Radiation of relativistic particles in single crystals

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The review treats the problem of radiation of relativistic charged particles in matter. The radiation of fast particles in an external field is considered from a unified point of view for an amorphous medium and for a single crystal. The basic attention is paid to the process of radiation in a single crystal where an enhancement of the radiation occurs as compared with an amorphous medium. This effect is shown to be due to the coherent and the interference mechanisms of radiation of relativistic particles in single crystals. First we outline the Born theory (quantum and classical) of coherent radiation of fast particles and show that this theory is valid if the particle propagates through the crystal far from the directions of channeling and if the scattering angle of the particle is small compared with the typical radiation angle of a relativistic particle. Then we show that a violation of these conditions leads to new effects in radiation such as, e.g., the effect of intense radiation of superbarrier and channeled particles, and the effect of suppression of coherent radiation. The comparison of theoretical and experimental results confirms the existence of new effects in the radiation. In conclusion, we review briefly new physical effects that must take place in single crystals at high energies in several other electrodynamical processes.

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CONTENTS

1. INTRODUCTION

The main effects of the electromagnetic interaction of electrons and photons with each other and with individual atoms are very well described by quantum electrodynamics.[1] These effects are modified and new effects arise in interaction of electrons and photons with groups of atoms—in passage of particles through condensed matter. The best known effects of this type are Cherenkov radiation[2-4] and the ionization loss of relativistic particles in matter.[5]

It is quite remarkable that the collective phenomena appear at arbitrarily high particle energies, although at first glance it seems that if the particle wavelength is less than the average distance between the atoms of the material, collective phenomena should not appear and the material should behave as a gas of independent atoms. This was clearly formulated for the first time by M. L. Ter-Mikaelyan[6] and by Landau and Pomeranchuk.[7] In Ref. 6 it was observed that the radiation of a charged relativistic particle in a crystal occurs over a great length (the coherence length) and it was shown

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that if in this length there is a large number of atoms, then the radiation of the particle can occur more intensely than in an amorphous medium.

Landau and Pomeranchuk[7] in turn showed that as a consequence of multiple scattering the intensity of radiation of a high-energy charged particle in an amorphous medium can be significantly less intense than radiation in a gas of independent atoms.

Special interest is presented by processes which arise in passage of fast charged particles through crystalline media, since in this case coherent and interference effects can play a role at arbitrarily high energies. These effects were first studied by Ferretti,[8] Ter-Mikaelyan,[6] and Überall[9] on the basis of the equivalent-photon method and the first Born approximation of perturbation theory. The condition of applicability of these approximations, however, is rapidly violated as the particle energy increases and as the angle between the incident beam and a crystallographic axis or plane decreases.[10-12] In particular it turns out that the Born approximation cannot be used to describe the motion of fast particles along crystallographic axes and planes.

In motion of fast charged particles along crystallographic axes and planes one can have particle channeling, in which the particles move in channels formed by the atoms located along crystallographic axes and planes, periodically deviating from the channel axis by small angles.[13-15]

A particle moving in a channel experiences the action of a strong electric field created by the atoms of the lattice, and therefore its radiation should be very intense.[16-20] However, even in the case of infinite motion with respect to the crystallographic axis or plane (or as it is called, superbarrier motion) the particle moves in a strong lattice field, and therefore a superbarrier particle, like a channeled particle, should radiate intensely.[20,21] The problem of radiation of a relativistic particle in a crystal in this case turns out to be closely related to the problem of radiation of a relativistic particle in strong macroscopic fields (for example, in accelerators, undulators, and so forth): the radiation behaves similarly in the two cases.

The present review is devoted mainly to the problem of radiation of relativistic particles in crystals. This problem, as we have just mentioned, is related to the problem of radiation of relativistic particles in strong external fields. Therefore we shall discuss these problems from a unified point of view, together with the radiation of particles moving in amorphous media.

The prediction of new features in the interaction of relativistic particles with crystals has served as the stimulus for extensive experimental study of the radiation of high-energy particles in crystals.[22-30] Therefore we give also a comparison of the the theoretical results with certain experiments on radiation in the case of motion of particles along crystallographic axes and planes.

2. RADIATION BY A RELATIVISTIC CHARGED PARTICLE IN AN EXTERNAL FIELD

a) Spectral density of radiation in classical electrodynamics

We shall begin with discussion of the radiation of a relativistic charged particle moving in external fields.

According to classical electrodynamics the spectral density of energy radiated by an electron is determined by the following general formula[31]:

$$\frac{\mathrm{d}E}{\mathrm{d}\omega}=\frac{e^2\omega^2}{4\pi^2}\int \mathrm{d}o\,|[\mathbf{nI}]|^2,\qquad \mathbf{I}=\int_{-\infty}^{\infty}\mathrm{d}t\,\mathbf{v}(t)\,e^{i\omega(t-\mathbf{nr}(t))},\tag{2.1}$$

where $\mathbf{v}(t)$ and $\mathbf{r}(t)$ are the velocity and position vector of the electron at a moment of time t, ω is the frequency of the radiated wave, do is the element of solid angle near the unit vector n which determines the direction of the radiation, and e is the charge of the electron (the velocity of light c is taken equal to unity).

The characteristic angles of scattering and radiation of a relativistic particle ($v\approx c$) in an external field are small, and therefore in Eq. (2.1) we can perform an expansion in terms of these angles. For this purpose we shall first carry out in (2.1) an integration over solid angle:

$$\frac{\mathrm{d}E}{\mathrm{d}\omega}=\frac{e^2\omega}{\pi}\int \mathrm{d}t_1\,\mathrm{d}t_2 e^{i\omega(t_1-t_2)}\left[\mathbf{v}_1\mathbf{v}_2-\frac{1}{\omega^2}\left(\mathbf{v}_1\frac{\partial}{\partial\mathbf{r}_1}\right)\left(\mathbf{v}_2\frac{\partial}{\partial\mathbf{r}_2}\right)\right]\frac{\sin\omega\,|\mathbf{r}_1-\mathbf{r}_2|}{|\mathbf{r}_1-\mathbf{r}_2|},\tag{2.2}$$

where $\mathbf{v}_1\equiv\mathbf{v}(t_1)$, $\mathbf{v}_2\equiv\mathbf{v}(t_2)$ and

$$(\mathbf{r}_1-\mathbf{r}_2)=\int_{t_2}^{t_1}\mathrm{d}t\,\mathbf{v}(t)=\int_0^{t_1-t_2}\mathrm{d}t\,\mathbf{v}(t_2+t).$$

Since $(\mathbf{v}_1\partial/\partial\mathbf{r}_1)=\mathrm{d}/\mathrm{d}t_1$ and $(\mathbf{v}_2\partial/\partial\mathbf{r}_2)=\mathrm{d}/\mathrm{d}t_2$, in Eq. (2.2) in the second term we can carry out integration by parts over t_1 and t_2. As a result we find that

$$\frac{\mathrm{d}E}{\mathrm{d}\omega}=\frac{e^2\omega}{\pi}\int \mathrm{d}t_1\,\mathrm{d}t_2\,(\mathbf{v}_1\mathbf{v}_2-1)\,\frac{\sin\omega\,|\mathbf{r}_1-\mathbf{r}_2|}{|\mathbf{r}_1-\mathbf{r}_2|}\,e^{i\omega(t_1-t_2)}.\tag{2.2'}$$

Using now the smallness of the particle-scattering angle, we can represent the quantity $\mathbf{v}(t_2+t)$ in the form

$$\mathbf{v}(t_2+t)\approx\mathbf{v}_2\left(1-\frac{1}{2}\mathbf{v}^{*2}(t)\right)+\mathbf{v}^*(t),\quad |\mathbf{v}^*|\ll|\mathbf{v}_2|,\tag{2.3}$$

where $\mathbf{v}^*\mathbf{v}_2=0$ [the asterisk on the vector $\mathbf{v}^*$ means that the components of $\mathbf{v}^*$ refer to the plane orthogonal to $\mathbf{v}_2$, i.e., to the plane whose position changes with the time t_2 (Fig. 1)]. Here it is obvious that

$$|\mathbf{r}_1-\mathbf{r}_2|\approx\mathbf{v}_2\tau-\frac{1}{2}\int_0^{\tau}\mathrm{d}t\,\mathbf{v}^{*2}(t)+\frac{1}{2\tau}\mathbf{r}^{*2}(\tau),$$

$$(1-\mathbf{v}_1\mathbf{v}_2)\approx\gamma^{-2}+\frac{1}{2}\mathbf{v}^{*2}(\tau),$$

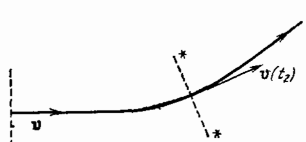

FIG. 1.

where $\tau = t_1 - t_2$, $\mathbf{r}^*(\tau) = \int_0^\tau dt\, \mathbf{v}^*(t)$ and $\gamma = (1 - v_2^2)^{-1/2}$. Using these formulas, we find[1)]

$$\frac{dE}{d\omega} \approx -\frac{e^2\omega}{\pi\gamma^2} \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} \frac{d\tau}{\tau} e^{i\omega\tau} \left(1 + \frac{1}{2}\gamma^2 \mathbf{v}^{*2}(\tau)\right) \times \sin\omega \left[v_2\tau - \frac{1}{2}\int_0^\tau dt\, \mathbf{v}^{*2}(t) + \frac{1}{2\tau}\mathbf{r}^{*2}(\tau)\right]. \tag{2.4}$$

The components of the vector $\mathbf{v}^*$ can be expressed at small scattering angles in terms of the components of the vector $\mathbf{v}_\perp(t)$ which is the projection of the particle velocity on a plane orthogonal to the incident particle velocity $\mathbf{v}$ [the location of this plane is fixed in time, in contrast to the plane* (see Fig. 1)]:

$$\mathbf{v}^*(t) \approx \mathbf{v}_\perp(t_2 + t) - \mathbf{v}_\perp(t_2). \tag{2.5}$$

At small scattering and radiation angles Eq. (2.1) can be represented also in the form

$$\frac{dE}{d\omega} = \frac{e^2\omega^2}{4\pi^2} \int do\, |\mathbf{I}_\perp - \mathbf{n}_\perp I_\parallel|^2, \quad \mathbf{v}(t) \approx \mathbf{v}\left(1 - \frac{1}{2}\mathbf{v}_\perp^2(t)\right) + \mathbf{v}_\perp(t), \tag{2.6}$$

where $\mathbf{I}_\perp$ and $I_\parallel$ are the components of the vector $\mathbf{I}$ orthogonal and parallel to $\mathbf{v}$, and $\mathbf{n}_\perp$ is the projection of the vector $\mathbf{n}$ on a plane orthogonal to $\mathbf{v}$.

b) Radiation on a small change of curvature of the particle trajectory

We shall consider first the important limiting case in which the time interval $\Delta\tau$ which makes the main contribution to the integral over τ in Eq. (2.4) is small in comparison with the time interval Δt in which the radius of curvature of the particle trajectory changes substantially. If this condition is satisfied the components of the vector $\mathbf{v}^*$ do not change greatly in the time interval $\Delta\tau$, and therefore according to Eq. (2.5) we have $\mathbf{v}^*(t) \approx t\dot{\mathbf{v}}^* \approx t\dot{\mathbf{v}}_\perp(t_2)$. Using this relation, we can express Eq. (2.4) in the form[3]

$$\frac{dE}{d\omega} \approx -\frac{2e^2\omega}{\sqrt{\pi}\gamma^2} \int dt_2 \left[\frac{\Phi'(u)}{u} + \frac{1}{2}\int_u^\infty dx\, \Phi(x)\right], \tag{2.7}$$

where $u = |\omega/\gamma^3 \dot{\mathbf{v}}_\perp(t_2)|^{2/3}$ and

$$\Phi(u) = \frac{1}{\sqrt{\pi}} \int_0^\infty d\xi \cos\left(u\xi + \frac{1}{3}\xi^3\right)$$

is the Airy function [the variable ξ is related to the variable τ which occurs in Eq. (2.4) by the relation $\xi = (\tau/2)(\omega\dot{\mathbf{v}}_\perp^2)^{1/3}$].

The mean contribution to the Airy function is from values $\xi \sim 1$ if $u \lesssim 1$, and from values $\xi \sim \sqrt{u}$ if $u \gg 1$, so that Eq. (2.7) is valid if $\gamma\vartheta(\Delta t) \times \min(1, \sqrt{u}) \gg 1$, where $\vartheta(\Delta t) \sim \Delta t \dot{v}_\perp$ is the scattering angle of the particle in the time interval Δt.

[1)]The method presented for obtaining the spectral density of radiation was first suggested by Landau and Pomeranchuk.[7] However, the formula obtained by them for $dE/d\omega$ differs from Eq. (2.4) in that instead of the factor $(1 + \frac{1}{2}\gamma^2\mathbf{v}^{*2})$ they have a factor $\gamma^2([(\mathbf{v}^*\mathbf{r}^*/\tau) - (\mathbf{r}^{*2}/\tau^2)])$. The difference is due to the fact that in Eq. (1) of Ref. 7 the factor $\sin\omega\, |\mathbf{r}_1 - \mathbf{r}_2|$ was differentiated and the factor $|\mathbf{r}_1 - \mathbf{r}_2|^{-1}$ was not differentiated, although the terms discarded in so doing are of the same order of magnitude as those which were retained.

We note that in the length $l_c \approx v_2\Delta\tau$ which the particle travels in a time $\Delta\tau$, the interference between the waves radiated by the particle from the different portions of its path is substantial. The quantity l_c is called the coherence length.

c) Radiation in the low-frequency region

We shall consider now the case in which the length in which the radiation is produced is large in comparison with the length in which the external field acts on the particle ($l_c \gg v_2\Delta t$). For this purpose we shall integrate the quantity $\mathbf{I}$ which appears in Eq. (2.1) over t by parts:

$$\mathbf{I} = \frac{i}{\omega} \int_{-\infty}^{\infty} dt\, e^{i\omega(t - \mathbf{n}\mathbf{r}(t))} \frac{d}{dt} \frac{\mathbf{v}(t)}{1 - \mathbf{n}\mathbf{v}(t)}. \tag{2.8}$$

The integrand in (2.8) is nonzero in the time interval Δt in which the external field acts on the particle. If the condition $l_c \gg v\Delta t$ is satisfied the argument of the exponential in (2.8) in the time interval Δt is small in comparison with unity, and therefore the exponential can be replaced by unity:

$$\mathbf{I} \approx \frac{i}{\omega}\left(\frac{\mathbf{v}'}{1 - \mathbf{n}\mathbf{v}'} - \frac{\mathbf{v}}{1 - \mathbf{n}\mathbf{v}}\right), \tag{2.8'}$$

where $\mathbf{v}$ and $\mathbf{v}'$ are the velocities before and after the scattering. Substituting this expression into (2.1), we obtain after integration over solid angle the following formula for $dE/d\omega$:

$$\frac{dE}{d\omega} \approx \frac{2e^2}{\pi}\left[\frac{2\xi^2 + 1}{\xi\sqrt{\xi^2 + 1}} \ln\left(\xi + \sqrt{\xi^2 + 1}\right) - 1\right], \tag{2.9}$$

where $\xi = \gamma\vartheta/2$ and $\vartheta \approx |\mathbf{v}' - \mathbf{v}|/v$ is the scattering angle ($\vartheta \ll 1$).

We shall show that the condition $l_c \gg v\Delta t$ is satisfied in the region of sufficiently low frequencies. For an ultrarelativistic particle the time interval $\Delta\tau$ in which oscillations of the exponential $\exp[i\omega(t - \mathbf{n}\mathbf{r}(t))]$ are important is determined in order of magnitude from the relation $|\omega\Delta\tau - \omega(v_2\Delta\tau - \Delta\tau\vartheta^2/2)| \sim 1$, from which it follows that Eq. (2.9) is valid if $(2\gamma^2/\omega)\min[1, (\gamma\vartheta)^{-2}] \gg \Delta t$, i.e., this formula is valid in the region of sufficiently low frequencies.

For $\xi \ll 1$ the spectral density of radiation (2.9) takes the form

$$\frac{dE}{d\omega} \approx \frac{8e^2}{3\pi}\xi^2. \tag{2.10a}$$

However, if $\xi \gg 1$, then

$$\frac{dE}{d\omega} \approx \frac{4e^2}{\pi}\ln 2\xi. \tag{2.10b}$$

d) Radiation in the dipole approximation

An important case is the dipole approximation, which is valid if $\omega\Delta\tau(\overline{\mathbf{v}_\perp^2})_{\Delta\tau} \ll 1$, where $(\overline{\mathbf{v}_\perp^2})_{\Delta\tau}$ is the average value of the square of the vector $\mathbf{v}_\perp$ in the time interval $\Delta\tau$. In this case it is necessary to carry out in Eq. (2.8) an expansion in the parameter $\omega\Delta\tau(\overline{\mathbf{v}_\perp^2})_{\Delta\tau}$. In the first approximation we obtain

$$\frac{dE}{d\omega} = \frac{e^2\omega}{2\pi} \int_\delta^\infty \frac{dq}{q^2}\left[1 - 2\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\right] |\mathbf{W}(q)|^2, \tag{2.11}$$

where $\delta = \omega/2\gamma^2$ and

$$\mathbf{W}(q) = \int_{-\infty}^{\infty} \mathrm{d}t\, \dot{\mathbf{v}}_\perp(t)\, e^{iqt}.$$

We note that Eq. (2.11) can be obtained after rather cumbersome manipulations also from Eq. (2.4) if in the latter an expansion in the parameter $\omega\Delta\tau(\mathrm{v}_\perp^2)_{\Delta\tau}$ is carried out.

