# Mechanism of induced transition radiation in fields of regular waves

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The elementary effect of induced transition interaction of a moving charge crossing the boundary between two dielectric half-spaces with the field of a monochromatic wave scattered at this boundary is studied. The quantitative relation between this effect and the corresponding collective process of excitation of charge density waves at the same nonuniformity, i. e., the modulation of the flux of charged particles by the field of a counter-propagating wave, is found. The contribution of the physical phenomena under investigation to the development of beam instabilities and electromagnetic wave generation is discussed.

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## CONTENTS



## 1. INTRODUCTION

The purpose of this note is the determination of the quantitative relation between elementary effects of transition radiation and absorption by a charge crossing a sharp boundary between two media in the presence of a monochromatic wave refracted at this boundary and the collective effect—the appearance of a modulation in the originally unmodulated beam passing through the same boundary in the presence of the counter-propagating regular wave.

Transition radiation discovered by V. L. Ginzburg and I. M. Frank more than thirty years ago[1] is one of a few fundamental effects of the interaction of moving charges with materials. Together with the Vavilov-Cherenkov radiation and normal and anomalous Doppler radiation of an oscillator the transition radiation is a part of the group known in classical physics of optical phenomena pertaining to uniformly moving radiation source in refractive media.[2] The main characteristic features of this group are that the interactions are brought about by the long range electromagnetic fields, and the resulting energy exchange of a charge with a medium is finite when the mass of the charge approaches infinity. Two consequences important for applications follow from this fact. First, a large number of particles of the medium are within the sphere of action of these fields and the corresponding radiation intensities are characterized by averaged macroscopic parameters such as density, temperature and so on. Second, due to the flow of charged particles there are present simultaneously many neighbors in the sphere of action of every charge. As a result, the radiation of the ensemble of particles is different from the radiation of an isolated charge. In particular, there are always present particles in a nonmodulated beam which radiate in opposite phase and, therefore, the intensity of spontaneous radiation of such beam is equal to zero (neglecting fluctuations). In contrast, in a beam that has been modulated beforehand the coherent summation of radiation fields leads to considerable increase of intensity of radiation of every particle. For example, effective field strengths of radiation fields of a beam in microwave amplifiers and generators based on the Vavilov-Cherenkov radiation effect achieve the order of kilovolts per centimeter (see Ref. 3, 4), i.e., they are nineten orders of magnitude higher than the radiation field strength for an isolated charge. Thus, the beam and the medium show their collective characteristics and, because of that, these interactions are called "cooperative."[5,6] It is important that the bunching of particles of the beam which is necessary for the amplification of the radiation intensity may be provided by the reciprocal influence of the radiation field on the movement of particles of the beam. In this sense the interaction process is called induced when the intensity depends

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upon parameters of the radiation field[7,8] in contrast to the spontaneous radiation which does not depend on these parameters.

The existence of induced emission and absorption effects was postulated by Einstein in his quantum theory of interaction of equilibrium radiation with matter. The corresponding quantitative characteristic of these processes—the induced transition probability—is proportional to the intensity of the radiation field divided by the energy of one quantum, i.e., proportional to the number of photons in the initial state of the field which induces the radiation of the quantum system (a radiator). Such a state in which values of the energy of the oscillator-radiator (the number of the initial level) and energy of the field (the number of photons) are completely determined corresponds to entirely indefinite phases of the field and the radiator. The mean values of the field and the radiator current in this state are equal to zero. It is important to note that only under these conditions is the intensity of induced exchange of energy between the field and the oscillator proportional to the square of the small interaction parameter (the product of the charge of the radiator and of the field amplitude, see Ref. 7).

At high levels of excitation of the oscillator and of the field when the energy change of each of these subsystems caused by radiation or absorption of one quantum is relatively small, the classical description of the field and of the radiator states in terms of oscillations with completely determined amplitudes and phases is possible. If, besides that, these oscillations are regular (monochromatic in frequency and their amplitudes and phases do not change in time), the mean values of the oscillator field and current in such states are different from zero. The intensity of the energy exchange of an isolated oscillator with a field in such a classical system is proportional to the first power of the small interaction parameter, i.e., linear in charge and field amplitude. The sign of the effect is determined in this case by the phase relationship between the charge and the field.[7]

Below we shall specifically limit our considerations to such an energy exchange between moving charges and the field stimulated by regular fields.

The induced interaction of regular fields with fluxes of charged particles leading to bunching of these fluxes into coherent bunches is one of the necessary elements of the physical mechanism of collective interaction which produces the formation of beam instabilities. The important role played by the collective interactions in applications is caused by the fact that they lead to the diffusion and heating of the plasma in natural and artificial systems of its confinement, to the generation and amplification of microradiowaves, and to the acceleration of charged particles. Therefore, the problem of the mechanisms governing the relationship between elementary effects of the spontaneous and stimulated radiation of moving charges and collective wave processes in charged particles fluxes is continues to be one of the most important problems in the theory of collective interactions during many years (see[2,9-28]). The understanding of these mechanisms may open the ways for the development of effective methods of controlling the corresponding wave processes.[14,17]

Up to present the quantitative relation between characteristics of spontaneous radiation of moving charges and increments of corresponding beam instabilities was rigorously determined only for kinetic Cherenkov and magnetic bremsstrahlung instabilities of relatively low current beams which have sufficiently broad distribution functions in velocity space.[11,15,19] It was shown that under these conditions (small flux intensities and large deviations) the increments are proportional to sums of the intensities of the separate radiators. The identification of hydrodynamic instabilities for which the coherent summation of fields of separate particles is important with corresponding elementary effects is made mainly by the comparison of the synchronism conditions for a particle and a field with the conditions at which the maximum of the increment occurs.[3,4,6,10,23]

It is natural to expect that the transition radiation of the moving charges at nonuniformities of a medium may lead to a collective interaction of the radiation with charged particle fluxes. It is necessary, however, to understand its special features in comparison with the Cherenkov and magnetic bremsstrahlung radiation.[1)] Indeed, the presence of an electrodynamical nonuniformity of the medium is a necessary condition for the generation of transition radiation, while Cherenkov radiation, for example, can occur also in a uniform dielectric, and magnetic bremsstrahlung occurs in a uniform magnetic field. Furthermore, due to the conditions of its generation, the transition radiation of one particle is always pulsed (it exists for a finite time) and, therefore, its intensity is characterized by total energy losses of a radiator at a given nonuniformity while the intensity of the Cherenkov and magnetic bremsstrahlung radiation are determined uniquely by losses of the particle energy per unit length of its path. Finally, due to absence of synchronism between the field and the moving charge, the transition radiation of the beam may be coherent only within the limits of the bunch of radiating particles. The spatial summation of fields of the periodic sequence of bunches which is characteristic for the Cherenkov and magnetic bremsstrahlung effects does not occur in the case of transition radiation.[2)]

Due to the special features of the transition radiation mentioned above, its role in the processes of collective interaction of charged particle fluxes continued to be unclear for a long time. In theoretical and experimental studies of the elementary transition radiation effect all attention was focussed only on spontaneous radiation. The problem of induced interaction for regular fields was not considered (see Ref. 26). In the present note we shall describe quantitatively the effect of the

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[1)]The results of the studies of characteristics of spontaneous transition radiation can be found in the monograph of Ref. 28 and in reviews of Refs. 2, 29–33.

[2)]The importance of the coherence of the transition radiation for applications was noted by V. I. Veksler.[34]

stimulated transition interaction of a moving charge with the field of a regular wave at a step-like nonuniformity of electrodynamical properties of the medium and consider the connection of this effect with the collective wave process, the transformation of a regular charge density wave in spatially nonuniform media.

## 2. INFLUENCE OF A REGULAR WAVE ON A CHARGE IN THE PROCESS OF ITS TRANSITION FROM FROM ONE MEDIUM INTO ANOTHER

In this section we shall determine the change of the energy of a charge $\Delta\mathscr{E}^i$ after passing through a step-like boundary between two media in the given field of an $E$-polarized wave refracted at this boundary according to the Fresnel law.

