Simulation of a gravitational lens in a lecture demonstration

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Although a systematic exposition of the general theory of relativity does not belong in a general physics course, a discussion of various general-relativistic effects is not only possible but actually very desirable and enlivens such a course with interesting modern data. One such effect is the curving of light rays in a gravitational field as predicted by Einstein as early as 1915. Moreover, a correct order-of-magnitude estimate of this effect can be obtained for the simplest situations (for example, in the case of a point mass) in the framework of simple notions (see, for example, Ref. 1).

In recent years, the interest in "gravitational lenses" has increased sharply because they can serve as one of the manifestations of black holes, and there have been recent reports of the discovery of such objects.[2] At the same time, gravitational lenses differ from the spherical lenses usually considered in optics in important features of the path of the rays and the nature of the obtained images, namely, there is strong astigmatism, multiple images, etc. In fact, it was the observation of two identical quasars (0957 + 561 A, B) that led to the suggestion that they are two images of the same source produced by a black hole gravitational lens.

In this connection, it is desirable in a discussion of gravitational lenses in a general physics course, first, to explain in detail these features of the formation of the images and, second, to illustrate these features by means of a specially chosen optical system with similar properties.

Suppose that a light ray from a distant source passes at a distance r from a massive object. The main difference between a gravitational lens and an ordinary ("spherical") converging lens arises because the rays passing near a point object (the center of the gravitational lens) are deflected through a larger angle than the more distant rays:

$$\alpha(r)=\frac{k}{r}, \quad k=\frac{4\gamma M}{c^2}, \tag{1}$$

where M is the mass of the object, γ is the gravitational radius, and c is the velocity of light[2] (we note that the elementary calculation gives for k half the value[1]). Therefore, a gravitational lens does not have a definite focal distance and does not form images in the usual sense of the word. In fact, one can only speak of an "image" formed by a gravitational lens because of the fact that not only the light source but also the gravitational lens itself are far from the terrestrial observer, so that the receiving instrument (eye, telescope objective) collects a beam of rays in a very small interval of angles α and the rays deflected through larger or smaller angles do not enter the aperture of the detecting instrument. The situation is approximately the same as in the observation of lines of equal inclination in optics.

As a result, when the source, the gravitational lens, and the observer are situated on a single straight line the apparent image of the point source takes the form of a uniformly illuminated ring. But if the center of the gravitational lens is shifted somewhat relative to the source–observed axis, then in the simplest case of a point mass one obtains a double image in the form of parts of arcs, these approximating in the limit to two point images of different brightness.

In the more general case (extended object, in particular a galaxy), the number of images may be greater. It is clear, for example, that for a spherically symmetric mass distribution the rays that pass strictly through the center will not be refracted, and the entire central part will "work" as a weak converging lens, so that the total number of images formed by the telescope objective will be not less than three.

It is readily seen that similar astigmatic properties will be possessed by a lens made, as usual, from a homogeneous transparent material with refractive index $n>1$ if its surface is given a pseudospherical shape (i.e., with negative Gaussian curvature) instead of a spherical form. To be specific, we shall assume that one of the surfaces of the lens is flat ($z=0$), and that the refracting surface is a surface of revolution of some curve $z=z(x)$ about the z axis (see Fig. 1). For brevity, we restrict ourselves to the case when the rays incident on such a lens are parallel to the z axis. Let β be the angle of incidence of the rays on the refracting

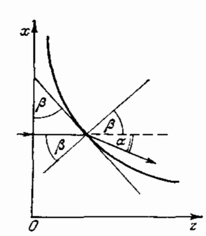

FIG. 1.

