much lower than the critical density ρ_c (that is, $\Omega \ll 1$):

$$\rho_c = \frac{3H_0^2}{8\pi G} = 1.9\cdot 10^{-29}\,h^2\ \text{g/cm}^3 \approx 10.7\,h^2\ \text{keV/cm}^3,\quad \Omega = \bar{\rho}/\rho_c \quad (1.4)$$

(G is the Newtonian gravitation constant), and the expansion rate will be almost independent of time. But if the density is near-critical ($\Omega \approx 1$) or supercritical ($\Omega > 1$), the braking effect of gravitation would have to be taken into account. In this event the initial expansion rates would have been considerably higher than today, so that the universe would be younger than the estimate (1.3). For a critical density ($\Omega = 1$), the age is

$$t_U = \frac{2}{3} H_0^{-1} = 6.7\,h^{-1}\cdot 10^9\ \text{yr}. \quad (1.5)$$

If the universe is closed, with $\Omega > 1$, it would be younger still. As an approximate formula we shall adopt the expression[75]

$$t_U \approx \frac{10h^{-1}}{1+0.5\sqrt{\Omega}}\cdot 10^9\ \text{yr}, \quad (1.6)$$

which provides a reasonably accurate estimate (±10%) for the age of the universe over a wide range in Ω.

If the mean density of the universe is determined by luminous matter, then[89-91] $\rho_b \approx 3\times 10^{-31}$ g/cm^3, $\Omega_b \approx 1.5\times 10^{-2} h^{-2}$, and the age of the universe would be $t_U \approx 9.5h^{-1}\times 10^9$ yr. In this event the universe would be open. But if the neutrino has a rest mass m_ν, the total density of matter in the universe would be higher and the universe would be younger.

Other estimates for the age of the universe are none too definite either. The earth is 4.7×10^9 yr old; the sun's age too is $(4.5-5)\times 10^9$ yr. Some very old stars are observed; theoretical models indicate that they are about $(14-16)\times 10^9$ yr old.[172]

Nuclear cosmochronology offers some interesting data on the age of the universe. Today uranium has the isotopic composition 0.7% U^{235} + 99.3% U^{238}. If equipartition prevailed at the time these isotopes were formed, then uranium would have been synthesized in the Galaxy about 7 billion years ago. Accordingly[173] $t_U > 7\times 10^9$ yr. From the rhenium–osmium method (the radioactive decay of rhenium into osmium) the universe is estimated[174] to be $(11-18)\times 10^9$ yr old.

All these estimates imply that the mean density of the universe probably is no higher than the critical density (with $h = 0.5$), so that the model of an open (hyperbolic, or perhaps parabolic) universe of infinite volume expanding without bound is evidently valid. These same estimates also restrict the value for the average mass m_ν of neutrinos.

Indirect arguments pointing to an open model universe and a comparatively low density $\Omega \approx 0.2-0.3$ emerge from analysis of the large-scale structure of the universe. Peebles[175] has arrived at similar values of Ω by estimating the density in systems of differing scale.

If we admit more complicated models, incorporating a cosmological Λ-term to describe the gravitation of vacuum, we can reconcile the evidence on the age of the universe with large values for the mean density.[75] With such models, however, the deviation from Friedmann models (in both the number and the properties of objects) would become appreciable for redshifts as small as $z \approx 0.3-0.5$, and could therefore be tested observationally.

In the event the mean neutrino density is so high that $\Omega \geqslant 1$, we would be faced with a closed model universe possessing a finite volume and a finite lifetime. In a closed model, gravitation would halt the expansion, which would give way to a contraction, causing the universe to collapse in 20–30 billion years. Closed model universes probably have certain advantages from the standpoint of the general theory of the singularity,[130] so the possibility does merit discussion, even though it would be decidedly difficult to reconcile a closed universe with current observational data.

2. NEUTRINOS IN A BIG BANG UNIVERSE

From the hot-universe theory one may infer that early in the cosmological expansion the neutrinos were in complete thermodynamic equilibrium with the electrons, protons, photons, and other particles. We thereby can calculate all the parameters of the neutrino distribution as they are today. As this topic has been thoroughly explained in a previous review,[10] we here give, without derivation, just a few equations and estimates that will be needed for the discussion to follow.

According to the theory of the big bang we should today observe neutrinos and antineutrinos with density n_ν and temperature T_ν:

$$n_\nu = n_{\bar{\nu}} = \frac{3}{22} n_\gamma \approx 75\, T_3^3\ \text{cm}^{-3}, \quad (2.1)$$

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma = 2.14\ T,\ ^\circ\text{K}; \quad (2.2)$$

here n_γ denotes the photon density, while $T_\gamma \equiv 3T_3$ °K is the temperature of the cosmic background radiation.

We shall start from the premise that three kinds of neutrinos exist: electron neutrinos ν_e, muon neutrinos ν_μ, and τ neutrinos ν_τ, to match the three species of charged leptons (e, μ, τ). If neutrino oscillations actually exist,[10] then the three separate kinds of neutrinos would not have any definite mass, and one should speak instead of three neutrino masses (m_1, m_2, m_3) each constituting a mixture of e-, μ-, and τ-type neutrinos.[4)] The quantity of consequence for cosmological problems, however, is not just the neutrino mass averaged over the different species, but the mass of each separate kind of neutrino, for that is the quantity which will determine the epoch when the neutrinos become nonrelativistic.

The experimental measurement of the mass of the electron neutrino[9] has yielded 46 eV $\geqslant m_{\nu_e} \geqslant$ 14 eV with a most probable value $m_{\nu_e} \approx$ 30 eV. For the other two kinds of neutrinos, upper limits $m_{\nu_\mu} \leqslant 1.5$ MeV and $m_{\nu_\tau} \leqslant 250$ MeV have been set. It therefore is of interest to consider several possible relationships among the

[4)] Data which Mössbauer reported to the Neutrino 82 conference held in Hungary in June 1982 fail to confirm oscillations in reactor experiments with ν_e's, thereby restricting the "mixing angles" and mass differences of neutrinos.

masses of the different kinds of neutrinos:

$$\left.\begin{array}{lll} \text{a)} & m_1 \approx m_2 \approx m_3 = m_0, & \bar{m} = m_0, \\ \text{b)} & m_1 \approx m_2 = m_0 \gg m_3, & \bar{m} = \frac{2}{3} m_0, \\ \text{c)} & m_1 = m_0 \gg m_2,\ m_3, & \bar{m} = \frac{1}{3} m_0. \end{array}\right\} \quad (2.3)$$

Our knowledge of the age of the universe places limits on its mean density:

$$\left.\begin{array}{l} n_\nu = \underset{e,\mu,\tau}{3} \times \underset{\nu,\bar{\nu}}{2} \times 75\, T_3^3\ \text{cm}^{-3} = 450\, T_3^3\ \text{cm}^{-3}, \\ \bar{\rho}_\nu = \overline{mn_\nu} = 2.5 \cdot 10^{-29}\, m_{30}\, T_3^3\ \text{g/cm}^3 = 13.5\, m_{30}\, T_3^3\ \text{keV/cm}^3, \\ \Omega_\nu = 1.25\, h^{-2}\, m_{30}\, T_3^3, \qquad m_{30} = \bar{m}_\nu / 30\ \text{keV}, \end{array}\right\} \quad (2.4)$$

and there by on the mean neutrino mass. In case (a), the mass m_0 would be the minimum value for a given mean density, and only one scale of inhomogeneity would be singled out, the scale associated with m_0; in case (c), the mass m_0 would take its maximum value, and in general the structure of the universe could contain several scales associated with the masses m_2 and m_3.

But it is also possible that the heavy particles of importance for cosmology have nothing to do with the known species of neutrinos. Supersymmetry theory in fact predicts a large family of particles, many of which have not been observed. Cosmological models involving hypothetical, possibly unstable,[176] particles can be developed.[88]

From the viewpoint of the formation of large-scale structure in the universe, the most interesting particles are those responsible for the mean density observed today (and in the relatively recent past, say when the redshift z had droped below 1000; when $z = 1000$ all dimensions in the universe would have been smaller by a factor of 1001). An there are two important parameters: the mass m_x of a particle and the contribution $\bar{\rho}_x = m_x n_x$ of such particles to the mean density of the universe. Actually it is more convenient to supplement the particle mass m_x by the dimensionless parameter $\alpha_x = n_x / n_0$, where n_0 denotes the combined number density of all massless particles and particles of mass $m < m_x$.

The constraints on the mean density of the universe place limits on the mass and number density of x-type particles:

$$m_x n_x \leqslant \Omega h^2 \cdot 1.9 \cdot 10^{-29}\ \text{g/cm}^3. \quad (2.5)$$

If as before we adopt $\Omega = 1$, $h = 0.5$ as a probable limit and allow for the contribution to n_0 from the radiation and two types of neutrinos, we will obtain in case (c) of the alternatives (2.3):

$$\alpha = \frac{3}{17} \approx 0.175, \quad \alpha m_0 \leqslant 3.15\, T_3^{-3}\ \text{eV}, \quad m_0 \leqslant 18\, T_3^{-3}\ \text{eV} \quad (2.6)$$

(which would imply that $m_0 \leqslant 25$ eV, since $T_\gamma = 3T_3 = 2.7$ °K).

In the general case of an arbitrary heavy, free x-particle (and antiparticle), if we include the contribution to n_0 from radiation and all three types of neutrinos we will have

$$\alpha_x m_x < 2.8\, T_3^{-3}\ \text{eV}, \quad m_x < 2.8\, \alpha_x^{-1} T_3^{-3}\ \text{eV}. \quad (2.7)$$

How these alternative cases are related to the large-scale structure of the universe will be considered below.

Whereas in previous $m_\nu = 0$ model universes the A- and E-theories differed fundamentally in the type of initial perturbations they predicted, in a neutrino universe the two theories will differ chiefly in the mass of the heavy particles. (Because of the low baryon density, classical entropic perturbations cannot play any role in themselves.) Thus with the development of model neutrino universes cosmology will now come to depend increasingly on the results gained from high-energy physics.

3. GRAVITATIONAL INSTABILITY OF THE UNIVERSE: LINEAR THEORY

A homogeneous and isotropic universe is a far-reaching idealization, tenable only on sufficiently large scales, in excess of 100–200 Mpc. On smaller scales matter is distributed very irregularly indeed: stars congregate in galaxies, galaxies in clusters.

On the other hand, the primordial disturbances should definitely have been small. That we know from the low amplitude of the fluctuations in the temperature of the microwave background radiation. We inescapably arrive, then, at the picture of a gravitational (perhaps not exclusively gravitational) enhancement of small primordial irregularities, leading to the growth of structure in the universe during the nonlinear stage.

The basic ideas behind the concept of gravitational instability had in fact been understood by Sir Isaac Newton. But further developments in gravitational-instability theory were to wait until 1902, when Sir James Jeans formulated the first mathematical theory.[177,178] In 1946 Evgeniĭ M. Lifshits solved the general-relativistic problem of the evolution of small perturbations.[15,16] Not until 1957 was an analysis of this problem in an expanding universe carried out in terms of Newtonian gravitation theory.[179]

Today the linear theory of gravitational instability has reached a good stage of development and is described in some detail in various monographs and review articles.[1,5,16,180] The laws for the evolution of irregularities in a neutrino-dominated universe are less well known, however. In this section we discuss briefly the main results achieved by the linear theory, perhaps repeating a few things that have long been understood. We limit our description to the growing modes of adiabatic and, less fully, entropic perturbations, because these are the only modes that enter into current theories for the formation of structure in the universe.

Let us take adiabatic perturbations first.

The early evolution of a big bang model universe will be dominated by radiation, which may be described in the hydrodynamic approximation of an ideal fluid having the ultrarelativistic equation of state (p_r denotes the pressure and ε_r the energy density of the radiation fluid):

$$p_r = \frac{\varepsilon_r}{3}. \quad (3.1)$$

We are entitled to adopt a hydrodynamic approach in describing the evolution of a radiation-filled universe since the mean free photon path, limited by free-electron scattering, will be small compared with the horizon and the other scales of interest in cosmology.

From the linear theory of gravitational instability we can make the following statements:

1. On large scales the pressure will not affect the development of irregularities; on these scales there will be a dominant perturbation mode growing with time:

$$\delta\rho/\rho \propto t, \tag{3.2}$$

and a decaying mode.

2. On small scales gravitation will be unimportant and the perturbations will turn into ordinary acoustic waves whose amplitude will remain constant despite the expansion (in accord with the general theory of adiabatic invariants).

3. The dividing line between large and small scales—the Jeans length—corresponds to the distance traversed by an acoustic wave during the characteristic hydrodynamic time in the expanding universe:

$$R_J \approx \frac{2ct}{\sqrt{3}}. \tag{3.3}$$

4. As was shown later on,[5,53,60,61] dissipative processes could play a major role in producing an oscillatory regime, for they would bring about decay of disturbances.

