Thermodynamic fireballs (state of the concept in the theory and in accelerator experiments)[1)]

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A peculiar situation which has persisted for a quarter of a century in the theory of the multiple production of high-energy hadrons is reviewed. On the one hand, cosmic-ray work established the concept of heavy intermediate formation—fireballs—a long time ago. The evolution and decay of these fireballs can be described thermodynamically (in particular, hydrodynamically). On the other hand, the advent of ultrarelativistic proton accelerators stimulated Feynman-diagram, peripheral, and Regge approaches to the description of few-particle events; later, multiperipheral, parton ("parton comb"), etc., approaches were taken to describe many-particle events. Over the past decade, the failure of Feynman scaling in the pionization region, the observation of clustering, etc., and the success of the thermodynamic interpretation of accelerator experiments, which has won new support from quantum chromodynamics, have marked a substantial convergence of the models which previously appeared mutually exclusive. A suitable approximate model describing this convergence is a model which has been under development for a long time now, peripheral or even multiperipheral in its essence but including thermodynamic subsystems. This convergence now seems even more pronounced than was thought only recently; the characteristics of thermodynamic entities are manifested time and time again in accelerator experiments. At the same time, for processes involving only a few particles (which thermodynamics cannot claim to describe), the picture is still dominated by both the phenomenological interpretation, based on reggeon exchange, and the quantum chromodynamics of point quarks for deep inelastic reactions involving the transfer of a large 4-momentum (the same is true for reactions involving the fragmentation of the incident particle or of the target).

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CONTENTS

1. INTRODUCTION AND CONCLUSION

Fireballs[2)] made their appearance in particle physics in 1958 when Miesovicz's group at Krakow,[1] Niu in Japan,[2] and Cocconi[3] concluded from emulsion experiments that the multiple production of hadrons at particle collision energies $E_L \sim 1$–10 TeV (in the laboratory frame) goes through an intermediate stage at least sometimes and perhaps always: First, two blobs of nuclear matter with masses[3)] $\mathfrak{M} \sim 2$–5 GeV form and move comparatively slowly in the center-of-mass frame; these blobs then decay, each into an average of $\langle n\rangle \sim 5$–10 particles, which are almost exclusively pions. These blobs have been named "fireballs." It was noted not much later that even at energies E_L an order of magnitude lower, where more sophisticated methods could be used (a Wilson cloud chamber in a magnetic field, in conjunction with an ionization calorimeter to determine the initial energy in each event), multiple production could be described in terms of the same fireballs, but an average of only one per event is formed.[4]

All this had been found in cosmic-ray work, at a time when high-energy physics was synonymous with cosmic-ray physics. The accelerators available at the time could reach only incomparably lower energies ($E_L = 10$ GeV at Dubna). Fireballs simply could not form at such energies.

The experimental difficulties in cosmic-ray work are well known, and it is not surprising that the fireball concept was received very skeptically by many physicists. The overwhelming majority of the cosmic-ray researchers, in contrast, adopted it immediately. The general picture which was drawn at international cosmic-ray conferences many years ago [Moscow (1959), Kyoto (1961), and Jaipur (1963)] remains intact in its general features today: Two initial colliding hadrons, e.g., nucleons, release some of their energy in the form of fireballs and then fly off in the forward direction, sometimes being excited; in this case they decay into a small number (2–4?) of final hadrons, called

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[1)] This article utilizes extensively the review paper presented at the XVIII International Cosmic Ray Conference, July 1981.

[2)] The Russian word is a translation of the English "fireball," which means "meteor," "bolide," or "ball lightning,"

[3)] We are using a system of units with $\hbar = c = k = 1$ everywhere.

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"isobaric" hadrons back in those days or "fragmentation" or "leading" hadrons more recently. The basic multiplicity is determined by the decay of the fireballs into particles which move comparatively slowly in the c.m. frame; these particles have come to be called "pionization" particles[4)] (Ref. 5). The intervening decades have served only to strengthen this entire concept in cosmic rays, and as we move on to study even higher values of E_L we have begun to see evidence for fireballs of even larger masses, up to $\mathfrak{M} \sim 100$ GeV, with $\langle n \rangle \sim 100-200$ (Ref. 6).

It was after the fireball concept appeared that accelerators became capable of producing nucleons with $E_L \approx 30$ GeV (CERN and Brookhaven) and then $E_L = 76$ GeV (Serpukhov). The breathtaking experimental opportunities which opened up understandably excited enthusiasm among the accelerator physicists, who eagerly plunged into research with these machines. They generated very detailed data, reliable beyond question. The fact that the energy was still incomparably lower than the energies which had been under study for a long time in cosmic-ray work was very frequently discounted. Many researchers believed that once we reached an energy $E_L \gg m_N$ (where m_N is the mass of the nucleon), and even a total c.m. energy $\sqrt{s} \approx \sqrt{2m_N E_L} \sim 8-12$ GeV $\gg m_N$, we would be in the asymptotic region. It seemed that all that was necessary was to study in scrupulous detail the simple few-particle reactions; then the mechanism would become clear, and we would be able to predict (in particular) what would happen at higher energies (a naive idea, of course). These very simple processes could be described quite successfully by single Feynman diagrams with adjustable parameters, single-meson or single-reggeon exchange, dispersion relations, and so forth. A model for many-particle processes was then constructed by the theoreticians by stringing together a series of few-particle Feynman diagrams. The threading together of a chain of single-meson exchanges, for example, led primarily to the "multiperipheral model" of Amati, Fubini, and Stanghellini[7] (see also the review in Ref. 8) and to the famous comb from each vertex of which a ρ meson was emitted; after the decay, each ρ meson provided two pions. This comb was subsequently replaced by a parton comb with approximately the same properties. For a while, this model fared well in comparison with experiment.

Since then, for nearly two decades, we may say that two schools of particle physics have existed. For the cosmic-ray physicists, the fireballs were real and extremely important entities, which figured prominently both in calculations of the development of extensive air showers (it was assumed that the fireball mass increases with energy) and in the interpretation of individual events observed in photographic emulsions. In contrast, for the accelerator physicists and for those theoreticians who worked exclusively from the accelerator data (this was the overwhelming majority of the theoreticians), fireballs simply did not exist: They were totally ignored. Only small groups of theoreticians (primarily in the USSR and Japan, but in ever-increasing number in many western countries in the past decade) continued to develop the thermodynamic theory of the decay of hadron clusters. In addition, since the early 1960s a peripheral and even multiperipheral hybrid model incorporating thermodynamic subsystems has been pursued continuously in the USSR (at the Lebedev Physics Institute, Moscow). We will discuss this model below (Sections 2 and 4), but at this point we simply wish to note that it is essentially a bridge between these two other schools, seemingly mutually exclusive; it might be more appropriate to say that a particular version of this bridge appeared to be, and indeed proved to be, useful in comparison with experiment. This work, however, was the exceptional case, outside the mainstream of research and researchers. Only in the mid-1970s did a multicluster model appear (essentially a greatly simplified version of this hybrid model). The appearance of the multicluster model, however, was a manifestation of a new stage, caused by the need to explain the clustering observed experimentally. Up to that time, all the West could offer was the extremely unusual thermodynamic model of Hagedorn, with its statistical bootstrap and the concept of a limiting possible temperature for any hadronic matter—a temperature of the order of the pion mass. We will have more to say about this model in Subsection 2a.

The situation began to change only 8–10 years ago, when accelerators finally began to nudge those energies at which fireballs were found in cosmic rays (the colliding beams at CERN, with $\sqrt{s} \lesssim 60$ GeV, with an equivalent laboratory energy $E_L \lesssim 2 \cdot 10^3$ GeV $= 2$ TeV). The simple former multiperipheral models of the comb type, where the teeth were light resonances or partons (there of necessity had to be a lot of teeth; at $\sqrt{s} \sim 60$ GeV, experiments yield an average multiplicity $\langle n \rangle \sim 18$), ran into conflict with accelerator experiments, at first not very seriously.[9,10,72] The most important point is that it finally came to be understood that, at any rate, not all the teeth of the comb were equal: There are fragmentation particles, and there are pionization particles[9] (these conclusions were reached at least one and a half decades after they had been reached in cosmic-ray work). It was then found that in the "central" pionization region ("central" in terms of the rapidity) there was nothing like the plateau predicted by the "comb." Instead there was a clearly defined Gaussian bell (which, however, continued to be called a "quasiplateau"[10b]).[5)] It was later found[11] that the height of even this "quasiplateau" in the inclusive distribution does not at all remain constant with increasing energy, despite the predictions of the comb, etc.

Attempts began to elaborate the comb models. Branching into two, three, or more combs was considered; the exchange of rather heavy resonances within

[4)]These are nearly the same as what are called "central particles" ("central" along the rapidity axis) in accelerator physics. The latter, however, also include the tails of the distribution functions of the fragmentation particles, which can be seen at energies which are not too high.

[5)]If we replace the rapidity distribution by a quasirapidity distribution, we find that the top of the bell is cut off because of the purely kinematic properties of such a transformation; a plateau or even a minimum can appear at the center (more on this below).

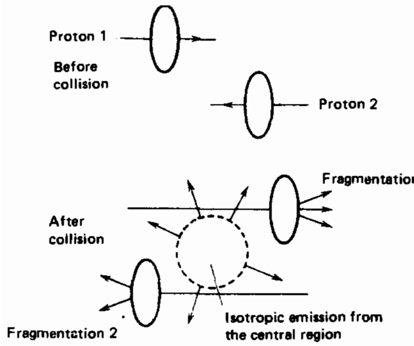

FIG. 1. Model for multiple production of hardons in the collision of high-energy protons (in the c.m. frame) which was formulated in Ref. 12a on the basis of CERN accelerator experiments at $\sqrt{s} \lesssim 60$ GeV (the effective laboratory energy is $E_L \lesssim 1$ TeV).

the comb was considered; etc. All this patching up, however, was only partly successful. Particular difficulties arose in attempts to describe particle correlations. The "naive" representations of the fireball type and the "old-fashioned" hydrodynamic model of Landau, which may be regarded as a model of the growth and decay of heavy fireballs, provided a surprisingly natural explanation for the various characteristics of the process.

Gradually, researchers who had previously not wished to hear about fireballs began to talk in terms of intermediate heavy formations, although out of caution (so as not to be suspected of believing in fireballs) these researchers usually called the entities something else: "clusters." The extreme position was taken by W. Jentschke, director of CERN, who (in his Annual Report for 1973) claimed as one of the major advances the new understanding of the mechanism for multiple production,[12a] which he described by the diagram in Fig. 1. This is of course nothing more than the single-fireball model, which had been formulated in cosmic-ray physics a decade and a half earlier. Two years later, in a five-year report, Jentschke expressed essentially the same model in quark–gluon terms[12b] (Fig. 2) in accordance with the ideas of Pokorski and Van Hove.[13] Finally, in 1973, came the long paper by Carruthers and Minh Duong-Van,[47] in which the authors were surprised to discover that Landau's simple original hydrodynamic

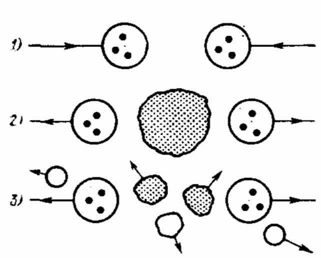

FIG. 2. The same as in Fig. 1, according to 12b, in terms of the quark–gluon model corresponding to Ref. 13. 1—Protons before the collision; 2—immediately after the collision (two excited nucleons and a gluon blob); 3—final state after the decay of the gluon cloud into pionization particles and after fragmentation of the excited nucleons into stable nucleons and mesons.

model gives an excellent description of both the rapidity distributions in various energies and the multiplicity of the production.[47] Curiously, Carruthers titled a summary paper in 1974 "Heretical models of multiple production." We have to salute the determination of all these physicists who rebelled against the dogma held dearly by researchers who worked exclusively from accelerator experiments.

This extreme interpretation of the accelerator data, however, was by no means universal. It was far more common to hear that hadrons formed in groups of three or four particles with a total mass $\mathfrak{M} \sim 1.5$–2 GeV/c (these entities were also called "clusters") or to hear renewed appeals to complicated combs.

In summary, after a decade and a half of sharp conflict we are beginning to see a convergence of the two schools. This convergence has in fact proceeded much further than is generally believed, as we will see below[6)] (Fig. 3).

By no means has this convergence been the result of simply the increase in accelerator energies. Equally important is the fact that after the traditional quantum-field approach had been rejected for nearly two decades (as early as 1959 an eminent theoretician had declared[131] that "the Hamiltonian method for strong interactions has outlived itself and should be buried, but of course with all the honors it deserves") attempts were made to replace quantum field theory by an axiomatic S-matrix theory, by a Regge-pole method, by dispersion relations, etc.), this theory was completely rehabilitated and restored to its rightful position. It was again recognized as the foundation for any theory of particles. Thus, a new theory which appeared—quantum chromodynamics—was constructed on precisely the same basis as quantum electrodynamics and the meson–nucleon field theory. The only changes were in the Lagrangian, and in the interval quantum numbers of the fields. Correspondingly, we can again use both the space–time picture of the development of the process and the semiclassical approximation, where necessary and justified. This is an exceedingly important point, as we will see, which allows us to retain the fireball concepts, in particular, the quasiclassical thermody-

[6)] This convergence, which was clearly recognized as early as the mid-1970s, was analyzed in detail in Ref. 120. The conclusions reached there remain valid today almost completely. [Unfortunately, on (Russian) page 61 of Ref. 120, in a table which strikingly compares the predictions of the two models with experiment, the columns are labeled inappropriately. This can lead to confusion unless the text proper is read carefully. According to the text proper, "statistics and thermodynamics" should be understood as the original hydrodynamic model of Landau with the formation of a common system and with a definite equation of state, while the heading "multiperipheral; Regge approach" actually refers to the "hybrid" multiperipheral model, which includes thermodynamic clusters (or fireballs) as subsystems which are generated at the nodes of the chain along with resonances and which gives a correct description of the pomeron, the ρ trajectory, and so forth (see Fig. 7 below)]. We refer the reader to the review in Ref. 120 for a more detailed discussion of many of the questions touched on here.

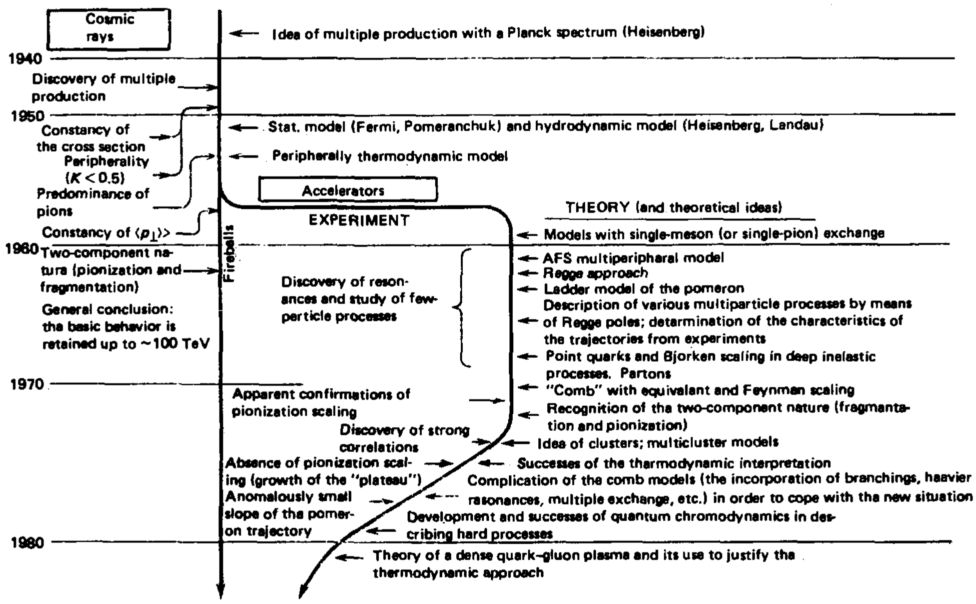

FIG. 3. Schematic representation of the history of research on *multiple* production of hadrons and the development of theoretical ideas (for simplicity we have not shown that the Regge phenomenology of *few*-particle processes and the theory of deep inelastic processes retain their independent significance.

namic approach. Quasiclassical models for collisions of hadrons and nuclei are appearing and being discussed right before our eyes; in some cases, they incorporate a quark–gluon structure as an added feature, but otherwise they are very similar to the models which were being discussed several decades ago. Many of the questions and uncertainties which arise in this connection are the same as those which have been discussed and resolved in the past. The reader should thus not be surprised to find that this review must reach back to some very old papers; they are of more than historical interest.

One of the most common arguments against fireballs was the assertion that they were exotic—some sort of a hazy concept, not amenable to interpretation in (for example) quantum field terms. It is now clear that the argument is erroneous. In any quantum field theory, as we will see, there is a natural place for unstable, nonresonant heavy entities. The only remaining question is how to describe their decay: thermodynamically or by some other approach. In particular, a completely specific physical structure can be proposed for them. One possibility is a quark–gluon bag which has an abnormally high density and is therefore unstable. Interest in formations of this sort picked up greatly over the past two years in connection with experiments which have already been begun on the collisions of ultrarelativistic heavy nuclei (with energies up to tens of GeV per nucleon) in colliding-beam situations, where the formation of a hot, dense quark–gluon plasma, which is progressively expanding and transforming into a hadron phase, is becoming extremely plausible.

Returning to fireballs in nucleon–nucleon collisions, we should emphasize that the mechanism for their formation, their production cross section, their mass spectrum, and many other questions are crucial and require further study.

Looking ahead to what we will be discussing, we need to emphasize two other points here.

1) We are interested primarily here in pionization particles and, in general, reaction products whose transverse momenta $p_\perp$ (transverse with respect to the collision axis) are not too large: $p_\perp \lesssim 1$ GeV and, only to some extent, $p_\perp \sim 1-5$ GeV. At present, particles with larger $p_\perp$ are described not badly as results of hard collisions of quarks and/or gluons in quantum chromodynamics. The latter processes are very rare and provide $\lesssim 10^{-7}$ of the particles produced.[7]

2) The list of fireballs which are understood as intermediate hadron formations described thermodynamically has recently been lengthening. In this list we should include both entities whose evolution should be described hydrodynamically because of their high initial energy density and certain other blobs of nuclear matter, perhaps even that unstable formation which arises during diffractive dissociation in collisions with nuclei, $\pi A \to (n\pi)A$, with $n = 3, 5, 7 \ldots$. While at $n = 3$ we can still see a governing effect of resonances in the final state, at $n \geqslant 5$ a nonresonant background seems to be playing an important role. The lighter clusters ($\mathfrak{M} \lesssim 2$ GeV) introduced by many investigators analyzing correlation accelerator experiments may also be fireballs (Section 4).

