Effects of double passage of waves in randomly inhomogeneous media

Yu. A. Kravtsov and A. I. Saichev

P. N. Lebedev Physical Institute, Academy of Sciences of the USSR and N. I. Lobachevskiĭ State University, Gor'kiĭ

Usp. Fiz. Nauk. 137, 501–527 (July 1982)

In this review, the effects stemming from double passage of reflected and backscattered waves through the same inhomogeneities of the medium, are discussed. The enhancement of phase and intensity fluctuations and of the average intensity and other effects, characteristic of double passage, are described in detail. Multichannel coherent effects, arising due to mutual coherence of different waves passing in opposite directions along identical channels, are investigated. The experimental work on double passage effects is reviewed. Possible practical applications of these effects are analyzed.

PACS numbers: 03.40.Kf

CONTENTS

1. INTRODUCTION

It is well known that when acoustic, electromagnetic and other waves propagate in real media, a variety of fluctuation effects, owing to the presence of random inhomogeneities of the medium, arise.[1,2] Recently, it was determined that qualitatively new fluctuation effects, originating from the double passage of waves through the same inhomogeneities, arise with backscattering.

Consider the simplest scheme of a radar experiment, leading to double passage of waves through a medium with large scale random inhomogeneities (Fig. 1). At first glance, it may appear that the fluctuation properties of the wave, reaching the receiver in a radar experiment (Fig. 1), are analogous to the properties of the wave passing twice the distance $2L$ along a straight path (Fig. 2), since in both cases the wave passes over a path with identical length $2L$ in the randomly inhomogeneous medium. In reality, however, the paths differ greatly. The point is that in the radar experiment (Fig. 1), *the reflected wave passes through the same inhomogeneities of the medium as the incident wave,* while along the direct path (Fig. 2), the wave propagates through different inhomogeneities. It is the double passage of the wave through the random inhomogeneities of the medium that gives rise to the specific fluctuation effects.

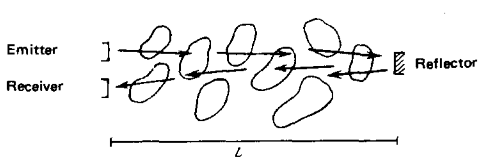

FIG. 1.

Some anomalies in the behavior of the reflected wave, owing to the correlation of inhomogeneities on the forward and reflected wave paths, were first pointed out by N.G. Denisov.[3] Then, N.G. Denisov and L.N. Erukhimov calculated[4a] the variance of the phase fluctuations of a normally incident wave reflected from the ionosphere (see also Ref. 4b). From these calculations follows the effect of doubling of the phase variance of a normally reflected wave compared to the phase variance of a wave reflected with oblique incidence. Another effect of double passage is "pure" enhancement of backscattering, i.e., enhancement of the average intensity of the backward reflected waves in large-scale randomly inhomogeneous medium, was discovered by Vinogradov *et al.*,[1),5] although the existence of specific interference effects accompanying backscattering was indicated by Watson[7] and De Wolf,[8] while a diagrammatic analysis of

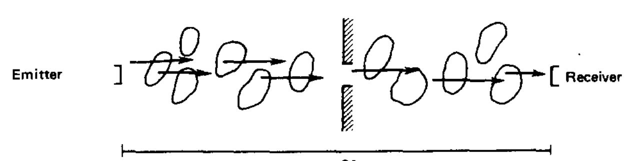

FIG. 2.

[1)] With the permission of the authors of Ref. 5, information on the enhancement of backscattering was also included in Ref. 6, which was published somewhat earlier than Ref. 5.

 0038-5670/82/070494-15$01.80

the perturbation series for the backscattered field was carried out by Yu.N. Barabanenkov.[9,10] Later, it was discovered that the backscattered wave has specific coherent and other, new (compared to the wave propagating without reflection) properties. This paper reviews the diverse fluctuation effects of double passage of waves through the same inhomogeneities.

2. DOUBLING OF THE PHASE VARIANCE ON BACKWARD REFLECTION OF WAVES IN A MEDIUM WITH LARGE-SCALE RANDOM INHOMOGENEITIES

a) Phase fluctuations on reflection from a mirror. Geometric optics approximation

The effect of doubling of the phase variance of the reflected wave was first discovered by Denisov *et al.*[4] for reflection from an ionospheric layer with a smooth dielectric permittivity profile. It is most convenient to explain the nature of this effect and its characteristic properties for a simple model of reflection of a wave from a mirror, placed in a randomly inhomogeneous medium.

Let a plane wave with wavelength $\lambda = 2\pi/k$

$$u_{\text{inc}}(\boldsymbol{\rho}, x) = \exp[ik(x - \rho_1\theta)]\ (\theta \ll 1)$$

be obliquely incident on a randomly inhomogeneous layer $0 < x < L$, bounded by an ideal mirror (Fig. 3). We shall assume that the inhomogeneities in the dielectric permittivity of the medium $\tilde{\varepsilon} = \varepsilon - \bar{\varepsilon}$ are small ($|\tilde{\varepsilon}| \ll 1$) and large-scale ($l_\varepsilon \gg \lambda$), while the path length L is such that the conditions for applicability of geometric optics are valid, namely, $L\lambda \ll l_\varepsilon^2$ and smallness of the fluctuations of the average intensity of the wave $\langle \chi^2 \rangle \sim \sigma_\varepsilon^2 L^3/l_\varepsilon^3 \ll 1$, which permits neglecting the bending of the rays of the incident and reflected waves. Since here we are interested only in the phase fluctuations of the reflected wave, we shall neglect the fluctuations in its average intensity and we shall write the reflected wave at the starting plane $x = 0$ in the form

$$u_{\text{ref}}(\boldsymbol{\rho}, 0) = \exp[ik(2L + \rho_1\theta + \varphi(\boldsymbol{\rho}, L))],$$

where, $\psi = k\varphi$ is the phase fluctuation and $\varphi(\boldsymbol{\rho}, L)$ is the random part of the eikonal of the reflected wave. In this case, the fluctuations of the eikonal $\varphi(\boldsymbol{\rho}, L)$ equal

$$\varphi(\boldsymbol{\rho}, L) = \frac{1}{2}\int_0^L \tilde{\varepsilon}(\rho_1 - 2L\theta + \theta x, \rho_2, x)\,dx + \frac{1}{2}\int_0^L \tilde{\varepsilon}(\rho_1 - \theta x, \rho_2, x)\,dx. \quad (2.1)$$

The first term in (2.1) corresponds to fluctuations of the eikonal of the incident wave on the section of the path from the beginning of the layer ($x = 0$) to the mirror ($x = L$), while the second term corresponds to fluctuations on the return path. Standard calculations of the variance of the eikonal of the reflected wave give (see Ref. 1)

$$\sigma_\varphi^2(\theta, L) = \frac{1}{2}\left[A(0)L + \int_0^L A(2\theta x)\,dx\right], \quad (2.2)$$

where

$$A(\mathbf{s}) = \int_{-\infty}^{\infty} \langle \tilde{\varepsilon}(\boldsymbol{\rho}, x)\tilde{\varepsilon}(\boldsymbol{\rho} + \mathbf{s}, x + \tau)\rangle\,d\tau. \quad (2.3)$$

We shall compare the scale of the function $A(\mathbf{s})$ to the scale of the inhomogeneities l_ε for single-scale inhomogeneities of the medium.

If the angle of incidence θ exceeds the "correlation angle" $\theta^* \sim l_\varepsilon/2L$, then the forward and return rays pass mostly through different inhomogeneities of the medium. This permits neglecting the second term in (2.2) compared to the first term, so that

$$\sigma_\varphi^2(\theta \gg \theta^*, L) \approx \frac{1}{2}A(0)L. \quad (2.4)$$

In this case, the variance of the eikonal of the reflected wave equals the variance of the eikonal of the wave passing without reflection through a randomly inhomogenous layer with thickness $2L$. If, on the other hand, the angle of incidence equals zero, then, according to (2.2),

$$\sigma_\varphi^2(0, L) = A(0)L. \quad (2.5)$$

Thus, the variance of the eikonal for backscattering is twice as large as the variance for a direct path with twice the length $2L$:

$$\sigma_{\varphi\,\text{return}}^2(0, L) = 2\sigma_{\varphi\,\text{direct}}^2(0, 2L). \quad (2.6)$$

This doubling of the variance of the phase with backscattering, evidently, is due to the double passage of the wave through the same inhomogeneities. We shall call the quantity

$$N_\varphi(\theta) = \frac{\sigma_\varphi^2(\theta, L)}{\sigma_\varphi^2(\theta \gg \theta^*, L)} \quad (2.7)$$

the enhancement factor of the phase variance. According to (2.4) and (2.5), for normal incidence on a mirror, i.e., for $\theta = 0$ we have

$$N_\varphi(0) = 2, \quad (2.8)$$

while for $\theta \gg \theta^*$, the enhancement factor equals unity. The smooth transition from the value $N_\varphi = 2$ to unity is shown in Fig. 4.

b) Phase fluctuations taking diffraction effects into account

If the condition for applicability of geometric optics $L\lambda \ll l_\varepsilon^2$ is not satisfied, then in calculating the eikonal of the reflected wave it is necessary to take diffraction effects into account. Such a calculation was carried out in the approximation of the method of smooth perturba-

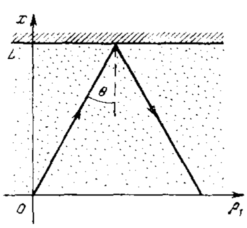

FIG. 3.

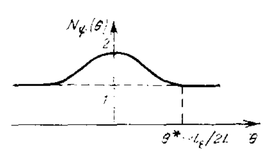

FIG. 4.

tions (MSP) in Ref. 11 for a gaussian beam normally incident on a randomly inhomogeneous medium and reflected by an ideal mirror in the plane $x = L$. According to Ref. 11, the enhancement of phase fluctuations of the reflected wave remains also when diffraction is included. In particular, it follows from Ref. 11 that the enhancement factor of the phase variance equals

$$N_\varphi = 2\,\frac{\int\limits_0^\infty \varkappa\Phi_\varepsilon(\varkappa)\cos^2\left(\frac{\varkappa^2 L}{2k}\right)\left[1+\left(\sin\left(\frac{\varkappa^2 L}{k}\right)\Big/\frac{\varkappa^2 L}{k}\right)\right]d\varkappa}{\int\limits_0^\infty \varkappa\Phi_\varepsilon(\varkappa)\left[1+\left(\sin\left(\frac{\varkappa^2 L}{k}\right)\Big/\frac{\varkappa^2 L}{k}\right)\right]d\varkappa}, \quad (2.9)$$

where $\Phi_\varepsilon(\varkappa)$ is the spectral density of fluctuations in the dielectric permittivity. For a turbulent medium with a von Karman spectrum

$$\Phi_\varepsilon(\varkappa) = 0.033 C_\varepsilon^2 (\varkappa^2 + \varkappa_0^2)^{-11/6} \exp\left(-\frac{\varkappa^2}{\varkappa_m^2}\right),$$

where $\varkappa_m = 5.92/l_0$, $\varkappa_0 = 2\pi/L_0$ (l_0 is the inner and L_0 is the outer scale of turbulence), it follows from (2.9) that for extended paths ($\varkappa_0^2 L/k \gg 1$), the variance of the fluctuations of the eikonal is twice the variance of the eikonal for oblique incidence ($N_\varphi \approx 2$). Together with this, due to the correlation of phases of the incident and reflected waves, there is an increase in the correlation length of the phase fluctuations of the reflected wave compared to the same quantity in the incident wave.