A prime task of the linear theory of gravitational instability is to calculate a transition function relating the perturbations at any time t to the primordial perturbations at initial time t_{in}. Since the undisturbed distribution of matter would have been homogeneous and isotropic, it is convenient to expand the perturbations in Fourier integrals:

$$\frac{\delta\rho}{\rho}=(2\pi)^{-3/2}\int \delta(k,\ t)\, e^{i\mathbf{kr}}d^3k \tag{3.4}$$

and to study how the function $c(k,t,t_{in})$, defined by

$$\delta(k,\ t)=c(k,\ t,\ t_{in})\,\delta(k,\ t_{in}), \tag{3.5}$$

will evolve with time. Here $\delta(k,t_{in})$ specifies the initial perturbations at time t_{in}, and $c(k,t,t_{in})$ is the required transition function, relating the perturbations to the wave vector k at the initial time t_{in} and at the running time t ($k=2\pi/\lambda\,|z=0$).

In a complete theory the initial perturbations $\delta(k,t_{in})$ ought to be dictated by processes operating close to the singularity. Ordinarily one assumes that the phases are uncorrelated and that the initial (random) disturbances are fully characterized by a spectrum $b^2(k)=\langle\delta^2(k,t_{in})\rangle$. Power spectra of the type $b^2(k)\propto k^n$ are considered here, as a rule. Certain indications as to the form of the spectrum can be gained by analyzing the observations (Sec. 4).

Many attempts have been made to analyze the evolution of small adiabatic perturbations in the absence of heavy free particles. It has been shown that the transition function $c(k,t,t_{in})$ will continue to vary only until the epoch of hydrogen recombination; at that stage the free electrons will disappear and the interaction between matter and radiation will effectively cease (see below). Thenceforth the perturbations will preserve their shape as they grow (during the linear era). By the close of recombination, at time $t=t_{rec}$, the transition function will have acquired the form[62,64] (assuming $\Omega h^2 \leqslant 0.3$)

$$c_A(k,\ t,\ t_{in})=\frac{t_{rec}}{t_{in}}\,\frac{\sin(kR_J)}{kR_J}\,e^{-kR_c/2}, \tag{3.6}$$

where the quantity R_J represents the characteristic Jeans scale at the epoch of recombination, while R_c denotes the scale of dissipative damping during recombination. For the scales R_J, R_c the estimates[62]

$$R_J\approx 90\ \text{Mpc},\quad R_c\approx 8\ \text{Mpc} \tag{3.7a}$$

have been obtained in the case $\Omega=1, h=0.5$, or

$$R_J\approx 220\ \text{Mpc},\quad R_c\approx 40\ \text{Mpc} \tag{3.7b}$$

if $\Omega=0.1, h=0.5$.

The evolution of entropic disturbances has been thoroughly discussed on several occasions.[5,17,62,64,161] It has been found that for large scales $l>R_J$ the entropic perturbation mode will go over to a decaying mode at the stage when the radiation density drops below the matter density. A growing perturbation mode, the one of interest from the standpoint of development of large-scale structure, will occur only with a small coefficient $(k^2R_J^2)$ and only subsequent to hydrogen recombination, at which time matter and radiation will have come into relative motion. The transition function for entropic perturbations may be written in the form

$$c_E(k,\ t,\ t_{in})\approx\frac{k^2R_J^2}{1+k^2R_J^2}\left[1+e^{-kR_c/2}\sin(kR_J)\right], \tag{3.8}$$

where the quantities R_J, R_c are the same as determined above [Eqs. (3.7)]. For the most part entropic perturbations will persist for values $kR_J>1$.

There have been a number of detailed investigations of how disturbances will develop in a neutrino-dominated universe.[65-70,77,78,82] According to Eqs. (2.1) and (2.2), about 63% of the energy density in the early universe can be allocated to radiation, and 37% to the three kinds of neutrinos—a very substantial fraction. But neutrinos cannot be treated in terms of hydrodynamics: from the epoch of cosmological nucleosynthesis onward, they will be virtually collisionless particles.[65] Hence disturbances will develop differently in the radiation and in the neutrinos. On large scales, for $\lambda>ct$, the difference will not be very significant, but overall it will be fundamental.

Two characteristic times should be singled out when considering the growth of disturbances in a neutrino universe:

$$\begin{aligned} &t=t_\nu, && 3kT_\nu\approx m_\nu c^2,\\ &t=t_{eq}, && \bar\rho_\nu=m_\nu\bar n_\nu=\bar\rho_\gamma. \end{aligned} \tag{3.9}$$

As soon as $t>t_\nu$ the neutrinos will become nonrelativistic. When $t>t_{eq}$, the neutrino density $\bar\rho_\nu$ will exceed the relativistic-particle density $\bar\rho_\gamma$, and the neutrinos will control both the general expansion and the evolution of inhomogeneties. In the standard version (2.3a) of a neutrino model universe ($m_1 \approx m_2 \approx m_3$) the epochs t_ν, t_{eq} will be roughly the same, but in the version involving massive, free χ-particles the difference between the two could be substantial.

After liberation of the neutrinos the inhomogeneities will evolve in the following way.

1. So long as $t < t_\nu$ the neutrinos will be relativistic. During this era:

a) For large scales of inhomogeneity the disturbances will, as before,[182] grow (the dominant mode) according to the law[5)] (for $kR_H < 1$)

$$\frac{\delta\rho}{\rho} \propto t; \tag{3.10}$$

b) small-scale disturbances will decay by mixing according to a law of the form

$$\frac{\delta\rho}{\rho} \approx \left(\frac{\delta\rho}{\rho}\right)_{kR_H=1} \frac{\sin kR_H}{\sqrt{kR_H}}; \tag{3.11}$$

c) the dividing line between large and small scales will be approximately the horizon scale R_H at the epoch in question:

$$R_H \approx 2ct. \tag{3.12}$$

2. For $t_\nu \leq t \leq t_{eq}$ the neutrinos will be nonrelativistic, but the density of the universe will continue to be dominated by radiation. During this era, neutrino disturbances will decay on scales $l \leq R_\nu$:

$$R_\nu \approx R_H(t_\nu). \tag{3.13}$$

But perturbations with scales in the range $R_\nu \leq l \leq R_H = 2ct$ will remain almost unchanged.

3. When $t > t_{eq}$ the relativistic-particle density will have become so low that neutrino disturbances with $l > R_\nu(1+z)^{1/2}(1+z_\nu)^{-1/2}$ will be able to grow according to the law

$$\frac{\delta\rho}{\rho} \propto t^{2/3} \propto (1+z)^{-1}. \tag{3.14}$$

On small scales, however, the perturbations will decay. Disturbances in the relativistic phase will retain a constant amplitude on scales within the horizon, and will grow by the law (3.14) on large scales.

Figure 1 illustrates all these results for case (2.3c), with $m_\nu = 20$ eV and $\Omega = 1$. In region I of the diagram, all the perturbations will grow; in region II, perturbations in the mixture of radiation and matter will take the form of acoustic waves; in region III, these perturbations will be dissipated by diffusion processes; and in region IV the neutrino perturbations will decay.

Discounting high-frequency oscillations, we obtain for the transition function $c(k, t, t_{in})$ the following approximate relation (for the relativistic and neutrino phases separately) at the hydrogen-recombination epoch $t = t_{rec}$ (see Fig. 2):

$$c_\nu = \frac{t_{eq}}{t_{in}} \cdot \left(\frac{t_{rec}}{t_{eq}}\right)^{2/3} (1 + k^2R_\nu^2)^{-7/2} (1 + k^2R_{eq}^2)^{-1}, \tag{3.15}$$

[5)]The neutrinos will manifest their collisionless character in that the pressure will become anisotropic, strongly affecting the form of the decaying modes.

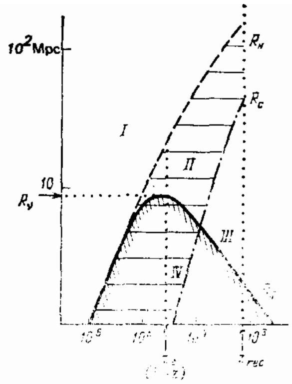

FIG. 1. The relation between redshift z and the characteristic scales R that determine how perturbations will evolve. All lengths are adjusted to present scales. Arrows mark the recombination epoch $z_{rec} \approx 1200$ and the epoch z_ν when the neutrinos cease to be relativistic. Neutrinos mass $m_\nu = 20$ eV, total density $\Omega_t = 1$.

$$c_r = \frac{c_\nu[1 - \cos(kR_J)]}{k^2R_J^2}, \tag{3.16}$$

where $R_{eq} = 2ct_{eq}$. On large scales (as $k \to 0$), $c_{r,\nu} \to$ const, and the perturbations will retain their form. According to numerical calculations by Bond and Szalay[69] (see also Peebles[70]), the function $c_\nu(k, t_{rec}, t_{in})$ will conform approximately to the law

$$c_\nu \propto (1 + k^2R_\nu^2)^{-6}, \tag{3.17}$$

which is similar to the relation (3.15) for $R_{eq} = R_\nu$.

A most important result from the linear gravitational-instability theory is the emergence of a natural scale R_ν on which neutrino perturbations will decay. This scale, in light of what we have said, specifies the maximum free path for neutrinos, and will be comparable with the horizon scale at the epoch when the neutrinos become nonrelativistic (when $3kT_\nu \approx m_\nu c^2$). The scale R_ν and quantities related to it will depend only on the neutrino mass (and the fundamental constants). There is an elegant expression for it[67] in terms of the Planck mass m_{Pl}, length l_{Pl}, and density ρ_{Pl}:

$$\left.\begin{aligned} \rho_\nu &= (Gt_\nu^2)^{-1} = c^3\hbar^3 m_\nu^4 = \rho_{Pl}\left(\frac{m_\nu}{m_{Pl}}\right)^4, \\ M_\nu &= \rho_\nu R_\nu^3 = m_{Pl}\left(\frac{m_\nu}{m_{Pl}}\right)^{-2}, \end{aligned}\right\} \tag{3.18}$$

with R_ν, ρ_ν, M_ν representing the characteristic values of the scale, density, and mass at epoch $t = t_\nu$ (when $3kT_\nu = m_\nu c^2$); $l_{Pl} = \sqrt{G\hbar/c^3}$, $\rho_{Pl} = c^3G^{-2}\hbar^{-1}$, $m_{Pl} = \sqrt{c\hbar/G}$ denote the Planck length, density, and mass; $\hbar$ is the Planck

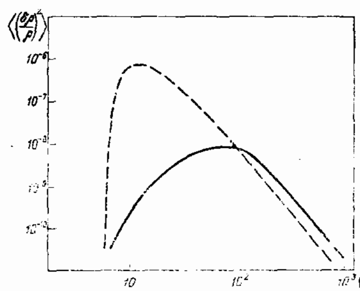

FIG. 2. The smoothed density-perturbation spectra (4.2) as functions of $\lambda = k^{-1}$. Dashed curve, the neutrino component; solid curve, the baryon component.

constant. For the case (2.3a), numerical estimates yield the following values for the damping scale today [$m_{30} \equiv m_\nu/(30\ \text{eV})$]:

$$R_\nu \approx 4.8\ m_{30}^{-1}\ \text{Mpc},\quad M_\nu \approx 10^{14} m_{30}^{-2} M_\odot. \tag{3.19}$$

In the general case of massive x-particles, the expressions (3.18) will be replaced[183] by

$$\begin{gathered} t_x = t_{\text{Pl}} \left(\frac{m_{\text{Pl}}}{m_x}\right)^2, \\ \rho_x = \rho_{\text{Pl}} \left(\frac{m_{\text{Pl}}}{m_x}\right)^{-4} \alpha_x, \quad M_x = m_{\text{Pl}} \left(\frac{m_{\text{Pl}}}{m_x}\right)^2 \alpha_x, \end{gathered} \tag{3.20}$$

$$\begin{gathered} t_{\text{eq}} = t_x \alpha_x^{-2}, \\ \rho_{\text{eq}} = \rho_x \alpha_x^{3}, \quad M_{\text{eq}} = M_x \alpha_x^{-2}. \end{gathered} \tag{3.21}$$

For three kinds of neutrinos with nearly equal masses [case (2.3a)], $\alpha_x > 0.3$ and there will be little difference between the quantities M_x and M_{eq}; but as α_x diminishes this difference will increase, and if $\alpha_x \ll 1$ it may be substantial:

$$R_x = 4.8\,(m_x/30\ \text{eV})^{-1}\ \text{Mpc},\quad R_{\text{eq}} = R_x \alpha_x^{-1}\ \text{Mpc}. \tag{3.22}$$

The characteristic mass M_{eq} will depend only on the mean density $\bar{\rho}_x = m_x \bar{n}_x$ at the present epoch, and will be close to the value (3.19). But the characteristic mass M_x, on the contrary, will diminish in proportion to m_x^{-3} (for a fixed density $\bar{\rho}_x$ today); if $m_x = 6$ keV we would have $M_x = 2 \times 10^7\ M_\odot$.

If smaller scales R_x, M_x should play the chief role in the process whereby objects are formed, then in view of the small values of the characteristic mass M_x and the scale R_x (for particles with $m_x \approx 3$ keV), perhaps we may look forward to models in the spirit of the gravitational-clustering theory[88] undergoing a revival in the context of a neutrino-dominated universe—models similar to those invoked in the E-theory for the formation of structure.