[7] Nevertheless, these are the processes which have been under particularly active study in recent years and are of special interest because they provide exceedingly important information about the internal quark–gluon structure of stable hadrons.

Here, however, we are primarily interested in traditional fireballs with masses $\mathfrak{M} \approx 2-5$ GeV, describable statistically, and heavier subsystems describable hydrodynamically.

The questions which we need to take up first are the theoretical arguments in favor of the appearance of thermodynamic systems (or subsystems) in high-energy collisions and the manifestations of such fireballs in accelerator experiments which have already been carried out. An examination of all these points should lead us to an answer to the basic question: Are fireballs real? As we will see, the conclusion can be summarized by saying that all the theoretical arguments and accelerator data presently available constitute evidence in favor of the fireball concept. A definitive solution of the problem, however, will probably have to await accelerator experiments at energies $E_L \gg 10$ TeV. Right now we are seeing accelerators moving into this energy range. In October 1981 a storage ring was started up at CERN in which proton–antiproton collisions occur with a total c.m. energy of $\sqrt{s} = 540$ GeV, which corresponds to a laboratory energy $E_L = 150$ TeV. The very first experiments have already provided some results of importance to our problem. It is thus particularly fitting to review here what is already known.

Section 2 of this review presents the theoretical case for the fireball concept and reviews certain results found by the thermodynamic and even hydrodynamic theories of fireball decay. Section 3 presents (extremely briefly) data on fireballs from cosmic-ray work. Section 4 analyzes the accelerator data at $E_L \leqslant 2$ TeV constituting evidence for the fireball concept. Section 5 is devoted to higher energies; in particular, it discusses the first colliding-beam experiments at an equivalent energy $E_L = 150$ TeV. That section of the review serves as a second conclusion section.

2. THEORETICAL FOUNDATIONS OF THE FIREBALL CONCEPT

a) Thermodynamic peripheral model

The cosmic-ray physicists were clearly prepared to accept the fireball concept from the theoreticians, who began, in 1936, to derive thermodynamic models for multiple production. The work began with Heisenberg, who attempted—even before the appearance of Bhabha and Heitler's cascade theory for electron—photon showers—to explain cosmic-ray extensive air showers (called "Auger showers" at the time) as the result of multiple production in a single event.[14] Heisenberg worked from the then-popular theory of beta decay proposed by Konopinski and Uhlenbeck.[15] Konopinski and Uhlenbeck replaced the Fermi Lagrangian for the four-particle interaction pneν (a proton, a neutron, an electron, and a neutrino) by a Lagrangian which was more complicated but which was believed at the time to give a better description of the experimental beta spectra of the Lagrangian type ($\varkappa$ is a nonzero integer; there is no summation over $\varkappa$)[8)]:

$$L_{\mathrm{KU}}^{\mathrm{int}} = g\,(\bar{\psi}_p \gamma_\mu \psi_n)\left(\frac{\partial^{\varkappa}\bar{\psi}_e}{\partial x_\alpha^{\varkappa}}\,\frac{\partial}{\partial x_\mu}\,\frac{\partial^{\varkappa}\psi_\nu}{\partial x_\alpha^{\varkappa}}\right). \tag{1}$$

Heisenberg pointed out that each extra differentiation of the wave function of a plane wave implied multiplication by the energy or momentum of the particle. At high energies and large numbers $\varkappa$, even if the coupling constant g is small, the interaction effectively becomes so strong that the perturbation theory breaks down, and many particles can be produced in a single event. Heisenberg noted that these particles, flying away from the scene, would continue to interact and multiply until their energy was reduced to the point that the interaction became effectively weak again. At this time, according to Heisenberg, a thermal equilibrium of the final particles is established, and they have a Planck spectrum.

After the Bhabha-Heitler cascade theory appeared, this mechanism was no longer necessary for explaining the electron–photon showers, and the discarding of the Lagrangian of the interaction with derivatives condemned Heisenberg's work to oblivion. However, it is not difficult to recognize prototypes of all the basic concepts both of the statistical model of Pomeranchuk, with its critical decay temperature T_{cr}, and of Landau's hydrodynamic model, which is a generalization of Pomeranchuk's (more on this below). Heisenberg himself returned to the derivation of a theory for multiple production in the case of strong interactions. He gave preference to a quasiclassical wave approach with an interaction which was highly nonlinear or contained derivatives. In his model, a collision of two nucleons is accompanied by the formation of a high-density wave packet, which spreads out until the density of the wave function falls to the limit at which the nonlinear or gradient interaction becomes weak. A Fourier expansion of the expanded packet at this time yields the momentum spectrum of the product particles.[17]

The quasiclassical thermodynamic approach thus has a long history. Some similar preliminary arguments were advanced by other workers,[19] but a special role

8) The history of the support of this version of the theory is very informative and worthy of note. The Fermi version, $L_F^{\mathrm{int}} = g(\bar{\psi}_p\gamma_\mu\psi_n)(\bar{\psi}_e\gamma_\mu\psi_\nu)$, immediately provided not only a solution of the problem in principle but also a fair description of experiment. Extensive study of the β spectra, however, began to generate discrepancies with the theory. It was at that time that Lagrangian (1) was proposed and received an enthusiastic welcome from experimentalists. A series of measurements seemed to support this Lagrangian, and it won many adherents. The skeptical warnings of certain theoreticians that $L_{\mathrm{KU}}^{\mathrm{int}}$ disrupts the basis of the ordinary quantum mechanical scheme went unnoticed. The appearance of higher time derivatives implied that in order to solve the wave equation it was necessary to specify not only the initial value of the ψ function but also the initial values of its time derivatives, which would not have been permissible. Sobriety was restored later, when Bethe, Hoyle, and Peierls[16] pointed out that deviations from the Fermi version might result from the superposition of several spectra in the cascade decay of a nucleus. This turned out to be the case. Version (1) has now been completely forgotten.

was played by Fermi's model,[18] which was fundamentally improved by Pomeranchuk.[20] This series of studies was crowned with Landau's derivation of his elegant hydrodynamic theory.[21]

In these models it is assumed that the interaction between the colliding high-energy nucleons is extremely strong—so strong that even with the very small, relativistically contracted thicknesses $l\sim m_\pi^{-1}\cdot 2m_N/\sqrt{s}$ ($m_\pi^{-1}\sim r_0$ is the radius of a nucleon at rest, and m_π is the pion mass) in the c.m. frame the nucleons have overlapping volumes and release all their energy in this overlap region. As a result, the vacuum becomes polarized and rises to a monstrous temperature T_0; the hadrons which are produced coalesce into a common blob of ultradense nuclear matter, which then expands. In Landau's theory, this expansion occurs in accordance with relativistic hydrodynamic laws with the equation of state

$$p = c^2\varepsilon; \tag{2}$$

where p is the pressure, ε is the energy density, and c is the sound velocity (which Landau took to be $c=1/\sqrt{3}$, as for a relativistic electron–photon gas). This stage lasts until the temperature T in a given volume element (in its rest frame) drops to the critical temperature T_{cr}, at which the energy density drops to that of normal hadrons, $\varepsilon\sim m_N/(4/3)\pi m_\pi^{-3}$ (or, equivalently, the distance between the hadrons which are produced begins to exceed the range of the forces, $r_0\sim m_\pi^{-1}$), so that a hadron gas with a temperature $T_{cr}\sim m_\pi$ appears. This estimate of T_{cr} had also been found by Pomeranchuk,[20] but in studying the expansion Pomeranchuk ignored the hydrodynamic pressure. For this reason, his statistical thermodynamic model applies only to blobs of modest size, for which the pressure can be ignored. For sufficiently small blobs, this model agrees well with experiment[22] (although this agreement is reached by departing from Pomeranchuk's original idea and assuming that the statistical system does not include "leading" particles, i.e., is a subsystem; more on this below). Landau's elegant theory is the most comprehensive.[9)]

A quasiclassical approach of this type is justified by the high production multiplicity n, i.e., by the large number of excited degrees of freedom and the large quantum numbers of the system. Correspondingly, the production probability is determined primarily by the volume of the multidimensional phase space at a given total energy. The situation is equivalent to a microcanonical distribution, and this equivalence leads us directly to thermodynamics.

All these models, however, apply exclusively to

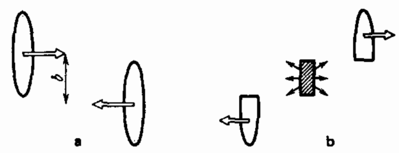

FIG. 4. Model for an inelastic peripheral collision of two high-energy nucleons according to Heisenberg.[25] a—Before the collision; b—after.

"head-on" or "central" collisions of nucleons, in which the initial particles are stopped (in the c.m. frame), and all their energy is transferred to the thermodynamic system. At the same time, analysis of data on extensive air showers demonstrated (as was shown by Zatsepin[23] and, later, by other investigators, who also analyzed the passage of individual hadrons through the atmosphere) that the dominant and perhaps only process is the so-called peripheral collision, in which the incident nucleon loses no more than half of its energy. Even from purely geometric considerations it was clear that the observed collision cross section $\sigma_0\sim\pi/m_\pi^2$ should result primarily from impact parameters $b\sim m_\pi^{-1}$ (that the cross section was of this order of magnitude and remained so up to $E_L\sim 10^6$ GeV followed from both Ref. 23 and papers by many other cosmic-ray workers). Not surprisingly, Fermi,[24] Heisenberg,[25] and other theoreticians attempted to take this circumstance into account, but their results were unsatisfactory. For example, Heisenberg assumed that at $b\neq 0$ the new particles acquire only that fraction ΔE of the total energy E (in the c.m. frame) which lies in the overlapping parts of the nucleons (Fig. 4) and that this assertion remains valid up to values of b so large that the energy ΔE is sufficient for the production of at least two pions. Since ΔE falls off with increasing impact parameter b, possibly exponentially, $\Delta E\sim\exp(-m_\pi b)$, we find the maximum effective impact parameter in an inelastic collision to be $b_{max}\sim m_\pi^{-1}\ln(E/\omega)$, where $\omega\sim m_N$ is the scale pion energy in the Heisenberg model. These arguments lead us to the absurdly large cross section

$$\sigma_{inel}\sim\frac{\pi}{m_\pi^2}\left(\ln\frac{E}{m_N}\right)^2\sim\frac{\pi}{4m_\pi^2}\left(\ln\frac{E_L}{m_N}\right)^2. \tag{3}$$

At $E_L\sim 10^6$ GeV, for example, this expression predicts a value 50–100 times the experimental value which was available even at the time (Heisenberg himself did not note this point).

The source of error, which was of fundamental importance for our topic, as we will see, was not difficult to trace.[26] At such large values of b, the quasiclassical approach must be abandoned. The duration of the collision, $\Delta t\sim l\sim m_\pi^{-1}\cdot 2m_N/\sqrt{s}$, is very short, and according to the uncertainty relation we can go over to the quasiclassical case only if $\Delta E\gg 1/\Delta t\sim m_\pi\sqrt{s}/2m_N$ or, essentially ($m_\pi/2m_N\approx 1/13$), only if

$$\Delta E\sim\sqrt{s}, \tag{4}$$

i.e., only if essentially all the energy is transferred to pions. In other words, Heisenberg's arguments were in

[9)] It should be noted that in this theory the decay into final hadrons does not occur simultaneously throughout the volume in the c.m. frame; instead, it occurs where the temperature has dropped to T_{cr} at a given time. We also note that in the West nothing was known about either the Pomeranchuk statistical model or the Landau hydrodynamic model until the 1970s. As early as 1976, many understood the "statistical model" to be only the Fermi model, while the "thermodynamic model" was the Hagedorn model,[28] based on it.

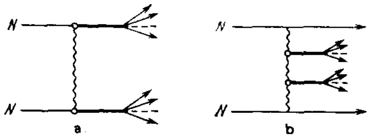

FIG. 5. Feynman diagram for the "peripherally-thermodynamic model" of multiple production. a—According to Ref. 26; b—Ref. 27. The thermodynamic systems arise as subsystems in peripheral collisions described as an exchange of field quanta.

conflict with the (Heisenberg) uncertainty relation.[10)] As we will see, the uncertainty relation allows a classical approach only for central collisions (or nearly central collisions), which either do not occur or are atypical.[11)]

We thus reach an important conclusion: *the primary interaction in a typical (peripheral) collision must be described by a quantum-field approach*, e.g., *in terms of the exchange of field quanta.* A quasiclassical entity evolving in accordance with thermodynamics can arise only as a subsystem. Correspondingly, *a peripherally thermodynamic model* was proposed[26] (Fig. 5). Here the exchange of a field quantum (a pion) causes the formation of subsystems which decay thermodynamically. At about the same time, the Takagi model,[27] which was to become quite well known, was proposed (the motivation for this model was not the situation being discussed here). This model may be included in the same group.[12)]

[10)]When, in the early 1970s, the increase in the nucleon collision cross section with increasing energy was discovered at $E_L > 100$ GeV, $\sigma=\sigma_0(1+0.5\,[\ln(E_L/100\text{ GeV})]^2)$, this old paper by Heisenberg was recalled and interpreted to some extent as predicting the observed growth of σ. This interpretation was of course incorrect. In Heisenberg's work, the term $(\ln)^2$ determined everything and, as mentioned earlier, gave a cross section exceeding σ_0 by a factor of 50–100 at $E_L\sim 10^6$ GeV. The growth of the cross section actually observed under these conditions is less than 100% of σ_0.

[11)]Fermi took a different approach.[24] He assumed that even in a peripheral collision, when nucleons flattened to a thickness l simply brush against each other, a common thermodynamic system is nevertheless formed over a time Δt, but it has an angular momentum perpendicular to the collision plane. For this to be true, however, the interaction would have to propagate a distance $\sim m_\pi^{-1}$ (along the vector b) in the time Δt and manage to thermalize the system. It is easy to show[26] that this would require an interaction transfer velocity $v\sim(m_\pi\Delta t)^{-1}\sim\sqrt{s}/2m_N \gg 1$, which also is not permissible.

[12)]We see that the question of the mechanism of fireball formation reduces to an extremely difficult problem of a quantum field transition from a dynamic system (two initial hadrons) to a stochastic system. Even in classical physics this is known to be a very difficult problem. Here, on the other hand, the exchange of field quanta (very probably gluons) must be considered anew. The initial Heisenberg-Fermi-Landau ideas ignored the "peripheral" nature of the process; i.e., they essentially ignored the quantum-field element of its mechanism. For this reason, they do not correspond completely to the physical nature of the pionization process. At present we can use only approximate and crude models for this process, of the nature of single-pion or single-gluon (or perhaps many-gluon) exchange.

Actually, Hagedorn's interesting thermodynamic model,[28] which seems quite distinct from other models, can also be included in this group. At first glance, the Hagedorn model would seem to drop completely out of the general picture, since (first) it is based on the Fermi statistical model (and thus ignores the expansion and cooling of each element of the system, the need for which was pointed out by Pomeranchuk[20] and which was mentioned even by Heisenberg[14]) and (second) it leans heavily on the concept of a limiting temperature of nuclear matter. This model is based on a so-called statistical bootstrap, which has received a very mixed response. This model could be reinterpreted without any change in its formal structure, however, if the Fermi statistical model in the Hagerdorn approach is replaced by the Pomeranchuk thermodynamic model and if the existence of a limiting temperature is discarded (§5.6 in Ref. 72). Specifically, it is assumed in this model that two subsystems result from the collision. One could be called the fragmentation subsystem and the other the pionization subsystem. Hagedorn later suggested that each element of these subsystems undergoes a Fermi decay at a temperature equal to the "limiting" temperature $T_H\approx 160$ MeV; these elements are moving in the overall c.m. frame, and their "macroscopic velocities" in the c.m. frame are specified by the two distribution functions F and F_0, which are essentially chosen by comparison with experiment. These distribution functions refer to the fragmentation and pionization parts, respectively. If we discard the concept of a limiting temperature but assume that the decay of an element occurs in the Pomeranchuk manner with $T\sim m_\pi \sim T_H$, then all the distinctive features of the model are retained in that the independently chosen functions F and and F_0 are used instead of describing the "macroscopic" motion hydrodynamically, as in the Landau model (revised to apply to the subsystems instead of to the entire system). Not surprisingly, the resulting description of the various aspects of multiple production agrees well with experiment in the particular energy range for which these distribution functions are chosen. Agreement is also achieved in terms of the composition of the product particles: a characteristic which is independent of the hydrodynamic motion and which is determined nearly exclusively by the decay temperature $T\sim m_\pi$.

It should also be emphasized that it has recently become possible to relate Hagedorn's "limiting temperature" (which is still slightly higher than that assumed in the Pomeranchuk model) to the temperature of the phase transition from the hot quark–gluon plasma to the hadronic phase. We then have no need for the statistical bootstrap, and the model becomes completely clear physically and remains useful.

In summary, the Hagedorn model may also be regarded as a "peripherally hydrodynamic model."

By switching from the original thermodynamic models of a central collision[17-21] to models in which a thermodynamic subsystem results from a quantum field interaction, we not only avoid the conflict with the uncertainty relation in terms of the time and energy in the peripheral collisions which actually occur but also eliminate three other problems. The reason is that now *the initial volume of the thermodynamic system does not have to be as small* ($l \sim m_\pi^{-1} \cdot 2m_N/\sqrt{s}$) *as it was assumed to be for a head-on collision.*

First, it was pointed out a long time ago[29] that in Landau's hydrodynamic theory[21] (for example) we are violating (or nearly violating) another uncertainty relation, $\Delta x \cdot \Delta p \gtrsim 1$, when we break up the volume of thickness l into progressively smaller volumes, as we must, for example, in applying the differential equations of hydrodynamics. This objection now vanishes for a subsystem which is generated through the exchange of quanta in a peripheral process.

Second, it is assumed for a head-on collision that nucleons of arbitrarily high energy stop instantaneously and are thermalized over a short distance $l \sim 1/\sqrt{s}$, and this distance becomes progressively smaller with increasing energy. This assumption means *an interaction of unbounded strength.* Landau himself recognized the strangeness of this theory and mentioned it in a discussion of the "null-charge" problem in quantum electrodynamics.[30] We might say now that this theory contradicts the present understanding of the asymptotic freedom of gluons and quarks. This difficulty again is obviously eliminated when we switch to a peripheral hydrodynamic model: The exchange of quanta can occur within the framework of quantum chromodynamics.

The third and final difficulty stems from the circumstance that in the Landau model we actually cannot assume that the expansion of a blob of thickness l is isentropic from the outset. Actually, the viscosity is high at first.[31,32] The system must expand extensively (by a factor of tens at $\sqrt{s} \sim 60$ GeV before the Reynolds number will become large and the expansion isentropic.[32] If the initial volume is large, on the other hand, this difficulty also is eased.