In the dipole approximation $\Delta\tau \sim \delta^{-1}$, and therefore the condition of applicability of the dipole approximation can be represented in the form $\gamma^2(\overline{\mathrm{v}_\perp^2})_{\Delta\tau} \ll 1$. This condition means that within the limits of the coherence length the particle-scattering angle must be small in comparison with the characteristic angle of radiation of a relativistic particle $\vartheta_n \sim \gamma^{-1}$.

e) Synchrotron radiation

Up to this time we have nowhere used a specific law of motion of the particle, and therefore the equations obtained can be used for determining the intensity of radiation of a particle moving in a specified macroscopic field and also of a particle moving in matter—both amorphous and crystalline.

We shall discuss first as an example synchrotron radiation, i.e., the radiation of a relativistic particle moving along a circle in a uniform external magnetic field. In the case of motion in a uniform magnetic field H the radius of curvature of the particle trajectory $\rho_H = m\gamma/eH$ does not change with time, and therefore to find the spectral density of radiation in this case we can use Eq. (2.7). Substituting into (2.7) $\dot{v}_\perp = eH/\varepsilon$, we obtain[31,32]

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} = \frac{\sqrt{3}}{2\pi}\frac{e^3 Hs}{m} F\left(\frac{\omega}{\bar{\omega}}\right), \quad F(x) = x\int_x^{\infty} \mathrm{d}y K_{5/3}(y), \tag{2.12}$$

where s is the pathlength, m is the electron mass,

$$\bar{\omega} = \frac{3eH\gamma^2}{2m}, \quad K_{5/3}(y) = 2\sqrt{3\pi}\,\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{\Phi'(u)}{u} + \frac{1}{2}\int_u^{\infty} \mathrm{d}z\,\Phi(z)\right],$$

and $u = (3y/2)^{2/3}$ [a plot of the function $F(x)$ is shown in Fig. 17 of Ref. 31].

It follows from Eq. (2.12) that the maximum of the synchrotron radiation spectrum occurs at $\omega \sim \bar{\omega}$; the spectral density of radiation in this frequency region is equal in order of magnitude to $\mathrm{d}E/\mathrm{d}\omega \sim se^2\gamma/\rho_H$. Assuming, for example, $H = 10^4$ G and $\varepsilon = 5$ GeV, we have for electrons $\rho_H \approx 17$ m and $\bar{\omega} \approx 17$ keV. The coherence length in this case for $\omega \sim \bar{\omega}$ is equal in order of magnitude to $l_c \approx 0.17$ cm.

3. BREMSSTRAHLUNG OF A RELATIVISTIC PARTICLE MOVING IN AN AMORPHOUS MEDIUM, AND THE LANDAU-POMERANCHUK EFFECT

a) The Landau-Pomeranchuk effect

In the previous section we obtained general formulas determining the radiation of a particle moving in an arbitrary external field. We shall now use these formulas to evaluate the spectral density of radiation in the case of motion of a relativistic particle in an amorphous medium.

We shall consider first the radiation in the dipole approximation, i.e., in the case in which $\gamma^2\vartheta^2(l_c) \ll 1$ and $l_c \sim \delta^{-1}$, where $\vartheta(l_c)$ is the scattering angle in a coherence length. In this case the quantity $|\mathrm{W}(q)|^2$ which enters into Eq. (2.11) can be represented in the form

$$|\mathbf{W}(q)|^2 \simeq \sum_{j,k} \vartheta_j\vartheta_k \exp[iq(t_j - t_k)],$$

where ϑ_j is the angle of scattering of the particle by the j-th atom of the medium and t_j is the moment of time when the collision of the particle with this atom occurs. After averaging this relation over the locations of the atoms in the material, we find that

$$\frac{\mathrm{d}\bar{E}}{\mathrm{d}\omega} \approx \frac{2e^2}{3\pi}\gamma^2\overline{\vartheta_a^2}(L),$$

where $\overline{\vartheta_a^2}(L)$ is the mean square angle of multiple scattering of the particle in a layer of matter of thickness L. In an amorphous medium[33]

$$\overline{\vartheta_a^2}(L) \approx 8\pi Z^2 e^4 nL\varepsilon^{-2}\ln(Ze^2R\sqrt{4\pi nL}), \tag{3.1}$$

where n is the density of atoms, $Z|e|$ is the charge of the nucleus of an individual atom, and R is the screening radius of the atom. Using this formula and the condition $\gamma^2\vartheta^2(l_c) \sim \gamma^2\overline{\vartheta_a^2}(l_c) \ll 1$, we find that

$$\frac{\mathrm{d}\bar{E}}{\mathrm{d}\omega} \approx \frac{4}{3}\frac{L}{L_R}\frac{\ln(Ze^2R\sqrt{4\pi nL})}{\ln(183Z^{-1/3})}, \qquad \omega \gg \frac{\varepsilon_s^2\varepsilon^2}{m^4 L_R}, \tag{3.2}$$

where $L_R = [(4Z^2e^6n/m^2)\ln 183Z^{-1/3}]^{-1}$ is the radiation length[10] and $\varepsilon_s^2 = 4\pi m^2/e^2$. Equation (3.2) coincides within logarithmic accuracy with the well known Bethe-Heitler formula[34]

$$\frac{\mathrm{d}E_{\mathrm{BH}}}{\mathrm{d}\omega} = \frac{4}{3}\frac{L}{L_R}\frac{\varepsilon'}{\varepsilon}\left[1 + \frac{3}{4}\frac{(\hbar\omega)^2}{\varepsilon\varepsilon'} - \frac{1}{12\ln 183Z^{-1/3}}\right], \tag{3.3}$$

if in the latter we neglect recoil accompanying radiation ($\varepsilon' = \varepsilon - \hbar\omega$ is the energy of the final electron). We see that in the low-frequency region the condition of applicability of the dipole approximation is violated.

Let us consider now the radiation in the case in which $\gamma^2\vartheta^2(l_c) \gg 1$. In this case the main contribution to the integral over τ in (2.4) is from the interval $\Delta\tau$ of variation of τ determined from the relation $l_c\delta\gamma^2\vartheta^2(l_c) \sim 1$. Here, according to Eq. (2.4), the spectral density of radiation in order of magnitude is

$$\frac{\mathrm{d}E}{\mathrm{d}\omega} \sim Le^2\delta\gamma^2\vartheta^2(l_c), \quad l_c \ll L.$$

Replacing $\vartheta^2(l_c)$ by $\overline{\vartheta_a^2}(l_c)$, in these relations, we find that $l_c \sim (\varepsilon/\varepsilon_s)\sqrt{L_R/\omega}$ and

$$\frac{\mathrm{d}\bar{E}}{\mathrm{d}\omega} \sim \frac{2\pi m^2}{\varepsilon\varepsilon_s}\frac{L}{L_R}\sqrt{L_R\omega}, \quad \omega \ll \left(\frac{\varepsilon_s\varepsilon}{m^2}\right)^2\frac{1}{L_R}. \tag{3.4}$$

This formula coincides up to a numerical coefficient[2)] with the result of Landau and Pomeranchuk[7]:

[2)] Note that Eq. (3.5) was obtained in Ref. 7 also with accuracy up to a numerical coefficient. The exact theory of the Landau-Pomeranchuk effect is given in articles by A. B. Migdal.[35]

$$\frac{dE_{LP}}{d\omega} \sim \sqrt{\frac{2\pi}{3}}\frac{m^2}{\varepsilon\varepsilon_S}\frac{L}{L_R}\sqrt{L_R\omega}. \tag{3.5}$$

Taking into account the Bethe-Heitler formula (3.3), we see that

$$E'_{LP} \sim \frac{\sqrt{3\pi}}{2\gamma\sqrt{\overline{\vartheta_a^2}(\delta^{-1})}} E'_{BH},$$

from which it follows that $E'_{LP} \ll E'_{BH}$. In other words, the presence of an amorphous medium leads to a significant decrease of the spectral density of radiation in comparison with the radiation of a relativistic particle in a rarified gas of atoms.

We note that in derivation of Eq. (2.4) we have neglected the influence of the polarization of the medium on the radiation,[3)] which is justified if $\omega \gg \gamma\omega_p$, where $\omega_p = \sqrt{4\pi n Z e^2/m}$ is the plasma frequency.

b) Radiation in a thin layer of matter

The Landau-Pomeranchuk result (3.5) applies to the case in which $L \gg l_c$. However, if $L \ll l_c$, then to find the spectral density of radiation it is necessary to proceed from Eq. (2.8) and average it over the scattering angle ϑ with the distribution function

$$f(\vartheta)\,d\vartheta = \frac{2}{\overline{\vartheta_a^2}(L)} \exp\left(-\frac{\vartheta^2}{\overline{\vartheta_a^2}(L)}\right)\vartheta\,d\vartheta.$$

As a result[4)] we obtain[37]

$$\frac{d\overline{E}}{d\omega} \approx \begin{cases} \frac{4}{3}\frac{L}{L_R}, & L \ll e^2 L_R, \quad (3.6a)\\ \frac{2e^2}{\pi}\ln\frac{L}{e^2 L_R}, & L \gg e^2 L_R. \quad (3.6b)\end{cases}$$

We see that in the case in which $l_c \gg L \gg e^2 L_R$, in contrast to the Bethe-Heitler and Landau-Pomeranchuk results $d\overline{E}/d\omega$ is almost independent of the target thickness L. This means that for $l_c \gg L \gg e^2 L_R$ the spectral density of radiation is practically independent of the number of collisions of the particle with atoms of the medium, i.e., in this case a suppression of the radiation occurs. We note that the essence of the Landau-Pomeranchuk effect and of the suppression of radiation in a thin layer of matter is the same—these effects arise if the scattering angle within a coherence length exceeds the characteristic angle of radiation of a relativistic particle. However, the conditions on the target thickness under which these effects occur are different, and therefore the spectral densities of radiation of relativistic particles in thin and thick layers of matter are different.

The nature of the dependence of $\overline{E}'$ on L is shown schematically in Fig. 2 [E'_L is determined by Eq. (3.6b)].

It is interesting to note that a phenomenon similar to the effect of suppression of radiation in a thin layer of matter was observed recently in study of the interaction of high energy hadrons with heavy nuclei.[39,40] This phenomenon consists of the fact that the average multiplicity of fast particles produced in interaction of a hadron with a nucleus depends very weakly on the number of nucleons in the nucleus if the process occurs over lengths exceeding the nuclear size.

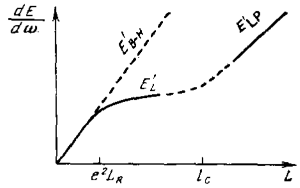

FIG. 2.

We note that all the effects mentioned in this section can be related to the phenomenon of temporary loss by a bare particle (a particle which is to a significant degree without its normal Coulomb field) of the ability to radiate in collisions with atoms of the medium, a phenomenon which was pointed out by E. L. Feĭnberg.[39] This phenomenon consists of the fact that after scattering a relativistic charged particle can move for a long time without a significant portion of its own normal Coulomb field (the Fourier components of the self-field of an electron with frequency ω appear at the electron only at a time interval $\Delta\tau \sim \delta^{-1}/v$ after the scattering), as a result of which the radiation of such a particle at the atoms of the medium will be decreased in comparison with the radiation in the case in which the self-field has been formed at the particle.[39,41]

The results obtained in this section apply to the case in which the interaction of a particle with the different atoms of the medium is random. However, in motion of a particle in the crystal, correlations between its successive collisions with lattice atoms may turn out to be important. As a result of these correlations the nature of the motion of particles in a crystal will differ from the nature of the motion in an amorphous medium. The radiation of particles in a crystal and an amorphous medium will also be different.

4. THEORY OF COHERENT RADIATION OF RELATIVISTIC PARTICLES IN A CRYSTAL IN THE BORN APPROXIMATION

a) Cross section for radiation in the Born approximation

We turn now to a discussion of the bremsstrahlung of relativistic particles moving in crystalline media. It is important in this case that in crystalline media coherent and interference effects can appear in the radiation. As a result of these effects the intensity of radiation of particles in a crystal can significantly exceed the intensity of radiation in an amorphous medium.

Coherent and interference effects in the radiation can arise at substantial photon energies ($\hbar\omega \sim \varepsilon$), and therefore in investigation of these effects a classical description is insufficient, and it is necessary, generally speaking, to use a quantum description of the radiation. The simplest quantum description is the description of

[3)] The theory of this effect has been developed by M. L. Ter-Mikaelyan.[10,36]

[4)] Similar results were obtained by F. F. Ternovskiĭ[38] by means of the method proposed by A. B. Migdal for description of the radiation of relativistic particles in an amorphous medium.

the radiation of relativistic particles in the first Born approximation, the discussion of which we shall now begin.[5)]

In the first Born approximation the differential cross section for bremsstrahlung is determined by the square of the modulus of the Fourier component of the potential energy of interaction of the electron with the external field $U(\mathbf{q})$, where $\mathbf{q}=\mathbf{p}-\mathbf{p}'-\mathbf{k}$ is the momentum transferred to the external field, $\mathbf{p}$ and $\mathbf{p}'$ are the momenta of the initial and final electrons, and $\mathbf{k}$ is the photon momentum. As an electron moves in a crystal its potential energy has the form $U(\mathbf{r})=\sum_k u(\mathbf{r}-\mathbf{r}_k)$, where $u(\mathbf{r}-\mathbf{r}_k)$ is the potential energy of interaction of the electron with the lattice atom located at the point $\mathbf{r}_k$. The square of the Fourier component of this expression is equal to

$$|U(\mathbf{q})|^2=|u_{\mathbf{q}}|^2\left|\sum_k e^{i\mathbf{q}\mathbf{r}_k}\right|^2, \qquad u_{\mathbf{q}}=\int d^3\mathbf{r}\, u(\mathbf{r})\, e^{i\mathbf{q}\mathbf{r}}. \tag{4.1}$$

Setting here $\mathbf{r}_i=\mathbf{r}_i^0+\mathbf{u}_i$, where $\mathbf{r}_i^0$ is the equilibrium position of the lattice atom and $\mathbf{u}_i$ is its thermal displacement, we obtain after averaging over the thermal vibrations and summation over the positions of the atoms in the lattice

$$|U(\mathbf{q})|^2=N_0|u_{\mathbf{q}}|^2\left[1-e^{-q^2\overline{u^2}}+e^{-q^2\overline{u^2}}\frac{(2\pi)^3}{\Delta}\sum_{\mathbf{g}}\delta(\mathbf{q}-\mathbf{g})\right], \tag{4.2}$$

where N_0 is the total number of atoms in the lattice, Δ is the volume of the unit cell, $\delta(\mathbf{q})$ is a delta function, $\mathbf{g}$ is the reciprocal lattice vector, and $\overline{u^2}$ is the mean square thermal displacement of the atoms in the lattice[10] (the bar designates averaging over the thermal vibrations). In the case of a screened Coulomb potential we have

$$u(r)=\frac{Ze^2}{r}e^{-r/R}, \qquad u_{\mathbf{q}}=\frac{4\pi Ze^2}{\mathbf{q}^2+R^{-2}}.$$

The last term in Eq. (4.2) determines the coherent effects, and the first two terms determine the incoherent effects in the radiation. Here the first term leads to the well known Bethe-Heitler formula for the cross section for radiation at N_0 independent centers. The cross section for radiation at an individual atom receives its main contribution from $\mathbf{q}$ values lying in the interval $R^{-1}\lesssim q\leqslant m$. On the other hand, the second and third terms will be important in the cross section if $q^2\overline{u^2}<1$. Since $m^2\overline{u^2}\gg 1$, the crystal lattice will affect the radiation in the case when $q\ll m$. In this region of momentum transfers the radiation cross section is determined by the formula[10,12]

$$d\sigma=\frac{e^2\delta^*}{4\pi^3m^2}\frac{d\omega}{\omega}\frac{\varepsilon'}{\varepsilon}\int_{\delta^*}^{\infty}dq_\parallel\int d^2q_\perp\left[1+\frac{\omega^2}{2\varepsilon\varepsilon'}-2\frac{\delta^*}{q_\parallel}\left(1-\frac{\delta^*}{q_\parallel}\right)\right]\frac{\mathbf{q}_\perp^2}{q_\parallel^2}|U(\mathbf{q})|^2, \tag{4.3}$$

where $\varepsilon'=\varepsilon-\omega$, $\delta^*=\omega m^2/2\varepsilon\varepsilon'$, and $q_\parallel$ and $q_\perp$ are the components of $\mathbf{q}$ parallel and orthogonal to $\mathbf{p}$ (the Planck constant $\hbar$ is taken equal to unity).