4. HYDROGEN RECOMBINATION IN THE BIG BANG UNIVERSE, AND BACKGROUND TEMPERATURE FLUCTUATIONS

According to the principles of the big band theory, when the temperature had dropped to $T \approx (3-4) \times 10^3$ °K the hydrogen in the universe would have recombined, so that the free electrons would swiftly have disappeared; matter and radiation would have ceased to interact.[6)] The radiation would have been able to pass freely through the neutral hydrogen, bringing us information on the state of the universe at that period. The small-scale fluctuations in the temperature of the cosmic background radiation are like a snapshot of the density and velocity perturbations in the plasma–radiation mixture during the era of recombination.

The recombining of hydrogen would have had a second consequence: the radiation would no longer have blocked the growth of irregularities in the neutral gas, and the perturbations in the baryon component would have become adjusted to the perturbations in the neutrino phase.[66,184] Let us examine these points more carefully.

At temperatures $T \approx (3-4) \times 10^3$ °K the equilibrium degree of ionization will be ≈ 0.5, so hydrogen ought to be able to recombine. But under the conditions prevailing in a homogeneous universe, the recombination process will have certain distinctive features. In conventional circumstances the $L_\alpha, L_\beta, \ldots, L_c$ photons resulting from the recombination would escape freely from the system. In the cosmological situation, however, the recombination will serve either to split up the energetic photons through two-photon $2S_{1/2}-1S_{1/2}$ transitions or, if a photon remains unabsorbed for very long time, to remove it to some low energy as a result of the general expansion.[5,185] It is these gradual processes which will draw out the recombination, in turn giving the perturbations more opportunity to dissipate, and weakening the fluctuations in the background temperature. Recombination will take place very slowly in a neutrino universe because of the low baryon density, and the dissipative scale R_c scale will become larger than the values (3.7). However, the scale R_c in a neutrino universe will affect only the background temperature fluctuations, since the neutrinos themselves will determine the spectrum of the density perturbations.

If $\Omega_b = 10^{-2}, \Omega = 1, h = 0.5$ we will have a Jeans scale $R_J \approx 160$ Mpc and a dissipative scale $R_c \approx 25$ Mpc, or expressed in angular variables, $\theta_J \approx 48'$, $\theta_c \approx 7'$. If instead $\Omega = 0.2$ (with $\Omega_b = 10^{-2}, h = 0.5$), then $R_J \approx 250$ Mpc, $R_c \approx 40$ Mpc, or in angular variables $\theta_J \approx 6°$, $\theta_c \approx 1°$. The large change in the angular scales as we pass from $\Omega = 1$ to $\Omega = 0.2$ reflects the strong dependence upon the mean density Ω of the "angular-size distance,"[5] the quantity which in curved space serves to convert linear scales into angular ones.

Three different processes could produce nonuniformities in the temperature of the cosmic background radiation:

a. On the very largest scales, $l > R_J$, the perturbations in the background-radiation temperature will reflect the gravitational influence of density fluctuations.[52]

b. On scales $l \approx R_J$, the temperature nonuniformities associated with perturbations in the radiation density[53] will predominate (since $\rho_r \propto T_r^4$):

$$\frac{\Delta T_r}{T_r} \approx \frac{1}{4}\frac{\delta\rho_r}{\rho_r}. \tag{4.1}$$

At smaller scales this effect would be annulled by the protracted span of the recombination era.

c. On small scales, $l < R_J$, the background temperature nonuniformities will depend on the velocities of moving matter, and will owe their origin to Compton scattering of photons by the moving electrons.[54] Since disturbances on these scales would take the form of acoustic waves, the velocity perturbations would be determined by the sound speed and by the amplitude of the disturbances.

In cosmological models with massless neutrinos, the background temperature fluctuations will be determined by Compton scattering. In a neutrino-dominated uni-

[6)]Nevertheless, the weak ($\approx 10^{-4}-10^{-3}$) residual ionization of matter would have been enough to keep the matter and radiation temperatures equal until levels $T \approx (1-2) \times 10^3$ °K were reached.

verse, however, the effects b and c will make only a minor contribution to the nonuniformities in the background temperature.[186] We will essentially be faced with a new scenario: the amplitude of the perturbations in the mean density of the universe will now be determined by fluctuations in the neutrino component extending to scales as great as $R_\nu = 4.8 m_{30}^{-1}$ Mpc, and it is the range of scales $l \approx R_\nu$ that will predominate in the amplitude of the nonuniformities in the neutrinos (but not in the radiation!). On scales $l \leq R_J$ the perturbations in the radiation and baryons will have a much smaller amplitude than the perturbations in the neutrino component. Thus the background temperature fluctuations will probably be dominated by the gravitational influence of the nonuniformities in the neutrino population, and the effect a will become the principal one.

Figure 2 displays smoothed spectra for the perturbations in the neutrino density (dashed curve) and in the density of a matter–radiation mixture (solid curve) at the epoch of recombination $t = t_{\rm rec}$:

$$\begin{aligned} \overline{\left(\frac{\delta\rho}{\rho}\right)_\nu^2} &= A (R_\nu k)^3 (1 + k^2 R_\nu^2)^{-9}, \\ \overline{\left(\frac{\delta\rho}{\rho}\right)_{\rm b}^2} &= \frac{64}{9} \overline{\left(\frac{\delta\rho}{\rho}\right)_\nu^2} [1 - \cos(kR_J)]^2 (kR_J)^{-4} \end{aligned} \tag{4.2}$$

for a "while noise" initial spectrum. We have here adopted $m_\nu = 20$ eV, $\Omega_b = 10^{-2}$, $R_\nu = 7.5$ Mpc, $R_J = 160$ Mpc, $A = 3 \cdot 10^{-5}$.

The background fluctuations associated with the gravitational influence of the density perturbations (effect a) will be determined by irregularities that existed at an epoch comparatively close to ours. The recombination epoch will have a decidedly weaker gravitational influence, because according to gravitational-instability theory fewer perturbations would have existed at that time. For the structure we observe in the universe today on scales $l \geq 100$ Mpc, these background temperature fluctuations should be minimal. Compton scattering effects and temperature nonuniformities at $t = t_{\rm rec}$ would only serve to reinforce the temperature fluctuations (in a statistical sense). It follows, then, that for the low baryon densities ($\Omega_b \approx 0.01$) existing in a neutrino universe we will in effect have a scenario that ensures almost the smallest possible fluctuations in the background-radiation temperature.

Large-scale fluctuation modes (quadrupole, octupole, ...) will reflect perturbations on scales comparable with the horizon,[5] $R_H \approx 6000 h^{-1}$ Mpc (if $\Omega = 1$). By studying nonuniformities on such scales we can secure direct information on the form of the initial spectrum, because for large scales the transition function in the A-theory is independent of k, so that the initial spectrum will not be distorted. In the E-theory, though, the situation will be quite different.

Measurements of the quadrupole anisotropy indicate[58,59] that

$$\left(\frac{\delta T}{T}\right)_Q \approx (1-3) \cdot 10^{-4}, \tag{4.3}$$

values 5–10 times higher than the upper limit on the fluctuation amplitude[187,188] for scales $\leq 1°$.

Using Eqs. (3.5), (3.15), (3.16), and specifying a perturbation amplitude

$$\sigma_0^2 = \sqrt{\frac{2}{\pi}} \int_0^\infty b^2(k, t_{\rm in}) c^2(k, t, t_{\rm in}) k^2 dk \tag{4.4}$$

for $b^2(k, t_{\rm in}) = b_0^2 \propto k^n$, we obtain the following expressions in the A-theory[81] for the jth harmonic $(\delta T/T)_j$ of the fluctuations and for the quadrupole ($j = 2$):

$$\left.\begin{aligned} n = 1:&\ \left(\frac{\delta T}{T}\right)_j \approx 75\sigma_0 \left(\frac{R_\nu}{R_H}\right)^2 \frac{1}{\sqrt{j(j-1)}} \xrightarrow[j=2]{} 2.2 \cdot 10^{-5} \sigma_0 h^2 m_{30}^{-2}, \\ n = 0:&\ \left(\frac{\delta T}{T}\right)_j \approx 112\sigma_0 \left(\frac{R_\nu}{R_H}\right)^{3/2} \sqrt{\frac{1}{(2j+3)(2j+1)(2j-1)}} \xrightarrow[j=2]{} 2.5 \cdot 10^{-4} \sigma_0 h^{3/2} m_{30}^{-3/2}, \\ n = -1:&\ \left(\frac{\delta T}{T}\right)_j \approx 19\sigma_0 \left(\frac{R_\nu}{R_H}\right) \frac{1}{\sqrt{(j+2)(j+1)j(j-1)}} \xrightarrow[j=2]{} 3.2 \cdot 10^{-3} \sigma_0 h m_{30}^{-1}; \end{aligned}\right\} \tag{4.5}$$

$R_H = 6000 h^{-1}$ Mpc represents the horizon scale at the present epoch, and $R_\nu = 4.8 h^{-1} m_{30}^{-1}$ Mpc.

For spectra having $n > 1$, the amplitude $\delta T/T$ of the large-scale fluctuations is nearly independent of the harmonic number, but in the case of the spectra (4.5) with $n \leq 1$ it diminishes with increasing j. For $n = -1$ the fluctuation amplitude is large, making the $n = 0$ or $n = 1$ spectrum preferable. In this respect we agree with the conclusions of Peebles.[187] But Peebles' attempts to ascribe the large-scale $\delta T/T$ fluctuations to the distribution of galaxies on scales $r \approx 30 h^{-1}$ Mpc seem unconvincing, because the effect a should predominate only in a neutrino universe.

In the E-theory, the same initial spectrum $b^2(k)$ will after recombination [by Eq. (3.8)] develop into the spectrum

$$\delta_E^2 \propto b^2(k) c_E^2(k, t, t_{\rm in}) \propto k^4 R_J^4 b^2 \propto k^{n+4}. \tag{4.6}$$

Taking the smallest possible (for power-law spectra) value $n = -3$, we obtain for the jth harmonic in the background-radiation irregularities the expression

$$\left(\frac{\delta T}{T}\right)_j \propto \frac{1}{\sqrt{j(j-1)}}. \tag{4.7}$$

If $n \geq -1$ the fluctuation amplitude will not diminish with increasing j. Since the estimate (4.7) does not allow for the Compton effect or temperature nonuniformities during the recombination era, we see that entropic perturbations with a power-law spectrum cannot account for the decline in the amplitude of the background temperature fluctuations with passage from the quadrupole anisotropy to scales of the order of degrees.

In discussing this problem Silk[64,188] has considered isothermal perturbations taking the form of an inhomogeneous distribution of baryons in the presence of constant-temperature radiation. These perturbations represent a superposition of adiabatic and entropic modes, the large-scale fluctuations being related to the adiabatic mode.

Recently some new and considerably lower estimates of $(\delta T/T)_Q$ have been given. Until this most important issue is resolved observationally, one cannot make any definite statements about the form of the large-scale asymptotics in the perturbation spectrum. If, for example, $(\delta T/T)_Q \approx 10^{-5}$, then in a neutrino universe a "fractal" spectrum with $n = 1$ [see Eqs. (4.5)] would become preferable to a white-noise spectrum with $n = 0$.

As we have pointed out, in the E-theory the amplitude $(\delta T/T)_Q$ probably should be no larger than the amplitude $\delta T/T$ on scales of several degrees. Thus the observational constraint $(\delta T/T)_Q \leq 10^{-5}$ cannot refute the E-theory for the formation of structure in the universe. One would have to perform a more thorough analysis of the whole array of empirical facts, examining, in particular, the angular dependence of $\delta T/T$ over a wide range of angles.

It has now been firmly established that the microwave-background temperature has a dipole anisotropy associated with the motion of the observer[57-59,188]:

$$\Delta T_D = 0.0033 \pm 0.0006\ {}^\circ\mathrm{K}.$$

By correcting for the motion of the solar system about the galactic center, one can also establish the velocity of the Galaxy with respect to the cosmic background:

$$V_G = 540 \pm 60\ \mathrm{km/sec}.$$

Theoretical estimates for the random velocities of clusters of galaxies yield values of 500–2000 km/sec, depending on the parameters of the cosmological model.[189] These values are consistent with indirect observational estimates.[50] Since the velocity of the Galaxy would have been determined not only by the primordial perturbation spectrum but also by the conditions under which the Galaxy was formed, dipole-anisotropy measurements will convey no information on the perturbation spectrum. To obtain such information one would have to determine the velocities of numerous clusters relative to the background radiation; for that purpose one can take advantage of the effect wherein the background photons are scattered by electrons in the hot intracluster gas.[190-193]

Figure 3 illustrates the observational constraints on the background temperature fluctuations (see Silk's review[188]). The dashed line corresponds to a $\delta T/T \propto \sqrt{\theta}$ law, and indicates how the $\delta T/T$ observations might be described in a neutrino-dominated universe.