In the general case the influence of the field on the process of interaction of a charge with a medium may occur through two channels: through a change of dielectric properties of the medium under the influence of the field and (or) through a change of the radiator characteristics (see Refs. 14, 17, 19). Later in our discussion we shall neglect the first of these effects, assuming that the stimulating fields are relatively weak. The most important result of the influence of the field on the charge motion in the case of regular fields considered by us is the change of the translational charge movement and corresponding deviation of its trajectory from the trajectory unperturbed by the field (retardation or acceleration together with the proper polarization field of the charge). We do not take into account the effect of the reciprocal influence of the spontaneous radiation field on the charge movement, as well as the bremsstrahlung of the charge in the wave field, because they are proportional to the square of the small parameter $r_0/\lambda$ ($r_0 \equiv e^2/mc^2$ is the classical radius of a radiator, $\lambda$ is the characteristic wavelength).

It is necessary to simplify as much as possible the mathematical model for the analytical description of characteristics of this phenomenon without changing its essential features. With this purpose in mind we shall consider the following assumptions to hold:

1. Electrodynamic properties of the medium in which the exchange of energy between the field and moving charges occurs change along the trajectory of these charges only once and by a step (one transition between two homogeneous semi-infinite media).

2. The energy exchange is stimulated only by waves with polarizations which are contained in the spectra of the spontaneous transition radiation of corresponding moving charges at the same inhomogeneities.

3. In absence of high frequency fields the moving charge crosses the boundary between dielectrics along the normal to the boundary surface from left to right.

4. The external waves are regular (monochromatic), are characterized by fixed phases and are incident on the boundary from the right.

These assumptions allow us to consider only the processes of interaction of charged particles (or of their fluxes) with fields of regular $E$-waves scattered in anisotropic (nongyrotropic) dielectric media with step-like nonuniformities of their parameters. The intensity of the corresponding energy exchange of the moving charge with the field is linear in the amplitude of the field, and the sign of the effect depends substantially on the phase of this field.[7] The results obtained in this way can be used in many cases for calculation of mean square characteristics in fields with random phases. For example, the dielectric permittivity of the medium determined by the regular field uniquely determines also the field fluctuations in this medium, according to the Callen-Velton theorem (see Ref. 7, 28).

Thus, let a charge $q$ with mass $m$ move along the positive direction of the $z$-axis in a medium filled with the transparent piecewise-uniform anisotropic dielectric $[\hat{\varepsilon}^- \equiv \hat{\varepsilon}(z<0) \neq \hat{\varepsilon}^+ \equiv \hat{\varepsilon}(z>0)]$ in the field of a plane $E$-wave coming from plus infinity. The charge has the velocity $V_0$ far from the boundary ($z \to -\infty$) where the radiation field is absent. We consider the dielectrics $\varepsilon^\pm$ being transparent for the moving charge. The above mentioned conditions may be most completely fulfilled in a plasma with a step-like nonuniform density placed in a strong external magnetic field parallel to the charge trajectory. In this case the tensor $\hat{\varepsilon}$ has the especially simple form[4,6]:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \quad \begin{array}{l} \varepsilon_\perp^\pm = 1, \quad \varepsilon_\parallel^\pm = 1 - \dfrac{\omega_{p\pm}^2}{\omega^2} \\ \omega_{p\pm}^2 \equiv \dfrac{4\pi e^2}{m} n_p \, (z \gtrless 0). \end{array}$$

Such plasma is used in experiments both for the transport of intense charged particle beams as well as a retarding medium providing the synchronism of the beam particles with the radiation emitted (absorbed) by them.[3-5,10,14,17,47,48] Because of this we consider an anisotropic dielectric, the transition to the corresponding isotropic dielectric being trivial. The necessary decrease of ionization losses can be provided by the channel the radius of which $R$ must be large in comparison with the characteristic size $\Lambda$ of the localization region of dielectric polarization fields: $R \gg \Lambda$.

The distribution of the wave field in the system is determined by the well-known formulas

$$H_y(x, z, t) = H_0 \exp(-i\omega t + ik_\perp x) \begin{cases} \exp(-ik_+ z) + R_E \exp(ik_+ z), & z > 0, \\ T_E \exp(-ik_- z), & z < 0, \end{cases} \tag{1(a)}$$

$$E_x = -\frac{i}{k_0 \varepsilon_\perp} \frac{\partial H_y}{\partial z}, \quad E_z = \frac{k_\perp}{k_0 \varepsilon_\parallel} H_y,$$

$$k_\pm^2 = \left(k_0^2 - \frac{k_\perp^2}{\varepsilon_\parallel^\pm}\right) \varepsilon_\perp^\pm, \quad \mathrm{Im}\, k_\pm > 0, \quad k_0 = \frac{\omega}{c}; \tag{1(b)}$$

where $R_E$ and $T_E$ are respectively the Fresnel coefficients of reflection and transmission for an $E$-wave:

$$R_E = \frac{k_+ \varepsilon_\perp^- - k_- \varepsilon_\perp^+}{k_+ \varepsilon_\perp^- + k_- \varepsilon_\perp^+}, \quad T_E = \frac{2k_+ \varepsilon_\perp^-}{k_+ \varepsilon_\perp^- + k_- \varepsilon_\perp^+}. \tag{1(c)}$$

The change of the charge energy $\Delta\mathscr{E}^i(\tau, t_0)$ induced by the field (1) after passage through the inhomogeneity may be determined from the first integral of the Lagrange equation for the motion of this charge

$$\frac{\partial \mathbf{V}_1}{\partial \tau}(\tau, t_0) = \frac{q}{m} \left\{ \mathbf{E}(\tau, t_0) + \frac{1}{c} [\mathbf{V}_L(\tau, t_0), \mathbf{H}(\tau, t_0)] \right\}, \tag{2}$$

where $\mathbf{V}_L(\tau, t_0)$ is the sought speed of the particle on the trajectory determined by the values of fields (1) at this trajectory with the initial condition $\mathbf{V}_L(\tau = -\infty, t_0) = \mathbf{V}_0$ and with the boundary condition $\mathbf{V}_L(\tau = +0, t_0) = \mathbf{V}_L(\tau = -0, t_0)$ ($\tau \equiv t - t_0$ is the proper time of a charge, $t_0$ is the Euler time of the charge crossing the nonuniformity at the plane $z = 0$). Integrating Eq. (2) in the first order approximation with respect to the small interaction parameter $\mu = qE_0 k_+[m(\omega - k_+V_0)^2]^{-1} \ll 1$, we find the speed $\mathbf{V}_L(\tau, t_0)$ and the energy change $\Delta\mathscr{E}^i$:

$$\Delta\mathscr{E}^i(\tau > 0,\ t_0) \equiv \frac{m}{2}[\mathbf{V}_L(\tau > 0,\ t_0) - \mathbf{V}_0]^2 = \Delta\mathscr{E}^i_{\text{inc}}(\tau,\ t_0) + \Delta\mathscr{E}^i_{\text{refl}}(\tau,\ t_0) + \Delta\mathscr{E}^i_{\text{trans}}(t_0), \tag{3}$$

where

$$\Delta\mathscr{E}^i_{\text{inc}}(\tau,\ t_0,\ k_+) \equiv qE_0L_+\frac{|\omega_-(k_+)|}{\omega_+(k_+)}\cos[\omega_+(k_+)\tau + \omega t_0], \tag{3a}$$

$$\Delta\mathscr{E}^i_{\text{refl}}(\tau,\ t_0,\ k_+) \equiv R_E\Delta\mathscr{E}^i_{\text{inc}}(\tau,\ t_0,\ -k_+),$$

$$\Delta\mathscr{E}^i_{\text{trans}}(t_0) \equiv -qE_0L_+T_E\frac{\omega|\omega_-(k_+)|\cos\omega t_0}{\omega_+(k_+)\,\omega_-(k_+)\,\omega_-(k_-)} \tag{3b}$$

$$\times\left[\omega(1-\beta_0^2\varepsilon^+_\perp)(\varepsilon^-_\parallel - \varepsilon^+_\parallel) + k_-V_0\varepsilon^-_\parallel\left(1 - \frac{\varepsilon^+_\perp}{\varepsilon^-_\perp}\right)\right], \tag{3c}$$

$$\omega_\pm(k_\alpha) \equiv \omega \pm k_\alpha V_0,\quad \beta_0 \equiv \frac{V_0}{c},\quad L_+ \equiv \frac{V_0}{|\omega_-(k_+)|},$$

$E_0 \equiv k_\perp H_0/k_0\varepsilon^+_\parallel$ is the amplitude of the electric component of the incident wave normal to the boundary plane.