Using Eqs. (4.2) and (4.3), we can represent the cross section for radiation of an electron in a crystal in the form

$$d\sigma=N_0(d\sigma_n+d\sigma_c), \tag{4.4}$$

where $d\sigma_n$ is the cross section for incoherent radiation (it does not differ greatly from $d\sigma_{BH}$) and $d\sigma_c$ is the cross section for coherent radiation:

$$d\sigma_c=\frac{2e^2\delta^*}{m^2\Delta}\frac{d\omega}{\omega}\frac{\varepsilon'}{\varepsilon}\sum_{\mathbf{g}}\frac{\mathbf{g}_\perp^2}{g_\parallel^2}\left[1+\frac{\omega^2}{2\varepsilon\varepsilon'}-2\frac{\delta^*}{g_\parallel}\left(1-\frac{\delta^*}{g_\parallel}\right)\right]|u_{\mathbf{g}}|^2e^{-g^2\overline{u^2}}, \tag{4.5}$$

where $g_\parallel\geqslant\delta^*$. In what follows we shall be interested mainly in the cross section $d\sigma_c$.

In Eq. (4.3) the main contribution to the cross section is from $q_\parallel\approx\delta^*$. These values of $q_\parallel$ correspond to distances of the order $l_c^*\sim 1/\delta^*$. The quantity l_c^* is called the coherence length[10] and is the length in which the radiation is formed (l_c^* differs from the length l_c introduced previously in that δ^* enters into it instead of δ).

We note that if we formally assume the lattice constants a_i $(i=1,2,3)$ to be very large, $a_i\to\infty$, the summation over $\mathbf{g}$ reduces to integration over $\Delta(2\pi)^{-3}d^3g$ and Eq. (4.4) takes the form $d\sigma=N_0 d\sigma_{BH}$. This case corresponds to $l_c^*\ll a_i$.

b) Cross section for radiation on motion of an electron near a crystallographic axis

If $a_i\ll l_c^*$, then within the limits of the coherence length there will be a large number of lattice atoms, and therefore coherent and interference effects will appear in the radiation. It is clear that the greatest number of atoms will occur in the length l_c^* in the case in which the particle moves near a crystallographic axis, and therefore we shall discuss this case first of all.

Assuming the angle ψ between the incident electron momentum $\mathbf{p}$ and the crystallographic axis z to be small, we have $g_\parallel\sim g_z+\psi(g_y\cos\alpha+g_x\sin\alpha)$, where α is the angle between the projection of the vector $\mathbf{p}$ on the (x,y) plane and the y axis (Fig. 3) (the lattice is assumed to be orthogonal, but not necessarily cubic). The main contribution to $d\sigma_c$ for $\psi\ll 1$ is from terms with $g_z=0$, so that

$$d\sigma_c=\frac{2e^2\delta^*}{m^2\Delta}\frac{d\omega}{\omega}\frac{\varepsilon'}{\varepsilon}\sum_{g_xg_y}\frac{\mathbf{g}_\perp^2}{g_\parallel^2}\left[1+\frac{\omega^2}{2\varepsilon\varepsilon'}-2\frac{\delta^*}{g_\parallel}\left(1-\frac{\delta^*}{g_\parallel}\right)\right]|u_{\mathbf{g}}|^2e^{-g^2\overline{u^2}}, \tag{4.6}$$

where $g_\parallel=\psi(g_y\cos\alpha+g_x\sin\alpha)\geqslant\delta^*$ and $\mathbf{g}_\perp^2=g_x^2+g_y^2$.

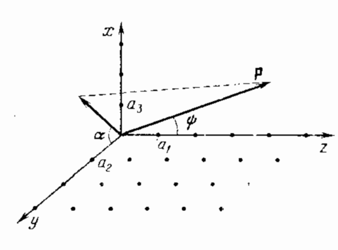

FIG. 3.

[5)] A clear and detailed exposition of the theory of coherent radiation of relativistic particles in crystals in the Born approximation is given by several authors.[10,44,45] In presenting this problem in the present review it has been our aim to focus attention on a number of questions which were not discussed in the previous reviews. These include, in particular, such questions as the region of applicability of the Born theory of coherent radiation of particles in crystals and various limiting cases of this theory.

It is important to note that in this case the combined potential energy, which depends on three coordinates, is replaced by a function $U(x, y)$ of the two coordinates x and y, which is the crystal energy averaged over the coordinate z: $U(x, y) = L^{-1} \int dz U(x, y, z)$, where L is the crystal thickness. This quantity can be considered as the potential energy of interaction of a particle with a set of continuous atomic strings of the crystal located along the z axis; see Eq. (6.1). In other words, as the principal element in interaction with which radiation occurs we now have not an individual atom, but a string of atoms.

We note that the main contribution to the cross section (4.6) is from $g_\perp \sim R^{-1}$.

If we formally assume that the lattice constants along the y and x axes are very large $(a_2, a_3 \to \infty)$—this corresponds to separation of the atomic strings of the crystal located parallel to the z axis by a large distance from each other, at which the interaction of a particle with the different strings of atoms can be considered to be independent—then in Eq. (4.6) we can replace the summation over g_x and g_y by an integration, and we arrive at the formula

$$d\sigma_c = \frac{e^2\delta^*}{2\pi^2 m^2 a_1} \frac{d\omega}{\omega} \frac{\varepsilon'}{\varepsilon} \int d^2 g \frac{g_\perp^2}{g_\parallel^2} \left[1 + \frac{\omega^2}{2\varepsilon\varepsilon'} - 2\frac{\delta^*}{g_\parallel}\left(1 - \frac{\delta^*}{g_\parallel}\right)\right] |u_g|^2 e^{-g^2\overline{u^2}} \quad (4.7)$$

with the restriction $g_\parallel \geqslant \delta^*$. From this formula it follows that for a screened Coulomb potential

$$d\sigma_c = N_c d\sigma_{BH}, \quad (4.8)$$

where $N_c = kR/a_1\psi$ if $2R\delta^*/\psi \ll 1$ and $N_c = k/\delta^* a_1$ if $2R\delta^*/\psi \gg 1$ $(k = \pi/2 \ln 183 Z^{-1/3})$.

We see that for $l_c^* \gg a_1$ the cross section for radiation in a crystal will be significantly greater than the radiation cross section determined by the Bethe-Heitler formula. In order of magnitude N_c represents the number of atoms in a coherence length.

The presence in the cross section of the additional factor N_c expresses the coherence effect which arises in a crystalline medium. As a result of this effect the radiation cross section per atom $d\sigma/N_0$ turns out to be $N_c \gg 1$ times greater than the radiation cross section in a rarefied gas of atoms or in an amorphous medium.

We obtained Eq. (4.7) on the assumption that the interaction of the electron with the different atomic strings is independent. This situation exists if a particle moves far from crystallographic planes closely packed with atoms [α and $(\pi/2 - \alpha)$ are not small in comparison with unity]. Indeed, in this case the main contribution to (4.6) is from $g_{x,y} \sim R^{-1} > 2\pi/a_{2,3}$. The function under the summation sign in this case will be smooth. Since the condition $g_\parallel = \psi(g_x \sin\alpha + g_y \cos\alpha) \geqslant \delta^*$ must be satisfied, then with an increase of δ^* the function will gradually drop out from the double sum one term at a time, and this will not greatly affect the double sum over g_x and g_y. Therefore the summation in Eq. (4.6) over g_x and g_y can be replaced by integration.

We shall consider now the case in which a particle is moving along a crystallographic plane, so that the condition $\alpha \ll 1$ is satisfied (in addition it is assumed that $\psi \ll 1$).

Here with change of δ^* for $\delta^* \sim \psi g_y$ entire sets of terms will drop out from the double sum over g_x and g_y, as a result of which the radiation cross section will change rapidly for values $\delta^* = 2\pi n\psi/a_2$ $(n = 1, 2, 3 \ldots)$. In this case the coherence length $l_c^* = 1/\delta^*$ will be of the order of a_2/ψ. The quantity a_2/ψ represents the path length traversed by a particle between two successive collisions with atomic strings of the crystal located in a crystallographic plane. The sharp peaks in the cross section are due to interference of waves radiated by the particle in interaction with different atomic strings. Here in a coherence length there will be $N_c \sim R/\psi a_1$ atoms of a single atomic string of the crystal, and all these atoms will radiate coherently.

In Fig. 4 we have shown the intensity of radiation as a function of the frequency ω for electrons with energy $\varepsilon = 1$ GeV moving in a silicon crystal near the $\langle 100 \rangle$ crystallographic axis. As ordinate we have plotted the quantity $(m^2/Z^2e^6)\,\omega d\sigma_c/d\omega$, and as abscissa—the frequency in MeV (it is assumed that $\overline{u^2} = 0$). Figure 4a corresponds to the case in which electrons are moving far from close-packed crystallographic planes and $\psi = 1$ mrad (curve 1), $\psi = 2$ mrad (curve 2), and $\psi = 4$ mrad (curve 3). Figure 4b corresponds to the case in which electrons are moving along the (011) plane at an angle $\psi = 2$ mrad to the $\langle 100 \rangle$ axis (the dot-dash curve corresponds to motion of an electron far from the crystallographic planes). The dashed curve in Fig. 4 corresponds to the Bethe-Heitler result (3.3). (In the case considered $a_1 = a$, where $a = 5.43$ Å is the lattice constant and the strings of crystal atoms near which the electron is moving form in the plane orthogonal to the $\langle 100 \rangle$ axis a two-dimensional lattice with $a_2 = a_3 = a/2\sqrt{2}$.)

We see that in the low-frequency region $\sigma_c' \gg \sigma_{BH}'$ and that in motion of an electron along a crystallographic plane the radiation cross section has sharp maxima. The smooth curve in Fig. 4b corresponds to coherent radiation of an electron in a single string of crystal atoms; the curve with the sharp maxima corresponds to the case in which there is a coherent effect in the radiation of the electron at the individual strings of atoms and an interference effect in the radiation at different strings of atoms.

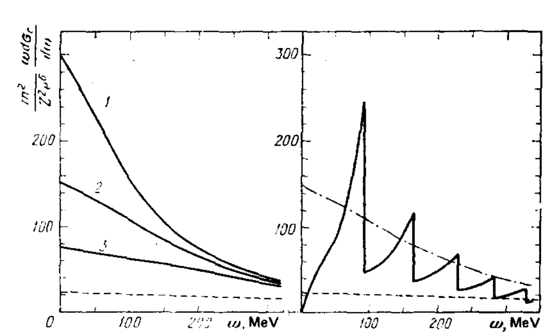

FIG. 4.

c) Cross section for radiation on motion of an electron near a crystallographic plane

We have discussed the radiation by an electron in its motion near a crystallographic plane, assuming that $\delta^* \sim \psi/a_2$. Now we shall assume that as before the particle moves near a crystallographic plane ($\alpha \ll 1$) and that we have the condition $\delta^* \sim \psi\alpha/a_3$ rather than the condition $\delta^* \sim \psi/a_2$.

In this case the mean contribution to the cross section for coherent radiation is from terms with $g_z = 0$ and $g_y = 0$, and the radiation cross section has the form[11]

$$d\sigma_c = \frac{2e^2\delta^*}{m^2\Delta\theta^2}\frac{d\omega}{\omega}\frac{\varepsilon'}{\varepsilon}\sum_{g_x \geq \delta^*/\theta}\left[1+\frac{\omega^2}{2\varepsilon\varepsilon'}-2\frac{\delta^*}{\theta g_x}\left(1-\frac{\delta^*}{\theta g_x}\right)\right]|u_{g_x}|^2 e^{-g_x^2\overline{u^2}}, \tag{4.9}$$

where $\theta = \psi\alpha$ is the angle between the incident-electron momentum and the (y, z) crystallographic plane. This formula shows that the entire plane as a whole near which the particle is moving acts as an elementary object and that the radiation occurs as a result of interaction with this object.

If we formally assume the distance between planes a_3 to be very large, then it is obvious that the summation over g_x can be replaced by integration over $(2\pi/a_3)^{-1}dg_x$. Here the radiation cross section takes the form

$$d\sigma_c = \frac{e^2\delta^*}{\pi m^2 a_1 a_2 \theta^2}\frac{d\omega}{\omega}\frac{\varepsilon'}{\varepsilon}\int_{\delta^*/\theta}^{\infty} dg_x\left[1+\frac{\omega^2}{2\varepsilon\varepsilon'}-2\frac{\delta^*}{\theta g_x}\left(1-\frac{\delta^*}{\theta g_x}\right)\right]|u_g|^2 e^{-g^2\overline{u^2}}. \tag{4.10}$$

In the case of a screened Coulomb potential the spectral distribution of the radiation given by Eq. (4.10) is shown in Fig. 5a. Curves 1 and 2 correspond to electrons with energy $\varepsilon = 1$ GeV moving at angles $\theta = 0.2$ and 0.4 mrad to the (011) plane of a silicon crystal ($a_1 = a = 5.43$ Å, $a_2 = a_3 = a/2\sqrt{2}$; thermal displacement of the atoms is not taken into account). In Fig. 5b we have shown the same quantity $\omega d\sigma_c/d\omega$ calculated from the exact formula (4.9) when the summation is not replaced by integration ($\theta = 0.4$ mrad). We see that here there are sharp maxima due to interference of different planes. In Eq. (4.10) these maxima have been integrated (the dot-dash curve in Fig. 5b corresponds to the quantity $\omega d\sigma_c/d\omega$ found from Eq. (4.10) at $\theta = 0.4$ mrad). The increase of the radiation cross section in the low-frequency region in comparison with the Bethe-Heitler result (the dashed curve in Fig. 4) is due to the coherence of the radiation of the electron in the crystal. In

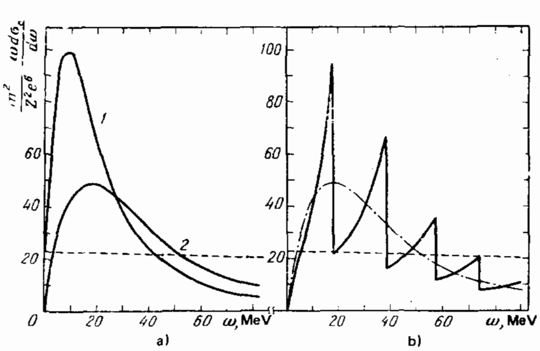

FIG. 5.

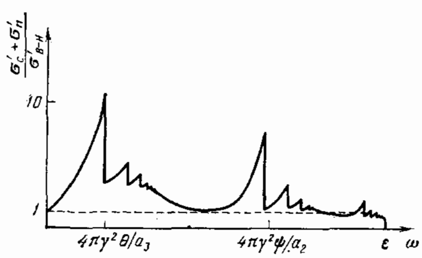

FIG. 6.

the case considered the electron radiates coherently, interacting with atoms located within the limits of an individual crystallographic plane.

The qualitative behavior of the spectral distribution of radiation over the entire frequency region for $\psi \ll 1$ and $\alpha \ll 1$ is shown in Fig. 6.[42] In the frequency region $\omega \sim 4\pi\gamma^2\psi/a_2$ the interaction of the electron with individual atomic strings of the crystal located along the z axis is important; on the other hand in the region $\omega \sim 4\pi\gamma^2\psi\alpha/a_3$ the interaction with individual planes of atoms is important. (The small maxima in the radiation spectrum at $\omega \approx \varepsilon$ are due to interference of waves radiated by the electron in interaction with individual lattice atoms. The locations of these maxima are determined from the condition $\delta^* = 2\pi n/a_1$, where $n = 1, 2, 3, \ldots$. The question of the radiation of electrons in this frequency region will not be considered in this article.)

We emphasize that since we are using the Born approximation the results are valid if the inequality $N_c Ze^2/\hbar c \ll 1$ is satisfied.[12]

d) Spectral density of radiation in classical electrodynamics in the first approximation in the potential[43]

If the opposite inequality is satisfied, $N_c Ze^2/\hbar c \gg 1$, then the Born approximation is inapplicable. However, if in addition to this inequality we have the condition $\hbar\omega \ll \varepsilon$, then it is possible to use the classical theory of radiation.

Radiation in the classical theory is determined by the particle trajectory, which is found from the equation of motion

$$\frac{d}{dt}\frac{m\mathbf{v}(t)}{\sqrt{1-\mathbf{v}^2(t)}} = -\nabla\sum_i u(\mathbf{r}-\mathbf{r}_i), \tag{4.11}$$

where $u(\mathbf{r}-\mathbf{r}_i)$ as before is the potential energy of interaction of the electron with the i-th atom of the crystal.

We shall show that if the potential energy can be considered as a small perturbation, then the spectral density of radiation given by the classical theory will coincide with the spectral density of radiation calculated in quantum electrodynamics in the Born approximation if recoil on radiation is neglected. Smallness of the potential energy means that the particle trajectory is close to rectilinear, $\mathbf{r}(t) \approx \mathbf{r}_0 + \mathbf{v}t$ ($\mathbf{r}_0$ is the point of entry of the particle into the crystal) and that the scattering of the particle is small.