One more result, obtained in the MSP approximation,[12] concerns the enhancement of the fluctuations in the average intensity and phase of a beam, whose angle of reflection from an ideal mirror is arbitrary. According to Ref. 12, the enhancement of the fluctuations in the average intensity disappears when the angle of incidence exceeds the value $2\sqrt{\lambda/L}$. On the other hand, the enhancement of phase fluctuations remains appreciable ($N_\varphi \sim 2$), even if the angle of incidence equals the angular size of the external scale of turbulence.

Doubling of the phase variance with double passage is characteristic also for more complex situations: with reflection of a spherical wave from a mirror, scattering by a point reflector (see below, Sec. 3) and by a cylinder,[13] and in many other cases.

c) Fluctuations in arrival angles and arrival times of signals

It is well known that fluctuations in the phase of a quasimonochromatic pulse are closely related to fluctuations in the time δt for the reflected pulse to return in a randomly inhomogeneous medium. In nondispersive media, we have $\delta t = \varphi/c$. Accordingly, the variance of the fluctuations in the arrival time of the sounding pulse $\sigma_t^2(\theta, L)$ is related to the variance of the eikonal (2.2) by the simple relation

$$\sigma_t^2(\theta, L) = \frac{1}{c^2}\sigma_\varphi^2(\theta, L).$$

Thus, doubling of the phase fluctuations with double-passage leads to doubling of the variance of the arrival time of the signal, which in its turn can increase the error in determining the distance to the objects being located.

Analogous results are also valid for the arrival

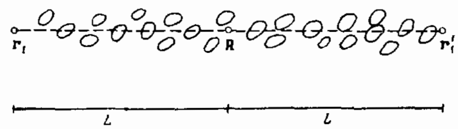

FIG. 5.

angles of a wave reflected in a randomly inhomogeneous medium.

3. ENHANCEMENT OF BACKSCATTERING

a) "Pure" enhancement of backscattering: point scatterer and point emitter

Propagation of a scalar monochromatic wave in a medium with dielectric permittivity $\varepsilon = 1 + \tilde{\varepsilon}$ ($\tilde{\varepsilon}$ is the fluctuating part of the dielectric permittivity) is described by the Helmholtz equation[2)]

$$\Delta u + k^2(1 + \tilde{\varepsilon})u = 0. \quad (3.1)$$

We shall assume that the random inhomogeneities are weak ($|\tilde{\varepsilon}| \ll 1$), statistically isotropic, and large scale ($l_\varepsilon \gg \lambda$). In addition, we shall temporarily assume that the inhomogeneities do not change in time (in what follows, we shall point out the permissible magnitude of the derivative $\partial\varepsilon/\partial t$; see subsection g).

Let a point isotropic emitter be located at the point $\mathbf{r} = \mathbf{r}_1$. The emitter can be taken into account by introducing on the right side of Eq. (3.1) the function $\delta(\mathbf{r}-\mathbf{r}_1)$. We shall denote the Green's function of Eq. (3.1) by $G(\mathbf{r}_1, \mathbf{r})$. We note the fundamental, for effects of double passage, reciprocity property of waves

$$G(\mathbf{r}_1, \mathbf{r}) = G(\mathbf{r}, \mathbf{r}_1). \quad (3.2)$$

Let us place at the point $\mathbf{r} = \mathbf{R}$ an isotropic point scatterer, which creates the scattered field

$$u_s(\mathbf{r}) = fG(\mathbf{r}_1, \mathbf{R})\,G(\mathbf{R}, \mathbf{r}); \quad (3.3)$$

here, f is the scattering amplitude. We shall be interested in the scattered field at the conjugate points $\mathbf{r} = \mathbf{r}_1$ and $\mathbf{r} = \mathbf{r}_1'$, separated by identical distances from the scatterer: $|\mathbf{r}_1 - \mathbf{R}| = |\mathbf{R} - \mathbf{r}_1'| \approx L$ (Fig. 5). According to (3.3) and (3.2), these fields are given by

$$u_s(\mathbf{r}_1) = fG^2(\mathbf{r}_1, \mathbf{R}), \quad u_s(\mathbf{r}_1') = fG(\mathbf{r}_1, \mathbf{R})\,G(\mathbf{R}, \mathbf{r}_1'). \quad (3.4)$$

We shall further assume that scattering by a single inhomogeneity (volume l_ε^3) is small, $l_\varepsilon \ll L$, while the direction of propagation of the multiply scattered wave along the path of length L remains practically unchanged. Then, the fluctuations in $G(\mathbf{r}_1, \mathbf{R})$ are caused by inhomogeneities of the medium in a narrow region around the straight line, connecting the emitter and scatterer $(\mathbf{r}_1, \mathbf{R})$, while fluctuations in $G(\mathbf{R}, \mathbf{r}_1')$ arise due to statistically independent inhomogeneities along the path $(\mathbf{R}, \mathbf{r}_1')$. For this reason, the functions $G(\mathbf{r}_1, \mathbf{R}')$ and $G(\mathbf{R}, \mathbf{r}_1')$ are statistically independent. The average intensities of the scattered wave at the conjuage points,

[2)] We note that the scalar approximation in media with large-scale inhomogeneities is justified also for electromagnetic waves, whose polarization is practically unchanged along the propagation path.

corresponding to them, equal:

$$\langle I_s(\mathbf{r}_1)\rangle = \sigma\langle I^2(\mathbf{r}_1, \mathbf{R})\rangle, \quad \langle I_s(\mathbf{r}_1')\rangle = \sigma\langle I(\mathbf{r}_1, \mathbf{R})\rangle\langle I(\mathbf{R}, \mathbf{r}_1')\rangle,$$
$$\sigma = |f|^2, \quad I_s(\mathbf{r}) = |u_s(\mathbf{r})|^2, \quad I(\mathbf{r}, \mathbf{R}) = |G(\mathbf{r}, \mathbf{R})|^2. \tag{3.5}$$

The enhancement of backscattering "in pure form"[5] follows from the first equality in (3.5) and from a trivial assertion of the theory of probability, namely, that the mean-square of a random quantity is always greater than the square of its average:

$$\langle I^2(\mathbf{r}_1, \mathbf{R})\rangle > (\langle I(\mathbf{r}_1, \mathbf{R})\rangle)^2. \tag{3.6}$$

Keeping in mind the undirected nature of the radiation and of the scattering, for a statistically isotropic and large-scale medium, from the law of conservation of energy flux, we have

$$\langle I(\mathbf{r}_1, \mathbf{R})\rangle = \langle I(\mathbf{R}, \mathbf{r}_1')\rangle = I_0(L),$$

where $I_0(|\mathbf{r}_1-\mathbf{r}|) = |G_0(\mathbf{r}_1-\mathbf{r})|^2$ is the intensity of the emitter field in a vacuum ($\tilde{\varepsilon} \equiv 0$). Substituting the last equality into the first relation in (3.5) and taking (3.6) into account, we arrive at a quantitative expression for the enhancement of backscattering:

$$\langle I_s(\mathbf{r}_1)\rangle = N I_{s0}(\mathbf{r}_1), \tag{3.7}$$

where $I_{s0}(\mathbf{r}_1) = \sigma I_0(L)$ is the intensity of the scattered wave in a vacuum, while

$$N = \frac{\langle I^2(\mathbf{r}_1, \mathbf{R})\rangle}{I_0^2(|\mathbf{r}_1-\mathbf{R}|)} > 1 \tag{3.8}$$

is the enhancement factor for backscattering in a randomly inhomogeneous medium. This factor exceeds unity all the more, the greater the fluctuations in the intensity of the wave incident on the scatterer. A clear geometric interpretation of the enhancement of the average backscattering intensity can be given by making use of the reciprocity theorem, which is valid both for the fields themselves and for geometric optics rays[15]: waves from the emitter to the scatterer and back again propagate along the same rays (Fig. 6). It is the correlation of fluctuations in the intensity of the incident and scattered waves, passing through the same inhomogeneities, that is responsible for the enhancement of the average intensity of the wave at the position of the emitter.

b) Spatial redistribution of the backscattering intensity

For the average intensity of the scattered field at an arbitrary point r, in analogy to (3.7), we have

$$\langle I_s(\mathbf{r})\rangle = N(\mathbf{r}_1, \mathbf{r}, \mathbf{R})\, I_{s0}(r), \tag{3.9}$$

where the quantity

$$N(\mathbf{r}_1, \mathbf{r}, \mathbf{R}) = \frac{\langle I(\mathbf{R}, \mathbf{r}_1)\, I(\mathbf{R}, \mathbf{r})\rangle}{I_0(|\mathbf{R}-\mathbf{r}_1|)\, I_0(|\mathbf{R}-\mathbf{r}|)} \tag{3.10}$$

now describes not only the enhancement, but also the spatial redistribution of the average backscattering intensity. Let the point of observation **r** be located on a sphere with center R and radius $|\mathbf{r}_1-\mathbf{R}| = L$. In this

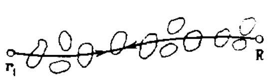

FIG. 6.

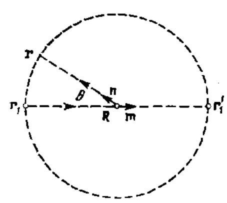

FIG. 7.

case, N becomes a function of the angle $\theta = |\mathbf{r}_1-\mathbf{r}|/L$: $N = N(\theta, L)$ (Fig. 7). From the law of conservation of the energy flux of the scattered wave, it follows that

$$\oint N(\mathbf{r}_1, \mathbf{r}, \mathbf{R})\, ds = 4\pi L^2, \tag{3.11}$$

where the integration is carried out over the sphere described above. Since for strict backscattering, the average intensity is enhanced $N(0, L) > 1$, then (3.11) is satisfied only under the condition that the enhancement at $\theta = 0$ is accompanied by a decrease in the average backscattering intensity at angles close to $\theta = 0$. As a result, the scattering indicatrix $N(\theta, L)$ assumes the characteristic form shown in Fig. 8. For sufficiently large θ, when the wave incident on the scatterer and scattered at an angle θ passes through different inhomogeneities, the intensities $I(\mathbf{R}, \mathbf{r}_1)$ and $I(\mathbf{R}, \mathbf{r})$ are statistically mutually independent and $N(\theta, L) = 1$.

c) Case of weak fluctuations

If the intensity fluctuations of the wave incident on the scatterer are sufficiently small, then it is possible to use the MSP approximation (see, for example, Ref. 16). In this approximation,

$$N(\theta, L) = 1 + 4B_\chi(\theta L, L), \tag{3.12}$$

where $B_\chi(\rho, L) = \langle\chi(\mathbf{r}_1, L)\chi(\mathbf{r}, L)\rangle$ is the correlation function for fluctuations in the average intensity of the spherical wave and $\rho = |\mathbf{r}-\mathbf{r}_1|$. In a statistically isotropic medium,

$$B_\chi(\rho, L) = 2\pi \int_0^\infty F_\chi(\varkappa, L)\, J_0(\varkappa\rho)\, d\varkappa, \tag{3.13}$$

where

$$F_\chi(\varkappa, L) = \frac{k^2\pi}{2} \int_0^L \Phi_\varepsilon\left(\varkappa \frac{L}{x}\right)\left(\frac{L}{x}\right)^2 \sin\left[\frac{\varkappa^2 L}{2kx}(L-x)\right] dx. \tag{3.14}$$

In the case of single-scale inhomogeneities of the medium, the correlation in the fluctuations of the level disappears for $\rho_N \sim l_\varepsilon$. For this reason, the redistribution of the average intensity of backscattering occurs in a cone of angles $\theta_N \sim l_\varepsilon/L$. In a turbulent medium (outer scale L_0, inner scale l_0) with $L_0 > \sqrt{\lambda L} > l_0$, the correla-

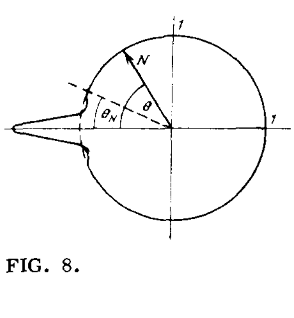

FIG. 8.

tion length is determined by the Fresnel scale $\rho_N \sim \sqrt{\lambda L}$. In this case, $\theta_N \sim \sqrt{\lambda/L}$.