As we will see in Section 2, an experimental determination of the temperature T_0 (with the hydrodynamic theory used to describe the experiment) yields values two or three times smaller than those which would be found at the corresponding energy ($\sqrt{s} \sim 60$ GeV) under the assumption that the entire energy is released in a volume of thickness l. Since the energy density ε depends very strongly on T_0 (for example, $\varepsilon \sim T^4$), the initial volume is actually tens of times larger (and ε is correspondingly smaller) than assumed by Fermi and Landau. Quantum chromodynamics shows why this is possible: When two nucleons collide, their gluons and quarks interact, but this interaction is not strong

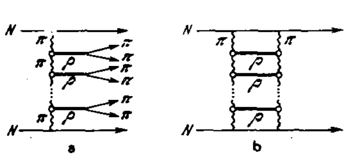

FIG. 6. Feynman diagram of the multiperipheral model of Amati, Fubini, and Stanghellini (AFS model).[7] a—Multiple production ("comb"); b—elastic scattering ("ladder").

enough for instanteous thermalization, and the process spreads out over space[13)] (more on this below).

b) Multiperipheral model with fireballs

However, as we have already pointed out, accelerator physics took a different path. If we restrict the discussion to few-particle processes (two or three particles, although they may also be resonances, each decaying into two or three final particles), we can describe events successfully as a consequence of the exchange of a meson and then of a reggeon with suitably chosen phenomenological parameters. We need to introduce a rather large number of these parameters: In the reggeon approach, these parameters are the intercept of each trajectory $\alpha_R(0)$ (the intersection with the ordinate axis in the plane of the square mass and the angular momentum), the slopes of the trajectories, the residues at the poles, and the residue form factors. In return we achieve an internal self-consistency in describing the voluminous experimental information on few-particle processes. This phenomenological model proved to be (and remains) extremely useful for describing few-particle hadron reactions, despite the difficulties regarding the question of the vacuum pole—the pomeron. This model furnishes experimentalists a clear and graphic language with substantial predictive value. Furthermore, this approach also turns out to be useful (even today, when we use quantum chromodynamics) for high-multiplicity processes for analyzing the fragmentation particles, if the latter can be distinguished.

We are interested here, however, in a really high multiplicity, such as we find at really high energies, where a governing role is played by pionization (or the "central region" along the rapidity scale), and we are also interested in pionization processes at modest energies. Attempts were immediately made to simply "extrapolate" the ideas and methods developed in the single-meson approach (later, the single-reggeon approach). It was suggested that we construct a chain of few-particle Feynman diagrams, with the interaction in each diagram occurring at a modest energy. This idea was raised by several authors,[7a] but it was embodied systematically in the *AFS multiperipheral model*,[7,8] which we have already mentioned. Here the exchange of a pion generates a Feynman diagram in the form of a *comb*, whose teeth are ρ mesons (Fig. 6a) (or, perhaps, some light resonances with suitable quantum numbers, etc.). For elastic scattering we again run into the equally well-known *ladder diagram* (Fig. 6b), in which the rungs are the same ρ mesons.

This model quickly won favor, because of three remarkable properties. However, an extremely careful analysis by some investigators who were familiar with experimental data on interactions and multiple produc-

[13)] The final equations of the hydrodynamic theory frequently contain the initial volume only in a logarithm and are thus comparatively insensitive to such an increase in this volume. We also note that a head-on hydrodynamic approach can be allowed in a "central" collision of heavy nuclei, although the question of the leading nucleons remains somewhat hazy.[45,55]

tion at high energies, of the order of 0.1–10 TeV, obtained from cosmic-ray work (data which were confirmed by accelerator experiments 10–15 years later), quickly revealed that all these properties have flaws[33,34] (see also Ref. 8).

1) According to this model, the energy dependence of the multiplicity asymptotically has a form suitable for describing experimental data at energies up to at least 1 TeV:

$$\langle n_\pi \rangle = a \ln \frac{E_L}{m} \tag{4'}$$

(although the cosmic-ray experimentalists preferred a power-law dependence at higher energies). The coefficient a, however, turns out to be two or three times smaller than experiment would require. This coefficient is actually determined by the number (K) of pions produced at one vertex of the multipheripheral chain ($K = 2$ in the AFS model with ρ mesons).

2. The ladder diagram for zero-angle elastic scattering (Fig. 6b), combined with the optical theorem, yields a total cross section σ_{tot} with the desired Regge form:

$$\sigma_{tot} \sim s^{\alpha_P(0)-1}. \tag{5}$$

The intercept of the pomeron trajectory calculated in this model, however, is inadmissibly small, $\alpha_P(0) \approx 0.3$ (Refs. 33d and 33e), so that $\sigma_{tot} \sim E_L^{-0.7}$ is predicted to fall off rapidly with energy (in sharp contradiction with experiment). An important point is that $\alpha_P(0)$ in this model is related to the spectrum $\bar{\sigma}(\mathfrak{M})$ of the masses $\mathfrak{M}$ generated by the teeth of the comb:

$$\alpha_P(0)(\alpha_P(0)-1) = \frac{1}{16\pi^2} \int \bar{\sigma}(\mathfrak{M})\, d\mathfrak{M}^2. \tag{6}$$

The spectrum $\bar{\sigma}(\mathfrak{M})$ represents the cross section for the production of an unstable particle of mass $\mathfrak{M}$ in the collision of two virtual pions that are being exchanged at the same vertex at which this mass is generated. The small value of $\alpha_P(0)$ reflects the presumed small interval of $\mathfrak{M}$ values over which the integral in (6) extends: If only ρ mesons with a mass $\mathfrak{M}_\rho \approx 0.77$ GeV were generated, we would have $\bar{\sigma}(\mathfrak{M}) - 0$ at $\mathfrak{M} > \mathfrak{M}_\rho$.

3) Finally, this model yields the following result (also with the desired Regge form) for small-angle elastic scattering:

$$\frac{d\sigma_{el}}{dt} \sim \exp\left(-\alpha_P'(0)|t| \ln \frac{E_L}{m_N}\right), \tag{7}$$

where t is the square of the 4-momentum transfer, and $\alpha_P'(0)$ is the slope of the pomeron trajectory. Experimentally, however, this slope is highly sensitive to the energy, and asymptotically, as $E_L \to 2$ TeV, it is very small, $\alpha_P'(0) \sim 0.2$ GeV^{-2}, as recent accelerator experiments have shown. As a result, this slope is puzzlingly different from the slopes of the other trajectories, for which the slope is about 1 GeV^{-2}.

All this could be said another way. The diagram in Fig. 6b is inspired by the dynamic pomeron model. The pomeron which results from this diagram, however, leads to elastic and total cross sections which are contradicted by experiment.

It turns out that all three of these flaws can be remedied,[33a-d] and all three advantages retained, if we alter the model by making the teeth heavier (or, equivalently, by making the rungs of the ladder heavier), giving them masses $\mathfrak{M} \sim 2$–4 GeV, so that the number of pions produced at a single vertex can also be substantial ($K \sim 4$–8 in the statistical decay of a blob), the integral in (6) increases, and the asymptotic value of $\alpha_P'(0)$ becomes small,[14)] 0.2, in accordance with observation.

The entities which appear at the vertices of a multiperipheral chain of this sort may be clusters or fireballs [becoming heavier as $\alpha_P'(0)$ becomes small and as a becomes larger], but in principle some as yet unknown resonances are also possible.

In addition to these three characteristics ($\langle n \rangle$, σ_{tot}, and α_P'), however, there are two more, which were also known from cosmic-ray experiments. First, there is the mean square 4-momentum transferred between the nodes of the chain, $\overline{\Delta^2}$. Experiment yields $\overline{\Delta^2} \gtrsim m_N^2$, while the AFS model predicts a much lower value. This discrepancy was stressed particularly persistently in Ref. 34 (and also Ref. 33a). It turns out that, here again, increasing $\mathfrak{M}$ to several times m_N eliminates the contradiction. Second, according to the AFS model all the particles produced would have to span an interval ~10 along the axis of the pseudorapidity $\eta = -\ln \mathrm{tg}(\theta/2)$ at $E_L \sim 1$–10 TeV, while the experimental interval is smaller by a factor of at least two; as was pointed out in Ref. 34, this problem again can be corrected by increasing $\mathfrak{M}$.

The situation is so simple, of course, only in the asymptotic behavior, where the number of vertices, $\mathfrak{N}$, is large. Actually, we conclude from purely kinematic considerations that for these masses $\mathfrak{M}$ the number $\mathfrak{N}$ could reach values of 3–4 only at $E_L \gtrsim 10$–100 TeV (and $\mathfrak{N} \sim 2$ at $E_L \sim 1$ TeV; Refs. 33 and 34). Only here can we speak in terms of a "multifireball asymptotic behavior." Whether a comb of this sort actually arises here or the fireballs get heavier while the number of fireballs remains the same will remain an open question until we see some experiments on the new accelerators. We will return to this point in Section 5.

That the teeth of the comb must be made heavier (and their number reduced at a given $\sqrt{s}$) can also be seen from the following physical considerations.[35]

We know from experiment that ~18 pions are produced at $\sqrt{s} \sim 60$ GeV (for example). If these pions result from the decay of ρ mesons, as they do in the AFS model,

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[14)] When this procedure was proposed, accelerators could reach energies of only $E_L \lesssim 30$ GeV. The slope measured at these energies was of the order of 1 GeV^{-2}, and it appeared that a contradiction with experiment had appeared (the diffraction cone was too wide; it was for this reason that Koba and Krzywicki, in a very detailed study,[33b] where all these details were explained, did not decide to propose simply to go over to heavier $\mathfrak{M}$. This question was not taken up in Ref. 33a). As accelerators moved to higher energies, however, the slope of the pomeron trajectory decreased, approaching ~0.2 TeV at 1 TeV, as it should for this particular increase in the mass of the steps.

there should be about nine ρ mesons. Experiments show that the products span an interval $Dy \approx 4-5$ along the rapidity (y) axis. This result means that the difference between the rapidities of adjacent teeth is $\Delta y \approx 0.5$. If we ignore the transverse motion of the teeth, we easily find that the Lorentz factor of the relative motion of adjacent ρ mesons is $\bar{\gamma} \approx \cosh \Delta y \approx 1.13$. Their relative motion is thus nonrelativistic. It is difficult to believe that under these conditions they will fly away from each other without undergoing multiple strong interactions. Furthermore, the interaction should involve not just a single pair of teeth but several teeth in the interval $\Delta y \sim 1$. For example, if we focus on four "adjacent" ρ mesons with an average distance $\Delta y = 0.5$ between teeth, then the outermost two of the ρ mesons lie a distance $\Delta y_{max} = 0.75$ from the common center and are thus moving with respect to their common center of mass with a Lorentz factor $\bar{\gamma} \approx \cosh 0.75 \approx 1.29$. This result means that all four of the ρ mesons in this group have nonrelativistic velocities in the frame of reference of the group. If we take into account the unavoidable fluctuations in the rapidity of the teeth, then we conclude that, since any fluctuations will cause some of the teeth to move closer together (along the y scale), it is nearly obvious that there should be a "coagulation" or clustering of several teeth. Even the multiple interaction of only four ρ mesons actually implies the formation of a nonresonant fireball, or cluster, which decays into $\gtrsim 8$ pions. We may say that the AFS model without a final-state interaction of the mesons is to some extent internally contradictory, since it does not incorporate this final-state interaction; this interaction should by no means reduce to simply an elastic rescattering which leaves the rapidity distribution of the ρ mesons unchanged. Rough estimates show that the volume of this cluster is not very small and may approach $\sim m_\pi^{-3}$.

Simultaneously and independently, Hasegawa[36] proposed a similar model after seeing suggestions of it in cosmic-ray emulsion experiments [he was not attempting to resolve the problems regarding $\langle n \rangle$, σ_{tot}, and $\alpha'_P(0)$]. He called the heavy entities produced at the nodes of the chain "H-quanta" (although these entities are not as heavy as in Ref. 33).

This model, however, should be regarded as oversimplified. Even if we work from a multicluster chain there are no grounds of any sort for completely eliminating other types of vertices, e.g., those corresponding to the production of light resonances, as in Ref. 7, or few-particle diffractive dissociation, etc. Correspondingly, a more general *hybrid* model (Fig. 7) was proposed; this model allows both processes of the types just listed and the scattering of two virtual pions accompanied by their conversion into real pions and baryon resonances N^* (Refs. 37–40; see also Ref. 8). In those papers this model is called the "multiperipheral cluster" model, the "multicluster" model, or the "multifireball" model. However, other multicluster models appeared subsequently (for example, the model of the independent emission of identical clusters, which we will be discussing in Section 5). For definiteness, therefore, we prefer to call the hybrid model of Fig. 7 the model of Dremin, Roĭzen, Uaĭt, and Chernavskiĭ

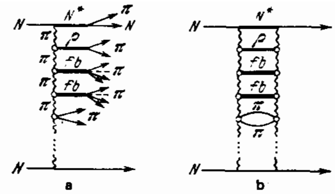

FIG. 7. The hybrid model of the Lebedev Physics Institute, Moscow, including the production at the nodes of the multiperipheral chain of both few-particle vertices (the production of ρ mesons, diffractive dissociation, the scattering of two virtual mesons, which converts them into real particles, etc.) and multiparticle fireballs. a—Inelastic process; b—elastic nucleon–nucleon scattering.

or, more briefly, the *Lebedev Institute Model* (LIM). While the discrepancy with the known experimental total cross section in the AFS model received no attention, and it was not considered at all in other multiperipheral models (the parton comb, etc.), in the LIM the experimental *energy dependence* $\sigma_{tot}(s)$ *is a given* among the initial phenomenological parameters [the LIM is based entirely on the circumstance that the integral in (6) should be extended to large values of $\mathfrak{M}$ if σ_{tot} is to remain approximately constant]. More precisely, it is required that these cross sections $\sigma_{NN}(s)$ and $\sigma_{\pi N}(s)$ be determined by the observed values of $\alpha_P(0)$ and $\alpha_{P'}(0)$—the intercepts of the P and P' trajectories—and by their approximately constant asymptotic cross sections $\sigma_{NN}(\infty)$ and $\sigma_{\pi N}(\infty)$ at high energies (four parameters). These conditions alone turn out to be very restrictive, leaving little latitude in the choice of the general features of the production spectum $\bar{\sigma}(\mathfrak{M})$ in (6), etc.

In summary, this Lebedev Institute Model (like the Regge model and others) is semiphenomenological in the sense that it borrows several parameters from experiment. Furthermore, it makes the assumption that the fireballs decay in accordance with the thermodynamic model (the statistical Pomeranchuk model if $\mathfrak{M}$ is not very large or the hydrodynamic Landau model if $\mathfrak{M} \gtrsim 8$ GeV). This model has been pursued analytically in some detail (for example, the Regge behavior of the cross sections has been demonstrated, and parameters of the ρ trajectories, the slopes of the P and P′ trajectories, etc., have been derived and found to agree with experiment). After all the arbitrary numerical parameters had been fixed (in 1973[39]), a computer program based on this model was used to generate a library of simulated events (about 70 000 events for NN and πN collisions over the E_L range from 28 to 400 GeV), in each of which all the exclusive data are known (see Ref. 41 for the calculation details). This library has been used for comparison with all the accelerator data, both the data available at the time and data which have become available since (including correlation data).[39-41] Nowhere has the discrepancy exceeded 10–15%. The agreement could be improved even further (and, for example, the fact that the cross sections are not asymptotically constant but instead exhibit a logarithmic behavior could be taken into account) by slightly varying the selected numerical parameters. However, the authors prefer to retain the parameters fixed in 1973 and instead increase the number of conclusions which can be compared with experiment (one deficiency of the fixed set of parameters which has already been identified is that the importance of light resonances is somewhat overestimated).

This agreement with experiment means that we may regard the world described by this model as a quite good approximation of the actual physical world, and we can use the model in attempts to learn about the mechanism of multiple production. An important point is that elastic and inelastic processes are described in a consistent manner.

There is a property of this model which deserves mention. As the energy E_L is raised from 28 to $\sim 10^3$ GeV, the effective *number of vertices* in the "chain" *remains small*, $\mathfrak{N} = 3-4$ (the four-vertex chains in Fig. 7a contribute no more than 3 mb to the total cross section at $E_L = 400$ GeV), but there is a change in the relative contributions of the light vertices (ρ resonances, etc.) and the fireball resonances; the effective mass of the fireballs itself also increases with E_L. At $E_L = 200$ GeV the average number of fireballs per event is ~1, while at $E_L \sim 10^3$ GeV it is ~2 (this point becomes significant in a comparison with old cosmic-ray data[1-4]; see Section 1). As a consequence, most of the properties of the products of the generation (the $p_\perp$ distribution, the relative numbers of pions and other hadrons, etc.) are determined primarily by the thermodynamic decay of one single fireball (in the energy interval $E_L \sim 200-400$ GeV which has been studied in particular detail).

Despite all this, we must emphasize that this model is of course very crude and elementary, if only because it is based on single-pion exchange (the case for completely discarding many-pion interactions is not very strong[15)]). Furthermore, this model has yet to be compared with experiment at $E_L \sim 1$ TeV, and as yet there is no evidence that it is applicable at these energies, and we will use it with caution.

We have mentioned earlier that the advocates of the multiperipheral model (the AFS model and the parton comb) did not take up the question of the total cross section. This assertion, however, is not completely correct. Attempts were made in the 1970s to achieve a constant cross section without fireballs. For this purpose the original AFS model was expanded in the sense that production of not only ρ mesons but also other light resonances (ω, f, A_2) was allowed at the nodes of the multipheripheral chain, and exchange of not only π-meson trajectories but also P, P′, ω, f, and A_2 trajectories was taken into account. This refinement of course required choosing many new numerical parameters. Levin and Ryskin,[42] for example, carried out a really ambitious study in which they examined ten possible sets of these parameters, and in some cases they were able to hold σ_{tot} approximately constant up to E_L $\sim 10^3$ GeV. Typically, however, the number of effective vertices decreased sharply, and the mass of many of them became quite large. Furthermore, when an attempt was made[43] soon thereafter to explain the observed two-particle rapidity correlations by the same approach (these correlations are easily explained in the LIM and in certain multicluster models; we will discuss correlations in detail in Section 4) it was found necessary to also incorporate a branching of combs and to make them "enhanced," i.e., to surround the vertices with auxiliary pomeron exchanges. As a result, the diagram transforms from a simple comb into a complex coil or set of coils of numerous entangled exchanges of very different trajectories. A coil of this sort may be regarded as a blob of nuclear matter or a blob of multiply interacting hadrons—as the same fireball, which can be "disentangled" and interpreted in terms of multiple interactions or in terms of "numerous exchanges of entangled trajectories of different types." The situation is reminiscent of the replacement of a liquid drop by a set of many molecules among which multiple interactions occur. This is essentially an attempt to derive a "microscopic theory" for a fireball.