The dynamics of the formation of the charge energy change stimulated by the field is especially clear in the case when the left medium is a conductor ($\varepsilon^- < 0$, $|\varepsilon^-_\parallel| \to \infty$):

$$\Delta\mathscr{E}^i_\infty(\tau,\ t_0) = qE_0L_+\left\{\frac{|\omega_-(k_+)|}{\omega_+(k_+)}\cos[\omega_+(k_+)\tau + \omega t_0] + \operatorname{sgn}\omega_-(k_+)\cos[\omega_-(k_+)\tau + \omega t_0] - \frac{2\omega}{\omega_+(k_+)}\cos\omega t_0\right\}. \tag{4}$$

As can be seen from this, the charge energy perturbation by the field of the wave increases from zero at the entrance to the dielectric ($\tau = 0$) to the sum of two oscillating and one constant terms at $\tau > L_+/V_0$. It is important for the further discussion, that the characteristic distance $L_+$, at which the formation of the constant term in the right side of Eq. (4) occurs, is precisely equal to the length of the formation of the spontaneous transition radiation field of the given current modulated at the frequency $\omega$: $L_+ \equiv V_0/|\omega_-(k_+)|$.[35,32]

Let us consider the physical meaning of these results. The first two terms in the right sides of Eqs. (3) and (4) describe the oscillations of the particle energy in the fields of the incident (+) and reflected (−) waves. Such oscillations could also take place in an infinite dielectric at the same fields (1). The last terms in Eqs. (3) and (4) do not depend on time, and, therefore, they describe the change of the kinetic energy of the charge after crossing the nonuniformity.

The physical origin of this change is determined by its dependence on the parameters of the particles, the field and the medium.

Thus, the linear dependence on the field amplitude and on the particle charge and also the dependence on the phase of the field, $\varphi = \omega t_0$, show that these terms independent of time $\tau$ describe the change of the charge energy stimulated by the field (1).[7]

Moreover, this part of the charge losses can be considered as caused by transition interaction of the charge with the wave field, because of a number of other features:

(a) becoming zero as the difference of dielectric permittivities approaches zero [see Eq. 3(b)];

(b) the absence of synchronism between the charge and the field characteristic of Cherenkov radiation as a necessary condition for the energy exchange;

(c) the absence of dependence on the mass of the charge (see Ref. 33):

(d) the coincidence of the characteristic size of the region of the formation of losses with the length $L_+$ of the formation of the field of the spontaneous transition radiation of the modulated current; this part of the energy lost by the charge should be regarded as being caused its transition interaction with the field of the wave.

Therefore, according to all the features listed above, the energy exchange of the charge with the wave field must be considered as the stimulated transition retardation ($\Delta\mathscr{E}^i_{\text{trans}} < 0$) or acceleration ($\Delta\mathscr{E}^i_{\text{trans}} > 0$).

It is necessary to mention especially the fact that, due to the law of energy conservation in the system, the energy increase occurs at retardation ($\Delta\mathscr{E}^i_{\text{trans}} < 0$) of the charge by the wave field and the energy decrease occurs at acceleration ($\Delta\mathscr{E}^i_{\text{trans}} > 0$).

The physical mechanism of the energy exchange between the beam and the field in the system under consideration can be made more clear by means of an analogy between the movement of a charge in the field of a reflected wave only and the rotation of a mathematical pendulum. This analogy is based on the identity of the equations for pendulum movement and charge movement in the coordinate system connected with the wave:

$$\ddot{\psi} + \Omega^2\sin\psi = 0,\quad \psi \equiv k_+z - \omega\tau,\quad \Omega^2 \equiv \frac{qE_0k_+}{m} \equiv \mu(\omega - k_+V_0)^2. \tag{4a}$$

As it follows from this equation, the processes of the charge retardation and acceleration stimulated by the field are periodically exchanged (the corresponding pendulum rotates around the point of suspension without change of the rotation sign). It is necessary to "switch off" the field at the moment when the energy change of the charge (pendulum) stimulated by the field is minimum or maximum in order for the result of such exchange to be finite. This problem is solved by using drift tubes[36] in ion accelerators and by limiting the region of the field concentration in beam microwave generators.[37] The effects of interaction of moving charges with periodically nonuniform dielectric media[38,39] and the effects of the radiation diffraction at the periodic boundary nonuniformities (similar to the Smith-Purcell effect, see Ref. 31) are based on the same principle.

The fact that the screening of the field which is necessary for the effective energy exchange between the moving charge and the field can occur only when the spatial uniformity of electrodynamical properties of the medium is destroyed is most essential for our problem. This nonuniformity follows from the initial conditions in the case of the mathematical pendulum and of the charge in the field of the reflected wave described by the Eq. (4a). The charge energy $mV_0^2/2$ unperturbed by the field and its phase $\varphi$ at the entrance into the field are respectively similar to the energy of the initial stroke and to the initial phase of pendulum oscillations, and the stimulated transition retardation and acceleration [Eq. 2(a)] are similar to the additional average kinetic energy which the pendulum acquires due to (positive or negative) elevation over the horizontal plane passing through the rotation axis. Such an analogy makes it possible, in particular, to explain why the maximum of the amplitude of the average energy change at the transition stimulated by the field (1) is achieved when the phases of the rise are integral multiples of $\pi$. It is specifically in this case that the increase of the charge energy preceding the change of sign of the field is a maximum:

$$m\frac{\partial V}{\partial \tau}=qE_0\sin[\omega_-(k_+)\tau],\quad V\left(\tau=\frac{\pi}{|\omega_-(k_+)|}\right)-V(0)=\frac{2qE_0}{m\omega_-(k_+)}.$$

Such initial conditions for the pendulum correspond to the beginning of the motion from the upper or lower equilibrium position, when the amplitude of its rise above the horizontal plane passing through the rotation axis is maximum.

It is necessary to note in conclusion of this section that in the right side of Eq. (2), we practically did not consider the influence of the spontaneous transition radiation fields on the charge movement in the external wave field (1). This approximation corresponds to the assumption that the effects caused by the spontaneous radiation (the renormalization of the charge mass amongst them[32]) which are proportional to $q^2$ are small in comparison with the effects of the stimulated interaction of the charge with the external field which are proportional to $qE_0L_+$.

## 3. INDUCED TRANSITION RADIATION OF A NONMODULATED BEAM OF CHARGED PARTICLES

The purpose of this section is the calculation and analysis of the expression for the time-averaged energy flux $\delta P_B^i$ of the radiation induced by the external Fresnel field (1) from the nonmodulated beam which is crossing a sharp boundary between two media.

The periodic dependence of the sign of the transition losses of the charge energy stimulated by the field (1) on the field phase at the moment of crossing the nonuniformity $\omega t_0$ shows that the particles of the nonmodulated beam which differ in phases will be retarded or accelerated by the field (1) on passing through the same boundary in a manner analogous to grid of klystron bunching. As a result, the beam will become modulated in density after passage through the nonuniformity, and the field of the wave reflected by the nonuniformity will be enhanced or reduced by the transition radiation of beam particles stimulated by this field. The quantitative analytical description of the dependence of the modulation intensity of the beam on the external parameters of the system (the amplitude and the frequency of the field, the current and the energy of the beam and also the amplitude of the dielectric permittivity jump of the medium) may be obtained using the following simplified physical model of such a modulator.

