Special features of properties of the "Shubnikov phases"

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1. HOMOGENEOUS SUPERCONDUCTORS OF THE SECOND KIND

Depending on their behavior in a magnetic field, all superconductors are divided into two major groups: superconductors of the first and second types. In a sufficiently strong magnetic field type II superconductors enter a special state which is referred to as the mixed state or the Shubnikov phase.[1] These superconductors are characterized by two critical magnetic fields: H_{c1} and H_{c2}. Superconductivity inside a specimen is destroyed in magnetic fields with $H > H_{c2}$. In the magnetic fields $H_{c1} < H < H_{c2}$ the longitudinal magnetic field partially penetrates a cylindrical specimen. The superconducting properties, for example the magnetic moment, fall off with increasing field strength.

In 1950, Ginzburg and Landau examined the superconducting transition as a phase transition of the second kind and introduced a formal order parameter Δ which characterizes this transition. In the case of a phase transition, a change in the free energy near the transation temperature may be expanded into a series in terms of the order parameter[2]

$$F = F_H + \nu \int \left[A|\Delta|^2 + \frac{B}{2}|\Delta|^4 + C\left|\left(\frac{\partial}{\partial \mathbf{r}} - 2ie\mathbf{A}\right)\Delta\right|^2\right] d^3r + \frac{1}{8\pi}\int \left((\mathrm{rot}\,\mathbf{A})^2 - 2\mathbf{H}_0\,\mathrm{rot}\,\mathbf{A}\right) d^3r, \tag{1}$$

where $\mathbf{H}_0$ is the external magnetic field, T_c is the superconducting transition temperature, $\mathbf{A}$ is the vector potential, and $\nu = mp_0/2\pi^2$ is density of states on the Fermi surface. The coefficients of expansion A and B are

$$A = -\frac{T_c - T}{T}, \quad B = \frac{7\zeta(3)}{8\pi^2 T^2}. \tag{2}$$

The coefficient C depends on the electron mean free path in a metal and, in the case of superconductors with a short mean free path

$$C = \frac{\pi D}{8T}, \quad D = \frac{v l_{tr}}{3}. \tag{3}$$

The equations for the order parameter Δ and vector potential $\mathbf{A}$ may be obtained by minimizing Eq. (1) for the free energy with respect to Δ^* and $\mathbf{A}$.

Analysis of Eq. (1) showed that the behavior of a superconductor in a magnetic field is determined by a dimensionless parameter $\varkappa$ which represents the ratio of the depth of penetration of the magnetic field into a superconductor to the correlation length

$$\varkappa^2 = \frac{63\zeta(3)}{2\pi^3 e^2 p^2 v l_{tr}^2}. \tag{4}$$

In the case $\varkappa > 1/\sqrt{2}$, the energy at the boundary between the superconducting state and normal metal in a magnetic field becomes negative. Ginzburg and Landau also obtained an expression for the critical field H_{c2} at which a superconducting nucleus first appears, and found its shape.[2] Nearly all pure metals are superconductors of the first kind with a small value of the Ginzburg-Landau parameter $\varkappa$. However, addition of admixtures increases the depth of penetration of the magnetic field into a superconductor and reduces the correlation length. As a result, $\varkappa$ increases and may attain values ~100 in alloys. The critical magnetic field H_{c2} increases sharply with decreasing electron mean free path. At present, values of 600 kOe have been attained. The high values of the critical magnetic field H_{c2} make type II superconductors important for practical applications.

In 1957, Abrikosov showed that in the Shubnikov phase the magnetic field penetrates a superconductor in the form of quantized flux lines.[3] Each vortex holds one magnetic flux quantum. The vortices are repelled and form a triangular lattice. However, the difference between quadratic and triangular lattices is only 2%. This condition plays an important role in the formation of the vortex structure in superconductors with defects in the crystal lattice.

The first successful experimental observation of a vortex lattice by direct methods occurred as late as 1968. To observe the vortex lattice directly Traüble and Essamann used the decorative method consisting of deposition of small iron particles on a superconductor in a magnetic field.[4] Their methodology enabled one to observe a very clear picture of vortex distribution. Direct count of the number of vortices showed that each vortex carries a single quantum of the magnetic flux.

The vortices always repel each other near the transition temperature. However, at low temperatures in a narrow range of values of $\varkappa$ close to $1/\sqrt{2}$, there exist regions of attraction between vortices.[5] This interesting phenomenon occurs as a result of the nonlocal coupling of the current density with the vector potential. A weak magnetic field is expelled from such a superconductor, although rescreening occurs in it: at a certain distance from the superconductor surface the magnetic field is directed in a direction opposite to the external magnetic field.