We shall also examine the problem of fluctuations in the average intensity $\chi_s(\mathbf{r})$ and phase $\psi_s(\mathbf{r})$ of the scattered wave. Their variances in the MSP method equal:

$$\sigma_\chi^2(\theta, L) = 2B_\chi(0, L) + 2B_\chi(\theta L, L), \tag{3.15}$$

$$\sigma_\psi^2(\theta, L) = 2B_\psi(0, L) + 2B_\psi(\theta L, L); \tag{3.16}$$

here, $2B_\chi(0,L)$ and $2B_\psi(0,L)$ equal the variances of the average intensity and phase of the scattered wave at the conjugate point $\mathbf{r}_1'(\theta=\pi)$, while the terms $2B_\chi(\theta L, L)$ and $2B_\psi(\theta L, L)$ are related to the double passage of the wave through the same inhomogeneities. For strict backscattering ($\theta = 0$), both the variances of the phase and of the average intensity are doubled compared to the variances at the conjugate point:

$$\sigma_\chi^2(0, L) = 4B_\chi(0, L), \quad \sigma_\psi^2(0, L) = 4B_\psi(0, L). \tag{3.17}$$

We note that it is the increase in the fluctuations of the average intensity in the backward direction that leads to the increase in the average intensity: the return wave is both focused ($\chi > 0$) and defocused ($\chi < 0$), but on the whole focusing dominates and, for this reason, $\langle I_s \rangle > I_{s0}$.

d) Case of saturated fluctuations

The main physical characteristic of a wave in a turbulent medium in the saturation regime is the transverse coherence radius $\rho_c(L)$ of a spherical wave, traversing a path of length L in a turbulent medium. It is determined from the equation[1]:

$$D_\psi(\rho_c, L) = 1, \quad D_\psi(\rho, L) = \frac{k^2}{4} \int_0^L D\left(\frac{\rho x}{L}\right) dx, \tag{3.18}$$

where

$$D(s) = A(0) - A(s), \tag{3.19}$$

while $A(s)$ is given by expression (2.3). The condition of saturability of the intensity fluctuations of the wave, traversing a path L, has the form $\gamma = L/k\rho_c^2 \gg 1$ (Ref. 17) and has a simple interpretation. The quantity γ equals the ratio of the mean-square transverse displacement of the ray $\sigma_\rho(L) \sim L/k\rho_c$ to the coherence radius ρ_c: $\gamma \sim \sigma_\rho/\rho_c$.[18] In other words, saturation of fluctuations occurs under conditions when the lateral displacement of the rays exceeds the coherence radius.

In the saturation region, the statistical properties of the incident wave field are asymptotically Gaussian.[17,19,20] This means that the probability density of the normalized intensity $J = I(\mathbf{r}, \mathbf{R})I_0(|\mathbf{r}-\mathbf{R}|)$ of the saturated ($\gamma \gg 1$) spherical wave asymptotically approaches

$$W_\infty(J) = e^{-J} \qquad (J > 0). \tag{3.20}$$

Its moments equal

$$\langle J^m \rangle = m!. \tag{3.21}$$

In using the distribution (3.20), it is necessary, however, to keep in mind the fact that the asymptotic approach of $W(J; L)$ to (3.20) is not uniform and becomes worse with increasing J. Physically, this is due to the presence of caustics and spikes in the intensity related to them, leading for large J to a slower decrease in the probability density of the intensity than predicted by (3.20). In the final analysis, the higher the moment, i.e., the greater m in (3.21), the more its true value differs for fixed γ from the value (3.21) and the greater γ must be in order to use Eq. (3.21).[17,19,20]

It follows from (3.21) and (3.8) that if the scatterer is located in a region of saturated fluctuations of the intensity of the incident wave, then

$$N_{\text{sat}} = 2. \tag{3.22}$$

We note that in the region of strong fluctuations (where $\gamma \sim 1$ and caustics, which are responsible for the strong fluctuations,[14] begin to appear), the enhancement factor N may turn out to be somewhat greater than 2.

e) Increase of the higher-order moments of the intensity on backscattering

The enhancement of the average intensity is accompanied by an enhancement of the higher moments of the intensity. If the fluctuations are weak, then in order to calculate the moments $\langle I_s^m \rangle$, it is possible to use MSP. In this approximation, $I = I_0 \exp(2\chi)$, where $\chi = \bar{\chi} + \tilde{\chi}$ is the average magnitude of the amplitude and, in addition, $\langle \tilde{\chi}^2 \rangle \equiv \sigma_\chi^2 = -\bar{\chi}$. Then, for the amplification factor of the m-th moment of the intensity $N^{(m)} = \langle I_s^m(\mathbf{r}_1) \rangle \times [I_{s0}^m(\mathbf{r}_1)]^{-1}$, we obtain

$$N^{(m)} = \exp[4(2m^2 - m)\sigma_\chi^2]. \tag{3.23}$$

From here follows the rapid increase in $N^{(m)}$ with increasing number m. The relative fluctuations $\Delta I_{m-s}/I_{s0}$, according to (3.23), equal

$$\frac{\sqrt{\langle I_s^2 \rangle - \langle I_s \rangle^2}}{I_{s0}} = \sqrt{\exp(24\sigma_\chi^2) - \exp(4\sigma_\chi^2)} \approx \sqrt{20}\,\sigma_\chi^2.$$

A similar result is also valid for saturated fluctuations: in accordance with (3.21), $N^{(m)} = \langle J^{2m} \rangle = 2m!$ In this case, $\Delta_{m-s}/I_{s0} = \sqrt{20}$ (see also Ref. 21).

f) Enhancement of the backscattering in the case of a phase screen

Let a phase screen, which modulates the passing wave immediately by a factor $\exp[i\Phi(\boldsymbol{\rho})]$, where $\Phi(\boldsymbol{\rho})$ is a two-dimensional random field, be present between the source and the scatterer. If the reflector is located in the focusing zone, then the enhancement factor N can be much greater than the limiting value $N = 2$ for a medium with volume fluctuations. A rough estimate of the enhancement factor in this case is given by $N \sim \ln \sigma_\Phi^2$, $\sigma_\Phi^2 = \langle \Phi^2 \rangle$.[22-24]

g) Condition for the existence of enhancement in nonstationary media

This reduces to the fact that over the time $\Delta t = 2L/c$ of propagation of the signal from the source to the scatterer and back again, the medium practically does not have time to change:

$$\Delta t = \frac{2L}{c} < \tau_\varepsilon \sim \varepsilon \left| \frac{\partial \varepsilon}{\partial t} \right|^{-1}; \tag{3.24}$$

here, τ_ε is the characteristic time of the fluctuations in the dielectric permittivity ε. In other words, the reflected signal must pass through the same inhomogeneities as the forward signal. If the change in the parameters of the medium along the propagation path occurs due to drift of inhomogeneities, then the reversal effect does not disappear, but is observed at a point displaced from the observer by a distance $2Lv_{\perp}/c$, where $v_{\perp}$ is the component of the drift velocity transverse to the beam.

h) Enhancement of backscattering in the case of extended sources, receivers, and reflectors

Above, we have discussed the enhancement effect in the simplest scheme of a point source and point scatterer. Under real conditions, scatters, emitters, and receivers have finite dimensions. Taking real factors into account is very important in order to develop recommendations for experimental observation and practical use of double passage effects. We shall examine some of the relevant problems here.

1) *Averaging action of a finite receiver aperture*. From what has been said above, it follows that if the characteristic size of the receiving aperture ρ_a exceeds the correlation radius ρ_N of the intensity fluctuations of the backscattered wave, then the receiver will not see the enhancement of the average backscattering intensity due to the averaging action of the receiver aperture.[5] Thus, the inequality $\rho_a < \rho_N$ serves as a condition for observing the enhancement effect. As an example, we shall examine the limitations on the aperture of an optical receiver under the conditions of a turbulent atmosphere. In the case of small intensity fluctuations of the incident wave, when the MSP approximation is valid and $\rho_N \sim \sqrt{\lambda L}$, we have $\rho_a < \sqrt{\lambda L}$. For laser radiation ($\lambda \approx 5\cdot 10^{-5}$ cm) and a path $L \sim 1$ km, we obtain $\rho_a \sim \sqrt{\lambda L} \sim 2$ cm. On longer paths $L \sim 5-10$ km, where the intensity fluctuations of the incident wave are usually already saturated, the aperture must be less than the spot with enhanced averaged intensity, whose radius is of the order of the coherence radius of the incident wave $\rho_a(L)$. Paths with $L \sim 5-10$ km correspond to a coherence radius of $\rho_c \sim 1$ cm. For this reason, $\rho_a < 1$ cm is necessary. Presently existing receiving apparatus can detect backscattering enhancement in a regime with both weak and strong intensity fluctuations of the wave incident on the scatterer.

2) *Influence of spreading of wave beams and of directional properties of scatterers on enhancement of backscattering*. It is evident that if the dimensions of the scatterer a are much less than the transverse coherence radius of the incident wave, $a \ll \rho_c(L)$, then near the scatterer the incident wave field $u_{\rm inc}(\mathbf{r})$ can be assumed to be locally planar and locally coherent.[5] In this case, in the region of Fraunhofer diffraction, the expression for the average backscattering intensity has the form

$$\langle I_s(\mathbf{r})\rangle = \sigma(\mathbf{q})\,\langle I_{\rm inc}(\mathbf{R})\, I(\mathbf{R},\mathbf{r})\rangle, \tag{3.25}$$

where, as before, $I(\mathbf{R},\mathbf{r})$ is the intensity of an undirected spherical wave, $I_{\rm inc} = |u_{\rm inc}(\mathbf{R})|^2$ is the intensity of the incident wave near the scatterer, $\sigma(\mathbf{q}) = |f(\mathbf{q})|^2$ is the scattering cross section, $f(\mathbf{q})$ is the scattering amplitude, $\mathbf{q} = k(\mathbf{n}-\mathbf{m})$ is the scattering vector, $\mathbf{m}$ is the direction of propagation of the incident wave near the scatterer, while $\mathbf{n}$ is the direction toward the point of observation $\mathbf{r}$ from the scattering point $\mathbf{R}$ (see Fig. 7).