If in massless-neutrino models one adopts densities $\Omega_t = \Omega_b = 10^{-2}$, then after hydrogen recombination the perturbations will grow[194,195] only by a factor of 5–7 (on scales $l < 200$ Mpc). The chief reason is that their growth will begin only when $\bar{\rho}_b > \bar{\rho}_r$, and will cease when $\Omega z < 1$. Hence the data now available on the background temperature fluctuations would (for Ω values of this order) be incompatible with the observed structure of the universe. Analogous estimates for $\Omega_t = \Omega_b = 1$ are far less convincing.[62]

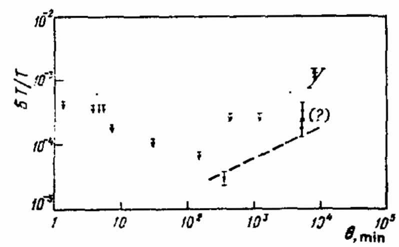

FIG. 3. Observational constraints on the temperature fluctuations of the microwave background radiation as a function of angular scale, after Silk[188] (angles in arc minutes). The dashed line represents the law $\delta T/T \propto \sqrt{\theta}$.

These arguments are exemplified by Fig. 4, which illustrates how perturbations of common scale would have grown in three cosmological models differing in their values of Ω_t and Ω_b. All the perturbations are normalized by the condition that $\delta\rho/\rho = 1$ when $z = 0$. The $\delta T/T$ fluctuations are several times smaller than $\delta\rho/\rho$. From Fig. 4 one can make a crude assessment of the capabilities of the various models. Even these rough estimates, though, are adequate to rule out models with $\Omega_t = \Omega_b = 0.01$ as inadmissible.

5. ADIABATIC THEORY FOR DEVELOPMENT OF STRUCTURE

In this section we offer a systematic outline of the adiabatic theory for the formation of structure in the universe, also called the "pancake" (*blin*) scenario, with subsequent fragmentation. As this scenario relies on the approximate nonlinear theory of gravitational instability, we begin with a rather full account of that theory.

Nonlinear gravitational-instability theory describes how perturbations whose scale much exceeds the Jeans scale $(M \gg M_J)$ will evolve by Newtonian gravitation. Under these circumstances the evolution of the disturbances will be wholly determined by gravitational forces. The nonlinear theory can describe the development of structure both in an $m_\nu = 0$ model universe and in a neutrino-dominated universe, and it also enables one to calculate the internal structure of dense regions.

In a neutrino universe, as Fig. 2 indicates, the perturbations in the neutrino and baryon components will differ strongly by the time the hydrogen-recombination epoch $z_{\mathrm{rec}} \approx 1200$ arrives. After recombination the perturbations will continue to grow in a linear regime for a considerable time. But just as in a multicomponent (pressure-free) medium there will be only a single growing mode; the baryon and neutrino perturbations will become adjusted to each other.[66,184] They will enter jointly into the ensuing nonlinear phase.

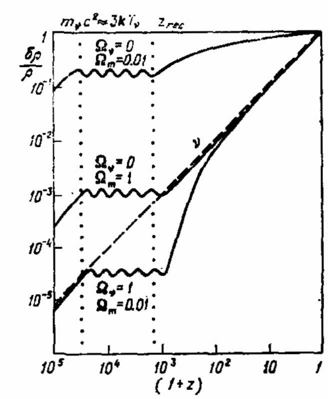

FIG. 4. Evolution of perturbations of equal present scale in three different cosmological models. Solid curves, evolution of fluctuations in the baryon component; dashed line, the neutrino component.

a) Nonlinear gravitational-instability theory in medium with $P=0$

In a first approximation, let us take a cold medium: $T=0, P=0$. We introduce Lagrangian coordinates q_i chosen to coincide with the particle coordinates in a perfectly homogeneous universe at some specified time. It is convenient to take that time to be the present epoch $z=0$. The evolution of perturbations will be described by the dependence of the Eulerian coordinates r_i of the particles, their velocities u_i, the density ρ, and other parameters upon the Lagrangian coordinates q_i and time t. Our analysis will be limited to the growing mode.

During the linear stage, that is, so long as $\delta\rho/\rho \ll 1$, the density perturbation at each particle (that is, for constant q_i) will grow in proportion to its initial value[196] $(\delta\rho/\rho)_{\text{in}}$:

$$\frac{\delta\rho}{\rho}(q_i,\ t)=\frac{B(t)}{B(t_{\text{in}})}\,\frac{\delta\rho}{\rho}(q_i,\ t_{\text{in}}); \tag{5.1}$$

here $B(t)$ represents the growing solution of the differential equation

$$a\ddot{B}+2\dot{a}\dot{B}+3\ddot{a}B=0. \tag{5.2}$$

If $\Omega=1$, then $B(t)=(t/t_0)^{2/3}=(1+z)^{-1}$. For arbitrary $\Omega<1$, the function B can be approximated to reasonable accuracy (15% or better) by the simple expression[5]

$$B(z)=(1+\omega z)^{-1},\qquad \omega=\frac{2.5\Omega}{1+1.5\Omega}, \tag{5.3}$$

which reduces to the correct laws both for $\Omega=1$ $[B=(1+z)^{-1}]$ and for $\Omega=0$.

An approximate nonlinear solution describing how the growing mode of potential (irrotational) perturbations will evolve may be written in the form[42]

$$r_i(\mathbf{q},\ t)=a(t)\,[q_i-B(t)\,s_i(\mathbf{q})]=(1+z)^{-1}\,[q_i-B(z)\,s_i(\mathbf{q})], \tag{5.4}$$

where the vector field $s_i(\mathbf{q})$ specified the initial perturbation. The potential character of the perturbations manifests itself in the requirement that $s_i(\mathbf{q})$ be a potential vector field; that is, a potential $\Phi(\mathbf{q})$ exists such that

$$s_i(\mathbf{q})=\frac{\partial\Phi}{\partial q_i}. \tag{5.5}$$

This condition ensures that the velocity field will be a potential field in r-space until the particle trajectories intersect.[42] Presently (Sec. 5b) we shall consider the accuracy of the approximate solution (5.4), but first let us describe its distinctive features.

Knowing the coordinates explicitly we can easily find both the velocity and the density as functions of t and $\mathbf{q}$:

$$u_i(\mathbf{q},\ t)=\frac{dr_i}{dt}=Hr_i-a(t)\,\frac{dB}{dt}\,s_i(\mathbf{q}), \tag{5.6}$$

$$\rho=\rho_0|D_{ik}|^{-1}=\bar{\rho}(t)|\delta_{ik}-B(t)\,\partial s_i/\partial q_k|^{-1}, \tag{5.7}$$

where $|D_{ik}|=(1+z)^{-3}|\delta_{ik}-B\,\partial s_i/\partial q_k|$ represents the Jacobian of the transformation from r to q, and $\bar{\rho}=\rho_0(1+z)^3$. In view of the potential property of $s_i(\mathbf{q})$ [Eq. (5.5)], we can reduce the tensor $\partial s_i/\partial q_k$ to the principal axes at each point and find the principal values $\alpha(\mathbf{q}), \beta(\mathbf{q}), \gamma(\mathbf{q})$, which for definiteness will be considered to be ordered as follows:

$$\alpha(\mathbf{q})\geqslant\beta(\mathbf{q})\geqslant\gamma(\mathbf{q}).$$

When written in terms of the principal values d_{ik} for each particle, Eq. (5.7) for the density will become

$$\rho(\mathbf{q},\ z)=\bar{\rho}(1-B\alpha)^{-1}(1-B\beta)^{-1}(1-B\gamma)^{-1}. \tag{5.8}$$

So long as the perturbations are small, with $B\cdot\alpha, B\cdot\beta, B\cdot\gamma<1$, Eq. (5.8) will expand to

$$\rho(\mathbf{q},\ z)\approx\bar{\rho}(z)\,[1+B(\alpha+\beta+\gamma)], \tag{5.9}$$

which agrees with the result (5.1) of the linear theory. At later stages, however, the density has to be determined directly from Eq. (5.8), which states that at "particles" where α reaches a positive maximum value $\alpha=\alpha_{\text{m}}>0$ at time $t=t_{\text{m}}$ such that

$$1-B(t_{\text{m}})\,\alpha_{\text{m}}=0, \tag{5.10}$$

the density will develop a singularity: $\rho\to\infty$.

These density singularities arise simply because we have neglected the initial temperature of the medium. Any small but finite temperature will remove the singularities, but the lower the temperature of the medium, the higher the density will be.

We return to our case of a cold medium. It is important to recognize that the density will become infinite because of a *one*-dimensional contraction along the principal axis $\mathbf{n}_\alpha$ corresponding to the eigenvalue α at the point $\mathbf{q}_{\text{m}}$. Along the other two principal directions $\mathbf{n}_\beta, \mathbf{n}_\gamma$, either an expansion or a contraction can take place at this stage, depending on the signs of β, γ at the "particle" in question.

Our solution (5.4) predicts, then, that during the nonlinear stage highly flattened clouds of compressed matter will be formed—pancake structures.

In Lagrangian space, ellipsoids with a finite axial ratio will develop in the neighborhood of the maxima $\alpha=\alpha_{\text{m}}$ on the isosurface $\alpha=\text{const}$. The density will become infinite concurrently at all particles lying on the surface of such an ellipsoid [see Eq. (5.8)]. When that happens the particles will form in Eularian space a pancake surface which will bound an extremely anisotropic body: shortly after the pancake has been created (when $t-t_{\text{m}}\ll t_{\text{m}}$) its thickness will be $d_1\sim(t-t_{\text{m}})^{3/2}$ whereas its transverse scales will be $D_{2,3}\sim(t-t_{\text{m}})^{1/2}$. The pancake will be born thin:

$$\frac{d_1}{D_{2,3}}\propto(t-t_{\text{m}})\to 0\quad\text{as}\quad(t-t_{\text{m}})\to 0. \tag{5.11}$$

On comparing Eqs. (5.8) and (5.9) we see that the anisotropic character of the contraction process will become distinctly apparent only during the nonlinear evolutionary phase. But it is worth emphasizing that even during the stage of small perturbations the deformation will have been anisotropic, described by the same principal values of the deformation tensor, α, β, γ. It is merely that during the linear phase this deformation anisotropy would not have manifested itself in the evolution of density perturbations, which would have depended [Eq. (5.9)] on the sum $\alpha+\beta+\gamma$.

b) Accuracy of approximate solution

Before examining the accuracy and range of applicability of the approximation (5.4) in the general case, let us consider two particular examples where it represents the exact solution.

First, it is easy to see that if the perturbations are small, with $B\alpha, B\beta, B\gamma \ll 1$, the solution (5.4) will possess the same accuracy that is given by the linear theory of small perturbations.

Second, one can show that if the perturbations are one-dimensional,[7)] with $s_1 = s_1(\mathbf{q})$ but $s_2 = s_3 = 0$, then Eq. (5.4) will remain the exact solution until the epoch when the density singularities have developed.[110,197]

Now let us turn to the general case of an arbitrary three-dimensional perturbation. As we have explained, the mass motions specified by Eq. (5.4) will determine the density distribution (5.8), which has been derived from the equation of continuity. But the density can be calculated in another way as well. By using the Euler and Poisson equations one can establish what density ρ_n would be needed to produce the motions described by Eq. (5.4):

$$\rho_n = -\frac{1}{4\pi G}\frac{\partial}{\partial r_i}\left\{\frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left[a\left(\frac{\mathrm{d}r_i}{\mathrm{d}t} - Hr_i\right)\right]\right\} + \bar{\rho} = \frac{\bar{\rho}(1 - B^2J_2 + 2B^3J_3)}{(1 - B\alpha)(1 - B\beta)(1 - B\gamma)},$$
$$J_2 = \alpha\beta + \alpha\gamma + \beta\gamma, \quad J_3 = \alpha\beta\gamma. \tag{5.12}$$

In arriving at Eq. (5.12) we have made use of Eq. (5.2). The quantity

$$\Delta = \frac{\rho_n - \rho}{\rho} = -B^2J_2 + 2B^3J_3, \tag{5.13}$$

which depends on the Lagrangian coordinates q_i and time, measures the accuracy of the solution (5.4). At the time when a pancake develops, when $B = 1/\alpha$ and $\rho \to \infty$, the value of Δ near its center will be

$$\Delta = -\frac{\beta}{\alpha} - \frac{\gamma}{\alpha} + \frac{\beta\gamma}{\alpha^2}. \tag{5.14}$$

Accordingly, the relative error in the density will remain finite even though the density becomes infinite.[197]

On analyzing Eq. (5.13) one finds that in the case of small perturbations ($B\alpha, B\beta, B\gamma \ll 1$) the error $\Delta \approx -B^2J_2 \to 0$, so that the solution (5.4) will possess first-order accuracy. But in the case of a one-dimensional perturbation ($\beta = \gamma = 0$) the error $\Delta \equiv 0$, and the solution (5.4) will be exact until the epoch when the density singularities are formed.

A pancake will be created at the epoch t_m when $\rho(q_m, t_m) \to \infty$. Its internal structure will depend on the properties of the matter from which it was formed. If that matter consisted of collisionless particles, such as stars or neutrinos, then there would be a region of three-stream flow inside the pancake, whereas its boundary would be an infinite-density surface—a caustic. However, even small thermal velocities would spread this infinitely thin surface into a layer in which ρ, while large, would remain finite. If the pancake instead were to be formed from gas, then at the time of its creation two shock waves would develop, diverging from each other and delineating its boundaries. The space between them would be filled with hot gas.

In neither case will the solution (5.4) apply inside the pancake. For a gaseous pancake one must allow for the pressure, which we have neglected. In a collisionless pancake a multistream flow should be considered, although since the error involved is not so sizable in this case, the solution (5.4) will continue to give a qualitatively correct result for some period after the pancake has been formed.