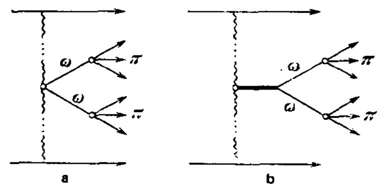

FIG. 8. a—Generalization of the multiperipheral model without fireballs, which allows production of two resonances at a single vertex[44]; b—interpretation of the same process as the production of a fireball which does not decay thermodynamically.

A completely analogous result was found in Ref. 44 in an attempt to explain the constancy of σ_{tot} for a multiperipheral chain with the emission of only known resonances. Here again, a positive result was achieved, but only through the introduction of heavy vertices, of the type in Fig. 8a, for example (with specially selected numerical parameters). Six pions are therefore produced at a vertex of this sort, and it may be regarded as a fireball (the mass of the entire vertex is not of a resonance nature; it belongs to the continuous spectrum). The decay of this fireball, however, is described in a special, nonthermodynamic way (Fig. 8b). Not surprisingly, one-, two-, and three-vertex diagrams are dominant. Here little has been left of the comb[16)] or the ladder, but much has come from the fireball. The multiplicity, of course, also turns out to agree better with experiment.

We thus see that the *basic experimental facts*—the approximate constancy of σ_{tot}, the absolute value of the

[15)]And in general, the exchange of a quark or gluon may be preferred to the exchange of a pion. However, many years of experiments of various investigators working with multiperipheral models have shown that such changes do not have any fundamental effect on the result (see the next paragraph).

[16)]A comb of this sort would probably have been suitable only for Bismarck's anecdotal barber, who asked his client on which side to make the part that day: two hairs to the left and one to the right or vice versa.

multiplicity, and the small slope of the pomeron trajectory (and also the large two-particle rapidity correlations; Section 4)—*urgently require that heavy intermediate (decaying) entities of the cluster or fireball type appear at* $E_L < 1$ TeV *in the peripheral interactions of hadrons* (the number of these entities is small, $\mathfrak{N} \sim 1-2$, at least at $E_L < 1$ TeV).

c) The miracle of the thermodynamic interpretation

We are indeed justified to speak of a miracle, or at the very least of an astonishing fact. The thermodynamic approach, supplemented when necessary (at high initial energy densities and at large masses of the blob of nuclear matter) by a hydrodynamic theory for the expansion and cooling of the blob, has been viable and unusually productive for 30 years. The corresponding theory has been pursued extensively; most significantly, some important aspects have been refined (the hydrodynamic equation has been solved more rigorously and more completely; a traveling wave has been taken into account; the possibility of various equations of state has been incorporated; the effect of thermal motion on the transverse momenta has been taken into account; and the role played by viscosity has been determined). The approach has been modified (the fact that the thermodynamically evolving blob is a *subsystem* which arises in a peripheral interaction has been taken into account). Its range of applicability has been expanded (account has been taken of hadrons and leptons with substantial values of $p_\perp$ and $m_\perp = \sqrt{m^2 + p_\perp^2}$ which "leak" from the surface or which are emitted from the interior of the blob before it cools; in general, the production of photons and leptons by mechanisms other than meson decay has been taken into account; etc.), and it has been strikingly successful in describing a variety of types of experimental data. This theory includes basically only two parameters. First, there is the critical temperature for the decay into the final hadrons, T_{cr}, which is known in order of magnitude at the outset, $T_{cr} \sim m_\pi$, and which differs from this value by only ±20%, as a comparison with experiment shows (interestingly, the deviations are slightly different for the different characteristics, but this circumstance may result from the common practice of using the hydrodynamic theory in the form which presupposes the occurrence of only head-on collisions). Second, it is necessary to choose an equation of state, more precisely, to choose a value for the square of the sound velocity c^2. Landau chose the value $c^2 = 1/3$, as for an ultrarelativistic electron–photon plasma. Analysis of experimental data and calculations from quantum chromodynamics suggest instead $c^2 \approx 1/5$ and, furthermore, a slight dependence on the temperature[17)] (Ref. 45).

The theory does not yet incorporate the actual process by which the thermodynamic subsystem is formed in the hadron–hadron collision. The appearance of this process—"thermalization"—has been the object of research for several years, but a convincing result has yet to be found. An extremely promising approach starts from the fact that the gluon scattering cross section ($gg \to gg$) is many tens of times larger than the quark-scattering cross section ($qq \to qq$) through large angles because of the large number of internal degrees of freedom of the gluon.[45] There is thus the possibility that the gluons in two colliding hadrons become thermalized comparatively rapidly, while the quarks move forward and produce the fragmentation particles in the spirit of the model of Pokorski and Van Hove[13] (Fig. 2). However, we still lack a convincing theory for the transition of the initial dynamic system into a statistical system.

We do not have space here to give anything approaching a complete picture of the successful applications of the thermodynamic theory, and for the most part we will simply refer the reader to other reviews. See Refs. 45 and 46 for reviews of the hydrodynamic treatment, and see Ref. 47 among the first papers which called attention in the West, in the 1970s, to Landau's hydrodynamic theory. See Ref. 22 regarding the statistical model. Let us examine the principal results.

1) In the old days it was believed that if the energy of the particles is far higher than the masses of stable hadrons then baryons and mesons would have to be produced in roughly equal number, in accordance with the numbers of internal degrees of freedom of these particles. For nucleons, kaons, and pions these ratios would be $n_N : n_K : n_\pi = 8:4:3$ (as is found, in particular, in the statistical Fermi model[18]). Experiments (with cosmic rays) carried out several decades ago showed that these ratios are actually completely different: 1:10:100, roughly speaking. The correct ratios (and their weak dependence on s), which have now also been furnished by accelerator experiments, are described completely successfully by the thermodynamic model. The ratios are determined primarily by the decay temperature T_{cr} and by the Boltzmann factors, which are small for large particle masses. This property is of course retained in the hydrodynamic theory of head-on collisions and also when we take into account the fact that the hydrodynamic system is actually a subsystem in a peripheral process.

2) We know that the average transverse momentum $p_\perp$ of the product pions is small and constant, $\langle p_\perp \rangle \approx 0.35$ GeV, for E_L up to nearly 100 TeV. It turns out that the distribution of $p_\perp$ values can be described excellently by a Planck thermodynamic distribution for a temperature ture[48a] $T_{cr} \approx m_\pi$. This striking fact (which had also been established in cosmic-ray work in the dim and distant past) has turned out to remain valid for the production of particles of other types (with corresponding changes in the values of $\langle p_\perp \rangle$) as the accelerator data have accumulated.[22,48b] This fact is reflected particularly clearly and in its general form in an $m_\perp$ distribution in the form[48b] $\exp(-\sqrt{m^2 + p_\perp^2}/T_{cr})$. This universal formula turns out to be correct (when leading particles are excluded) for a wide variety of product hadrons: $\pi^\pm$, p,

[17)] For example, in a detailed study of the distributions in the transverse and longitudinal momenta incorporating transverse expansion, *based on the picture of a head-on collision*,[121] a rather large spread was found in both the suitable values of c^2 and of T_{cr}.

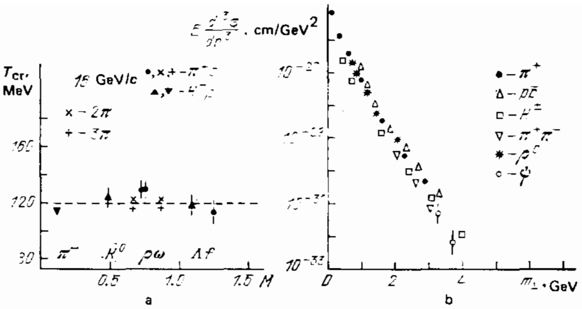

FIG. 9. a—Determination of the temperature T_{cr} from the $\langle p_\perp \rangle$ distributions for the particles (and also for pairs and trios of pions) of various masses M at $E_L = 16$ GeV (Ref. 122); b—universal distribution in the transverse mass $m_\perp = \sqrt{M^2 + p_\perp^2}$ for various reaction products at energies of the order of hundreds of GeV (Fig. 26 from Ref. 45).

$\bar{p}$, $K^\pm$, ρ, ω, f, Λ, and ψ; for $\pi^+\pi^-$ pairs; etc. (Fig. 8). This property remains in force even when there is a hydrodynamic motion, since a transverse hydrodynamic expansion is of minor importance in comparison with thermal motion, as was shown a long time ago,[49a] at energies up to $E_L \sim 1$ TeV. The same is true, of course, when the thermodynamic system is a subsystem in a peripheral collision: The transverse momentum which it acquires upon its formation is small and is furthermore distributed among the many products of its decay. At $E_L \sim 1$ TeV, however, hydrodynamics predicts a slight increase in $\langle p_\perp \rangle$ with $\sqrt{s}$, which had been observed sometime ago in cosmic rays[49b] and which is apparently already observable in accelerator experiments at $\sqrt{s} = 540$ GeV (Section 5). It would be difficult to imagine that some other, nonthermodynamic theory could describe the transverse-momentum distributions of hadrons differing so widely in mass and internal quantum numbers in a natural way, and by means of a single parameter—the decay temperature $T_{cr} \approx m_\pi$.

3) It was mentioned a long time ago[50] that during the expansion, while the blob was still quite hot, hadrons could leak from its surface at $T \gg T_{cr}$; under these conditions, these hadrons would have "anomalously large" $p_\perp$ and would contain relatively more heavy hadrons than in the final decay, at $T = T_{cr}$. These results were in fact observed in an analysis of experimental data, and there is even the suggestion that it may be possible to distinguish cases in which it is a quark rather than a fully-formed hadron which leaks out. In such a case, the quark should produce a jet with a large value of $p_\perp$ in the course of hadronization (see Refs. 45, 51, and 52; see also the references cited there).

4) Long before its decay at $T = T_{cr}$, the hot blob of nuclear matter should emit photons (blackbody emission) whose frequency is higher, the higher the temperature at which they are emitted, and it should also emit dileptons whose invariant mass is higher, the higher this temperature.[53] Especially hard photons and heavy dileptons are thus emitted in the initial stage. A detailed analysis of the experimental data shows[51,52,45] that in the mass interval $1 \lesssim m \lesssim 5$ GeV the dilepton pairs, and in the interval $1 \lesssim p_\perp \lesssim 4$ GeV the photons, can be explained in a consistent way by a mechanism of this type. The absolute intensity agrees well with the quantum chromodynamics calculation for a quark–gluon plasma [at $m \gtrsim 5$ GeV, hard collisions of quarks and gluons in the plasma play a more important role; this is the Drell-Yan mechanism, which is completely inadequate at $m_\perp < 4$ GeV (see Fig. 30 in Ref. 45 and also Ref. 123)].

5) The leakage of hadrons and quarks [see point 3) above] and the emission of photons and dileptons [point 4)] in a consistent manner make it possible to determine the effective temperature in various stages of the expansion (see Fig. 31 in Ref. 45 and also Ref. 52). According to other, earlier work (cited in Refs. 45 and 53b), even at $E_L \sim 1$ TeV the temperature does not exceed[18)] 0.4–0.5 GeV, while in a head-on collision and at $c^2 = 1/3$ the temperature is always estimated to exceed 1 GeV. As mentioned earlier, since the energy density is proportional to a high power of T ($\varepsilon \sim T^4$ at $c^2 = 1/3$), we conclude that the *initial energy density is lower by a factor of tens* (and the initial volume is larger) than expected in a head-on collision.[21] All this is evidence that the hydrodynamic system is a *subsystem*.

However, the phenomena described in points 1) and 2) of this list are insensitive to the hydrodynamic process and can occur for fireballs which are not very heavy, whose decay can be described statistically.[20]

6) The inclusive rapidity distribution in the central region according to the hydrodynamic theory agrees well with experiment.[47,52,55,11] The same is true of the production multiplicity (especially when we take into account the circumstance that only a part of the total energy goes into the hydrodynamic subsystem[56]). The multiplicity which is found agrees with experiment for very different pairs of initial hadrons.

7) One more result: We know that hydrodynamics works with average multiplicity values. How are we to explain (for example) the multiplicity fluctuations, which are far larger than the Poisson fluctuations ($D \sim \sqrt{\langle n \rangle}$, where $D = \sqrt{\langle n^2 \rangle - \langle n \rangle^2}$ is the dispersion), having values $D \approx 0.5 \langle n \rangle$? It turns out that if we take into account the huge fluctuations in the inelasticity which we have mentioned (there is essentially a uniform probability for the primary nucleons to carry off between ~0.2 and ~0.9 of the initial energy) then it is a simple matter to explain the multiplicity fluctuations at $E_L = 50$, 65, 100, and 300 GeV (Ref. 56).

In addition to the phenomena in which the predictions of the thermodynamic model have already been definitely confirmed, we could also discuss some possibilities which arise at energies above accelerator energies if the mass of the hydrodynamic subsystem increases with the energy of the process.

We will also point out that the emission of "forward" photons (essentially, blackbody emission) during the early stages of the expansion has been suggested for

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[18)] An even lower initial temperature ($T_{eff} \approx 0.3$ GeV at $\sqrt{s} \sim 50$ GeV) was found in a detailed analysis in Ref. 54.

diagnostics (temperature measurements) of the hot quark–gluon plasma. This process has another aspect, however: If the initial mass of the blob is very large, then it will expand for a long time, and the photons which are emitted may carry off a substantial fraction of the energy of the blob [furthermore, if the mass is large enough the photons may be reabsorbed and reach an equilibrium with the hadronic matter; in this case, if we assume for simplicity that the hadrons produced are pions, one-fifth of the total energy will go to each degree of freedom (π^+, π^-, π^0) and to the two polarizations of the direct photons; after the decay of the π^0 mesons, the electromagnetic component will carry off not one-third of the entire energy, as usual, but three-fifths]. Rough estimates based on Landau's "canonical" model yield $n_\gamma/n_\pi \sim Cn_\pi^{1/3} \sim C'E_L^{1/12}$ as the ratio of the number of direct photons to the number of pions; at $E_L \sim 0.1$–1.0 TeV, this ratio is of the order of 10%. Halzen and Scott,[124] however, pointed out that the equations extracted from cosmic-ray experiments predict that the hadron multiplicity will increase more rapidly than $\sim E_L^{1/4}$ at $E_L \gtrsim 100$ TeV. This effect will be seen in a corresponding increase in the number and energy of the photons produced. Indications of such a "gammazation" were seen a long time ago in data on extensive air showers.[125]

We could continue this discussion for a long time. It would be better for us to refer the reader to the existing reviews and the literature cited there for the details and to move on to the question of how are we to understand the success of the hydrodynamic approach even at comparatively low energies, beginning at a few tens of GeV, how is such a miracle possible? For a long time this success was puzzling, because when the hydrodynamic theory was being worked out it was believed to be applicable at best at $E_L \gtrsim 1$ TeV ($\sqrt{s} \gtrsim 60$ GeV), when many particles are produced—enough particles to allow a macroscopic statistical approach. The initial temperature, on the other hand, had to be high enough ($T_0 > m_N$) that the colliding nucleons could "be crushed" and transformed into some sort of continuous nuclear matter. It turned out, on the other hand, that good results were attained even at $E_L > 20$–50 GeV, where $\langle n_\pi \rangle \lesssim 5$ and where the number of degrees of freedom is not very large, $3\langle n_\pi \rangle \lesssim 15$.

This mystery was solved by quantum chromodynamics. We now know that a pion contains two valence quarks and that half of its momentum is carried by quark pairs and gluons, which are furthermore concentrated at small momenta and are thus numerous. Consequently, *the total number of degrees of freedom of the system is many times larger than* $3\langle n_\pi \rangle$; the quasiclassical approach is legitimate; and *thermodynamics is possible in principle.*

We thus have an elegant situation: In contradiction of the commonplace (and unscientific) assertion that "the laws of classical physics break down in the microworld," the behavior of nuclear matter in a volume of the order of that of a stable hadron can be described at a sufficiently high energy density by a nonquantum theory: classical (relativistic) hydrodynamics. This is

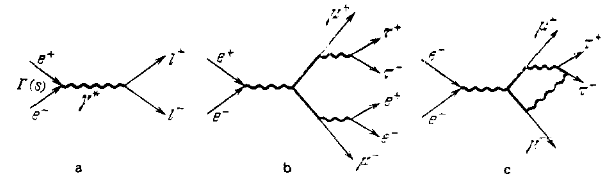

FIG. 10. Electron-positron annihilation into leptons. a—Lowest order of perturbation theory in the coupling constant; b, c—some higher-order amplitudes.

not some crude approximation but an approach completely justified by the short range involved and the short wavelengths of the (quasi) particles in such a plasma in comparison with the dimensions of the system (phonons with a wavelength $\sim 1/T$, where T is the temperature, may be regarded as quasiparticles, while quarks and gluons are particles).

d) Just what is a fireball in quantum field theory?

Is it true, as we sometimes hear even today, that a nonresonant massive blob of nuclear matter—a fireball—is something exotic, lying outside the general picture of elementary particles and quantized fields? Absolutely not. An entity of this sort, which decays into stable particles, is a completely ordinary and necessary element in *any* quantum field theory.[57]

We will first discuss this question without appealing to quantum chromodynamics, remaining within the framework of quantum electrodynamics, which is the quantum field theory which has been studied most thoroughly; and we will ignore hadrons for the moment. Let us consider electron–positron annihilation into a lepton pair, $e^+e^- \to l^+l^-$, e.g., into $\mu^+\mu^-$, $\tau^+\tau^-$, or back into an e^+e^- pair (Fig. 10a). Here the heavy γ ray γ^* (whose mass is $\mathfrak{M}_{\gamma*} = \sqrt{s}$) is an intermediate, nonresonant, decaying entity. It decays only into two particles, because we restricted the discussion to the lowest order of perturbation theory in the small coupling constant $e^2 \ll 1$. But, for example, in the sixth order (very rare) many-particle decays can occur, Fig. 10b, c. It is clear that here γ^* is a very real fireball, but its decay can be calculated dynamically because of that same smallness of e^2. The mass spectrum of the γ^* fireballs which are generated is described by the s dependence of the total annihilation cross section, $\sigma_{tot}^{annih}(s)$

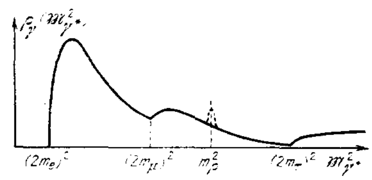

FIG. 11. Schematic representation of the spectral function $\rho\nu$ of the photon propagator associated with the mass spectrum of the virtual photons which are produced, $\mathfrak{M}_\gamma^*$, in quantum electrodynamics (solid curve). The dashed peak shows the contribution from the production of a ρ meson [the abscissa scale is greatly distorted to show in a single figure the various production thresholds: the thresholds corresponding to the production of many pairs at $(4m_e)^2$, $(6m_e)^2$, etc. are not shown].