A monoenergetic beam of charged particles of uniform density $N_0$ propagates with equilibrium velocity $V_0$ along the axis of a sectionally uniform anisotropic dielectric placed along a strong magnetic field (the presence of the strong magnetic field allows one to leave out of consideration transverse deflecting forces acting on the beam and to limit consideration to the one-dimensional movement of beam particles along this field). The medium is an anisotropic dielectric transparent for beam particles the dielectric properties of which have a discontinuity in the plane $z=0$. The beam approaches this plane from the left and the modulating $E$-wave (1) arrives from the right.

The intensity of the energy exchange between the beam and the field in the system under consideration characterizes the energy flux extracted by the field from the beam. In principle, this flux can be calculated by summing the energy losses of every particle in the beam, i.e., by phase averaging all these losses over the phases $\varphi=\omega t_0$ of beam particles entering the medium on the right (see Appendix 1). In the case of low currents $(N_0\to 0)$ it is simpler to use for this purpose the method of a given field and to describe the influence of this field on the beam by hydrodynamic (Euler) variables for beam particles. Under these conditions, the influence of the beam on the wave field is unimportant and it may be considered as being the same as in the absence of the beam [see Eq. (1)]. The corresponding expression averaged over a field period for the energy flux of the radiation from the beam particles induced by the external field and determined by the work of the field (1) on the flux of charged particles is, in the general case, a complex function of system parameters (see Appendix 2). Independently of specific values of these parameters, the intensity of the energy exchange between the beam and the field is identically zero in the half-space from which the beam emerges and is proportional to the density of the field energy and to the density of the flux of beam particles; it becomes zero when the discontinuity in $\hat{\varepsilon}$ approaches zero and depends also on the distance traversed by the beam after the discontinuity $(z=0)$:

$$\delta P_b^i(z)\equiv\frac{1}{2}\left\langle \mathrm{Re}\left\{-\int_{-\infty}^{z}dz'j_z(x,z',t)\,\tilde{E}_z^*(x,z',t)\right\}\right\rangle = N_0V_0P_0\begin{cases}0, & z<0,\\ F(z), & z>0;\end{cases}\tag{4b}$$

where $\tilde{E}_z$ is the longitudinal component of the field (1), $j_z$ is the perturbation of the beam current stimulated by the field, $P_0$ is the density of the energy flux of the incident wave

$$2P_0=\frac{c}{4\pi}\mathrm{Re}[\tilde{\mathbf{E}},\tilde{\mathbf{H}}^*]_z=\frac{k_+c}{4\pi k_0\varepsilon_-^+}|H_0|^2,$$

$F(z)$ is a complex function of the coordinate $z$ (see Appendix 2).

The right side of Eq. (4b) is most simple when the medium on the left ($z < 0$) is a conductor ($|\varepsilon_{\parallel}^{-}| \to \infty$, $\varepsilon_{\parallel}^{-} < 0$), and the speed of beam particles $V_0$ is comparable to the phase speed of the reflected wave ($|\Delta_-| \ll \Delta_+$), so that the interaction of the beam with this wave becomes dominant:

$$\delta P_{\mathrm{b}}^{i}(z) = \frac{k_{\perp}\omega_{\mathrm{b}}^{2}\varepsilon_{\perp}^{+}}{V_{0}^{2}\varepsilon_{\parallel}^{+}k_{+}} P_0 \left\{ \frac{\mathrm{d}}{\mathrm{d}\Delta_-} \left[ \frac{1-\cos\Delta_- z}{\Delta_-^2} \right] \right\}, \quad \Delta_{\pm} \equiv k_{\pm} - k_{\mathrm{b}}, \quad k_{\mathrm{b}} \equiv \frac{\omega}{V_0}; \tag{4c}$$

where $\omega_{\mathrm{b}}$ is the plasma frequence of the beam: $\omega_{\mathrm{b}}^2 \equiv 4\pi q^2 N_0/m$.

As can be seen from Eq. (4c), the stimulated energy exchange of the nonmodulated flux with the field of the wave travelling in the opposite direction changes sign passing through zero at the point of the Cherenkov synchronism of beam particles with this wave ($\Delta_- = 0$). The corresponding energy losses of beam particles oscillate, as functions of the coordinate $z$, growing linearly in amplitude, the characteristic period being of the order of the length of the formation of the spontaneous radiation field of the current modulated at the wave frequency $\omega$: $L_+ \equiv |\Delta_-|^{-1}$.

It should be noted that the method of the given field was developed more than forty years ago in Ref. 40 where for the first time an expression proportional to the right side of Eq. (4c) was obtained for the beam increment of the axially uniform oscillations of a resonator. At present the method of the given field is widely used in theoretical microwave electronics and leads to expressions for beam conductivity of a resonator with conducting walls proportional to the right side of Eq. (4c).[18,20,22,41]

But the problem of the physical processes underlying the energy exchange between the beam and the field described by Eq. (4c) has not yet been considered. An analysis of this problem using the results obtained above allows one to interpret uniquely the flux $\delta P_{\mathrm{b}}^{i}(z)$ determined by the Eq. (4c) as a result of the induced transition interaction of the beam with the field.

Indeed, it follows from Eq. (4c) that the interaction of the beam with the field takes place with arbitrary values of the parameters $\hat{\varepsilon}^{+}$ including the case when the conditions for the Cherenkov radiation of beam particles are not satisfied in either of the two uniform half-spaces ($z \gtrless 0$). It means that the interaction considered is not a Cherenkov radiation (this question is discussed in more details in section 4).

Moreover, the intensity of this interaction becomes zero when either the difference ($\hat{\varepsilon}^{+} - \hat{\varepsilon}^{-}$) approaches zero (i.e., $R_{E} \to 0$) or the energy flux of the incident wave $P_0$ approaches zero.

Further, independently of the discontinuity in $\hat{\varepsilon}$ and of the wave parameters this intensity is identically equal to zero in the region $z < 0$, where there is no nonuniformity in the medium (see appendix 2).

The elementary mechanism of the modulation of the beam by the wave field leading to the interaction (4c) different from zero is uniquely related to retardation and acceleration of beam particles by the field (1). This conclusion follows from the analysis of the origin of different terms in the right side of Eq. (4c) by the Lagrange variable method (see appendix 1). Under these conditions, the only sources of the contribution of the beam to the energy change of the reflected wave are perturbations of trajectories of the beam particles by the field (1) which are caused, as well as in the case (3c), by the presence of the nonuniformity in the medium (the initial conditions for the trajectories of beam particles in the plane $z = 0$).

The right side of Eq. (4c) becomes zero at the point of the Cherenkov resonance in the right half-space ($z > 0$, $\Delta_- = 0$) where the intensity of the Cherenkov interaction of a specific charge with the reflected wave field is maximum. This means that the Cherenkov interaction of the beam with the field (1) does not occur under the conditions considered.

Therefore, the set of properties enumerated above of the energy exchange between the beam and the field allows one to define it uniquely as a consequence of the induced transition interaction of the beam with the field of a regular wave.

The existence of the relationship between the spatial distribution and the intensity of the induced transition radiation of the unmodulated beam (4c) and the corresponding characteristics of the spontaneous transition radiation of the given modulated current

$$j_z(x, z, t) = j_0 \exp(-i\omega t + ik_{\mathrm{b}}z - ik_{\perp}x) \tag{5a}$$

of the same intensity ($j \equiv qN_0V_0$) and with the same nonuniformity of the medium is the proof of the reliability of this conclusion. Indeed, the density of the field energy flux excited by the current (5a) is

$$2\delta P_{\mathrm{b}}^{\mathrm{sp}}(z) \equiv \mathrm{Re}\left\langle -\int_0^z j_z E_z^* \,\mathrm{d}z' \right\rangle = 4\pi j_0^2 \frac{\varepsilon_{\perp}^{+} k_{\mathrm{b}} k_{\perp}^2}{\varepsilon_{\parallel}^{+} k_{+}\omega(k_{+}+k_{\mathrm{b}})} \cdot \frac{1-\cos(k_{+}-k_{\mathrm{b}})z}{(k_{+}-k_{\mathrm{b}})^2}, \tag{5b}$$

where $E_z$ is the total field of the current (5a) in the region $z > 0$.