We return to Eq. (2.11) and in it set $\dot{\mathbf{v}}_\perp = -\varepsilon^{-1}\nabla_\perp \sum_k u(\mathbf{r}-\mathbf{r}_k)$ and $\mathbf{r}(t)=\mathbf{r}_0+\mathbf{v}t$. As a result we obtain

$$\frac{dE}{d\omega}=\frac{e^2\delta}{\pi m^2}\int_\delta^\infty \frac{dq}{q^2}\left[1-2\frac{\delta}{q}\left(1-\frac{\delta}{q}\right)\right] \times\left|\sum_k \int dt e^{iqt}\nabla_\perp u(\mathbf{r}_0+\mathbf{v}t-\mathbf{r}_k)\right|^2. \tag{4.12}$$

Equation (4.12) must be averaged over the points of entry $\mathbf{r}_0$ of the particles into the crystal. Using for this purpose a Fourier expansion of the potential energy

$$u(\mathbf{r})=\frac{1}{(2\pi)^3}\int d^3q u_q e^{i\mathbf{q}\mathbf{r}}, \tag{4.13}$$

we find that

$$\frac{d\overline{E}}{d\omega}=\frac{e^2\delta}{4\pi^3 m^2}\int d^3q \frac{\mathbf{q}_\perp^2}{q_\parallel^2}\left[1-2\frac{\delta}{q_\parallel}\left(1-\frac{\delta}{q_\parallel}\right)\right]|u_q|^2\left|\sum_k e^{i\mathbf{q}\mathbf{r}_k}\right|^2, \tag{4.14}$$

where $q_\parallel \geqslant \delta$. Then performing in (4.14) an averaging over the thermal vibrations of the lattice atoms and noting that the spectral density of radiation $d\overline{E}/d\omega$ is related to the differential cross section for radiation of a particle in the crystal field $d\sigma/d\omega$ by the realtion $d\overline{E}/d\omega = nL\omega d\sigma/d\omega$, we arrive at Eq. (4.4) if in the latter we neglect recoil during radiation. [We note that in this case the incoherent part is reproduced with logarithmic accuracy; see also Eq. (3.2).] Thus, we have arrived at a formula which is valid in the Born approximation, but our derivation does not require the condition of applicability of the Born approximation. It is necessary only that the scattering angle within the limits of a coherence length be sufficiently small.

We see that the formula for the radiation cross section obtained in the Born approximation has in reality a region of applicability much greater than the region of applicability of the Born approximation. Here it is important that coherent and interference effects in the radiation are described both by the quantum theory and the classical theory of radiation.

It is interesting to note that almost all experiments on verification of the predictions of the Born theory of coherent radiation of relativistic electrons in a crystal have been carried out under conditions in which the Born approximation is not valid. At the same time good agreement has been observed between the results of the experiments and the results of the first Born approximation.[10,44,45] The results obtained in this section show that the reason for the agreement lies in the agreement of the formulas of the quantum and classical theories of radiation of fast electrons in a crystal in the case in which the crystal field perturbs the electron motion only slightly.

5. RADIATION WHEN RELATIVISTIC PARTICLES MOVE NEAR A CRYSTALLOGRAPHIC PLANE

a) Spectral density of radiation of superbarrier and channeled particles[20,42]

We shall now turn to a more detailed discussion of radiation when a relativistic electron or positron moves near a crystallographic plane or axis, assuming that the radiation can be described by classical electrodynamics. For this purpose it is necessary in particular that the photon energy be substantially less than the electron energy ($\hbar\omega \ll \varepsilon$) and that the condition[12] $N_c Ze^2/\hbar c \gg 1$ be satisfied, where N_c is the number of atoms in a coherence length.[6)]

We shall begin with a discussion of the motion of a particle near a crystal plane ($\alpha \ll 1$). We shall be interested in the radiation in the region of frequencies for which the coherence length $l_c \sim \delta^{-1}$ is greater than or of the order of R/θ, where R is the screening radius.

In the Born approximation, as we have seen, in this case the action of a crystallographic plane with many discretely located atoms is equivalent to the action of a plane with continuously distributed atoms. In other words, in this case the crystal potential can be replaced by an averaged potential $U(x) = (1/L_y L_z)\int dydzU(x,y,z)$, where L_y and L_z are the linear dimensions of the plane in the y and z directions. The potential energy $U(x)$ can be represented in the form

$$U(x)=\sum_n U_p(x-x_n),\quad U_p(x)=\frac{1}{a_1 a_2}\int dy\, dz u(x,y,z), \tag{5.1}$$

where $U_p(x)$ is the averaged potential energy of interaction of an electron with a crystallographic plane and x_n is the coordinate of the point of intersection of the x axis with the n-th plane. The same substitution can also be used in a classical discussion of the radiation of relativistic particles. This is due to the fact that in the relativistic case when $l_c \sim \delta^{-1} \gtrsim R/\theta$ the particle will interact within a coherence length with a large number of atoms of a crystal plane. Since the radiation is formed in a length of order l_c and for small θ the change of the impact parameter between successive collisions of the particle with the atoms of the plane located within a length l_c is small, then the nonuniformity of the potential distribution in the plane is unimportant for the radiation.

The velocity and acceleration of a particle in the field (5.1) will obviously be periodic functions of the time. If T is the period of oscillation of these quantities, then

$$\mathbf{v}_\parallel=\frac{1}{T}\int_0^T d t\mathbf{r}(t)$$

will be the velocity of the translational motion of the particle. Since $\mathbf{v}(t+T)=\mathbf{v}(t)$, then $\mathbf{r}(t+T)=\mathbf{v}_\parallel T+\mathbf{r}(t)$ and $\mathbf{r}_\perp(t+T)=\mathbf{r}_\perp(t)$, where $\mathbf{r}_\perp(t)$ is the component of $\mathbf{r}(t)$ orthogonal to $\mathbf{v}_\parallel$.

We shall now determine the spectral density of the radiation according to Eq. (2.6), taking into account the periodicity of the electron motion in the direction orthogonal to $\mathbf{v}_\parallel$. In this case

$$|\mathbf{I}_\perp-\mathbf{n}_\perp I_\parallel|^2=\frac{\sin^2(NqT/2)}{\sin^2(qT/2)}|\mathbf{I}_\perp^*-\mathbf{n}_\perp I_\parallel^*|^2,$$

$$\mathbf{I}_\perp^*=\int_0^T d t\mathbf{v}_\perp(t)e^{i\omega(t-\mathbf{n}\mathbf{r}(t))},\quad I_\parallel^*=\int_0^T dte^{i\omega(t-\mathbf{n}\mathbf{r}(t))},$$

[6)] References 83 and 84 give a detailed discussion of the relation between the results of the classical and quantum theory of radiation of relativistic particles in an external field.

where $q=\omega(1-\mathbf{n}\mathbf{v}_{\parallel})$ and N is the number of oscillations which the electron makes in traversing the crystal. For large N

$$\frac{\sin^2(NqT/2)}{\sin^2(qT/2)} \rightarrow N\frac{2\pi}{T}\sum_{\nu}\delta(q-\nu),$$

where $\nu=2\pi k/T$, $k=0,\pm 1,\pm 2,\ldots$, and the expression for $\mathrm{d}E/\mathrm{d}\omega$ takes the form

$$\frac{\mathrm{d}E}{\mathrm{d}\omega}=N\frac{e^2\omega}{2\pi T}\sum_{\nu\geqslant(1+\gamma^2\overline{\mathbf{v}_{\perp}^2})\delta}\int\limits_0^{2\pi}\mathrm{d}\varphi\,|\mathbf{I}_{\perp}^{*}-\mathbf{n}_{\perp}I_{\parallel}^{*}|^2; \tag{5.2}$$

here

$$\overline{\mathbf{v}_{\perp}^2}=\frac{1}{T}\int\limits_0^T \mathrm{d}t\,\mathbf{v}_{\perp}^2(t)$$

and φ is the angle between $\mathbf{n}_{\perp}$ and $\mathbf{v}_{\perp}$. Since $q=\omega(1-\mathbf{n}\mathbf{v}_{\parallel})$, then $\nu=(1+\gamma^2\overline{\mathbf{v}_{\perp}^2}+\gamma^2 v_{\mathrm{n}}^2)\delta$. In the dipole approximation when $\gamma^2\overline{\mathbf{v}_{\perp}^2}\ll 1$ we have

$$\frac{\mathrm{d}E}{\mathrm{d}\omega}=N\frac{e^2\omega}{T}\sum_{\nu\geqslant\delta}\frac{|\mathbf{W}(\nu)|}{\nu^2}\left[1-2\frac{\delta}{\nu}\left(1-\frac{\delta}{\nu}\right)\right],$$
$$\mathbf{W}(\nu)=\int\limits_0^T \mathrm{d}t\,\dot{\mathbf{v}}_{\perp}e^{i\nu t}. \tag{5.3}$$

It follows from the equation of motion (4.11) that in the field (5.1)

$$\ddot{x}=-\frac{1}{\varepsilon}\frac{\partial}{\partial x}U(x). \tag{5.4}$$

Here

$$\frac{\varepsilon\dot{x}^2}{2}+U(x)=\varepsilon_{\perp}(x_0),\qquad \varepsilon_{\perp}(x_0)=\frac{\varepsilon\theta^2}{2}+U(x_0),$$

where x_0 is the coordinate of the point of entry of the particle into the crystal and $\dot{x}=v_{\perp}(t)$. From this it is easy to find the particle trajectory:

$$t=\sqrt{\frac{\varepsilon}{2}}\int\limits_{x_0}^{x}\frac{\mathrm{d}x}{\sqrt{\varepsilon_{\perp}(x_0)-U(x)}},$$

where the integration is carried out along the path traveled by the particle in the crystal along the x axis.

For $\varepsilon_{\perp}(x_0)>U_{\max}$, where $U_{\max}$ is the maximum value of $U(x)$ (for electrons $U_{\max}=0$, and for positrons $U_{\max}=|U(x)|_{\max}$) the energy of the transverse motion of the particle is greater than the height of the potential barrier. The motion of the particle along the x axis will be infinite. A particle moving under these conditions is called superbarrier. The period of oscillation of such a particle obviously is

$$T_u=2\int\limits_0^{d/2}\mathrm{d}x\,|\dot{x}|^{-1}, \tag{5.5}$$

where d is the distance between planes.

If $\varepsilon_{\perp}(x_0)<U_{\max}$, then the motion of the particle in the direction perpendicular to the plane will be infinite. Here the particle is called channeled. We see that a necessary condition of channeling is that the inequality $\theta<\theta_c$ be satisfied, where $\theta_c=\sqrt{2|U|_{\max}/\varepsilon}$ is the critical angle of channeling.[13] For $\theta>\theta_c$ the motion will be superbarrier (Fig. 7; the trajectory 1 corresponds to motion of a positron in a crystal in the case in which $\theta\gg\theta_c$; trajectory 2 corresponds to the case in which $\theta\sim\theta_c$ *and the motion is infinite; trajectory 3 corresponds*

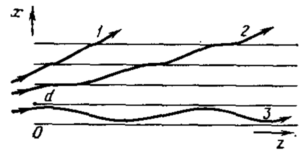

FIG. 7.

to motion of a channeled positron). The period of oscillation of a channeled particle is

$$T_c=4\int\limits_0^{d/2}\frac{\mathrm{d}x}{|\dot{x}|}\Theta(\varepsilon_{\perp}(x_0)-U(x)), \tag{5.6}$$

where $\Theta(\xi)$ is a step function.

Note that the trajectories and periods of oscillation of particles moving in a crystal under conditions of channeling and superbarrier motion are different, and therefore the spectral distributions of their radiation will also be different.

Since the trajectory of a particle depends on x_0, the quantity $\mathrm{d}E/\mathrm{d}\omega$ must be averaged over x_0. Assuming the distribution of particles over x_0 at the entrance to the crystal is uniform, we have

$$\frac{\mathrm{d}\overline{E}}{\mathrm{d}\omega}=\frac{1}{d}\int\limits_0^d\mathrm{d}x_0\frac{\mathrm{d}E}{\mathrm{d}\omega}. \tag{5.7}$$

b) Radiation of superbarrier particles for $\theta\gg\theta_c$

If the entry angle θ is sufficiently large ($\theta\gg\theta_c$), then $x(t)\approx x_0+\theta t$ and $T_u\approx d/\theta v$. Here in the dipole approximation ($\gamma^2 v_{\perp}^2\ll 1$) we have $|W(\nu)|^2=(g_x/\varepsilon\theta)^2|U(g_x)|^2$, where

$$U(g_x)=\int\limits_0^d\mathrm{d}x\,U(x)\exp(i\nu x),$$

$\nu=\theta g_x$, and the formula for the spectral density of radiation takes the form[20]

$$\frac{\mathrm{d}\overline{E}}{\mathrm{d}\omega}=L\frac{2e^2\delta}{m^2d^2\theta^2}\sum_{g_x\geqslant\delta/\theta}\left[1-2\frac{\delta}{\theta g_x}\left(1-\frac{\delta}{\theta g_x}\right)\right]|U(g_x)|^2. \tag{5.8}$$

It is easy to see that this formula leads to the same result as Eq. (4.10). Indeed, noting that $\overline{E}=nL\omega\sigma'$ (L is the thickness of the crystal) and $U(g_x)=ndu_{(g_x)}$, where $u_{(g_x)}$ is the Fourier component of the potential energy of an individual atom with $g_y=g_z=0$, for the case $\overline{u^2}=0$ and $\hbar\omega\ll\varepsilon$ we arrive at Eq. (4.10) for σ_c'.

We emphasize that Eq. (5.8) is not valid for $\theta\lesssim\theta_c$. In this case the motion of the particle in the crystal already cannot be considered rectilinear, and in order to find $x(t)$ it is necessary to know the specific form of the function $U(x)$.

c) Radiation of electrons and positrons for $\theta\leqslant\theta_c$

Let us see how the spectral distribution of the radiation changes with decrease of the angle θ. For this purpose we shall write $\mathrm{d}\overline{E}/\mathrm{d}\omega$ in the form

$$\frac{\mathrm{d}\overline{E}}{\mathrm{d}\omega}=LAf(\zeta,\beta),\quad f(\zeta,\beta)=f_c+f_u, \tag{5.9}$$

where $A=32e^2U_0^2/\pi^4m^2\theta_c d$ and $f(\zeta,\beta)$ is a dimensionless function of the parameters $\zeta=\delta d/2\theta_c$ and $\beta=\theta/\theta_c$; $f_c=f_c(\zeta,\beta)$ refers to channeled particles and $f_u(\zeta,\beta)$ refers to superbarrier particles.

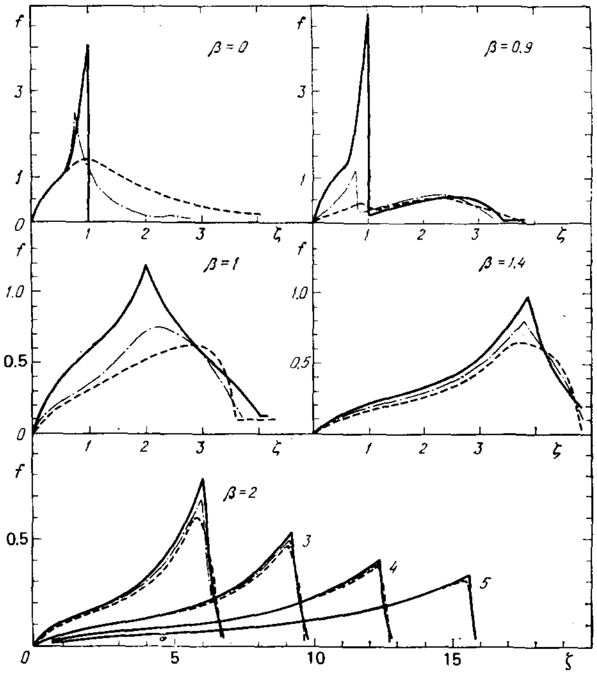

FIG. 8.

The functions f_c and f_u can be found by numerical methods from Eqs. (5.2)–(5.7) if the interplanar potential is given. For light elements the potential energy $U(x)$ can be approximated with high accuracy by a parabolic function[14]:

$$U(x) = \pm U_0 \frac{4x^2}{d^2}, \quad |x| \leqslant \frac{d}{2}, \tag{5.10}$$

where the + and − signs refer respectively to positrons and electrons, $U_0 = 2\pi\eta nRdZe^2$, n is the density of atoms, and η is a numerical coefficient of the order of unity determined from the condition of best approximation of the interplanar potential by the parabolic function.

In Fig. 8 we have shown the results of calculation of the function $f(\zeta, \beta)$ in the dipole approximation for various values of the parameter β. The solid curves refer to positrons moving in a field with the parabolic potential distribution (5.10), and the dashed curves are for electrons moving in the same potential. In the same figure we have shown the results of calculations[7)] (the dot-dash curves) of the function $f(\zeta, \beta)$ for electrons and positrons moving in a field with a linear potential distribution[42]

$$U(x) = \pm U_0 \frac{2|x|}{d}, \quad |x| \leqslant \frac{d}{2}. \tag{5.11}$$

In the field of this form the function $f(\zeta, \beta)$ is the same for electrons and positrons.

These results show how the spectral distribution of the radiation depends on the value of the angle θ, on the sign of the particle charge, and on the form of the approximation of the interplanar potential.