$\sim |\Gamma_{ee\gamma}(s)|^2 \rho_\gamma(s)$, where $\Gamma_{ee\gamma}(s)$ is the vertex function (a form factor; a constant for point leptons), and $\rho_\gamma(s)$ is the so-called spectral propagator of the photon (not to be confused with the ρ meson!). This function is calculated directly in quantum electrodynamics, and it is shown by the somewhat schematic sketch in Fig. 11. We now take into account the possibility of annihilation into a resonant hadronic state, e.g., the production of a ρ^0 meson, $e^+e^- \to \gamma^* \to \rho^0 \to \pi^+\pi^-$. In this case a resonance peak appears at $s = m^2$ against a smooth background (the dashed peak in Fig. 11).

All these arguments can be extended in a qualitative way to any quantum field theory. Quantum chromodynamics, for example, adds a (*nonresonant*) quark–antiquark pair to the final state.

If, instead of e^+e^- annihilation, we consider collisions of hadrons at a large value of s, then the final states with different numbers of hadrons have matrix elements which are comparable in magnitude (in contrast with quantum electrodynamics, in which the amplitude in Fig. 10a is far larger than those in Figs. 10b and 10c). Figure 12 shows, as an example, a possible case in quantum chromodynamics, for a πN collision; in this particular case, a "pion fireball" π^* is produced. The mass spectrum of such a fireball is again determined by the product of the square of the modulus of the vertex function $\Gamma_{q q \pi}(\mathfrak{M}^2_{\pi*})$ and the spectral propagator of the pion, $\rho_{\pi*}(\mathfrak{M}^2_{\pi*})$. This spectral function will have an isolated peak at $\mathfrak{M}_{\pi*} = m_\pi$ corresponding to the real pion, and beginning at values of $\mathfrak{M}_{\pi*}$ sufficient for the emission of at least three pions (the $\pi \to 2\pi$ transition is not possible) there is a continuous spectrum. On the curve there will again be jumps at the threshold for the production of each new hadronic entity [for example, at $\mathfrak{M}^2_{\pi*} = (nm_\pi)^2$, where $n = 3, 4, \ldots$; at at $\mathfrak{M}^2_{\pi*} = (m_\pi + n \cdot 2m_K)^2$, where $n = 1, 2, \ldots$]. Superimposed are resonance peaks at values of $\mathfrak{M}_{\pi*}$ equal to the masses of all the possible hadron resonances having the same quantum numbers as the pion. As a result, this continuous curve will not have very large smooth regions. It will be ragged, with some significant bumps where particularly strong resonances appear, etc. (Actually, Fig. 11 is an idealization even for γ^*, since for clarity we have not shown the bumps on the curve at the thresholds for the production of each additional electron pair, muon pair, and τ pair. But already in the interval from $\mathfrak{M}^2_{\nu*} = (2m_e)^2$ to $\mathfrak{M}^2_{\nu*} = (2m_\mu)^2$ there are $m_\mu/m_e \approx 200$ such jumps, so that the entire curve is ragged here also. Since e^2 is small, however, each bump is much smaller than the preceding one.)

In summary, the nonresonant background (with superimposed resonance peaks which are numerous in the physics of hadrons, and which have breaks at the thresholds) and thus the *nonresonant heavy entities* which decay into many final particles *are natural elements in any quantum field theory.* An entity of this sort is simply a particle (with corresponding quantum numbers) which has been displaced far from its mass shell in the time-like direction. Those who assume that heavy hadronic clusters are *only* resonances (possibly not yet discovered) are actually introducing an extremely special hypothesis: the hypothesis that the spectral propagator does not have a smooth background and consists exclusively of resonance peaks. This hypothesis is incorrect in quantum electrodynamics (Fig. 11), and there is no reason of any sort to believe that it will hold in any other quantum field theory, particularly in hadron physics. The probability for the production of a nonresonant cluster will of course also depend on the form factor $\Gamma(\mathfrak{M}^2_i)$. It may be very small at large values of $\mathfrak{M}^2_i$ for some reason or other, but the same is equally true of resonant entities.

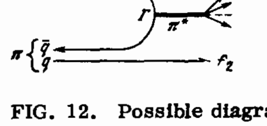

FIG. 12. Possible diagram for an inelastic πN collision which leads to the production of a π^* fireball in quantum chromodynamics. f_1 and f_2 are fragmentation jets.

We have thus seen that a nonresonant heavy hadronic blob is nothing at all unusual in any field theory. Quantum chromodynamics, however, introduces some new and significant features.

In the particle physics based on quantum chromodynamics, the idea of stable hadrons as "bags" containing valence quarks, their gluon fields, and "sea quark" (pairs of quarks from the polarized vacuum) has already taken root quite firmly. This idea, introduced by a group of theoreticians from the Massachusetts Institute of Technology (the "MIT bag"),[58] does not explain quark confinement but adopts it as a phenomenological fact, attributable to some pressure on the surface of the bag. Inside the bag the interaction of the quarks and gluons is weak (weaker, the greater the compression of the bag), so that we can use perturbation theory in quantum chromodynamics. Already at this level it is possible to explain many aspects of the properties of hadrons. But this theory has by now advanced much further.

In the first place, the existence of such a pressure can be linked in a qualitative way to the difference between the fluctuations of the quantum chromodynamics vacuum in free space (where the fluctuations are large) and in the presence of matter—the quarks—inside a bag (where the fluctuations are suppressed).[59,60]

Second, when a highly fluctuating vacuum of this type was taken into account in a semiphenomenological way it was found possible to determine the masses of the low-lying resonances, and this approach has in general been used successfully to analyze the spectral propagators in quantum chromodynamics.[61] As an example we might cite Shifman's paper,[61b] where it is concluded that there is a "glueball"—a resonance with the quantum numbers of a gluon—and where its mass is estimated. The approach taken is to calculate the spectral function of the gluon current correlator, which can be done only quite crudely. This function does not have a simple resonance shape, but on the curve there are some broad rises which may be attributed to formations of the type sought with masses of 1.2–1.3 and 2.5 GeV.

Third and finally, a more detailed bag model has been developed[62] in which it is assumed that the valence quarks are not bare; instead, each produces around itself a sheath of quark pairs and a gluon field which has spatial dimensions much smaller than that of the entire hadron. These "constitutent quarks" or "valons"[63] represent an intermediate rank in the spatial-structure hierarchy of matter. This model successfully explains certain aspects of high-energy interactions.

At any rate, we are naturally led to the idea that there exists at high densities and temperatures a quark–gluon plasma within which (due to the asymptotic freedom) perturbation theory can be applied to the interaction of quarks and gluons. In particular, we can find an equation of state for such matter[45,64] (incidentally, it is at very high temperatures that things get hazy regarding the applicability of perturbation theory[65]). If we write this equation in the form $p=c^2\varepsilon$, where c is the sound velocity, and p is the pressure, then it turns out that c^2 itself depends on T and takes on the ultrarelativistic value of 1/3 only asymptotically, at large values of ε. If the compression is not too pronounced, we have $c^2 < 1/3$.

A plasma of this type is interesting, on the one hand, as a specific case of a fireball or, in general, of a thermodynamically (hydrodynamically) evolving subsystem in high-energy hadron collisions and, on the other, in connection with the study of certain superdense (neutron) stars.[45] The collisions of high-energy heavy nuclei represent a nearly ideal mechanism for producing a plasma of this sort in the laboratory. A substantial effort is presently being made in this direction. Experimentally, collisions of nuclei with energies of many tens of GeV per nucleon have already been arranged in colliding beams at CERN, and there are plans to go to the heaviest of nuclei. Corresponding theoretical work is also being carried out. It is planned to make use of the very leakage of photons and leptons mentioned above as a diagnostic tool for determining the plasma temperature in the early stages of the plasma evolution. Only in these stages can the emission of these particles be calculated reliably, through the use of perturbation theory (such calculations have already been carried out in an analysis of direct-production dileptons and photons with $p_\perp \sim 1$–4 GeV).

As it expands and cools, a quantum chromodynamics plasma may (or, rather, should) undergo a phase transition, turning into nuclear matter at some critical T_c. (In the "valon" picture we can expect to find two phase-transition temperatures: At a certain T_{c_1} the gas of free gluons and quarks transforms into a gas of valons, and later, at $T_{c_2} \equiv T_{cr} < T_{c_1}$, it transforms into the hadronic phase.[66])

There have been particularly persistent efforts to derive a phase transition of this type from quantum chromodynamics itself in the past two years. This cannot be done, of course, by perturbation theory, so that the problem is intimately related to the problem of the confinement of color quantum chromodynamics entities. A special technique is used: numerical solution of the quantum chromodynamics equations in a space–time having a lattice structure. Definite progress has been made in various simplified versions of the problems. For a gluon field [the Yang–Mills equation with SU(2) symmetry], for example, a phase transition of this type in fact emerges from the calculations: There exists a temperature T_c such that at $T \gg T_c$ the specific heat of the system is the same as that for blackbody emission (a gas of gluons), $c_V \sim T^3$, while at $T \ll T_c$, c_V is the same as for a hadronic gas with mass spectrum of the real hadrons known from experiment (and embodied in the calculation). At $T \sim T_c$, the $c_V(T)$ curve has the shape typical of a phase transition (a "λ point").[67–69]

This is, of course, an idealized example, and furthermore the absolute value of T_c can be determined only if we made additional assumptions; nevertheless, *the general spirit of the approach serves as a reason*able basis for a hydrodynamic theory.

The phase-transition temperature T_c must be slightly higher than the critical temperature T_{cr} for the decay into final hadrons, which is determined experimentally from the momentum spectrum of these hadrons ($T_{cr} \approx m_\pi$), because of a rather trivial factor: The hadronic phase which arises at T_c cannot arise immediately as a gas of completely free hadrons. Initially, at $T=T_c$, there is a hadronic liquid or, at any rate, a nonideal gas of hadrons. With further expansion the gas undergoes a further cooling, which gives rise to an ideal gas at $T=T_{cr} \approx m_\pi$. Since the hadrons of various types (nucleons, pions, etc.) have significantly different interaction cross sections, the transition to the final state of an ideal gas for them may occur at slightly different temperatures. There are experimental indications of differences of this type.[70]

In summary, while quantum chromodynamics does not yet explain why a quasiclassical subsystem (a quark–gluon plasma) appears in a high-energy collision of two hadrons, it does give us our first "microscopic" explanation for the hydrodynamic theory of multiple production and, in general, thermodynamic systems or subsystems, e.g., fireballs.[19)] But also concerning the question of the formation of such systems, we see indications of an explanation, as we mentioned earlier (Subsection 2c). The explanation lies in the circumstance that the scattering cross section for gluons is far larger than that for quarks. The result may be a more rapid thermalization and coalescence into a common system of the gluons from the two colliding hadrons than occurs for the corresponding quarks. We are still, of course, left with the question of how rapidly does this thermalization occur (it is of course accompanied by an increase in entropy) and the question of the size of the initial volume for the subsequent isentropic

[19)] New calculations on a lattice for quantum chromodynamics taking into account both gluons and fermions support the existence of *two* phase transitions (the higher-temperature transition corresponds to a violation of chiral symmetry)[126,127] and thus the concept of two scales.[62,63,66] expansion of the subsystem in accordance with the hydrodynamic theory. We have seen that experimental data (measurements of the initial temperature T_0, which, as it turns out, does not exceed ~0.4 GeV at E_L ~ 1000 GeV) put this size at a comparatively large value, tens of times greater than the volume assumed by Fermi and Landau, $V_F \sim m_\pi^{-3} \cdot 2m_N/\sqrt{s}$ [see point 5) of the preceding subsection]. But despite the comparatively large gg scattering cross section, its *absolute value* is still small (otherwise, we could not calculate it by perturbation theory). It becomes large only when, as a result of the cooling, we reach the range of forces responsible for the confinement of quarks and gluons. Accordingly, *the initial size of the thermodynamic subsystem cannot be very small*.[20]

The appearance and development of quantum chromodynamics thus breathes new life into the theory of multiple production based on thermodynamic subsystems, in particular, fireballs, and gives this concept a sounder theoretical foundation.

e) Conclusions of Section 2

Let us summarize the theoretical case for fireballs (and the related experimental facts).

1) Quasiclassical thermodynamic systems can arise in extremely high-energy hadron collisions of a peripheral nature only as *subsystems* formed as a result of a quantum field interaction.

2) Heavy clusters are required in any multiperipheral model compatible with the basic known properties of multiple production: the approximate constancy of the cross section, the significant multiplicity, and the small slope of the pomeron trajectory.

3) In principle, these clusters might be heavy, as yet unknown, resonances or nonresonant fireballs. Nonresonant fireballs are nothing at all exotic and fit naturally into any quantum field theory. In fact, the assumption that they do not exist is a special hypothesis which is not borne out by the well-studied quantum electrodynamics.

4) The possibility that these entities evolve in a thermodynamic way, even if their masses are not very large, receives new support from quantum chromodynamics: The number of excited degrees of freedom in such a system is very large because of the many quarks and gluons which may by virtue of interactions reach a local thermodynamic equilibrium (a "quark-gluon plasma").

5) The initial spatial dimension of a subsystem of this sort, as is shown by a determination of its temperature through the observation of hadrons, photons, and dileptons at $m_\perp \sim 1$–4 GeV, is far larger (by a factor of tens at $E_L \sim 10^3$ GeV) than is assumed in the thermodynamic theories for head-on collisions (the Fermi and Landau theories). This situation is natural for the thermodynamic subsystems which arise as a result of a quantum exchange of virtual particles or of an interaction of the individual constituents of hadrons (gluons, for example).

6) An unstable blob of a hot quark-gluon plasma which cools on expansion, undergoes a phase transition to a hadronic phase, and ultimately transforms into a gas of hadrons is a good candidate for the role of a physical fireball.

3. FIREBALLS IN COSMIC RAYS

To distinguish fireballs experimentally primarily requires selecting groups of particles which are distributed (nearly) isotropically in some Lorentz frame of reference. For this purpose, a remarkable property of isotropic decay is used (and has been used since the early cosmic-ray emulsion studies). Specifically, the distribution of decay products in the rapidity y or the pseudorapidity $\eta = -\ln\tan(\theta/2) \approx y$ can be approximated quite well by a Gaussian curve:

$$\frac{dN}{d\eta} = \frac{K}{\sqrt{2\pi}\delta} \exp\left(-\frac{(\eta-\eta_{cl})^2}{2\delta^2}\right) \equiv G(\eta-\eta_{cl};\ \delta),\ \delta \approx 0.8; \quad (8)$$

here K is the total number of particles in the "cluster" (K is replaced by K^{ch} or K^-, etc., if we are considering only charged particles or only negatively charged particles, etc., and their distributions $dN^{ch}/d\eta$, $dN^-/d\eta$, etc.); and η_{cl} is the pseudorapidity of the cluster as a whole. The products of the fireball decay, regardless of their total number K, thus span an interval $(Dy)_{fb}$ ~1.5–2 along the rapidity or pseudorapidity scale. According to accelerator experiments, the total interval actually spanned by the particles at $E_L \sim 1000$ GeV is Dy ~ 4, and if this interval increases at all with E_L it does so very slowly, perhaps logarithmically (see Fig. 22 below). *This gives rise to the primary difficulty in attempts to single out fireballs*: Their products overlap greatly along the rapidity scale with the decay products of other fireballs and resonances.

Figure 13 illustrates the situation with distributions

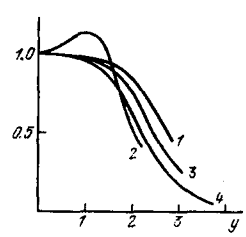

FIG. 13. Rapidity distributions. 1—Experimental (at $\sqrt{s}$ = 54.4 GeV) and for various models incorporating identical fireballs; 2—two fireballs with center-of-mass rapidities $y_{c1} = -1$ and $y_{c2} = +1$; 3—three fireballs with $y_{c1} = -1.6$, $y_{c2} = 0$, and $y_{c3} = +1.6$; 4—nine isotropically decaying clusters arranged in an equidistant manner between $y_c = -2.0$ and $y_c = +2.0$ (the distance between adjacent clusters is $\Delta_{yc} = 0.5$), which approximates the explanation of experiment by curve (1) in the AFS model[7] ("ρ mesons" producing 18 pions). All results are shown for the c.m. frame. All the curves are normalized at $y = 0$.

[20] Here, of course, models of the type in Figs. 5 and 7, with a subsystem arising upon the exchange of a single quantum, are a crude approximation. The collisions of gluons are multiple collisions, but with respect to quarks the process is still peripheral—of the type in Fig. 2 and in the model of Pokorski and Van Hove.[13]

resulting from the decay of various combinations of isotropically decaying fireballs (along with the inclusive distribution which is actually observed in the c.m. frame at 1 TeV). All the distributions are normalized to the same ordinate at $y=0$. It is obviously impossible to identify anything suggesting fireballs here (except, perhaps, curve 2), especially since their rapidities y_{cl} fluctuate, causing a further spreading of the curves.

If, however, we assume that as E_L increases the number of fireballs does not, but instead remains constant, while the mass of the fireballs and the number of particles resulting from the decay of each do increase, these events should be reflected in *an increase in the density* of the particles along the rapidity axis: If this density at $E_L \sim 1$ TeV is ~2 charged particles per unit rapidity interval near $y=0$, corresponding to $\mathfrak{M}_t \approx 4$ GeV when two fireballs are produced, the density will be correspondingly higher at $\mathfrak{M}_t \gg 4$ GeV.

On top of this we have another difficulty. As we mentioned earlier, and as was known in cosmic-ray work more than a quarter of a century ago, for a given energy of the colliding hadrons, E_L, the characteristics of the inelastic collision fluctuate sharply from case to case (in particular, we mentioned the scatter in the inelasticities and the multiplicities, for which the dispersion is far larger than the Poisson dispersion, $D \sim \sqrt{\langle n \rangle}$, having a value $D \approx 0.5 \langle n \rangle$). This entire situation is intimately related to the *superposition of different production mechanisms* which have comparable probabilities. For example, the diffractive dissociation of one (or both) of the initial particles gives us a small group (or two small groups) greatly separated along the rapidity scale from the other initial particle (or the other group). The multiperipheral chain describes a quasiplateau in the central region, etc. Correspondingly, the rapidity distributions of the individual events have irregular, sharply varying shapes. Only a superposition of many events, i.e., the inclusive distribution, has something approaching the standard shape in Fig. 13 (curve 1). It is difficult, however, to learn anything about fireballs from these results. Two approaches can be taken here.