Let us write (3.25) in a form similar to (3.7):

$$\langle I_s(\mathbf{r})\rangle = \langle I_s(\mathbf{r})\rangle_0 N; \tag{3.26}$$

here, the first factor $\langle I_s(\mathbf{r})\rangle_0 = \sigma(\mathbf{q})\langle I_{\rm inc}(\mathbf{R})\rangle I_0(|\mathbf{R}-\mathbf{r}|)$ represents the average intensity of the scattered field neglecting the correlation between the incident and scattered fields. This quantity is customarily used in estimates of the power, scattered by bodies in a randomly inhomogeneous medium. The second factor in (3.26) is the scattering enhancement factor

$$N = \frac{\langle I_{\rm inc}(\mathbf{R})\, I(\mathbf{R},\mathbf{r})\rangle}{\langle I_{\rm inc}(\mathbf{R})\rangle I_0(|\mathbf{R}-\mathbf{r}|)}. \tag{3.27}$$

In contrast to the factor N, studied in Subsection a, here N characterizes not the absolute (compared to the vacuum), but the relative enhancement of scattering, since in the case of narrow beams, the intensity of the incident wave can decrease due to the spreading of the beam in a randomly inhomogeneous medium. We shall return to the case of narrow beams in Subsection m. Here we note that if the scatterer is located in the Fraunhofer zone of the emitter and if the directivity pattern $F(\mathbf{m})$ is so wide that its spreading due to random inhomogeneities in the medium can be neglected, then it follows from (3.26) and (3.27) that absolute enhancement of the average backscattering intensity occurs.[5] Actually, in this case, the wave incident on the scatterer can be represented in the form $u_{\rm inc}(\mathbf{R}) = F(\mathbf{m})G(\mathbf{r}_1,\mathbf{R})$. In this case, the enhancement factor (3.27) coincides with (3.10), while the quantity

$$\langle I_s(\mathbf{r})\rangle_0 = \sigma(\mathbf{q})\,|F(\mathbf{m})|^2 I_0(|\mathbf{r}_1-\mathbf{R}|)\, I_0(|\mathbf{R}-\mathbf{r}|) \tag{3.28}$$

equals the backscattering intensity in a vacuum. Therefore, in this case, absolute enhancement also occurs.

3) *Restrictions on the dimensions of smooth convex reflectors*. The stringent restriction $a \ll \rho_c$ on the dimensions of the scatterer is greatly weakened if the scattered field is formed not by the entire scatterer, but only by a small part of its surface, namely, by the bright spot.[5] Thus, for a smooth sphere with radius $a \gg \lambda$, the size of the bright spot $\sim\sqrt{\lambda a}$, so that the inequality $a < \rho_c$ should be replaced by a much weaker one: $\sqrt{\lambda a} < \rho_c$. In this case, if the wavelength $\lambda \approx 5\cdot 10^{-5}$ cm, while $\rho_c \sim 1$ cm, from here, we have $a < \rho_c^2/\lambda \sim 2\cdot 10^4$ cm $= 200$ m. Thus, the restrictions on the dimensions of smooth reflectors greatly decrease.

4) *Weakening of scattering by extended bodies under conditions of strong phase fluctuations*. If at least one of the radii of curvature of the body is infinite (disc, strip, cylinder), while the phase fluctuations are large ($\langle\psi^2\rangle \gg 1$), then, as established in Ref. 71, enhancement of backscattering is replaced by weakening, since the coherent superposition of synchronous fields, scattered by the elements of the body, breaks down. According to Ref. 71, the weakening of scattering by a body of length b occurs when the following three conditions are satisfied: a) the point of observation is located in the

geometric optics zone ($\lambda L \ll l_\varepsilon^2$); b) phase fluctuations are large; and, c) the coherence radius of the field $l_E \sim l_\varepsilon/\sqrt{\langle\psi^2\rangle}$ is quite small: $l_E \ll \lambda L/b$.

i) Enhancement in the case of scattering by a rough surface and by a collection of discrete inclusions

Backscattering is also enhanced in the case of scattering of a weakly directed spherical wave by a rough surface. It is necessary that the coherence radius of the incident wave ρ_c be much greater than the dimensions of the areas (bright spots), forming the effective scattering indicatrix $\sigma_s(\mathbf{q})$ of the surface.[5,25] Under these conditions, the observer sees the surface as a collection of uncorrelated scatterers (bright spots). The characteristics of the spatial redistribution of the average backscattering intensity for the case of a rough surface are described in Ref. 25. Similar phenomena also occur with scattering by discrete inclusions, placed in a large-scale medium.

j) Enhancement of backscattering by small-scale inhomogeneities in the presence of large inhomogeneities

Let us separate the spatial spectrum of the inhomogeneities of the medium into two independent parts: $\Phi_\varepsilon(\varkappa) = \Phi_\nu(\varkappa) + \Phi_\mu(\varkappa)$, were $\nu(\mathbf{r})$ is the large-scale ($\varkappa < \varkappa^*$), while $\mu(\mathbf{r})$ is the small-scale ($\varkappa > \varkappa^*$) part of the fluctuations in the dielectric permittivity $\varepsilon = \nu + \mu$; here, $\varkappa^* \ll k$ is the separation boundary in wave number space. Wave propagation in such a two-scale medium is described by the Helmholtz equation (3.1), which can be solved in the single-scattering approximation with respect to the small-scale component μ, if the random field $u_\nu(\mathbf{r})$, which is affected only by the large inhomogeneities ν, is taken as the zeroth order approximation. The statistical independence of both components permits averaging independently with respect to ν and μ, and in so doing, the result within certain limits does not depend on the cutoff of $\varkappa^*$. Such a hybrid approach was realized in Ref. 26 and led to the following result for the intensity of the field backscattered to the source:

$$\langle I_s(\mathbf{r}_1)\rangle = \int\limits_V \frac{\langle I_{\text{inc}}(\mathbf{R})\rangle\, \sigma_\mu(q)\, N(\mathbf{R})}{|\mathbf{r}-\mathbf{R}|}\, d^3\mathbf{R}; \tag{3.29}$$

here, $\sigma_\mu(q) = (\pi k^4/2)\Phi_\mu(q)$ is the Born cross section for scattering by the small-scale component ($q = 2k\sin\theta/2$), $I_{\text{inc}}(\mathbf{R})$ is the intensity of the incident wave, while the enhancement factor $N(\mathbf{R})$ is given by expression (3.8). Enhancement of scattering is manifested in the fact that the scattering cross section σ_μ appears in the integrand in (3.29) with the factor $N(\mathbf{R}) > 1$, which describes the effect of large inhomogeneities.

It is important that the single backscattering approximation has a much larger range of applicability within the scope of the hybrid method than the standard Born approximation, whose validity is limited by the condition that the total cross section $L\int \sigma_\varepsilon d\Omega \ll 1$ be small, while within the scope of the hybrid method it is only required that the cross section for the small-scale component be small: $L\int \sigma_\mu d\Omega \ll 1$.[26]

We also note that the distribution of the scattering inhomogeneities over a large volume can significantly

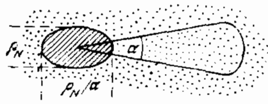

FIG. 9.

reduce the region near the emitter, where enhancement of backscattering is manifested. If α is the width of the directivity pattern of the emitter, then the dimensions of the enhancement region are $\sim\rho_N$ across the main lobe of the diagram $\sim\rho_N/\alpha$ along it (Fig. 9).

k) Enhancement of backscattering as an effect not taken into account in the theory of radiation transfer

In analyzing the statistical and, in particular, the coherent properties of waves multiply scattered in randomly inhomogeneous media, the equation of radiation transfer (ERT) is often used.[27,28] A "statistical-wave" derivation of ERT is given in Refs. 29 and 30 (see also Ref. 1) with the help of selective summation of a series with respect to the scattering multiplicity and its diffraction content is established. A calculation of the average backscattering intensity within the scope of ERT, taking into account multiple scattering by large-scale components of inhomogeneities of the medium and single scattering by the small-scale components, does not lead to the enhancement of backscattering.[31-33] As a diagramatic analysis of the applicability of ERT showed,[9,10,34] enhancement of backscattering is not manifested within the scope of ERT due to the fact that the ladder approximation of the Bethe-Salpeter equation used in its derivation neglects the contribution of the so-called cyclical diagrams. Figure 10 shows schematically the propagation paths of waves from the point $\mathbf{r}_0'$ to the point $\mathbf{r}'$ and from the point $\mathbf{r}_0''$ to $\mathbf{r}''$ in the approximation of double and triple scattering, and the corresponding cyclical diagrams are also presented.

Essentially, ERT is not capable of describing the enhancement of backscattering because in the ERT, the scattered wave intensities are summed incoherently, while for backscattering, in general, the coherence of waves passing through the same inhomogeneities cannot be neglected. Apparently, the specific interference effects accompanying backscattering were first pointed out in a paper by Watson.[7] Watson noted that the path $(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n, \mathbf{r}_1)$ and the reverse path $(\mathbf{r}_1, \mathbf{r}_n, \ldots, \mathbf{r}_2, \mathbf{r}_1)$ (Fig. 11) give identical, i.e., coherent, fields. This coherence is responsible for the enhancement of backscattering. A coherent calculation of the average intensity of waves in a small-scale plane-layered medium,

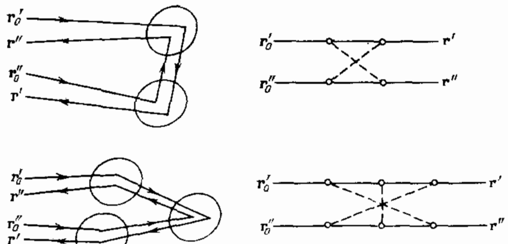

FIG. 10.

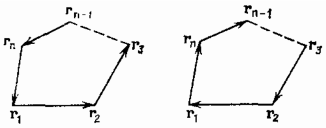

FIG. 11.

taking into account multiple backscattering, also confirms the inapplicability of ERT in describing backscattering (see, for example, Refs. 35–40).

The discussion above might raise the warning that the ERT cannot generally be used to describe backscattering. However, as noted above (Subsection j), in the presence of scattering in a medium with non-one-dimensional volume random inhomogeneities, the backscattering enhancement effect is almost everywhere smeared out except for a small region around the emitter. In this sense, three dimensional scattering problems are qualitatively different from one-dimensional problems, where the smearing effect is absent. It is the smearing of the enhancement of backscattering in the three-dimensional case that leads to the fact that deviations from incoherent superposition of scattered fields, on which the ERT is based, are usually small.

l) Methods for analyzing double passage

In calculating double-passage effects, methods that differ in their range of applicability and degree of justification are used. As long as the intensity fluctuations of the incident wave are weak, we shall use MSP, which was widely used in calculating the fluctuations of the reflected wave field (see, for example, Refs. 5, 11, 12, 26, and 41–44). A detailed calculation of double-passage effects for strong fluctuations was carried out based on the phase approximation of the Huygens-Kirchhoff method (PAHKM) and its modifications.[6,45-48] However, the applicability of PAHKM for quantitative calculations of waves in a turbulent medium is not always sufficiently justified.[17,47] A functional method for analyzing the statistics of waves reflected in a randomly inhomogeneous medium was developed in Refs. 38 and 49. Since in a turbulent atmosphere, the direction of propagation of the incident wave practically does not change, while double-passage effects are manifested only in a small cone of backscattering angles, these effects are reliably described by the parabolic equation method (PEM). In this case, as demonstrated in Refs. 50 and 51, the Markov approximation is applicable in the statistical analysis of the reflected waves. A series with respect to the multiplicity of backscattering along the direction of propagation of the wave is obtained in Ref. 52. It is shown in Refs. 53 and 54 that the Markov approximation is valid in the analysis of such series. We note that the equations of single backscattering[52-54] practically coincide with the equations of the hybrid method.[26] In Ref. 64, the boundary value problem for backscattering is reduced to the Cauchy problem, which is more convenient for statistical analysis.