The vortex lattice under deformation behaves like an elastic medium. Its properties are characterized by three elastic moduli: shear modulus C_{66}, bending modulus C_{44} and multiaxial compression modulus $C_L = C_{11} - C_{66}$. The theory of elasticity applies in the normal elastic media if the size of a deformed area is large in comparison with the lattice period. For a vortex lattice there are physical reasons for which the C_{11} and C_{44} moduli exhibit a strong spatial dispersion at comparatively small wave vectors.[6] This is related

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to the fact that the longwave displacements of the lattice drag the magnetic field along with them and the elastic energy is determined by the change in the energy of the magnetic field. However, the latter cannot change over distances smaller than the effective depth of penetration δ_{eff}. Therefore, in the case of deformations of the vortex lattice with wave vectors K greater than δ_{eff}^{-1}, the magnetic field separates from the vortex lattice. In this case the elastic energy does not increase with an increase in K. As the critical field H_{c2} is approached, the effective penetration depth δ_{eff} increases as $(H_{c2} - H_0)^{-1/2}$ (H_0 is an external magnetic field). The nonlocal effects are, therefore, especially strong near the critical field H_{c2}. For superconductors with a small electron mean free path the elastic moduli near the transition temperature T_c and critical field H_{c2} are given by[6,7]

$$C_{44} = \frac{B^2}{4\pi}\frac{k_h^2}{K^2 + k_h^2} + \frac{B(H_0 - B)}{4\pi}, \quad C_{11} - C_{66} = \frac{B^2 k_h^2}{4\pi}\left(\frac{1}{K^2 + k_h^2} - \frac{1}{K^2 + k_\psi^2}\right). \tag{5}$$

$$k_\psi^2 = 4e(H_{c2} - B), \quad k_h^2 = \frac{32\pi^4 \nu D e^2}{7\zeta(3)\beta_A}\,\frac{(T_c - T)\,[1 - (H_0/H_{c2})]}{1 - (1/2\varkappa^2)}.$$

The coefficient β_A depends on the form of the vortex lattice and, in the case of a triangular lattice, $\beta_A = 1.1596$. The spatial dispersion of the elastic modulus C_{36} is small.

2. EFFECT OF INHOMOGENEITIES ON A VORTEX LATTICE

In a conventional solid point defects do not disturb the long range order. This is not the case for a vortex lattice; the long range order disappears with even weak inhomogeneities.[8] This is associated with the fact that the energy of interaction between defects and lattice in a solid is independent of a homogeneous displacement and is determined by deformation, since the defects in a solid shift together with the lattice. The vortex lattice does not drag along with itself defects of the crystalline structure. Therefore, a displacement of vortices caused by such defects slowly decreases with distance from a defect. Displacements caused by various defects accumulate and lead to disappearance of long range order in the vortex lattice. The size of a region with short range order is determined by the elastic moduli and by the force of interaction $\mathbf{f}_{pin}$ between the vortex lattice and a defect.

The flow of current in ideal type II superconductors causes a motion of the vortex lattice as a whole and is accompanied by dissipation of energy. Defects of different kinds—ever present in a superconductor—lead to the pinning of the vortex lattice. As a result of this, nondissipative current of a finite density may flow in a superconductor. The maximum possible value of current density at which the vortex lattice does not move as a whole is called the critical density. The value of the latter is substantially determined by the force of interaction between a single defect and the vortex lattice. Moreover, two basically different cases are possible: strong and weak pinning. In the case of strong pinning, the critical current density j_c is proportional to the defect concentration n. However, occurrence of strong pinning necessarily requires the fulfillment of a rather rigid criterion[9]: the magnitude of displacement of the vortex lattice at a defect site should be of the order of the range of the pinning forces. The fulfillment of this criterion makes possible the formation of metastable states on a defect. The variation of the free energy in the course of transition from one metastable state to another thus determines the value of the critical current.[10] This criterion is very rigid and cannot be fulfilled for small size defects.[7,11]

However, in addition to metastable states with smooth deformation a sharp change in the condition of the vortex lattice is possible at a defect site of the structural transition type. The resulting state cannot be described by the theory of elasticity. There is a physical reason for the relative facility with which this transition occurs: the vortex lattice is very loose and the energy of the triangular and quadratic lattices near the critical field H_{c2} differs by only 2%. Such a change in the state occurs even at weak defects with a small range.[12] In contrast with the Labusch deformation instability, for which the interaction sign is unimportant, a change in the lattice structure near a defect occurs at a numerically weak interaction only in a case where the vortices are repelled by the defect. Closer to H_{c2}, $|\Delta|^2$ falls off as $1 - (H_0/H_{c2})$. Clearly, a condition for the occurrence of a structural transition in the case of repulsion between a vortex and defect may be fulfilled near H_{c2} even for small-size weak defects.[12] The same sensitivity to the sign of interaction is preserved also for large-scale defects.[10] The fact that the structural transition becomes easier closer to H_{c2} permits one to explain the so-called "peak effect," a sharp rise in the critical current as H_{c2} is approached. Moreover, along the ascending portion of the dependence of j_c on $1 - (H_0/H_{c2})$ defect size distribution plays a significant role, and this function does not exhibit a universal nature. Progressively closer to H_{c2} the critical current density drops off and its dependence on the parameter $1 - (H_0/H_{c2})$ follows a universal law which is independent of the force of interaction between vortices and a defect.[12]
In the case of defects in the form of small size pores (a vortex is attracted to a defect) a very similar situation follows[13]; however, a condition for the formation of metastable states becomes, in this case, considerably more rigid.[12,13]

If metastable states fail to form on a single defect, an average pinning force appears only as a result of collective effects when a large volume V_c enters a metastable state inside which short range order is preserved. In the case where the dispersion of elastic moduli is negligible, the critical current is[7]

$$Bj_c \sim \frac{n^2 f_{pin}^4}{a^3 C_{44} C_{66}^2}, \tag{6}$$

where a is the vortex lattice period, n is defect concentration, f_{pin} is the force of interaction between a defect and the vortex lattice.