One is to analyze carefully the individual events by special methods developed in the physics of cosmic rays (Duller-Walker F-plots; the selection of rare events with a large total width of the Dy distribution, in which we can expect the two fireballs—if they exist—to lie apart, in a random way, and to be manifested individually; and there are many other possibilities[71,72]). The data which have been extracted from analyses of this type around the world were recently collected by Lattes *et al.*[6] They emphasize their own personal conclusions, according to which there are three types of fireballs: small fireballs [given the Old Indian name "Mirim" or referred to as H quanta (heavy quanta)], with an average mass $\mathfrak{M} \sim 2$–3 GeV and a decay temperature $T_{cr} \approx 0.13$ GeV; intermediate fireballs ("Açu" or "superheavy" fireballs—SH), with $\mathfrak{M} \sim 15$–30 GeV and $T_{cr} \approx 1$ GeV; and giant fireballs (Guaçu" or "ultraheavy" fireballs—UH), with $\mathfrak{M} \sim 100$–300 GeV and with even higher values of T_{cr} [by a factor of 2–4 (?)]. This sharp distinction among three classes of fireballs basically reflects the point of view of one school of workers which has not yet been adopted by other cosmic-ray physicists. All the mass distributions are very blurred. There are, however, some weaker assertions, which seem to be accepted quite widely: 1) The mass spectrum of the fireballs at $E_L \sim 10^3$–10^7 GeV stretches up to $\mathfrak{M} \sim 100$ GeV; 2) the average transverse momentum of their decay products increases slightly with E_L; 3) at higher values of E_L the appearance of higher values of $\mathfrak{M}$ is more probable. We should recall here that a giant "star" containing something of the order of 100 charged particles was detected and carefully analyzed 20 years ago. The angular and momentum distributions of the particles in it agreed well with the interpretation of this event as the decay of one fireball. It even received its own name: the Texas Lone Star, after the place where it was detected.[73] In many cases, evidence for the decay of one heavy fireball comes from simply the high density of particles along the pseudorapidity axis.[21)]

Another approach is to study the various correlations in inclusive and semi-inclusive distributions. This approach can be followed if the statistical base is large enough. It is thus applicable for analyzing accelerator data and has been used in some very different versions. Although the energies involved here so far are $E_L < 2000$ GeV, we will examine these results in more detail.

4. CLUSTERS AND FIREBALLS IN ACCELERATOR EXPERIMENTS BELOW 2 TeV

The general principle for studying correlations is clear: If we select two particles with rapidities y_1 and y_2 in an interval $|y_1 - y_2| \leq 2\delta \delta \approx 1$–2 [see (8)], there is a fair probability that they will have derived from the same isotropically decaying cluster (although there is unavoidably some overlap of the decay products of different clusters). If instead we have $|y_1 - y_2| > 2\delta$, then it is nearly certain that the particles belong to different clusters. In the former case, if we see some sort of "short-range ordering" (SRO) it will reflect a correlation within a cluster. In the latter case, if we observe a "long-range ordering" (LRO) it will be evidence of a mutual correlation of clusters. These correlations may have either a kinematic origin (conservation of energy, momentum, charge, etc.) or a dynamic origin (an additional interaction between particles or clusters).

It would be totally impossible to describe, in one paper, all the theoretical and experimental research which has been carried out on correlations of this type without omitting some studies and underrating the importance of others. I will have to express my regrets regarding this matter and direct the reader to published reviews[74,80,57] for the most part. We will restrict the discussion in the present review to two impressive examples.

1) The simplest type of correlation is the *two-particle*

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21) In the giant fireballs which we were discussing above, there are tens of charged particles per unit rapidity interval, not two as in the accelerator experiments at $E_L \lesssim 1$ TeV.

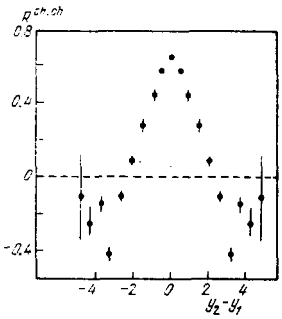

FIG. 14. Two-particle rapidity correlations of charged particles $R^{ch,ch}(y_1, y_2)$ at $E_L=200$ GeV (pp collisions, experimental).

rapidity correlation. A measure of this correlation is the correlation function

$$C(y_1, y_2)=\frac{1}{\sigma_{in}}\frac{d^2\sigma(y_1, y_2)}{dy_1dy_2}-\frac{1}{\sigma_{in}^2}\frac{d\sigma(y_1)}{dy_1}\frac{d\sigma(y_2)}{dy_2} \qquad (9)$$

or the correlation coefficient, found from the correlation function,

$$R(y_1, y_2)=\frac{C(y_1, y_2)}{(1/\sigma_{in}^2)(d\sigma(y_1)/dy_1)(d\sigma(y_2)/dy_2)}=\frac{\sigma_{in}d^2\sigma/dy_1dy_2}{(d\sigma/dy_1)\,d\sigma/dy_2}-1. \qquad (10)$$

These measures vanish if the particles are emitted independently, and we have $d^2\sigma/dy_1dy_2=(d\sigma/dy_1)\,d\sigma/dy_2$. These values have been measured in accelerator work, and they reveal an unexpectedly strong correlation when pairs of particles of identical charge are selected (R^{++} or R^{--}) and also when pairs of arbitrarily charged particles are selected ($R^{ch\cdot ch}$). Figure 14 illustrates the situation with a plot of $R^{ch\cdot ch}$. At $y_1=y_2=0$ we find $R^{ch\cdot ch}\approx 0.6$. This value is a direct indication that there are many correlated particles in the decaying entity. The simple multiperipheral model not only fails to explain this result; it in fact predicts *a negative sign* for the effect: that the adjacent vertices will tend to repel each other in rapidity space and that there should be a minimum, rather than a maximum, at $y_1=y_2=0$ (Ref. 43). We mentioned earlier (Subsection 2b) that a major effort had been undertaken to change this theoretical situation. It has become necessary to admit into the group of exchangeable particles (in the comb in Fig. 6) and in the group of product particles (the teeth of the comb)—in addition to pions—all the light resonances: ρ, ω, f, and A_2. For exchange, their reggeized trajectories have to be included. Understandably, however (the repulsion of vertices is retained!), this did not help. A branching of combs (and trajectories) was considered next. Again, however, the situation was not changed. Only when an "enhancement" was introduced in the branching of trajectories—i.e., only when each branching vertex was additionally surrounded by many pomeron lines—was it possible to achieve $R^{ch\cdot ch}(0,0)\approx 0.6$.

All these alterations in the method of complex angular momenta can of course be defended and are not illegal. As a result, however, the diagram transforms from a simple comb into a heavy coil (or set of coils) of greatly entangled trajectories and may be regarded (as we have already mentioned) as a reggeon model of a fireball, its "microscopic theory." A fireball, on the other hand, gives us a large value of $R(0,0)$ in a completely

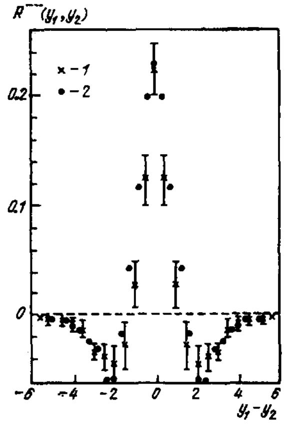

FIG. 15. Example of the description of two-particle correlations in the Lebedev Institute Model[37-41] (from Ref. 40): $pp\to\pi^-\pi^-X$ (69 GeV). 1—Experimental; 2—model.

natural way; everything is determined by simply the number of particles generated in the fireball and by the overlap along the rapidity scale with other decaying entities—resonances or fireballs—which arise in parallel, for example, at other vertices of a multiperipheral chain. In particular, the Lebedev Institute Model has yielded an accurate description of experimental data.[40] Figure 15 shows an example.

A multicluster model with independent emission of clusters, regarded on the average as identical [the "independent cluster emission model (ICEM), which might be termed more accurately the "independent identical cluster emission model" (IICEM)], is based on the experimental fact that long-range correlations are small, so that the mutual correlation between clusters as a whole is totally ignored. Here $C(y_1, y_2)$ can be expressed analytically in terms of the characteristics of the clusters. As was shown in Ref. 81, we can write

$$C(y_1, y_2)=A_0\frac{1}{\sigma_{in}}\left(\frac{d\sigma}{dy}\right)G(y_1-y_2;\,\delta\sqrt{2}), \qquad (11)$$

where $d\sigma/dy$ is taken at $y=(y_1+y_2)/2$, G is the Gaussian function (8), with δ replaced by $\delta\sqrt{2}$, and the coefficient A_0 characterizes the clusters:

$$A_0=\frac{\langle K(K-1)\rangle}{\langle K\rangle}. \qquad (12)$$

Depending on the charges of the pairs of particles being analyzed (− −, + +, or ch ch) we should take K to represent the number of correspondingly charged particles in the cluster (K^+, K^-, or K^{ch}; the total number of particles in the cluster is K).

Experimentally the function $C(y_1, y_2)$ in (9) is measured, and this function is in fact described well by expression (11) if, considering charged particles only, we assign the coefficient A_0 a value of 2.4–2.7 (at $E_L\sim 0.4$–2 TeV). According to the ICEM, or expression (12), this would mean that there are $\langle K^{ch}\rangle\approx 3$–4 charged particles in the cluster, on the average, while if a third of the particles are π^0 mesons the estimate would be $\langle K\rangle\approx 5$–6.

It seems more reasonable, however, to apply this method not to all events taken together (for the given

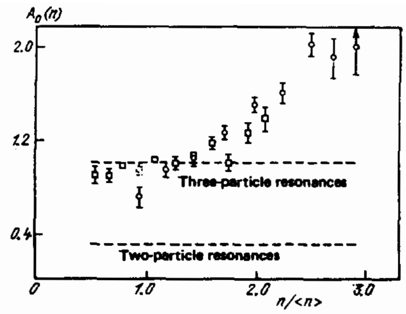

FIG. 16. Clusterization parameters $A_{0,n}$ (charged particles) obtained from experimental data with the help of Eq. (13) for pp collisions at $E_L = 200$ GeV (Ref. 83), plotted as a function of $n/\langle n\rangle$ (in the independent cluster emission model, ICEM, we have $A_{0,n} = \langle K(K-1)\rangle/\langle K\rangle$). Dashed lines—Values expected for clusters which decay into only two or only three particles.

E_L) but to events selected on the basis of the total multiplicity n; i.e., it seems more appropriate to analyze semi-inclusive data. Assigning the corresponding quantities a subscript "n" we find the following expression from the ICEM[82]:

$$C_n(y_1, y_2) = A_{0,n}\frac{1}{\sigma_n}\frac{d\sigma_n}{dy}G(y_1 - y_2; \delta\sqrt{2}) - \frac{1 + A_{0,n}}{n}\frac{1}{\sigma_n^2}\frac{d\sigma_n}{dy_1}\frac{d\sigma_n}{dy_2} \qquad (13)$$

[again, $d\sigma_n/dy$ is taken at $y = (y_1 + y_2)/2$]. Experimental data for $E_L = 200$ GeV give us values of $A_{0,n}$ (as a function of n), as shown in Fig. 16 (for charged particles). We immediately see an interesting circumstance: At $n/\langle n\rangle > 1.5$ the coefficient $A_{0,n}$ begins to increase sharply. If we again use expression (12) for the region in which $A_{0,n}$ remains constant, we find only $\langle K^{ch}\rangle \approx 2$ or $\langle K\rangle \approx 3$, i.e., clusters far smaller than those emerging from the inclusive experiment. These new values at $n < 1.5\langle n\rangle$ make it possible to identify clusters with three-particle resonances of the ω-particle type. However, we are then left with the completely baffling total elimination of ρ mesons; furthermore, at $n > 1.5\langle n\rangle$ it is clear that clusters far heavier than three-particle clusters are involved. It is probably the averaging over all n in the inclusive analysis, mentioned earlier, which gives rise to the larger values of $\langle K\rangle$. By itself, the small value $\langle K\rangle \approx 3$ is not yet very indicative: Clearly, at $E_L = 200$ GeV there should not be as many very heavy fireballs. This is only part of the story, however. *These* ICEM *estimates substantially reduce the actual size of the fireballs*. The primary reason for this result is that all the clusters are assumed identical. On the other hand, many investigators understood even a long time ago that it is necessary to take into account the presence in multiple production of at least two components with distinct properties. A two-component model was worked out in a series of studies; one component of this model is a few-particle diffractive dissociation (D), while the other is a many-particle nondiffractive pionization (ND). In some cases, this model is assigned to a multiperipheral comb, while in others it is assigned to heavier clusters (Refs. 81, 84–86, etc.). Harai and Rabinovici,[86] for example, have shown that by working from these two components, even ignoring their mutual interference, one can explain the set of inclusive and semi-inclusive distributions by fitting seven adjustable parameters (see also Ref. 87). The really remarkable point, however, is that although the D component introduces a far smaller multiplicity than the ND component its presence is enough to cause marked changes in certain measurable characteristics of the process. One such characteristic is the so-called second Mueller moment f_2: the integral of $C(y_1, y_2)$ over the rapidity. It turns out that if the D component is present with a relative probability c (so that the relative probability of the ND component is $1 - c$), then we have[79,84]

$$f_2 = cf_2^D + (1-c)f_2^{ND} + c(1-c)(\langle n^D\rangle - \langle n^{ND}\rangle)^2, \qquad (14)$$

where f_2^D and f_2^{ND} are the Mueller moments of the two components, each considered separately, and $\langle n^D\rangle$ and $\langle n^{ND}\rangle$ are their average multiplicities. Accordingly, in spite of the condition $\langle n^D\rangle \ll \langle n^{ND}\rangle$, and even if the admixture of the D component is small, $c \ll 1$, the mixed term strongly affects the total value of f_2 because of the large value of the pionization multiplicity $\langle n^{ND}\rangle$.

Such a two-component model may be regarded as a simplified version of, for example, the Lebedev Institue Model, shown in Fig. 7, where the few-particle component includes not only diffractive dissociation but also the excitation of baryon resonances and other processes. The two-component role in determining the sizes of the clusters or fireballs can thus be determined by making use of this model and comparing the predictions with those of the single-component ICEM multicluster model.

As mentioned earlier, there is an extensive library of computer-simulated events based on the Lebedev Institute Model.[39,40] The various distributions extracted from this library, including correlations, agree well with experiment. This library has also been used to determine the size of fireballs. It is important to note that there are *two different ways* to do this.

First, the simulated events could be regarded as experimental raw data and processed as experimentalists would process their own data (for example, by the procedure which led to the results in Fig. 16). Specifically, we could extract from the library data on exclusively 1) $d\sigma_n/dy$ and 2) $C_n(y_1, y_2)$, i.e., the two low-order moments of the rapidity distributions and then work with these moments to find $A_{0,n}$ from (13). These values are shown by the filled squares in Fig. 17 for 200-GeV pp collisions. The hatched bands in Figs. 17a and 17b show—within the uncertainties due to the experimental errors and the small statistical base—what an actual accelerator experiment would yield for pp collisions at this energy and under these conditions. The fact that the values extracted from the library of model-based data fall in the hatched band again confirms that the model agrees well with these experimental results also—that the model is realistic. If, continuing to interpret this ICEM "experiment," we again appeal to (12), we should understand the values of $A_{0,n}$ extracted from the model to be the values of $\{\langle K^{ch}(K^{ch}-1)\rangle/\langle K^{ch}\rangle\}_n$ for the given value of n.

However, this library contains far more-detailed in-

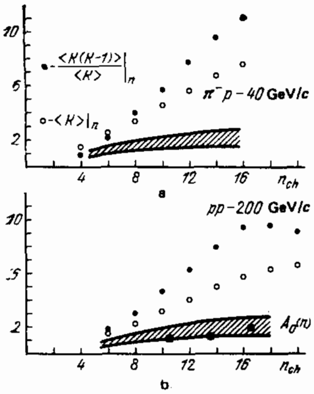

FIG. 17. Clusterization parameter $A_{0,n}$ for charged particles in π^-p collisions at 40 GeV and in pp collisions at 200 GeV found from experimental data with the help of (13) (the hatched bands, which reflect the experimental uncertainty) and calculated from the library of exclusive events simulated on the basis of the Lebedev Institute Model (Fig. 7), also with the help of (13) (filled squares). Filled circles—values of $\langle K(K-1)\rangle/\langle K\rangle$ calculated from the same library of events directly [without the use of Eq. (13)]; open circles—values of $\langle K\rangle$ (Ref. 40).

formation. In particular, from the very process by which *each event* is simulated we know whether there is a fireball present and just how many pions appear during the decay of this fireball. We can therefore directly determine both $\langle K^{ch}\rangle_n$ and $\{\langle K^{ch}-1)\rangle/\langle K^{ch}\rangle\}_n$ for the same kind of charged particles in events of any given total multiplicity *without appealing* to Eq. (13), which is based on the single-component multicluster model. The corresponding results[40] are also shown in Fig. 17. The actual cluster sizes are obviously far larger than those found from the analysis of the ICEM data; that model ignores that there are two types of mechanisms: few-particle and many-particle.

A second reason why the ICEM underestimates the size of the clusters is the complete independence in this model of the emission of clusters with arbitrary rapidities (in particular, the conservation laws are ignored). In any multiperipheral field model there is something resembling repulsion between adjacent clusters in rapidity space: The difference between the rapidities of different clusters cannot be very small. In particular, as Levin and Ryskin have shown[43] it is for this reason that the two-particle short-range rapidity correlation coefficient turns out to be negative in the multiperipheral model without clusters instead of positive, as is observed. To pursue this effect, Orlov[117] studied an analytically solvable multicluster model "with repulsion" in which the clusters are assumed to be identical, but the difference between the rapidities of different clusters cannot be smaller than a certain value. This model gives a good description of the correlation characteristics in the central region, but the size of the cluster has to be assumed much larger than the ICEM predicts: $K^{ch}\approx 5-8$.

A remarkable fact has been found here: The *actual* sizes of the fireballs are far larger than those which emerge from the analysis of the ICEM events. For events with $n_{ch}\sim 16$ (in the cases used, this condition means $n_{ch}/\langle n_{ch}\rangle \sim 2$) the fireball contains an average of 6–7 charged particles, i.e., only ~10 pions. This result is evidence that the single-component multicluster model with independent emission of clusters is not a valid model for determining the cluster size. This model also sharply underestimates the cluster masses.