Qualitatively it is apparent that the solution (5.4) will work best near the center of the pancake. So long as the perturbations remain small, this solution will be at least as good (of the same order of accuracy) as the solution given by the linear theory. And when the perturbations have grown to be of order unity and the one-dimensional contraction stage sets in, the solution (5.4) will continue to provide a good description.

c) Statistical aspects of the nonlinear theory

As indicated in Sec. 5a, the dominant role in the nonlinear theory for the formation of structure by gravitational instability is played by the principal values α, β, γ of the tensor $d_{ik} = \partial s_i/\partial q_k$. It therefore is of great interest to inquire how likely a given set α, β, γ is to occur. Let us suppose, in the spirit of the de Moivre–Laplace limit theorem, that the small density perturbations $\delta\rho/\rho$ are normally distributed. Then it can be shown[198] that a given set α, β, γ will be realized with probability

$$\mathrm{d}P(\alpha, \beta, \gamma) = \frac{27}{8\sqrt{5}\pi\sigma^6}(\alpha - \beta)(\alpha - \gamma)(\beta - \gamma) \times \exp\left(-\frac{3}{5\sigma^2}J_1^2 + \frac{3}{2\sigma^2}J_2\right)\mathrm{d}\alpha\,\mathrm{d}\beta\,\mathrm{d}\gamma$$
$$\infty > \alpha > -\infty, \quad \alpha \geqslant \beta > -\infty, \quad \beta \geqslant \gamma > -\infty, \tag{5.15}$$

where $J_1 = \alpha + \beta + \gamma$, $J_2 = \alpha\beta + \alpha\gamma + \beta\gamma$, and σ^2 denotes the dispersion of the diagonal components of the tensor $d_{ik} = \partial s_i/\partial q_k$. The factors $(\alpha - \beta), (\alpha - \gamma), (\beta - \gamma)$ are of particular interest here, for they demonstrate that the coincidence of two (or three) principal values has an especially low probability, far lower than that of the coincidence of two independent quantities.

Upon integrating Eq. (5.15) one finds that only about 8% of the matter should contract along all three axes ($\alpha > \beta > \gamma > 0$); 42% will contract along two axes ($\alpha > \beta > 0 > \gamma$), and 42% along just one axis, while 8% of the matter will expand along all three axes. But the condition for matter to collapse into a pancake does not depend on α alone. Since the pancake may be considered thin, we can obtain a good estimate of its mass, and thereby of the total fraction w of the mass that will be contained in pancakes, by treating a pancake as an infinitely thin surface coincident with the central layer of the pancake:

$$w(\eta) \approx \sqrt{3}\,P\ (\alpha > \eta\sigma). \tag{5.16}$$

The derivation of Eq. (5.16) makes use of the condition that $\alpha(\mathbf{q})$ reach an extremum at the center of the pancake. If more than (20–30)% of all matter should go into pancakes, the expression (5.16) would give an overestimate. For large values of w one may adopt the approximation[189]

$$w(\eta) \approx \left[1.12\eta + 0.92(1 - 1.2\eta + 0.5\eta^2)\exp\left(-\frac{\eta^2}{4}\right)\right]\exp\left(-\frac{\eta^2}{2}\right), \tag{5.17}$$

[7)] A homogeneous Hubble flow will continue to be maintained along the two other, undisturbed directions.

where $\eta=\alpha/\sigma$, $\sigma^2=\langle(\delta\rho/\rho)^2_{in}\rangle/5$; the quantity $\langle(\delta\rho/\rho)^2_{in}\rangle$ should be calculated from the equations of the linear theory at initial time t_{in}.

The expression (5.17) will agree with the estimate (5.16) if $\alpha\gg\sigma$, and it will retain acceptable accuracy (≈10%) for values $\alpha<\sigma$. In the linear theory half the matter will expand under the assumptions we have made, and half will contract. In the nonlinear regime one finds that no less than 92% of the matter $[P(\alpha>0)=0.92]$ will eventually turn into pancakes. By the present epoch the condition (5.10), $1-B(t_0)\alpha=0$, has probably been satisfied for all "particles" for which $\alpha\approx\sigma$, and that, by Eq. (5.17), would correspond to the condensation of about 80% of all matter into pancakes.

The $w(\eta)$ relation is a universal one, but if we are interested in the dependence of the fractional mass in pancakes upon time t (or upon redshift), then we will have to use the explicit form of the function $B(t)$. The function $w(t)$ will therefore depend on the parameters of the cosmological model as well.

The statistical aspects of the nonlinear gravitational-instability theory have been discussed more fully by two of us elsewhere.[198,189]

d) Structure of an individual pancake

Thorough studies have been made of the internal structure of a pancake, both analytically and numerically, beginning with the early work by one of us[42] on the nonlinear theory of gravitational instability.

It was established at that time that inside a gaseous pancake there should be a thin layer of adiabatically compressed gas.[42] Adjoining it on either side would be layers of gas compressed and heated in the shock wave that forms the boundary of the gaseous pancake.[110] The gas located closest to the adiabatic layer should have been able to cool to a temperature $T\approx10^4$ °K, but farther out there would be a layer of hot ($T\approx10^6-10^7$ °K) gas bounded by the shock on the outside.

Pancakes consisting of neutrinos and of gas should be formed simultaneously and be located in the same places, but the neutrino pancakes will be some 1.5 times as thick[199] (Fig. 5). In the cold-medium approximation the neutrino pancakes will be bounded by caustics–infinite-density surfaces. Three-stream flows will initially develop between the caustics, followed by multistream flows. But the original thermal velocity

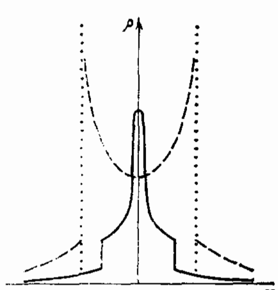

FIG. 5. Schematic density profile across a two-component pancake. Solid curve, the varyong density; dashed curve, the neutrino density; dotted lines, the position of the caustics.

dispersion of the neutrinos will spread out the pancake boundary, limiting the maximum density to a level[200] $\rho_{max}\approx(10-20)\bar{\rho}_\gamma$.

The theory of the structure of a neutrino universe contains a sole small parameter[24]: the amplitude of the metric perturbations, $h_\nu\approx10^{-4}$. This parameter determines the relative role played by gravitation and by the neutrino thermal velocities during the transition to the nonlinear stage in the growth of irregularities. It also will determine the maximum neutrino density that can be achieved at the time a pancake is formed, and later on its boundary[200]:

$$\rho^{(c)}_{max}\approx h_\nu^{-1/3}, \tag{5.18}$$
$$\rho^{(s)}_{max}\approx h_\nu^{-1/4}. \tag{5.19}$$

These dependences of ρ_{max} on h_ν are weak, because the "gas" of collisionless particles will have the very rigid equation of state $P\propto\rho^3$ during the one-dimensional contraction.

e) Formation of galaxies and clusters of galaxies

As yet there has been little study of the processes whereby a pancake would break up into separate clouds, with stars forming in them. We therefore shall merely discuss the scenario in general outline; in that way we will in fact be able to refine certain constraints that are imposed on the theory.

The analysis of the gas-dynamical and thermal processes taking place in a pancake has little to do with the presence of neutrinos, so the conclusions drawn without reference to neutrinos will largely remain valid. This class of phenomena has been examined in a series of papers[110-113,201]; in summarizing the results we can single out the following processes as most important.

1. For real three-dimensional pancakes, as matter crosses the oblique shock wave bounding a gaseous pancake fairly powerful vortex flows will develop in the compressed gas.

2. Kelvin-Helmholtz instability will set in the cooled central layer, causing it to fragment into separate clouds and turbulizing the hot gas.

3. In the cooled pancake gas thermal instability will develop,[202,203] and in the pancake gravitational field, in the presence of vortical tangential velocities, it will serve to mix and turbulize the flows within the pancake. This turbulence will be sustained by the energy flux crossing the shock wave.

4. Explosions of the earliest stars will help to turbulize the gas in the pancake, perhaps inducing fragmentation, and they will also enrich the gas with heavy elements. Probably these explosions will be of considerably smaller scale than some authors believe.[129]

We arrive, then, at the concept of a three-component pancake: clouds of dense, cool ($T=10^4$ °K) gas moving within hot ($T=10^6-10^7$ °K) gas, all embedded within a neutrino pancake. However, to pass from the picture of an ideal one-dimensional pancake whose different components are arranged in layers to the cloud structure in a three-component pancake, it is not enough to point to mechanisms of thermal and hydrodynamic instability. Small-scale seed perturbations are also required. On the scales of interest to us, however, adiabatic perturbations will not reach sufficient amplitude. In a neutrino-dominated universe one possible way to preserve perturbations of requisite scale would be through classical entropic perturbations of the baryon component which result from certain phase transitions in the early universe.[8)]

The initial perturbation spectrum will rapidly be effaced under the conditions prevailing inside a pancake, and a quasistationary regime will be established in which perturbations occur on all scales, with the outcome determined not so much by the seed perturbations as by features inherent in the nonuniformity growth processes. While this state of affairs does not remove the need to specify suitable initial small-scale inhomogeneities, it does mitigate the demands on their amplitude and characteristic scale.

In the scheme we are discussing, the clouds of cool, dense gas that develop will themselves be inhomogeneous: they may contain inclusions of smaller-scale gas clouds which do not undergo shock compression and which possess their primordial entropy. Under these circumstances the first stars, of mass $M_* \approx 10-100\ M_\odot$, would probably be able to form[205,206] within gravitationally bound clouds of mass $M_{c1} \approx 10^6-10^7\ M_\odot$. These complexes of gas and stars may resemble dwarf galaxies in their properties, or perhaps the globular clusters in galaxies. Both possess a very low heavy-element abundance[207-209] and undoubtedly were formed from primordial matter not contaminated with heavy elements (or hardly so).

The formation of the first stars would have enriched the pancake gas with heavy elements through supernova explosions in the dwarf galaxies, thereby producing inhomogeneities in the chemical composition of the pancake gas. The chemical inhomogeneity would in turn have helped to trigger thermal instability, and would have led to the formation of novae and so on. Perhaps we are today observing the culminating stage of these processes, in the form of dwarf blue galaxies.[207,209]

X-ray observations of the hot gas in clusters of galaxies have revealed a high (near-solar) heavy-element abundance.[101,210] At the same time a markedly lower heavy-element content is observed not only in certain dwarf galaxies and globular clusters but in many stars of our Galaxy. This disparity can be resolved in a natural way by the scheme we are proposing, because: a) many of the stars (including those that belong to the Galaxy) would have been formed during the early evolution of a pancake even before the large galaxies developed; b) nucleosynthesis of heavy elements in dwarf galaxies would naturally have produced a high heavy-element abundance in the gas outside the large galaxies.

The influence of explosions of fully evolved objects upon the formation of new ones is much weakened in the pancake picture, since the highly flattened pancake shape will aid the shocks to breach the layer of cool gas, and the energy from the explosions will quickly break out into the surrounding hot gas. But these explosions probably will not only enrich the pancake gas with heavy elements: they should help to sustain the turbulence in the gas and may also promote fragmentation of an individual pancake, or supercluster of galaxies, into separate groups of galaxies. In addition, this process presumably would expedite a steep increase of the dispersion in the orientation of the angular-momentum vectors of the member spiral galaxies, thereby accounting for the indeterminacy of the observational data on this point.[211-216]

Galaxies and galaxy clusters would, if this scheme is correct, be created through gravitational aggregation of dwarf galaxies, star–gas complexes, and galaxies. Now, however, one can no longer ignore the neutrinos, whose mean density in a pancake will be at least that of the "visible" matter. This problem of star–gas complexes on differing scales and neutrinos (which, by the way, would possess a sharply anisotropic velocity distribution) agglomerating under the conditions prevailing in the highly anisotropic pancakes warrants a comprehensive three-dimensional numerical analysis. Some simplified models have already been studied, though, and they have yielded very promising results.[124,127,217-223]

In particular, it has been shown that by simulating the collapse of an ensemble of point masses initially distributed in a highly flattened ellipsoidal configuration (axial ratio 1:10 or 1:20), one can interpret the following properties of elliptical galaxies and rich clusters:

a. The anisotropy of the particle distribution and velocity dispersion, reflecting the fact that many (but not all) particles have an auxiliary integral of motion.[162]

b. The similarity of the surface-density profile to the laws observed, even though in phase space the particles may have very complicated distribution functions.[222]

Carnevali *et al.*[223] have analyzed numerically the evolution of groups comprising 10 or 20 systems of particles (galaxies) and simulating the formation and evolution of sparse clusters of galaxies. They demonstrate that groups can develop resembling the ones observed, including a giant cD-type galaxy at the center.

Calculations of galaxy formation processes should naturally incorporate the hydrodynamics of the gaseous phase; that will much complicate the problem. A full numerical simulation of these processes is therefore a task for the future. Still, even the results obtained thus far are in good accord with a variety of observations.

Several authors have shown[224-227] that the oblateness of elliptical galaxies has nothing to do with their rotation. Accordingly it has been the practice to assume that the flattening of elliptical galaxies is attributable to the anisotropy of their stellar velocity dispersion,[124,127,217-222] in the spirit of the scheme outlined above.