In fields with parabolic and linear distributions of the potential the dominant contribution to the radiation is from the first harmonic of the radiation—the first term of the sum over ν in Eq. (5.3). The maximum of the spectral distribution of the radiation of this harmonic corresponds to frequencies $\omega \sim 4\pi\gamma^2/T$. For $\theta \sim \theta_c$ in the region of the maximum of the radiation $f(\zeta, \beta) \sim 1$ (see Fig. 8), and consequently in this frequency region $\overline{E}'$ is determined in order of magnitude by the crystal thickness L and the value of the factor A; see Eq. (5.9). Comparing $\overline{E}'$ in the frequency region corresponding to the maximum of the radiation with the value $E'_{\rm BH}$ corresponding to the radiation of a particle in an amorphous medium, we find that

$$\frac{d\overline{E}}{d\omega} \sim N_c \frac{dE_{\rm BH}}{d\omega}, \tag{5.12}$$

where $N_c \sim nR^2d/\theta_c$ is the number of lattice atoms which are located within a coherence length. We see that for $\theta \lesssim \theta_c$ the enhancement of the radiation is due to the coherent mechanism of radiation of relativistic particles in interaction with the atoms of a crystallographic plane. The sharp peaks in the radiation spectrum (see Fig. 8) are due to interference of waves radiated by the particle from periodically repeated portions of the path.

The curves given in Fig. 8 show that the difference in the spectral distributions of the radiation between electrons and positrons are especially great for small angles θ ($\theta \lesssim \theta_c$). Beginning with θ of the order $2\theta_c$ the radiation spectra of electrons and positrons practically coincide.

For $\theta > \theta_c$ only superbarrier particles exist. The period of oscillation of these particles increases with decrease of θ ($T_u \sim d/\theta$).

Here the region of characteristic frequencies of radiation $\omega \sim 4\pi\gamma^2\theta/d$ decreases, and the intensity of the radiation at the maximum increases.

For $\theta \leqslant \theta_c$ in addition to superbarrier particles, channeled particles appear, whose oscillation period is about twice that of superbarrier particles; therefore the characteristic frequencies of radiation of channeled particles are a factor of two lower than those of superbarrier particles. In Fig. 8 the sharp peak in the spectral density of radiation in the region $\zeta \sim 1$ is due to channeled particles, and the maximum in the region $\zeta \gtrsim 2$ is due to superbarrier particles.

We note that the intensity of radiation of channeled and superbarrier particles near their radiation maxima are of the same order of magnitude.

For $\theta = 0$ the intensity of radiation of positrons in the region $\zeta \approx 1$ is less than that of positrons for $\theta = 0.9\theta_c$. This is due to the fact that for $\theta = 0.9\theta_c$ the fraction of channeled positrons whose oscillation amplitude is large is greater than at $\theta = 0$, and their contribution to the spectral distribution of the radiation is dominant.

[7)] For a number of other approximations of the interplanar potential the results of calculations of the radiation spectra of electrons and positrons moving near a crystallographic plane are given in Refs. 42 and 46.

d) Influence of divergence particles in the beam on radiation[42]

Up to this time we have assumed that the particles enter the crystal strictly at a given angle θ to one of the crystallographic planes. In practice, however, a particle beam always has some divergence, and therefore it is important to determine how the beam divergence affects radiation. It is clear that the spectral density of radiation in this case will be determined not by Eq. (5.9), but by the formula

$$\frac{d\overline{E}}{d\omega}=LA\int d\theta g(\theta)f(\zeta,\beta), \tag{5.13}$$

where $g(\theta)$ is the distribution function over the angle θ of the particles which have entered the crystal. In the simplest case when the particles enter the crystal parallel to crystallographic planes and the particles in the beam are distributed uniformly in an interval of angles $\theta_0 \geq \theta \geq 0$, the function $g(\theta)$ has the form $g(\theta) = \frac{1}{\theta_0}\Theta(\theta_0-\theta)$, where $\Theta(x)$ is a step function. Here

$$\frac{d\overline{E}}{d\omega}=LA\overline{f}(\zeta,\beta_0),\quad \overline{f}(\zeta,\beta_0)=\frac{1}{\beta_0}\int_0^{\beta_0} d\beta f(\zeta,\beta), \tag{5.14}$$

where $\beta_0=\theta_0/\theta_c$. In Fig. 9 we have shown plots of the function $\overline{f}(\zeta,\beta_0)$ for positrons (Fig. 9a) and for electrons (Fig. 9b) in the case in which the particles are moving in a field with a parabolic distribution of the interplanar potential. The numbers on the curves correspond to various values of the quantity $\beta_0=\theta_0/\theta_c$. In addition to these curves we have shown plots of the function $\overline{f}(\zeta,\beta_0)$ in the case of a linear interplanar potential (Fig. 9c).

If the particle beam has a divergence, then only part of the beam particles will satisfy the conditions of channeling. Here the channeled particles will make the main contribution to the radiation in the frequency region $\omega\sim 4\gamma^2\theta_c/d$ (i.e., $\zeta\sim 1$), while the superbarrier particles will make the main contribution in the frequency region $\omega\gtrsim 8\gamma^2\theta_c/d$ (i.e., $\zeta\gtrsim 2$).

It is evident from the graphs that in the case of a positron beam with high divergence there is a sharp peak in the radiation spectrum at $\zeta\sim 1$. This result does not depend on the specific form of the interplanar potential (see Figs. 9a and c). This is due to the fact that all channeled positrons have similar oscillation periods, and consequently all these particles give a major contribution to the radiation at $\zeta\sim 1$. In regard to superbarrier positrons, for them there is no sharp peak in the radiation spectrum, since superbarrier positrons with different values of θ contribute to the radiation in different frequency regions (see Fig. 8).

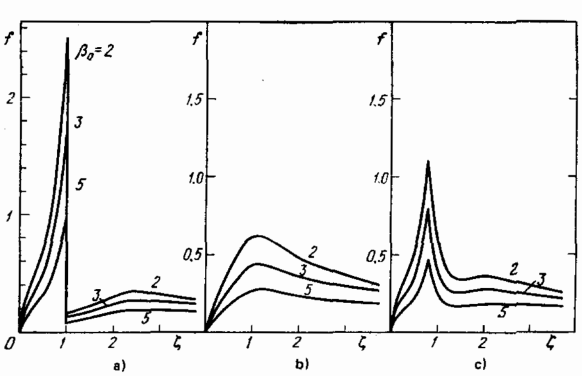

FIG. 9.

For electrons, in contrast to positrons, the radiation is affected to a greater degree by the form of the interplanar potential (see Figs. 9b and c). For electrons moving in a parabolic potential, under channeling conditions the particle-oscillation period will depend strongly on the value of the angle θ and on the coordinate of the point of entry of the particle into the crystal, and therefore there is no sharp peak in their radiation spectrum.

The existence of the sharp peak in the positron radiation spectrum shows that experimental study of the radiation of channeled particles can be carried out most simply by means of positrons.

Up to this time we have not taken into account multiple scattering of particles, which plays an important role if the mean square angle in multiple scattering $\overline{\vartheta_a^2}$ exceeds the square of the beam divergence angle θ_0^2. In this case the distribution function $g(\theta)$ entering into Eq. (5.13) must be considered to depend not only on the angle θ, but also on the particle penetration depth into the crystal: $g(\theta)=g(\theta,z)$. Taking into account that $f(\zeta,\beta)$ does not depend on z, in determining the radiation spectrum we can replace $g(\theta)$ by

$$\overline{g}(\theta)=\frac{1}{L}\int_0^L dz\, g(\theta,z).$$

Here the spectral density of radiation $\overline{E}'$ will be determined by Eq. (5.13) with this distribution function.

e) Inclusion of the nondipole nature of the radiation

Equation (5.3) for the spectral density of the radiation is valid in the case of the dipole approximation, i.e., under the condition $\gamma^2\overline{v_\perp^2}\ll 1$. However, if we have instead the condition $\gamma^2\overline{v_\perp^2}\gtrsim 1$, then the dipole approximation cannot be used and the radiation spectrum must be determined on the basis of Eq. (5.2).

Violation of the dipole radiation condition occurs earliest for channeled particles. In fact, for these particles $\overline{v_\perp^2}\sim\theta_c^2$, and therefore $\gamma^2\overline{v_\perp^2}\sim\gamma^2\theta_c^2\sim 2U_0\gamma/m$; for sufficiently high energies the latter quantity is greater than unity. In the case of superbarrier particles ($\theta>\theta_c$) the value of $\overline{v_\perp^2}$ is of the order of θ_c^4/θ^2, i.e., $\overline{v_\perp^2}<\theta_c^2$, and therefore for superbarrier particles the value of $\gamma^2\overline{v_\perp^2}$ is significantly smaller than for channeled particles (the factor θ_c/θ reflects the decrease of the time of action of forces for a superbarrier particle in comparison with a channeled particle). For this reason we shall consider the influence of the nondipole nature of the radiation only for channeled particles, assuming that the beam particles enter the crystal parallel to crystallographic planes.

The spectral density of radiation averaged over the point of entry will be determined by Eqs. (5.2) and

(5.7). In the general case it is not possible to calculate the integrals occurring in these formulas analytically, and therefore to find $\mathrm{d}\overline{E}/\mathrm{d}\omega$ we have used numerical calculations[47] performed on a computer.[8)] (If the function $x(t)$ is known, then to fine $\overline{E}'$ it is necessary to calculate a triple integral over the time t, the azimuthal angle φ, and the coordinate x_0 of the point of entry of the particle into the crystal. Such calculations can be carried out with various approximations of the interplanar potential.)

The quantity $\overline{E}'$ is conveniently written in the form

$$\frac{\mathrm{d}\overline{E}}{\mathrm{d}\omega}=LAf_{\varkappa}(\zeta), \tag{5.15}$$

where $\varkappa=\gamma\theta_c$ is the parameter of the nondipole nature of the radiation.

For positrons moving in a diamond crystal along the (110) plane the function $f_{\varkappa}(\zeta)$ is shown in Fig. 10 [the interplanar potential is assumed to be parabolic with $U_0 = 20$ eV; curve 1 corresponds to a positron energy $\varepsilon = 6$ GeV ($\varkappa = 0.96$), and curve 2 corresponds to $\varepsilon = 14$ GeV ($\varkappa = 1.47$)]. In the same figure we have shown the function $f_0(\zeta)$, which corresponds to the result of the dipole approximation (curve 0).

It follows from Eqs. (5.2) and (5.7) that for $\varkappa \lesssim 1$ the location of the maximum of the radiation spectrum of channeled positrons is determined from the condition $2\pi/T_c = (1+\gamma^2\overline{v_\perp^2})\delta$; for a parabolic potential this leads to the following value of the frequency corresponding to the maximum of the spectral density of radiation:

$$\omega_{\max}=\frac{\omega_0}{1+(1/2)\varkappa^2}, \tag{5.16}$$

where $\omega_0 = 4\gamma^2\theta_c/d$ is the position of the maximum of the radiation spectrum in the dipole approximation. Therefore this value of $\omega_{\max}$ corresponds to $\zeta_{\max} = [1+(1/2)\varkappa^2]^{-1}$.

It can be seen from Fig. 10 that for $\zeta < \zeta_{\max}$ the dipole and nondipole approximations give similar results.

For $\zeta > 1$ ($\omega > \omega_0$) in the dipole approximation $\overline{E}' = 0$, while inclusion of the nondipole nature leads to $\overline{E}' \neq 0$.

A simple formula for the spectral density of radiation can be obtained in the frequency region $\omega < \omega_{\max}$ if the channeled positrons are moving in a parabolic interplanar potential. For this purpose it is necessary to use Eq. (2.4). In the case that positrons move in a parabolic potential we have $x(t) = x_0 \cos\Omega t$, where $\Omega = 2\theta_c/d$ and $|x_0| \leq d/2$. Substituting this expression for $x(t)$ into Eq. (2.4) and noting that

$$\gamma v^*(t)=\gamma(v_\perp(t_2+t)-v_\perp(t_2))=-2\varkappa\xi\sin\frac{\tau_1}{2}\cos\left(\tau_2+\frac{\tau_1}{2}\right),$$

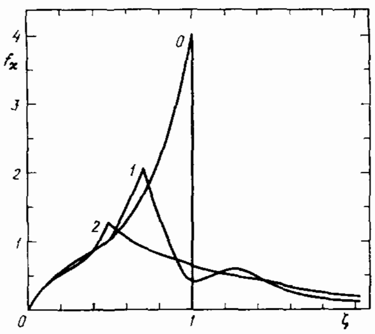

FIG. 10.

where $\varkappa=\gamma\theta_c$, $\xi=2x_0/d$, $\tau_1=\Omega t$, and $\tau_2=\Omega t_2$, we obtain after integration over τ_2

$$\frac{\mathrm{d}E}{\mathrm{d}\omega}=4Ne^2\zeta\left\{\int\limits_0^\infty\frac{\mathrm{d}\tau_1}{\tau_1}\left[\left(1+(\varkappa\xi)^2\sin^2\frac{\tau_1}{2}\right)\sin CJ_0(D)\right.\right.$$
$$\left.\left.-(\varkappa\xi)^2\sin^2\frac{\tau_1}{2}\cos CJ_1(D)\right]-\frac{\pi}{2}\right\}, \tag{5.17}$$
$$C=\zeta\left[\left(1+\frac{(\varkappa\xi)^2}{2}\right)\tau_1-2(\varkappa\xi)^2\frac{\sin^2\tau_1/2}{\tau_1}\right],$$
$$D=\frac{\zeta(\varkappa\xi)^2}{2}\left(\sin\tau_1-\frac{4\sin^2(\tau_1/2)}{\tau_1}\right).$$

Equation (5.17) is valid for any frequency. We see that the nondipole parameter $\varkappa$ enters into E' in the form of the combination $\zeta\varkappa^2\xi^2/2$. In the low-frequency region this quantity will be small regardless of the value of $\varkappa$. In the first approximation in this parameter $\overline{E}'$ takes the form

$$\frac{\mathrm{d}\overline{E}}{\mathrm{d}\omega}=N\pi e^2\varkappa^2\zeta\left[1-2\zeta(1-\zeta)\right]\int\limits_0^1\mathrm{d}\xi\xi^2\Theta\left(1-\zeta\left(1+\frac{\varkappa^2\xi^2}{2}\right)\right). \tag{5.18}$$

This simple formula satisfactorily describes the positron radiation spectrum in the frequency region $\omega \lesssim \omega_{\max}$.

f) Comparison of theoretical results with experiment

On entry into the crystal, due to the imperfections of the surface of the lattice, a redistribution of the particles in angle can occur, as a result of which the spectral distribution of radiation of particles moving in the crystal will differ from that in the case in which the surface of the crystal lattice is ideal.

The results obtained above show that in the case of positrons for $\theta < \theta_c$ the main contribution to the radiation in the low-frequency region $\omega < \omega_{\max}$ is from channeled particles (see Figs. 8 and 10), and therefore, by comparing the results of theory and experiment in this frequency region one can determine the fraction of channeled particles in a specific experiment. For this purpose we present the spectral density of radiation of particles moving in a crystal in the low-frequency region in the form

$$\frac{\mathrm{d}\overline{E}}{\mathrm{d}\omega}=\eta\frac{\mathrm{d}E_c}{\mathrm{d}\omega},\quad \omega\leqslant\omega_{\max}, \tag{5.19}$$

where η is the fraction of channeled particles ($\eta \leq 1$) and $\mathrm{d}E_c/\mathrm{d}\omega$ is the spectral density of radiation of channeled particles; we shall determine the value of η from comparison of theory and experiment in the low-fre-

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[8)] We note that the radiation of channeled particles with allowance for the nondipole nature has been investigated also in Refs. 46 and 48–50 in the framework of undulator theory. The results obtained in these studies apply to positrons moving in a parabolic interplanar potential. The necessity of taking into account the nondipole nature for description of the radiation of channeled particles was pointed out by Bazylev and Zhevago.[18]

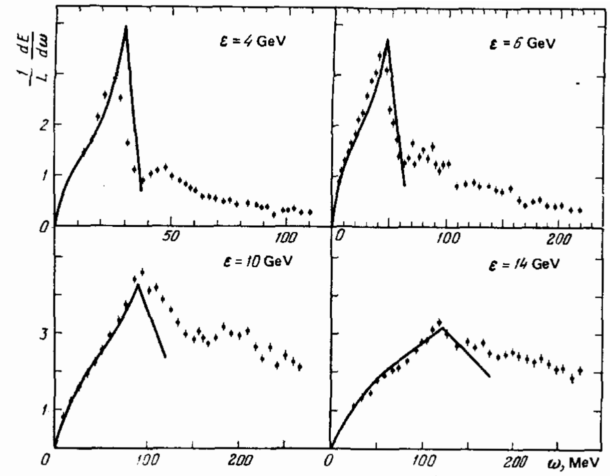

FIG. 11.

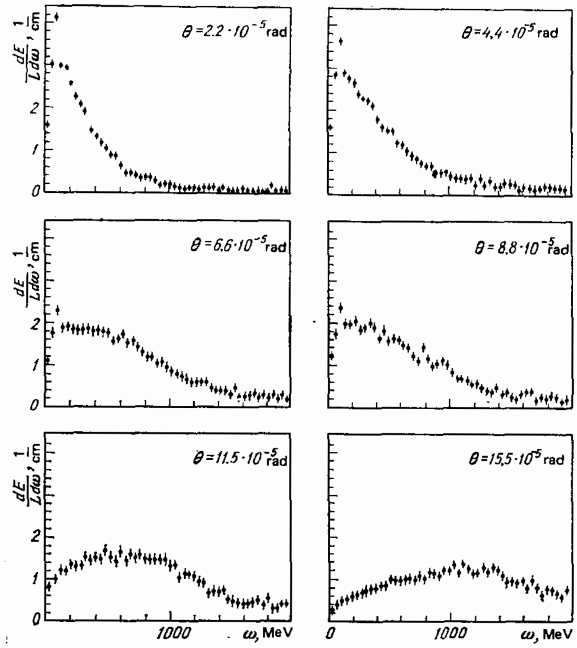

FIG. 12.

quency region.