It follows, hence, that the right sides of Eqs. (4c) and (5b) are related by the relation

$$\delta P_{\mathrm{b}}^{i} = k_{\mathrm{b}} \left[ \frac{d}{d\Delta_-} \delta P_{\mathrm{b}}^{\mathrm{sp}} \right] \frac{H_0^2 \,\mathrm{sgn}\, \varepsilon_{\parallel}^{+}}{2\pi\beta_0^2 N_0 m V_0^2}. \tag{6a}$$

Dividing both parts of Eq. (6a) by the density of the particle flux of the beam ($N_0V_0$), we obtain the relation between induced and spontaneous losses of a specific charge under the considered conditions

$$\delta\mathscr{E}_1^{i} = \frac{\delta P_{\mathrm{b}}^{i}}{N_0 V_0} = \frac{H_0^2 k_{\mathrm{b}}}{2\pi\beta_0^2 N_0 m V_0^2} \left[ \frac{\mathrm{d}}{\mathrm{d}\Delta_-} (\delta\mathscr{E}_1^{\mathrm{sp}}) \right], \quad \delta\mathscr{E}_1^{\mathrm{sp}} \equiv \frac{\delta P_{\mathrm{b}}^{\mathrm{sp}}}{N_0 V_0}. \tag{6b}$$

The dimensionless factor on the right side of the Eq. (6b) is substantially larger than one ($H_0^2 \gg 2\pi N_0 m V_0^2$, see Ref. 42) in the considered approximation ($N_0 \to 0$), from which it follows that the intensity of the induced radiation of a specific charge (at equal flux densities) is considerably larger than its spontaneous radiation intensity.

Thus, due to the transition interaction of every particle with the field of a regular wave described by the energy change $\Delta\mathscr{E}_{\mathrm{trans}}^{i}$ in the presence of a discontinuity in the dielectric permittivity, the initially unmodulated flux of charged particles passing across this discontinuity acquires perturbations of the equilibrium density $N_0$ and of the velocity $V_0$ which are the cause of the energy flux of induced radiation (absorption) with an average value different from zero on the right side from the discontinuity [Eq. (4b) and in a simplified form Eq. (4c)]. Induced losses per particle in this case are considerably larger than spontaneous losses for equal equilibrium fluxes $N_0V_0$.

The two approaches to the calculation of these losses—the summation of transition energy losses of the separate charges $\Delta\mathscr{E}^{\,i}_{\mathrm{trans}}$ (in Lagrange variables) and the calculation of the work of the field on the particle flux crossing the same boundary between two media (in Euler variables) give the same result.

## 4. RECIPROCAL TRANSFORMATION OF REGULAR WAVES AT A STEP-LIKE NONUNIFORMITY OF A DIELECTRIC IN THE PRESENCE OF A FLUX OF CHARGED PARTICLES

The energy flux of the induced transition radiation of beam particles (4b), determined by the work of the field on the beam, grows linearly with the coordinate $z$. It means that the assumption of the small influence of the beam on the field (1) on which the derivation of the formula (4c) is based is justified only at relatively small distances from the discontinuity limited by the inequality $|F(z)| \ll 1$. It is necessary to reject the method of the given field in the problem discussed above in order to describe correctly the interaction of the beam with the field of the counter-propagating wave in the whole region $z > 0$. Therefore, the purpose of this section is to obtain an expression for the energy flux of the transition radiation of the originally unmodulated beam taking into account the reciprocal influence of the beam and of the field of the counter-propagating incident wave as well as of the fields of the waves excited at the nonuniformity, i.e., to obtain this expression by means of a self-consistent theory. A self-consistent theory allows one to reject the assumption of the smallness of the equilibrium particle flux ($N_0 \to 0$), to take into account, using the wave approach, the high frequency polarization fields of this flux (collective fields of bunches formed by beam particles) and to consider the possibility of the Cherenkov amplification of these fields in the medium volume. Moreover, the self-consistent theory allows one to establish limits of applicability of the approximation of the given (unaltered by the beam) field of the external wave. This more general approach based on a self-consistent description of the dynamics of interaction of the radiation field with the monoenergetic flux of charged particles is described below.

In each spatially uniform part of the medium we represent the solutions of the self-consistent system of the equations of motion, i.e., the continuity equations for the beam particles and the Maxwell equations for the field as a superposition of plane waves of the type

$$E_x(x,\ z,\ t) = \sum_{\alpha=1}^{4} E_\alpha \exp\left[-i\omega t + ik_\alpha(\omega)\, z + ik_\perp x\right]. \tag{7a}$$

The longitudinal wave numbers of these waves $k_\alpha(\omega)$ are determined from the conditions for the absence of trivial solutions of the system of algebraic equations for the $E_\alpha$ coefficients which is obtained from the initial system of equations for each dielectric. In the case under consideration of an anisotropic dielectric and a magnetized beam the equation for $k_\alpha$

$$k_\perp^2 = (k_0^2\varepsilon_\perp - k_\alpha^2)\left[\frac{\varepsilon_\parallel}{\varepsilon_\perp} - \frac{\omega_b^2}{\varepsilon_\perp(\omega - k_\alpha V_0)^2}\right] \tag{7b}$$

has four solutions (for every fixed $k_\perp$). Two of them approach the constant $k_b = \omega/V_0$ and describe waves of the charge density of the beam when the beam density approaches zero ($\omega_b^2 \to 0$). The other two solutions correspond to quasitransverse waves of the system (modified by the presence of the beam) which may also exist in the absence of the beam.

In such a self-consistent approach each of the partial waves (7a) propagates along the uniform dielectric independently of the others. Interaction between these waves can take place only in the presence of a nonuniformity at which the nonelastic scattering of these waves occurs. Such scattering causes a transformation of the wave approaching the nonuniformity into the waves receding from this nonuniformity. The determination of the scattering intensity consists of the calculation of amplitudes of waves receding from the nonuniformity using the known amplitudes of arriving waves.[20] At low beam currents the charge density waves can propagate only along the flux of beam particles.[43] Because of this, beam waves in our problem are incoming in the left semi-infinite dielectric ($z < 0$) and outgoing in the right dielectric ($z > 0$).

Analytical calculations of amplitudes of the proper waves of the system receding from the discontinuity are most simple in the limiting case when the left dielectric ($z < 0$) is a conductor ($|\hat{\varepsilon}^-| \to \infty$), the right one is filled by magnetized plasma [$\varepsilon^+ = 1,\ \varepsilon_\parallel^+ = 1 - (\omega_p^2/\omega^2)$], and the system parameters in general are such that the there is no Cherenkov amplification of the slow charge density wave in the dielectric volume ($k_b < k_* \equiv k_\perp \times\sqrt{\varepsilon_\perp^+/|\varepsilon_\parallel^+|}$). In this case all proper waves of the system are propagating ($\mathrm{Im}\,k_\alpha = 0$), and their amplitudes are uniquely determined by the boundary conditions at the end $z = 0$. These conditions consist of the requirements that the tangential component of the total electric field should become zero ($E_x = 0$) and also that there should be no perturbations of the density and of the speed of the beam stimulated by high frequency fields ($\tilde{N} \equiv N - N_0 = \tilde{V} \equiv V - V_0 = 0$). All the aforementioned quantities can be easily found knowing one of them from the initial system of equations, for example, knowing the amplitude of the magnetic field $H_0$ of the incident wave:

$$H_y(x,\ z > 0,\ t) = H_0 \exp(-i\omega t + ik_\perp x) \times \left\{\exp(ik_4 z) + \sum_{\alpha=1}^{3} S_{\alpha 4} \exp(ik_\alpha z)\right\}; \tag{7c}$$

where $k_\alpha(\omega)$ are the solutions of Eq. (7b) which are the fast charge density wave ($\alpha = 1$), the slow charge density wave ($\alpha = 2$), the direct electromagnetic wave ($\alpha = 3$) and the counter-propagating electromagnetic wave ($\alpha = 4$) which is an incident wave in this case. The components $S_{\alpha 4}$ of the scattering matrix of the counter-propagating wave (4) at the end of a waveguide where the beam enters the interaction region that are determined by the boundary conditions at this end and the corresponding general expression for the energy flux of reflected waves are given in Appendix 3.