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8)Some role may perhaps also be played by entropic perturbations arising from the dissipation of the adiabatic perturbations.[204]

That scheme in fact brings us to a very interesting conclusion: galaxies evidently never constituted single quasistationary clouds—protogalaxies, which is not to rule out, however, that galaxies may have experienced stages in their evolution when (10–20)% of their mass went through a gaseous phase. In spiral galaxies, with their prominent disk component, gas-dynamical processes presumably would have played a more important evolutionary role than in elliptical systems.

Quite possibly the scenario we have been discussing can serve to explain why efforts to observe young galaxies have not met with success. In the traditional approach[228] a protogalaxy would transform into a galaxy to the accompaniment of a powerful burst of star formation. It has been estimated[228,229] that such an object—a young galaxy—would be observable at very great distances. But searches have failed to reveal any objects of this kind.[230,231] As a result, some of the strongest supporters of the concept of protogalaxies and bright young galaxies have lately given up that idea.[232]

In the model considered above much of the supernova activity would have taken place prior to or in the course of the galaxy formation process. The energy of the supernovae would have been injected into a substantially larger volume, greatly diminishing the brightness of the "young" galaxies. From the point of view that we have developed, the genuinely young galaxies would be blue, active dwarf systems.[207,209] The count of quasars seems to fall off rapidly[233] for redshifts $z>3.5$, suggesting that all the processes of structure formation probably occurred comparatively recently, when z had dropped below 5–6, and were not too violent.

f) Cluster formation and the missing-mass problem

Even the earliest measurements of the velocities at which galaxies move within clusters showed that the visible mass could not suffice to keep the clusters in a steady state.[93] The visible mass is determined from the combined luminosity of the galaxies making up a cluster, and from a galaxy mass–luminosity relation established independently (such as by studying the rotation of galaxies, or from double galaxies). By applying the virial theorem to the observed motion of the galaxies in a cluster one can evaluate its dynamical mass:

$$\frac{GM}{R}=\frac{v^2}{2}, \tag{5.20}$$

where M, R denote the mass and radius of the cluster and v^2 is the velocity dispersion of the member galaxies. For well-investigated, steady-state clusters such as the Coma cluster, the dynamical mass is 10–50 times the visible mass.[1,94-100]

In 1974 evidence emerged that hidden mass probably is also present in certain individual galaxies.[103,104] Subsequent measurements of the rotational velocities of galaxies both optically and from the neutral-hydrogen radio line (beyond the optical disk of the system) have borne out this conclusion.[90-94] In galaxies the unseen mass has turned out to be located chiefly beyond the visible contour. Lately, however, it has been argued[102] that many pairs of galaxies fail to show evidence of hidden mass. The problem is a complicated one and is far from being fully solved.

The first suggestions that the missing mass might be attributable to finite-mass neutrinos seem to have been put forward[71,72] in the early 1970s, but a systematic resolution of this issue calls for analysis of the complete picture of formation and evolution of density fluctuations in the nonlinear regime. Such a treatment could be carried through today on the basis of the nonlinear gravitational-instability theory (Sec. 5a) and concepts regarding rapid relaxation processes. Having outlined above the general manner in which nonuniformities should evolve, we would here simply reemphasize that a treatment of galaxy formation cannot avoid allowing for dissipative processes such as collisions among agglomerating clouds of gas and stars, which would cause the clouds to merge, with their kinetic energy being radiated away and their matter settling toward the center of the condensation. Only by considering processes of this type would one be able to account for the exceptionally high densities that are encountered in the central parts of galaxies[234] (up to 10^{-20} g/cm^3).

A study has recently been made[235] of dissipative effects in the baryon component and their possible influence on the growth of the neutrino density during the galaxy formation process. It has been shown that the rise in the density ρ_b of colliding particles will cause the gravitational field to vary with time, leading in turn to a rise in the density ρ_ν of the collisionless particles, except that it would be more gradual:

$$\rho_\nu \propto \rho_b^{3/4}. \tag{5.21}$$

The collisionless-particle density will diminish more slowly with radius than the density of ordinary matter; and if the ratio ρ_b/ρ_ν can be $\approx 3-10$ in the central regions, then near and beyond the boundaries of the galaxies one may expect to find $\rho_\nu>\rho_b$, while at a distance of several optical radii of the galaxy the neutrino density should be close to the background neutrino density in the pancake—several times the density of the visible matter in the same region of space.

If neutrinos are indeed responsible for the missing mass, then in rich clusters it should manifest itself very differently from what it does in groups or individual galaxies. The reason lies in the differing gravitational potentials of rich clusters and superclusters and of galaxies. Clusters have a high gravitational potential, corresponding to velocities of the order of 10^8 cm/sec; tidal forces are strong as well, giving rise to common extended envelopes around galaxies. Hence the neutrinos in a cluster probably will not be associated with the individual galaxies. In clusters it would be more natural to find a general neutrino background, analogous to a missing-mass background consisting of hypothetical dwarf stars, black holes, and so on. Probably the background of massive neutrinos in a cluster would not manifest itself in any special way. On the contrary, in individual galaxies, hypergalaxies, and groups of galaxies the contribution of the neutrino background to the missing mass could be somewhat smaller than in clusters and giant galaxies.

One can formulate a general principle: if the evolution of a system is dictated by gravitational forces (the formation and evolution of a pancake, and thereby a cluster of galaxies), the neutrinos and the matter will evolve in such a way that the missing-mass density is proportional to the density of luminous matter. But in those cases where a major role is played by dissipative processes, thermal instability, and radiative energy losses (galaxy formation on all scales, star formation, and the like), the matter will become detached from the neutrinos, forming high-density complexes. Meanwhile the neutrinos will persist, in accord with the law (5.21), as a halo surrounding the system.

The principle formulated above evidently will not be applicable to neutrinos alone. The evolution of any mixture of particles interacting only gravitationally, such as a mixture of ordinary and mirror matter,[236] will follow the same scheme: a common formation of pancakes and of clusters and superclusters of galaxies, with a separate formation of galaxies, stars, and so on.

In dissipation-free collapse, according to the Liouville theorem, the phase density of the particles would be preserved. However, as Lynden-Bell pointed out some years ago,[237] in a collisionless clustering process the phase density will become a very complicated function of the coordinates and momenta, filling only a small fraction of the phase volume. This behavior has repeatedly been confirmed by numerical experiments for one-dimensional problems.[44,238-240] Only an appropriately averaged phase density has true physical significance, but the averaging process itself will seriously diminish the phase density, leading in fact to a growth in the effective "entropy" of the contracting matter. When allowance is made for the highly nonmonotonic character of the true phase function (or for the effective entropy, if an average is taken), the Liouville theorem will reduce to an inequality restricting the maximum value of the phase density. This inequality can be rewritten as a constraint on the size or mass of a stationary object formed through dissipation-free collapse[74,79,80]:

$$\frac{R}{l\,\mathrm{kpc}} > 4.6 m_{\nu 0}^{-2} \left(\frac{v}{100\ \mathrm{km/s}}\right)^{-1/2}. \tag{5.22}$$

Just as in $m_\nu = 0$ models of the universe, the true density (or size) and velocity dispersion in galaxies and clusters of galaxies will here be determined solely by the perturbation parameters during the initial contraction of the nonuniformities and will be practically independent of the primordial neutrino distribution function.

g) Development of lattice-cell structure

In the past few years a new concept, that of lattice-cell structure in the universe, has been formulated in light of all the observational evidence. Concurrently a theory for the origin of large-scale structure in the universe has been worked out analytically and numerically on the basis of the nonlinear theory of gravitational instability. Let us briefly review the basic principles of that theory.

Turning to the approximate nonlinear solution, let us trace the evolution of the collisionless component—the neutrinos. We shall neglect thermal velocities. In a formal sense Eq. (5.4) represents a one-parameter family of mappings of Lagrangian space into Eulerian space, with time as the parameter. The function $s_i(\mathrm{q})$ is a random but *smooth* function of the coordinates q_i. Over short time intervals, while the perturbations remain small, the map of q into r will be regular. But as time passes, singularities will begin to emerge: the density ρ will become infinite on certain surfaces.

The mappings (5.4) are Lagrangian (see the books by Arnol'd and his colleagues[162,164] for a discussion of the properties of Lagrangian maps), so that we can make use of the classification that has been developed for the singularities of Lagrangian maps and their rearrangements.[23,159-164] Although the solution (5.4) is not exact, its qualitative behavior and the type of singularities and their rearrangements that it predicts are strictly the same as in the case of the unknown exact solution. All the laws for the growth of density in the neighborhood of the singular surfaces, lines, and points will remain in force, as well as the topological structure of each type of singularity.

The singularities discussed here, those predicted by the approximate solution (5.4), have a more general character than that solution. For example, the very same singularities will develop in a collisionless medium of noninteracting particles, each moving at constant velocity. Another example would be the formation of caustics as a parallel light beam passes through a plate having a "random" smooth surface; accordingly one can apply optical techniques to simulate the evolution of two-dimensional perturbations in a collisionless medium.[165]

Pancake-type singularities are always the first to develop; in catastrophe theory their two-dimensional analog has received the name "lips." As time passes, the pancakes that have already been created will grow in size; new ones will appear, individual pancakes will merge and intersect, and structure will develop: regions of compressed matter will form comparatively narrow walls separating regions of lower density. Both the pancake formation process and the merger or intersection of two pancakes constitute a perfectly definite type of "catastrophe": singularities of the mapping (5.4).

Equation (5.4) establishes a relation between the Lagrangian and Eulerian coordinates of the particles. Hence to each singularity in Eulerian space there will correspond a definite structure in Lagrangian space. An important part is played here by the distribution in q-space of the principal values α, β, γ of the tensor $d_{ik} = \partial s_i/\partial q_k$. We have previously stated that pancakes will be created at the maximum points of the largest principal value, $\alpha = \alpha_{\max}$. The rim of a pancake will move in q-space[9)] along a surface on which the eigenvector $\mathbf{n}_\alpha$ corresponding to the principal value of α is tangent to the level surfaces:

$$(\mathbf{n}_\alpha \nabla \alpha) = 0. \tag{5.23}$$

[9)] The map of q into r will give the motion in r-space.

The pancakes will merge at saddle points of α. In the formation of structure a major role will be played by "swallowtail"-type singularities, which in two dimensions will have the appearance of triple points—nodal clumps of particles from which three "rays" extend, tracing out regions of high particle density (Fig. 6).

Once the singularities associated with the largest eigenvalue of α have emerged, analogous singularities associated with β and then with γ will successively develop; they will be realized within pre-existing pancakes, forming denser linear and compact structures probably will manifest themselves as rich clusters of galaxies and chains of clusters. The overall structure will become cellular: compact regions will be interconnected by a system of lines (Fig. 6).

The more compact the singularity—that is, the smaller the dimensions of the space in which it develops—the steeper will be the law by which the density grows in its neighborhood. This behavior is supported by a general conclusion[45]: compact formations will be the most prominent structural elements, lineations will be less prominent, and surfaces will be still more weakly defined. As time passes, the pancakes which have initiated the process of structure formation will be pushed aside by the "younger competitors"—clusters and filaments—into second (or better, third) place. The general trend will be such that with passage of time neither two-dimensional nor one-dimensional structures will remain; the future will belong to compact formations—to rich clusters of galaxies!

The picture of the formation and evolution of structure that we have described relies on the general theory of Lagrangian singularities, which constitute the set of elements—a comparatively small set—from which the structure is built. These singularities, it is worth emphasizing, will arise under initial conditions of general type, that is, conditions which preclude any type of special organization. Hence these singularities will remain stable: a small random perturbation will slightly alter the place and time that the singularities come into being, but it will be unable to change their type or to liquidate them. There is nothing that resembles spherically symmetric or even homogeneous collapse among these singularities, so the attempts that have been made to apply spherically symmetric or homogeneous ellipsoidal models for constructing a theory of large-scale structure[242] would seem to be an overidealization.

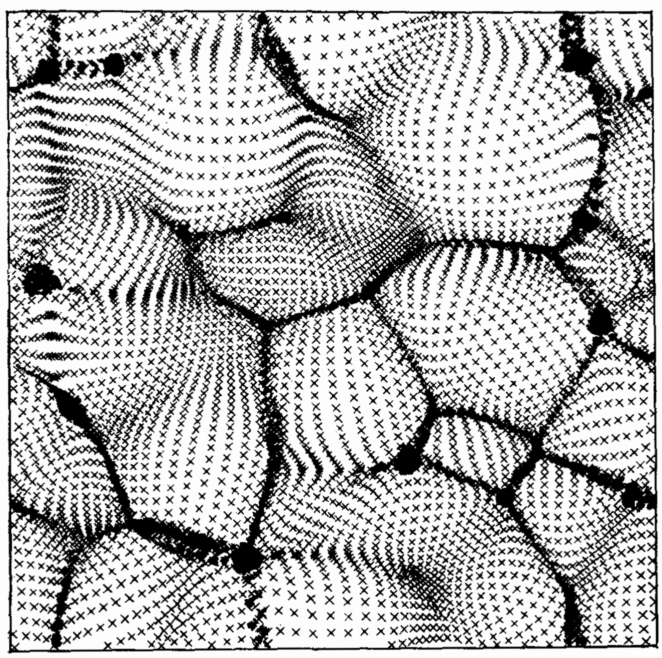

FIG. 6. Particle distribution obtained by Melott[241] from a two-dimensional numerical simulation based on the adiabatic theory for the origin of structure in the universe.