In 1978 an experiment was performed at Stanford in which the radiation spectra of positrons with energies $\varepsilon = 4$, 6, 10, and 14 GeV were measured when the particles moved in a diamond crystal near the (110) crystallographic plane.[23] In Fig. 11 we have shown the results of this experiment for the case in which positrons entered the crystal parallel to the crystallographic plane.[23,47] As ordinate we have plotted $(1/L)d\bar{E}/d\omega$ in cm^{-1}, and as abscissa—the photon energy in MeV. The beam divergence was less than 10^{-5} rad, and the crystal thickness was $L = 80$ μm. In the same figure we have shown by the solid curves the results of calculation of the radiation spectrum of channeled particles in the low-frequency region $\omega \lesssim \omega_{max}$. The calculations were carried out using Eqs. (5.18) and (5.19). The value of U_0 was taken as 20 eV—a good approximation of the interplanar potential by a parabolic function is obtained with this value of U_0. Values of η were chosen from the condition of agreement of theory and experiment in the low-frequency region. For $\varepsilon = 4$, 6, 10, and 14 GeV these values are respectively $\eta = 0.7$, 0.7, 0.85, and 0.6.

We see that in the experiment considered, even at $\theta = 0$, in addition to channeled positrons there is a rather large group of superbarrier particles.[9)] The appearance of this group of particles at $\theta = 0$ may be due to an imperfection of the crystal lattice (surface imperfection, mosaic structure, or extended defects). This is indicated also by a series of measurements of the radiation spectra of positrons with $\varepsilon = 10$ GeV made in the case in which $\theta \neq 0$ (Fig. 12; Ref. 47). Indeed, in the experiment with increase of θ from $\theta = 0$ to $\theta = \theta_c$ ($\theta_c = 6.8 \cdot 10^{-5}$ rad) the rise noted above (at the end of Section c) in the intensity of radiation of channeled particles was not observed; the experimental results show that even for $\theta > \theta_c$ in the low-frequency region $\omega < \omega_{max}$ channeled particles contribute to the radiation; for $\theta > \theta_c$ there is no sharp peak in the radiation spectrum of superbarrier particles, which indicates a large spread of these particles in angle.

In the high-frequency region ($\omega > \omega_{max}$) both channeled and superbarrier particles make important contributions to the radiation. The radiation spectrum of the latter depends very strongly on the spread of the particles in angle. Since the distribution in angle of the particles which had left the crystal was not studied in the experiment discussed, and the defects of the crystal lattice were not investigated, we shall not analyze the experimental results for this frequency region. For the same reason in the present work we shall not dwell on comparison of theory with the results of experiments[24-26] in which the radiation spectra were measured under conditions of collimation of the radiation.

The results obtained in this section can be used also to determine the spectral density of radiation of relativistic particles in an undulator,[52-55] i.e., in a device in which the transverse motion of a particle is periodic. The basic difference between the radiation of particles in a crystal and in an undulator is that in an undulator the radiation of all particles occurs identically, while in a crystal it is necessary to distinguish the radiation of channeled and superbarrier particles. To this we must add that in motion of particles in a crystal near a crystallographic axis the condition of periodicity of the transverse motion generally cannot occur (see the next section).

6. MOTION OF RELATIVISTIC ELECTRONS AND POSITRONS NEAR A CRYSTALLOGRAPHIC AXIS

a) The approximation of continuous strings

We turn now to investigation of the interaction of relativistic particles with a crystal in the case in which the particle is moving near a crystallographic axis. In contrast to motion near a crystallographic plane, which

[9)] The same conclusion was reached by the authors of Ref. 51.

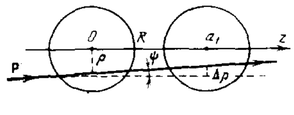

FIG. 13.

was essentially one-dimensional (the effective potential energy depended only on the distance to the plane), we are now dealing with a two-dimensional problem of motion and radiation of a particle in a field created by strings of crystal atoms located parallel to a crystallographic axis near which the particle is moving. We shall assume that the interactions of a particle with the different strings are independent. This assumption is justified if the particle moves far from close-packed planes or if the angle of scattering of a particle by an individual string of atoms is large in comparison with the angle between the incident beam and the crystallographic plane.

The angle of scattering of a relativistic particle by an individual atom of a lattice is small ($\vartheta_1 \sim 2Ze^2/pv\rho$, where ρ is the impact parameter); therefore if the particle enters the crystal at a small angle ψ to one of the crystallographic axes (the z axis), then the change of the impact parameter $\Delta\rho$ at neighboring atoms of the lattice will be small in comparison with the impact parameter, $\Delta\rho \sim a_1(\psi + \vartheta_1) \ll \rho$ (Fig. 13). It is clear that under these conditions for description of the motion of a particle in a crystal the nonuniformities of the potential along the z axis in the transition from one atom to another are unimportant; therefore the potential energy of a particle moving in a crystal can be represented in the form

$$U(\rho) = \sum_n U_r(\rho - \rho_n), \quad U_r(\rho) = \frac{1}{a_1} \int_{-\infty}^{\infty} dz\, u(\rho, z), \tag{6.1}$$

where $U_r(\rho)$ is the effective potential energy of interaction of a particle with a string of atoms located parallel to the z axis, and ρ_n characterizes the location of a string of atoms in the (x, y) plane orthogonal to the z axis.[10]

In the field (6.1) it is evident that the component of the particle momentum parallel to the z axis is conserved. However, in the (x, y) plane the motion of the particle is described by the equation

$$\frac{d^2\rho}{dt^2} = -\frac{1}{\varepsilon} \frac{\partial}{\partial\rho} U(\rho). \tag{6.2}$$

The first integral of this equation has the form

$$\frac{\varepsilon}{2} \left(\frac{d\rho}{dt}\right)^2 + U(\rho) = \varepsilon_\perp(\rho_0), \tag{6.3}$$

where $\varepsilon_\perp(\rho_0) = (\varepsilon\psi^2/2) + U(\rho_0)$ is the energy of the transverse motion of the particle and ρ_0 is the point of entry of the particle into the crystal.

[10] The approximation of continuous strings was introduced by J. Lindhard[13] in discussion of the motion of nonrelativistic channeled positively charged particles in a crystal. In the relativistic case this approximation can be used both for channeled particles and for unchanneled particles. It is necessary only that the angle ψ be sufficiently small.

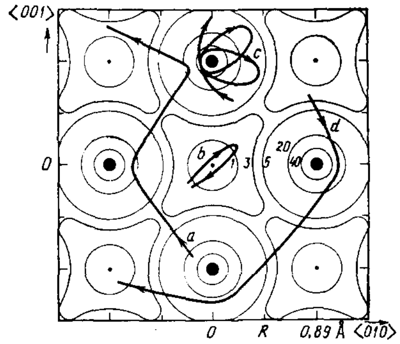

FIG. 14.

In Fig. 14 we have shown equipotential surfaces $U(\rho)$ = const for diamond when the z axis is parallel to the ⟨100⟩ axis of the crystal. [For the continuous potential of an individual string of atoms we used in the calculations the formula $U_r(\rho) = (2Ze^2/a_1)\sum_{i=1}^{3} \alpha_i K_0(\beta_i \rho/R)$, where $K_0(x)$ is a Bessel function, $\alpha_i = (0.1, 0.55, 0.35)$, and $\beta_i = (6, 1.2, 0.3)$]. The numbers on the lines with equal potential correspond to values of $U(\rho)$ = const in eV. The value of $U(\rho)$ at the center of the cell is taken as zero. The dots in Fig. 14 designate centers of cells; the heavy circles are the axes of atomic strings.) In the same figure we have shown typical trajectories in the crystal of positrons (curves a and b) and electrons (curves c and d) in the (x, y) plane orthogonal to the crystallographic axis.

We see that the motion of positrons and electrons in the (x, y) plane can be both finite and infinite. The infinite motion [$\varepsilon_\perp(\rho_0) > U^*$, where U^* is the value of the potential energy at the central point between the closest atomic strings] corresponds to superbarrier particles; the finite motion [$\varepsilon_\perp(\rho_0) < U^*$] corresponds to particles which are called hyperchanneled (Ref. 14).[11]

Taking into account that the fraction of hyperchanneled particles is small and that as the result of dechanneling the number of these particles decreases, we shall consider further the motion and radiation of only superbarrier particles. (We note that it is always possible to create conditions in which all particles moving in a crystal will be superbarrier. This requirement is satisfied, for example, if $\psi > \psi_c$.) Here we shall not especially separate the group of channeled positrons, since this group of particles belongs to the superbarrier particles.

[11] In the case of motion near a crystallographic axis the term "channeled particle" is used for positively charged particles whose energy $\varepsilon_\perp = \varepsilon\psi^2/2$ satisfies the condition $U^* < \varepsilon_\perp < \varepsilon\psi_c^2/2$, where $\psi_c^2 = \sqrt{4Ze^2/\varepsilon a_1}$ is the critical channeling angle.[13,14] It is clear that these particles execute infinite motion, in contrast to hyperchanneled particles. The term "channeled particle" in the case of motion near a crystallographic axis was introduced in order to distinguish particles which do not approach closely to nuclei and, in particular, do not take part in nuclear reactions.

b) Scattering of a particle by an individual string of atoms

We shall discuss first the motion of a particle in the field of a continuous potential of an individual atomic string $U_r(\rho)$. The trajectory of a particle incident on the string is determined in the (x, y) plane by the equation

$$\frac{\varepsilon}{2}\left(\frac{d\rho}{dt}\right)^2 = \varepsilon_\perp - U_r(\rho), \qquad \varepsilon_\perp = \frac{\varepsilon\psi^2}{2}, \tag{6.4}$$

from which it follows that

$$t \equiv t(\rho) = \pm\frac{1}{\psi}\int_{\rho_0}^{\rho}\frac{d\rho'}{\sqrt{1-[U_r(\rho')/\varepsilon_\perp]-(b^2/\rho'^2)}}, \tag{6.5}$$

where b is the impact parameter of the string (Fig. 15), ρ_0 is the distance of closest approach of the particle to the string, and the signs + and − correspond to the portions of the trajectory in which the particle is traveling away from the string and approaching the string. The azimuthal scattering angle of the particle by the string (see Fig. 15) is

$$\varphi(b) = \pi - 2b\int_{\rho_0}^{\infty}\frac{d\rho/\rho^2}{\{1-[U_r(\rho)/\varepsilon_\perp]-(b^2/\rho^2)\}^{1/2}}, \tag{6.6}$$

and the angle of scattering of the particle by the string $\vartheta(b)$ is related to the azimuthal angle $\varphi(b)$ as follows:

$$\vartheta(b) = 2\psi\sin\frac{\varphi(b)}{2}. \tag{6.7}$$

If $\varepsilon_\perp \gg |U_r|$, i.e., if $\psi \gg \psi_c$, then $|\varphi(b)| \ll 1$ and

$$\vartheta(b) = \psi\varphi(b), \quad \varphi(b) = -\frac{1}{2\varepsilon_\perp}\frac{\partial}{\partial b}\int_{-\infty}^{\infty}dx\,U_r(\rho), \tag{6.8}$$

where $\rho = \sqrt{x^2+b^2}$ [the x axis is parallel to the component of momentum of the incident particle in the (x, y) plane].

c) Multiple scattering of fast particles by atomic strings in a crystal [56,57]

The formulas given above characterize the scattering of a particle by a single atomic string. In the case in which the crystal thickness L is large, $L \gg \bar{a}^2/R\psi$ [$\bar{a} = (na_1)^{-1/2}$ is the average distance between strings], it is necessary to take into account multiple scattering of particles by different strings. For this purpose we shall introduce the particle-distribution function $f(\varphi, z)$ which shows how the particles are distributed in the angle φ at a depth z. This function satisfies the kinetic equation[56-59]

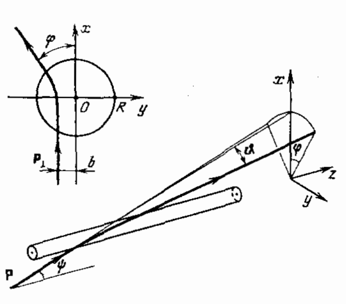

FIG. 15.

$$\frac{d}{dz}f(\varphi, z) = na_1\psi\int_{-\pi}^{\pi}d\chi\left|\frac{db}{d\chi}\right|[f(\varphi+\chi, z) - f(\varphi, z)], \tag{6.9}$$

which corresponds to a one-dimensional nature of the scattering—the scattering occurs only in the angle φ. The elementary scattering process here is scattering by an individual atomic string, and not scattering by individual atoms as occurs, for example, in an amorphous medium. For relativistic particles Eq. (6.9) is valid regardless of the relation between ψ and ψ_c.[56]

The solution of Eq. (6.9) with the boundary condition $f(\varphi, 0) = \delta(\varphi)$ has the form

$$f(\varphi, z) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}e^{ik\varphi}\exp\left(-2na_1z\psi\int_{-\pi}^{\pi}d\chi\left|\frac{db}{d\chi}\right|\sin^2\frac{k\chi}{2}\right). \tag{6.10}$$

An important quantity which characterizes the scattering of particles in a crystal is the mean square scattering angle of the particles. According to Eq. (6.7) it is given by the formula

$$\overline{\vartheta^2}(L) = 4\psi^2\int_{-\pi}^{\pi}d\varphi f(\varphi, L)\sin^2\frac{\varphi}{2}. \tag{6.11}$$

Using Eq. (6.10), we have

$$\overline{\vartheta^2}(L) = 2\psi^2\left[1-\exp\left(-2na_1L\psi\int_{-\pi}^{\pi}d\chi\left|\frac{db}{d\chi}\right|\sin^2\frac{\chi}{2}\right)\right]. \tag{6.12}$$

If $\psi \gg \psi_c$, then the characteristic values of the angle φ of scattering by an individual atomic string are small. Here, if

$$\overline{\varphi^2} = 2na_1L\psi\int_0^{\infty}db\,\varphi^2(b) \ll 1,$$

then for $\varphi \lesssim \sqrt{\overline{\varphi^2}}$ the sum in Eq. (6.10) can be replaced by an integral. In this case the distribution of particles will be Gaussian:

$$f(\varphi, L) = \frac{1}{\sqrt{2\pi\overline{\varphi^2}}}e^{-\varphi^2/2\overline{\varphi^2}}. \tag{6.13}$$

Here

$$\overline{\vartheta^2}(L) \approx \psi^2\overline{\varphi^2} = \frac{8na_1L}{\varepsilon^2\psi}\int_0^{\infty}db\left(\frac{d}{db}\int_0^{\infty}dx\,U_r(\rho)\right)^2. \tag{6.14}$$

If, in particular, the potential of an individual atomic string is taken as the Lindhard potential[13]

$$U_r(\rho) = \frac{Ze^2}{a_1}\ln\left(1+\frac{3R^2}{\rho^2}\right),$$

then according to (6.14) we have

$$\overline{\vartheta^2}(L) = \sqrt{3}(4-\pi)\frac{4\pi^2Z^2e^4nRL}{\varepsilon^2\psi a_1}. \tag{6.15}$$

Comparing this value of $\overline{\vartheta^2}(L)$ with the mean square scattering angle of the particles in an amorphous medium $\overline{\vartheta^2_a}$ [see Eq. (3.1)], we find that

$$\frac{\overline{\vartheta^2}(L)}{\overline{\vartheta^2_a}} \sim \frac{R}{\psi a_1}. \tag{6.16}$$

We see that for sufficiently small values of the angle ψ the mean square particle-scattering angle in the crystal can significantly exceed the mean square particle-scattering angle in an amorphous medium. We recall that Eq. (6.16) is valid if $\psi \gg \psi_c$. In Fig. 16 we have shown the dependence of $\sqrt{\overline{\vartheta^2}(L)/\overline{\vartheta^2_a}}$ on $\psi \times \psi_c^{-1}$ for an ar-

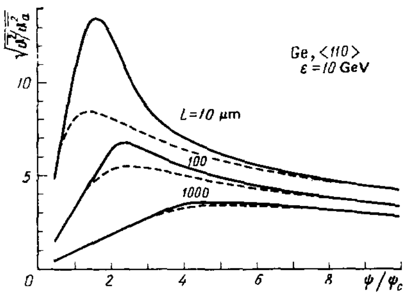

FIG. 16.

bitrary relation between ψ and ψ_c. This dependence is obtained by numerical calculation in a computer according to Eq. (6.12). As the continuous potential of a string of atoms in the calculations we used the Lindhard potential. The curves shown apply to positrons (solid curves) and electrons (dashed curves) with $\varepsilon = 10$ GeV moving in germanium crystals with thickness $L = 10$, 100, and 1000 μm at a small angle ψ to the $\langle 110 \rangle$ crystallographic axis.