m) Description of reflected waves by the parabolic equation method

Let a wave, given by $u_0(\rho)$ in the emission plane $x = 0$ (ρ denotes the transverse coordinates), propagate along the x axis. We shall place a reflector at the end of the path with a local coefficient of reflection $f(\rho)$ on the underlying surface $x = L$. Then, the complex amplitude of the reflected wave in the emission plane ($x = 0$) according to PEM equals

$$u_s(\boldsymbol{\rho}) = \int f(\boldsymbol{\rho}')\, u_{\rm inc}(\boldsymbol{\rho}', L)\, G(\boldsymbol{\rho}, 0, \boldsymbol{\rho}', L)\, d\boldsymbol{\rho}', \tag{3.30}$$

where $u_{\rm inc}(\rho, x)$ is the complex amplitude of the incident wave:

$$u_{\rm inc}(\boldsymbol{\rho}, x) = \int u_0(\boldsymbol{\rho}_0)\, G(\boldsymbol{\rho}_0, 0, \boldsymbol{\rho}, x)\, d\boldsymbol{\rho}_0.$$

The Green's function entering here satisfies the Cauchy problem:

$$2ik\frac{\partial G}{\partial x} + \Delta_\rho G + k^2\tilde{\varepsilon}(\boldsymbol{\rho}, x)\, G = 0,$$
$$G(\boldsymbol{\rho}_0, 0, \boldsymbol{\rho}, 0) = \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_0). \tag{3.31}$$

We shall present some exact consequences of (3.30) and (3.31). For a plane incident wave ($u_0 = 1$) and statistically homogeneous $\tilde{\varepsilon}$, it follows from (3.30) and (3.31) that[48,50,51]

$$\langle u_s(\boldsymbol{\rho})\rangle = \langle u_{\rm inc}(\boldsymbol{\rho}, L)\, u_f(\boldsymbol{\rho}, L)\rangle, \tag{3.32}$$

where $u_f(\rho, x)$ is the incident wave, whose complex amplitude in the plane $x = 0$ equals $f(\rho)$. If the reflector is an ideal mirror, then $f = 1$ and we have from (3.32) $\langle u_s(\rho)\rangle = \langle u^2_{\rm inc}(\rho, L)\rangle$. For weak intensity fluctuations, this equality is a result of the enhancement of phase fluctuations of the reflected wave described in Sec. 2.

The total intensity flux $I_s(\rho) = |u_s(\rho)|^2$ of the reflected wave, as follows from (3.30) and (3.31), equals[51]:

$$\int I_s(\boldsymbol{\rho})\, d\boldsymbol{\rho} = \int |f(\boldsymbol{\rho})|^2 I_{\rm inc}(\boldsymbol{\rho}, L)\, d\boldsymbol{\rho}. \tag{3.33}$$

It is evident from here that if the dimension of the reflector is less than the intensity correlation radius of the incident wave, then the fluctuations of the total flux of the reflected wave reproduce the fluctuations in the intensity of the incident wave, i.e., the averaging action of the receiving aperture saturates (see Refs. 55–57). It is also evident from (3.33) that the average reflected flux equals the average flux of the wave incident on the reflector. The latter is a result of the smearing of the enhancement of the average intensity of the reflected wave by the receiving aperture, indicated in Subsec. h. For a plane incident wave and an ideal mirror ($u_0 = f = 1$), the equality[50] $\langle I_s(\rho)\rangle = 1$, indicating the absence in this case, of enhancement of the average reflected wave intensity, is valid.

The expression for the reflected wave in the form (3.30) is convenient for use of the Markov approximation.[47,50,51] However, due to the correlation of the inhomogeneities along the paths of the forward and reflected waves, the equations for the moment functions of the reflected waves are more complicated than the equations for the corresponding moment functions of the incident waves. Thus, the coherence function of the reflected wave $\Gamma_s = \langle u_s(\rho_+ + \rho/2)u_s^*(\rho_+ - \rho/2)\rangle$ in the simplest case of a plane incident wave and an ideal mirror equals

$$\Gamma_s(\boldsymbol{\rho}) = \int M(\boldsymbol{\rho}_-, 0, 0; L \mid \boldsymbol{\rho})\, d\boldsymbol{\rho}_-, \tag{3.34}$$

where $M(\rho_-, \rho_1, \rho_2; L/\rho)$ satisfies the equation

$$\frac{\partial M}{\partial L}=\frac{i}{k}\,(\nabla_1\nabla_2)\,M-\frac{k^2}{4}\,F\,(\rho_1,\ \rho_2,\ \rho_-)\,M,$$
$$M\,(\rho_-,\ \rho_1,\ \rho_2;\ 0|\rho)=\delta\,(\rho_--\rho-\rho_2), \tag{3.35}$$
$$F\,(\rho_1,\ \rho_2,\ \rho_-)=D\,(\rho_1\pm\rho_-)+D\,(\rho_2\pm\rho_-)-D\,(\rho_1\pm\rho_2).$$

The exact solution of this equation is not known. An asymptotic analysis has shown that for saturated fluctuations in the intensity of the incident wave ($\gamma\gg 1$), its stochastic Green's function G is asymptotically Gaussian.[17-20] Therefore, in the regime of saturated fluctuations, the averages of the products of stochastic Green's functions of the forward waves, entering into the expression for the moment functions of the reflected waves, can be separated according to the laws of Gaussian statistics. This significantly simplifies the statistical analysis of the reflected wave.

We shall discuss the special features of the manifestation of the enhancement of the average reflected wave intensity in the regime of saturated fluctuations using the example of a beam with radius d, collimated in the plane $x=0$, incident on a plane mirror with radius a in the plane $x=L$.[45,58] If $d<\rho_c(L)$, i.e., if diffraction divergence of the incident beam exceeds the turbulent divergence, relative enhancement occurs of the average reflected wave intensity in a spot with radius $\rho_c(L)$ in the vicinity of the emitter with enhancement factor $N\approx 2$. In addition, for sufficiently small ($a<\rho_c(L)$) and sufficiently large ($a>\sigma_\rho(L)$) reflectors, the enhancement effect becomes absolute. For $d>\rho_c(L)$, the enhancement effect vanishes. Analogous results are also valid in more general cases (diverging beams, rough reflectors, and so on[45,58]). We note that the behavior of the reflected wave in the regime of saturated fluctuations is satisfactorily explained by the multichannel coherent effects examined in Sec. 4.

n) Enhancement effect on reflection from a rough surface in the presence of shading

I.M. Fuks discovered an effect analogous to the enhancement of the average backscattering intensity in a randomly inhomogeneous medium[59]: for small glancing angles for a wave incident on a rough surface, backscattering is weakened by shading of the incident and scattered rays. For almost all azimuthal scattering angles, the incident and reflected rays move along different reliefs of the rough surface and their shading is statistically independent. However, in a narrow sector of azimuthal backscattering angles, when the incident and reflected rays pass above the same irregularities, the correlation of the shading of the incident and reflected rays turns out to be significant. Due to this correlation (if the incident ray was reflected strictly backwards, then the reflected ray is not shaded), there is an enhancement of the scattering cross section in a narrow range of azimuthal angles with maximum enhancement factor $N\approx 2$.[59]

4. MULTICHANNEL COHERENT EFFECTS WITH BACKSCATTERING

a) Coherent effects, related to multichannel propagation

Let the wave proceed from the source (point $\mathbf{r}_1$) to the scatterer (point $\mathbf{R}$) along two paths, which corre-

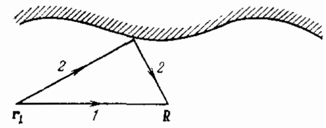

FIG. 12.

sponds to the total Green's function

$$G\,(\mathbf{r}_1,\,\mathbf{R})=G_1\,(\mathbf{r}_1,\,\mathbf{R})+G_2\,(\mathbf{r}_1,\,\mathbf{R}). \tag{4.1}$$

This situation arises, for example, in the presence of a good reflecting interface boundary (Fig. 12). According to (3.4) and (4.1), the backscattered field is written in the form

$$u_s(\mathbf{r}_1)=f\,(G_1+G_2)^2=f\,(G_1^2+G_1G_2+G_2G_1+G_2^2)=u_{11}+u_{12}+u_{21}+u_{22}. \tag{4.2}$$

The terms $u_{11}=fG_1^2$ and $u_{22}=fG_2^2$ correspond to scattering along paths 11 and 22, while the terms $u_{12}=fG_1G_2$ and $u_{21}=fG_2G_1$ correspond to the crossed propagation channels 12 and 21 (Fig. 13). According to the reciprocity theorem, these cross channels are coherent: $u_{12}=u_{21}$, leading to an enhancement of the corresponding intensity $I_{12+21}=|u_{12}+u_{21}|^2$ by a factor of 4 compared to the intensity in a single channel $I_{12}=|u_{12}|^2$, which equals the geometric average of $I_{11}=|u_{11}|^2$ and $I_{22}=|u_{22}|^2$:

$$I_{12+21}=4\,|u_{12}|^2\equiv 4I_{12}=4\,|f|^2\,|G_1|^2\,|G_2|^2=4\sqrt{I_{11}I_{22}}. \tag{4.3}$$

The total intensity of the scattered field in the vicinity of the emitter equals

$$I_s=|u_s|^2=I_{11}+I_{22}+4I_{12}+(G_1^2G_2^{2*}+2G_1^2G_1^*G_2^*+2G_2^2G_2^*G_1^*+\text{c.c. }). \tag{4.4}$$

The terms in parenthesis oscillate rapidly as the source or scatterer is displaced. Having in mind averaging over some region of variation of $\mathbf{r}_1$ and $\mathbf{R}$ (sufficiently large so that there would be many oscillations in it), we shall drop these terms, writing

$$I_s=I_{11}+I_{22}+4I_{12}. \tag{4.5}$$

This expression exceeds the sum of the intensities

$$I_{\text{incoh}}=I_{11}+I_{22}+I_{12}+I_{21}=I_{11}+I_{22}+2I_{12}, \tag{4.6}$$

corresponding to incoherent superposition of the wave intensities over all channels. In essence, I_{incoh} can be measured at some distance from the emitter, where channels 12 and 21 already lose coherence (see Subsection b below). In a randomly inhomogeneous medium, all quantities in (4.5) and (4.6) must be viewed as averaged over ensembles of inhomogeneities:

$$\langle I_s\rangle=\langle I_{11}\rangle+\langle I_{22}\rangle+4\,\langle I_{12}\rangle,\qquad \langle I_{\text{incoh}}\rangle=\langle I_{11}\rangle+\langle I_{22}\rangle+2\,\langle I_{12}\rangle. \tag{4.7}$$

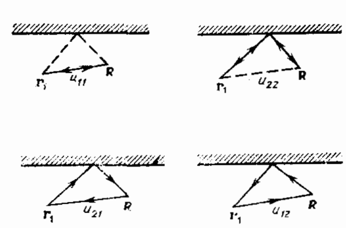

FIG. 13.

In this case, the ratio

$$N = \langle I_s \rangle \langle I_{\mathrm{incoh}} \rangle^{-1} \tag{4.8}$$

characterizes the enhancement of the average intensity due to coherent effects with multichannel (in this case, two channel) propagation. If I_{11}, I_{12} and I_{22} are approximately equal, the enhancement factor equals $N \approx 6/4 = 1.5$. As the number of channels M increases, the enhancement factor N increases approximately as $N \sim 2 - 1/M$.