2) Another correlation example which we will consider is the *two-particle azimuthal correlation.*

The rapidity y is basically a measure of the longitudinal motion of the particles. But for the products of a fireball decay the transverse momenta should also cancel out. Therefore, the azimuthal emission angles φ_i of the products in the transverse plane, should be correlated to some extent, more closely, the smaller is the difference between their rapidities y_1 and y_2. In fact, if $|y_1-y_2|\ll 2\delta\sim 1-2$, then there is a good chance that the two particles will belong to the same fireball. We might also take into account yet another qualitative consideration: As the fireball becomes heavier, the number of particles which appear from it increases, the importance of a single particle in the overall cancellation process fades, and the azimuthal correlation of a pair of particles should therefore become less pronounced. We will see that all these arguments stand up under analysis.

We choose a particle with a certain rapidity y_1 as a reference point, and we adopt the direction of its emission in the transverse plane as the origin for measuring the emission angles of the other particles, φ_i. For a certain $y_2=y_1+\Delta y$ the particles will be emitted with angles φ_i (Fig. 18). The number of particles, $N(\Delta y;\ |\varphi_i| < \pi/2)$, which are emitted with $|\varphi_i|<\pi/2$ (we denote this number of $N_<$) should be slightly smaller—say one particle smaller—than the number of particles, $N(\Delta y;\ |\varphi_i| > \pi/2)\equiv N_>$, which are emitted into the opposite hemisphere (with respect to the reference particle). The ratio

$$B(\Delta y) = (N_> - N_<)/(N_> + N_<) \qquad (15)$$

can be taken as a measure of the azimuthal correlations. For $\Delta y=0$ and large values of K we can therefore expect $B(0)\sim 1/\langle K\rangle$ (more likely, $B(0)\lesssim 1/\langle K\rangle$, because of the superposition of particles from another fireball).

In fact, experiment shows that this effect occurs: $B(\Delta y)$ is greater than zero. The effect can be seen particularly clearly if we single out multiparticle events, $n_{ch}\geqslant 6$ (Fig. 19), i.e., if we apply the same selection rule which was used back in 1958 to single out fireballs in cosmic-ray physics.[1,71] If we adopt the estimate $B(0)$

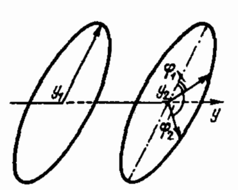

FIG. 18. Notation for determining azimuthal correlations.

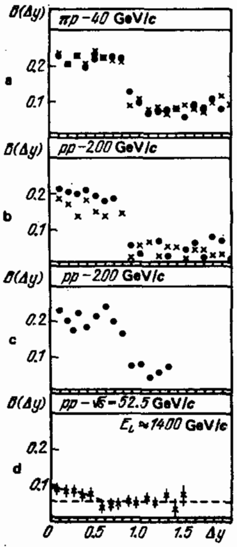

FIG. 19. The azimuthal correlation parameter $B(\Delta y)$ in (15) for events with $n_{ch} \geq 6$ and $\Delta p_{\perp} > 0.2$ GeV/c. ×—Experimental; ●—from the library of events simulated on the basis of the Lebedev Institute Mode.[40] a—$\pi^- p$, 40 GeV, taken from the data of Ref. 88; b—pp, 200 GeV, taken from the data of Ref. 80; c—pp, 200 GeV, from the Lebedev Institute Model incorporating only the contribution of fireballs; d—pp, $\sqrt{s} = 52.5$ GeV (corresponding to $E_L = 1400$ GeV) (Ref. 89).

$\approx 1/\langle K \rangle$, we find the effective clusters to be large, $\langle K^{ch} \rangle \sim 5$–10; they were called "superclusters" in a CERN paper.[89] The model of Fig. 7 gives a completely satisfactory description of this effect. It is important to note that the effective size of the cluster for events with $n_{ch} \geq 6$ and $\Delta p_{\perp} > 0.2$ GeV/c increases with increasing E_L; $-B(0)$ falls off from 0.22 at $E_L = 40$ GeV to 0.093 at $\sqrt{s} = 52.5$ GeV ($E_L \sim 1000$ GeV). This effect undoubtedly requires further research, however.

As we mentioned earlier, it is not possible to present the results found in a study of multiple correlations. For example, a detailed study has been made of the distribution in the magnitude of the rapidity interval between adjacent[90] and nonadjacent[91,92] (along the rapidity scale) particles and of the distribution of the lengths of the maximum intervals in a given event[93]; a fluctuation analysis has been carried out[94]; studies have been made of the rapidity dispersion,[95] charge transport between the forward and rear hemispheres,[96] and charge transport through various points on the rapidity scale[97]; the two-particle rapidity correlation has been studied as a function of the invariant mass of the two particles[98]; and the correlation between the lengths of adjacent rapidity intervals (a short interval following a short interval or a long interval following a short interval).[99,100] Special statistical characteristics were invented for all this work, and in fact a special statistical method was developed.[101,102] This work is continuing today. Of the further results which have now been obtained we will mention only the estimate of the total electric charge of a cluster, $Q|e|$: There are indications that $|Q|$ is equal to zero or one.[77]

The independent cluster emission model (ICEM) has been the starting point for very many of these studies. There is the predominant conclusion that multiple production in fact involves the formation of intermediate clusters, which are too large to be reduced to simply light meson resonances ($\rho, \omega, f, \eta, A_2$), since the average number of particles per cluster is $\langle K \rangle \approx 3$–4 or $\langle K^{ch} \rangle \approx 2$; on the other hand, these intermediate clusters are too small to be identified with cosmic-ray clusters, with $\langle K \rangle \sim 6$–10.

Some investigators, however, believe that we must also include a significant component consisting of four- and five-particle resonances (p. 180 in Ref. 74). Analysis of the distribution of rapidity gaps also points to large clusters,[100] $\langle K \rangle \sim 6$–10. We have already mentioned the contradiction between the cluster sizes found from inclusive rapidity correlations and semi-inclusive rapidity correlations, etc.

This is not the whole story, however. In the first place, in cosmic-ray work clusters are found by selecting events with $n_{ch} \geq 6$, not by averaging over all events. Second, the events under discussion there have energies $E_L \gtrsim 1000$ GeV, while the study of clusters in accelerators is based almost entirely on lower energies. Third and finally—and most importantly—the classification of all clusters as being of a common type (as is done, for example, in the ICEM) apparently results in a severe underestimate of the actual size of the fireballs or clusters which are primarily responsible for the pionization component, as we mentioned earlier. A joint analysis of the involvement of different components would, on the other hand, complicate the analysis considerably.

The Lebedev Institute Model, which seems quite realistic, shows that the so-called average cluster emerges from an averaging over different entities. We could write symbolically

$$\text{"average cluster"} = \langle \alpha \cdot \text{resonances} + \beta \cdot \text{(diffractive dissociation)} + \ldots + \varepsilon \cdot \text{heavy fireballs} \rangle,$$

where $\alpha, \beta, \ldots, \varepsilon$ are certain coefficients, generally different in studies of different correlation effects. These are the differences which can explain the differences (mentioned above) in the values found for $\langle K \rangle$ in the analysis of (first) rapidity intervals, (second) two-particle inclusive correlations, and (third) semi-inclusive correlations. The analysis of accelerator data incorporating the presence of various components, which we mentioned earlier (Fig. 17), shows that even at $E_L = 200$–1400 GeV the fireballs are actually quite heavy, $\langle K \rangle \sim 6$–10, especially if we single out events with $n_{ch} \geq 6$ or $n_{ch}/\langle n_{ch} \rangle \gtrsim 1.5$–2.0. We should emphasize again that in the multiperipheral approach of Fig. 7 in this energy range the number of vertices in the chain is still very small, $\mathfrak{N} \sim 2$–3 at $E_L \sim 200$ GeV or $\mathfrak{N} \sim 3$–4 at $E_L \sim 2000$ GeV. Therefore, an "average cluster" $\langle K \rangle = 3$ can appear if, in a collision of two nucleons, the two retain their individuality, and in addition a single ρ meson and a single fireball with $K = 8$ form.

In summary, the accelerator data at $E_L \sim 200$–2000 GeV *are consistent with the concept of cosmic-ray fire-*

balls with masses $\mathfrak{M} \sim 2-5$ GeV *and in fact provide weighty arguments for this concept.* As for heavier fireballs (Achu, SH and Guachu, UH in the terminology of Ref. 6), it would be premature to discuss them at these accelerator energies.

5. WHAT DOES THE FUTURE HOLD?

We have attempted to show the following: a) The accelerator data at $E_L < 2$ TeV fundamentally contradict a multiperipheral model with many light comb teeth. b) These teeth tend to coalesce in nonresonant clusters or fireballs.[22)] c) The customary and widely adopted estimates of cluster sizes from accelerator data—based on correlation analysis and the assumption that all the clusters are of the same type (the ICEM)—severely underestimate the size of the clusters. The correlations actually show that a cluster/fireball decays into an average of ~6–10 pions. d) The thermodynamic model for the decay of fireballs (or the hydrodynamic model if the fireballs are heavy) explains, in a surprisingly natural way, a wide variety of characteristics of multiple production. e) Nonresonant heavy hadron clusters are not at all exotic and instead find a natural place in quantum field theory. f) In real peripheral collisions a thermodynamic system can arise only as a subsystem, as a result of a quantum field process. g) Quantum chromodynamics has explained many problems in the thermodynamic picture and gives it a new foundation and adds to it.

We thus see that two seemingly contradictory interpretations of multiple production, one based on fireballs and confirmed by cosmic-ray work and the other based on a multiperipheral comb with many light teeth, have actually converged substantially as accelerator energies have increased. This convergence may be described (extremely schematically) as a transition from the diagrams of Fig. 6 to those of Fig. 7 with a governing influence of fireballs (Fig. 3).

It should be noted, however, that the thermodynamic decay of a fireball, in such good agreement with many experimental results (see Subsection 2c), cannot be regarded as obligatory in all cases. For example, a quark or gluon jet can undergo hadronization only after a stage of cascade multiplication (extremely popular at the moment).[103,104] In the dual-resonance model one deals with the sequential emission of pions; this approach leads to a momentum spectrum very similar to that found by the thermodynamic approach,[105] etc.

We naturally wonder what the future holds: How will this picture evolve as we move to higher energies?

The first question which arises if we adopt the fireball model is this: As s increases, will the number of fireballs in each collision increase, while the size of the fireballs remains constant, having already reached a limit, or will the small number of fireballs be conserved, while the fireballs themselves become heavier?

The first possibility is expressed in particular by a multifireball chain. In this case the multiplicity must increase logarithmically with s in the asymptotic region (the region of many fireballs, $\mathfrak{N} \gg 1$), according to (4), but the coefficient a will be established at a corresponding rather large value. As we mentioned earlier, however, $\mathfrak{N}$ could reach values of even 3–4 only at $E_L \sim 100$ TeV for purely kinematic reasons. In the inclusive approach a Feynman scaling will be established, precisely as in the AFS model (Fig. 6), in the inclusive approach, because of the superposition of many fireballs spread out slightly along the rapidity axis.

The second possibility—a further increase in the mass of the fireballs while their number remains constant, $\mathfrak{N} \sim 1-2$—would mean a simple continuation of the behavior observed (in the interpretation based on fireballs) at $E_L < 2$ TeV. This is the preferred possibility among the cosmic-ray workers, since in this case the multiplicity is a power-law function of E_L. This is the behavior of the thermodynamic decay. In the hydrodynamic picture, for example (which is mandatory at a cluster mass $\mathfrak{M} \gtrsim 5-10$ GeV), a single cluster yields $\langle n\rangle \sim \mathfrak{M}^{(1-c^2)/(1+c^2)}$, where c is the sound velocity (as we mentioned earlier, in Subsection 2c, $\mathfrak{M}$ is by no means identical to $\sqrt{s}$, and when the broad distribution of values of $\mathfrak{M}/\sqrt{s}$ is taken into account we find that the E_L dependence of the total number of particles in the process, $\langle n\rangle$, while remaining an approximately power-law dependence will contain c^2 in a slightly different way[23)]).

Observations of processes in cosmic rays furnish direct examples of extremely heavy fireballs (up to $\mathfrak{M} \sim 100$ GeV; see Section 3), but even more important are the confirmation, based on analysis of the development of extensive air showers, that at $E_L \gtrsim 10^2-10^3$ TeV the exponent α in the dependence $\langle n\rangle \sim E_L^\alpha$ is greater than even 1/4, lying between 1/3 and 1/2. The onset of this behavior might be noticeable even at $p\bar{p}$ collisions in the CERN colliding-beam apparatus, with an effective energy $E_L = 150$ TeV.

Actually, however, there is also an intermediate possibility. We recall, for example, that in the "hybrid" model of the Lebedev Physics Institute (Fig. 7) the mass spectrum of the fireballs is determined by the function $\bar{\sigma}(\mathfrak{M})$ in (6). The behavior of this function at very large values of $\mathfrak{M}$ cannot be specified unambiguously in the model, whose parameters are selected with reference to an energy $E_L < 2$ TeV. The only point which is clear is that $\bar{\sigma}(\mathfrak{M})$ must be a function which falls off rapidly enough that all the integrals in the theoretical model converge. In this model we will therefore have the direct production of both resonances (ρ and heavier reso-

[22)] We again recall that we are using the term "fireballs" to apply also to those heavy subsystems which can be described hydrodynamically.

[23)] In this connection there are some recent measurements of the multiplicity during the diffractive production of hadronic systems of large mass, $\mathfrak{M} \sim 1-5$ GeV, in $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ collisions at 100 and 200 GeV. It turns out that $\langle n\rangle \approx 2\sqrt{\mathfrak{M}}$ (Ref. 128; $\mathfrak{M}$ is in GeV) gives an excellent description of the data everywhere.

nances) and fireballs with an entire spectrum of masses.[24)]

All this refers to hadron–hadron collisions, which are in fact the subject of the entire review. As we have already pointed out, however, collisions of heavy ultrarelativistic nuclei—a rapidly developing research field—provide a favorable situation for the appearance of thermodynamic (or hydrodynamic) systems. The mechanism for the production of such systems may prove closer to the picture of head-on collisions originally envisaged by Heisenberg, Fermi, and Landau. There is a solid basis for hoping that research on these processes will permit a study of dense quark–gluon plasmas.

But let us return to hadron–hadron collisions.

We have attempted to show that the concept of fireballs as quasiclassical nonresonant systems which arise as an intermediate step in ultrarelativistic collisions is not only consistent with but also substantially supported by accelerator experiments at $E_L < 2$ TeV and the theoretical picture which emerges from modern quantum field theory (including quantum chromodynamics). Final conclusions about this entire concept, however, must await experiments in the new generation of accelerators at an effective energy $E_L \sim 10^2$–10^3 TeV.

These experiments ($p\bar{p}$ collisions at $\sqrt{s} = 540$ GeV) have already begun, and the first, extremely fragmentary, results have proved unusually significant from our standpoint. We will discuss these results here, although we recognize that by the time this review is published far more extensive information will undoubtedly be available, and some of the early results will have to be changed. We will discuss the work by two experimental groups, UA1 (Ref. 106) and UA5 (Ref. 107 and 108).

So far, the UA1 data[106] cover only a small pseudorapidity interval, $-1.3 < \eta < 1.3$, in the c.m. frame, so that at the moment no data are available on either the total multiplicity or the total cross section. Nevertheless, an extremely important result has been obtained: The density of charged particles at the pseudorapidity axis in the central (pionization) region, which was found to increase even in colliding-beam experiments[11,110] in the interval $23 < \sqrt{s} < 62$ GeV, continues to increase, in accordance with the same law. Figure 20 shows the number of charged particles per unit pseudorapidity interval, $\Delta\eta = 1$, in this region. We see that it is increasing at least linearly with ln$s$, and instead of a value of the order of 2 at $\sqrt{s} \approx 24$ GeV it reaches 3.9 ± 0.3 at $\sqrt{s} = 540$ GeV (in both cases, events with at least one charged particle in these intervals were selected; an estimate which also includes events without charged particles reduces the value to 3.6 ± 0.3).

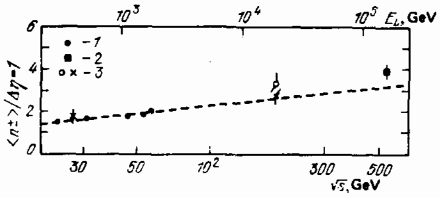

FIG. 20. Number of charged particles per unit pseudorapidity interval ($\Delta\eta = 1$) at $\eta \approx 0$ (in the c.m. frame) for pp collisions (1) (Ref. 110) and $p\bar{p}$ collisions (2) (Ref. 106), along with cosmic-ray emulsion data (3) (Refs. 111, 112), according to Ref. 106.

A corresponding result was obtained in the UA5 experiment,[108] where it was primarily the inclusive cross section at the point $\eta = 0$ which was measured (Fig. 21). Here again we see an increase with s; the measured value again increases by a factor of about 2.

These results are unmistakable evidence against Feynman scaling in the pionization region—evidence that nothing even resembling this scaling is present. We can thus discard the simple multiperipheral model of Fig. 6. We might note that the cosmic-ray data obtained through the exposure of an emulsion chamber carried to the "very top" of the atmosphere on balloons had even earlier yielded data[111,112] in agreement with these accelerator data.

A second important result is the increase in the multiplicity. A value $\langle n_{ch} \rangle = 27.4 \pm 2.0$ was found in the UA5 experiment[107] over an interval $-5 < \eta < +5$. This η distribution (Fig. 22) furnishes evidence that nearly all the particles are included, but it should be recalled that this value was obtained through an extrapolation in the region $|\eta| > 5$. Various assumptions regarding the behavior of the curve (in particular, special incorporation of diffractive dissociation) may reduce this figure, which cannot be considered very accurate. Nevertheless, we see from Fig. 23 that it does conform well to the curve

$$\langle n_{ch} \rangle = a + b \lg s + c (\lg s)^2, \qquad (16)$$

proposed in Ref. 110, with $a = 0.88 \pm 0.10$, $b = 0.44 \pm 0.05$, and $c = 0.118 \pm 0.006$; it is also in agreement with cosmic-ray data.[111,112] The deviation from the $\langle n_{ch} \rangle \sim \lg s$

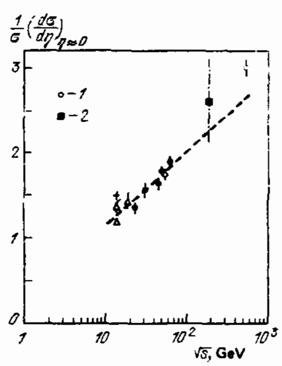

FIG. 21 Inclusive cross section of charged particles at $\eta = 0$. Summary of data from Ref. 108 [including the measurements of Ref. 108 (1)]; 2—cosmic rays.