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FIG. 2.

surface from within the lens; then in accordance with Snell's law

$$\frac{\sin(\alpha+\beta)}{\sin\beta}=n. \qquad (2)$$

Since, on the other hand, $\tan\beta=-dz/dx$, Eq. (2) relates the angle of refraction $\alpha(x)$ of the rays passing through the lens at distance x from its axis to the local angle of inclination of the surface. It follows from (2) that to obtain a decreasing dependence $\alpha(x)$ it is necessary to take a curve with derivative dz/dx that decreases with increasing x, i.e., a pseudospherical refracting surface. If the angles α and β are assumed to be sufficiently small, so that $\sin(\alpha+\beta)\approx\alpha+\beta$, $\sin\beta\approx\tan\beta\approx\beta$, then from (2) we obtain the simple relation

$$\alpha(x)=-(n-1)\frac{dz}{dx}. \qquad (3)$$

Integrating (3), we can in principle find the profiles $z(x)$ corresponding to different given functions $\alpha(x)$ corresponding to particular axisymmetric distributions of the mass in the gravitational lens and vice versa. In particular, substitution of the asymptotic law (1), $\alpha=k/r$, corresponding to a point mass, leads to an exponential profile $x(z)$:

$$x(z)=x_0\exp\left[-\frac{n-1}{k}(z-z_0)\right] \qquad (4)$$

(here we have taken $z=z_0$ for $x=x_0$).

For our experiments, the pseudospherical lenses (Fig. 2) were machined from sheet plastic of thickness 4–5 cm and diameter 8–10 cm. The lens in Fig. 2a with a point at the center obviously simulates a point mass. The profile $x(z)$ was tested by a specially prepared template, and a sufficiently transparent surface was obtained by subsequent polishing. Of course, for qualitative simulation under the conditions of a lecture demonstration there is no need to maintain the profile (4) with high accuracy, especially at small x, where this would have been difficult, since for a more or less distant observer only a small sector Δx of the lens

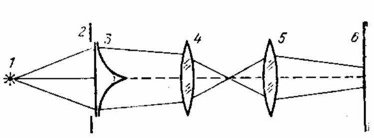

FIG. 3.

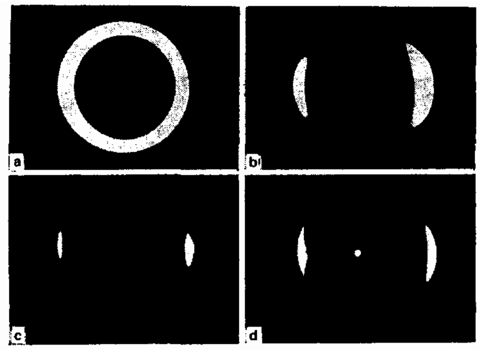

FIG. 4.

"works," and the rays passing through the central region of the lens do not enter the detector. Therefore, it is quite adequate to take the profile $x(z)$ and the corresponding template in the form of an arc of a circle. *Moreover, reasonable results can be obtained, for* example, by using the bottoms of broken-off stems of wineglasses, which usually have a pseudospherical surface.

To obtain the lens shown in Fig. 2b, which corresponds to an extended object with smoothly decreasing density, the central projection was cut off and smoothed. Then the peripheral sections of the lens operate, as above, like a pseudospherical lens, and the central region as an ordinary spherical lens.

Having such a lens, the virtual images characteristic for a gravitational lens can be readily seen directly by the eye by examining a more or less distant light source through the lens. However, in a lecture demonstration it is preferable to obtain real (and fairly large) images on a screen. For this, it is necessary to use an additional objective (Fig. 3). Here, 1 is the light source (any sufficiently bright lamp), 2 is a stop (which is desirable to eliminate unnecessary illumination of the screen and the room), 3 is the lens that simulates the gravitational lens, and 4 and 5 are objective lenses giving a magnified image on the screen 6. Displacing the lens 3 relative to the optic axis of the system, it is easy to obtain successively the images shown in Fig. 4 (Figs. 4a–4c were obtained with the lens in Fig. 2a, and Fig. 4d with the lens in Fig. 2b).

We note finally that these experiments to simulate gravitational lenses by means of light refracting systems are not in fact as formal as they might appear, since as is well known, it follows from the generally covariant properties of Maxwell's equations that the influence of the gravitational field on the propagation of electromagnetic waves is equivalent to the action of some dielectric medium.[3,4]

[1]Berkeley Physics Course, Vol. 1, Mechanics, McGraw-Hill (1965) [Russian translation published by Nauka, Moscow (1975)].
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