Let us return to the process of structure formation. We shall suppose that galaxies can be formed only within pancakes, and that the space between the pancakes is filled with ionized gas whose density is several times lower than the mean density of matter in the universe. Galaxies will never be created within this gas. In other words, "black regions" will exist—voids containing no objects (or hardly any) that emit light.

The total volume within which the galaxy density is high will evidently be many times smaller than the volume of the black regions. Rather unexpectedly, then, the "bright" volume (in which galaxies are born) will form a continuous family of surfaces and lines (in the limit, a lattice structure), rather than individual patches (see Fig. 6).

If the two types of regions—bright and dark—are randomly distributed, one would expect the regions occupying the smaller volume to be strewn like isolated drops within the continuous bulk taking up the larger volume.[24] An everyday example: milk comprises 6% fat and 94% water (by volume), with individual blobs of fat being embedded in a continuous mass of water.

In the universe roughly 80% of all matter (by mass) would occur in the bright regions, where the density is about 10 times the average. Hence the bright regions would occupy $\approx 8\%$ of the whole volume. The remaining 20% of matter would occupy $\approx 92\%$ of the whole, so that the density of the matter in the voids would be ≈ 0.22 of the average density. Thus the bright regions, it would seem, ought to be disconnected and surrounded by a continuous dark region. But the cell structure implies that, surprisingly enough, the actual situation is quite different.

Let us again consider the motion of matter in Lagrangian coordinates, but now we shall not insist on a solution of particular form. We shall take the position r of a particle to be a function of its initial position q and of time t:

$$\mathbf{r} = \psi(\mathbf{q}, t). \tag{5.24}$$

We choose the initial position early enough, at time t_{in}, for the density ρ of matter still to have been little perturbed, and everywhere approximately equal to the value $\bar{\rho}_{in}$.

In coordinates q, consider a bright region, wherein matter will be compressed and galaxies will be formed. In these coordinates, since the initial density is constant the proportionate volume occupied by bright regions will be equal to the proportionate mass, because $dm = \rho_{1u} d^3q$. Hence if the bright regions contain more than half of the matter (≈80% in the example given above), it will be natural for them to join up into a single continuous region. The dark regions, which occupy ≈20% of **q**-space, will take the form of separate blobs in **r**-space.

The central point of this whole argument is the continuous dependence of **r** on **q**: the vector function $\psi(\mathbf{q}, t)$ nowhere has any discontinuities. Physically, this property means that two neighboring points will always remain neighbors. In **q**-space, within the bright region, we take some continuous closed surface containing a dark blob inside. This continuous surface will remain continuous even after transformation to **r**-space. But then the bright regions will necessarily be connected with one another, which is what we had to show.

At the same time one should not forget that the whole picture of structure formation is statistical in character. Several different stages of the process may therefore coexist in space. Furthermore, we have considered an idealized pattern for the evolution of the mean density distribution, whereas what we actually observe is the pattern of the luminosity distribution. Although these are not quite identical, they will undoubtedly bear a qualitative similarity.

The next important circumstance that ought to be remembered is the process of nuclear energy release as galaxies and stars are formed. The energy liberated as supernovae explode in a galaxy will be capable of significantly affecting the subsequent evolution of the matter surrounding that galaxy.

Evidently the process of explosive energy release[17,129,243] might have some effect upon the galaxy formation process, but it hardly is likely to exert significant influence on the large-scale structure. The explosions are even more unlikely to have any effect on the distribution of the bulk of the mass in the universe, if it resides in the form of massive neutrinos. We discuss this question more fully in Sec. 6.

By explicating the qualitative aspects of the formation of lattice structure in the universe, we have lately been able to begin a quantitative analysis of this structure,[120] relying on a study of the parameters of regions defined by constant-density surfaces. It has been shown that the perturbation spectrum will define a characteristic scale, and that the observed structure parameters can be expressed in terms of that scale: the number of superclusters per unit volume, the average size of an individual pancake/supercluster, and the mean distance between pancakes/superclusters along the line of sight. In turn several expressions relate these parameters to one another and to the fractional mass contained in pancakes. Verifying whether these relationships actually hold represents an important test for the A-theory of the large-scale structure of the universe.

In linear gravitational-instability theory one often measures scales by the mass of the particles contained in a sphere whose radius (or diameter) is equal to the wavelength $\lambda = 2\pi/k$ (or inverse wave number $\lambda = 1/k$) having the scale in question. Estimates of this kind are sometimes invoked to draw inferences regarding the agreement or conflict between theory and the structure observed.[63,188] But the quantitative results of such order-of-magnitude estimates depend heavily on the definitions adopted. The uncertainties (fully two orders of magnitude in the estimated mass) can be eliminated only by passing from dimensional estimates to accurate determinations of the observable parameters.[120] Thus it is only through systematic application of the nonlinear theory that well-founded quantitative conclusions can be reached concerning the amount of consistency between theory and observation.

As mentioned earlier, we live in an era when the structure has not yet decayed. This fact restricts the possible amplitude of the nonuniformities. On the other hand, we do not see any signs of active star or galaxy and cluster formation out to redshifts $z = 1$. It has been proposed[244] that peculiarities observed in two clusters located at $z \approx 0.5$ testify to evolutionary processes, but the suggestion has not been corroborated.[245] Probably these observations represent evidence for a comparatively small change in the amplitude of inhomogeneity (a factor of 5–10 from the effective start of formation to the period of structural decay) over a fairly long time interval (and a large redshift interval). That could happen only for sufficiently low values of the density. Probably $\Omega \approx 0.3$ is the best choice; this value would be in good accord with the steep falloff in the quasar density[233] observed at $z = 3.5$.

h) Evolution of intergalactic medium

The ultraviolet radiation and x rays emitted by the compressed gas will ionize and heat the rarefied gas which has not yet been incorporated into pancakes.[246] This heated gas should give rise to pancakes of a new type. The original and most massive pancakes will have been formed from cool gas having a primordial entropy determined by the coupling of radiation and matter during the hydrogen-recombination era. Those pancakes will contain ultracool ($T \approx 100-500$ °K) gas which has experienced only adiabatic contraction. In such gas conditions will be most suitable for formation of the first stars. The pancakes which develop from the gas that has been ionized and heated by the radiation of the original pancakes will be devoid of an ultracool layer. Nevertheless, the central parts of these later pancakes will cool down to temperatures $T \approx 10^4$ °K, and this cooled and recombined gas also can presumably serve as the birthplace of galaxies and stars. Finally, in pancakes that have been formed at a still later stage the gas will no longer have had an opportunity to cool off, even by our present epoch. Neither galaxies nor stars will form in such pancakes. The authors[246] as well as Oort[247] have looked into the possibility that these last pancakes might be discoverable from their weak ultraviolet and x radiation or from absorption lines in quasar spectra.

It is important to recognize that while the heating of external gas by the original pancakes cannot inhibit the formation of new pancakes, it will cut off the processes of star and galaxy formation in the new pancakes.

The same remark applies not only to different pancakes but to different parts of an individual pancake: if the central part of a pancake observed today as an element of lattice structure corresponds to pancakes of the first type, then the outlying parts would correspond to pancakes of the second type.

The low-density regions in which no galaxies are formed will contain about (20–30)% of all matter but occupy up to ≈90% of all space [see Eq. (5.17)]. Neutral gas, even if of low density, could have been detected from its redshifted Lyman-α absorption. In continuous gas the cosmological expansion would spread the Lα line into a band. Analysis of the spectra of distant quasars has set an upper limit on the mean density of neutral hydrogen[248]:

$$n_{\mathrm{H}} \leqslant 10^{-11}(1+z)\,\mathrm{cm}^{-3}. \tag{5.25}$$

If we suppose that even in the low-density regions the total gas density is not excessively small, then the constraint (5.25) would mean that this gas should be strongly ionized. Radiation emitted by the original class of pancakes could have furnished the requisite ionization.

6. ENTROPIC THEORY FOR DEVELOPMENT OF STRUCTURE

All versions of the entropic theory (the E-theory) for the formation of structure in the universe presuppose that objects of comparatively low mass developed first, namely 10^6–$10^7\,M_\odot$, values similar to the Jeans mass in the recombining hydrogen. As we have pointed out, entropic perturbations would not dissipate on these scales. Accordingly one can devise a systematic scheme for the formation of primordial low-mass objects, with subsequent escalation in the scales of the structure.

In the earliest version[17] it was conjectured that the objects which are formed would evolve into superstars and explode, leading to successive escalation up to galactic scale. In a later version[121] it was remarked that the evolution of such objects would result not in explosions but more likely in the development of complexes of gas and stars such as globular clusters or dwarf galaxies.

The most fully elaborated version of the E-theory[1,121,122,127] reduces the ensuing evolution to a successive agglomeration of objects into ever more massive complexes (the hierarchical clustering model). Theoretical analysis and numerical simulation of this process[131-140] has shown that many of the phenomena observed can successfully be interpreted in terms of the concepts that have been worked out. Other versions of E-theories[125,129,249] have served chiefly for analyzing particular topics in galaxy and cluster formation; no systematic evolutionary scheme has been developed.

Peebles[1] and Gott[127] have reviewed the questions of galaxy and cluster formation in the context of the successive clustering theory. In this theory no preferred scale would emerge in the initial perturbation spectrum. At any given time there would exist only one characteristic scale, for which the perturbation amplitude would be unity. On larger scales the perturbations would be small, growing in accord with the laws for the growth of small perturbations; on smaller scales relaxation of bound systems would become important. The evolution of the system as a whole would be determined by gravitation only, and may conveniently be described by the language of correlation functions. If the perturbations initially depend on mass by the law

$$\frac{\delta\rho}{\rho} \propto M^{-(1/2)-(n/6)}, \tag{6.1}$$

then it has been shown[1,127,250] that asymptotically, for large t, the correlation function is

$$\xi(r) \propto r^{-\gamma}, \tag{6.2}$$

with $\gamma = n+3$ in the domain where the linear theory is applicable, whereas $\gamma = (9+3n)/(n+5)$ in the nonlinear domain.[250] The numerical experiments[131-140] have yielded results in general agreement with these authors' expectations. A correlation function has indeed been obtained similar to the one observed.[1,251] Figure 7 portrays a typical particle distribution derived from numerical experiments simulating the E-theory.[132,133,137]

With respect to the theory of galaxy and cluster formation, the E-approach relies on models of dissipation-free collapse[10)] and rapid relaxation.[217-219,221,222,237] In the earlier versions of the E-theory, spherical collapse models were considered[124,127] and the ellipticity of galaxies was ascribed entirely to rotation. Now these models have been generalized, but the distension of galaxy clusters and particularly the correlation between protuberances and the positions of neighboring clusters has been left unexplained by these versions of the E-theory.

In the E-theory the rotation of galaxies is attributed to tidal effects. The theory consequently runs into certain difficulties, because numerical models domonstrate

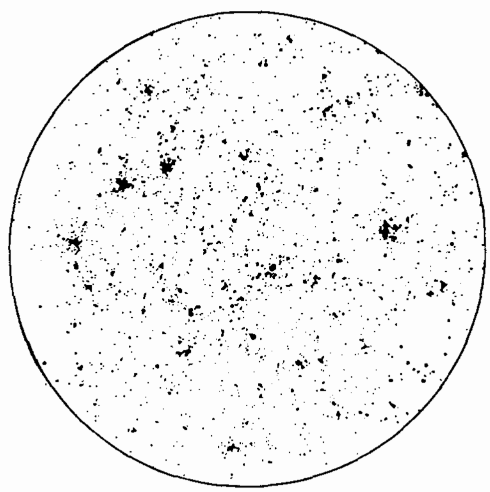

FIG. 7. The particle distribution at the present epoch obtained by Aarseth *et al*.[132] in a three-dimensional numerical experiment based on an entropic theory for the origin of structure in the universe.

[10)]Dissipative processes should be important during the formation of the dense nuclei of galaxies, and probably the flat components of spiral galaxies.

that the rotation effect will be small: one obtains for the specific angular momentum μ the relation[127,134,139,252]

$$\mu=\lambda\sqrt{GMR}, \quad \lambda\approx 0.06\pm0.03, \tag{6.3}$$

where M is the mass of the object and R is its size. That is not enough angular momentum to account for the observed rotation of spiral galaxies. Gott[127] therefore proposes a model which includes a large, invisible halo. He believes that such a model can explain the observations.

Another type of E-theory has been developed[125,126,141-147] which provides for earlier ($z\approx 100$) star formation. According to this model, which Carr and Rees[141] call the "tepid" universe, the radiation from early stars, after being reemitted by dust, molecules, and so on, would be observed in the form of the microwave background, thereby explaining the observed entropy of the universe. The mass contained in the earliest stars (stars belonging to the "third" population type) would provide the observed missing mass if those stars are regarded as having a particularly low luminosity.