Indications of the existence of such effects already existed a long time ago (see, for example, Ref. 60); however, their importance was recognized only recently.[61,62]

b) Condition for the existence of coherent effects

It is significant that coherent effects in themselves have a nonstatistical nature. They can be observed in an arbitrary smoothly inhomogeneous medium in the presence of arbitrary (but smooth) interfaces and so on. It is only necessary that the wave move from the source to the scatterer along several independent paths (channels) and that over the propagation time $\Delta t \sim 2L/c$, the medium (or its boundary) does not have time to change appreciably. In the particular case of a variable interface (Fig. 12), for the existence of coherent effects, it is necessary that the difference $t'-t''$ between the times that the rays 21 and 12 touch the surface be sufficiently small that over this time no appreciable (with respect to $\lambda/2$) path difference $\Delta l \sim v_z(t'-t'')\cos\theta$ would appear between channels 12 and 21.[62]

The spatial region for observation of coherent multichannel effects is limited by the condition that the waves, arriving at the point of observation along the intersecting channels, be in phase. When the observation $\mathbf{r}$ and emission $\mathbf{r}_1$ points are separated, the wave arrives at point $\mathbf{r}$ along channels 12′ [Fig. 14(a)] and 21′ [Fig. 14(b)]. Coherent superposition disappears when the difference between paths 12′ and 21′ equals $\lambda/2$.

c) Long-range correlation in crossed scattering channels

Another coherent effect, related to multichannel propagation, can be observed with simultaneous scattering of two waves, emitted by mutually synchronized (coherent) sources (Fig. 15).[63] The scattered field, evidently, equals

$$u_s(\mathbf{r}) = fG(\mathbf{R}, \mathbf{r})\,[G(\mathbf{r}_1, \mathbf{R}) + G(\mathbf{r}_2, \mathbf{R})]. \tag{4.9}$$

Being interested in the mutual correlation of the scattered field at points at which the emitters are located, we shall find the coherence function $\Gamma_s(\mathbf{r}_1, \mathbf{r}_2) = \langle u_s(\mathbf{r}_1) u_s^*(\mathbf{r}_2)\rangle$. Assuming that the phase fluctuations of the emitted waves G are large, while the distance between the emitters $|\mathbf{r}_1-\mathbf{r}_2|$ is greater than the coherence scales of the waves G, so that $\langle G(\mathbf{r}_1, \mathbf{R}) G^*(\mathbf{r}_2, \mathbf{R})\rangle = 0$, we obtain

$$\Gamma_s(\mathbf{r}_1, \mathbf{r}_2) = |f|^2 I_0(|\mathbf{r}_1 - \mathbf{R}|)\, I_0(|\mathbf{r}_2 - \mathbf{R}|). \tag{4.10}$$

Thus, there is a strong correlation between the values of the scattered wave field at the emitter locations separated by a large distance caused by the fact that the contribution to the scattered wave at the point $\mathbf{r}_2$ from the wave of an emitter situated at the point $\mathbf{r}_1$ passes along the same path $\mathbf{r}_1 \to \mathbf{R} \to \mathbf{r}_2$, along which the wave, emitted from point $\mathbf{r}_2$ and scattered at the point $\mathbf{R}$ along the direction to $\mathbf{r}_1$, propagates.

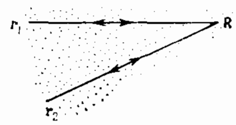

FIG. 15.

d) Coherent effects on reflection from a rough surface

Conditions, under which a wave is scattered strictly backwards after a double reflection, can appear with the reflection of waves from a statistically irregular surface (Fig. 16). This leads to mutual coherence of waves, scattered in a different sequence by the same sections of the surface. As a result, an appreciable enhancement of the scattering cross section will occur (calculated per unit area) in a narrow range of backscattering angles. Thus, if the irregularities of the surface are sufficiently steep, then coherent effects with multiple rereflection must play an important role in backscattering.

e) Relations between enhancement and multichannel coherent effects

There is no sharp boundary between these effects. This can be verified by interpreting the backscattering enhancement effect from the point of view of the theory of multiple scattering (see Subsection k in Sec. 3 and Fig. 11). The difference between the two effects reduces, in essence, to the number of coherent scattering channels: with backscattering in a randomly inhomogeneous medium, the number of channels is infinite, while for multichannel propagation effects, examined in this section, this number is finite.

Thus, both effects are related to one another in the same way that diffraction and interference are related: in both cases, we are talking about the superposition of waves, but diffraction involves superposition of an infinite number of waves, while interference involves superposition of a finite number of waves. The arbi-

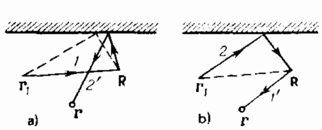

FIG. 14.

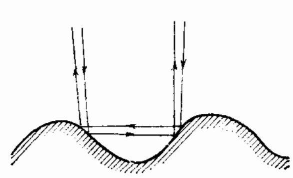

FIG. 16.

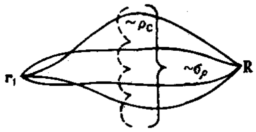

FIG. 17.

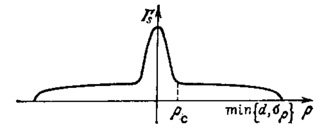

FIG. 18.

trariness of the separation of the two effects being studied becomes especially obvious in those cases when the infinite number of scattering paths in a randomly inhomogeneous medium can be reduced to a finite number of independent channels. We shall examine one such example.

f) Interpretation of the backscattering enhancement effect under conditions of saturated fluctuations from the point of view of multichannel propagation

The value of the enhancement factor $N=2$ in the saturated region can be obtained as a result of coherent effects accompanying propagation of waves along many channels. Indeed, the condition of saturation $\sigma_\rho(L) \gg \rho_c(L)$ (see Subsection 3d) indicates, in ray language, that the incident wave hits the scatterer approximately along $M\sim\gamma^2$ rays, where $\gamma\sim\sigma_\rho/\rho_c\gg 1$ is the multiray parameter (Fig. 17). The Green's function corresponding to this problem can be represented in a form analogous to (4.1) only for a large ($M\gg 1$) number of channels:

$$G(\mathbf{r}_1,\mathbf{R})=\sum_{m=1}^{M}G_m(\mathbf{r}_1,\mathbf{R}),\quad G_m=A_m e^{i\psi_m};\qquad(4.11)$$

here, A_m and ψ_m are the amplitude and phase of the Green's function of the m-th channel. Assuming that pairs $\{A_m,\psi_m\}$ with different numbers are mutually independent, for $M\gg 1$, it follows from (4.11) and the central limit theorem that $G(\mathbf{r}_1,\mathbf{R})$ is Gaussian and then we immediately find that $N=2$ (see Subsection 3d). We emphasize that the same results remain in force if fluctuations in amplitudes are neglected in (4.11) and only the phases ψ_m are assumed to be random and mutually independent.

The interference of backscattered waves vanishes for $\theta L>\rho_c(L)$. For this reason, the enhancement effect occurs only in the spot $\theta_0\sim\rho_c(L^{-1})$. The attenuation for $\theta>\theta_0$, ensuring that the conservation law (3.11) is satisfied, is described by diagonal terms of the type

$$\langle|G_m^2(\mathbf{r}_1,\mathbf{R})|\,|G_m^{*}(\mathbf{r},\mathbf{R})|\rangle=\langle A_m^{*}(\mathbf{r}_1,\mathbf{R})\,A_m^2(\mathbf{r},\mathbf{R})\rangle.$$

Here, fluctuations in the intensities of the incident and scattered waves are already important. Their correlation radius is of the order of $\sigma_\rho(L)$. Thus, in the saturation regime, the enhancement effect is distinctly manifested in the sector $\theta\geqslant\rho_c/L$, while the attenuation is locally small and spread out over a large region $\theta\sim 1/k\rho_c$.

Similar ray considerations lead to the conclusion that enhancement of backscattering disappears when the point of observation is separated by a distance $\sim k\rho_c^2$ *from the emitter along a straight line connecting the* emitter and scatterer.

g) Partial reversal of the wave front on reflection in a randomly inhomogeneous medium

Coherent effects accompanying multichannel propagation in a regime with saturated fluctuations leads to one more interesting effect: partial reversal of the wave front of the incident wave.[58,63] We shall discuss it within the scope of the PEM (see Subsection 3m). Let a beam with radius $d>\rho_c$ in the plane $x=0$ and complex amplitude $u_0(\rho)$ be incident on a reflector at the point $(x=L,\boldsymbol\rho=0)$. Then, *in the Gaussian approximation* for the Green's function of the incident wave, the coherence function of the reflected wave in the plane $x=0$ is given by the expression[58]

$$\Gamma_s(\boldsymbol\rho_+,\boldsymbol\rho)=\left\langle u_s\left(\boldsymbol\rho_++\frac{1}{2}\boldsymbol\rho\right)u_s^{*}\left(\boldsymbol\rho_+-\frac{1}{2}\boldsymbol\rho\right)\right\rangle=\Gamma_1+\Gamma_2,\qquad(4.12)$$

where

$$\Gamma_1(\boldsymbol\rho_+,\boldsymbol\rho)=|f|^2\left(\frac{k}{2\pi L}\right)^2\exp\left[\frac{ik}{L}(\boldsymbol\rho\boldsymbol\rho_+)-D_\psi(\rho,L)\right]\int|u_0(\boldsymbol\rho)|^2W(\boldsymbol\rho,L)\,d\boldsymbol\rho,$$

$$\Gamma_2(\boldsymbol\rho_+,\boldsymbol\rho)=|f|^2u_0\left(\boldsymbol\rho_+-\frac{1}{2}\boldsymbol\rho\right)u_0^{*}\left(\boldsymbol\rho_++\frac{1}{2}\boldsymbol\rho\right)\times W\left(\boldsymbol\rho_++\frac{1}{2}\boldsymbol\rho,L\right)W\left(\boldsymbol\rho_+-\frac{1}{2}\boldsymbol\rho,L\right),$$

$$W(\boldsymbol\rho,L)=\left(\frac{k}{2\pi L}\right)^2\int\exp\left[\frac{ik}{L}(\boldsymbol\rho\mathbf{q})-D_\psi(q,L)\right]d\mathbf{q}$$

is a function with width $\sim\sigma_\rho(L)$, which describes the statistical broadening of the beam.

In (4.12), Γ_1 is the coherence function of the reflected wave with coherence radius $\sim\rho_c(L)$, neglecting correlations along the incident and reflected wave paths, while Γ_2 takes into account double-passage effects. It is evident from (4.12) that for $d>\rho_c(L)$ there is practically no enhancement of the average intensity of the reflected wave ($I_2\ll I_1$). However, multichannel coherent effects lead, in this case, to the appearance of a small, but wide base in Γ_2 with coherence radius of the order of $\min\{d,\sigma_\rho(L)\}$ (Fig. 18) in the coherence function of the reflected wave. The restriction of the radius of the base by the initial radius of the beam d is explained by the fact that only rays, leaving in the plane $x=0$ from the aperture of the incident beam, contribute to coherent multichannel effects.

We emphasize especially that Γ_2 is proportional to $u_0(\rho_+-\frac{1}{2}\rho)\times u_0^{*}(\rho_++\frac{1}{2}\rho)$, the complex conjugate of the coherence function of the incident wave. Thus, due to multichannel coherent effects, the reflected wave contains a component whose wave front is reversed with respect to the front of the incident wave.[3)] We shall clarify the mechanism of reversal due to coherence effects for an example similar to that of Subsection 4c. Let the first source (Fig. 15) emit a wave with phase ψ_1, while the second source emits a wave with phase ψ_2.

[3)] The idea of the presence of a weak reversed component in the reflected wave was first stated by B. Ya. Zel'dovich.