In the limiting case of low currents of the beam, the asymptotic forms of these matrices are equal to ($\varepsilon_{\parallel}^{+} \equiv \varepsilon_{\parallel}$, $\varepsilon_{\perp}^{+} \equiv \varepsilon_{\perp}$)

$$\underset{l=1,2}{S_{l4}} = -(-1)^l \Omega_{\text{B}} \Gamma^{1/2} \frac{\varepsilon_{\perp} \beta_0^2 k_{\perp}^2}{\varepsilon_{\parallel} k_0^2 (1-\beta_0^2 \varepsilon_{\perp})(1-\beta_0^2 \varepsilon_{\perp} C^2)} \times \left\{1-(-1)^l \frac{2\Omega_{\text{B}} \varepsilon_{\parallel} \Gamma^{3/2}}{1-\beta_0^2 \varepsilon_{\perp}} - (-1)^l \frac{\Omega_{\text{B}} \Gamma^{1/2} (1+\beta_0^2 \varepsilon_{\perp} C^2)}{1-\beta_0^2 \varepsilon_{\perp} C^2}\right\}, \tag{8a}$$

$$S_{34} = 1 + \frac{2\Omega_{\text{B}}^2 k_{\perp}^2 \beta_0 \sqrt{\varepsilon_{\perp}}\, C}{\varepsilon_{\parallel}^2 k_0^2 (1-\beta_0^2 \varepsilon_{\perp} C^2)} - \frac{4\Omega_{\text{B}}^2 (1-\varepsilon_{\parallel} \Gamma)(1+\beta_0^2 \varepsilon_{\perp} C^2)}{\varepsilon_{\parallel} \beta_0 \sqrt{\varepsilon_{\perp}}\, C (1-\beta_0^2 \varepsilon_{\perp} C^2)}, \tag{8b}$$

$$\Omega_{\text{B}}^2 \equiv \frac{4\pi q^2 N_0}{m_{\parallel} \omega^2}, \quad m_{\parallel} \equiv \frac{m}{(1-\beta_0^2)^{3/2}}, \quad C^2 \equiv 1 - \frac{k_{\perp}^2}{k_0^2 \varepsilon_{\parallel}}, \tag{8c}$$

$$\Gamma \equiv \left[\varepsilon_{\parallel} + \frac{k_{\perp}^2 \beta_0^2 \varepsilon_{\perp}}{k_0^2 (1-\beta_0^2 \varepsilon_{\perp})}\right]^{-1}.$$

The asymptotic expression for the energy flux of the transition radiation of the beam for low currents is

$$\delta P_{\text{B}}^{i} = P_0 \left\{8\Omega_{\text{B}}^2 |\varepsilon_{\parallel}|^{3/2} \sqrt{\varepsilon_{\perp}}\, \Gamma^3 \frac{k_{\perp} V_0}{\omega} \left(1 + \frac{\varepsilon_{\perp} k_{\perp}^2 V_0^2}{|\varepsilon_{\parallel}| \omega^2}\right) \times (1-\cos\theta_t \cos\theta_l) + 4\Omega_{\text{B}} \Gamma^{3/2} \sqrt{\varepsilon_{\perp} |\varepsilon_{\parallel}|}\, \frac{k_{\perp} V_0}{\omega} \sin\theta_t \sin\theta_l\right\}, \tag{9}$$

$$\theta_t \equiv \left(k_{\text{B}} - \frac{\omega}{c} \sqrt{\varepsilon_{\perp}}\, C\right) z, \quad \theta_l \equiv k_{\text{B}} z \Omega_{\text{B}} \Gamma^{1/2}.$$

At small distances from the boundary ($\theta_l \ll 1$) this expression coincides with the corresponding result (4b) obtained by the method of the given field. Hence it follows that the method of the given field is applicable in the region of small distances from the boundary, $z \ll L_{+} \equiv V_0^2/\omega^2 \Omega_{\text{b}} \sqrt{|\varepsilon_{\text{H}}| \Delta_{+} | \Delta_{-} |}$, where the beam particles do not participate in the oscillatory movement under the influence of the Coulomb charge of the beam. At large distances from the boundary these oscillations must be taken into account and the self-consistent approach is necessary for the correct description.

The self-consistent theory proves also that the collective process of the transformation of the counter-propagating wave at the nonuniformity of the nonequilibrium dielectric with a propagating beam considered by us is different in principle from the Cherenkov amplification of charge density waves in the volume of this dielectric. In order to show this, it is sufficient to compare the conditions for the volume Cherenkov radiation and for the surface transition radiation in the system under consideration. Indeed, it follows from the self-consistent equation of the spectrum (7b), in the simplest non-relativistic case ($\beta_0^2 \varepsilon_{\perp} \ll 1$) that the Cherenkov amplification of the slow charge density wave ($l=2$) takes place in the region $V_0 < V_{+} \equiv \omega/k_{+}$,

$$(\omega - k_2 V_0)^2 - \frac{\omega_{\text{B}}^2 k_{+}^2 k_{\text{B}}^2}{k^2 \varepsilon_{\perp} (k_{+}^2 - k_{\text{B}}^2)} < 0, \tag{10}$$

and is absent $[(\omega_{\text{B}} - k_2 V_0)^2 > 0]$ in the region $V_0 > V_{+}$. The result of the transition interaction of the beam with the field of the counter-propagating wave in the region of the first maximum of the right sides of Eqs. (4c) and (9) ($z \approx z_1 = 4.5 L_{+}$) is completely different under the same conditions: the amplification ($\delta P_{\text{b}}^{i} > 0$) is observed for $V_0 > V_{+}$.

Thus, the conditions for the volume Cherenkov radiation and the surface transition radiation are incompatible,[3)] and hence it follows that the induced Cherenkov interaction between the field and the beam can not be responsible for the energy flux increase of the counter-propagating wave given by Eqs. (4b) and (9).

## 5. CONCLUSION

It follows from the above discussion that the application range of the transition radiation is sufficiently large.[4)] In particular, this interaction is responsible for self-excitation of those auto-oscillating systems[20,24,40,41,46] in which the beam both enters and leaves its interaction region with the electromagnetic field—the resonator cavity. From the point of view of the possibility of generation by the beam of high frequency fields by means of transition radiation, this resonator performs the following most important functions.

a) On the beam entering the cavity its modulation by the field of the counter-propagating wave is produced by means of the induced transition interaction, i.e., feedback between the beam and the field occurs.

b) At the exit from the resonator an increase in the field amplitude due to the spontaneous transition radiation of the bunches formed by beam particles occurs when the feedback is positive.

c) In the volume of the resonator accumulation of energy of the field occurs as a result of coherent summation of radiation fields of a large number of passing bunches of the beam.

These processes lead to self-excitation of oscillations in autogenerators even in the absence of synchronism conditions for the beam and the radiation field. The corresponding increment is proportional to the right side of Eqs. (4b) and (9). It is necessary to emphasize especially that the increments of the absolute instabilities caused by the transition interaction depend linearly on the beam current in the limiting case of low currents, independently of the conditions for the Cherenkov synchronism of beam particles in the volume of the resonator. And only when the dimensions of the system are comparable with the Cherenkov relaxation length of the beam in an infinite system these increments are proportional to the cube root of the beam current.[24] In the presence as well as in the absence of this synchronism, the induced transition interaction of the beam with the field at the entrance end of the resonator is necessary for the existence of feedback in auto-oscillating systems.[20,24,47] In the absence of this interaction the induced radiation of beam particles in the volume of the resonator structure leads only to the spatial growth

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[3)]It is easy to show that this conclusion is correct also for the case of an isotropic dielectric in which the Cherenkov amplification is observed in the region $1 < \beta_0^2 \varepsilon < 1 + k_{\perp}^2 V_0^2/\omega^2$, and the surface radiation at $\beta_0^2 \varepsilon > 1 + k_{\perp}^2 V_0^2/\omega^2$.