Measurements of distortions in the millimeter-wavelength cosmic background spectrum[148,149] have played a large part in the tepid model, but other observations[150] have not confirmed these data. The tepid model universe faces major difficulties in seeking to generate an acceptable spectrum for the cosmic background at long wavelengths. Furthermore, it seems unlikely that the model would be compatible with the heavy-neutrino hypothesis.

The A- and E-theories for the development of large-scale structure are very similar to each other with regard to problems of galaxy and cluster formation by successive agglomeration of dwarf galaxies and clouds of gas and stars. But when it comes to the formation of large-scale structure, the two theories take diametrically opposite points of view. According to the A-theory, the large-scale structure would form first; in a second stage it would break up into dwarf galaxies and star–gas complexes, and in a third step the dwarf objects would congregate into galaxies and clusters of galaxies. In the E-theory, on the contrary, the process of large-scale structure formation would culminate with the development of galaxies and clusters, with the agglomeration taking place by successive escalation of the scale of the gravitationally bound objects.

What might seem a rather insignificant rearrangement in the order of clustering implies a fundamental difference between the observable (in principle) predictions: if the A-theory is correct, then neither galaxies, stars, nor heavy elements[11)] should be observed in the voids separating the pancakes—dense objects would never have developed there; but if the E-theory is right, there should not be any chemically pure space (regions devoid of elements heavier than helium) in the universe at all, or space free of stars and dwarf galaxies.

In the E-theory the low-density regions would still contain more or less conventional galaxies, but in lesser abundance. It is intuitively hard to imagine a complete emptying of any region of space due to gravitational instability (and this belief is supported by numerical simulations). Hence the observations of large voids containing no bright galaxies more likely argue that the physical conditions in those regions inhibited galaxy formation despite a modest density of matter. The notion that every galaxy would have departed from some fairly large region of space seems very implausible—especially because in order for a galaxy to leave a 50–100 Mpc region during a cosmological time span it would have to travel at ≈5000 km/sec, and velocities as high as this are not observed even in rich clusters of galaxies! Thus the detection of huge voids probably lends support to the fragmentation theory (the A-theory) and militates against the theory of successive clustering (the E-theory).

It has just lately been announced[253] that in the central part of one of the voids, in Boötes, several galaxies have been found. This announcement simply underscores the tremendous and fundamental importance of studying the voids in detail—recording ever fainter galaxies, identifying truly isolated galaxies, perhaps estimating their peculiar velocities, and so on. That in fact will be the only way to determine the true structural parameters of the universe, and to delineate bridges and chains of faint galaxies.

In massive-neutrino cosmologies, the E-theory for the formation of structure is not applicable in the orthodox form discussed above, since the baryons—the carriers of entropic perturbations—would constitute only a minor fraction of the mean density of the universe. However, Bond *et al.*[88] have now proposed a model which in its main features revives the hierarchical-clustering version of the E-theory. They posit the existence of exceptionally massive particles ($m\approx 3$ keV), which would serve to diminish the characteristic masses of objects to $M\approx 10^8$–$10^{10}\,M_\odot$. Through hierarchical escalation of scales, this scheme might regenerate the E-theory with all its advantages and shortcomings.

Ostriker and Cowie[129] (see also Ikeuchi[243]) have again drawn attention to the explosive E-theory version mentioned above.[17] Most of their emphasis is placed on the escalation problem accompanying the coordinated explosion of numerous supernovae that congregate in complexes of successively larger scale. In this model the maximum scale of inhomogeneity would be set by the cooling rate of the intergalactic medium; its value would be around 100 Mpc. The supernova outbursts would become synchronized through successive triggering of star formation in some neighborhood (as protostars are compressed by explosion products), followed by a fairly rapid evolution of those stars until they explode—a supernova cascade.[254] Structure formation

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11) To some extent the regions between pancakes would be enriched with heavy elements through supernova outbursts. If, however, the heavy elements were to account for about 2% of all matter (the solar heavy-element abundance), extensive regions having the primordial chemical composition ought to be preserved.

processes would largely have been completed by redshifts $z \approx 4$, and that, in these authors' opinion, would explain why the processes fail to manifest themselves to the observer.

On this scenario we essentially have an attempt to derive the basic results of the A-theory from the small-scale initial irregularities typical of the E-theory. Along with the advantages of the A-theory outlined above, this plan offers a natural way out of the problem of ionizing the gas left over in "holes," the problem of supercluster fragmentation, and so on. Through nongravitational amplification processes this scheme could start out with particularly small initial perturbations, compatible with the fluctuations we observe in the microwave background temperature.

Further discussion of this version of the E-theory is hindered at present by the fact that not much detail has yet been worked out. But some critical comments are brought to mind by the concept itself.

This E-theory version is not likely to be consonant with views concerning the massive neutrino. As we have indicated, nongravitational processes would serve to redistribute matter without affecting the neutrinos. Hence in a neutrino-dominated universe the neutrinos would, on this model, be distributed quasihomogeneously or would collect together through the action of perturbations of some other kind, whereas the visible matter would by virtue of the explosions come to form an independent "pattern" against the neutrino background. Since the mean density of matter is not very high, there would also be little entrainment of neutrinos.

From the standpoint of this proposed theory, the heavy-element enrichment of the medium would promote the formation of new galaxies and stars. However, while the hot gas in galaxy clusters does exhibit a near-solar heavy-element abundance, in dwarf galaxies and in the stars of the galactic spherical population the heavy-element abundance is 10–100 times lower than the solar value. The objects we observe were evidently formed prior to the heavy-element enrichment of the intergalactic medium, or perhaps in places where enrichment did not occur.

Of course one should not overlook the influence that supernova explosions will have on the properties of galaxies, the intergalactic medium, and the structure of the universe. But their influence should not be overstated either. As pointed out above, in the A-theory the first supernova outbursts along with the creation of heavy elements would cause additional turbulization of the interval motions in pancakes as well as fragmentation of pancake/superclusters. Perhaps these processes might result in some swelling of the pancakes, with heavy elements being thrown out into the interpancake medium. Nevertheless, since the explosions would not affect the neutrino phase and would have little influence on cool, dense clouds, their overall effect on the pancakes probably would not be very great.

7. NUMERICAL EXPERIMENTS

A most significant role in working out ideas for the development of structure in the universe is currently being played by numerical experiments, which enable one to simulate various structure formation processes. Depending on the initial conditions that are specified, these experiments fall into either of two classes. If the initial data provide for small-scale perturbations, the calculations will simulate the E-theory.[2,131-140] If instead only large-scale irregularities are present in the initial data, the calculations will model A-theory processes.[43-45,120,241,255-258] One investigation, by Miller,[259] stands apart: although the initial data are chosen according to the E-model, singularities in the calculation procedure bring about a decay of the small-scale perturbations during the linear stage, and the results obtained for the basic parameters are similar to those for the A-model.

In the E-models, attention has centered mainly on analyzing the processes whereby particles collect into groups. At various elapsed times one either calculates the two-point correlation function $\xi(r)$ or uses other parameters to describe the extent to which the points have become bunched. In particular, in some experiments clusters have been identified and later sorted by mass.[2,132,133,137]

Efforts to interpret the results of these model calculations are impeded mainly by the small number of particles used (in simulations allowing for all two-body interactions[2,132,133,137] the sample size $N \approx 1000$–4000). On small scales ($l \sim N^{-1/3}$) the discrete character of the particle distribution will have a strong effect; on large scales, the decisive factor will be the limited size of the sample. The results inevitably can be trusted only within a small range of spatial scales, although data are ordinarily quoted covering a much wider range. These factors become especially significant when the initial perturbations differ from "white noise," because the accumulation of error will distort the influence of the initial spectrum.

The numerical calculations involved in A-models rely on a different ideology. Since the A-theory has been more fully elaborated, numerical experiments can be used to test and refine (or to establish) various fundamental principles of the theory whose implications later can be checked anew against the model material.[44,120] Both two- and three-dimensional models have been employed successfully.[43-45,120,241,256-258] Comparison of the theory against two-and three-dimensional models has yielded the following basic results.

1. Well-developed lattice structure will arise during the nonlinear stage only if small-scale perturbations are suppressed from the initial data.

2. The lattice structure will be fully determined by the field of the deformation tensor $d_{ik} = \partial s_i / \partial q_k$ (Sec. 5).

3. As time passes the lattice structure will decay; the particles will collect into separate complexes, which will successively blend into bigger aggregates. The outward structural differences between the A- and E-models will disappear at this stage.

We would emphasize that in attempting to ascertain how the initial conditions are related to the parameters of the computed pattern, it is vital to assess correctly both the properties of the initial perturbations and their evolution during the course of the calculation.

8. COMPARISON WITH OBSERVATIONS

Observers have been showing more and more interest in problems of large-scale structure. Certain data have been alluded to above, in discussing the theoretical models; others have not yet been mentioned.

In the observational analysis of structure two general trends can be discerned, which may be called the descriptive and the statistical.[1]

Taking the descriptive approach, Abell[260] has been able to point out 2712 rich clusters of galaxies. Several superclusters have been identified and described,[26,261] and black regions devoid of galaxies have been discovered. The observational material as well as the problems entailed in studying superclusters and voids have recently been surveyed by Oort,[41] to whose review we refer the interested reader.

The alternative, statistical, approach to the study of large-scale structure has gained much recognition and come into wide use in the past few years, thanks largely to the work of Peebles and his colleagues. The most complete and exhaustive account both of procedural matters and of the results obtained by the statistical approach will be found in the book that Peebles has recently published.[1]

Of the various statistical methods of analysis, the most popular is to calculate correlation functions for the distribution of galaxies. The two-point correlation function $\xi(r)$ is defined by the relation

$$dP = \bar{n}(1+\xi(r))\,dV, \tag{8.1}$$

where dP is the probability of finding a galaxy within a small volume dV separated by distance r from a galaxy selected at random, and $\bar{n}$ denotes the mean number density of galaxies. Three-point and other correlation functions can be defined similarly.

The outstanding results obtained from correlation analysis may briefly be summarized as follows.

1. The two-point correlation function computed from the galaxy distribution has a simple power-law form over a wide range of distances[1,46,251,262]

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-1.77}, \quad r_0 \approx 4h^{-1}\ \mathrm{Mpc}, \quad 0.1h^{-1}\ \mathrm{Mpc} < r < 10h^{-1}\ \mathrm{Mpc}. \tag{8.2}$$

2. On large scales $r \geqslant 10h^{-1}$ Mpc, $\xi_1(r)$ diminishes rapidly, until when $r > 50h^{-1}$ Mpc one can only set an upper limit[1] $|\xi(r)| \leqslant 0.025$.

3. The two-point correlation function derived from rich (Abell) clusters differs from the galaxy correlation function both in characteristic scale (r_0 now exceeds $20h^{-1}$ Mpc) and in amplitude.[49-51] Analyses of the distribution of other objects[263-268] (quasars, radio sources) have not revealed any perceptible correlation.

Along with correlation analysis, other statistical methods have been applied to investigate the distribution of galaxies.[47,48] These all confirm that the galaxy distribution does manifest correlation, with a characteristic scale of 5–10 Mpc.

Any theoretical scheme that may be proposed will clearly have to interpret the results of correlation analysis. But different aspects of that analysis will come to the fore in different theories. In the context of E-theory the fact that a unique functional relation $\xi(r)$ holds within wide limits may be regarded as evidence to support the successive clustering idea (escalation of scales). On the other hand the disparity between the correlation functions for galaxies and for galaxy clusters has not yet been explained. This disparity, it would appear, can be interpreted most simply by the A-theory, for clusters would correspond to preferred points of the initial perturbations.[269] But the unique function $\xi(r)$ over a broad range of scales is here viewed as evidence for relaxation processes in clusters, and, to some extent, in superclusters. When this question is examined on the basis of numerical models[45,258] one finds that as structure develops, the correlation function computed for the particles will evolve, growing steeper. This effect probably results from the formation of two- and one-dimensional dense regions. In fact, for a two-dimensional (homogeneous) region $\xi(r) \propto r^{-1}$ (the fractional area of a sphere cut by a plane), while for a one-dimensional region $\xi(r) \propto r^{-2}$ [if $\xi(r) > 1$], and the distinction arises solely from geometrical factors.[270]

One of the authors has just proposed a new quantitative technique,[25] based on the concepts of percolation theory, which can resolve the issue of the topology of the large-scale structure: Do superclusters comprise a bound, unified system, or are they isolated from one another?

Among the numerous other observed facts we believe it is especially worth singling out the correlation that has been found between the elongation of galaxy clusters and the elongation of the brightest member galaxy, as well as the correlation observed between the mutual position of close clusters and their orientation.[41,271,272]

In the spirit of dissipation-free collapse these findings suggest that clusters and their brightest members were formed *after* (or during) the formation of superclusters (or of even larger-scale structure).

In efforts to discover vast regions of the universe containing no galaxies[2,28-38,40]—observations of the most acute interest—the sensitivity thus far achieved is adequate to exclude only luminous galaxies. It is a pressing task indeed to lower the threshold on the limiting brightness. At the current level of sensitivity the results are not yet good enough to give precedence to the A-theory.

For many enlightening discussions of the topics treated in this review, the authors are grateful to V. I. Arnol'd, M. Yu. Khlopov, A. A. Klypin, and R. A. Syunyaev.

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Translated by R. B. Rodman

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