These results show that the maximum of $\sqrt{\overline{\vartheta^2}(L)/\vartheta_a^2}$ is reached at $\psi \sim \psi_c$; the value of $\overline{\vartheta^2}(L)$ decreases with decrease of ψ if $\psi < \psi_c$.

7. RADIATION FROM A RELATIVISTIC PARTICLE NEAR A CRYSTALLOGRAPHIC AXIS

a) Radiation in a thin crystal for $l_c \gg L$[56]

In the previous section we showed that the motion of a particle near a crystallographic axis differs substantially from motion in an amorphous medium. The scattering of the particles also differs greatly in the two cases, and therefore the radiation will also be quite different.

Let us turn now to discussion of the radiation of particles in motion near one of the crystallographic axes of the crystal (the z axis), and as in the preceding section we shall use classical electrodynamics. The main fraction of particles from the incident beam executes infinite motion in the plane orthogonal to the z axis (see Chapter 6), and therefore we shall restrict the further discussion to the radiation of only this group of particles.

We shall begin with the simplest case of radiation of low frequency, assuming that the coherence length is large in comparison with the crystal thickness, $l_c \gg L$, and further that $L \gg \overline{a}^2/R\psi$, where $\overline{a}$ is the average distance between atomic strings. In this case a particle in passing through the crystal will interact with a large number of different atomic strings located parallel to the z axis; the scattering by the individual atomic strings will be assumed to be independent.

The spectral density of radiation E' for $l_c \gg L$ is determined, according to Eq. (2.9), by the particle-scattering angle ϑ. Since in the case considered $\vartheta = \vartheta(\varphi)$ and the angle φ can take on various values, it is necessary to average E' over φ with a distribution function $f(\varphi, L)$:

$$\frac{d\overline{E}}{d\omega} = \int_{-\pi}^{\pi} d\varphi f(\varphi, L) \frac{dE(\vartheta(\varphi))}{d\omega}. \tag{7.1}$$

We shall consider several very interesting cases. In the dipole approximation when $\gamma^2\overline{\vartheta^2} \ll 1$, where $\overline{\vartheta^2} = \overline{\vartheta^2}(L)$ is the mean square angle of scattering of the particles by the crystal (6.12), Eq. (7.1) takes the form

$$\frac{d\overline{E}}{d\omega} = \frac{2e^2}{3\pi} \gamma^2 \overline{\vartheta^2}(L). \tag{7.2}$$

If $\psi \gg \psi_c$, then in accordance with (6.14)

$$\frac{d\overline{E'}}{d\omega} = L \frac{16e^2 n a_1}{3\pi m^2 \psi} \int_0^\infty db \left(\frac{d}{db} \int_0^\infty dx\, U_r(\rho) \right)^2. \tag{7.3}$$

The spectral density of radiation in this case increases rapidly with decrease of the angle ψ.

Comparing the expression found for $\overline{E}'$ with Eq. (3.3) for E'_{BH}, we find that $\overline{E}' \sim N_c E'_{BH}$, where $N_c \sim R/a_1\psi$ is the number of atoms from an individual string which are located within a coherence length. We see that for $l_c \gg L$ the enhancement of the intensity of radiation in motion of particles in a crystal in comparison with the radiation in an amorphous medium is due to the effect of coherence.

For $\psi \lesssim \psi_c$, as was shown in Section 6, $\overline{\vartheta^2}(L)$ decreases with decrease of ψ (see Fig. 16). Therefore in the low-frequency region $\overline{E}'$ does not depend on ω and reaches a maximum value at $\psi \sim \psi_c$.

If $\gamma^2\overline{\vartheta^2} \gg 1$, then in accordance with Eq. (2.10b)

$$\frac{d\overline{E}}{d\omega} \approx \frac{2e^2}{\pi} \ln \gamma^2 \overline{\vartheta^2}(L). \tag{7.4}$$

The radiation spectrum in the crystal in this case, as in an amorphous medium [see Eq. (3.6b)] is almost independent of the crystal thickness L. Thus, for multiple scattering of particles by strings of atoms of the crystal, as in an amorphous medium, a suppression of the radiation is possible, but now the radiation is coherent. We note that since the condition $\overline{\vartheta^2}(L) \gg \overline{\vartheta_a^2}$, is satisfied over a wide range of the angles ψ, then more favorable conditions can be created in a crystal for study of the radiation-suppression effect than in an amorphous medium.

In derivation of Eq. (7.1) we did not take into account transition radiation, which is a valid procedure for $\omega \gg L\omega_p^2$, where ω_p is the plasma frequency, and therefore the formulas given above can be used if $l_c \gg L$ and $\omega \gg L\omega_p^2$.

b) Radiation in the dipole approximation for $l_c \ll L$

We shall consider now radiation under conditions in which $\overline{a}^2/R\psi \gg l_c \gg a_1$, assuming as before that the interaction of particles with different atomic strings occurs independently. In this case the radiation is formed in smaller lengths than the distance traveled by the particle between successive collisions with atomic strings, and therefore the radiation will be determined by the interaction of the particle with the field of the individual atomic string.

The spectral density of the radiation is determined by the trajectory of the particle in the field of the atomic string (6.5), which in turn depends on the modulus of the string impact parameter $|b|$, i.e., the distance between the string axis and the projection of the incident-particle trajectory on the plane orthogonal to the z axis. The spectral density of radiation $\mathrm{d}E(b)/\mathrm{d}\omega$ must obviously be averaged over this impact parameter with a probability $\mathrm{d}b/\bar{a}$, $|b| \leqslant \bar{a}/2$. Since the particle interacts with $L\psi/\bar{a}$ atomic strings of the crystal, then for the average spectral density of the radiation we obtain the expression[21]

$$\frac{\mathrm{d}\bar{E}}{\mathrm{d}\omega} = L\psi n a_1 \int_{-\infty}^{\infty} \mathrm{d}b \frac{\mathrm{d}E(b)}{\mathrm{d}\omega} \tag{7.5}$$

[we have made use of the fact that $\bar{a}^2 = 1/na_1$ and have taken into account that $E'(b)$ falls off rapidly with increase of $|b|$].

Let us consider first the dipole approximation. The spectral density of the radiation in this case, according to Eq. (2.10), is determined by the square of the Fourier component of the acceleration $|\mathbf{W}|^2 = |\int \mathrm{d}t \dot{\mathbf{v}}_{\mathrm{I}} e^{iqt}|^2 = W_x^2 + W_y^2$. Using Eqs. (6.4) and (6.5), we obtain

$$W_x = \frac{2}{\varepsilon} \int_0^{\infty} \mathrm{d}t \frac{x}{\rho} \frac{\partial U_r}{\partial \rho} \cos qt(\rho), \quad W_y = \frac{2}{\varepsilon} \int_0^{\infty} \mathrm{d}t \frac{y}{\rho} \frac{\partial U_r}{\partial \rho} \sin qt(\rho)$$

[the (x, y) plane is located perpendicular to the axis of the string, and the x axis is the symmetry axis of the trajectory (Fig. 17)]. Since $x/\rho = \cos\varphi^*(\rho)$, $y/\rho = \sin\varphi^*(\rho)$ and $\mathrm{d}t = \mathrm{d}\rho/\dot{\rho}$, where

$$\varphi^*(\rho) = b\psi \int_{\rho_0}^{\rho} \frac{\mathrm{d}\rho}{\rho^2 \dot{\rho}}, \quad \dot{\rho} = \psi \sqrt{1 - \frac{U_r(\rho)}{\varepsilon_\perp} - \frac{b^2}{\rho^2}},$$

the final formula for $\mathrm{d}\bar{E}/\mathrm{d}\omega$ in the dipole approximation has the form

$$\frac{\mathrm{d}\bar{E}}{\mathrm{d}\omega} = L \frac{8e^2 n a_1 \delta}{\pi m^2 \psi} \int_{\delta}^{\infty} \frac{\mathrm{d}q}{q^2} \left[1 - 2\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\right] \int_0^{\infty} \mathrm{d}b\,(A^2 + B^2), \tag{7.6}$$

where

$$A = \psi \int_{\rho_0}^{\infty} \frac{\mathrm{d}\rho}{\dot{\rho}} \frac{\partial U_r}{\partial \rho} \cos\varphi^*(\rho) \cdot \cos qt(\rho), \quad B = \psi \int_{\rho_0}^{\infty} \frac{\mathrm{d}\rho}{\dot{\rho}} \frac{\partial U_r}{\partial \rho} \sin\varphi^*(\rho) \cdot \sin qt(\rho).$$

In the limiting case of low frequencies, i.e., for $l_c \gg 2R/\psi$, we have $A \to -2\varepsilon_\perp \cos\varphi^*(\infty)$ and $B \to 0$, and therefore

$$\frac{\mathrm{d}\bar{E}}{\mathrm{d}\omega} \to L \frac{4e^2 n a_1 \gamma^2 \psi}{3\pi} \int_0^{\infty} \mathrm{d}b\,\vartheta^2(b), \tag{7.7}$$

where $\vartheta(b) = 2\psi \sin\varphi(b)/2$ is the angle of scattering of the particle by the string $(\varphi(b) = \pi - 2\varphi^*(\infty))$. We note that this formula can be obtained also from Eqs. (7.6) and (2.10a) if in the latter we set $\vartheta = \vartheta(b)$.

For $\varepsilon_\perp \gg |U_r|$, i.e., for $\psi \gg \psi_c$, it is possible to carry

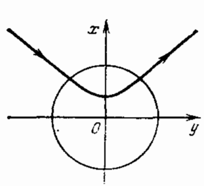

FIG. 17.

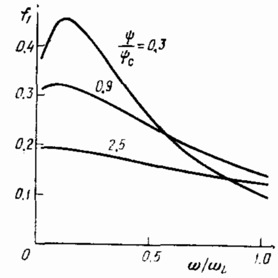

FIG. 18.

out in Eq. (7.6) an expansion in the parameter ψ_c/ψ, as a result of which we obtain

$$\frac{\mathrm{d}\bar{E}}{\mathrm{d}\omega} = L \frac{e^2 n a_1 \delta}{2\pi^2 m^2 \psi^2} \int_{\delta/\psi}^{\infty} \mathrm{d}g_x \int_{-\infty}^{\infty} \mathrm{d}g_y \frac{g_x^2 + g_y^2}{g_x^2} \left[1 - \frac{2\delta}{\psi g_x}\left(1 - \frac{\delta}{\psi g_x}\right)\right] |U_r(\mathbf{g})|^2, \tag{7.8}$$

where

$$U_r(\mathbf{g}) = \int \mathrm{d}^2\rho\, U_r(\rho) \exp(i\mathbf{g}\rho) \tag{7.9}$$

is the Fourier component of the potential energy $U_r(\rho)$.

If, using the relation $\bar{E} = Ln\omega\sigma'$ we go over to the cross section for radiation of particles and use the definition of the continuous potential energy of the string (6.1), then as can be easily verified Eq. (7.8) leads to the Born expression for the radiation cross section.

The case $\psi \gg \psi_c$ corresponds to an almost rectilinear particle trajectory. However, if $\psi \lesssim \psi_c$, then it is necessary to take into account the curvature of the trajectory. The value of $\bar{E}'$ in this case must be found directly from Eq. (7.6).

In Fig. 18 we have shown the dependence of the spectral density of radiation of electrons on frequency for $\psi = 0.3\psi_c$, $\psi = 0.9\psi_c$, and $\psi = 2.5\psi_c$, found with Eq. (7.6) by means of a computer[60] (as ordinate we have plotted $f_1 = (4e^2 n a_1 R U_1^2 L/\pi m^2 \psi_c)^{-1} (\mathrm{d}\bar{E}/\mathrm{d}\omega)$, and as abscissa we have plotted ω/ω_L, where $\omega_L = 2\gamma^2\psi_c/R$). The calculations were carried out for $U_r(\rho) = U_1 \exp(-\rho/R)$. We see that for $\psi < \psi_c$, in contrast to the case $\psi > \psi_c$, in the region of frequencies ω for which $l_c \sim 2\gamma^2/\omega \sim R/\psi_c$ there is a maximum of the radiation. The presence of the maximum is due to the curvature of the particle trajectory in the field of the atomic string.

c) Radiation in the low-frequency region

Up to this time we have assumed the radiation to be dipole. Let us now investigate the effect of a nondipole nature of the radiation. This must be taken into account for $\gamma\vartheta \gtrsim 1$ (ϑ is the angle of scattering of the particle by the atomic string). If $\psi \sim \psi_c$, then $\vartheta \sim \psi_c$, and therefore the condition $\gamma\vartheta \sim \gamma\psi_c \gtrsim 1$ can be satisfied for sufficiently large values of ε (in a germanium crystal, for example, this condition is satisfied for $\varepsilon \gtrsim 1$ GeV).

For low frequencies at which $\bar{a}^2/R\psi > l_c > 2R/\psi$, the change of the radius of curvature of the particle trajectory within a coherence length l_c is large; therefore to find the quantity E' which enters into Eq. (7.5) we can make use of Eq. (2.9). Here the spectral density of the radiation E' is determined by the angle of scattering of the particle by the string $\vartheta(b)$ and

$$\frac{d\overline{E}}{d\omega}=2La_1n\psi\int_0^\infty db\,\frac{dE(\vartheta(b))}{d\omega}. \tag{7.10}$$

In the dipole approximation this formula goes over into Eq. (7.7). However, if $\gamma\overline{\vartheta}\gg 1$, then

$$\frac{d\overline{E}}{d\omega}\approx Lna_1\psi\,\frac{8e^2R}{\pi}\ln\gamma\overline{\vartheta}, \tag{7.11}$$

where $\overline{\vartheta}$ is the average value of the angle of scattering of the particle by the string, $\overline{\vartheta}\sim\vartheta(R)$.

d) Radiation in the high-frequency region

In the high-frequency region where $2R/\psi\gg l_c\gg a_1$, the change of the radius of curvature of the particle trajectory within a coherence length is small, and therefore to find $E'(b)$ we can use Eq. (2.7). Replacing the integration over t_2 in this equation by integration over ρ by means of the relation $dt_2=d\rho/\dot{\rho}$, where $\dot{\rho}=\psi\sqrt{1-(U_r/\varepsilon_\perp)-(b^2/\rho^2)}$, and using the equation of motion $\dot{v}_\perp=-\varepsilon^{-1}\partial U_r/\partial\rho$, we write the spectral distribution of radiation (2.7) in the form

$$\frac{dE(b)}{d\omega}=-\frac{8e^2\delta}{\sqrt{\pi}}\int_0^\infty\frac{d\rho}{\dot{\rho}}\left(\frac{\Phi'(u)}{u}+\frac{1}{2}\int_u^\infty dx\,\Phi(x)\right)\Theta\left(1-\frac{U_r}{\varepsilon_\perp}-\frac{b^2}{\rho^2}\right), \tag{7.12}$$

where

$$u=\left|\frac{\omega m}{\gamma^2\partial U_r/\partial\rho}\right|^{2/3}$$

and $\Theta(x)$ is a step function. Then, substituting this expression into (7.5), we obtain after integration over b the following expression[21] for $d\overline{E}/d\omega$:

$$\frac{d\overline{E}}{d\omega}=\frac{4e^2}{\sqrt{3}}Lna_1\delta\int_{\rho'}^\infty\rho\,d\rho\int_{\omega/\omega_c}^\infty dx\,K_{5/3}(x),\quad \omega_c=\frac{3\gamma^2}{2m}\left|\frac{\partial U_r}{\partial\rho}\right|, \tag{7.13}$$

where $K_{5/3}(x)$ is a modified Bessel function [see Eq. (2.12)] and $\rho'=0$ for electrons and is determined from the condition $\varepsilon_\perp=U_r(\rho')$ for positrons.

e) Radiation by electrons

We see that the spectral distributions of the radiation of electrons and positrons are different. Let us consider first the radiation of electrons. For them, according to Eq. (7.13), $\overline{E}'$ does not depend on the entry angle ψ [we note, however, that the angle ψ governs the region of applicability of Eq. (7.13): $2R/\psi>l_c$].

For $\omega>\omega_c(R)$ the main contribution to the integral in (7.13) over ρ is from $\rho\lesssim R$. Here we can assume $U_r(\rho)\approx U_1\ln(R/\rho)+\text{const}$, where $U_1=Ze^2\eta/a_1$ and η is a numerical coefficient of the order of unity determined from the condition of best approximation of the continuous potential energy of the string by a function of the form $U_1\ln(R/\rho)+\text{const}$. The spectral density of the radiation of electrons in a field of this sort[21] according to Eq. (7.13) has the form[21)]

$$\frac{d\overline{E}}{d\omega}=L\frac{2\pi e^2U_1^2na_1}{3m^2\delta}. \tag{7.14}$$

This formula differs from the corresponding formula

[12)] In Ref. 21 an error was made: in Eq. (3) a factor e^2 has been omitted, and in Eq. (4) the factor in front of the function $F(\xi)$ should contain the quantity (U_0/m) to the first power.