[24)]Resonances may be produced either directly, at the vertices of the chain, or upon the decay of a thermodynamic fireball. Depending on which path has the higher probability, the characteristics of the decay products ($p_\perp$, for example) will or will not be describable by means of the thermodynamic equations.

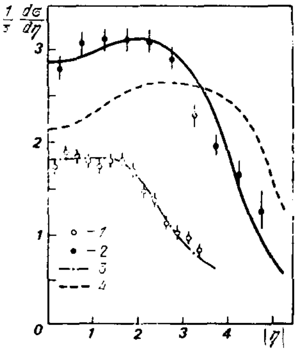

FIG. 22. Quasirapidity distribution of charged particles in the central region. 1—At $\sqrt{s}=53$ GeV; 2—at $\sqrt{s}=540$ GeV; 3, 4—calculated from a model with a phase volume bounded with respect to $p_\perp$ with $\langle p_\perp\rangle = 0.35$ GeV for 53 and 540 GeV, respectively; solid curve—the same calculation, for $\sqrt{s}=540$ GeV, but with $\langle p_\perp\rangle = 0.50$ GeV (Ref. 108).

law is further evidence that a simple comb model with light teeth is not valid.

A third and final result is the η distribution in the pionization region. We see from Fig. 22 that this distribution is comparatively narrow. While at $\sqrt{s}=540$ GeV we have $\ln s = 12.6$, the inclusive cross section decreases to half the height of the "quasiplateau" at $\eta=\pm 4$ (at $\sqrt{s}=62$ GeV we have $\ln s = 8.3$, and the half-height is reached at $\eta=\pm 3$). The minimum at $\eta=0$ is of kinematic origin; it may transform into a plateau or even a maximum when we transform to a distribution with respect to y. Such a transformation can be carried out, however, only if we know the $p_\perp$ distribution or at least the average value $\langle p_\perp\rangle$. This average has not yet been measured in these experiments. It is important to note, however, that if we assume the same values as at lower energies, $\langle p_\perp\rangle \approx 0.35$ GeV, the quasiplateau y distribution (in the model with a $p_\perp$-limited phase volume) leads to an η distribution which agrees with experiment for $\sqrt{s}=62$ GeV (the lower curve in Fig. 22) but sharply contradicts experiment at $\sqrt{s}=540$ GeV (the middle curve). If we assume $\langle p_\perp\rangle = 0.5$ GeV, however, we find

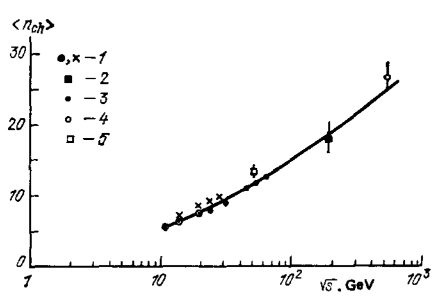

FIG. 23. Average charged-particle multiplicty $\langle n_{ch}\rangle$ as a function of the energy in the c. m. frame. Cure—Empirical dependence $a + b\ \lg s + c(\lg s)^2$ found in Ref. 110. 1—Data from various Fermilab measurements (—all inelastic processes, including diffractive; ×—nondiffractive processes exclusively); 2—cosmic rays[112]; other data—CERN, colliding pp beams (3) and colliding $p\bar{p}$ beams (experiment UA5–4) (4, 5—diffractive processes have been eliminated, according to Ref. 107).

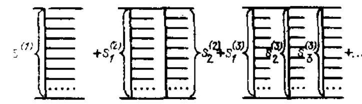

FIG. 24. Summation of the branching combs in the model of Refs. 109 and 130.

a very good agreement (the upper curve[25)]).

This result also agrees with the conclusion reached in Ref. 106, where the estimate $\langle p_\perp\rangle \sim 0.5$ GeV was reached on the basis of other considerations (from measurements of the energy flux in the transverse direction).

We will not go into the other results found in Ref. 107 (adherence to KNO scaling, etc.), but we do wish to point out that there are cases with a very high multiplicity ($n_{ch}\sim 100$); if we classify the events at $\sqrt{s}=540$ GeV on the basis of the values of n_{ch} we find that the inclusive cross section for $\eta=0$ increases linearly with increasing n_{ch}.

Granted that these early results will certainly be refined and perhaps changed in part, we would like to see what they imply for the problem of interest here.

As we mentioned earlier, these results completely rule out the simple comb with light teeth in Fig. 6. Attempts have accordingly been undertaken to change this model by adding complications to it[130] (these attempts have been analogous to, but quite different from, the approach in Refs. 42 and 43). A theory with a "supercritical bare pomeron" was proposed sometime ago.[109] This theory, in other words, has an initial pomeron intercept $\alpha_P^{(0)}(0) = 1+\Delta$, where $\Delta \approx 0.07 > 0$; it transforms into a reasonable pomeron if we allow branching into combs of a special type, specifically, if we use in the inelastic amplitude the sum of amplitudes in Fig. 24, where the energy in each comb naturally decreases as the division becomes finer, so that each comb contributes a plateau to the rapidity distribution. The width of each plateau decreases (in proportion to $\ln s_i^{(N)}$) as the division becomes finer. The overall distribution is as shown in Fig. 25, with an increasing inclusive cross section at the center (required for agreement with experiment; the authors adopt, for example, $\sqrt{s_i^{(N)}} = \sqrt{s}/N$, etc.). The fitting of certain parameters results in a very good description of the experimental data on the rapidity distribution, on the multiplicity distribution, and on the average multiplicity for the data available up to[130] $E_L = 150$ TeV. The total cross section is predicted to increase in accordance with $\sigma \sim s^\Delta \approx s^{0.07}$.

It should be expected that attempts to explain the correlations—primarily the rapidity correlations—run into the same difficulties which arise in models with branching combs[42,43]: In order to achieve large, positive rapidity correlations it is necessary to take into account both the production of all possible resonances and the

[25)]Admittedly a special assumption regarding the shape of the distribution is used in the conversion: the so-called model with a cylindrical phase volume.

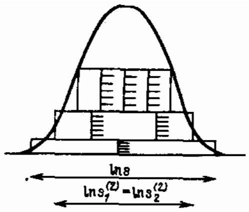

FIG. 25. Resultant inclusive rapidity distribution found by combining the branching diagrams in the model of Ref. 130 (for the model in Fig. 24).

exchange of all possible trajectories in the amplification of vertices, etc. In other words, it is necessary to allow an entangling of the comb elements of the diagrams, which reduces the number of teeth and makes them heavier (Subsection 2b). We are essentially led to models of the peripherally fireball type.

Furthermore, we cannot ignore the fact that here the fundamental question of whether we can ignore the interaction of the emitted hadrons is even more crucial (Subsection 2b). On a unit rapidity interval in the central region we must now have not 2 but 3–4 charged particles; i.e., 5–6 particles when we include neutrals. The rapidity distance between adjacent particles is thus $\Delta y \approx 0.2$, and all these 5–6 product hadrons per unit interval have nonrelativistic longitudinal velocities in the common c.m. frame of this group. The "adjacent" hadrons, on the other hand, move with respect to each other with a longitudinal-motion Lorentz factor $\bar{\gamma} = E/m = \cosh \Delta y \approx 1 + (1/2)(0.2)^2 = 1.02$ (we are ignoring the transverse motion). It is difficult to believe that under these conditions these hadrons will not undergo repeated collisions and would not coalesce, forming a heavy blob of nuclear matter.

What do the new results imply in the thermodynamic approach?

First, experiment shows that there is a marked increase in the density of the rapidity distribution. This is as it should be if the mass of the fireball is increasing, rather than the number of fireballs at a fixed mass. In particular, the hydrodynamic evolution of a single heavy fireball gives rise to a Gaussian y distribution with a width $\langle y^2 \rangle \sim \ln s$, i.e.,

$$\frac{dN}{dy} = \frac{N_0}{\sqrt{2\pi L}} e^{-y^2/2L}, \quad L = \text{const} \cdot \ln s, \tag{17}$$

where the constant depends on the particular equation of state which is used.[113,114,46] At $y=0$ (and $\eta = 0$) the distribution density is thus proportional to $N_0/\sqrt{\ln s}$, where $N_0 = N_0(s)$ is the total number of particles in the hydrodynamic subsystem, which increases with s, as we know, more rapidly than $\sqrt{\ln s}$. Consequently, the derivative $(dN/dy)_{y=0}$ should increase with s [in accordance with (16)] as the sum of terms proportional to $(\ln s)^{1/2}$ and $(\ln s)^{3/2}$.

Second, the multiplicity increases more rapidly than $\ln s$. Alpgard *et al.*[108] emphasize that the $s^{1/4}$ dependence is ruled out. They apparently hold the common opinion that the hydrodynamic theory predicts something of this sort; actually, however, this opinion is erroneous for several reasons. This question deserves some discussion.[26)]

First, I would like to emphasize that there is no single-term formula at all ($\sim \ln s$ or $\sim s^\alpha$) which will describe a process which involves different mechanisms. The multiplicity of fragmentation particles, for example, has an s dependence completely different from that of the multiplicity of pionization particles. The fact that the positive excess of muons in the cosmic-ray flux in the atmosphere remains constant ($J_{\mu^+}/J_{\mu^-} \approx 1.25$, where J_{μ^+} and J_{μ^-} are the fluxes of positive and negative muons) over a markedly wide energy range of the protons which produce them, from ~10 GeV to ~10 TeV, has been under discussion for a long time now and interpreted as the result of the constancy of the number ($n_{f,ch}$) of fragmentation particles in this energy range (see Ref. 72, for example). Accordingly, in studying the multiplicity of pionization particles in the hydrodynamic subsystem we need to study, not the s dependence of n_{ch}, but the dependence on the mass of the hydrodynamic fireball ($\mathfrak{M}$) of the difference $n_{ch} - n_{f,ch}$ for a fixed target and $n_{ch} - 2n_{f,ch}$ in experiments with colliding beams (the fragmentation of both protons gives rise to relativistic particles). Also, as we mentioned earlier, $\mathfrak{M}$ undergoes extreme fluctuations at a constant value of s, while the fraction $\langle \mathfrak{M} \rangle / \sqrt{s}$, which is smaller than the average inelasticity, may vary slightly with s. All this distorts the observable multiplicity, and the function $n_{ch}(s)$ does not come close to directly reflecting the multiplicity in the hydrodynamic process.

According to Refs. 110 and 108, for example (we are adopting the figure given at the moment), we have the following approximate results for pp and $\bar{p}$p collisions in colliding beams:

E_L:	100 [110]	1000 [110]	150 000 [108] GeV
$\langle n_{ch} \rangle$:	6.2	11	27.4
$\langle n_{ch} \rangle - 2$:	4.2	9	25.4

We will take into account only one factor: The fragmentation beams from each p (or $\bar{p}$) must carry off approximately one charge (or perhaps more). In the hydrodynamic subsystem the multiplicity thus does not exceed $\langle n_{ch} \rangle - 2$. If we parametrize the pionization multiplicity by

$$\langle n_{\pi^\pm} \rangle = \langle n_{ch} \rangle - 2 \sim E_L^\alpha,$$

we find the following values for α for the specified energy intervals

$$E_{L1} - E_{L2} \qquad \alpha = \ln\left(\frac{\langle n_{ch}\rangle_2 - 2}{\langle n_{ch}\rangle_1 - 2}\right)\left(\ln\frac{E_{L2}}{E_{L1}}\right)^{-1}:$$

100–1000 GeV	0.33
100–150 000 GeV	0.24
1000–150 000 GeV	0.19.

[26)] It was demonstrated recently that the η distribution in Fig. 22 is in excellent agreement with the predictions of the hydrodynamic theory in the quasi-one-dimensional approximation.[49a] The value $\langle p_\perp \rangle = 480$ MeV is predicted.[129]

We thus see that a power law with an exponent of about 0.25 is by no means ruled out. Adopting $\alpha=0.25$, and moving from $E_{L1}=1$ TeV to $E_{L2}=150$ TeV, we should have, according to this (highly simplified) model,

$$\langle n_{ch}\rangle_2=\langle n_{\pi^\pm}\rangle+2=9\cdot 150^{1/4}+2\approx 33$$

If we work from $E_{L1}=0.1$ TeV by the same law we find $\langle n_{ch}\rangle_2=4.2\cdot 1500^{1/4}+2=28$, instead of the experimental value[108] of 27.4 ± 2. When we furthermore take into account the circumstance that the fragmentation jet can carry off more than one charged particle, that the inelasticity may change slightly as we go to $E_L=150$ TeV, that the sound velocity may differ from $c=1/\sqrt{3}$ (which leads to $\alpha=1/4$), and that the sound velocity may change with increasing temperature (accompanying the increase in $\sqrt{s}$),[45] it becomes clear that it is at best premature to speak in terms of a disagreement of the multiplicity with the hydrodynamic law. Instead, we may say that there is a fair agreement.

Finally, we return to the conclusions regarding $\langle p_\perp\rangle$. We have already mentioned that, as was shown a long time ago,[49] the transverse momenta of the product particles at $E_L<1$ TeV are determined by the thermal motion during the decay of the system, and the hydrodynamic expansion of the fireball can be ignored. At higher energies, however, hydrodynamics takes the leading role and determines that the transverse momentum will have an exceedingly slow growth[115]: $\langle p_\perp\rangle\sim E_L^{1/12}$ to $E_L^{1/14}$. As we go from $E_L=1$ TeV to $E_L=150$ TeV, we would thus have an increase by a factor of 1.52–1.43, i.e., from $\langle p_\perp\rangle\approx 0.35$ GeV to $\approx 0.35\cdot 1.5\approx 0.5$ GeV. This change agrees with the estimates[27)] in Refs. 106 and 108.

There is a question to be answered here, however: Just why does a significant number of physicists cling so tightly to this groundless denial of the thermodynamic interpretation? It may be that a general answer to this question should run as follows. For a decade and a half the accelerator experiments have been confined almost exclusively to few-particle processes of comparatively modest energy. Successful descriptions were generated on the basis of very simple exchanges of field quanta and, later, reggeons. As a rule, heavy clusters could not arise at all under these circumstances, while light clusters could be reduced to light meson resonances (with a mass less than 1.5 GeV). This success inspired an ideology, and the adherents to this ideology have persistently attempted (and, in several cases, continue to attempt) to extend it to large and genuinely high multiplicities. These adherents have been content when they have, on occasion, been successful in describing one or two characteristics of the multiple production process. As we emphasized earlier, a discussion of the production mechanism must include *all* the wide variety of characteristics (so far, this test has been successfully passed only by the Lebedev Institute Model, which has been compared in detail with experiment at energies up to 200 GeV).

As an example we consider the paper by Jancso *et al.*[116] Here, at high energies, 2 TeV, a study was made of the distribution of the emitted pions with respect to the mass of the pair of particles, $C(M_{\pi\pi})$. A smooth curve was found with a broad maximum at $M_{\pi\pi}\approx 0.5$ GeV. This curve could of course be explained easily by assuming that the pions are produced thermodynamically at a decay temperature $T_{cr}\sim m_\pi$ for the system. In this case the mass of the particle pair would have the highest probability for falling at the position of the maximum found. The authors, however, take a different approach: They construct a model consisting of some "cocktail" of five resonances (ρ, ω, f, A_2, η). They choose suitable weight factors to describe the participation of the various resonances, and they achieve agreement with the experimental curve. They leave unexamined all the other characteristics which are ordinarily studied, such as the various correlation coefficients, the distribution of intervals, and much more—characteristics which must be studied in order to draw conclusions about the production mechanism, in particular the reality of heavier clusters (see Section 4 of this review, especially the list of criteria which have been studied). In the eyes of many readers of Ref. 116, its conclusions were sufficient for this paper to be taken as evidence that the multiple-production mechanism reduces to a multiperipheral production of meson resonances exclusively. We mentioned earlier some papers[42–44] which pointed out the difficulties which this simplified representation encounters in an attempt to achieve an at least approximately constant total cross section and, especially, a large short-range correlation coefficient: It becomes necessary to assume a complicated entanglement of Regge poles being exchanged which may actually be regarded as a microscopic description of a few heavy clusters. Here again, however, no explanations are offered for such surprising results as the universality of the transverse-mass distribution for all the product particles, resonances, and particle pairs (Fig. 9), which arises in a completely natural way in the thermodynamic picture.

The conservative approach (a multiperipheral model in which only light resonances and a parton comb with Feynman scaling are produced) received heavy blows when it was shown,[11] with great accuracy, that at energies of the order of 2 TeV in the central region the rapidity distribution of the pions of various signs is described by the hydrodynamic theory with a common decay temperature, that Feynman scaling does not hold (there is no plateau, and the quasiplateau rises sharply with the energy[11]; cf. Fig. 22), and that correlations require the introduction of heavy clusters[83,89] (we showed above that these estimates of the cluster masses are far too low). Not surprisingly, an active theoretician at CERN noted as early as 1976 that if these results had been known a few years earlier the "scaling in the central region" or even the "multiperipheral model" would not have been so extremely popular as they were at the time and that, on the contrary, a better

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27) The result of a direct measurement, $\langle p_\perp\rangle=420$ MeV, was reported in June 1982 (at the Symposium on Multihadron Processes in Holland).

understanding of everything could have been extracted from, for example, Landau's hydrodynamic model.[118]

Since then, the first colliding-beam experiments at $\sqrt{s}=540$ GeV have simply underscored these remarks. A definitive explanation will probably emerge from detailed correlation and other experiments on the new generation of accelerators, but it is already clear that the thermodynamic (and hydrodynamic) interpretation cannot be rejected out of hand no matter how skeptical one may be of it. It continues to merit the same attention that it has long ago deserved.

The rapidly developing research on the quantum chromodynamics of plasmas[45,64-69] and the theoretical evidence for the existence of a phase transition from a quark-gluon plasma into a hadronic phase put a solid foundation under the hydrodynamic and, in general, the thermodynamic concept of multiple production. We are, however, left with one serious problem: Just how, in the collision of hadrons (or nuclei), as a result of the mutual scattering of the constituent gluons (and quarks, but their collision cross sections are far smaller), does a thermalized, thermodynamic subsystem arise and then undergo hydrodynamic expansion and cooling with a phase transition to a hadronic phase, and ultimately with a decay into stable hadrons? What is the initial volume of this subsystem? What is its temperature? An so forth.

This is an unusually difficult theoretical problem, as is any problem involving the conversion of a dynamic process to a stochastic one, but the situation is even more difficult than usual because of the complexity of the structure of the dynamic entities involved. Attempts have been made to approach the solution of this problem at a meson-nucleon level.[119] It is more likely, however, that, as in analogous problems in macroscopic physics, we should not wait for a complete theoretical solution. For the time being we can content ourselves with a qualitative or a semiquantitative understanding of this thermalization process and carry out research based on a phenomenological determination of the initial characteristics of the thermodynamic subsystem.

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