In this case, the coherent components of the scattered field at the source points equal

$$u_s(\mathbf{r}_1) = fe^{i\psi_2}G(\mathbf{r}_2, \mathbf{R})\,G(\mathbf{r}_1, \mathbf{R}),$$
$$u_s(\mathbf{r}_2) = fe^{i\psi_1}G(\mathbf{r}_1, \mathbf{R})\,G(\mathbf{r}_2, \mathbf{R}).$$

Their phases equal, respectively, $\psi_2 + \psi_{21}$ and $\psi_1 + \psi_{12}$, where ψ_{12} is the phase increment over the path $\mathbf{r} \to \mathbf{R} \to \mathbf{r}_2 (\psi_{12} = \psi_{21})$. But, $\psi_2 + \psi_{21} = \varphi_0 - \psi_1$ and $\psi_1 + \psi_{12} = \varphi_0 - \psi_2$ ($\varphi_0 \equiv \psi_1 + \psi_2 + \psi_{12}$). Thus, to within φ_0, the phases of the coherent components of the scattered wave at the points $\mathbf{r}_1$ and $\mathbf{r}_2$ equal, respectively, $-\psi_1$ and $-\psi_2$, i.e., they are reversed with respect to the initial phases of the emitted waves.

For $d > \rho_c(L)$, the reversed component in the reflected wave does not give rise to enhancement of the average intensity of backscattering. However, its high coherence compared to the incoherent part of the reflected wave permits separating the reversed component with the help of focusing of the reflected wave. Let the reflected wave be incident in the plane $x = 0$ on a lens, whose aperture coincides with the aperture of the collimated incident beam $u_0(\rho)$. Then, the field at the center of the focal plane of the lens is proportional to[16]

$$v_s = \int u_0(\rho)\, u_s(\rho)\, d\rho = f u^2_{\text{inc}}(0, L).$$

Here, the equations in Subsection 3m and the fact that the reflector is a point reflector are taken into account. Correspondingly, the average intensity of the reflected wave at the center of the focal plane is proportional to $\langle |v_s^2| \rangle = |f|^2 \langle I^2_{\text{inc}}(0, L) \rangle$. From here follows, in particular, the fact that for a plane incident wave $(d > \sigma_\rho(L))$, when it is known that there is no enhancement of the average intensity of the reflected wave in the plane $x = 0$, enhancement occurs in "pure form" at the center of the focal plane of the lens. This enhancement effect is explained by the presence of a small, but well-focused reversed component, whose appearance is a result of the mutual coherence of waves passing in opposite directions through the same inhomogeneities.

5. EXPERIMENTAL OBSERVATIONS AND SOME APPLICATIONS OF DOUBLE PASSAGE EFFECTS

In this section, we shall discuss the known experimental data on double-passage effects, and we shall point out some possible applications of these effects. As is evident from the preceding sections, double-passage effects can greatly affect the results of radar ranging measurements, while the correct interpretation and taking into account the characteristics of scattering with double-passage could provide a foundation for new methods for measuring the characteristics of a scattering medium, discovery of optimum conditions for range determination and sounding, and so on.

a) Applications of the doubling effect in ionospheric measurements of phase variance

The difference between the phase variances for normally and obliquely reflected waves can be used to measure the parameters of randomly inhomogeneous media. We shall demonstrate the possibilities arising here on the example of reflection of radio waves from the ionospheric layer. The scheme for a possible experiment, using the difference between the phase fluctuations for normal and inclined reflection from the ionospheric layer, is shown in Fig. 19. An estimate of the scale l_ε is the distance $y = Ob$, for which the phase variance along the inclined path Oab decreases by a factor of 2 compared to the variances with vertical sounding (path cac). An experiment on simultaneous vertical and oblique sounding of the ionosphere[70] confirmed reliably the enhancement effect for phase fluctuations with vertical sounding compared to oblique sounding. Analogous experiments are possible in optics and acoustics.

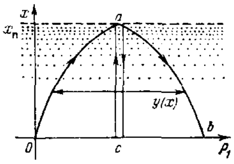

FIG. 19.

We shall point out one more result of the enhancement effect for fluctuations with double passage. Let the inhomogeneities of the medium drift across the path along the ρ_1 axis with constant velocity V. Then, the variance of the fluctuations in the eikonal of the reflected wave, found under the same assumptions as (2.2), will assume the form

$$\sigma^2_\varphi(0, L) = \frac{1}{2}\left\{A(0)\,L + \int_0^L A\left[2x\left(0 - \frac{v}{c}\right)\right] dx\right\}.$$

From here it follows that the drift of inhomogeneities leads to an angular displacement (by an angle $\theta_0 = V/c$) of the enhancement effect. This angular displacement can, in principle, be used to determine the velocity of the medium or the relative velocity of the emitter.

b) Observations of backscattering enhancement in optics

The work described in Ref. 72, which was undertaken at the suggestion of one of the authors of the present review, concerned the study of the average intensity of light reflected backwards in the laboratory in an artificial randomly inhomogeneous medium (turbulent convective air flow, arising when air is blown through a heated grating). The measurements were carried out with a helium–neon laser and a prismatic coupler was used to reveal the spatial redistribution of the scattered radiation against the background of the primary laser beam. The receiving aperture $\rho_a = 0.06$ mm was much less than $\rho_N \approx 1-2$ mm. The scatterers consisted of the following: a convex spherical mirror, whose radius of curvature satisfied the condition $\sqrt{\lambda a} < \rho_c$, (cf. Subsection 3i3) a rough surface, namely, a sheet of writing paper; and, for a control, a flat mirror. The observed absolute enhancement of the average power of the received scattered signal agreed well with theory for the spherical mirror and the rough surface,[5] while enhancement was not observed at all in measurements with the flat mirror. The measurements of the spatial distribution of the average intensity of the scattered radiation agreed with the calculation using (3.12). The enhancement of backscattering under natural conditions was studied in Ref. 73 for path lengths $L = 650$ and 1300 m. A flat scatterer made up of a large number of convex mirrors was used in this work. The investigations in Ref. 73 revealed the possibility of measuring the parameters of turbulence from optical measurements of the scattered radiation. This method of performing the measurements has important advantages over the usual scheme involving transmission, when the source and receiver of the radiation are situated at opposite ends of the path. Measurements of fluctuations in the intensity of the scattered radiation are reported in Ref. 74, wherein values $\langle I_s^2\rangle/\langle I_s\rangle^2 = 12$, in agreement with the estimates in Subsec. 3d, are noted.

Fluctuations in the intensity and phase of the backward reflected wave in a regime with weak intensity fluctuations, as well as the averaging action of the receiving aperture, were studied experimentally in Refs. 56, 57, and 65. It was noted in Ref. 55 that due to the residual correlation (spatial correlation coefficients for amplitude and phase fluctuations of the reflected wave do not vanish when the observation points are far apart), it is possible to measure the intensity fluctuations of a wave incident on a quite small reflector for arbitrary dimensions of the receiving aperture. This is related to the fact that the receiving aperture of the reflected wave cannot average the fluctuations in the flux of the wave incident on the reflector [see also (3.33)]. Saturation of the averaging action of the receiving aperture on the intensity fluctuations of the reflected wave for small effective dimensions of the reflector permits, for example, determining the structural characteristic of turbulence C_ε^2 from measurements of the reflected wave flux without restricting the dimensions of the receiving aperture. In this sense, the measurement of the parameters of turbulence from data on fluctuations in the reflected wave flux may turn out to be preferable to measurements using direct transmission, for which the dimensions of the receiving aperture must be small.[55,66]

Results of measurements of fluctuations in the intensity of backward reflected laser beams in the regime of strong fluctuations are presented in Ref. 67. The starting dimensions of the incident beams d and the effective dimensions of the reflectors a in Ref. 67 greatly exceeded the coherence radius of the incident wave, so that the average intensity of the reflected wave was not enhanced. However, double passage through the turbulent medium strongly increased the intensity fluctuations of the backward reflected wave compared to the incident wave, which agrees with the calculations carried out in Ref. 48. The difference between the variances of the intensity fluctuations of the reflected and incident waves vanished only for wide incident beams $L/kd^2 < 1$ and large reflectors $L/ka^2 < 1$.

It is clear that up to the present time, all possibilities of double passage have by no means been used in optics. Thus, when receiving backscattered waves, the quantity C_ε^2 can be obtained from the measurements of the average intensity in the focal plane of the lens, just as this is done in transmission optics.[68] The presence of a component of a partially reversed incident wave in the reflected wave (see Subsection 4g) leads to enhancement of the average intensity in the focal plane. In the simplest case of an incident plane wave, the enhancement (with factor $N \approx 2$) occurs in a spot with size $\sim\rho_c(L)F/L$, independent of the properties of the reflecting surface.[58] By measuring the radius of this enhancement spot, it is possible, in principle, to measure $\rho_c(L)$.

c) Influence of backscattering enhancement on the determination of the electron concentration in the ionosphere by the incoherent scattering method

In the incoherent scattering method,[69] the electron concentration n_e is determined from the magnitude of the average intensity of the backscattered field. The appearance of the enhancement factor N in backscattering due to the passage of the electromagnetic wave through the randomly inhomogeneous ionosphere could be the reason for the overestimation of the values of $n_{e\,\text{meas}}$ by a factor of N compared to the true value $n_{e\,\text{true}}$. This overestimate can turn out to be appreciable for high layers of the ionosphere ($H \sim 500-1000$ km), where strong fluctuations of radio waves, passing through the ionosphere, which plays the role of a phase screen, can be observed.[5]

d) Double-passage effects in backward-oblique sounding of the ionosphere and in other remote-sensing experiments

The enhancement of the average intensity and of the intensity fluctuations accompanying backscattering, compared to scattering sideways, and other characteristics of double passage must be taken into account in choosing the conditions for optimal reception of signals from long-range sounding, in particular, signals from backward-oblique sounding of the ionosphere in the short wavelength range ($\lambda \sim 10-30$ m). Thus, separated reception of sounding signals has some advantages over coincident reception, since in this case intensity fluctuations decrease and, therefore, the signal is reproduced more accurately. If, on the other hand, we are talking about receiving very weak signals, then coincident reception is preferable, since in this case there appears the additional possibility of observing the signal due to strong swings above the average intensity.

e) Multichannel effects with scattering of radio waves in the ionosphere

Radio waves with frequencies below the critical frequency are reflected from the ionosphere and form a caustic, in the vicinity of which the multichannel scattering effects, described in Sec. 4, are possible. The magnitude of the effect, taking into account the proximity of the caustic, is analyzed in Ref. 61.

f) Multichannel effects in hydroacoustics

Taking into account the coherence of fields in crossed channels leads to some reevaluation of the scattering cross section of different inclusions (primarily bubbles), located near the sea surface. The first indication of enhancement of scattering occurs in

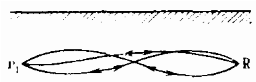

FIG. 20.

Ref. 60, while a detailed analysis of the problem is given in Ref. 62, where the transition from coherent to incoherent superposition with the transition to higher frequencies or with intensification of wind-driven waves on the ocean surface is observed. This transition can be used as an indication for determining the height of waves.

Multichannel effects can also be important in scattering of sound in underwater sound channels. In the presence of m rays, passing from the source to the scatterer (Fig. 20), m forward and $m(m-1)/2$ crossed channels are possible, i.e., $m(m+1)/2$ channels in all. Analysis of the average intensity with backscattering could be useful for diagnostics of regular and random characteristics of the ocean, in particular, the parameters of large scale inhomogeneities of the ocean. The possibilities of this method of diagnostics increase with the use of an active transponder as the scatterer.

g) Double-passage effects on reflection of waves from phase-front reversal mirrors

The adaptive properties of systems that reverse the phase front (PFR mirrors) are fully revealed in turbulent media: automatic compensation of phase distortions, acquired by the incident wave in the medium, occurs only under conditions when the reflected wave returns precisely through those inhomogeneities through which the incident wave passed.