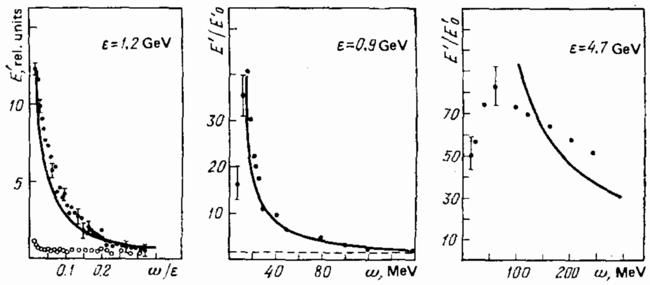

FIG. 19.

for an amorphous medium (3.3) by a factor $k(\delta^{-1}/a_1)$, where $k=\pi\eta^2/8\ln(183Z^{-1/3})$ and (δ^{-1}/a_1) is equal in order of magnitude to the number of atoms in a coherence length.

We note that Eq. (7.14) remains valid not only for $\gamma\overline{\vartheta}\gg 1$, but also in the case when $\gamma\overline{\vartheta}<1$. In fact, substituting $U_r=U_1\ln(R/\rho)+\text{const}$ into Eq. (7.6) and noting that in the region of high frequencies $\omega>\omega_c(R)$ the main contribution to the radiation is from small values of ρ, $\rho\lesssim R$, for which in the quantities $\dot{\rho}$, $\varphi^*(\rho)$, and $t(\rho)$ the slowly varying function $U_1\ln(R/\rho)$ can be replaced by a constant, after all the integrations we obtain Eq. (7.14).

If the electrons fall on the crystal parallel to the crystallographic z axis, then the particles which have entered near the center of a unit cell will be superbarrier and the particles which have entered near the string axis will be hyperchanneled. Hyperchanneled electrons pass through the region $\rho\lesssim\sqrt{\overline{u^2}}$, in which the nuclei of the atoms of the string are located. Here as a result of multiple scattering by the nuclei these electrons will leave the hyperchanneling regime and be converted to superbarrier particles. Thus, the fraction of superbarrier particles will increase with depth of penetration of the particles into the crystal. Since the radiation spectrum of superbarrier electrons (7.14) does not depend on the value of the angle ψ, by comparing the spectral distribution of radiation (7.14) with the experimental radiation distribution of electrons moving in a crystal, it is possible to estimate the fraction of superbarrier particles.

At the Khar'kov Physico-technical Institute,[27] at the Tomsk Institute of Nuclear Physics,[28] and at the Erevan Physics Institute[25] radiation spectra of electrons with energy $\varepsilon=1.2$ GeV, 0.9 GeV, and 4.7 GeV have been measured with the electrons moving parallel to the $\langle 110\rangle$ axis[27,28] and the $\langle 100\rangle$ axis[25] of a diamond crystal. The results of these experiments are shown in Fig. 19 (the hollow circles and E'_0 correspond to the radiation in a disoriented crystal, measured in relative units; E' is the radiation spectrum in the case in which the beam enters the crystal parallel to the crystallographic axis). In the same figure we have shown $\overline{E}/E'_{BH}$ (the solid curves) found from Eq. (7.14) and Eq. (3.3).[13)] Compar-

[13)] For the $\langle 110\rangle$ axis of diamond U_1 was taken as 31 eV; with this U_1 a good approximation of the potential energy $U_r(\rho)$ by the function $U_1\ln(R/\rho)+\text{const}$ is achieved for $\rho\lesssim R$. For the $\langle 100\rangle$ axis $U_1\approx 26$ eV.

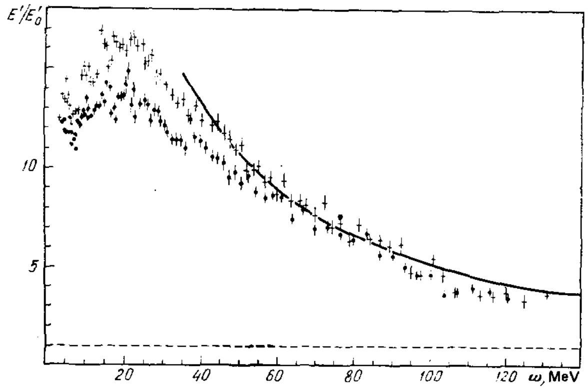

FIG. 20.

ison of the theoretical results with experiments in the region of applicability of Eq. (7.14) shows that in these experiments almost all electrons are superbarrier.

At the Khar'kov Physico-technical Institute an experiment was performed[30] in which the radiation spectra of electrons with energy $\varepsilon = 1.2$ GeV were measured in a silicon crystal under conditions in which hyperchanneling of electrons is possible ($\psi \approx 0$) and also when this phenomenon is clearly absent ($\psi > \psi_c$).[14)] The electrons entered the crystal near the ⟨110⟩ axis. The divergence of the incident beam was less than the critical channeling angle. In contrast to Refs. 25–29, the measurements in this experiment were made under conditions in which there was no collimation of the radiation. The results of the experiment are shown in Fig. 20: the crosses refer to the case in which $\psi = 0$, and the dots to the case in which $\psi = 0.5$ mrad. The dashed line at the bottom corresponds to the measured spectrum of radiation from a disoriented crystal. The heavy line corresponds to the radiation spectrum obtained with Eq. (7.14) with the value $U_1 = 51$ eV.

We see that disorientation of the crystal to an angle ψ of the order of ψ_c does not affect the intensity of radiation of electrons in the high-frequency region.[15)] This indicates that the group of superbarrier particles plays a dominant role in the radiation of an electron beam passing through a crystal.

In the low-frequency region, $\omega < \omega_c(R)$, the intensity of radiation depends greatly on the distribution of particles in angle; see Eqs. (7.6) and (7.10). This distribution changes with depth of penetration of the electrons into the crystal. Since the distribution in angle of the particles leaving the crystal was not studied in the experiments of Refs. 25–30, we shall not analyze here the results of measurements relating to the low-frequency region.

[14)] The need of performing such an experiment was pointed out in Ref. 21.

[15)] Similar results were obtained in the experiment of Ref. 29 [see Fig. 1(a) of that article]. In this experiment, however, the measurements were made with collimation of the radiation, which greatly affects the shape of the radiation spectrum in the low-frequency region.

f) Radiation by positrons

In contrast to electrons, the quantity $\bar{E}'$ for positrons depends on the angle ψ (or on the distance ρ_0 of closest approach of the positron to the string axis). Generally speaking, averaging must be carried out over these quantities. In particular, if the beam enters parallel to a crystallographic axis, then the spectral density of radiation averaged over ρ_0 is determined by the formula

$$\frac{d\bar{E}}{d\omega} = na_1 \int d^2\rho_0 \, \frac{d\bar{E}(\rho_0)}{d\omega}, \tag{7.15}$$

where the integration over ρ_0 is carried out within a unit cell in the plane orthogonal to the string axis.

Positrons which have entered at the center of the cell will execute finite motion in the (x, y) plane (see Fig. 14)—they move in a region with a small potential gradient and therefore the intensity of their radiation is low. Positrons which have entered near the string axis (superbarrier positrons) will move in a region with a high potential gradient and therefore the intensity of radiation by these positrons will be high.

Substituting into Eq. (7.15) the expression (7.13) for $\bar{E}'(\rho_0)$, we obtain the radiation spectrum of superbarrier positrons in the high-frequency region. In particular, if $U_r(\rho)$ is approximated by a function of the form $U_r = U_1 R/\rho$,[61] then

$$\frac{d\bar{E}}{d\omega} = L \frac{\pi^2 e^2 (na_1)^2 U_1^2 R^2}{m^2 \delta}. \tag{7.16}$$

This formula differs from the corresponding formula for an amorphous medium by a factor $k^*(nR^2/\delta)$, where $k^* = \pi^2\eta^2/16 \ln 183 Z^{-1/3}$ and $nR^2/\delta \sim (\delta^{-1}/a_1)(R^2/a_1^2)$ (the first factor is the number of atoms in a coherence length; the second factor determines the fraction of particles moving in a region with a large potential gradient).

If the potential of the string is approximated by the function $U_r(\rho) = U_1\left(1 - \frac{\rho}{R^*}\right)\Theta(R^* - \rho)$, where $\Theta(R^* - \rho)$ is a step function and U_1 and R^* are adjustable parameters determined from the condition of best approximation of $U_r(\rho)$ by the function used, then according to Eqs. (7.13) and (7.15),[21]

$$\frac{d\bar{E}}{d\omega} = \frac{L}{4m} \pi \sqrt{3} U_1 (na_1)^2 R^{*3} F(\zeta), \quad \zeta = \frac{4}{3} \frac{\delta m R^*}{U_1}, \tag{7.17}$$

where $F(\zeta)$ is a function used in the theory of synchrotron radiation; see Eq. (2.12). It follows from Eq. (7.17) that the maximum of $\bar{E}'$ exceeds $E'_{\rm BH}$ by approximately a factor $(mna_1R^{*3}/10Ze^2)$.

At Stanford an experiment was carried out in which the radiation spectra of positrons with energies $\varepsilon = 4$, 6, 10, and 14 GeV were measured when the particles entered a diamond crystal along the ⟨100⟩ axis. The results of this experiment are shown in Fig. 21. The ordinate is the quantity $(\omega/L_R)\, dN/d\omega$, where $dNd\omega$ is the measured number of photons in the frequency interval $(\omega, \omega + d\omega)$. The dashed curve corresponds to Eq. (7.16) with $U_1 = 34$ eV; the solid curve corresponds to Eq. (7.17) with $U_1 = 50$ eV and $R^* = 1.6R$.

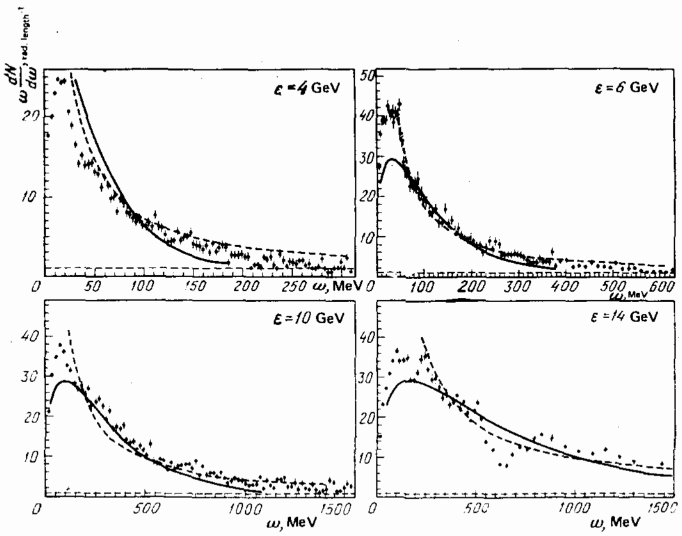

FIG. 21.[16)]

We see that in the experiment discussed above the dominant role is played by superbarrier positrons, i.e., positrons which execute infinite motion in the plane orthogonal to the ⟨100⟩ axis.

8. CONCLUSION

We see that the radiation of a relativistic particle in matter is formed over a large region of space along the particle trajectory. If within this length there are a large number of atoms of the material, the radiation will occur in a different manner than for motion of a particle in a rarefied gas of atoms.

In an amorphous medium at high energies a suppression of radiation can occur (the Landau-Pomeranchuk effect and the effect of decrease of the radiation in a thin layer of matter): for this it is necessary that the mean square angle of scattering of the particle within a coherence length exceed the characteristic angle of radiation of a relativistic particle.

For motion of fast electrons and positrons near a crystallographic axis or plane, coherent and interference effects can occur which lead to an enhancement of the radiation intensity in comparison with that for an amorphous medium.

In motion near a crystallographic plane there are both channeled and superbarrier particles. Both these groups of particles radiate intensely but in different frequency regions.

The greatest enhancement of the radiation is achieved for particle motion near a crystallographic axis. The nature of the particle motion in the crystal in this case differs substantially from the nature of the motion in an amorphous medium. In the motion of a particle near a crystallographic axis, as in the case of motion near a crystallographic plane, both hyperchanneled and superbarrier particles exist. It is important to note that the fraction of superbarrier particles in motion near an axis is always high (in motion near a plane it can be either a large or small fraction depending on the orientation of the plane with respect to the incident-particle momentum). Comparison of existing experimental data with the results of the theory shows that the group of superbarrier particles makes the main contribution to the radiation of a particle beam passing through a crystal.

In motion of relativistic particles in a crystal a suppression effect similar to the Landau-Pomeranchuk effect is possible. In contrast to an amorphous material, in a crystal the coherent radiation is suppressed. This effect is due to the large scattering of particles in the crystal in a coherence length. The effect arises at energies significantly lower than in the case of an amorphous medium.

As in the case of bremsstrahlung, in production of electron-positron pairs in a crystal by a photon or a charged particle there also can be coherent and interference effects.[9,10,62] As a result of these effects, pairs will be produced more intensely in a crystal than in an amorphous medium. This effect is due to the fact that the effective length in which a high-energy pair is formed can reach macroscopic size and can include a large number of atoms of the crystal lattice.

As the result of coherent effects in radiation and in production of electron-positron pairs, electromagnetic showers in crystals can be formed in lengths which are one to two orders of magnitude smaller than in an amorphous medium.[63] These effects, however, we shall not discuss here (in this connection see Refs. 9 and 63–66). Discussion of these effects, and also of such questions as the angular distribution of the radiation, radiation in the high-frequency region,[22,42,67–69] orientation dependence in the total radiation yield,[20,70–72] motion and radiation of electrons with low energy,[73,74] motion of fast particles in a curved crystal,[75,76] the influence of thermal vibrations of lattice atoms on the motion and radiation of relativistic particles,[10,42] quantum effects in interaction of fast particles with a single crystal,[26,77–81] and the development in space and time of electrodynamic processes at high energies in single crystals,[39,41,82] all require special reviews.

LIST OF PRINCIPAL NOTATIONS

$Z\|e\|$	nuclear charge
$\varepsilon, \mathbf{p}, \mathbf{v}, \gamma=\varepsilon/m$	energy, momentum, velocity, and Lorentz factor of the incident particle
$\varepsilon', \mathbf{p}', \mathbf{v}'$	energy, momentum, and velocity of the electron after radiation of a photon
$\omega, \mathbf{k}=\omega\mathbf{n}$	frequency and wave vector of the photon
$\mathbf{v}_\perp(t)$	projection of the particle velocity at a time t on the plane orthogonal to $\mathbf{v}$
l_c	coherence length
R	screening radius of an atom

[16)] The experimental data presented in Fig. 21 are published with the kind consent of I. I. Miroshnichenko. These data were obtained in a joint Soviet-American experiment by the group of I. I. Miroshnichenko, J. J. Murray, G. Kh. Figut, and R. O. Avakyan. The experiment and its results have been described briefly in Ref. 23.

n	density of atoms
ω_p	plasma frequency
L_R	radiation length
$E' = dE/d\omega$	spectral density of radiation of a particle moving along a trajectroy $\mathbf{r}(t)$
$\bar{E}' = d\bar{E}/d\omega$	spectral density of radiation of a particle in matter, averaged over various trajectories
$\mathbf{q}=\mathbf{p}-\mathbf{p}'-\mathbf{k}$	momentum transferred in radiation to an external field
$q_\parallel, q_\perp$	components of the vector $\mathbf{q}$ longitudinal and transverse to $\mathbf{p}$
$\delta^* = \omega m^2/2\varepsilon\varepsilon'$	minimum value of $\mathbf{q}_\parallel$
g	reciprocal lattice vector
$a_i (i=1,2,3)$	lattice constants along the z, y, and z axes
Δ	volume of unit cell
$\overline{u^2}$	mean square amplitude of thermal vibrations of lattice atoms
N_0	number of atoms in the crystal
N_c	number of crystal atoms in a coherence length
$\mathbf{r}_k$	position of the kth atom in the material
$\mathbf{r}_0$	point of entry of the particle into the crystal
$u(\mathbf{r}), u_\mathbf{q}$	potential energy of interaction of an electron with an individual atom of the material and its Fourier component
$U(\mathbf{r}), U(\mathbf{q})$	potential energy of interaction of an electron with the crystal lattice field and its Fourier component
$U_p(x), U_r(\rho)$	continuous potential energy of interaction of an electron with the atoms of an individual crystallographic plane and of an individual atomic string near which an electron is moving
T_c, T_u	periods of oscillation of channeled and superbarrier particles
ψ	angle between $\mathbf{p}$ and the crystallographic z axis.
α	angle between the projection of the vector $\mathbf{p}$ on the (x, y) plane and the y axis
θ	angle between $\mathbf{p}$ and the (y, z) crystallographic plane
θ_c, ψ_c	critical angles of planar and axial channeling
d	distance between crystallographic planes near which an electron is moving
$d\sigma_c, d\sigma_n$	coherent and noncoherent lengths of the radiation
$\varkappa = \gamma\theta_c$	nondipolarity parameter of channeled-particle radiation
$\Theta(\zeta)$	$=\begin{cases}1, & \zeta \geq 0 \text{ (step functions)} \\ 0, & \zeta < 0\end{cases}$
$g(\theta)$	distribution function in angle of particles which have entered a crystal
b	impact parameter of atomic string
ρ	distance to axis of string
$\bar{a}$	average distance between atomic strings
$\varphi(b)$	azimuthal angle of scattering of a particle by an atomic string
$f(\varphi, z)$	distribution function of particles in the angle φ at depth z
$\overline{\vartheta_a^2}(s)$	mean square angle of multiple scattering of particles in a length s in an amorphous medium
$\overline{\vartheta^2}(s)$	mean square angle of multiple scattering of particles by atomic strings of a crystal in a length s

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