In this case, the critical current density is low.

3. NONLINEAR EFFECTS ASSOCIATED WITH VORTEX MOTION

No simple generalization of the Ginzburg-Landau equation to the nonstationary case exists, even near the transition temperature. The description of the effect of a variable electromagnetic field on superconductors requires the use of a more complex system of equations for the Green's function.[14,15] In the case of slowly-varying fields, these equations may be reduced in a number of instances to kinetic equations for two distribution functions and an equation for the order parameter Δ. The energy relaxation in the kinetic equations occurs due to electron-electron or electron-phonon collisions. The energy relaxation times are long, τ_ε of the order of ε_F/T^2 or θ_D^2/T^3, where ε_F is the Fermi surface energy and θ_D is the Debye temperature. In view of this, nonlinear effects associated with electric fields rapidly become significant in superconductors. They are especially noticeable near the transition temperature.

The flow of current in ideal type II superconductors in a magnetic field is accompanied by motion of the vortex structure as a whole. Moreover, in a weak electric field the vortex velocity and current density are proportional to the electric field

$$\mathbf{j} = \sigma \mathbf{E}. \tag{7}$$

The conductivity σ in Eq. (7) is a function of temperature, magnetic field strength and electron mean free path. In a normal metal, σ is determined by the total number of electrons and is independent of the specifics of their energy distribution. The dependence of conductivity on the electric field intensity in a normal metal is, therefore, very weak. The same is not true in superconductors. Significant changes occur in the conductivity and order parameter Δ near the transition temperature in a weak electric field at a time when the perturbation distribution function varies little. This is due to the fact that the nonequilibrium correction to the distribution function for the order parameter Δ is of the same magnitude as the terms proportional to $T_c - T$.

The physical picture of a vortex moving in a magnetic field $B \ll H_{c2}$ is as follows: normal excitations inside a vortex are activated and scatter into a region of energies greater than Δ. The generation of new excitations in the vortex center is insignificant. As a result, the effective pressure of excitations on vortex walls decreases and the vortex contracts. The conductivity in this case is[16]

$$\frac{\sigma}{\sigma_N} = \frac{4}{\sqrt{1-(T/T_c)}} \frac{H_{c2}}{B} \frac{1}{1+(E/E^*)^2}, \quad B \ll H_{c2}, \tag{8}$$

where

$$(E^*)^2 = \frac{DB^2 \sqrt{1-(T/T_c)}}{\tau_\varepsilon}.$$

Measurements carried out on films,[17] are in a good quantitative agreement with Eq. (8).

As the magnetic field grows, both cooling of electrons entrained in a vortex and heating of electrons with energies greater than Δ become significant. The heating of electrons and the resultant decrease in T_c and H_{c2} are the most significant effects in a magnetic field close to H_{c2}.[16]

The nonequilibrium electron energy distribution which occurs due to vortex motion leads to emission of nonequilibrium phonons. The most interesting phenomena occur at a low temperature. The electric field, rather than temperature, determines the excitation distribution function in relatively weak fields. The excitations remain within the vortex core as long as the effective temperature is low compared to the order parameter Δ, and a broad spectrum of phonons is generated with an energy comparable to the energy of nonequilibrium excitations. In a sufficiently strong electric field, electrons, having heated up in the vortex region, attain energies $\varepsilon = \Delta$. Subsequently, excitations leave the vortex core and, if the vortex density is low, further heating strongly abates. Excitations become accumulated with an energy ε near Δ, and as they recombine phonons are emitted with a frequency near 2Δ. In a weak magnetic field $B \ll H_{c2}$ the width of the phonon energy distribution is proportional to the small parameter $(B/H_{c2})^{1/3}$.[15]

Type II superconductors discovered by Shubnikov have found broad application in science and technology. Numerous experimental and theoretical papers are devoted to investigation of the properties of superconductors in the mixed state, i.e., the Shubnikov phase. The structure of the mixed state in ideal superconductors of the second kind has been fully investigated. The dependence of conductivity on temperature and magnetic field amplitude has been studied. As a rule, experimental data are in a good agreement with theory.

There is a large body of experimental work that deals with the critical current and current-voltage characteristics of inhomogeneous superconductors. However, these data are poorly systematized as a result of the lack of a comprehensive theory of pinning. To date, this theory is sufficiently well developed for those cases in which the lattice deformation may be assumed elastic. However, in many other cases the theory of elasticity is, clearly, inapplicable. The investigation of the structure of a mixed state in this case is in fact only just beginning.

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