The study of the problem of the effectiveness of focusing the reversed wave on a source under the conditions of a turbulent medium is still in the initial stages. It was shown in Ref. 75 that if the radius of the PFR mirror a exceeds the mean-square displacement of the beam σ_ρ, then the distortions of the phase front of the reflected wave are practically completely compensated, i.e., diffraction spreading, characteristic of a homogeneous medium, is realized in the turbulent medium. In addition, some improvement in the adaptive properties for $a<\sigma_\rho$ is noted in Ref. 75.

As far as the temporal fluctuations of the medium are concerned, their effect on the operation of PFR systems can be neglected, if conditions (3.24), requiring that the parameters of the medium remain practically constant over the double-passage time, are satisfied.

In conclusion, we are deeply grateful to N.G. Denisov and A.N. Malakhov for useful discussions, which stimulated this review, as well as to A.S. Gurvich and S.A. Akhmanov for valuable remarks, made while reading the manuscript.

[1]S. M. Rytov, Yu. A. Kravtsov, and V. I. Tatarskiĭ, Sluchaĭnye polya [Random Fields], Nauka, Moscow (1978); Vvedenie v statisticheskuyu radiofiziku [Introduction to Statistical Radiophysics], Pt. II.
[2]A. M. Prokhorov, F. V. Bunkin, K. S. Gochelashvili, and V. I. Shishov, Usp. Fiz. Nauk **114**, 415 (1975) [Sov. Phys. Usp. **17**, 826 (1975)].
[3]N. G. Denisov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **7**, 378 (1964).
[4]a)N. G. Denisov and L. M. Erukhimov, Geomagn. Aéron. **6**, 695 (1966); b) Yu. A. Kravtsov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **8**, 876 (1965).
[5]A. G. Vinogradov, Yu. A. Kravtsov, and V. I. Tatarskiĭ, *ibid.* **16**, 1064 (1973).
[6]M. S. Belen'kiĭ and V. L. Mironov in: Kvantovaya élektronika [Quantum Electronics], Sov. Radio, Moscow (1972), No. 5 (11), p. 38.
[7]K. M. Watson, J. Math. Phys. **10**, 688 (1969).
[8]D. A. De Wolf, IEEE Trans. **AP-19**, 254 (1970).
[9]Yu. N. Barabanenkov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **16**, 88 (1973).
[10]Yu. N. Barabanenkov, Usp. Fiz. Nauk **117**, 49 (1975) [Sov. Phys. Usp. **18**, 673 (1975)].
[11]V. P. Lukin, Kvant. Elektron. (Moscow) **4**, 923 (1977) [Sov. J. Quantum Electron. **7**, 522 (1977)].
[12]S. S. Koval', G. Ya. Patrushev, A. I. Petrov, and V. V. Pokasov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **23**, 326 (1980).
[13]V. V. Tamoĭkin, *ibid.* **9**, 1124 (1966).
[14]Yu. A. Kravtsov, Zh. Eksp. Teor. Fiz. **55**, 798 (1968) [Sov. Phys. JETP **28**, 413 (1969)].
[15]Yu. A. Kravtsov and Yu. I. Orlov, Geometricheskaya optika neodnorodnykh sred [Geometric Optics of Inhomogeneous Media], Nauka, Moscow (1980).
[16]V. I. Tatarskiĭ, Rasprostranenie voln v turbulentnoĭ atmosfere [Propagation of Waves in a Turbulent Atmosphere], Nauka, Moscow (1967).
[17]V. U. Zavorotnyĭ, V. I. Klyatskin, and V. I. Tatarskiĭ, Zh. Eksp. Teor. Fiz. **73**, 481 (1977) [Sov. Phys. JETP **46**, 252 (1977)].
[18]I. G. Yakushkin, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **19**, 384 (1976).
[19]R. Dashen, J. Math. Phys. **20**, 894 (1979).
[20]I. G. Yakushkin, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **21**, 1194 (1978).
[21]V. P. Aksenov, V. A. Banakh, and V. L. Mironov in: II-nd All-Union Conf. on Atmospheric Optics, Abstracts of Reports, Tomsk (1980), Pt. II, p. 131.
[22]E. E. Salpeter, Astrophys. J. **147**, 433 (1967).
[23]V. A. Alimov and L. M. Erukhimov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **16**, 1541 (1973).
[24]V. I. Shishov, *ibid.* **13**, 85 (1970).
[25]A. G. Vinogradov, *ibid.* **17**, 1584 (1974).
[26]A. G. Vinogradov and Yu. A. Kravtsov, *ibid.* **16**, 1055 (1973).
[27]S. Chandrasekhar, Transfer of Radiant Energy [Russian translation], Inostr. Lit., Moscow (1953).
[28]A. Isimaru, Proc. IEEE **65**, No. 1, 46 (1977).
[29]Yu. N. Barabanenkov, A. G. Vinogradov, Yu. A. Kravtsov, and V. I. Tatarskiĭ, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **15**, 1852 (1972).
[30]A. L. Apresyan, *ibid.* **16**, 461 (1973).
[31]B. V. Ermakov and Yu. A. Il'inskiĭ, *ibid.* **12**, 694 (1969).
[32]K. M. Watson, Phys. Fluids **13**, 2514 (1970).
[33]L. S. Dolin and M. M. Savel'ev, Izv. Akad. Nauk SSSR Fiz. Atmos. Okeana **7**, 5 (1971).
[34]Yu. N. Barabanenkov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **17**, 113 (1974).
[35]Yu. L. Gazaryan, Zh. Eksp. Teor. Fiz. **56**, 1856 (1969) [Sov. Phys. JETP **29**, 996 (1969)].
[36]J. B. Keller in: Intern. Symp. on EM Wave Theory, Tbilisi (1971), p. 9.
[37]Yu. A. Ryzhov, Radio Sci. **11**, 121 (1976).
[38]V. I. Klyatskin, Stokhasticheskie uravneniya i volny v sluchaĭno-neodnorodynykh sredakh [Stochastic Equations and

Waves in Randomly Inhomogeneous Media], Nauka, Moscow (1980).
[39]D. Marcuse, IEEE Trans. MTT-20, 541 (1972).
[40]J. A. Morrison, *ibid.* MTT-22, 126 (1974)
[41]Z. I. Feĭsulin, Radiotekh. Elektron. **15**, 1390 (1970).
[42]J. Smith, J. Opt. Soc. Amer. **63**, 1095 (1973).
[43]J. Smith and T. Pries, Appl. Optics 14, 1165 (1975).
[44]G. Ya. Patrushev, Kvant. Elektron. (Moscow) **5**, 2342 (1978) [Sov. J. Quantum Electron. **8**, 1315 (1978)].
[45]V. P. Aksenov and V. L. Mironov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **22**, 141 (1979).
[46]V. P. Aksenov, V. A. Banakh, and V. L. Mironov in: Rasprostranenie opticheskikh voln v sluchaĭno-neodnorodnoĭ atmosfere [Propagation of Optical Waves in a Randomly Inhomogeneous Atmosphere], Nauka, Siberian Branch, Novosibirsk (1979), p. 47.
[47]V. P. Aksenov and V. L. Mironov, J. Opt. Soc. Am. **69**, 1609 (1979).
[48]V. P. Aksenov, B. A. Banakh, and V. L. Mironov, Kvant. Elektron. **3**, 2266 (1976) [Sov. J. Quantum Electron. **6**, 1233 (1976)].
[49]V. I. Klyatskin, Zh. Eksp. Teor. Fiz. 65, 234 (1973) [Sov. Phys. JETP **38**, 27 (1974)].
[50]V. I. Gel'fgat, Akust. Zh. **22**, 123 (1976) [Sov. Phys. Acoust. 22, 65 (1976)].
[51]A. I. Saichev, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **21**, 1290 (1978).
[52]V. E. Ostashev and V. I. Tatarskiĭ, *ibid.* **21**, 714 (1978).
[53]A. N. Malakhov and A. I. Saichev, *ibid.* **22**, 1325 (1979).
[54]A. I. Saichev *ibid.* **23**, 1163 (1980).
[55]M. S. Belen'kiĭ and V. L. Mironov, Kvant. Elektron. (Moscow) **1**, 2253 (1974) [Sov. J. Quantum Electron. **4**, 1256 (1975)].
[56]M. S. Belen'kiĭ, V. L. Mironov, A. A. Makarov, and V. V. Pokasov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 21, 299 (1978).
[57]V. P. Lukin, and V. V. Pokasov in: I-st All Union Conf. on Atmospheric Optics, Abstracts of Reports, Tomsk (1976), Pt. I, p. 134.
[58]A. B. Krupnik and A. I. Saichev, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **24**, 1234 (1981).
[59]I. M. Fuks, Radiotekh. Elektron. **21**, 625 (1976).
[60]V. P. Glotov and Yu. P. Lysanov, Akust. Zh. **14**, 371 (1968).
[61]Yu. A. Kravtsov and S. A. Namazov, Radiotekh. Elektron. **25**, 459 (1980).
[62]Kh. G. Akhunov and Yu. A. Kravtsov in: Volny i difraktsiya: Korotkie tezisy dokladov na VII Vsesoyuznom simpoziume po difraktsii i rasprostraneniyu voln (L'vov, 1981) [Waves and Diffraction: Abstracts of Reports at the VII All-Union Symposium on Diffraction and Propagation of Waves (L'vov, 1981)], Moscow (1981), Vol. 3, p. 145.
[63]A. B. Krupnik and A. I. Saichev, *ibid.*, Vol. 2, p. 137.
[64]G. I. Babkin, V. I. Klyatskin, and L. Ya. Lyubavin, Dokl. Akad. Nauk SSSR **250**, 1112 (1980).
[65]V. V. Pokasov, G. Ya. Patrushev, V. P. Lukin, A. I. Petrov, and O. N. Emaleev, in: XII-th All-Union Conference on Propagation of Radio Waves, Abstracts of Reports, Tomsk (1978), Pt. II, p. 122.
[66]M. Subramanian, J. Opt. Soc. Am. **62**, 667 (1972).
[67]V. P. Aksenov, N. Ts. Gomboev, *et al.*, in: Rasprostranenie opticheskikh voln v sluchaĭno-neodnorodnoĭ atmosfere [*Propagation of Optical Waves in a Randomly Inhomogeneous* Medium], Nauka, Moscow (1979), p. 40.
[68]A. K. Artem'ev and A. S. Gurvich, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **14**, 734 (1971).
[69]Incoherent Scattering of Radio Waves [translation edited by V. A. Rudakov], Mir, Moscow (1965).
[70]V. A. Alimov, L. M. Erukhimov, L. F. Mironenko, *et al.*, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **18**, 894 (1975).
[71]V. V. Tamoĭkin, Izv. Vyssh. Uchben. Zaved. Radiofiz. **23**, 380 (1980).
[72]A. S. Gurvich and S. S. Kashkarov, *ibid.* 20, 794 (1977).
[73]S. S. Kashkarov, "Experimental investigation of the enhancement of backscattering in a turbulent atmosphere," Preprint IFA AN SSSR, Moscow (1978).
[74]S. S. Kashkarov, T. N. Nesterova, and A. S. Smirnov in: Abstracts of Reports at the VI-th All Union Symposium on Propagation of Laser Radiation, Tomsk (1981).
[75]A. V. Polovinkin and A. I. Saichev, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 24, 433 (1981).

Translated by M. E. Alferieff