[4)]Apparently, the effect of the induced transition absorption of the regular field by the beam was observed in the "most pure" form in the experiments on the modulation of the beam by the field of light waves in thin transparent dielectric films (see Refs. 44, 45).

(amplification) of the field amplitude of the corresponding charge density wave of the beam.[5)]

Thus, in addition to the diagnostics of relativistic charged particles[27-32] and to the energy dissipation of waves with a finite amplitude in a collision-free plasma[19,32], the transition radiation provides also collective energy exchange between charged particles fluxes and the electromagnetic radiation, which causes the generation of the transition radiation.

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## APPENDICES

### 1. Power losses in Lagrange variables

The perturbation of the beam current in Eq. (4) stimulated by the field (1) is determined in Euler variables from the hydrodynamic equations of motion and continuity for beam particles which in the case under consideration (magnetized beam) are

$$\begin{gathered}-i\omega\tilde{V}+V_0\frac{d\tilde{V}}{dz}=\frac{q}{m}E_z,\\ -i\omega\tilde{N}+V_0\frac{d\tilde{N}}{dz}+N_0\frac{d\tilde{V}}{dz}=0.\end{gathered} \tag{A1.1}$$

Solving these equations with continuity conditions at the discontinuity in the dielectric ($z=0$) and zero conditions as $z\to-\infty$ we obtain the expression for the perturbation of the density of the current $j=q(N_0\tilde{V}+V_0\tilde{N})$ stimulated by the field:

$$\left.\begin{aligned}j(z<0)&=-\frac{q^2}{m}N_0\frac{k_\perp cH_0T_E}{\varepsilon_\parallel^- V_0^2\Delta_+^2(k_-)}\exp(-i\omega t-ik_-z+ik_\perp x),\\ j(z>0)&=\frac{q^2}{m}N_0\frac{k_\perp cH_0}{\varepsilon_\parallel^+V_0^2}\exp(-i\omega t+ik_Bz+ik_\perp x)\\ &\times\left\{-\frac{\varepsilon_\parallel^+}{\varepsilon_\parallel^-\Delta_+^2(k_-)}+\frac{1-\exp[-i\Delta_+(k_+)z]}{\Delta_+^2(k_+)}+R_E\frac{1-\exp[-i\Delta_-(k_+)z]}{\Delta_-^2(k_+)}\right.\\ &\left.\qquad+iz\left(\frac{\varepsilon_\parallel^+T_E}{\varepsilon_\parallel^-\Delta_+(k_-)}-\frac{1}{\Delta_+(k_+)}-\frac{R_E}{\Delta_-(k_+)}\right)\right\},\\ \Delta_\pm(k_\alpha)&=k_B\pm k_\alpha,\end{aligned}\right\} \tag{A1.2}$$

from which follows the increment of the field energy flux (4c).

The motion of beam particles in the Lagrange description is given by the equations

$$\frac{\partial t}{\partial\tau}=1,\quad\frac{\partial\mathbf{R}_L}{\partial\tau}=\mathbf{V}_L,\quad\frac{\partial\mathbf{V}_L}{\partial\tau}=\frac{q}{m}\left(\mathbf{E}+\frac{1}{c}[\mathbf{V}_L,\mathbf{H}]\right), \tag{A1.3a}$$

which are characteristics of the Vlasov kinetic equation (see Ref. 50)

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial\mathbf{r}}+\frac{q}{m}\left(\mathbf{E}+\frac{1}{c}[\mathbf{V},\mathbf{H}]\right)\frac{\partial f}{\partial\mathbf{V}}=0. \tag{A1.3b}$$

[5)]The last statement refers only to autogenerators with external beam injection. Absolute instabilities can develop without participation of the transition radiation in spatially periodical fluxes similar to relativistic rings in a magnetic field.[48] The feedback is provided in this case by the corresponding induced magnetic bremsstrahlung absorption of the beam radiation field in the interaction region.[18,25,26]

The trajectories-characteristics differ from each other by the values of entrance parameters of beam particles (time of entrance $t_0$, entrance velocities $V_0$, radial $r_0$ and azimuthal $\theta_0$ coordinates). The combination of these characteristics describes the beam, with the number of particles being conserved according to Eq. (A1.3b) along each characteristic:

$$\begin{gathered}dn(\Phi_0)=J(\Phi_0)\,d\Phi_0,\\ d\Phi_0=dV_0\,dt_0r_0\,dr_0\,d\theta_0;\end{gathered} \tag{A1.3c}$$

where $J(\Phi_0)$ is the flux density of beam particles at the entrance into the interaction region which is a function of entrance parameters. The density of the beam current is uniquely determined by the Lagrange trajectories of beam particles[50] (A1.3a)

$$j(\mathbf{r},t)=q\int v_\parallel f(v_\parallel,\mathbf{r},t)\,dv_\parallel=q\int v_\parallel\,dv_\parallel\int d\Phi_0J(\Phi_0)\,\delta(\mathbf{r}-\mathbf{R}_L)\,\delta(\mathbf{V}-\dot{\mathbf{R}}_L). \tag{A1.4}$$

Substitution of this current into the right side of Eq. (4a) taking into account that in our problem the beam is monoenergetic and uniform in the cross section plane,

$$J(\Phi_0)=j_0\delta(v_0-V_0),\qquad j_0=qN_0V_0,$$

gives us the explicit dependence of the increment of the radiation energy flux stimulated by the field on the velocity $V_0$ and the density $N_0$ of beam particles and also on the time of entrance of these particles into the interaction volume

$$\begin{aligned}\delta P_B=&-4\pi qj_0\frac{k_\perp H_0}{k_0\varepsilon_\parallel^+}\sin\omega t\int_0^a r_0\,dr_0J_0(k_\perp r_0)\\ &\times\int_{t-(z/V_0)}^{t_0}dt_0\{\dot{\Delta}_L\cos k_+V_0\tau-k_+V_0\Delta_L\sin k_+V_0\tau\};\end{aligned} \tag{A1.5}$$

where $\Delta_L$ and $\dot{\Delta}_L$ denote respectively the perturbation of the Lagrange trajectory of a particle stimulated by the field:

$$\begin{aligned}\Delta_L(r,V_0,t_0,\tau)=&\frac{qk_\perp H_0J_0(k_\perp r_0)}{mk_0\varepsilon_\parallel^+}\left\{\tau\left[\frac{1}{\omega_+(k_+)}+\frac{1}{\omega_-(k_+)}\right]\cos\omega t_0\right.\\ &\left.-\frac{\sin\omega_+(k_+)\tau}{\omega_+^2(k_+)}-\frac{\sin\omega_-(k_+)\tau}{\omega_-^2(k_+)}+\left[\frac{1-\cos\omega_+(k_+)\tau}{\omega_+^2(k_+)}+\frac{1-\cos\omega_-(k_+)\tau}{\omega_-^2(k_+)}\right]\sin\omega t_0\right\}.\end{aligned} \tag{A1.6}$$

and the corresponding rate ($\dot{\Delta}_L=\partial\Delta_L/\partial\tau$).

Integration on the right side of Eq. (A1.5) gives a result coincident with the result obtained in the hydrodynamic approximation [see Eq. (4c)] in the limit of flat geometry. The main advantage of the kinetic approach in this case is that the important role of the boundary in the energy exchange between the beam and the field is especially clearly seen in this form of representation of Eq. (4c). In fact, the contribution to the right side of Eq. (A1.5) different from zero is given only by those terms in Eq. (A1.6) which are due to the presence of the boundary. The term on the right side of Eq. (A1.6) growing linearly with $\tau$ and describing the displacement of a beam particle in relation to its entrance phase caused by induced transition interaction with the wave field (see section 2) gives the main positive contribution to the flux (4c)—the first term ($\Delta_-z\sin\Delta_-z$) and half of the second term ($1-\cos\Delta_-z$). The second half of the term proportional to ($1-\cos\Delta_-z$) in Eq. (4c) originates from the terms on the right side of Eq. (A1.6) which do not depend on $\tau$. The terms in

Eq. (A1.6) oscillating with the time $\tau$ and describing oscillations of a charge in the fields of direct and counterpropagating waves in the dielectric volume do not give any contribution to the expression for the average energy flux of the induced radiation from the beam particles.

### 2. Calculation of losses of an unmodulated beam at a density discontinuity in an anisotropic dielectric in the field of a Fresnel $E$ wave

Let an unmodulated beam with equilibrium values of the density $N_0$ and of the velocity $V_0$ cross the boundary between two half-spaces of anisotropic dielectrics in the field of the wave (1). Let us calculate the energy losses of the beam particles induced by the field neglecting the influence of proper polarization fields of the beam on the dispersion and the wave field pattern in the case of low values of the beam current. The change of the energy flux through a unit area is given by Eq. (4b). Inserting the current density $j$ from (A1.2) we obtain

$$\langle \delta P_{\mathrm{B}}(z)\rangle = \begin{cases} 0, & z<0, \\ P_0 F(z), & z>0, \end{cases} \tag{A2.1}$$

where $F(z)$ is

$$\begin{aligned} F(z) = \frac{\varepsilon_\perp^+ k_\perp^2 \omega_{\mathrm{B}}^2}{\varepsilon_\parallel^+ k_+ V_0^2} \Bigg[ & \frac{\cos \Delta_+(k_+) z - 1}{\Delta_+(k_+)} \left\{ \frac{1}{\varepsilon_\parallel^- \Delta_+^2(k_-)} - \frac{1}{\varepsilon_\parallel^+ \Delta_+^2(k_+)} \right. \\ & \left. + \frac{R_{\mathrm{E}}}{\varepsilon_\parallel^+ \Delta_-^2(k_+)} - \frac{Q_{\mathrm{E}}}{\varepsilon_\parallel^+ \Delta_-^2(k_+)} \right\} + \frac{\cos \Delta_-(k_+) z - 1}{\Delta_-(k_+)} R_{\mathrm{E}} \\ & \times \left\{ \frac{1}{\varepsilon_\parallel^- \Delta_+^2(k_-)} - \frac{1}{\varepsilon_\parallel^+ \Delta_+^2(k_+)} - \frac{R_{\mathrm{E}}}{\varepsilon_\parallel^+ \Delta_-^2(k_+)} + \frac{Q_{\mathrm{E}}}{\varepsilon_\parallel^+ \Delta_-(k_+)} \right\} \\ & + \frac{R_{\mathrm{E}}(\cos 2k_+ z - 1)}{\varepsilon_\parallel^+ \; 2k_+} \left( \frac{1}{\Delta_-^2(k_+)} - \frac{1}{\Delta_+^2(k_+)} \right) + \frac{z \sin \Delta_+(k_+) z}{\varepsilon_\parallel^+ \Delta_+(k_+)} Q_{\mathrm{E}} \\ & + \frac{z \sin \Delta_-(k_+) z}{\varepsilon_\parallel^+ \Delta_-(k_+)} R_{\mathrm{E}} Q_{\mathrm{E}} \Bigg], \quad Q_{\mathrm{E}} \equiv \frac{\varepsilon_\parallel^+ T_{\mathrm{E}}}{\varepsilon_\parallel^- \Delta_+(k_-)} - \frac{1}{\Delta_+(k_+)} - \frac{R_{\mathrm{E}}}{\Delta_-(k_+)}. \end{aligned} \tag{A2.2}$$

### 3. Derivation of the scattering matrix and the energy flux leaving the boundary between an anisotropic dielectric and a conductor

The wave with the known amplitude $H_y^{\mathrm{inc}} = H_0 \exp(-i\omega t + ik_4 z + ik_\perp z)$ is incident on the boundary ($z = 0$) between an anisotropic dielectric and a conductor. Emerging from the conducting half-space is an unmodulated beam in which the incident wave excites charge density waves and a reflected wave. Let us determine the amplitudes of all the waves excited at the dielectric-conductor boundary and the energy flux leaving this boundary.

We introduce the notation

$$H_y^{\mathrm{refl}} = \sum_{\alpha=1}^{3} S_{\alpha 4} H_0 \exp(-i\omega t + ik_\alpha z + ik_\perp x), \tag{A3.1}$$

where $k_\alpha$ ($\alpha = 1, 2, 3$; 4 is for the incident wave) satisfy the dispersion equation (7b). Requiring equality to zero of the electric field component $E_x$ tangent to the surface of the conductor and of the high frequency perturbations of the velocity and of the density of the beam at the dielectric-conductor boundary, we obtain from the system of inhomogeneous equations for the coefficients $S_{\alpha 4}$ from which we find

$$S_{14} = \frac{\varepsilon_1}{k_2 - k_1} \left( S_{34} \frac{k_3 - k_1}{\varepsilon_3} + \frac{k_4 - k_2}{\varepsilon_4} \right), \tag{A3.2a}$$

$$S_{24} = -\frac{\varepsilon_2}{k_2 - k_1} \left( S_{34} \frac{k_3 - k_1}{\varepsilon_3} + \frac{k_4 - k_1}{\varepsilon_4} \right), \tag{A3.2b}$$

$$\varepsilon_\alpha \equiv \varepsilon_\parallel^+ (\omega - k_\alpha V_0)^2 - \omega_{\mathrm{B}}^2,$$

$$S_{34} = -\frac{k_4 + \dfrac{k_1(k_4 - k_2)\varepsilon_1 - k_2(k_4 - k_1)\varepsilon_2}{(k_2 - k_1)\varepsilon_4}}{k_3 + \dfrac{k_1(k_3 - k_2)\varepsilon_1 - k_2(k_3 - k_1)\varepsilon_2}{(k_2 - k_1)\varepsilon_3}}. \tag{A3.2c}$$

The energy flux leaving the boundary normalized to the energy flux of the incident wave can be expressed through the components of the matrix $S_{\alpha 4}$ in the following manner:

$$\begin{aligned} \frac{\delta P_z^{\mathrm{refl}}}{|P_0|} = \Bigg\{ & \frac{k_3}{|k_4|} S_{34}^2 + \sum_{l=1}^{2} \frac{2k_l}{k_4} S_{l4}^2 [1 - \cos(k_1 - k_2) z] \\ & + \sum_{l=1}^{2} \frac{2k_l}{|k_4|} S_{34} S_{l4} \cos(k_l - k_3) z \Bigg\}. \end{aligned} \tag{A3.3}$$

In the limiting case of low currents we find asymptotic values of the wave numbers

$$\underset{l=1,2}{k_l} = k_{\mathrm{B}} \left\{ 1 + (-1)^l \Omega_{\mathrm{B}} \Gamma^{1/2} \left[ 1 + (-1)^l \Omega_{\mathrm{B}} \Gamma^{3/2} \frac{\varepsilon_0 + \beta_0^2 k_\perp^2 / k_0^2}{(1 - \beta_0^2 \varepsilon_\perp)^2} \right] \right\}, \tag{A3.4a}$$

$$k_{3,4} = \pm k_0 \sqrt{\varepsilon_\perp}\, C \left\{ 1 - \frac{\Omega_{\mathrm{B}}^2 k_\perp^2 / k_0^2}{2\varepsilon_\parallel^2 C^2 (1 \mp \beta_0 \sqrt{\varepsilon_\perp}\, C)^2} \right\}. \tag{A3.4b}$$

Substituting Eq. (A3.4) into Eqs. (A3.2) and (A3.3) we obtain formulas (8) and (9).


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